Non-Periodic Co-Cycles: Real Estate and Stock Markets

Abstract. **The literature is not clear on whether there are co-dependencies domestically across real estate and stock markets, nor whether there are international co-dependencies for these asset classes, despite the importance of this question for portfolio diversification strategies. In this article, we use a non-linear technique to search for co-dependence over the long term. We find no evidence to suggest long co-memories between stock and property markets in the United States and the United Kingdom, but some evidence of this in Australia. In an international context, if we take whole of sample period data, we find no evidence of long co-memory effects, however if we sample on either side of the 1987 market correction we find evidence of long co-memory.**

Introduction

Investment portfolio managers aspire to optimize the return from their portfolios at given levels of risk. Conventionally this strategy requires the allocation of resources across a number of asset classes such as property, stocks and bonds as well as across international boundaries. An underlying assumption of this conventional wisdom is that the given assets are not close substitutes for each other since, if this were the case, there would be little to gain in terms of risk reduction by holding such substitutable assets in the portfolio. The underlying motivation for the present research is to consider whether securitized property should be held in a portfolio that contains general stocks, or whether securitized property from different countries should be held in an investment portfolio. In other words, it is of interest to know whether the markets for these assets are integrated or segmented in either a domestic or an international context.

A search of the literature reveals that there is no consensus on whether property should be included in domestic portfolios, or whether investment portfolios should extend across international boundaries. This article will contribute further to the debate on portfolio diversification by considering the question of whether securitized property markets and stock markets have very long-term ‘co-memories’ in both a domestic and

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international context. It is clearly important to discover if asset markets have some kind of long co-memory or co-cycle since this will have important ramifications for portfolio diversification strategies in the short to medium versus the long term. In a conventional sense, we would not expect such a co-cycle to be periodic, but we would expect such a co-cycle to be governed by the broader economic forces. If assets markets are found to have a long term relationship (*i.e.*, to be co-integrated over the long term) then this would suggest that there may be little long term gain in terms of risk reduction by holding such assets jointly in a portfolio. Moreover, if long co-memory exists in asset returns series, then statistical inferences on asset pricing models that are based on standard testing procedures may no longer be valid. Consequently, the primary objective of this article is to look for evidence of long term co-memories among domestic asset classes (specifically property and stocks) as well as long term co-memories between property asset classes across international boundaries. Specifically, we are seeking evidence of long-term dependencies and long run non-periodic cycles and co-cycles. Since the finding of fractional cointegration implies the existence of long run co-dependencies and long run non-periodic cycles in the residual series, we are seeking evidence of fractional integration and fractional cointegration in securitized property markets and stock markets within and between countries.

Literature Review

The literature remains unclear as to whether real estate and stock markets are segmented or integrated either in the short run or the long run. For instance, Liu, Hartzell, Greig and Grissom (1990) found evidence of market segmentation between real estate and stock markets when using appraisal based returns. These results were also supported by Geltner (1990) who found the noise component of real estate and stock returns were different and concluded that the two markets were probably segmented. However, studies by Ambrose, Ancel and Griffiths (1992) and Gyourko and Keim (1992) have produced opposite results. Ambrose, Ancel and Griffiths employed a rescaled range analysis to test deterministic nonlinear trend in the return series. Their results showed that mortgage and equity real estate investments trusts (REITs) displayed similar return generating characteristics to the stock market and they concluded that the real estate and stock markets might be somehow integrated. Gyourko and Keim provided evidence that the stock market contains important information about real estate fundamentals. In their study, they regress equity REIT returns against returns on the S&P 500 and find that S&P 500 returns have significant explanatory power in predicting equity REIT returns. Furthermore, Meyer and Webb (1993) found that the returns on equity REITs appeared to be much like the returns on common stocks, hence indicating some degree of integration between these markets. To further confuse the issue Liu, Hartzell, Greig and Grissom also produced opposite evidence that the equity REIT and stock market were integrated when compared to using appraisal-based returns.

The limited research that has been undertaken on the question of whether securitized property markets (property investment trusts and the like) are integrated internationally has generally tended to support market segmentation, although there is some divergence of opinion. Giliberto (1990), Asabere, Kleiman and McGowan (1991), Sweeney (1993) and Eichholtz and Lie (1995) all provide a certain amount of evidence of the potential risk reduction and return benefits to be derived from international diversification in property investments. Ziobrowski and Curcio (1991), on the other hand, point out that the benefits to be gained from international diversification in real estate are illusory once exchange rate fluctuations are taken into account.

It is apparent from the above discussion that the matter is still unclear whether the real estate and stock markets are segmented or integrated. We contribute to the debate by using a technique that provides a test for long co-memory effects between real estate and stock markets in a domestic context and property markets in an international context. Whether there are long memory effects in asset returns has important implications for many of the models used in financial and real estate economics since, as Cheung and Lai (1995) suggest, any series having long memory is characterized by long term dependence and non-periodic long cycles. Knowledge of the possible existence of such long cycles in financial assets markets (including securitized and physical real estate markets) is important. For instance, in terms of strategic asset allocation, portfolio diversification decisions may become extremely sensitive to the investment horizon if asset returns were long-range dependent, but not short-range dependent. Such sensitivity would depend on the speed of mean reversion. That is, if mean reversion is very slow (long -range dependent), then there may be diversification benefits in the short to medium term, but not if the assets are held together over the long term.[1]

This study examines the issue of a possible relationship between real estate and stock markets that incorporates the notion of cointegration by Engle and Granger (1987) and of fractional differencing put forward by Granger and Joyeux (1980) and Hosking (1981). The traditional Engle and Granger cointegration analysis tests for a long-term *linear* relationship between economic variables. To implement the procedure it is necessary to test each series to determine the order of integration. This is accomplished via unit root tests that presume the order of differencing is integer. Typical empirical work on cointegration analysis between, say, two series will first test if the series are both integrated to the same order [conventionally I(1)] and, assuming this to be true will then test if the error term in the cointegrating regression is I(0). Finding the error term to be I(0) implies that the error term exhibits *mean reverting* behavior and that there exists a long run equilibrium relationship between the series in the cointegrating regression. On the other hand, finding the error term to be I(1) implies this is non-stationary and hence there cannot be a long run equilibrium relationship between the series in the cointegrating regression.

Numerous studies, however, have suggested that the strict I(1) and I(0) condition is too restrictive and that allowing the differencing operator to be non integer allows for a much richer class of mean reverting processes. As Cheung and Lai (1993, 1995) point out the strict I(1) and I(0) differentiation is, in analytical terms, purely arbitrary and the equilibrium error does not have to be I(0) exactly for it to be mean reverting. It has been shown that if the differencing operator is between zero and one the process will still be mean reverting. In fact, there is a growing body of evidence that supports the idea that some macroeconomic data are generated by fractionally differenced models. For example, Diebold and Rudebusch (1989) found evidence of fractional integration when analyzing different measures of aggregate economic activity. Similarly, Cheung and Lai (1993) have shown that purchasing power parity holds in the long term under fractional cointegration (while the traditional Engle and Granger (1987) cointegration tests could not offer similar support). Geweke and Porter-Hudak (1983) also provide some evidence that consumer and wholesale price indices are fractionally integrated series. Interestingly, Granger (1980) has shown that for a particular class of AR(1) processes the aggregation of these series leads to a fractionally integrated process. This may have important implications in relation to aggregate stock market and real estate indices if one is trying to determine whether the two markets are related.

As suggested, the literature has shown that fractionally cointegrated series are slowly mean reverting and exhibit significant persistence in the long term. If domestic property and stock markets are fractionally co-integrated, or if these markets are fractionally cointegrated in an international setting, then this implies the existence of an equilibrium relationship between these markets that may not be apparent if standard ARMA processes are used. The reason is that conventional ARMA models assume that the underlying data generating process is *linear*, whereas fractionally integrated/co-integrated models are not restricted to data generating processes that are linear. While the underlying data generating process may be *non-linear*, the process may still be mean reverting, but the return to equilibrium may be very slow. Failure to recognize equilibrium relationships that only return to equilibrium very slowly may yield less than optimal portfolio diversification strategies over the long term. In terms of risk diversification, the hope is to find no long run co-dependence between asset classes that have been included in the same portfolio.

Much of the literature in relation to the integration of real estate and stock markets, using conventional cointegration tests, supports the view that the two markets are segmented. However, these results may be biased as Diebold and Rudebusch (1989) and Cheung and Lai (1993) have shown that standard cointegration tests have low power against fractionally integrated alternatives. Since the literature has shown that the equilibrium error for a cointegrating regression can show slow mean reversion that is not captured by the usual I(0) process, then any broad test for cointegration must incorporate fractional cointegration.

The format of the rest of the paper is as follows. The next section briefly outlines the concept of fractional differencing and its inter-relationship with long memory as well as with conventional ARIMA models. Empirical results of tests on domestic and international property markets are presented in the following section. The final section summarizes the findings of the article, while the appendices contain more technical detail on estimation procedures.

Fractional Cointegration

A series is said to be integrated of order d if it is stationary after differencing d times. The use of the differencing operator is to induce stationarity in an otherwise nonstationary series. It has been suggested that integer differencing may sometimes be inappropriate given the findings of Diebold and Rudebusch (1989), Cheung and Lai (1993, 1995) and Baillie and Bollerslev (1994). These authors submit that some economic time series cannot be satisfactorily modeled using standard ARIMA techniques, and thus it becomes necessary to use techniques that can capture the long-term memory feature displayed by some economic time series.

To commence discussion consider the following standard ARIMA model defined by:

$$\phi(L)(1 - L)^d Y_t = \theta(L)e_t, \tag{1}$$

where L is the lag operator, $e_t \sim (0, \sigma_e^2)$ and $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 \ldots - \phi_p L^p$, $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 \ldots - \theta_q L^q$, all roots of $\phi(L)$ and $\theta(L)$ are on or outside the unit circle and d is an integer. On the other hand, if d is allowed to be real the standard ARIMA model can be extended to permit fractional differencing. In Equation (1), for instance, a binomial expansion on $(1 - L)d$ yields the following (fractional) filter:

$$\begin{aligned}(1 - L)^d &= \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k \\ &= 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 + \ldots\end{aligned} \tag{2}$$

If the model specified by Equations (1) and (2) is fractionally integrated then the series is referred to as an autoregressive fractionally integrated moving average model (ARFIMA). The ARIMA model is a special case of this fractional model with d taking on integer values.[2] For values of $0 < d < 0.5$, the autocorrelations decay at a hyperbolic rate, whereas the rate of decay of the autocorrelations of an ARMA process are geometric. The slower rate of decay of the fractionally integrated series displays the characteristics of a long memory process—long memory processes show significant dependence between observations widely separated in time. For $0.5 \leq d < 1$, the process is nonstationary as the variance is infinite, however a shock does not have a permanent effect since the process is slowly mean reverting.

Autocorrelation analysis reveals information about serial dependence in the data set. Now, to provide some insight into how the autocorrelations vary between the ARMA (short memory) and ARFIMA (long memory) models we reproduce a table (Exhibit 1) from Diebold and Rudebusch (1989), which shows the difference between the autocorrelations for a pure AR(1,0,0) and pure fractional noise ARFIMA(0,d,0).

Suppose the data in Exhibits 1 and 2 was generated from quarterly observations on a series that we were interested in examining for some internal dependence (memory). It is evident from the ARIMA (1,0,0) process that the autocorrelations die off fairly quickly (no autocorrelation in the series after five quarters—short memory process), whereas for the ARFIMA (0,d,0) process there is still an autocorrelation of 0.11 at lag 100. It can be seen that the ARFIMA model has the potential to capture processes with long memory characteristics, which display significant dependence between

Exhibit 1
Autocorrelations Functions for ARIMA (1,0,0) and ARFIMA (0,*d*,0)

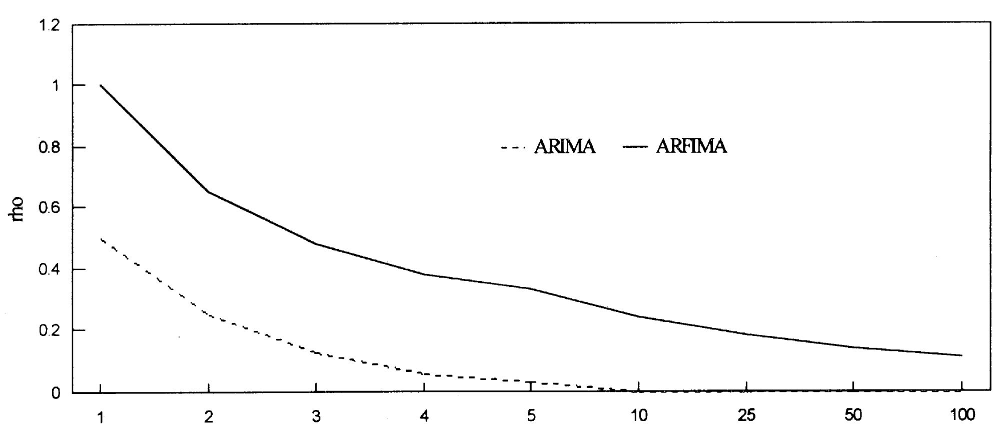
	Lag(ζ)								
	1	2	3	4	5	10	25	50	100
$(1 - 0.5L)Y_t = e_t, \rho(\zeta)$	0.50	0.25	0.13	0.06	0.03	0.00	0.00	0.00	0.00
$(1 - L)^{0.3}Y_t = e_t, \rho(\zeta)$	0.50	0.40	0.35	0.32	0.30	0.24	0.18	0.14	0.11

observations widely separated in time. On the other hand, a stationary ARMA process typically displays significant dependence up to lag ζ and then the dependence between observations falls off rapidly as time increases. Now, if we can estimate *d*, and show this to be fractional rather than integer, then we will have produced evidence of long term dependence in the series as illustrated in Exhibit 2.[3]

Implementation of the fractional cointegration technique follows along similar lines to the standard cointegration technique (Cheung and Lai, 1993). Initially, each series is tested to determine the order of integration. The order of integration of each series needs to be the same, as is the case with the Engle and Granger technique. Once this condition is satisfied (and assuming $d > 0$ in the conventional unit root test on each individual series) we perform the cointegrating regression and then check the residuals for fractional integration.

The next step in the fractional integration test on the residual series requires estimation of the parameter *d* along with tests of significance on this. In this paper, this parameter

Exhibit 2
Autocorrelation Functions
ARIMA (1,0,0) and ARHMA (0,*d*,0)



is our primary interest because evidence of fractional integration (*i.e.*, $0 < d < 1$) in the error series provides evidence of long term co-memory between the two series. That is, a value of d between zero and one in the residual series from the cointegrating regression provides evidence of a long-term equilibrium relationship. The estimation is undertaken using a spectral regression technique developed by Geweke and Porter-Hudak (1983) (GPH).[4] Estimates of d are based on the low frequency (*i.e.*, long cycle) order of the spectral density function (*i.e.*, near $\omega = 0$ where ω is the angular frequency measured in radians). The statistical procedure involves estimating d using the following spectral regression (see Appendix A for development of this equation):

$$\ln(I(\omega_j)) = \beta_0 - \beta_1 \ln\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + \eta_t, \tag{3}$$

where: $I(\omega_j)$ denotes the periodogram at ordinate j; $\beta_1 = (d - 1)$; $\omega_j = 2\pi j/T(j = 0, 1, 2, \ldots T - 1)$ are the harmonic frequencies of the sample; $\eta_t = \ln(I(\omega_j)/f_x(\omega_j)$ are iid across the harmonic frequencies; $f_x(\omega_j)$ is the spectral density of the first differenced series, X_t.

Thus, all of our series information is contained in Equation 3 (see Appendix A). However, the amount of information that is used in this spectral regression in Equation (3) is crucial. Geweke and Porter-Hudak have shown that the number of low frequency periodogram ordinates used in the spectral regression is a function of sample size and that reasonable estimates of the fractional differencing coefficient, d, can be obtained from β_1 if the number of ordinates used varies according to $K = g(T) = T^{0<\tau<1}$. Clearly some subjectivity is involved in the selection of the number of low frequency ordinates to include. Both Diebold and Rudebusch (1989) and Cheung and Lai (1993) point out that if K is too large then the estimate of d will be contaminated due to the inclusion of medium and high frequency components (*i.e.*, the inclusion of short memory effects). On the other hand, a value of K that is too low will lead to imprecise estimates of d due to limited degrees of freedom in estimation. These authors have found that reasonable estimates of the fractional differencing coefficient can be obtained when $K = g(T) = T^{0.5\leq\tau\leq0.6}$. In line with this earlier research, we will also estimate the fractional differencing coefficient, d, for τ in a somewhat similar range, namely $0.4 \leq \tau \leq 0.6$. For instance, if our data set has 300 observations, then the number of low frequency periodogram ordinates (*i.e.*, ordinates near $\omega = 0$) included in the regression will be somewhere between ten and thirty.

Empirical Evidence

The Data

The securitized property data employed for the United States were monthly observations of the All REIT, Hybrid REIT, Equity REIT and Mortgage REIT indices published by the National Association of Real Estate Investment Trusts and which reflected different aspects of the real estate market in the U.S. The Standard and Poors Composite Price Index (S&P 500) was used to reflect movements in the stock market.

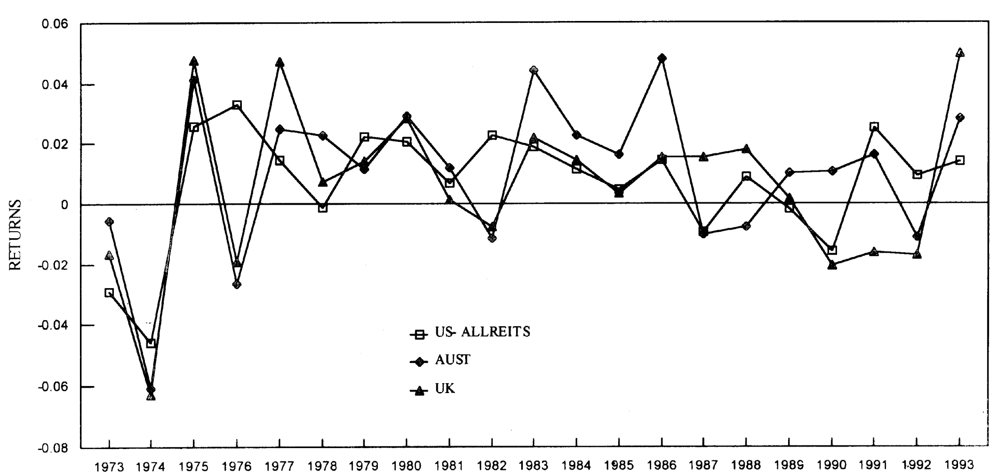
We also used the S&P Small Cap Index as a measure of movements in the stock market since Liu, Hartzell, Greig and Grissom (1990) have suggested that REITs movements are similar to small capitalized stocks rather than a large cap index such as the S&P 500. The sample period was from December 1971 to December 1993, with the base period set to March 1980. In the United Kingdom, we used the Financial Times Stock Exchange (FTSE) index to represent the stock market and the Financial Times All Properties (FTAP) index to represent the property market over the period 1969 to 1993. Finally, for Australia, the All Ordinaries (ALLORDS) index represented stock market movements, while the securitized property market was represented by a Property Index obtained from Datastream International for the period 1973 to 1993.[5] In all cases, the base period was set to March 1980.

The reader can obtain a visual sense for the data from Exhibit 3, which shows the approximate annual returns from securitized property in each of the study countries (All REITs was used for the U.S.). Approximate returns were obtained from each of the above property indices by differencing the natural logarithm of the monthly data and averaging this for the year.[6]

Long Co-Memories in Domestic Property and Stock Markets

Research by Perron (1989) and Zivot and Andrews (1992) amongst others suggested that consideration of structural breaks form an important part of any cointegration analysis. Wilson, Okunev and Webb (1998) used a variety of linear techniques to examine the above data sets for cointegration under conditions of known and unknown structural breaks. The results from these linear tests showed that the real estate and equity markets were not cointegrated. Using the Zivot and Andrews and Tsay (1986)

Exhibit 3
Approximate Returns
Securitized Property



techniques Wilson, Okunev and Webb identified, not unexpectedly, October 1987 as a structural break common to each series. Consequently, a number of sample periods were used for the purpose of testing for fractional cointegration in the relevant series for each country, namely: (1) whole of period sample for each country; and (2) sub-samples on either side of the October 1987 market correction. While undertaking conventional cointegration tests, Wilson, Okunev and Webb showed that the above individual series were all I(1) and these results are summarized briefly in Appendix B.[7]

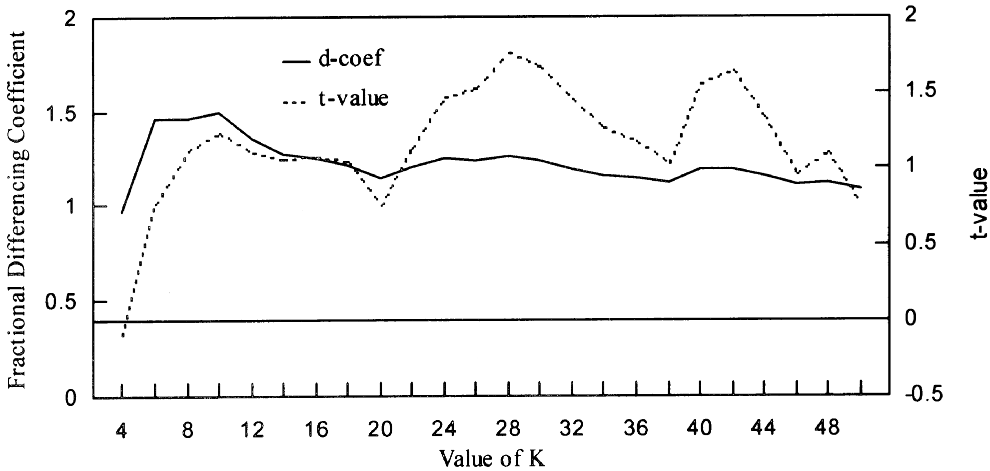
It was pointed out earlier that there is a degree of subjectivity involved in the analysis regarding the number of low frequency periodogram ordinates, K, to use in estimating the fractional differencing coefficient, d. Exhibit 4 shows the sensitivity of *d-coefficient* estimates to the number of low frequency periodogram ordinates included in the GPH spectral regression. This is presented by way of example for the U.S. (All REITs vs. S&P 500) and a visual impression (for the range $K = 4..50$) is shown in Exhibit 4. Exhibit 5 clearly demonstrates the difficulties associated with estimation of the fractional differencing coefficient—namely, the number of low frequency ordinates (K) to use in the spectral regression.

Now, what number of low frequency ordinates should be used in the spectral regression? In estimation of d over a number of individual series, Geweke and Porter-Hudak (1983) chose the value of K that minimized the mean square error of 20-step-ahead, in-sample forecasts. Using this procedure, these authors found that the number of ordinates selected was between $T^{0.55}$ and $T^{0.6}$. Other authors have used a wider range for K (see Diebold and Rudebusch (1989); and Cheung and Lai (1993, 1995). For this article, to select the number of low frequency ordinates to use in the spectral regression, we have decided to use that value of τ that produced the highest t-value

Exhibit 4
Sensitivity of Fractional Differencing Coefficient Estimate to Number of Low Frequency Ordinates used in Spectral Regression

# of Ordinates		
K	*d-coef.*	*t*-value
10	1.452	1.30
12	1.319	1.15
14	1.257	1.12
16	1.268	1.25
18	1.233	1.22
44	1.155	1.35
46	1.109	0.97
48	1.122	1.11
50	1.081	0.46

Exhibit 5
Sensitivity of *d-Coefficient* to Number of Low Frequency Ordinate Used



for the *d-coefficient* when using τ values in the approximate range found to be appropriate by other researchers, namely $0.4 \leq \tau \leq 0.6$.

At this point, it is important to bear clearly in mind what information our estimate of the fractional differencing coefficient will convey (and, perhaps more importantly, what information it will not convey). A *d-coefficient* between zero and one indicates that there *is* long memory within the series. If we are working with residual series from cointegrating regressions, such a *d-coefficient* would be indicative of long co-memory between the two series. While a series having long memory is characterized by *non-periodic long cycles* (Cheung and Lai, 1995) the value of the fractional differencing coefficient itself does not reveal any information about average wavelength or amplitude of such cycles, that is the value of the *d-coefficient* is not defining the cycle. The relevance of our work is that, within the confines of our data set, a value of *d* between zero and one tells us whether we should be looking for long co-cycles or not.[8] If two assets, say property and stocks, are held in the same portfolio then it would be desirable for these not to have long co-memory (*i.e.*, not to have a *d-coefficient* in this range) since it would reduce the long term risk reduction benefits associated with diversification. For individual series estimation of the *d-coefficient* provides the first, but most important step, in the estimation of an ARFIMA model that may be later used for forecasting purposes. That is, once *d* has been estimated, the filter shown at Equation (2) may be applied to (fractionally) difference the series. The differenced series may be then used to estimate the autoregressive and moving average components of the ARFIMA model. This ARFIMA model may then be compared with the original series to estimate how well we have fared in predicting turning points etc.[9]

Exhibit 6 shows the estimated values for the *d-coefficient* (along with associated *t*-values) for tests of fractional cointegration between U.S. property and stock markets.

Exhibit 6
Fractional Differencing Coefficient for Residual Series U.S. Property and Stock Markets

		K		*d-coefficient*		*t*-value	
LHS Series	Period	S&P[a]	SC[b]	S&P[a]	SC[b]	S&P[a]	SC[b]
All REITs	1972–1993	28	30	1.27	1.21	1.77	1.78
	1972–1987	22	26	1.23	1.37	1.71	1.73
	1987–1993	10	14	1.33	0.72	1.41	−1.66
Equity REITs	1972–1993	12	28	1.43	0.86	0.86	−1.76
	1972–1987	18	22	1.37	1.45	1.53	1.43
	1987–1993	14	16	0.94	0.67	−0.32	−0.94
Mortgage REITs	1972–1993	30	30	1.26	1.31	2.03	1.95
	1972–1987	20	22	1.35	1.25	1.60	1.67
	1987–1993	14	16	1.63	1.42	1.49	1.70
Hybrid REITs	1972–1993	30	28	1.29	1.28	1.77	2.85
	1972–1987	22	22	1.32	1.30	1.62	1.78
	1987–1993	14	14	1.65	1.24	1.53	0.94

Note: Since we are looking for evidence of fractional integration, the test is $d = 1$ against the one-sided alternative $d < 1$. The *d-coefficient* has a 5% CV: 1972–1993 = −1.87, 1972–1987 = −1.87 and 1987–1993 = −1.85. The *t*-value has a 10% CV: 1972–1993 = −1.60, 1972–1987 = −1.59 and 1987–1993 = −1.58.
[a]S&P is the S&P 500 and is RHS variable.
[b]SC is the S&P Small Cap Index and is RHS variable.

The exhibit also indicates how many low periodogram ordinates (K) were used in each spectral regression to estimate the *d-coefficient*. A number of researchers have clearly identified October/November 1987 as representing a structural break period in stock market and securitized property market series (see Wilson, Okunev and Webb, 1998). Other research has shown that it is important to consider the impact of structural breaks on tests of cointegration (see Perron, 1989; Zivot and Andrews, 1992; and Perron and Vogelsang, 1992). In view of this, Exhibit 6 uses three clearly defined periods to estimate the fractional differencing coefficient viz. a whole of period data set along with subsets on either side of the 1987 markets crash. Since conventional *t*-tables cannot be used for tests of significance in the spectral regression shown at Equation 3, (Cheung and Lai, 1993:107) the critical *t*-values for each of the data set and subsets in this and other tables were obtained by Monte Carlo simulation as outlined in Cheung and Lai (1993) for sample values of 265, 180, 167 and 74 with 50,000 replications each.

Even allowing for the difficulties associated with estimation of the fractional differencing coefficient as indicated earlier, Exhibit 6 provides strong evidence to suggest that the residuals of the cointegrating regression between property and stock markets in the U.S. are nonstationary and that these series do not have long co-memories. Irrespective of whether the S&P 500 or the S&P Small Cap series was used to represent the stock market, in only three cases was the *d-coefficient* found to

Exhibit 7
Fractional Co-integration Coefficient
U.K. Property against Stock Markets

Period	K	*d-coefficient*	t-value
1969–1993	26	1.12	0.72
1969–1975	10	1.28	0.99
1975–1987	10	1.33	1.02
1987–1993	20	0.91	−0.58

Note: The *d-coefficient* has a 5% CV: 1969–1993 = −1.87, 1969–1975 = −1.85, 1975–1987 = −1.87 and 1987–1993 = −1.85. The t-value has a 10% CV: 1969–1993 = −1.60, 1969–1975 = −1.58, 1975–1987 = −1.59 and 1987–1993 = −1.58.

be less than one, and two of these were found to be significant only at the weak 10% level.[10]

In like manner, the results presented in Exhibit 7 offer very little support for long co-memory between property and stock markets in the U.K. However, results presented in Exhibit 8 for Australia do offer some support for fractional cointegration between property and stock markets. The fractional differencing coefficient for the full sample period and the period prior to the 1987 market correction were both less than one, although only the estimate for the full sample period was significant. A possible explanation for these Australian results is that, since only ten or twelve companies make up the securitized property index constructed by Datastream International, it may take only a relatively small number of buyers/sellers in this securitized property market to move the index. If such players enter/leave securitized property as a reaction to events in the broader stock market then this may be sufficient to generate the long co-memory effects found for Australia. The implication for both the U.S. and the U.K. is that securitized property and other stocks are not substitutable assets over the long run and these assets may be held together in a portfolio for diversification purposes.

Exhibit 8
Fractional Co-integration Coefficient
Australian Securitized Property against Stock Market

Period	K	*d-coefficient*	t-value
1973–1993	26	0.69	−2.51
1973–1987	12	0.63	−1.08
1987–1993	18	1.37	1.48

Note: The *d-coefficient* has a 5% CV: 1973–1993 = −1.87, 1973–1987 = −1.85 and 1987–1993 = −1.87. The t-value has a 10% CV: 1973–1993 = −1.60, 1973–1987 = −1.58 and 1987–1993 = −1.58.

The proportions in which they are held, of course, will depend directly on the risk reduction benefits traded against portfolio returns that accrue.

Long Co-Memories in International Property and Stock Markets

As suggested, the literature is not entirely clear on the question of whether international property markets are integrated or not. Prior to testing for international fractional co-integration, all series were expressed in U.S. dollars according to the following procedure:

$$P_{adj} = \frac{P_{it}}{S_{it}}. \tag{4}$$

Where P_{adj} is the foreign price index adjusted for domestic currency, P_{it} represents the given price index of country i at time t and S_{it} represents the spot exchange rate expressed in units of the foreign currency for one unit of the domestic currency at time t. Expressing international series in a common currency unit reduces the likelihood of any indicator of co-movements of the series being contaminated by currency fluctuations.

As was the case for the previous section, research by Wilson, Okunev and Webb (1998) on these data sets showed that all of the individual series were I(1) and that October 1987 represented a structural break even in the presence of exchange rate differences (see Appendix B). The results for the tests of fractional cointegration between the U.S. and each of the U.K. and Australia for property markets are shown in Exhibit 9. There is marginal support for the notion that international property markets may have long co-memory. In tests of long co-memory for the U.S./U.K. international property markets two of the fractional differencing coefficients were less

Exhibit 9
Fractional Co-integration Test of International Property Markets

Series	Period	*K*	*d-coefficient*	*t*-value
U.K./U.S.	1972–1993	16	0.84	−0.84
	1972–1987	28	0.54	−2.02
	1987–1993	10	1.29	0.57
Aust./U.S.	1972–1993	16	1.21	1.07
	1972–1987	16	0.84	−0.61
	1987–1993	20	0.83	−1.03
Aust./U.K.	1972–1993	26	0.75	−1.15
	1972–1987	24	0.52	−2.98
	1987–1993	20	0.87	−0.81

Note: The *d-coefficient* has a 5% CV: 1972–1993 = −1.87, 1972–1987 = −1.87 and 1987–1993 = −1.85. The *t*-value has a 10% CV: 1972–1993 = −1.60, 1972–1987 = −1.59 and 1987–1993 = −1.58.

than one, although only one of these was significant whereas for the U.S./Australian property markets, while two of the *d-coefficients* were also less than one, neither of these was significant. In tests of long co-memory for the Australian/U.K. markets (in Australian dollars), all of the *d-coefficients* were less than one, although only one of these was significant. Both cases where there was a significant result occurred in the sample period prior to the 1987 markets correction. An implication of this is that important world economic events, such as the 1987 markets upheaval, can upset the status quo. In the period prior to the 1987 correction, the U.S. and U.K. property markets appeared to have some co-dependence over the long term. After the correction, however, this co-dependence appeared to disappear, perhaps as a result of the particular policies pursued by each country as it sought to avoid a recurrence of the events leading to the 1987 correction. While the coefficient over the longer 1972 to 1993 period is less than one, although not significant, this does not imply that as time unfolds and the effects of 1987 are dissipated, that the pre-1987 co-dependency will not re-emerge. Hence, there is a need for constant monitoring of these international asset classes.

Rolling and Recursive Estimation of Fractional Differencing Coefficient

Exhibits 6–8 produced little evidence to suggest long term co-integration between domestic property and stock markets, while Exhibit 9 produced results that suggested the possibility of international property markets having long co-memories in the period prior to the 1987 market correction, but such co-dependence being affected by the market correction. This latter result raises the question of whether estimates of the fractional differencing coefficient may be sensitive to important economic events occurring during the estimation period. Under these circumstances, it was thought appropriate to examine how robust the estimates would be to both different sampling periods and sample sizes. This was done via rolling and recursive (*i.e.*, sequential) estimation procedures. With rolling estimation the sample size is held constant as the sample moves forward through the data set and the *d-coefficient* is re-estimated on each roll through the series. In this analysis, the sample size was arbitrarily set at 100 months (a window size of 100), with the oldest month dropping off and a new month being added as the sample selection moved forward in time.

On the other hand, with recursive estimation, an initial sample size is nominated (in the current analysis this was arbitrarily set at 100 months) and the *d-coefficient* is estimated. From that point, a new observation is added and the *d-coefficient* is re-estimated. This sequential addition of observations and re-estimation of the *d-coefficient* continues until the full data set is exhausted. Hence, unlike the rolling estimation procedure, the sample size with recursive estimation continually grows. For both the recursive and rolling estimation procedures, the fractional differencing coefficient was extracted according to the same rule as was applied earlier.

Both rolling and recursive estimation procedures were undertaken for fractional differencing in domestic markets and the results are presented in Exhibit 10. In the U.S. domestic market, we used the All REITs, S&P 500 as market indicators, and for neither rolling nor recursive estimates did the fractional differencing coefficient fall

Exhibit 10
Fractional Differencing Coefficient
Domestic Markets

Markets	Proportion[a] (%)
U.S. Property/Stock Rolling	0
U.S. Property/Stock Recursive	0
U.K. Property/Stock Rolling	6
U.K. Property/Stock Recursive	0
Aust. Property/Stock Rolling	10
Aust. Property/Stock Recursive	72

Note: The number of estimates produced varied according to whether the procedure was rolling or recursive. In all cases the number of estimates produced exceeded 100.
[a]Proportion is <1 and significant at the 10% level or higher.

below a value of one. This clearly demonstrated that there did not appear to be any long term co-memory effects between property and stock markets using these indicators to represent the U.S. markets, and these results agreed with the earlier one-off (*i.e.*, single sample) results.[11]

A similar analysis was undertaken for the U.K. with only marginally different results—none of the recursive estimates were both less than one and significant, while only 6% of all rolling estimates were both less than one and significant (at the 10% level or better). As was the conclusion with the one-off (single sample) data, these rolling and recursive results offer little support for fractional co-integration between property and stock markets in the U.K.

These results provide positive reinforcement to portfolio diversification strategists in both the U.S. and the U.K. That is, across a variety of sample periods and sample sizes, the evidence suggests that property and stock markets do not appear to have long-term co-memory. In other words, there does not appear to be any long-term persistence in stock market behavior being transferred to, or reflected in, behavior in securitized property markets. The results are consistent with the one-off (single sample) outcomes for the sampling periods shown earlier in Exhibits 6–8. In contrast, a portfolio manager may not be quite so content with the results obtained for Australia (which are also consistent with earlier one-off results). Here 10% of the rolling estimates were both less than one and significant, while 72% of the recursive estimates were both less than one and significant. As suggested earlier, these results may be influenced by the relatively small number of companies making up the securitized property index in Australia.

Both rolling and recursive estimates of the fractional differencing coefficient were also undertaken for international property markets (in the common currency units described earlier) and the results are presented in Exhibit 11. In rolling estimates of the U.K./U.S. property markets, 9% of the *d-coefficient* estimates were both less than

Exhibit 11
Fractional Differencing Coefficient
International Markets

Markets	Proportion[a] (%)
Panel A: International Property	
U.K./U.S. Rolling	9
U.K./U.S. Recursive	30
Aust./U.S. Rolling	3
Aust./U.S. Recursive	2
Panel B: International Stock	
U.K./U.S. Rolling	40
U.K./U.S. Recursive	20
Aust./U.S. Rolling	52
Aust./U.S. Recursive	34

Note: The number of estimates produced varied according to whether the procedure was rolling or recursive. In all cases the number of estimates produced exceeded 100.
[a]Proportion is <1 and significant at the 10% level or higher.

one and significant (at the 10% level or better). For the recursive estimates, 30% of the coefficients were both less than one and significant. All of these significant statistics fell prior to the 1987 market correction, as we would expect from the earlier one-off sample results. Also, as we might expect from the earlier one-off rolling and recursive estimates of the Australian and U.S. securitized property markets, there was very little evidence to suggest long term co-memory effects between these markets. Only 3% of the *d-coefficient* rolling estimates were both less than one and significant, while only 2% of recursive estimates were both less than one and significant. These results are interesting in that the suggestion that there was co-dependence between U.S. and U.K. securitized property markets during part of the study period, while co-dependence occurs rarely between the U.S. and Australia, further support s the results reported in Wilson and Okunev (1996). There, using conventional mean-variance analysis to maximize return at the given risk level, it was suggested that a suitable international property diversification strategy by a U.S. investor would have about 63% of property investments in the U.S., about 30% in Australia, with only 7% in U.K. securitized property holdings.

Conclusion

Large pension funds seek to optimize the return from their investment portfolios at given levels of risk. Conventionally, such a strategy requires the allocation of resources across a number of asset classes such as stocks, bonds and property and across international boundaries. A search of the literature revealed that there is no consensus on whether property should be included in domestic portfolios, or whether property investment portfolios should extend across international boundaries.

The technique of fractional co-integration can provide evidence on whether there is long-term co-dependence (long run non-periodic cycles) across asset classes. The presence of fractional co-integration between pairs of assets would suggest that diversification across the relevant asset classes may not produce the risk reduction benefits expected. Fortunately, the research in this article showed that there was no support for long co-memory effects between property and stock markets in either the U.S. or the U.K., although the results were less clear for Australia.

There was some evidence of long term co-memory in property markets across international boundaries when the 1987 market correction was used as a breakpoint. These results indicated that estimates of fractional co-integration might be sensitive to important economic events occurring during the period sampled for analysis. As a further check on both domestic and international results, one slice rolling and recursive estimates to observe stability of the fractional co-integration coefficient were undertaken. In line with the earlier one-off results, these rolling and recursive estimates did not support long co-memory in the domestic U.S. and U.K. markets while, once again, there did appear to be co-memory effects between securitized property and stock markets in Australia. Also in-line with earlier results, rolling and recursive coefficient estimates found some evidence to indicate that there was some international co-dependence in securitized property markets between the U.S. and the U.K. when samples were rolled and expanded up to and through the 1987 structural break. A possible explanation for this outcome is that these international property markets may be non-linearly related with slow mean reversion between the two. The shock of 1987 disrupted this long run cyclical relationship and the number of years to the end of the study period has been insufficient for this long run cyclical relationship to be re-established. That is, the effects of the 1987 market correction operate on the estimate of the fractional differencing coefficient much like an outlier in a data set in conventional regression—the slope of the line is pulled in the direction of the outlier. As more data are added (especially data that is more spread out) the effects of this pull are reduced. The true relationship is always there, but the data attempting to reveal this relationship are influenced by the vagaries of the market.

The outcome from this article, and other studies by the authors (Wilson, Okunev and Webb, 1998), provides support for the conventional wisdom of diversifying across the equity and securitized property asset classes domestically. In addition, the whole of study period results lead us to support the notion of diversification across property classes internationally, although we must add one rider to this conclusion, namely the evidence also implies the need for constant monitoring of the international investment climate in the wake of important economic events such as the 1987 market correction.

Appendix A

Following the approach demonstrated by Geweke and Porter-Hudak (1983), Diebold and Rudebusch (1989) and Cheung and Lai (1993) the relevant series ($\{Y_t\}$) is first differenced *i.e.*, $X_t = (1 - L)Y_t$ and d is then estimated from the model:

$$(1 - L)^{(d-1)} X_t = \phi^{-1}(L)\theta(L)e_t = u_t, \tag{A.1}$$

where it is now assumed that u_t is stationary. It is known that the spectral density of X_t is given by:

$$f_x(\omega) = 2 \sin \left(\frac{\omega}{2}\right)^{-2(d-1)} f_u(\omega), \tag{A.2}$$

where $f_u(\omega)$ is the spectral density of the stationary error process u_t (Cheung and Lai, 1993). From the spectral representation theorem (see Hamilton, 1994) we know that the area under the spectral density function gives the variance of X_t, while the area under this function for any ω_j between 0 and π may be interpreted as the variance of X_t that is associated with frequencies ω that are less than ω_j (in absolute value). Taking logarithms of Equation (A.2) and evaluating at the harmonic frequencies $\omega_j = 2\pi j/T$ ($j = 0, 1, 2, \ldots T - 1$) and adding and subtracting $\ln(f_u(0))$ to the right hand side it can be shown that (see Diebold and Rudebusch, 1989:197):

$$\ln(f_x(\omega_j)) = \ln(f_u(0)) - (d - 1) \ln\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + \ln\left(\frac{f_u(\omega_j)}{f_u(0)}\right). \tag{A.3}$$

When the frequency is very low, say ω near 0, the last term in Equation (A.3) can dropped as it is negligible.

Now, letting $I_u(\omega)$ denote the periodogram at ordinate j where, for sample size T:

$$I_u(\omega) = \frac{1}{2\pi}\left[\hat{\gamma}_0 + 2\sum_{j=1}^{T-1} \hat{\gamma}_j \cos(\omega_j)\right], \tag{A.4}$$

and the γ_j is the sample autocovariances calculated using:

$$\hat{\gamma}_j = T^{-1} \sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y}) \qquad \text{for } j = 0,1, \ldots T - 1. \tag{A.5}$$

Then adding and subtracting $I_u(\omega)$ to both sides of Equation (A.3) and re-arranging yields:

$$\ln(I(\omega_j)) = \ln(f_u(0)) - (d - 1)\ln\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + \ln\left(\frac{I(\omega_j)}{f_x(\omega_j)}\right), \tag{A.6}$$

for $j = 1,2, \ldots, K$, where K refers to the number of low frequency periodogram ordinates used in the spectral regression and we estimate sequentially for $j \leq K \leq T$. Equation (A.6) can be estimated by ordinary least squares when we note that:

$$\ln(I(\omega_j)) = \beta_0 - \beta_1 \ln\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + \eta_t, \tag{A.7}$$

where $\beta_1 = (d - 1)$ and the $\eta_t = \ln(I(\omega_j)/f_x(\omega_j))$ are iid across the harmonic frequencies.

Appendix B

Perron Unit Root Test incorporating structural breaks. Models A, B and C refer to Perron's (1989) classification of ''crash'' model, ''breaking slope'' model and ''crash with breaking slope'' model. Exhibit B1 is a summary of Table 6 in Wilson, Okunev and Webb (1998) around the October 1987 market correction. Exhibit B2 provides values for Perron Unit Root Tests on exchange adjusted price indices.

Exhibit B1
Perron Unit Root Test Before and After Differencing

			t-Value		Lag	
Series	Model	Breakpoint	Before Diff.	After Diff.	Before	After
S&P 500	C	Oct. 1987	−3.78	−12.28	1	1
	B	Oct. 1987	−2.91	−12.36	1	0
	A	Oct. 1987	−3.67	−12.28	1	0
Small Cap	C	Oct. 1987	−1.91	−14.35	1	1
	B	Oct. 1987	−1.94	−14.09	1	0
	A	Oct. 1987	−1.81	−14.23	1	0
All REITs	C	Oct. 1987	−2.41	−14.76	8	1
	B	Oct. 1987	−2.46	−14.83	8	0
	A	Oct. 1987	−2.38	−14.77	8	0
FTSE	C	Oct. 1987	−2.64	−8.15	3	4
	B	Oct. 1987	−2.57	−8.24	3	4
	A	Oct. 1987	−2.67	−8.17	3	4
FTAP	C	Oct. 1987	−2.42	−12.78	2	1
	B	Oct. 1987	−2.44	−12.84	2	1
	A	Oct. 1987	−2.42	−12.79	2	1
ALLORDS	C	Oct. 1987	−1.82	−9.85	1	1
	B	Oct. 1987	−1.88	−9.81	1	0
	A	Oct. 1987	−1.90	−9.73	1	0
PTRSTS	C	Oct. 1987	−2.27	−6.55	2	6
	B	Oct. 1987	−2.11	−7.25	2	4
	A	Oct. 1987	−2.33	−7.16	2	4

Perron 5% min. CV; Model A = −3.76, Model B = −3.96 and Model C = −4.24.
The critical value varies according to the breakpoint relative to the total sample size. We only report here the minimum critical value. Furthermore, these are asymptotic critical values as Perron (1989) argued that the only manageable analytical distribution theory was asymptotic in nature.

Exhibit B2
Perron Unit Root Tests on Exchange Adjusted Price Indices All in U.S. Dollars with a Common Breakpoint of October, 1987

		t-Value		Lag	
Series	Model	Before Diff.	After Diff.	Before	After
FTSE	C	−2.60	−13.87	0	0
	B	−2.31	−13.94	0	0
	A	−2.57	−13.81	0	0
FTAP	C	−1.41	−12.56	0	0
	B	−1.27	−12.69	0	0
	A	−1.25	−12.60	0	0
ALLORDS	C	−2.92	−10.11	1	0
	B	−2.67	−10.17	1	0
	A	−2.85	−10.11	1	0
PTRSTS	C	−2.98	−6.74	0	4
	B	−3.03	−6.85	0	4
	A	−2.98	−6.76	0	4

Perron 5% min. CV; Model A = −3.76, Model B = −3.96 and Model C = −4.24.
Extracted from Wilson, Okunev and Webb (1998).

Endnotes

[1] Which, in turn, would add acomplicating dimension to asset allocation models. In a strict economic sense, we could conceive of the short, medium and long term as when none, some and all of the factors of production are variable. It may be more convenient, however, to conceive of these periods in terms of Kitchin cycles (about forty months), Juglar cycles (about nine to ten years) and Kuznets cyles (about sixteen to twenty-two years).

[2] If d is an integer, then Equation (2) reduces to the conventional formula for the binomial coefficient: $d!/k!(d-k)!$

[3] Although not pursued here, once d has been estimated a suitable ARFIMA model may then be developed for forecasting purposes.

[4] Using maximum likelihood methods Sowell (1992a, b) demonstrates that, if the correct ARMA model specification is known, it is possible to obtain more accurate estimates of d than by using the GPH method. However, when the correct ARMA model is unknown it is debatable as to which estimation procedure is superior. The difficulty here is that the ARMA model is estimated on a stationary series, but if fractional differencing is required to obtain stationarity then clearly it is necessary to know d before the ARMA model is estimated.

[5] A previous (conference) version of this article used a Property Trust Index developed by the Australian Stock Exchange. The difficulty with this index is that it only commenced in 1980. Consequently, it was decided that, for this article, we would use the longer Datastream Securitized Property Index, which commenced in 1973.

[6] Note that all analyses were undertaken with monthly data.

[7] In their study on purchasing power parity, Cheung and Lai (1993) also applied the GPH test to the individual series as a check against the ADF unit root test.

[8] This is an important point. It would be ideal to have very long data sets. For instance, to identify a Kondratieff type cycle (which we are not attempting to do here) we would need data for at least one complete cycle—about fifty-four years. However, lets suppose we only have twenty-five years of data—say 300 monthly observations. Estimation of the *d-coefficient* will still provide information about possible cycles in this data since a *d-coefficient* of between zero and one implies non-periodic long cycles in the series that may be indicative of perhaps Kondratieff or Juglar or possibly other long cycles.

[9] This is the subject matter of a later article.

[10] If $d \geq 1$ conventional integer differencing is undertaken.

[11] It should be emphasized that these results are only limited to the stated market indicators and that similar analyses were not undertaken using Small Cap stock market indicators nor other forms of REITs.

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The authors thank two anonymous referees whose comments brought about a significant improvement in this work, Adam Kucera for his programming expertise and James Webb for data provision. This research is supported by various UTS Research Grants.