Evaluating the Interest-Rate Risk of Adjustable-Rate Mortgage Loans

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Abstract. **This paper evaluates the interest-rate risk inherent in an adjustable-rate mortgage (ARM) with sporadic rate adjustments and possibly binding periodic and life-of-loan rate change constraints. Simulation analysis forecasts ARM cash flows, determines the probability that constraints will hold, and partitions the loan into fixed and variable components. Simulation parameters are then altered to measure the impact of changes in contract terms and market conditions on the interest-rate risk of a typical ARM loan. Interest-rate sensitivity is found to be significantly less than that of fixed-rate loans and remarkably *insensitive* to changes in loan margins or initial loan rates after the first few years of an ARM's life. Therefore, it is not surprising that lenders have used these features to lure borrowers to ARMs. Periodic rate change limits and volatility in the underlying index are the only factors that influence the interest-rate risk of an existing ARM in a substantive way.**

Introduction

Adjustable-Rate Mortgages (ARMs) were first offered by federally chartered depository institutions in 1981. Since that time they have become a widely accepted alternative to the fixed-rate, long-term mortgage loan. The ARM loan transfers *some* of the interest-rate risk of fixed-rate mortgage lending from the lender to the borrower. Under a fixed-rate loan, the lender must hold a below-market rate asset if market rates rise after the loan closes. It does not earn an above-market rate if interest rates fall, however, because borrowers can refinance the existing loan at the new, lower market rate.

The adjustable-rate loan minimizes unprofitable exposure to both upward and downward interest-rate movements by creating a debt instrument whose interest rate is seldom *substantially* different from the market rate. The ARM loan rate is adjusted periodically by adding a prespecified margin to the current value of a market-determined index. ARM lending is not free of interest-rate risk, however, because loan payments are fixed for the length of the adjustment period. Furthermore, periodic rate changes in either direction are subject to a prespecified limit while a prespecified premium added to the initial rate places a cap on the loan rate throughout the loan's life. Future rate adjustments are complicated further by the fact that the initial rate on most ARMs is significantly less than its fully indexed value (current index level plus stipulated margin), and caps and limits work off of stated rates, not fully indexed rates.

Caps and limits that temporarily constrain rate adjustments and fix loan payments during an adjustment period mean that ARMs are not purely variable rate debt and that

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ARM lenders are exposed to interest-rate risk. Despite the volume of ARM loans in existence, scant attention has been paid to the quantification of this interest-rate risk.

This paper assesses the interest-rate risk of an ARM loan by simulating cash flows under constrained and unconstrained rate changes and calculating the probability that each regime will hold. The expected value and duration of the constrained cash flows can then be calculated and used to determine the interest-rate sensitivity of the entire ARM loan.

The next part of the paper reviews previous academic research on the risk/return trade-offs inherent in ARM lending. Section three demonstrates the interest-rate risk assessment methodology. Section four analyzes the relationship between an ARM's interest-rate sensitivity, its adjustment parameters and the volatility of the underlying index. Results suggest that the interest-rate risk of an ARM is relatively insensitive to the initial loan rate or the margin but fairly sensitive to the periodic rate change limit and to volatility in the underlying index. This result provides an economic justification for the fact that ARM lenders have relied on modifications in margins and teasers instead of smaller rate change limits to attract borrowers to ARM loans. The final section summarizes the paper's conclusions.

Risk/Return Trade-Offs in ARM Lending

The typical ARM loan has five parameters that govern the manner in which its interest-rate path evolves through time. These are:

(1) the market index which initiates rate changes on the ARM;
(2) the margin that is added to the current value of the adjustment index to determine the fully indexed loan rate;
(3) the teaser, or discount off the fully indexed rate, that attracts borrowers interested in reducing loan payments in the early years of their mortgage;
(4) the annual rate change limit which prevents an ARM loan rate from rising or falling dramatically from year to year; and
(5) the life-of-loan cap which places a ceiling on the ARM rate throughout the loan's life, regardless of movements in the underlying index.

The interaction of these parameters as the ARM ages can create complicated relationships that underlie the cash flows on ARM loans. Since the value of an ARM is contingent on its actual cash flow, many authors have adapted contingent-claims models based on the seminal work of Cox, Ingersoll and Ross (1985) to the valuation of ARMs.[1] Two of the most comprehensive analyses to date were performed by the same authors (Kau, Keenan, Muller, and Epperson, 1990, 1993) who solved a series of partial differential equations to partition an ARM into four components. These are: (1) the value of the loan's cash flows to the lender; (2) the value of the prepayment option to the borrower; (3) the value of the default option to the borrower; and (4) the value of insurance against that default.

Kau et al. also consider fixed-rate loans and provide simulations that are powerful enough to allow comparisons of the two types of funding. For example, the borrower's prepayment option is typically worth more on a fixed-rate loan than an ARM because the fixed-rate loan rate does not drop with market rates. In contrast, insurance is typically worth more on an ARM than on a fixed-rate loan because rising payments on the adjustable-rate debt may lead to increased default risk.

In addition, the Kau et al. results quantify the interest-rate risk trade-offs faced by lenders who must select some combination of adjustment parameters when quoting ARMs. For example, a higher margin (which raises future loan rates and payments) is required to offset the effects of a higher initial year teaser, a tighter annual rate change limit or a lower life-of-loan cap. Furthermore, caps and limits have a much more dramatic effect on ARM value when the underlying index is volatile; periodic limits influence ARM cash flows much more strongly than life-of-loan caps do; and upward adjustment limits are more constraining than downward adjustment limits because borrowers will prepay ARMs when rate drops are constrained.[2]

Given the recent attention focused on valuing adjustable-rate loans at origination and partitioning that value into cash flow and default and prepayment option components, it is puzzling to note that little effort has been made to quantify the interest-rate risk that teasers, limits and caps inpute to ARM loans as the loans age.[3] Only one study (Ott, 1986), specifically addresses the relative interest-rate sensitivity of various ARM loans, but Ott limits his attention to interest-rate risk on the first interest-rate adjustment date.

A wide variety of adjustment indices and parameters appeared in the market when adjustable-rate mortgages were first introduced. Ott demonstrated that interest-rate sensitivity was positively related to the length of the adjustment period (the time for which payments are fixed), and to the choice of the index. For example, loans indexed to Treasury rates with terms longer than the adjustment period, to Federal Home Loan Bank Board mortgage rates, or to cost-of-fund indices, were found to lag market interest rates, increasing exposure to interest-rate risk.

The passage of time and market developments have resulted in a need to reexamine the interest-rate risk of an ARM. First, the sets of adjustment parameters typically offered on ARM loans have been reduced. One-year adjustment frequencies predominate and most ARMs are indexed directly to a corresponding Treasury rate. Second, as ARMs age, the focus of ARM lenders and researchers has extended beyond the initial rate adjustment date. From this perspective, the impact of periodic or life-of-loan rate adjustment constraints over time is much more important than behavior on the first adjustment date.

The methodology described in detail in the next section of the paper adopts the traditional finance concept of duration first proposed by Macauley (1938) as a measure of interest-rate risk. Recognizing that the elasticity of an ARM's price to changes in interest rates is non-zero when loan payments are constrained (e.g., during the current rate adjustment period and when rate change limits or caps hold in future periods), the approach uses simulation analysis to determine the probability of constrained payments, the value of constrained payments, and their duration. These are then used to measure the overall interest-rate risk of a given ARM loan.

Measuring the Interest-Rate Risk of an Adjustable-Rate Mortgage Loan

Given a change in the discount rate, the approximate price elasticity of a stream of fixed cash flows is measured by the security's duration (DUR), a statistic first proposed by Macauley (1938). While the original construct has since been enhanced by many authors, Bierwag, Kaufman and Toevs (1983), among others, note that Macauley's measure performs reasonably well when compared with more sophisticated formulations. Furthermore, as noted in Anderson, Barber and Chang (1993), the correlation between the loan rate and the market rate determines the applicability of the conventional (Macauley) duration formula to mortgage cash flows. Given the existence of a market-based rate adjustment process, it is clear that conventional duration is a more appropriate choice for ARMs than for fixed-rate loans.

Duration

The general form for the Macauley duration is

$$DUR = \sum_{t-1}^{N} \frac{tC_t}{(1+Y)^t} / P \,, \tag{1}$$

where:

C_t = the payment the security is scheduled to make in time period t;
N = the number of time periods until the maturity time period;
Y = the security's yield-to-maturity; and
P = the security's price.

It is evident from (1) that duration is calculated by weighting each expected cash flow by the time it occurs, discounting each weighted cash flow to present value at the security's yield, and dividing the sum of the discounted cash flows by the security's price. The beauty of the statistic is that duration multiplied by a given change in yield provides a good approximation for the expected percentage price change or return on a security. As such, it is an absolute measure of interest-rate risk that can be used to compare the price sensitivity of various contracts. The duration measure in (1) cannot be applied directly to adjustable-rate mortgage loans, however, because the C_t in equation (1) varies over time in an unknown fashion.

The Cash Flows on an ARM

Consider the rate adjustment process of an ARM with annual and lifetime caps. After the initial year, the mortgage rate can fall into one of three states each year: *S*1: the lifetime and annual caps are not binding; *S*2: the lifetime or annual cap is binding on the upside; or *S*3: the lifetime or annual cap is binding on the downside. The mortgage rate for the coming year is set by the current index level on each adjustment date. For each state, the rate for year *t*, R_t, is:

$$\begin{aligned} S1\colon R_t &= I_{t-1}+M \\ S2\colon R_t &= \mathrm{Min}\,[R_{t-1}+AC,\ R_1+LC] \\ S3\colon R_t &= \mathrm{Max}[R_{t-1}-AC,\ R_1-LC]\,, \end{aligned} \tag{2}$$

where $t-1$ refers to the previous time period, I_{t-1} is the index at the end of period $t-1$, M is the margin, AC is the annual cap, and LC is the lifetime cap.[4]

Estimating the future values of the R_t's using equation (2) is not an easy task in a realm of interest-rate uncertainty. The difficulty arises from the recursive nature of the process in that year t's rate is conditional upon the mortgage rate from year $t-1$, but the $t-1$ rate is conditional upon $t-2$, etc. As a result, all the rates starting from the inception of the mortgage are relevant in determining R_t. The historical rates and the entire distribution of the index must be taken into consideration to find the anticipated mortgage rate for any given future period.

Once the expected mortgage rate is known, the expected mortgage payment depends upon a complicated relationship because mortgage amortization is a nonlinear process. Thus, the outstanding balance on an adjustment date *t*, B_{t-1}, is based on the values in the loan rate history and the order in which the preceding loan rates occurred. For $t \geq 2$, the anticipated monthly mortgage payment conditional on B_{t-1} is:

$$\begin{aligned} E(C_t\,|B_{t-1}) \;=\; & *_{S1_t}\,(C_t|I_{t-1}+M)\,f(I_{t-1})\,dI_{t-1} \\ & +(C_t\,|\text{ Min }[R_{t-1}+AC,\ R_1+LC])\text{ Prob. }(S2_t) \\ & +(C_t\,|\text{ Max }[R_{t-1}-AC,\ R_1-LC])\text{ Prob. }(S3_t)\ , \end{aligned} \tag{3}$$

where:

$$B_{t-1} = \frac{C_{t-1}}{r_{t-1}}\left[1-\frac{1}{(1+r_{t-1})^{(N-(t-1)12)}}\right],$$

$f(\cdot)$ = the density function of the index;
C_t = the monthly mortgage payment (per \$1 of initial mortgage) in year t;
r_{t-1} = the monthly equivalent of R_{t-1}.

The terms on the right-hand side of equation (3) are the contribution to the conditional expected mortgage payment made by the different index rates when the caps are non-binding (top), when the up cap is binding (middle), and when the down cap is binding (bottom). The last two terms are comparable to mortgage payments from fixed-rate mortgages because they represent instances where rate adjustment constraints hold.

In year 2 there is only one possible prior year mortgage ending balance, B_{t-1}, but there are infinite possibilities for the prior ending balances for the more distant periods ($t \geq 3$). As a result, the unconditional anticipated monthly mortgage payment is:

$$E(C_t)=E(C_t\,|B_{t-1}) \text{ for } t=2\ , \tag{4a}$$

and

$$E(C_t)=*_{B_{t-1}}\ E(C_t\,|B_{t-1})\ g(B_{t-1})dB_{t-1} \quad \text{for } t \geq 3\ . \tag{4b}$$

The density function of B_{t-1}, $g(B_{t-1})$, can be expressed in terms of all the density functions of the index back to period $t=2$.[5] Each year's calculation in equation (3) is conditional upon prior years, so the ability to estimate the $E(C_t)$ directly can quickly be hampered by limited computational capacity. Simulation analysis of an array of interest-rate paths provides a reasonable alternative approach because the historical probability distribution of the market index is used in the specification of the simulation parameters and the simulation output captures the entire evolving history of the mortgage rate.

Yield on an ARM

Once the mechanism for computing the expected ARM payments is specified, duration can be calculated if the discount rate is known. Yield calculations require an iterative procedure where the set of expected cash flows is discounted over a universe of interest rates until the rate that equates the present value of the cash flows to the price is isolated.

If capital markets are efficient, the initial price of the mortgage, P_0, must equal the initial loan balance (\$1), at the beginning of the loan. Therefore, in the absence of prepayment, the monthly yield of a new ARM, y, can be computed via:

$$1 = \sum_{t=1}^{N} \sum_{i=1}^{12} \frac{E(C_t)}{(1+y)^{(t-1)12+i}}, \tag{5}$$

where i is i^{th} month in year t.[6]

Alternative Assumptions

Equations (3), (4) and (5) do not incorporate three features that could generate a more realistic representation of expected ARM cash flows because the equations assume that the ARM remains outstanding for its entire life. An alternative specification would imbed a distribution of termination probabilities in the cash flow simulation to cover: (1) non-interest-dependent prepayments, (2) interest-dependent prepayments, and (3) default.

The first two factors are omitted because previous research shows they are likely to have a minimal effect on the expected cash flows. Secondary mortgage market studies (see McConnell and Singh, 1991, on ARMs or Richard and Roll, 1989, on fixed-rate loans) find that non-interest-dependent prepayments are an increasing function of loan age that level out to a small percentage of the outstanding balance after the first few years of the loan's life. Interest-dependent prepayments, which are a significant risk for fixed-rate lenders, are minimized by the interest-rate adjustment process on ARMs because downward-rate adjustment constraints are temporary. Therefore, it is not surprising that Cunningham and Capone (1990) find that portfolios of ARM loans exhibit substantially smaller interest-dependent prepayments than pools of fixed-rate loans or that simulation results in Kau et al. (1993) show that the prepayment option has significantly greater value on fixed-rate debt than on ARMs.[7]

Loan defaults present a more significant problem. The Cunningham and Capone (1990) study and later work by the same authors (Capone and Cunningham, 1992) shows that many of the same factors drive default on fixed-rate loans and ARMs. Specifically, default probabilities are positively related to loan size and the loan-to-value ratio at origination. Defaults are more likely to occur early in a loan's life, when the loan rate exceeds the market rate, and when the term structure is upward sloping. Furthermore, borrower and economic characteristics such as age and the local unemployment rate are positively related to default. These results suggest that foreclosure occurs when borrowers are unable to afford payments on existing loans.

Results in the Cunningham and Capone (1990) study show that ARMs do experience greater default risk than fixed-rate loans. Marginal increases in credit risk are especially acute for ARMs that have short adjustment frequencies and large caps, and the probability of foreclosure increases when the yield curve has a steep upward slope. These findings imply that the possibility of sustained payment increases generates the increased default risk on ARMs.

The model presented in equations (3) through (5) could easily be extended to incorporate default by adding a constant survival rate as in the method used to calculate the constant prepayment duration in Anderson, Barber and Chang (1993). Specifics could be derived from the current risk-free rate and an assumed recovery percentage (given the existence of private mortgage insurance) in the manner described by Altman (1988), but there are three compelling reasons not to do so. First, ARM lenders can diversify away the default risk on single loans because defaults should not be correlated across borrowers in a national loan portfolio. Second, results in Capone and Cunningham (1992) show that default and prepayment factors virtually offset each other in the derivation of termination probabilities for ARMs. Third, and most important, the inclusion of default, interest-dependent prepayments and/or non-interest-dependent prepayments shortens the expected maturity of the ARM over all but the shortest expected life assumptions. This decreases the interest sensitivity of the loan.[8] Therefore, the omission of terminations caused by these sources provides a conservative assessment of the interest-rate risk of an ARM loan.

The Duration of an ARM

The components of the duration calculation process have now been specified so it is possible to partition the expected cash flows on an ARM into fixed- and variable-rate components. Recall that ARMs are not pure adjustable-rate loans because of the presence of upward and downward rate change constraints and the practice of fixing the loan rate and payments for the length of the adjustment period. The parts of an ARM that are comparable to a fixed-rate mortgage are the current year payments and the later period payments that are constrained by the caps ($S2$ and $S3$). The rest of the ARM is the portion unrestricted by the caps ($S1$), and is equivalent to a pure variable-rate loan.

Thus, an ARM's value is composed of:

$$P = \sum_{t=2}^{T}\sum_{i=1}^{12}\frac{E(C_t^n)}{(1+y)^{(t-1)12+i}} + \sum_{t=1}^{T}\sum_{i=1}^{12}\frac{E(C_t^b)}{(1+y)^{(t-1)12+i}}, \tag{6}$$

where:

$$E(C_t^n) = *_{S1_t}(C_t \,|I_{t-1}+M)\, f\,(I_{t-1})\; d\,(I_{t-1}) \text{ for } t \geq 2\ ,$$

and

$$E(C_t^b) = \begin{cases} C_1 \quad \text{for } t=1 \\ (C_t \big| \text{Min}\left[R_{t-1}+AC,\, R_1+LC\right] \text{Prob.}(S2_t) \\ +(C_t \big| \text{Max}\left[R_{t-1}-AC,\, R_1-LC\right] \text{Prob.}(S3_t) \end{cases} \Big\} \text{ for} \geq 2\,.$$

C_t^n and C_t^b are the portion of monthly payments in year t from non-binding and binding caps, respectively. The first term on the right-hand side of equation (6) is the value of all the expected payments when the caps are non-binding. The second term represents the mortgage value under interest-rate caps. This is the portion that is comparable to a fixed-rate mortgage and gives rise to interest-rate risk.

The distribution between fixed or fully adjustable components depends on the volatility of the index and the severity of the caps. If the caps are tighter, the constraints will be more binding. Thus S2 and S3 will be more probable and the proportion of the fixed-rate mortgage will be larger. In the extreme case, the caps approach zero and the ARM becomes a fixed-rate mortgage.

Another important aspect of the typical ARM is that the rate adjustment process has a one-period lag. The monthly payment for the current year is always fixed and those fixed payments create interest-rate exposure. Ott (1986) shows that the fixed-rate component of an ARM is a positive function of the interval between successive adjustment periods. As the adjustment interval approaches the term-to-maturity (T), the fixed-rate proportion approaches one. The model above differs from that proposed by Ott in one major respect, however. Here, the fixed-rate portion of the ARM loan will never approach zero, even if the adjustment period is instantaneous, because of the presence of periodic and life-of-loan rate change constraints. Once the fixed-rate loan component of the ARM is specified, the interest-rate risk of the fixed-rate portion, DUR_B, can be calculated as:

$$DUR_B = \sum_{t=1}^{T-1}\sum_{i=1}^{12}\frac{((t-1)12+i)E(C_t^b)}{(1+y)^{(t-1)12+i}} \Big/ \sum_{t=1}^{T}\sum_{i=1}^{12}\frac{E(C_t^b)}{(1+y)^{(t-1)12+i}}. \tag{7}$$

Equation (7) describes the elasticity of the value of the fixed-rate component of a given ARM to a change in the discount rate. Assuming that the pure variable-rate component has a duration of zero, the duration of the entire loan, DUR_A, can be calculated as:

$$DUR_A = DUR_B \sum_{t=1}^{T}\sum_{i=1}^{12}\frac{E(C_t^b)}{(1+y)^{(t-1)12+i}} \Big/ P. \tag{8}$$

In equation (8), the duration of the entire ARM loan is the value-weighted average of the durations of its two components (the duration of the non-binding portion is zero).

The Simulation Parameters

An actual application of the interest-rate risk assessment model above requires the specification of a stochastic process to describe the dynamics of the adjustment index. This interest-rate process generates possible paths of the index throughout the life of the mortgage. The corresponding mortgage payment and mortgage balance are calculated at each adjustment point for each index path and the average of all the simulations is the expected value. The frequency distribution of the mortgage paths (non-binding caps, binding annual caps and binding life caps) measures the probabilities of $S1$, $S2$ and $S3$.

A simple mean reverting process is chosen to model the index innovations.[9] The process is given by:

$$1\text{n}\ (I_t/I_{t-1})=a+bI_{t-1}+e_t\ ,$$

where:

$$
\begin{aligned}
I_t &= \text{the index level in period } t; \\
I_{t-1} &= \text{the index at last adjustment;} \\
e_t &= \text{a random disturbance term; and} \\
1\text{n}(\bullet) &= \text{natural logarithm function.}
\end{aligned}
$$

This model was estimated using historic one-year Treasury index data over the period 1985–1990.[10] The regression gave the following results:

$$
\begin{aligned}
\hat{a} &= .875432; \\
\hat{b} &= -.116802; \\
\hat{\sigma}_e &= .134829; \\
R^2 &= 53.23\%; \text{ and} \\
\text{n} &= 263 \text{ observations.}
\end{aligned}
$$

Illustration

Twenty thousand independent index paths were constructed to perform the simulation analysis for a new thirty-year mortgage under the following conditions:

Initial rate	=	8.00%	Lifetime cap	=	6.00%, and
Margin	=	2.75%	Initial index	=	6.00%
Annual cap	=	2.00%			

The results are summarized in Exhibit 1. The third column of the exhibit is the expected monthly mortgage payment when the constraints are not binding while the fourth column is the expected monthly mortgage payment when one of the constraints is binding. The last three columns represent the frequencies of non-binding constraints, binding upward constraints, and binding downward constraints, respectively.[11] The upward constraint is binding a significant number of times during the early years because there is a first-year teaser in the example (the initial rate is less than the index plus the margin). It is interesting to note that the effect of the teaser disappears after the third year. From then on, the probability of binding constraints shows little variation from year to year and is relatively small under simulation parameters which represent recent market conditions. The economic justification for this result lies in the mean reverting tendency of the adjustment index because the long-run stability simulated in the interest-rate process negates the effect of rate adjustment constraints in the distant future.

Various combinations of the columns in Exhibit 1 are needed to compute the duration of the ARM. The overall average monthly payment (column 2) is the $S1$ payment times the $S1$ frequency plus the $S2$ and $S3$ payment times the sum of the $S2$ and $S3$ frequencies. Given that the original loan balance is \$100,000, these expected cash flows imply an annual yield to maturity of 10.38%.

The product of the "under $S2$ and $S3$" payments and the sum of the $S2$ and the $S3$ frequencies is the expected constrained cash flow. Its present value at 10.38% is \$21,735. Time-weighting the expected constrained cash flows, discounting at 10.38% and dividing by \$21,735 shows that the duration of the fixed-rate portion of this ARM loan is equal

Exhibit 1

Simulation Results: (20,000 paths) — $100,000 ARM with an 8% Initial Rate, 2% Annual Cap 6% Lifetime Cap and 2.75% Margin

	Average Monthly Payment ($)			Frequencies (in %)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	Overall*	Under *S*1	Under Constraints *S*2 and *S*3	*S*1	*S*2 (up limit and cap)	*S*3 (down limit)
1	733.77					
2	846.37	823.90	875.11	56.12	43.88	.00
3	885.36	877.33	961.67	90.47	9.35	.18
4	890.80	877.91	914.30	89.05	6.36	4.59
5	892.86	889.79	917.33	88.85	5.83	5.32
6	892.50	889.92	913.03	88.82	5.54	5.64
7	891.28	888.83	910.60	88.74	5.41	5.85
8	891.90	889.29	913.25	89.10	5.50	5.40
9	892.40	889.77	913.14	88.72	5.79	5.49
10	892.15	889.63	912.26	88.89	5.45	5.66
11	891.79	889.51	909.66	88.72	5.48	5.80
12	891.70	889.04	912.13	88.50	5.84	5.66
13	891.30	889.17	907.94	88.64	5.52	5.84
14	891.60	889.52	907.55	88.49	5.75	5.76
15	891.92	889.66	909.55	88.67	5.72	5.61
16	891.87	889.92	907.43	88.87	5.60	5.53
17	890.98	889.28	904.46	88.80	5.49	5.71
18	891.30	889.22	906.99	88.26	5.91	5.83
19	890.70	888.90	904.94	88.78	5.40	5.82
20	890.98	889.01	905.59	88.09	6.22	5.69
21	891.12	889.51	903.40	88.41	5.67	5.92
22	891.01	889.60	901.97	88.62	5.59	5.79
23	890.69	889.24	901.98	88.61	5.73	5.66
24	890.84	889.71	899.56	88.60	5.67	5.73
25	890.60	889.60	898.36	88.56	5.67	5.77
26	890.49	889.60	897.45	88.62	5.50	5.88
27	890.58	889.66	897.60	88.41	5.96	5.63
28	890.40	889.68	895.80	88.30	5.73	5.97
29	890.48	889.85	895.39	88.44	5.85	5.59
30	890.44	890.03	893.75	88.78	5.55	5.67

*column 2=(column 3×column 5)+(column 4×(column 6+column 7))
Source: derived by the Authors

to 5. Given that the fixed-rate portion is equal to 21.35% of the value of the whole ARM loan, the implied duration of the entire loan is 1.087 years.

This duration is considerably greater than the duration of a one-year balloon loan with a coupon rate of 8%, which was calculated by Ott (1986) to be .96 years. The increase represents the cumulative effect of the expected constrained payments. The ARM duration is also substantially less than that of a thirty-year, fixed-rate 8% loan (9.56 years), indicating that the rate adjustment mechanism removes more than 80% of the price sensitivity in long-term mortgage lending. Furthermore, ARMs issued in the early to mid-1980s, when the loan was first introduced, have passed into the stable portion of

$S2$ and $S3$ frequencies where interest-rate exposure is minimal compared to the first few years of the loan where the probabilities of constrained payments are much higher.

The Impact of Contract Terms and Market Conditions on an ARM's Interest-Rate Risk

An ARM's adjustment parameters (annual limit, lifetime cap, margin and teaser) and the stochastic dynamics of the market index interact to determine the loan's value and interest-rate risk. The interest-rate risk model in this paper allows an examination of how the size of the fixed-rate component of the loan and that component's duration change as one feature of the rate adjustment mechanism is altered. The overall effect on the ARM's duration is the product of these two inputs.

Exhibit 2 summarizes the impact of changes in each of the important adjustment parameters and index volatility. The interest-rate process is identical to the one used in the previous example. As before, each entry is the result of the simulation of 20,000 index paths.

Exhibit 2
Relationship between Rate Adjustment Characteristics and ARM Interest-Rate Risk

Panel A: Sensitivity to Change of Initial Rate/Teaser

Teaser (%)	Initial Rate (%)	Yield (%)	*Fixed-Rate Equivalent* Portion of Loan Viewed as Fixed Rate (%)	Duration of Fixed Portion	Duration of Entire ARM
1.75	7.00	10.18	24.90	4.74	1.18
1.50	7.25	10.23	24.12	4.78	1.15
1.25	7.50	10.29	23.33	4.84	1.13
1.00	7.75	10.33	24.49	4.92	1.11
.75	8.00	10.38	21.73	5.00	1.09
.50	8.25	10.42	21.06	5.08	1.07
.25	8.50	10.46	20.46	5.15	1.05
.00	8.75	10.50	20.03	5.20	1.04

Panel B: Sensitivity to Margin Changes

Margin (%)	Yield (%)	*Fixed-Rate Equivalent* Portion of Loan Viewed as Fixed Rate (%)	Duration of Fixed Portion	Duration of Entire ARM
1.25	8.98	18.56	5.95	1.10
1.75	9.45	19.09	5.70	1.09
2.25	9.92	20.09	5.38	1.08
2.75	10.38	21.73	5.00	1.09
3.25	10.82	23.82	4.63	1.10
3.75	11.25	26.02	4.35	1.13

Exhibit 2 (continued)
Relationship between Rate Adjustment Characteristics and ARM Interest-Rate Risk

Panel C: Sensitivity to Change in Volatility of the Index

Index Volatility (%)	Yield (%)	*Fixed-Rate Equivalent* Portion of Loan Viewed as Fixed Rate (%)	Duration of Fixed Portion	Duration of Entire ARM
−.6	10.41	11.45	0.82	.09
−.4	10.40	12.88	1.55	.20
−.2	10.39	16.55	3.39	.56
.0	10.38	21.73	5.00	1.09
+.2	10.36	27.13	6.05	1.64
+.4	10.34	32.20	6.73	2.17
+.6	10.31	36.80	7.17	2.64

Panel D: Sensitivity to Annual Rate Change Limits

Annual Limit (%)	Yield (%)	*Fixed-Rate Equivalent* Portion of Loan Viewed as Fixed Rate (%)	Duration of Fixed Portion	Duration of Entire ARM
.0	8.30	100.00	9.56	9.56
.5	9.94	71.01	8.09	5.74
1.0	10.22	47.97	7.19	3.45
1.5	10.33	32.08	6.19	1.98
2.0	10.38	21.73	5.00	1.09
2.5	10.41	15.36	3.68	.57
3.0	10.42	11.78	2.44	.29
3.5	10.43	9.89	1.54	.15
4.0	10.43	9.01	1.02	.09

Source: derived by the Authors

Initial Loan Rates and First-Year Teasers

The top panel of the exhibit covers a variety of initial loan rate/teaser combinations and shows that the fixed-rate component of the ARM declines as the teaser falls (the first-year loan rate rises). A larger initial rate means that the upward adjustment constraint is less likely to hold in the early years (smaller *S*2 frequencies in Exhibit 1). Loan cash flows will increase, generating a higher yield. The decline of the *S*2 frequencies in the early years places a greater relative weight on the constrained cash flows in the later years, which serves to increase the duration of the fixed-rate component. The increasing duration of the fixed-rate component and the decreasing weight of that component exert minimal influence on the ARM's overall duration however. The use and the size of teaser rates have increased substantially since ARMs were first originated.[12] Teasers of 2% or more are common in today's market. This is not surprising given that research suggests that borrowers are attracted to ARMs by the initial interest-rate savings (see Brueckner and Follain 1988; Dhillon, Shilling and Sirmans 1987). Teasers appear to be a convenient vehicle that allow lenders to attract ARM borrowers without affecting the long-term interest-rate risk of their positions in a substantial way.

Margins

The second panel in the exhibit covers a variety of possible margin amounts. Cash flows over the loan's life rise as the margin rises so the increased margin has a dramatic effect on yield. The higher margin also increases the probability that the upward rate change limit holds in the early years so the fixed-rate component of the loan increases as well. The increase in the *S*2 frequencies gives greater weight to the loan's early fixed-rate cash flows while those in the later years remain constant. This shift in relative cash flows from back-loading to front-loading means that the duration of the fixed-rate component falls as the component's percentage of value rises. As before, the net effect on overall ARM duration is minimal.

ARM margins have increased steadily since the loans were first offered in the early 1980s and Houston, Sa-Aadu and Shilling (1991) present empirical evidence that ARM lenders are trading higher initial teaser rates for larger margins. For example, values of 1.5% to 1.6% were popular when ARMs were introduced while current advertisements are dominated by the 2.75% amount used in the base example. This response by lenders is entirely consistent with the offering of increased teaser rates since margins affect earnings in all years, not just the early adjustment periods. Our results suggest that these increased margins allow lenders to recoup initial discounts without significantly increasing the interest-rate risk of the loan since increased margins and teasers combine to affect the loan cash flows in the early years but not the later years. In addition, they act to increase the upward limit binding frequency while mitigating the effect of the downward limit constraint. As such, the size of the fixed-rate component and its duration move in opposite directions and total ARM duration remains stable.

The top two panels in Exhibit 2 also demonstrate how changes in margins and teasers impact the expected yield or return to the mortgage lender. Logically, expected yield rises as the margin rises or the teaser falls. The relative impact of each change highlights an additional result of recent ARM lender behavior.

Note that yield decrease suffered as the teaser rises is much smaller than the yield increase earned when the margin increases by an identical amount. This result occurs because the margin affects all of the future ARM cash flows while the impact of the teaser is felt only in the early years. Thus, results suggest that lenders who trade lower teasers for higher margins benefit from increased mortgage returns with little impact on overall interest-rate risk.

Index Volatility

Adjustments to the volatility level and the periodic rate change limit have effects that are different from those in the preceding analysis in two important ways. First, they increase the *S*2 and *S*3 (upward and downward constraint) frequencies equally. Second, they impact all years of the loan's life.

As shown in Panel C of Exhibit 2, when index volatility increases, these developments enlarge the value of the fixed-rate component and give greater relative weight to its later cash flows, increasing duration. Therefore, since the two components of total ARM duration are moving in the same direction, small increments in index volatility cause substantial increases in the interest-rate risk of an ARM loan.[13] This analysis suggests that lenders should be especially concerned about protecting the value of their adjustable-rate loan portfolios when short-term rates are erratic, and corroborates the findings of Kau et al. (1990, 1993) that volatility decreases the value of an ARM loan. Notice also that increased volatility decreases expected ARM returns (yield) in Exhibit 2.

Annual Rate Change Limits

As the annual rate change limit rises, Panel D of Exhibit 2 shows that the frequencies with which both upward and downward constraints hold diminish rapidly. This decreases the size of the fixed-rate component and its duration so the net effect of increased limits on the interest-rate risk of the total loan is a dramatic reduction. Expected ARM yield also rises.

Typical ARM limits moved from 1.5% to 2% early in the origination history of the loans. That effort by lenders appears to have decreased their interest-rate exposure by almost 50%. This finding is consistent with earlier work applying option pricing models to ARMs. For example, Hendershott and Shilling (1985) found that doubling an ARM's adjustment limit from 1% to 2% increased the risk-neutral required coupon rate premium from 1% to only 1.25%.

Recent History of the Adjustment Index

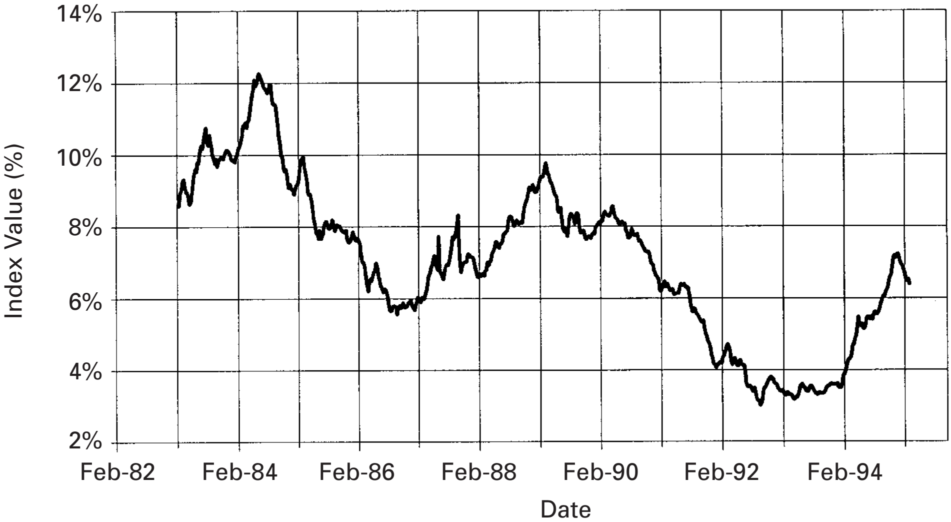
Simulation parameters that generated the results in Exhibits 1 and 2 were estimated from weekly values of the one-year constant maturity Treasury yield from 1985 to 1990. As shown in Exhibit 3, the level and volatility of the adjustment index has varied dramatically over the past decade, which raises the question of the applicability of results estimated from earlier data.

Conclusions drawn from a reestimation of the simulation parameters (available from the authors) over the 1984 to 1995 time period can be summarized as follows. Estimation over the entire 577 weekly observations decreases the intercept and the slope coefficient, and increases the standard deviation of the error term. As a result, the explanatory power of the model drops to 17.2%. The slope coefficient is still some eleven times its standard error, however, indicating that the mean reversion component—which appears in the theoretical models of Cox et al. (1985) and Kau et al. (1990, 1993) and which forms the basis of our simulation process—is justified by the recent empirical behavior of the ARM adjustment index.

The most obvious difference between the estimated parameters for the entire sample period and the earlier period is the increase in volatility in the interest-rate process. The effect of this increase in volatility is evident if the results in Exhibit 1 are reestimated using parameters developed over the entire sample period. These results, which are available from the authors, are strikingly similar to those in Exhibit 1 in that the probabilities of constrained upward adjustment are significantly greater in the first two yeas of the loan's life than in the later periods, and that the "steady-state" $S2$ and $S3$ frequencies are both stable and fairly low (they exceed the values in Exhibit 1 by only 2.5% to 3%, on average).

Under the extended-sample simulation parameters, the duration of the ARM used as an example in this study would double. This growth is consistent with the findings in

Exhibit 3
One-Year Constant Maturity Treasury Index



Sources: *Federal Reserve Bulletin*, various issues; *Wall Street Journal*, various issues

Panel C of Exhibit 2 where relatively small increases in index volatility lead to large increases in ARM duration.

The extended sample estimation results discussed above highlight the importance of the relationship between index volatility and the interest-rate risk exposure inherent in a portfolio of ARMs. They also serve to reaffirm the general applicability of the findings documented in Exhibits 1 and 2.

Conclusions

This paper evaluates the interest-rate risk of adjustable-rate mortgage loans via the traditional duration-based approach to price sensitivity. Simulation analysis using parameters estimated from market data generates index paths that are used to compute ARM cash flows and to divide those cash flows into periods where rate change constraints are binding and periods where they are not binding. The expected value of both sets of cash flows generates a yield estimate that is used to compute the value of the fixed-rate component and its duration. The fixed-rate component weight and duration provide an estimate of the duration of the entire ARM loan.

Simulation results using popular values for an ARM's rate adjustment parameters suggest that:

- life-of-loan rate caps have minimal impact;
- upward adjustment constraints often hold in the early years of a loan's life, the probability that either constraint holds is balanced and fairly small after the early years, but the constraints do imply that ARMs are not free of interest-rate risk; and

- ARM loans (especially aged loans) have significantly less interest-rate risk than fixed-rate loans.

Simulation results that address the variation in interest-rate sensitivity that occurs as one of the features of the adjustment mechanism is altered suggest that:

- increasing teasers and margins have a minimal effect on interest-rate sensitivity so the former can be used to attract ARM borrowers initially while the latter allows income lost by the teaser to be recouped over the loan life (actual trends in ARM parameters suggest that lenders have recognized the beneficial risk/return characteristics of this strategy);
- volatility in the adjustment index and changes in the periodic constraints do have a significant impact on the interest-rate risk of an ARM loan;
- the stochastic process and the results presented in this study are general enough to be applicable to both the time period used to generate the simulation parameters (1985–90) and the entire past decade, despite the fact that these markets were characterized by different distributions of interest rates. This suggests that the conclusions developed from Exhibits 1 and 2, and the managerial inferences that arise from them, are applicable in future interest-rate environments.

The model advocated in this paper as a means of assessing the interest-rate risk of ARM loans has provided interesting insights into the dynamic behavior of the loan position as interest rates move over time. As ARM loans increase in popularity, an understanding of the interest-rate exposure they generate becomes crucial to the successful management of assets held in these mortgages.

Notes

[1]Kau et al. capture the dependence of a current ARM rate on its past values by the inclusion of an additional state variable, the current ARM rate. This allows them to use a theoretically superior backward-solving approach to construct and discount an ARM's cash flow. See Kau et al. (1990, p. 1417) and Hendershott and Shilling (1985, p. 318) for a discussion of the problems encountered in applying limited backward-solving option approaches to ARM valuation.

[2]Earlier word by Buser, Hendershott and Sanders (1985) and Hendershott and Shilling (1985) includes a similar analysis of the initial contract rates required on ARMs to compensate lenders for rate adjustment constraints. These authors corroborate the volatility, limit vs. cap and upward vs. downward adjustment findings documented in the two Kau et al. papers (1990, 1993).

[3]Given the recent drop in interest rates, the duration of fixed-rate loans containing prepayment options has attracted more academic interest. See Haensly, Waller and Springer (1993) for a traditional interest-rate risk analysis of a fixed-rate loan with exogenous prepayments, and Anderson, Barber and Chang (1993) for a duration specification that allows the relationship between the existing loan rate and the current market rate to spur prepayment.

[4]A teaser exists at origination if R_t is lower than the initial index level plus the margin.

[5]A corresponding transformation would be performed on dB_{t-1}.

[6]The rate adjustment occurs only once every year. Consequently, the year t has a bearing on the monthly mortgage payment but the month, i, has no impact. Origination points, which reduce the amount the lender disburses on the loan by a small fraction, are ignored here in the interest of simplicity although it would be a trivial matter to incorporate them into the model. Points would raise the yield slightly, reducing the numerator in the eventual duration calculation. They would also lower the denominator by lowering price so the net effect is likely to be minimal.

[7]An anonymous reviewer has brought to light an important point. Prepayments on ARMs may be influenced by interest rates on fixed-rate loans because borrowers may switch out of ARMs when fixed-rate loan rates fall to attractive levels. The ARM prepayment model of McConnell and Singh (1991) includes a component specifically designed to incorporate the actions of “switchers” and finds their role to be significant. As explained below, our model assumes a given ARM remains outstanding for its entire life in the interest of conservatism. Secondary results that consider shorter expected lives are discussed in note 8.

[8]Our model has been reestimated assuming prepayment in a given year and assuming that the outstanding balance is part of the restricted component. Under reasonable assumptions, the estimated duration of the ARM declines as expected life increases for the nearest few payoff dates. This occurs because more and more unconstrained cash flows are being added to a loan that was dominated by the constrained cash flows in year 1. Once this trend reverses (usually when expected maturity is four years or more), ARM duration increases with maturity and the rate of increase is almost identical to the rate of increase in the duration of a similar fixed-rate loan as its expected life rises. Estimates of the duration of an ARM with a given expected prepayment date are available from the authors.

[9]The methodology for interest-rate risk analysis developed in this paper is not dependent upon the interest-rate process adopted but our specification has a precedent because it is the same as the one adopted by Tucker (1991) to compare expected interest costs of fixed-rate loans and ARMs. The process also has a mean reversion component similar to those found in the original stochastic interest-rate model proposed by Cox et al. (1985) and the option valuation models of Kau et al. (1990, 1993). Our particular formulation is chosen because it is well-behaved in that negative values for the underlying index are impossible and extreme positive values are unlikely. Furthermore, recent comparative empirical work by Chan, Karolyi, Longstaff, and Sanders (1992) suggests that none of the competing complicated stochastic alternatives advocated by continuous time research performs well in explaining the behavior of short-term interest rates. Finally, the specification above has the additional attractive property that volatility in the adjustment index is positively related to the mean long-run level of interest rates. This feature is one of the few interest-rate regularities for which Chan et al. were able to document empirical support.

[10]ARMs first became a significant component of the residential mortgage market beginning in the mid-1980s so the estimation period for the index dynamics begins then. The effect of recent values of the adjustment index is covered in a later section of the paper.

[11]The frequencies in the $S2$ column are dominated by the effect of the periodic constraints instead of the lifetime constraint because the lifetime up cap is rarely binding. Out of the 20,000 simulations of thirty rate adjustments, the lifetime cap was binding only twelve times. The lack of importance of the life-time cap is consistent with the loan valuation results in Kau et al. (1990, 1993), Buser et al. (1985) and Hendershott and Shilling (1985).

[12]Heuson (1989) contains summary statistics for the size and time pattern of teasers, margins and rate change limits offered by a variety of ARM lenders while Houston et al. (1991) document the adjustment parameter trade-offs accepted by ARM lenders.

[13]It is interesting to note that federally insured lenders were first allowed to offer ARMs in the early 1980s. Although this was considered to be a period of high interest-rate volatility, volatility in the adjustment index used in our simulations was only 30% higher in the early 1980s than it is in the period of this study. This increase is well within the bounds covered in Panel C, Exhibit 2.

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