DISCUSSION PAPER SERIES

Discussion paper No.46

What role should public enterprises play in free-entry markets?

Hiroaki Ino
Kwansei Gakuin University

Toshihiro Matsumura
University of Tokyo

June 2009

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

What role should public enterprises play in free-entry markets?

Hiroaki Ino*
School of Economics, Kwansei Gakuin University
and
Toshihiro Matsumura†
Institute of Social Science, University of Tokyo

June 3, 2009

Abstract

We investigate a desirable role of public enterprise in mixed oligopoly in free-entry markets. We compare the following three cases: (i) a public firm produces before private firms (public leadership), (ii) all firms produce simultaneously (Cournot), (iii) a public firm produces after private firms (private leadership). We find that private leadership is best and public leadership is worst, in contrast to the cases without entries and exits of private firms. We also investigate the welfare implication of privatization. We find that some important results shown by existing works do not hold under private leadership.

JEL classification numbers: H42, L13

Key words: free-entry market, Stackelberg, Cournot, mixed oligopoly, commitment

*The corresponding author. Address: School of Economics, Kwansei Gakuin University, 1-155 Uegahara Ichiban-cho, Nishinomiya 662-8501, Japan. E-mail: hiroakiino@04.alumni.u-tokyo.ac.jp.

†E-mail: matsumur@iss.u-tokyo.ac.jp

1 Introduction

We have observed a worldwide wave of privatization of public enterprises. Nevertheless, many state-owned public firms still continue to exist and many of them compete against private firms in developed, developing, and former communist transitional countries. Public and private firms compete in a range of industries including the airline, rail, telecommunications, natural gas, electricity, steel, and overnight-delivery industries as well as in services including banking, home loans, health care, life insurance, hospitals, broadcasting, and education. In addition, we observe many cases of public involvement in private firms facing financial problems, and the firms that obtain public funds compete against pure private firms.

Recently studies on mixed oligopoly involving both private and public enterprises, have become increasingly popular.[1] Many existing works assume that the public firm maximizes social welfare (the sum of consumer surplus and firms' profits) whereas the private firm maximizes its own profits.[2] Some works on mixed oligopoly analyzed Cournot-type simultaneous-move games. Others analyzed Stackelberg-type sequential-move games. Whether the public firm assumes a leadership role is important because an alternative order of moves often gives rise to different results. De Fraja and Delbono (1989) show that in quantity-setting simultaneous-move games, welfare-maximizing behavior by a public firm is not always more efficient than profit-maximizing behavior. Thus, the

[1] This interest in mixed oligopolies is due to their importance to the economies of Europe, Canada, and Japan, as well as China, Russia, Brazil and India. Although they are less significant in the United States, there are some examples of mixed oligopolies such as the packaging and overnight-delivery industries. In addition, in the US the government recently have provided public funds for firms in financial and automobile industries, as well as in many European and Asian countries. Recently, the literature on mixed oligopoly has become richer and more diverse. For example, Mujumdar and Pal (1998) consider tax effects. Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Fjell and Heywood (2002), Dadpay and Heywood (2006), Matsushima and Matsumura (2006), Chao and Yu. (2006) and Fujiwara (2006) investigate international competition. Gil-Moltó and Poyago-Theotoky (2008) examine adoption of FMS. Ishida and Matsushima (2008) investigate wage bargaining. Bárcena-Ruiz and Garzón (2003) discuss a merger problem. Bárcena-Ruiz and Garzón (2006) and Ohori (2006) analyze environmental policies. Cremer et al. (1991), Matsushima and Matsumura (2003), Lu and Poddar (2007), and Heywood and Ye (2008a,b) analyze endogenous product differentiation and spatial competition. Lee (2006) discusses mixed markets under vertical relationship. Lee and Hwang (2003) and Heywood and Ye (2009) investigate agency problem.

[2] The assumption of welfare-maximizing public firms is popular in the literature. See, among others, Anderson et al. (1997), Cremer et al. (1991), De Fraja and Delbono (1989), Merrill and Schneider (1966), and Pal (1998).

privatization of a public firm may improve welfare even without improving the managerial efficiency of the public firm. Furthermore, Matsumura (1998) shows that under moderate conditions, welfare-maximizing behavior by the public firm is not optimal if we allow partial privatization, and also explains that partial privatization usually improves welfare. By contrast, in sequential-move games where a public firm is the leader, welfare-maximizing behavior is always better than profit-maximizing behavior by the public firm. Thus, the effect of privatization of a public firm depends on whether each firm moves simultaneously or sequentially.[3]

In all of the papers mentioned above, the number of private firms is given exogenously.[4] This assumption was very natural because many mixed markets were highly regulated, with both explicit and implicit restrictions on entry. These entry restrictions have, however, been significantly weakened in recent years. We now observe new entry of private firms in many mixed markets such as the banking, insurance, and transportation markets in Japan. Thus, we should regard the models with fixed number of firms as short-run models.

In this paper, we conduct a long-run analysis by examining free-entry markets. We consider a game where private firms choose whether to enter the market, and then face quantity-setting competition. We investigate the following three cases: (i) a public firm produces before private firms (public leadership), (ii) all firms produce simultaneously (Cournot), (iii) a public firm produces after private firms (private leadership). We find that private leadership is best and public leadership is worst. This is in sharp contrast to the short-run case. In the short-run (when the number of firms is given exogenously), public leadership is always better than the Cournot case and private leadership is either better or worse than public leadership.[5] Our result contrasts with

[3] For the discussion on sequential move games and endogenous timing games in mixed oligopoly, see Beato and Mas-Colell (1984), Pal (1998), Matsumura (2003a,b), Lu (2006), and Bárcena-Ruiz (2007).

[4] Using monopolistic competition models, Anderson et al. (1997) discuss free-entry markets. See also Matsumura and Kanda (2005) and Brandão and Castro (2007) for homogeneous goods oligopoly and Fujiwara (2007) for another formulation of product differentiation in free-entry mixed markets.

[5] Pal (1998) shows that private leadership dominates public leadership under linear demand and constant marginal costs. As Matsumura (2003a) shows, this result does not depend on the linear demand. However, it depends on the assumption of constant marginal cost. It is possible that public leadership is better under increasing marginal costs. See Tomaru and Kiyono (2006).

the existing one with fixed number of firms in the following two respects.[6] (1) public leadership yields the smaller welfare than Cournot, which never appears in the case with a fixed number of firms and (2) private leadership is always best.

Our result indicates that public firm should be the follower. In other words, the public firm plays a complementary role of private firms as a potential competitor. If the public firm cannot commit to being the follower, it should play the same role as the private firms under conditions of equal footing and should commit to the Cournot role at an earlier stage. Commitment is important because the firm has an incentive to be the leader after the entry of private firms.

We also investigate the welfare implication of privatization. We find that private leadership yields a starkly different welfare implication from existing works on simultaneous move games. If firms face simultaneous move competition, the public firm running a deficit should be shut down in the long run.[7] This holds true when the public firm is the leader but not when it is the follower. In the latter case, the public firm can be desirable for the society even if it operates with a deficit in the long run.

The paper is organized as follows. Section 2 formulates the model. Section 3 investigates the equilibrium outcomes when the number of private firms is given. Section 4 investigates how the free entry of private firms affects the equilibrium welfare and presents the main results. Section 5 examines the effect of privatization. Section 6 concludes the paper.

2 The Basic Setting

Firms produce perfectly substitutable commodities for which the market inverse demand function is given by $p(q) : \mathbb{R}_+ \to \mathbb{R}_+$ (price as a function of quantity).

Firm 0 is a public firm and it is assumed to maximize social welfare.[8] Firm i ($i = 1, 2, ..., n$) is

[6] Similar contrasts are intensively discussed in the context of international trade. Lahiri and Ono (1995) show that some well-known theorems under perfect competition hold true in a Cournot-type oligopoly under free entry, but not when the number of firms is given exogenously. For the discussions on competition policy and industrial policy, see Etro (2004,2006,2007,2008), Davidson and Mukherjee (2007), and Marjit and Mukherjee (2008).

[7] See Anderson et al. (1997) and Matsumura and Kanda (2005).

[8] The assumption that the public firms maximize welfare implies that the owner (government) of the public firms

a private firm maximizing its own profits. Social welfare W is the sum of consumer surplus and firms' profits, and is given by

$$W = \int_0^X p(q)dq - pX + \sum_{i=0}^{n} \Pi_i = \int_0^X p(q)dq - \sum_{i=0}^{n}(c_i(x_i) + f_i),$$

where $\Pi_i \equiv px_i - c_i - f_i$ is firm i's profit, x_i is firm i's output, $c_i(x_i) : \mathbb{R}_+ \to \mathbb{R}_+$ is firm i's production cost, f_i (> 0) is firm i's entry cost $(i = 0, 1, ..., n)$, and X is the total output given by

$$X \equiv \sum_{i=0}^{n} x_i.$$

We make the following assumptions on the cost and demand functions.

Assumption 1. $p(X)$ is twice differentiable and $p' < 0$ $\forall X$ such that $p(X) > 0$.

Assumption 2. $c_i'' > 0$ $\forall x_i > 0$ and c_i $(i = 0, 1..., n)$ is strictly increasing $\forall x_i \geq 0$.[9]

We employ three models: the first depicts the case where the public firm is the leader (we call it L-model), the second one is the model wherein all firms face Cournot competition (C-model), and the last is the case where the public firm is the follower (we call it F-model). All are complete information games. In the first stage, private firms independently choose whether

is a welfare-maximizer and there is no agency problem in the public firms. This assumption is adopted intentionally to stress our purpose, which is to show that even under ideal situations for public firms above, public firm's leadership impairs welfare at free entry markets under plausible conditions. In this paper we do not allow the government to nationalize more than one firm. As pointed out by Merrill and Schneider (1966), the most efficient outcome is achieved by the nationalization of all firms in the case where nationalization does not change firms' costs (i.e., there is no X-inefficiency in the public firm). The need for an analysis of mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without competitors public firms may lose the incentive to improve their costs, resulting in a loss of social welfare. Thus we neglect the possibility of nationalizing all firms. Our result holds true for cases of more than one public firm unless the number of private firms is zero.

[9] We do not allow c_i'' to be non-positive. If $c'' \leq 0$, then average cost $(c_i + f_i)/x_i$ is decreasing (because $f_i > 0$) for any output level. If a public firm is as efficient as private firms, it is obvious that a monopoly by the public firm is desirable. In order to eliminate such an obvious case we assume $c_i'' > 0$, which induces the U-shaped average cost curve. Some readers may think that public firms are usually less efficient than private firms. However, not all studies show that public enterprises perform more poorly than private (See Stiglitz (1988) ch. 7). Thus, we think that it is important to investigate the case where a public firm is as efficient as each private firm, although we allow differences between the cost functions of public and private firms. For discussion of endogenous cost differences, see Ishibashi and Matsumura (2006), Matsumura and Matsushima (2004) and Nett (1994).

to enter the market. In the second stage, the public firm (firm 0) competes against private firms entering the market. In the L-model, firm 0 chooses x_0 first. After observing x_0, each private firm i simultaneously chooses x_i. In the C-model, all firms, including both private and public firms, choose their outputs independently. In the F-model, all private firms entering the market choose their outputs independently. After observing the total output of private firms, firm 0 chooses x_0.

3 Short-run analysis given the number of firms

We investigate the equilibrium outcome in the three models. We use the subgame perfect Nash equilibrium as the equilibrium concept and solve the game by backward induction. Thus, we discuss the second-stage game, given the number of private firms n.

3.1 The C-model

First, we discuss the C-model where all firms independently choose their outputs. The first-order condition of each private firm i ($i = 1, 2, ...n$) is given by[10]

$$p'(X)x_i + p(X) - c_i'(x_i) = 0. \tag{1}$$

The first-order condition of firm 0 is given by

$$p(X) - c_0'(x_0) = 0. \tag{2}$$

We make an assumption that guarantees the second-order conditions and the stability condition of the Cournot model.

Assumption 3 (Stability condition). $p''x_i + p' < 0$ for $i \in \{1, 2..., n\}$.[11]

Let $R_i(X_{-i})$ ($i = 0, 1, \ldots, n$) denote the firm i's reaction function in the second-stage game, where $X_{-i} \equiv \sum_{i \in \{0,1,\ldots,i-1,i+1,\ldots,n\}} x_i$.

[10] We focus on the cases where each model has a unique and interior solution throughout the paper.

[11] $p'' \leq 0$ is a sufficient but not a necessary condition for Assumption 3. Note that this assumption allow linear demand functions, which are commonly used in the literature.

From hereon, we assume that all private firms are identical, although we allow that the cost of the public firm (firm 0) is different from that of each private firm.

Assumption 4 (Identical private firms). $c_i(x) = c_1(x)$ and $f_i = f_1$ $\forall x \geq 0$ and $i \in \{1, 2..., n\}$.

Henceforth, we restrict our attention to symmetric equilibria where each private firm entering the market chooses the same output. In other words, we assume that $x_1 = x_2 =, ..., = x_n$ in equilibria. We can show that the unique equilibrium is symmetric in all second-stage (production stage) subgames.

3.2 The L-Model

We discuss the L-model where all private firms independently choose their outputs after observing x_0 . The first-order condition of private firm 1 is given by (1). The first-order condition of firm 0 is given by

$$p(X) - c_0'(x_0) + nR_1'(X_{-1})(p(X) - c_1'(x_1)) = 0. \tag{3}$$

We assume that the second-order condition is satisfied.

R_1 is derived from (1) and thus satisfies

$$R_1'(X_{-1}) = \frac{\partial R_1(X_{-1})}{\partial x_0} = -\frac{p' + p''x_1}{n(p' + p''x_1) + (p' - c_1'')} \in (-1/n, 0).$$

Therefore, the impact of x_0 on the total output of the private firms $nR_1'(X_{-1})$ lies in $(-1, 0)$. This is a well-known result in the standard quantity-setting games. An increase in the output of the leader decreases the total output of the followers and increases the total output of all firms (including the leader and the followers).

3.3 The F-Model

We discuss the F-model where firm 0 chooses x_0 after observing all private firms' outputs. Given n, the first-order condition of the public firm is the same as that in the C-model. The first-order condition of firm 1 is

$$(1 + R_0'(X_{-0}))p'(X)x_1 + p(X) - c_1'(x_1) = 0. \tag{4}$$

We assume that the second-order condition is satisfied.

R_0 is derived from (2) and thus satisfies

$$R_0'(X_{-0}) = \frac{\partial R_0(X_{-0})}{\partial x_1} = -\frac{p'}{p' - c_0''} \in (-1, 0).$$

Therefore, even if a follower is the public firm, an increase in the output of a leader decreases the output of the follower and increases the total output of all firms.

3.4 Comparison among three models

Let $x_i^L(n)$, $x_i^C(n)$, and $x_i^F(n)$ respectively denote firm i's equilibrium output in the L-model, C-model, and F-model, given the number of private firms n.

Lemma 1: *Suppose that Assumptions 1–4 are satisfied. Then, (i) $x_0^L(n) < x_0^C(n)$ and $x_1^L(n) > x_1^C(n)$; (ii) $x_0^F(n) < x_0^C(n)$ and $x_1^F(n) > x_1^C(n)$.*

Proof. See Appendix. **Q.E.D.**

Lemma 1 (ii) states that the leadership of profit-maximizing firms (private firms) evokes aggressive behavior as in the standard quantity-setting games. On the other hand, Lemma 1 (i) states that the welfare-maximizing firm (public firm) gets accommodative if it assumes leadership. The intuition behind this result is as follows. In the C-model, firm 0 chooses x_0 so as to maximize the total social surplus given the outputs of private firms. In the L-model, a decrease in x_0 increases x_1. In other words, production substitution takes place from firm 0 to all private firms. Since $p = c_0' > c_1'$ in the C-model, the above production substitution saves the production cost, resulting in an improvement in welfare.[12] Thus, firm 0 chooses a smaller output in the L-model than in the C-model.

Let $W^j(n)$ ($CS^j(n)$) denote the equilibrium welfare (equilibrium consumer surplus) in the j-model ($j = \{L, C, F\}$), given the number of firms n.

Proposition 1: *Suppose that Assumptions 1–4 are satisfied. Then, (i) $W^L(n) > W^C(n)$ and $CS^L(n) < CS^C(n)$; (ii) $W^F(n) > W^C(n)$ and $CS^F(n) > CS^C(n)$.*

[12] For clear discussion on this welfare-enhancing production substitution effects, see Lahiri and Ono (1988,1997).

Proof. See Appendix. **Q.E.D.**

Proposition 1(i) states that the leadership of the public firm improves welfare in the short run. In the L-model, the public firm can choose the same output level as in the C-model. Thus, by definition, welfare is never smaller in the L-model than in the C-model. The strict inequality is derived from Lemma 1(i).

Proposition 1(ii) states that the leadership of the private firm also improves welfare. In the F-model, all the firms' behaviors become closer to marginal cost pricing than in the C-model since each private firm chooses a higher output level in the L-model than in the C-model and the public firm engages in marginal cost pricing in both models. This results in welfare improvement.

As for consumer surplus, Proposition 1(i) (Proposition 1(ii)) states that leadership of the public firm reduces (whereas that of the private firms enhances) consumer surplus. As discussed above, firm 0 (firm 1) chooses a smaller (greater) output when it is the leader, resulting in a smaller (greater) total output and higher (lower) price. Note that in the L-model, firm 0 must abandon marginal cost pricing to urge the private firms toward marginal cost pricing. This factor negatively affects consumer surplus.

4 Long-run analysis with free entry

We investigate the first stage. We neglect the integer problem of the number of private firms. At free-entry equilibrium, each private firm's profit must be zero.[13] Thus, we have

$$p(X)x_1 - c_1(x_1) - f_1 = 0. \tag{5}$$

Let x_0^j, x_1^j, n^j, and X^j respectively denote the equilibrium output of firm 0, the equilibrium output of each private firm, the equilibrium number of private firms, and the equilibrium total output in the j-model ($j = \{L, C, F\}$).

[13] If each private firm needs capital, c_1 includes capital costs. The zero-profit condition means that excess profit of each firm is zero.

Lemma 2: *Suppose that Assumptions 1–4 are satisfied. Then, (i)* $x_1^C = x_1^L < x_1^F$*, (ii)* $X^C = X^L < X^F$*, (iii)* $x_0^L < x_0^C$*, (iv)* $x_0^F < x_0^C$*, and (v)* $n^L > n^C$*.*

Proof. See Appendix. **Q.E.D.**

Lemma 2(i) contains a key result. First, we explain the intuition behind the result $x_1^C = x_1^L$. In both the L-model and the C-model, the average cost curve of each private firm must be tangent to the "residual demand curve" of each private firm at the long-run equilibrium. As is discussed above, firm 0 is less aggressive in the L-model than in the C-model and chooses a smaller output given n. This shifts firm 1's residual demand curve upward, but in the long run it induces new entries of private firms, yielding a downward shift in firm 1's residual demand curve. Eventually, the upward shift in firm 1's residual demand curve caused by a smaller output of firm 0 is canceled out by new entries, resulting in an unchanged curve. This is why two models yield the same equilibrium output of each private firm at the long-run equilibrium (See Figure 1).

****
Insert Figure 1 here
****

Next, we explain the intuition behind the result $x_1^C < x_1^F$. As we explained above, the average cost curve of each private firm must be tangent to the residual demand curve of each private firm. In the F-model, the residual demand curve of each private firm is different from those in the L- and the C-models. An increase in the private firm's output decreases the public firm's output. Thus, given the output of other private firms, the slope of the residual demand curve becomes less steep than those in the other two models. This yields a higher output of each private firm at the long-run equilibrium (See Figure 2).

****
Insert Figure 2 here
****

We present our main result. Let W^j (CS^j) denote welfare (consumer surplus) at the long-run equilibrium in the j-model ($j = \{L, C, F\}$).

Proposition 2: *Suppose that Assumptions 1–4 are satisfied. Then, (i)* $W^L < W^C < W^F$ *and (ii)* $CS^L = CS^C < CS^F$.

Proof. See Appendix. **Q.E.D.**

As for the comparison between the C- and the L-models in the long-run case, the public firm's leadership does not change consumer surplus. Nevertheless, the public firm's leadership is not beneficial in the long run. As discussed above, the public firm is more aggressive in the product market in the C-model than in the L-model. This leads to a decrease in the number of private firms n (Lemma 2(v)) without affecting the equilibrium price (Lemma 2(ii)). A larger output of the public firm induces production substitution from the new entrants to the public firm. Let ac_1 denote the private firm's average cost. Since $p = ac_1 > c_0'$ in the L-model, the above production substitution improves welfare. This is why the public firm's leadership is harmful.

Proposition 2 states that the public firm should not assume leadership in the output market. This is in sharp contrast to the implication suggested by Proposition 1, which states that the public firm's leadership improves welfare after the entry of private firms. These results indicate that the problem of time inconsistency takes place: before the entries of private firms, the public firm should not assume leadership, but after their entries, it has an incentive to be the leader.

Proposition 2 crucially depends on the assumption that the public firm becomes the leader after the private firms' entries. Alternatively, if we assume that the public firm can commit to its output level before the private firms' entries, this result does not hold. In such a case, the logic of a leader being able to replicate the Cournot outcome works as in Proposition 1, and thus, the public firm's leadership must improve welfare. However, entry decision usually requires longer time than choosing an output level. Thus, the timeline in our paper is at least as plausible as the abovementioned alternative one.

Proposition 2 also states that the public firm should be the follower both from the viewpoint of consumer surplus and total social surplus.[14] In other words, private firms should act first and

[14] A similar result is presented in Pal (1998) in a model with a fixed number of firms (a short-run model). He shows that public followership yields higher welfare under the assumption of constant marginal costs. However, this

the public firm should play a complementary role as the follower if possible. As discussed above, each private firms' output is greater in the F-model than in the C-model. Thus, the behavior of all the firms (including the public firm engaging in marginal cost pricing in both models) becomes closer to marginal cost pricing. This results in welfare and consumer-surplus improvement in the long-run model as well as in the short-run model.

5 Privatization

In this section, we investigate the long-run effect of privatization. We assume that if the public firm is privatized, no asymmetry exists between private and privatized firms in the long run. That is, if the public firm is privatized, all the firms, including the privatized firm, have the same cost function, $c_1 + f_1$, and they move simultaneously. Therefore, in the model where the public firm is privatized, the ordinary symmetric Cournot competition with free entry emerges: each firm's first-order condition is (1) and the number of firms is determined by (5). In this setting, we have the following proposition regarding privatization.

Proposition 3: *Suppose that Assumptions 1–4 are satisfied. (i) In the L- and the C-models, privatization does not affect consumer surplus; (ii) in the L- and the C-models, privatization improves welfare if and only if firm 0's profit is negative; (iii) in the F-model, privatization always reduces consumer surplus; and (iv) in the F-model firm 0's deficit is a necessary (not sufficient) condition that privatization improves welfare.*

Proof. Similar to Lemma 2(i) and (ii), the C-model and the model where the public firm is privatized must have the same total output; thus, Proposition 3(i) holds. (Note that in both the models, (5) and (1) are satisfied.)

Profits of private firms are zero in all models. Consumer surplus in the L- and the C-models is the same as that after privatization. Thus, privatization improves welfare if and only if firm 0's profit is negative before the privatization. This implies Proposition 3(ii).

result does not hold true under increasing marginal costs. The authors can provide a counterexample upon request.

Propositions 2(i) and 3(i) imply that privatization raises the price in the F-model. Thus, Proposition 3(iii) holds.

Consumer surplus in the F-model is larger than that after the privatization. Thus, in the F-model, it is possible that privatization does not improve welfare even if firm 0's profit is negative before privatization, whereas privatization never improves welfare if firm 0's profit is nonnegative. This implies Proposition 3(iv). **Q.E.D.**

A result similar to that in Proposition 3(ii) is obtained in Anderson et al.(1997) in a monopolistic competition model. Proposition 3(iv) indicates that their principle cannot apply to the case where the public firm is a follower. The public firm can be beneficial for the society even if it operates with a deficit in the long run. We now emphasize that in our C-, L-, and F-models, the public firm earns strictly positive profits as long as it has the same cost function as the private firms do[15], which is not true in Anderson et al. (1997).

6 Concluding Remarks

We investigated the desirable role of a public firm in mixed oligopoly in the long run. We obtained a clearcut result. The public firm should be the follower since it yields the highest social welfare and consumer surplus. The public firm should not be the leader since it yields the worst social welfare and consumer surplus.

Note that in our timing structure, private firms' entry decisions are made before the public firm's production decision. Thus, the public leader chooses its output given the number of entering firms. In this timing structure, even if the public firm intends to be a leader, it is not necessary for the public firm to be concerned with the long-run issue of private firms' entries. The public firm can easily be (or sometimes, possibly without intention) the leader by making its production decisions more inflexible than private firms' production decisions (not entry decisions) through, for example, budgetary control of public finance[16] or rigid administrative procedures. Our result

[15] This is because $x_0^j > x_1^j$ for $j = \{L, C, F\}$ since the public firm engages in marginal cost pricing while each private firm reduces its output under the marginal cost-pricing level.

[16] For example, the Japanese government can choose to limit the supply of housing loans of the Public House

indicates that such leadership is harmful in free-entry markets. Thus, once the market has been transited from a regulated to free-entry one, the government should reform the public firm to make it a more flexible one and, additionally, enhance the flexibility so as to accommodate the public firm at least with competition on equal-footing (Cournot) and preferably with a complementary role as the follower (private leadership).

Loan Corporation through budget control. If the government sets a binding limit, the public firm commits to its output.

APPENDIX

Proof of Lemma 1: We prove this by contradiction. Let $X^j(n)$ denote the short-run equilibrium total output in the j-model ($j = \{L, C, F\}$).

(i) Suppose that $x_0^L(n) \geq x_0^C(n)$. Since $n\partial R_1/\partial x_0 > -1$, the total output in the L-model ($X^L(n) = x_0^L(n) + nR_1$) is never smaller than that in the C-model ($X^C(n) = x_0^C(n) + nR_1$). Compare the first-order conditions in the two models. The first term in the LHS of (3) is never larger than that in the LHS of (2) since the total output in the L-model is never smaller than that in the C-model and $p' < 0$. The second term in the LHS of (3), $-c_0'$, is never larger than that in the LHS of (2) since $c_0'' > 0$. The third term in the LHS of (3) is negative. Thus, if (2) is satisfied, the LHS of (3) must be negative, which is a contradiction. Therefore, we obtain $x_0^L(n) < x_0^C(n)$ and this immediately implies that $x_1^L(n) > x_1^C(n)$ by $R_1' < 0$.

(ii) Suppose that $x_0^F(n) \geq x_0^C(n)$. Then, from the first-order condition (2) and $c_0'' > 0$, $p(X^F(n)) = c_0'(x_0^F(n)) \geq c_0'(x_0^C(n)) = p(X^C(n))$. Thus, we must have $X^F(n) \leq X^C(n)$ by $p' < 0$. This implies that $x_1^F(n) \leq x_1^C(n)$ since we have supposed that $x_0^F(n) \geq x_0^C(n)$. Then, a contradiction is induced as follows:

$$\begin{aligned}
0 &= p'(X^C(n))x_1^C(n) + p(X^C(n)) - c_1'(x_1^C(n)) \quad (\because (1)) \\
&\leq p'(X^C(n))x_1^F(n) + p(X^C(n)) - c_1'(x_1^F(n)) \quad (\because x_1^F(n) \leq x_1^C(n),\ p' < 0,\ c_1'' > 0) \\
&\leq p'(X^F(n))x_1^F(n) + p(X^F(n)) - c_1'(x_1^F(n)) \quad (\because X^F(n) \leq X^C(n), \text{ Assumption 3}) \\
&< (1 + R_0')p'(X^F(n))x_1^F(n) + p(X^F(n)) - c_1'(x_1^F(n)) \quad (\because -1 < R_0' < 0,\ p' < 0).
\end{aligned}$$

Thus, if (1) is satisfied, the LHS of (4) must be positive, which is a contradiction. Therefore, we obtain $x_0^F(n) < x_0^C(n)$ and this immediately implies that $x_1^F(n) > x_1^C(n)$ by $R_0' < 0$. **Q.E.D.**

Proof of Proposition 1(i): Firm 0 can choose $x_0 = x_0^C(n)$ in the L-model. Thus, by definition, $W^L(n) \geq W^C(n)$, and the equality is satisfied only when $x_0^L(n) = x_0^C(n)$. From Lemma 1(i), we obtain $W^L(n) > W^C(n)$.

Since $n\partial R_1/\partial x_0 > -1$, $X^L(n) < X^C(n)$ if and only if $x_0^L(n) < x_0^C(n)$. Thus, $CS^L(n) < CS^C(n)$ is derived from Lemma 1(i). **Q.E.D.**

Proof of Proposition 1(ii): Since $x_0^F(n) < x_0^C(n)$ by Lemma 1(ii), $p(X^F(n)) = c_0'(x_0^F(n)) < c_0'(x_0^C(n)) = p(X^C(n))$ from (2) and $c_0'' > 0$. Thus, we must have $X^F(n) > X^C(n)$ by $p' < 0$, implying that $CS^F(n) > CS^C(n)$.

We can show that $W^F(n) > W^C(n)$ through the following manipulation:

$$\begin{aligned}
W^F(n) - W^C(n) &= \int_{X^C(n)}^{X^F(n)} p(q)dq - \left(c_0(x_0^F(n)) - c_0(x_0^C(n))\right) - n\left(c_1(x_1^F(n)) - c_1(x_1^C(n))\right) \\
&> \int_{X^C(n)}^{X^F(n)} p(q)dq - n(c_1(x_1^F(n)) - c_1(x_1^C(n))) \quad (\because x_0^F(n) < x_0^C(n),\, c_0' > 0) \\
&> (X^F(n) - X^C(n))p(X^F(n)) - n\int_{x_1^C(n)}^{x_1^F(n)} c_1'(q)dq \quad (\because X^F(n) > X^C(n),\, p' < 0) \\
&> n\int_{x_1^C(n)}^{x_1^F(n)} \left(p(X^F(n)) - c_1'(q)\right) dq \quad (\because X^F(n) - X^C(n) > n\left(x_1^F(n) - x_1^C(n)\right)) \\
&> 0 \quad (\because x_1^F(n) > x_1^C(n)),
\end{aligned}$$

where the last inequality comes from the fact that $p(X^F(n)) > c_1'(q)$ for all $q \in [x_1^C(n), x_1^F(n)]$ since $p(X^F(n)) > c_1'(x_1^F(n))$ by (4), $x_1^F(n) > x_1^C(n)$, and $c_1'' > 0$.

Proof of Lemma 2(i): First, we prove that $x_1^L = x_1^C$. Note that we prove this by considering the fact that in both the C- and L-models, (5) and (1) are satisfied.

Let x^* denote the output-minimizing average cost of the private firm $(c_1 + f_1)/x_1$. We show that $x_1^L \leq x^*$. Suppose, to the contrary, that $x_1^L > x^*$, further, suppose that one private firm (firm 2) deviates from the equilibrium strategy and chooses $x_2 = x^*$. Since the firm 0's output is constant in the second stage, the deviation reduces the total output, resulting in the rise in price (Note that in the L-model, firm 0 is the leader and chooses its output before observing the private firms' outputs). Firm 2's profit is zero before the deviation by (5). The deviation raises the price and reduces firm 2's average cost, and thus, firm 2 obtains positive profits after the deviation, which is a contradiction. For the same reason, we have $x_1^C \leq x^*$.

We prove that $x_1^L = x_1^C$ by contradiction. Suppose that $x_1^L < x_1^C$. The equilibrium price in each model is equal to the private firm's average cost. Since $x_1^L < x_1^C \leq x^*$, the equilibrium price in the L-model is higher than that in the C-model. Since $p' < 0$, it implies that $X^L < X^C$. We compare the LHS of (1) in the L-model with that in the C-model.

$$\begin{aligned} p'(X^C)x_1^C + p(X^C) - c_1'(x_1^C) &< p'(X^L)x_1^C + p(X^L) - c_1'(x_1^C) \quad (\because X^L < X^C \text{ and Assumption 3}) \\ &< p'(X^L)x_1^L + p(X^L) - c_1'(x_1^C) \quad (\because x_1^L < x_1^C \text{ and } p' < 0) \\ &< p'(X^L)x_1^L + p(X^L) - c_1'(x_1^L) \quad (\because x_1^L < x_1^C \text{ and } c_1'' > 0). \end{aligned}$$

Thus, if (1) is satisfied in the C-model, the LHS of (1) in the L-model must be positive–a contradiction. Similarly, we suppose $x_1^L > x_1^C$ and derive a contradiction.

Next, we prove that $x_1^F > x_1^C$. We can show that $x_1^F \leq x^*$ in almost a similar manner as in the second paragraph of this proof except in using (2). Suppose, to the contrary, that $x_1^F > x^*$ and one private firm (firm 2) chooses $x_2 = x^*$ (deviation from the equilibrium strategy). Since $-1 < R_0' < 0$ from (2), the deviation increases the output of firm 0 but reduces the total output, resulting in the rise in price. Therefore, for a similar reason as in the second paragraph, firm 2 obtains positive profits after the deviation, which is again a contradiction.

We prove that $x_1^F > x_1^C$ by contradiction. Suppose that $x_1^F \leq x_1^C$. The equilibrium price in each model is equal to the private firm's average cost. Since $x_1^F \leq x_1^C \leq x^*$, the equilibrium price in the F-model is never lower than that in the C-model. Since $p' < 0$, it implies that $X^F \leq X^C$. We compare the LHS of (4) with that of (1).

$$\begin{aligned} p'(X^C)x_1^C + p(X^C) - c_1'(x_1^C) &\leq p'(X^F)x_1^C + p(X^F) - c_1'(x_1^C) \quad (\because X^F \leq X^C \text{ and Assumption 3}) \\ &\leq p'(X^F)x_1^F + p(X^F) - c_1'(x_1^C) \quad (\because x_1^F \leq x_1^C \text{ and } p' < 0) \\ &\leq p'(X^F)x_1^F + p(X^F) - c_1'(x_1^F) \quad (\because x_1^F \leq x_1^C \text{ and } c_1'' > 0) \\ &< (1 + R_0')p'(X^F)x_1^F + p(X^F) - c_1'(x_1^F) \quad (\because -1 < R_0' < 0). \end{aligned}$$

Thus, if (1) is satisfied, the LHS of (4) must be positive–a contradiction. **Q.E.D.**

Proof of Lemma 2(ii): The equilibrium price in the j-model ($j = \{L, C, F\}$) is equal to $ac_1(x_1^j)$ (the average cost of firm 1). Since $x_1^C = x_1^L < x_1^F \leq x^*$ from the proof of Lemma 2(i), we have $ac_1(x_1^F) < ac_1(x_1^L) = ac_1(x_1^C)$. Therefore, $X^C = X^L < X^F$ is derived from $p' < 0$. **Q.E.D.**

Proof of Lemma 2(iii). From (3), we have $p > c_0'$ in the L-model. From (2), we have $p = c_0'$

in the C-model. From Lemma 2(ii), we have that the equilibrium price in the L-model is equal to that in the C-model. Since $c_0'' > 0$, x_0^L must be smaller than x_0^C. **Q.E.D.**

Proof of Lemma 2(iv): From (2), we have $p = c_0'$ in both the F- and the C-models. From Lemma 2(ii), we have that the equilibrium price in the F-model is lower than that in the C-model. Since $c_0'' > 0$, x_0^F must be smaller than x_0^C. **Q.E.D.**

Proof of Lemma 2(v). This is derived from Lemma 2(i)–(iii). **Q.E.D.**

Proof of Proposition 2: First, we compare the L-model and the C-model. Profits of all private firms are zero in the two models. In the Proof of Lemma 2(ii), we have already shown that $p(X^L) = p(X^C)$. This implies that $CS^L = CS^C$. Since $p - c_0' > 0$ in the L-model and $p - c_0' = 0$ in the C-model, firm 0 earns higher profit in the C-model than in the L-model given the same price $p(X^L) = p(X^C)$. These imply that $W^C > W^L$.

Next, we compare the F-model and the C-model. Profits of all private firms are zero in the two models. In the Proof of Lemma 2(ii), we have already shown that $p(X^F) < p(X^C)$. This implies that $CS^C < CS^F$. In both the F- and C-models, $p = c_0'$. Thus, firm 0's profit is maximized given p. Since $p(X^F) < p(X^C)$, firm 0 earns higher profit in the C-model than in the L-model. The consumer surplus in the F-model is larger than in the C-model by at least $(p(X^C) - p(X^F))X^C$ with $X^C < X^F$, and firm 0's profit in the C-model is larger than in the F-model by at most $(p(X^C) - p(X^F))x_0^C$ with $x_0^F < x_0^C$. Since $X^C > x_0^C$, the welfare is larger in the F-model than in the C-model. **Q.E.D.**

References

Anderson, S. P., A. de Palma, and J. -F. Thisse. "Privatization and Efficiency in a Differentiated Industry." *European Economic Review*, 41(9), 1997, 1635–54.

Bárcena-Ruiz, J. C. "Endogenous Timing in a Mixed Duopoly: Price Competition." *Journal of Economics*, 91(3), 2007, 263–72.

Bárcena-Ruiz, J. C. and M. B. Garzón. "Mixed Duopoly, Merger and Multiproduct Firms." *Journal of Economics*, 80(1), 2003, 27–42.

Bárcena-Ruiz, J. C. and M. B. Garzón. "Mixed Oligopoly and Environmental Policy." *Spanish Economic Review*, 8(2), 2006, 139–60.

Beato, P. and A. Mas-Colell. "The Marginal Cost Pricing Rule as a Regulation Mechanism in Mixed Markets." In: M. Marchand, P. Pestieu and H. Tulkens (Eds), The Performance of Public Enterprises. North-Holland: Amsterdam; 1984, 81–100.

Brandão, A., and S. Castro. "State-Owned Enterprises as Indirect Instruments of Entry Regulation." *Journal of Economics*, 92(3), 2007, 263–74.

Chao, C.-C. and E. S. H. Yu. "Partial Privatization, Foreign Competition, and Optimum Tariff." *Review of International Economics*, 14(1), 2006, 87–92.

Corneo, G. and O. Jeanne. "Oligopole Mixte dans un Marche Commun." *Annales d'Economie et de Statistique*, 33, 1994, 73–90.

Cremer, H., M. Marchand, and J.-F. Thisse. "Mixed Oligopoly with Differentiated Products." *International Journal of Industrial Organization*, 9(1), 1991,43–53.

Dadpay, A. and J. S. Heywood. "Mixed oligopoly in a Single International Market." *Australian Economic Papers*, 45(4), 2006, 269–80.

Davidson, C. and A. Mukherjee. "Horizontal Mergers with Free Entry." *International Journal of Industrial Organization*, 25(1), 2007, 157–72.

De Fraja, G. and F. Delbono. "Alternative Strategies of a Public Enterprise in Oligopoly." *Oxford Economic Papers* 41(2), 1989, 302–11.

Etro, F. "Innovation by Leaders." *Economic Journal*, 114, 2004, 281–303.

Etro, F. "Aggressive Leaders." *Rand Journal of Economics*, 37(1), 2006, 146–54.

Etro, F. *Competition, Innovation, and Antitrust: A Theory of Market Leaders and Its Policy Implications*, 2007, New York: Springer.

Etro, F. "Stackelberg Competition with Endogenous Entry." *Economic Journal*, 118, 2008, 1670–97.

Fjell, K. and D. Pal. “A Mixed Oligopoly in the Presence of Foreign Private Firms.” *Canadian Journal of Economics*, 29(3), 1996, 737–43.

Fjell, K. and J. S. Heywood. “Public Stackelberg Leadership in a Mixed Oligopoly with Foreign Firms.” *Australian Economic Papers* 41(3), 2002, 267–81.

Fujiwara, K. “Trade Patterns in an International Mixed Oligopoly.” *Economics Bulletin*, 6(9), 2006, 1–7.

Fujiwara, K. “Partial Privatization in a Differentiated Mixed Oligopoly.” *Journal of Economics*, 92(1), 2007, 51–65.

Gil-Moltó, M. J. and J. Poyago-Theotoky. “Flexible versus Dedicated Technology Adoption in the Presence of a Public Firm.” *Southern Economic Journal* 74(4), 2008, 997–1016.

Heywood, J. S. and G. Ye. “Mixed Oligopoly, Sequential Entry, and Spatial Price Discrimination.” forthcoming in *Economic Inquiry*, 2008a, doi:10.1111/j.1465-7295.2008.00134.x.

Heywood, J. S. and G. Ye. “Mixed Oligopoly and Spatial Price Discrimination with Foreign Firms.” forthcoming in *Regional Science and Urban Economics*, 2008b, doi:10.1016/j.regsciurbeco.2009.03.004.

Heywood, J. S. and G. Ye. “Delegation in a Mixed Oligopoly: the Case of Multiple Private Firms.” *Managerial and Decision Economics* 30(2), 2009, 71–82.

Ishibashi, I. and T. Matsumura. “R&D Competition between Public and Private Sectors.” *European Economic Review*, 50(6), 2006, 1347–66.

Ishida, K. and N. Matsushima. “Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach.” *Journal of Public Economics*, 2008, dpi:10.1016.j.jpubeco.2008.11.004.

Lahiri, S. and Y. Ono. “Helping Minor Firms Reduces Welfare.” *Economic Journal*, 98, 1988, 1199–202.

Lahiri, S. and Y. Ono. “The Role of Free Entry in an Oligopolistic Heckscher-Ohlin Model.” *International Economic Review* 36(3), 1995, 609–24.

Lahiri, S. and Y. Ono. “Asymmetric Oligopoly, International Trade, and Welfare: A Synthesis.” *Journal of Economics*, 65(3), 1997, 291–310.

Lee, S.-H. “Welfare-Improving Privatization Policy in the Telecommunications Industry.” *Contemporary Economic Policy*, 24(2), 2006, 237–48.

Lee, S.-H. and H.-S. Hwang. “Partial Ownership for the Public Firm and Competition.” *Japanese Economic Review*, 54(3), 2003, 324–35.

Lu, Y. “Endogenous Timing in a Mixed Oligopoly with Foreign Competitors:The Linear Demand Case.” *Journal of Economics*, 88(1), 2006, 49–68.

Lu, Y. and S. Poddar. “Firm Ownership, Product Differentiation and Welfare.” *Manchester School*, 75(2), 2007, 210–17.

Marjit, S. and A. Mukherjee. “International Outsourcing and R&D: Long-Run Implications for Consumers.” *Review of International Economics*, 16(5), 2008, 1010–22.

Matsumura, T. “Partial Privatization in Mixed Duopoly.” *Journal of Public Economics*, 70(3), 1998, 473–83.

Matsumura, T. “Endogenous Role in Mixed Markets: A Two Production Period Model.” *Southern Economic Journal*, 70(2), 2003a, 403–13.

Matsumura, T. “Stackelberg Mixed Duopoly with a Foreign Competitor.” *Bulletin of Economic Research*, 55(3), 2003b, 275–87.

Matsumura, T. and O. Kanda. “Mixed Oligopoly at Free Entry Markets.” *Journal of Economics*, 84(1), 2005, 27–48.

Matsumura, T. and N. Matsushima. “Endogenous Cost Differentials between Public and Private Enterprises: A Mixed Duopoly Approach.” *Economica*, 71, 2004, 671–88.

Matsushima, N. and T. Matsumura. “Mixed Oligopoly and Spatial Agglomeration.” *Canadian Journal of Economics*, 36(1), 2003, 62–87.

Matsushima, N. and T. Matsumura. “Mixed Oligopoly, Foreign Firms, and Location Choice.” *Regional Science and Urban Economics*, 36, 2006, 753–72.

Merrill, W. and N. Schneider. “Government Firms in Oligopoly Industries: A Short-Run Analysis.” *Quarterly Journal of Economics*, 80(3), 1966, 400–12.

Mujumdar, S. and D. Pal. “Effects of Indirect Taxation in a Mixed Oligopoly.” *Economics Letters*, 58(2), 1998, 199–204.

Nett, L. “Why Private Firms are More Innovative than Public Firms?” *European Journal of Political Economy*, 10(4), 1994, 639–53.

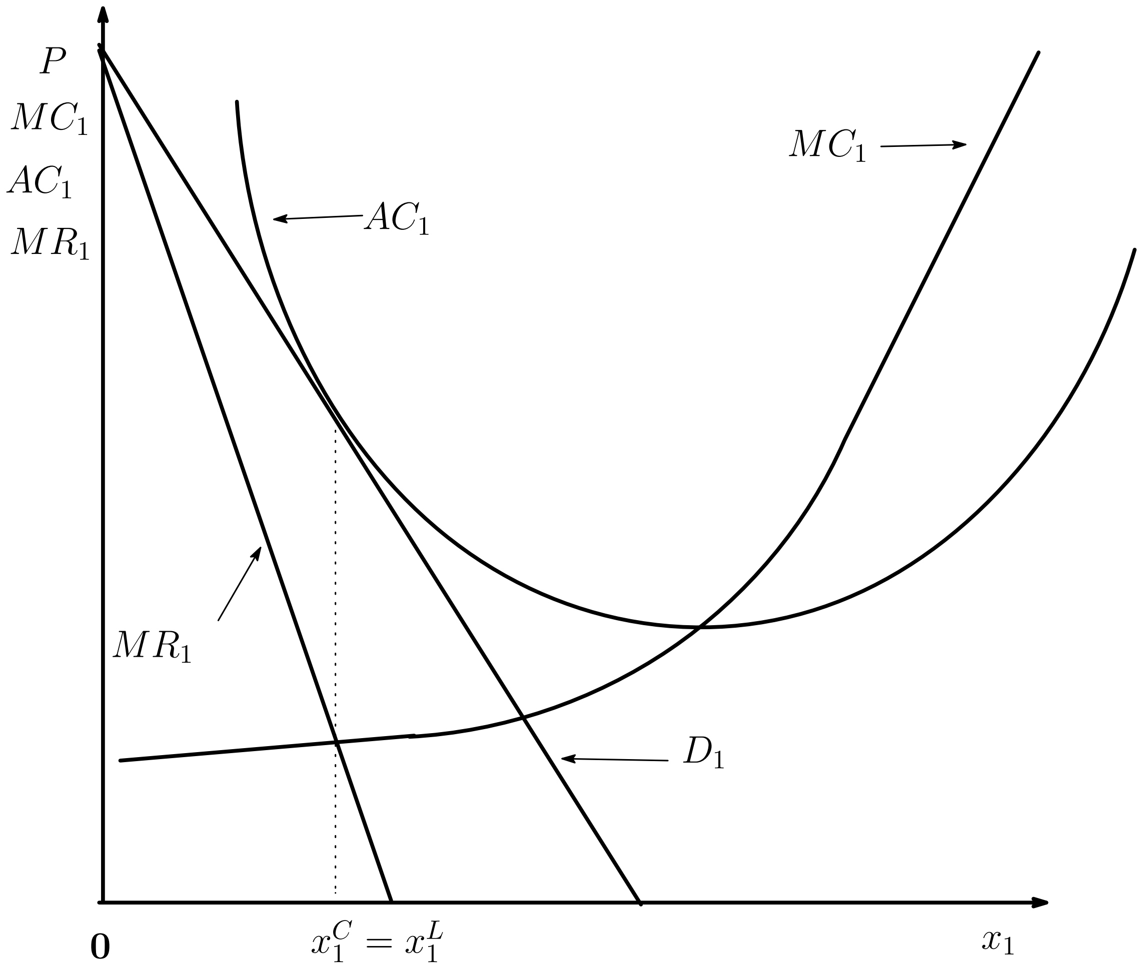
Ohori, S. “Optimal Environmental Tax and Level of Privatization in an International Duopoly.” *Journal of Regulatory Economics*, 29(2), 2006, 225–33.

Pal, D. “Endogenous Timing in a Mixed Oligopoly.” *Economics Letters*, 61(2), 1998, 181–5.

Pal, D. and M. D. White. “Mixed Oligopoly, Privatization, and Strategic Trade Policy.” *Southern Economic Journal*, 65(2), 1998, 264–81.

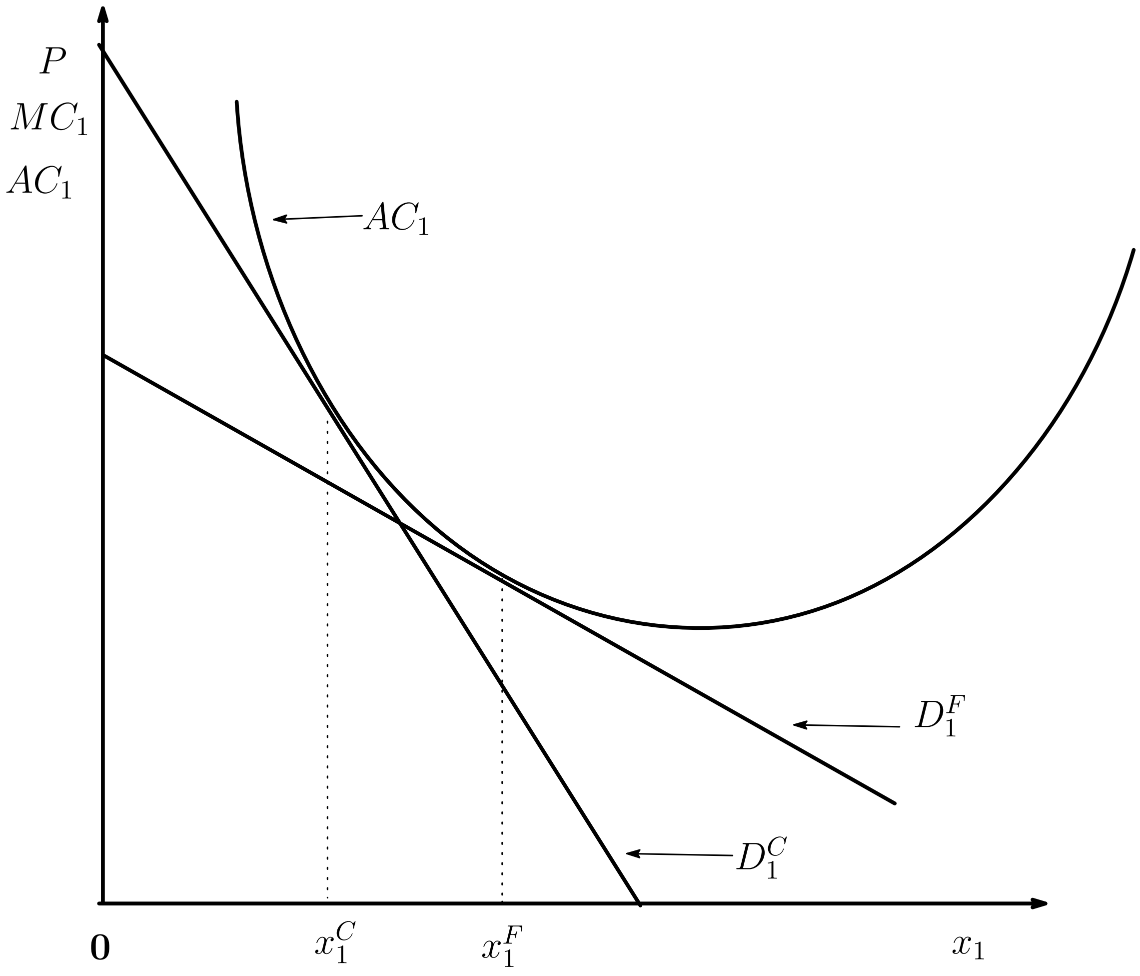
Stiglitz, J. E. *Economics of the Public Sector*, 2nd Edition. Norton: New York; 1988.

Tomaru, Y. and K. Kiyono. “Endogenous Timing in Mixed Duopoly.” 2006, paper presented at JEA.



D_1: **residual demand of a private firm**
AC_1: **average cost of a private firm**
MR_1: **marginal revenue of a private firm**
MC_1: **marginal cost of a private firm**

Figure 1



D_1^C: residual demand of a private firm in the C-Model
D_1^F: residual demand of a private firm in the F-Model
AC_1: average cost of a private firm

Figure 2