An Elasticity Approach to Equity Risk Evaluation

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Abstract. **This study defines and derives a measure of risk for real estate investment decisions using the concept of elasticity. Specifically, the elasticity of the after-tax equity yield with respect to the before-tax net operating cash flow growth rate is derived from a discounted cash flow equity valuation model. Also, an illustration of the use and interpretation of this elasticity measure is provided.**

Introduction

Rational investment decisions in real estate must incorporate all anticipated benefits and risks from ownership of the asset. Such an informed decision requires careful estimation of all expected cash flows from the asset over the investment holding period and a systematic determination of the value of these cash flows. While little or no disagreement exists over how to determine these cash flows and the expected return to equity, disagreement continues to exist over the appropriate procedure to systematically evaluate and incorporate investment risk into the investment performance evaluation. The purpose of this article is to develop a measure for real estate investment risk based on the concept of elasticity which is similar in nature, assumptions, and interpretation to the corporate risk measure degree of leverage and the security risk measure duration.

The article is organized as follows. First, current methods of return and risk evaluation are reviewed. Next, a measure for equity investment risk is developed. Finally, an application of this risk measure is illustrated in the examination of the investment risk for different financing alternatives for a commercial real estate property.

Assessing the Investment Value of Equity

Probably the most prevalent objective in real estate financial analysis is to design better procedures for determining the value of investors' equity. The notion that an investor's objective is expected utility maximization suggests that rules to establish the value of investors' equity must be based on discounted cash flow analysis. Conceptually an investor determines the value of the following stream of expected cash flows:

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$$\text{Equity Investment Value} = \sum_{i=1}^{n} \frac{\text{Net Cash Flow to Equity}_i}{(1+r)^i} + \frac{\text{Net Asset Reversion Cash Flow to Equity}}{(1+r)^n}$$

where

Equity Investment Value = purchase price minus loan amount
n = number of years in investment holding period
r = equity yield-to-investment holding period.

By equating the present value of the equity investment to the value of all expected cash flows to equity, the expected equity yield can be determined. The equity yield or internal rate of return is a measure of expected equity investment return since it is based on projected future cash flows determined from most likely estimates for asset income and reversion values.

In a world of uncertainty, investment risk is of concern in addition to the expected return from the investment. Investment risk, therefore, must be assessed and included in the valuation of equity.[1] In recent years asset pricing theory has made significant progress in dealing with investment risk. In particular, two distinct avenues have been pursued for defining, measuring, and/or incorporating risk into investment performance analysis: probabilistic modeling and deterministic modeling. Probabilistic modeling approaches, such as risk-adjusted discount rates, decision tree analysis, or simulation, are based on presupposed probability distributions of the unknown variables necessary to evaluate future cash flows from the asset. Such probability distributions are typically based on past estimates of probability distributions and on assumptions of distribution stationarity over time. Probabilistic risk modeling is successful in cases where assets trade frequently and return distributions and correlations of returns between assets can be estimated and stationarity of these distributions is testable. Its usefulness in real estate valuation is limited due to the fact that individual assets trade infrequently and as a result historical return distributions cannot be estimated and hence their stability cannot be tested.

Deterministic modeling approaches to risk analysis are based on presupposed changes in a variable that affects the asset's future cash flows without assessing probabilities of such changes. One widely recognized form of deterministic risk modeling is sensitivity analysis.[2] The objective of this approach is to measure the magnitude of the impact which presupposed variable changes have on return to equity. For a discussion on the limitations of sensitivity analysis, see Wofford [8].

A second deterministic risk modeling approach that has gained acceptance in corporate risk analysis and bond risk analysis is based on a measure of elasticity between risk variables and profitability measures such as equity return. Examples of these are the degree of corporate leverage and duration. In contrast to sensitivity analysis, these approaches do not depend on assumptions of hypothetical changes in the risk variables but measure risk in the form of a single value elasticity coefficient. In the following section, a similar approach is taken to develop a risk measure for real estate equity performance evaluation.

Assessing the Investment Risk of Equity

In order to include all variables which affect the asset's future cash flow and therefore the risk and return of the investor's equity, the general equity valuation model presented earlier is extended as follows:

$$V_0 - L_0 = \left\{ \frac{V_n - [(V_n - V_0 + nDEP)TRATE] - (L_0 - \sum_{i=1}^{n} PRIN_i)}{(1+r)^n} \right\}$$

$$+ \sum_{i=1}^{n} \left\{ \frac{[([(1+GRATE)^{i-1}(EGI_0 - VOER\ EGI_0 - FOE_0)] - DEP - COUP_i)(1-TRATE) + DEP - PRIN_i]}{(1+r)^i} \right\} \quad (1)$$

where

$$V_n = \left\{ \frac{(1+GRATE)^n (EGI_0 - VOER\ EGI_0 - FOE_0)(1-TRATE)}{(r - GRATE)} \right\}$$

$$+ \sum_{m=n+1}^{\infty} \left\{ \frac{DEP\ (TRATE) - COUP_m\ (1-TRATE) - PRIN_m}{(1+r)^{m-n}} \right\} + (L_0 - \sum_{i=1}^{n} PRIN_i)\ ,\ r > GRATE; \quad (1a)$$

and

V_0 = value of the asset
L_0 = value of the loan
EGI = effective gross income
$VOER$ = variable operating expense ratio
FOE = fixed operating expenses excluding depreciation
$GRATE$ = net before-tax operating cash flow growth rate
DEP = depreciation amount (assuming straight line)
$COUP$ = interest portion on mortgage payment
$PRIN$ = principle portion on mortgage payment
$TRATE$ = ordinary income tax rate
V_n = sales price at end of holding period[3]
r = after-tax expected equity yield.

The net operating cash flow growth rate is a composite of the gross income growth rate, the vacancy growth rate, and the growth rate of variable and fixed operating expenses excluding depreciation. Asset investment risk is defined as the responsiveness of the equity yield to changes in the net operating cash flow growth rate. A single value measure of this elasticity is developed as follows using the concept of point elasticity.[4] First, differentiating equation (1) with respect to the equity yield, (1+*r*), and with respect to the operating cash flow growth rate, (1+*GRATE*), gives:

$$\frac{\partial(V_0-L_0)}{\partial(1+r)} = -n\left\{\frac{V_n - [(V_n-V_0 + nDEP)TRATE]-(L_0-\sum_{i=1}^{n} PRIN_i)}{(1+r)^{n+1}}\right\} + \left\{\frac{(1 - TRATE)\left[\frac{\partial V_n}{\partial(1+r)}\right]}{(1+r)^n}\right\}$$

$$+ -\sum_{i=1}^{n} i\left\{\frac{[([(1+GRATE)^{i-1}(EGI_0-VOER\ EGI_0-FOE_0)]-DEP-COUP_i)(1-TRATE)+DEP-PRIN_i]}{(1+r)^{i+1}}\right\} \quad (2)$$

where

$$\frac{\partial V_n}{\partial(1+r)} = \left\{\frac{-[(1+GRATE)^n(EGI_0-VOER\ EGI_0-FOE_0)(1-TRATE)]}{(r-GRATE)^2}\right\}$$

$$+ \sum_{m=n+1}^{\infty}\left\{\frac{[DEP(TRATE)-COUP_m(1-TRATE)-PRIN_m](m-n)}{(1+r)^{m-n+1}}\right\} \quad (2a)$$

and

$$\frac{\partial(V_0-L_0)}{\partial(1+GRATE)} = \sum_{i=1}^{n} (i-1)\left\{\frac{(1+GRATE)^{i-2}\,(EGI_0-VOER\ EGI_0-FOE_0)(1-TRATE)}{(1+r)^i}\right\} + \left\{\frac{(1-TRATE)\left[\dfrac{\partial V_n}{\partial(1+GRATE)}\right]}{(1+r)^n}\right\} \quad (3)$$

where

$$\frac{\partial V_n}{\partial(1+GRATE)} = \left\{\frac{(1+GRATE)^n\,(EGI_0-VOER\ EGI_0-FOE_0)(1-TRATE)}{(r-GRATE)^2}\right\}$$

$$+\, n\left\{\frac{(1+GRATE)^{n-1}\,(EGI_0-VOER\ EGI\text{-}FOE_0)(1-TRATE)}{(r-GRATE)}\right\} \quad (3a)$$

Then using equations (2) and (3), the sensitivity of the equity yield with respect to changes in the operating cash flow growth rate is expressed as the following elasticity coefficient:

$$^{n}(1+GRATE) = -\,\frac{\left\{\dfrac{\partial(V_0-L_0)}{\partial(1+GRATE)}\right\}}{\left\{\dfrac{\partial(V_0-L_0)}{\partial(1+r)}\right\}}\left\{\frac{(1+GRATE)}{(1+r)}\right\} \quad (4)$$

where $^{n}(1+GRATE)$ is the coefficient of elasticity of the equity yield with respect to the operating cash flow growth rate.

This coefficient of the operating cash flow growth rate is a single value measure of real estate investment risk. Its interpretation is similar to that of the degree of total leverage which measures the responsiveness of stockholder yield to changes in corporate revenues and to that of duration which measures the responsiveness of bond returns to changes in the interest rate. The elasticity coefficient of the operating cash flow growth rate measures the responsiveness of investor equity yield to changes in asset cash flow growth rate. Also, similar to the degree of total leverage and duration, this risk measure does not depend on assumptions about future corporate revenue or future interest rates; the coefficient of the operating cash flow growth rate makes no assumption about the magnitude of future changes of the asset's cash flow growth rate.

Evaluating Equity Risk and Performance

This section illustrates an application of the equity risk measure specified in equation (4). The Appendix depicts three financing alternatives for a hypothetical real estate asset. A priori Alternative 3 seems to be placed at a disadvantage over the other alternatives based on risk as measured by the mortgage constant. Using equation (1), the equity yield was calculated. Next using equation (4), the elasticity coefficient of the operating cash flow growth rate for each financing alternative was calculated. These are presented in Exhibit 1. The elasticity coefficients suggest that with Financing Alternative 1, the investor's equity yield is more sensitive to changes in the net operating cash flow growth rate. Whereas, with Financing Alternative 2 the investor's equity yield is less sensitive. To the extent, therefore, that measures of elasticity have become acceptable definitions of risk and that such definitions can be applied to real estate, the above analysis suggests that Financing Alternative 1 has the greatest degree of equity risk whereas Financing Alternative 2 has the least degree of equity risk. It should also be noted that in the above simple example all variables were held constant with the exception of term and coupon. A priori comparison of investment risk of several assets with differing operating characteristics and financing alternatives becomes more difficult and in such cases can be made using the elasticity coefficient calculated from equation (4).

Conclusive equity performance analysis requires the reconciliation of the expected equity yield and the equity risk. While this analysis generally is based on the investor's utility function, a number of one-parameter equity performance evaluation models have been developed and employed in the past. Representative examples of these are the Sharpe, Jensen, and Treynor measures of performance in terms of investment risk (see Copeland and Weston [1], p. 339). Using the single value risk measure of equation (4) and the expected equity yield of equation (1), and following the general notion of other one-parameter equity performance evaluation models, the expected performance of the equity investment for each financing alternative

Exhibit 1
Equity Yields and Coefficients of Elasticity

	Financing Alternative 1	Financing Alternative 2	Financing Alternative 3
r	18.85%	18.65%	19.00%
n	1.3984	1.3894	1.3949
r/n	13.48%	13.42%	13.62%

Source: Equity yields and elasticity coefficients were calculated using equation (1) and (4) and the information from the Appendix.

was calculated and presented in the lower part of Exhibit 1. The standardized equity yields suggest that Alternative 3 offers the best equity performance, whereas Alternative 2 offers the worst equity performance.

Summary and Conclusions

Real estate investment analysis must consider investment risk in addition to investment return in order to provide useful information to investors. In this article a measure of real estate investment risk was developed based on the concept of point elasticity. Unlike other existing approaches to investment risk analysis such as probabilistic and deterministic risk modeling, this risk measure does not depend on presupposed future changes in variables or probability distributions but is simply based on the assets operating and financial leverage. It determines the sensitivity of the equity return to changes in the return-determining variables. Therefore, similar to total corporate leverage or duration, investment risk is defined as the elasticity of the equity return with respect to changes in the independent variables. This risk measure is a single numerical value based on the forecast of operating cash flows from the real estate asset.

In comparison to current practices of risk analysis in real estate, this single value equity risk measure offers several significant advantages to the investor. First, it does not require assumptions about future events and their probabilities other than the projection of the expected operating cash flows. Second, it allows for the comparison of investment risk of assets with different operating and financial characteristics. And finally, it provides a definition of investment risk and investment return and thus offers the investor the possibility to analyze both dimensions of the investment decision simultaneously.

The properties of such a single value equity risk measure suggest interesting applications to real estate equity risk analysis, particularly in cases where the equity performance of investments must be compared which differ in operating and financial characteristics.

APPENDIX
Terms of Purchase, Operating Projections, and Financing Alternatives of a Hypothetical Real Estate Asset

Purchase Price	$300,000
Building Value	170,000
Land Value	130,000
Loan-to-Value Ratio	80%
Annual Gross Income	50,000
Variable Expense Ratio	35%
Annual Fixed Cost	4,000
Annual Vacancy Rate	3%
Annual Net Operating Cash Flow Growth Rate	6%
Investment Holding Period	3 Years
Marginal Tax Rate	28%

	Financing Alternative 1	Financing Alternative 2	Financing Alternative 3
Coupon	9.5%	9%	10%
Term	30 Years	25 Years	40 Years
Mortgage Constant	.101681	.101806	.102259

Source: Hypothetical Investment Profiles developed by the authors for illustration of the risk measure.

Notes

[1]See for example Jaffe [3] or Jaffe and Sirmans [4].

[2]For a comprehensive presentation of the use of subjective sensitivity analysis see G. E. Greer and M. D. Farrell [2]. For an example see Walters [7].

[3]Assuming a normal operating cash flow growth rate, the asset reversion value can be modeled according to the normal growth Gordon valuation model in equation (1a). For a presentation of the Gordon growth model and the assumptions and derivation of the formula see Copeland and Weston [1], pages 21, and 705-06. The asset appreciation rate and, therefore, reversion value are assumed to be a function of the asset's operating cash flow.

[4]The use of elasticities as measures of risk has been employed in finance and economics. Three of the best known are duration [see Macaulay [5]], and degree of operating and degree of financial leverage [see Schall and Haley [6]].

References

[1] E. T. Copeland and J. F. Weston. *Financial Theory and Corporate Policy*. Reading, Mass.: Addison-Wesley, 1983.

[2] G.E. Greer and M. D. Farrell. *Investment Analysis for Real Estate Decisions*. New York: Dryden Press, 1984.

[3] A. J. Jaffe. *Property Management in Real Estate Investment Decision-Making*. Lexington, Mass.: Lexington Books, 1979.

[4] A. J. Jaffe and C. F. Sirmans. *Real Estate Investment Decision Making*. Englewood Cliffs, N.J.: Prentice-Hall, 1982.

[5] F. R. Macaulay. Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856. New York: National Bureau of Economic Research, 1938, 44-53.
[6] L. D. Schall and C. W. Haley. *Introduction to Financial Management*. St. Louis: McGraw-Hill, 1983, 451-59.
[7] D. W. Walters. Just How Important is Property Management. *Journal of Property Management* 38 (July/August 1973), 164-68.
[8] L. E. Wofford. A Simulation Approach to the Appraisal of Income Producing Real Estate. *Journal of the American Real Estate and Urban Economics Association* 6 (Winter 1978), 371-94.

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