A Note on the Valuation of Mortgage Loan Commitments: Incorporating the Commitment Cost in the Mortgage Rate

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Abstract. **Kutner and Seifert [4] recently found that mortgage loan commitments have significant financial value. This note presents a procedure that may be used to incorporate this value in the nominal lending rate. The findings indicate that typical lending rates should be augmented by approximately 8 to 44 basis points in order to adequately capture this additional cost.**

In a recent issue of this *Journal*, Kutner and Seifert [4] estimate values of mortgage loan commitments using the option pricing theory developed by Black and Scholes [2] and Merton [5]. FNMA fixed-rate mortgage loan commitments were found to have significant value, averaging over $1300 and ranging from $500 to over $3000 on a $100,000 thirty-day commitment over a two-year period from 1985 to 1987.

As Kutner and Seifert [4] point out, these estimates must be interpreted carefully because of the severe restrictions embodied in the Black-Scholes methodology. These restrictions include no allowance for early exercise of the option, no transactions costs, and the requirement that interest rates follow a specific stochastic process. Furthermore, the model assumes that the option is marketable which is not the case for a mortgage loan commitment. In spite of these difficulties, the simple tractable methodology provides appealing results.

Some of the estimates discussed above appear large compared to the typical explicit fee charged by lending institutions at the time a commitment is extended. However, the commitment cost can be embedded in the nominal loan rate rather than appearing as an explicit charge. The purpose of this note is to present a procedure that may be used to incorporate the commitment cost in the nominal mortgage rate. Estimates are presented which indicate the size and nature of the adjustment.

Consider a mortgage commitment made by a lending institution at a rate r_o for M years for a loan of amount L. The value of this commitment, U_o, can be estimated using equation

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(8) from Kutner and Seifert [4]. The payment per unit time, c, can be found as follows. Recall that:

$$L = \int_0^M c \exp(-r_o t)\, dt = c\,(1 - \exp(-r_o M))/r_o \tag{1}$$

where exp(·) is the natural number (2.71828 · · ·) raised to the argument. Solving equation (1) for c, we get:

$$c = r_o\, L/(1 - \exp(-r_o M)) \tag{2}$$

Suppose the lender desires to charge for the commitment as well as the loan keeping the rate at r_o, then the new payment per period, c', is:

$$c' = r_o\,(L + U_o)\,/\,(1 - \exp(-r_o M)) \tag{3}$$

Since the borrower is only borrowing an amount L, this new payment stream increases the effective nominal rate on the mortgage. Letting r_1 be the new effective loan rate, then using equations (2) and (3), r_1 must satisfy the following condition:

$$r_1\, L/(1 - \exp(-r_1 M)) = r_o\,(L + U_o)\,/\,(1 - \exp(-r_o M)) \tag{4}$$

However, $U_o = L\,\exp(-rT)(2N(.5\sigma\sqrt{T} - 1))$ where r is the risk-free rate, σ is the volatility of the indebtedness, T is the length of the commitment, and $N(\cdot)$ is the cumulative normal density function.[1] Equation (4) then becomes:

$$(1 - \exp(-r_1 M))\,/\,r_1 = (1 - \exp(-r_o M))\,/\,[r_o(1 + u_o)] \tag{5}$$

where $u_o = U_o/L$ is the value of a commitment for a one-dollar loan. Notice that the rate r_1 is independent of the loan size. To find the new mortgage rate r_1 which incorporates the full charge for the mortgage commitment, equation (5) must be solved iteratively for r_1.

A fruitful numerical procedure for finding r_1 is the Newton-Raphson procedure (of first-order). For the details of this procedure see Isaacson and Keller [3] or Avriel [1]. To find r_1 we proceed as follows. After estimating u_o for a given commitment and letting $f(r_1) = (1 - \exp(-r_1 M))\,/\,r_1$, we want $r_1{}^*$ such that:

$$f(r_1{}^*) = (1 - \exp(-r_o M))\,/\,[r_o(1 + u_o)] \tag{6}$$

within a desired tolerance.

The Newton-Raphson procedure involves choosing r_{ck} as follows:

$$r_{ck} = r_{ck-1} - \varepsilon(r_{ck-1})\,/\,(df\,/\,dr_1) \qquad k = 2,3,\cdots \tag{7}$$

where $df\,/\,dr_1$ is the derivative of $f(r_1)$ and r_{ck} is the kth choice of r_1. $\varepsilon(r_{ck-1}) = f(r_{ck-1}) - [(1 - \exp(-r_o M))\,/\,(r_o(1 + u_o))]$ is the error in the $(k-1)$th choice and $r_{c1} = r_o$. Calculating $df\,/\,dr_1$, equation (7) becomes:

$$r_{ck} = r_{ck-1} - \varepsilon(r_{ck-1})\,/\,[M\exp(-r_1 M)\,/\,r_1 - (1 + \exp(-r_1 M))\,/\,r^2{}_1] \tag{8}$$

Exhibit 1
Estimated Adjustment in Nominal Thirty-Year Mortgage Rate*
(basis points)

	Nominal Mortgage Rate (r_o)					
Volatility (ε^2)	0.06	0.08	0.10	0.12	0.14	0.16
0.005	8.31	9.36	10.62	11.90	13.36	14.86
0.006	10.02	11.30	12.74	14.28	16.03	17.83
0.007	11.69	13.18	14.86	16.65	18.70	20.80
0.008	13.35	15.06	16.97	19.03	21.36	23.77
0.009	15.01	16.74	19.09	21.40	24.03	26.74
0.010	16.68	18.81	21.21	23.77	26.69	29.70
0.011	18.34	20.69	23.32	26.14	29.33	32.67
0.012	20.00	22.56	25.43	28.51	31.99	35.64
0.013	21.65	24.43	27.55	30.87	34.65	38.60
0.014	23.31	26.30	29.66	33.24	37.31	41.56
0.015	24.97	28.17	31.76	35.60	39.96	44.52

*30-day commitment

Within a tolerance of ± 0.00000001, r_1* can be found usually in less than five iterations using this procedure. Exhibit 1 presents representative adjustments for thirty-day mortgage commitments on thirty-year loans at a rate r_o and volatility ε. These adjustments were found in three to seven iterations with the above tolerance. The range of volatilities (0.005 to 0.015) presented in Exhibit 1 is consistent with those found by Kutner and Seifert [4] on FNMA mortgage commitments from 1985 to 1987.[2] Notice that as either the mortgage rate or volatility increases, the size of the adjustment increases. The adjustments range from 8 to more than 44 basis points depending upon interest-rate conditions.

For comparison, Kutner and Seifert [4] found that over the period 1985 to 1987 the average volatility on FNMA mortgage commitments was 0.0075 and the average FNMA mortgage rate was 0.0993. As Exhibit 1 indicates this translates into a rate adjustment of approximately 15 basis points. Thus, the average commitment value of $1354, found by Kutner and Seifert [4], is equivalent to a 15-basis-point rate adjustment on a thirty-year mortgage.

This note provides the details of a simple iterative procedure that may be used to properly set a mortgage loan rate that incorporates the commitment cost. These findings suggest that lenders should augment their mortgage rates accordingly if they desire to incorporate the commitment cost in their mortgage loan pricing.

Notes

[1]See Kutner and Seifert [4].
[2]Private communication.

References

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