IZA

DISCUSSION PAPER SERIES

IZA DP No. 3377

A Million Answers to Twenty Questions: Choosing by Checklist

Michael Mandler
Paola Manzini
Marco Mariotti

February 2008

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

A Million Answers to Twenty Questions: Choosing by Checklist

Michael Mandler
Royal Holloway, University of London

Paola Manzini
Queen Mary, University of London
and IZA

Marco Mariotti
Queen Mary, University of London

Discussion Paper No. 3377
February 2008

IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

IZA Discussion Paper No. 3377
February 2008

ABSTRACT

A Million Answers to Twenty Questions: Choosing by Checklist*

Many decision models in marketing science and psychology assume that a consumer chooses by proceeding sequentially through a checklist of desirable properties. These models are contrasted to the utility maximization model of rationality in economics. We show on the contrary that the two approaches are nearly equivalent. Moreover, the length of the shortest checklist as a proportion of the number of an agent's indifference classes shrinks to 0 (at an exponential rate) as the number of indifference classes increases. Checklists therefore provide a rapid procedural basis for utility maximization.

JEL Classification: D01

Keywords: bounded rationality, procedural rationality, utility maximization, choice behavior

Corresponding author:

Marco Mariotti
Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
United Kingdom
E-mail: m.mariotti@qmul.ac.uk

* We are grateful to Ariel Rubinstein and Chris Tyson for comments and discussions.

1 Introduction

You go to a used car lot. You first state your maximum price, then ask if any cars with a manual transmission are available, then if any sport cars are available, then any Italian sport cars ... and you end up driving away in a red Alfa Romeo.

In this example you make your decision when facing a set of alternatives using only *properties* of the alternatives. You go through your 'checklist' of properties – each property is a subset of alternatives, e.g., all sports cars – until you are able to narrow down the set sufficiently. At each step you eliminate the alternatives that do not have the specified property, or, if no alternative has the property, you do not eliminate any options and move on to the next property. No maximization of utility or of preferences is invoked. All that is required is an ordered list of desirable attributes. An unordered list does not qualify: in a checklist earlier properties always trump later properties. If the car buyer checks car color only with his final property, then color can never take precedence over the properties earlier in the checklist.

The sequential elimination of alternatives by whether or not they possess properties underlies several decision making models in psychology[1] and marketing science.[2] Any decision procedure that follows a flowchart of 'yes or no' questions can be written as a checklist. Checklists can also serve as normative guides in fields such as clinical medicine that do not make economic decisions. For example, Fischer et al. [7] develop a simple rule to decide whether to prescribe a certain antibiotic to treat pneumonia in young children. Because resistance can develop, this drug should be prescribed only in specific cases. The rule is (1) if the patient has had fever for less than two days, do not prescribe, (2) otherwise, and if the patient is less than three years old, do not prescribe, and (3) otherwise, prescribe. We will translate the car and antibiotic examples into the language of our model in section 2, where we incorporate deal-killing properties that a decision must obey.

[1] E.g. from the classic Elimination by Aspect model by Tversky [18], to the more recent Bereby-Meyer, Assor and Katz [1], Brandstätter, Gigerenzer and Hertwig [3] and Katsikopoulos and Martignon [13].

[2] See e.g. Yee et al. [19]. The term 'non-compensatory choice models' is used in these fields to underscore the lack of 'tradeoffs' between earlier and later properties.

Decision-making with a checklist is considered basic precisely because it eschews the use of preference relations over alternatives, the hallmark of economic analysis. Its attraction is its simplicity: in the language of Gigerenzer and Todd [10], it generates 'fast and frugal' heuristics, appropriate when time, knowledge and computational power are scarce. Gigerenzer and Todd indeed emphasize the contrast between such heuristics and 'demonic rationality', by which they mean preference or utility maximization.

In this paper we explore the connection between checklists and the economic model of maximization. As the psychologists' position illustrates, it is not clear at first sight that there is a connection. Moreover, the sequential elimination feature of checklists means that discriminations among alternatives made by a property can never be overturned by later properties: perhaps therefore the only maximizing agents that the model can capture are those who do not make trade-offs among different types of goods. We will see that the reverse is the case: agents who choose with a checklist always maximize a preference relation, and, when agents choose among commodity bundles, checklists are tractably short if and only if agents *do* display the trade-offs of classical utility maximizers. In particular, agents with a tractable checklist cannot have lexicographic preferences (where, e.g., agents prefer more of good 1 and good 2 quantities are decisive only when good 1 quantities are tied).

Our first result reports that an agent who uses a checklist, no matter how long, always chooses as if he or she has a preference relation. Whatever goes on in the minds of checklist users they act like preference maximizers. While a converse to this result also holds – if a choice function maximizes some preference relation then it has a checklist – this conclusion is less satisfying: the checklist might be intractably (uncountably) long and therefore impractical.

Much of the rest of the paper is devoted to showing that in the important economic settings rational maximizers can use the short checklists that define tractable choice procedures.

First, when an agent has n (a finite number) indifference classes the agent can make do with a checklist with only a small number of properties relative to n. Agents with a checklist can in effect perform a binary search, and the ratio of the number of properties to n will converge to 0 at an exponential rate. For example, an agent who makes a 1,000,000 preference discriminations needs a checklist that is only 20 properties long.

Second, the prototypical economic agent who chooses among commodity bundles is endowed with a utility function on $\mathbb{R}_+^n$ that defines uncountably many indifference curves. Despite this large set of discriminations, such an agent can make decisions with a tractable checklist. For any finite set of alternatives, the agent will need to go through only finitely many properties on his or her checklist before coming to a decision: the checklist 'finitely terminates.' On arbitrary domains, including budget sets, the agent's decision for any given choice problem will be well-approximated by a finite number of checklist properties.

There is in fact a full equivalence between choosing by checklist and utility maximization: not only will any utility-maximizer have a checklist that finitely terminates but any agent with a checklist that finitely terminates will have a utility function. This result requires a domain restriction, but without a domain restriction an alternative equivalence holds: an agent maximizes utility if and only if there is a checklist that approximates his or her behavior arbitrarily closely. The procedural model of checklists thus nearly coincides with the economic model of rationality.

That rational agents can use a short checklist is important on two grounds. If we take the procedural view of agents – checklists are the primitive – then we can conclude not only that checklist agents are rational but also that they can finely discriminate among alternatives. Alternatively if we take agents' preferences as primitive then we conclude that checklists are a concise way to translate preferences into choice behavior.

We end up near the Gigerenzer and Todd [10] point of view but with a caveat. Checklists are indeed 'fast and frugal': they are a fast and frugal way to maximize utility.

2 Checklists

2.1 Standard checklists

Fix a nonempty set of alternatives X. Given a set Σ of nonempty subsets of X, a choice function on Σ is a map c that associates with each $S \in \Sigma$ a nonempty set $c(S) \subset S$ (the agent's selection from S). Following tradition, we call c a function but each $c(S)$ is a set.

The decision maker may have a large pool of properties to discriminate among alternatives, but we require that for every decision problem a final selection is reached in a finite number of steps.

Let I be either the finite set $\{1, ..., n\}$ or the entire set of natural numbers $\mathbb{N}$. A *checklist* is a map P that associates with each $i \in I$ a set of alternatives $P(i) \subset X$. Each $P(i)$ is a *property*. We say that 'alternative x has property $P(i)$' whenever $x \in P(i)$.

Given a set $S \subset X$ and a checklist P, define inductively the following sets $M_i(S)$:

$$M_0(S) = S$$

$$M_i(S) = \begin{cases} M_{i-1}(S) \cap P(i) & \text{if } M_{i-1}(S) \cap P(i) \neq \varnothing \\ M_{i-1}(S) & \text{otherwise} \end{cases}$$

This sequence describes an elimination procedure applied to S, where at each step i the agent checks whether the surviving alternatives have the ith property. If some alternatives do have the property, the alternatives that do not are thrown away. Otherwise, all alternatives survive the application of the property. In both cases the agent moves to the next property.

Definition 1 *A choice function $c : \Sigma \to X$* ***has a (standard) checklist*** *if and only if there exists a checklist P such that, for all $S \in \Sigma$, there is a property $j \in I$ with*

$$\begin{aligned} M_i(S) &= M_j(S) \text{ for all } i \geq j \\ c(S) &= M_j(S) \end{aligned} \tag{1}$$

If I is finite the checklist is **finite** and in the remaining case where $I = \mathbb{N}$ the checklist is **countable**.

Thus a choice function that has a checklist satisfies two features. First, the procedure 'finitely terminates': for any choice set S there exists a property in the checklist such that the procedure generates the same set of alternatives for all later properties.[3] Second, this selected set of alternatives coincides with what the choice function selects from S.

[3]For the agent, after reaching $P(j)$ in Definition 1, to execute a decision the agent must conclude that it would be pointless to consider any further properties. The agent can make this inference in two prominent cases: if M_j is a singleton or if M_j is a subset of a single indifference class (taking preferences as primitive in the latter case). The remaining cases are more problematic and 'finite termination' must be understood as an approximate description, as we will explain in section 6.

We call a complete and transitive binary relation on X a *preference relation* and say that a choice function c with domain Σ *maximizes a preference relation* $\succsim$ if $c(S) = \{x \in S : x \succsim y \text{ for all } y \in S\}$ for all $S \in \Sigma$.

Example 1 In the car example of the introduction, we can model the option of not choosing any car by letting some or all of the attributes be 'deal killers,' i.e. attributes that a car must have for a purchase to go through. For any car lot, let an object of choice be either a vehicle v_i in the lot, or the option w of walking away without buying anything. A choice set S (a car lot) then has the form $\{v_1, v_2, .., v_n, w\}$. For the consumer in the introduction, with an ordered set of desirable attributes, the first s attributes will be deal killers if the first s properties all include w. For example, if attribute 1, say having price less than \$30,000, and attribute 2, having a manual transmission, are deal killers then $w \in P(1)$ and $w \in P(2)$. A S that has no manual transmission car cheaper than \$30,000 will then lead the consumer to walk. If every attribute is a deal killer, let w be in each $P(i)$ and add an extra property that repeats the final $P(i)$ but omits w. Then if there is a car in S with every desirable attribute it is chosen, and w is eliminated by the extra property; otherwise, every car in S is eliminated and w survives as the only option. ■

Example 2 In the medical example in the introduction, an object of choice is a child who has had a fever for f days and is y years old, and who receives either the treatment $T = A$ if the antibiotic is prescribed or $T = NA$ if the antibiotic is not prescribed, hence a triple of the form (f, y, T). A choice set S is a $\{(f, y, A), (f, y, NA)\}$: any given child either does or does not receive the antibiotic. The checklist described in the introduction is then $P(1) = \{(f, y, T) : f < 2, T = NA\}$, $P(2) = \{(f, y, T) : y < 3, T = NA\}$, $P(3) = \{(f, y, T) : T = A\}$ which, as desired, ensures that only a child who has had a fever for two or more days and who is three or older receives the drug. There is a shorter checklist that delivers the same decision rule, the single property $Q(1) = P(1) \cup P(2) \cup \{(f, y, T) : f \geq 2, y \geq 3, T = A\}$. Evidently, because some alternatives do not group together naturally in the minds of decision-makers, the shortest possible checklist may not be the easiest to use. ■

Example 3 Suppose an agent has a preference relation $\succsim$ with n indifference classes, labeled $X(n), ..., X(1)$ going from best to worst. Let c be a choice function that maximizes

$\succsim$ on some domain Σ. Then $P(1) = X(n)$, $P(2) = X(n-1)$, ..., $P(n-1) = X(2)$ is a finite checklist for c. ■

Example 3 is a worst case scenario: the checklist has only one fewer property than the number of indifference classes. An agent with a checklist of this sort could spend a long time eliminating alternatives before coming to a decision. Luckily, as we will see in section 4, the Example 3 checklists fail to be minimal when $n > 1$.

2.2 Extended checklists

We now generalize the checklists in section 2.1 to allow uncountably many properties. Readers uninterested in these details can skip to section 3, noting only that any checklist in section 2.1 qualifies as an one of the 'extended checklists' that we now define.

In our earlier elimination procedure, each set of survivors $M_h(S)$ is a subset of its immediate predecessor $M_{h-1}(S)$. Since therefore $M_{i-1}(S) = \bigcap_{k<i} M_k(S)$, we could equivalently define the elimination by

$$M_0(S) = S$$

$$M_i(S) = \begin{cases} \bigcap_{k<i} M_k(S) \cap P(i) & \text{if } \bigcap_{k<i} M_k(S) \cap P(i) \neq \varnothing \\ \bigcap_{k<i} M_k(S) & \text{otherwise} \end{cases}$$

for each $i > 0$. This specification has the advantage that it can be applied to an uncountable set of properties. We can therefore weaken the assumption that the indices I in a checklist are a set of natural numbers and suppose instead that I is well-ordered by some $\leq$, setting 0 as the least element of I.[4] The assumption that I is well-ordered implies that each $i \in I$ has an immediate successor (the least element of $\{k \in I : i < k\}$); thus the procession through the checklist of properties remains orderly. Since some of the $M_i(S)$ need not have immediate predecessors, the above specification uses a variant of standard induction (transfinite induction) to define each $M_i(S)$ using its entire set of predecessors and $P(i)$.

[4] A set A is *well-ordered* by $\leq$ if $\leq$ is a linear order (a complete, transitive, and antisymmetric relation) on A such that every nonempty subset of A has a least element a: $a \leq x$ for all $x \in A$. See Halmos [11] for the set theory concepts we use in this section.

We say that a choice function c has an **extended checklist** if c satisfies Definition 1 except that the $M_i(S)$ are defined as above and I is permitted to be any well-ordered set whose least element is 0. Since any finite set of integers or the entire set of natural numbers is well-ordered by $\leq$, any c that has a checklist has an extended checklist. Although the terminal step j continues to be defined as in Definition 1, j now need not be finite.

If we apply an arbitrary well-ordered set of properties to a choice set S, it could happen that $M_i(S)$ is empty for some i. This occurs when $\bigcap_{k<i} M_k(S) = \varnothing$ and hence $\bigcap_{k<i} M_k(S) \cap P(i) = \varnothing$. Since $c(S) \neq \varnothing$ for any $S \in \Sigma$, this possibility cannot arise when a c has an extended checklist. (It cannot even in principle happen with a finite or countable set of properties since in that case each $i \in I$ is finite.)

If c has an extended checklist that 'finitely terminates' – for each $S \in \Sigma$, the index j identified in Definition 1 is finite – then c has a standard checklist since then we can excise all but the properties with finite indices.

3 Checklists always maximize preference relations

We first show that any choice function that has a checklist maximizes a preference relation. This conclusion holds for extended checklists (and hence for standard checklists) and in this setting an exact converse obtains. The result is valid no matter what the domain of the choice function, for example, it applies equally to budget sets in consumer theory and to finite sets.

Theorem 1 *A choice function has an extended checklist if and only if it maximizes a preference relation.*

All proofs are in the appendix, but the arguments for Theorem 1 are easy. When a choice function c has a standard checklist, we can identify each $x \in X$ with a sequence of 'ins' and 'outs' that indicate in any coordinate i whether x is in or is not in property $P(i)$, and declare $x \succsim y$ if the x and y sequences are identical or if there is a first coordinate where the sequences differ and x scores an 'in' there. This $\succsim$ defines a preference relation on X and c must maximize $\succsim$: if x is chosen from some S that also contains y then y could not score an 'in' before x does (this would eliminate x), and conversely if x is

$\succsim$-maximizing on S then x can never be eliminated by any $y \in S$ since if there is a first property that has one of x and y but not both it must be y that is missing and is eliminated. This reasoning is unchanged if the checklist for c is extended.[5]

In the other direction, we begin with a preference relation $\succsim$ on X that some c maximizes and let the properties be the (weak) upper contour sets: for each $x \in X$, set a property P_x equal to $\{y \in X : y \succsim x\}$ (ignoring the duplicates that arise when $P_x = P_{x'}$ because $x \sim x'$). If $\succsim$ has a finite or countable number of indifference classes, then we can write down these P_x in a finite or countable list. When applying this standard checklist to some S, the agent will eventually hit a property P_x where $x \succsim y$ for all $y \in S$, whereupon no further eliminations can occur. If $\succsim$ has uncountably many indifference classes, we have to write down the P_x in a well-ordered list, and the agent will again hit the upper contour of the best available option in S.

The case where an agent has uncountably many indifference classes is the primary model of consumer theory. Unfortunately the checklists we have constructed in this case are problematic: we have had to resort to a pool of properties with cardinality beyond the natural numbers. Such checklists need not finitely terminate and therefore have no claim to tractability or procedural realism. The problem shows up in the proof of Theorem 1 when we well-order the agent's indifference classes, a nonconstructive step.[6]

The conclusion in Theorem 1 that a preference-maximizing choice function has a checklist is therefore satisfying only when the preference relation has a finite or countable number of indifference classes; then we can generate checklists with, respectively, a finite or countable number of properties (or in the finite case, recall Example 3). To use the checklist model when an agent has uncountably many indifference classes, we must look for cases where the agent can nevertheless make do with a standard checklist, i.e., a very

[5] A less general argument works via the weak axiom of revealed preference (WARP). A choice function with an extended checklist must satisfy WARP since if x is chosen when y is available it must be that if there is a first property $P(i)$ that contains either x or y but not both then $P(i)$ contains x, hence if y is chosen from any S then x must be chosen too. So on any domain where WARP implies that a choice function maximizes some preference relation, for example the domain of finite subsets of X, a choice function with a checklist must also maximize a preference relation.

[6] In standard set theory, the principle that any set can be well-ordered relies on the axiom of choice (see Halmos [11]).

small number of properties relative to the number of indifference classes. Since a standard checklist must finitely terminate, this might seem too ambitious a goal.

The underlying trouble with long checklists also arises when checklists are finite. To be useful, a checklist must be short. An agent with n indifference classes who turns to the Example 3 checklist with $n-1$ properties could end up with a procedure that is plodding and profligate, not fast and frugal.

The rest of the paper addresses these points. Can an agent with finitely many indifference classes use a reasonably short checklist? And can the agents of consumer theory use a standard checklist at all?

Before proceeding, we refine the half of Theorem 1 stating that a choice function with a checklist maximizes a preference relation. For the more important case, standard checklists, we can strengthen this to 'maximizes a utility function.' This conclusion is obvious if a checklist is finite since then there can be only finitely many of the sequences of 'ins' and 'outs' described earlier. Consequently any choice function with a finite checklist maximizes a utility function with finite range. For countable checklists, the number of sequences remains manageable.

A choice function $c : \Sigma \to X$ *maximizes a utility function* if there exists a function $u : X \to \mathbb{R}$ such that $c(S) = \{x \in X : u(x) \geq u(y) \text{ for all } y \in X\}$ for all $S \in \Sigma$.

Theorem 2 *If a choice function has a standard checklist then it maximizes a utility function.*

Since lexicographic preferences cannot be represented by a utility function, we conclude that an agent who chooses with a standard checklist cannot have such preferences.[7] Checklist users, who at first glance seem not to make trade-offs, turn out to fit the textbook ideal of an economic consumer.

[7] On $\mathbb{R}^2_+$, for example, lexicographic preferences are defined by $x \succsim y$ if and only if $x_1 > y_1$ or ($x_1 = y_1$ and $x_2 \geq y_2$).

4 Finite checklists can always be short

Suppose an agent maximizes a preference relation with n indifference classes (a finite number): what is the shortest checklist the agent can use? These indifference classes might be deduced from some c that has a checklist. If the preference relation that c implicitly maximizes has n indifference classes then our question is, 'what is the shortest checklist for c?'.

Consider an example with four indifference classes

$$X = \{1, 2, 3, 4\}$$

where the choice function c, defined on all subsets of X, maximizes the usual order $\geq$ on integers. It is easy to see that $P(1) = \{4, 3\}$, $P(2) = \{4, 2\}$ is a checklist for c.

Next, consider

$$X = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

with c again maximizing $\geq$. Define the checklist $P(1) = \{8, 7, 6, 5\}$, $P(2) = \{8, 7, 4, 3\}$, $P(3) = \{8, 6, 4, 2\}$. Again, it is easy to verify that this is a checklist for c. (It suffices to consider just the two-element subsets of X.)

Notice how the first example is nested in the second: the last two properties $P(2)$ and $P(3)$ of the second example treat $\{5, 6, 7, 8\}$ and $\{1, 2, 3, 4\}$ just as P in the first example treats $\{1, 2, 3, 4\}$, with the additional first property $P(1)$ serving only to separate the two chains. So, we have provided a checklist with 2 properties for a preference relation with 4 levels, and a checklist with 3 properties for a preference relation with 8 levels. This conclusion extends inductively:

Theorem 3 *If c maximizes a preference relation with n indifference classes, then c has a checklist with k properties, where k is the smallest integer such that $2^k \geq n$. If in addition the domain of c includes all the two-element sets then the minimum number of properties in a checklist for c is k.*

Theorem 3 shows how checklists become more and more efficient as the number of indifference classes increases. Not only will the required number of properties as a proportion of the number of indifference classes n fall to zero as n increases, but it will do so at an exponential rate. Since $2^{20} \geq 1{,}000{,}000$, Theorem 3 explains the claim in the introduction that a 1,000,000 preference discriminations require only 20 checklist properties.[8]

The pertinent feature of a choice set is its highest indifference class; in the notation of the above examples, a decision maker needs to identify, given $S \subset \{1, ..., n\}$, the largest integer in S. The solution of this problem via 'yes or no' questions is a classic illustration of a binary search algorithm: first ask 'does S contain an integer between $\lceil \frac{n}{2} \rceil$ (the least integer $\geq \frac{n}{2}$) and n?', and then, if yes, ask 'does S contain an integer between $\lceil \frac{3n}{4} \rceil$ and n?' and, if no, ask 'does S contain an integer between $\lceil \frac{n}{4} \rceil$ and $\lceil \frac{n}{2} \rceil$?', and so on. That a recursive computer program, where the choice of the ith question depends on earlier answers, can execute this algorithm in $\lceil \log_2 n \rceil$ steps is hardly news.[9] What is notable about a checklist is that it executes the algorithm nonrecursively. A property $P(i)$ does not change as a function of the eliminations that occur prior to i, and every property is used for every S. To do without input from earlier steps, each property in effect encodes a set of questions. Consider again $X = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and let m denote $\max S$. Then $P(1)$ 'asks' one question, 'is $m \in \{8, 7, 6, 5\}$?', $P(2)$ 'asks' two conditional questions, 'if $m \in \{8, 7, 6, 5\}$ then is $m \in \{8, 7\}$?' and 'if $m \notin \{8, 7, 6, 5\}$ then is $m \in \{4, 3\}$?, and $P(3)$ 'asks' four conditional questions. For $i > 1$, the eliminations prior to i ensure that only one of the antecedents of the $P(i)$ questions is satisfied. Property $P(i)$ therefore asks the right question, and without recursive instructions or an exhaustive tree of $n - 1$ 'if then' commands (where each answer to a command leads to a distinct subsequent command).

We can compare the efficiency of a checklist relative to an optimal tree of 'yes or no' questions. If we can ask questions of the form 'does S intersect $Y \subset \{1, ..., n\}$?', then, depending on the probabilities that particular integers lie in S, the minimum expected number of questions can be less than $\lceil \log_2 n \rceil$. For example if it highly likely that $m = \max S = 4$, then one can first ask 'does S intersect $\{5, 6, 7, 8\}$?' and if no 'does S intersect $\{4\}$?'. But if each $x \in X$ is equally likely to be m then $\lceil \log_2 n \rceil$ is the minimum expected number of questions: the optimal tree does no better than the optimal

[8] If c always selects a singleton, then Theorem 3 can be rephrased using the number of alternatives in X rather than the number of indifference classes.

[9] See, e.g., Knuth [14], chapter 6, Theorem B.

checklist.[10]

5 Utility maximizers can have short checklists

Finite checklists are appealingly concrete: there is a uniform upper bound on the number of properties the decision maker has to examine before the choice procedure terminates. In an arbitrary standard checklist, it remains true that each choice set needs to be checked against only finitely many properties but there might not be any bound on the number of properties that serves simultaneously for *all* choice sets. This small difference makes standard checklists much more powerful.

As we will now see, an agent who makes uncountably many preference discriminations can sometimes use a standard checklist (which therefore is 'short' relative to the number of discriminations). Classical commodity consumers can thereby fit under the umbrella of the checklist model.

Example 4 Let the choice function c be defined on all finite subsets of the real line, and let c maximize the usual order $\geq$ of the real line. Define the standard checklist P by letting, for each rational number z, a property P_z equal the weak upper contour set of z, $\{x \in X : x \geq z\}$, and then enumerate these properties so that exactly one is identified with each natural number. Given any finite choice set S, the checklist defined by P eliminates in a finite number of steps all alternatives except the highest number in S. ■

This simple example goes some way towards showing the reach of standard checklists. Under certain conditions, they can mimic preference maximization even when a preference relation admits a continuum of indifference classes. Relative to this large set of preference discriminations, a standard checklist makes do with a small number of properties (indeed a finite number for any specific choice set).

[10] If questions of the form 'is $m \in Y$?' are permitted, which is exactly the game 'Twenty questions,' Huffman coding [12] generates the optimal tree. See also Zimmerman [20], and Gilbert [9] for the connection to our problem.

It is 'almost' true that any utility maximizer can choose using a standard checklist. The claim holds precisely if the domain of choice is restricted to some family of finite subsets of X, the universal set of alternatives. Note that no restriction is imposed on the cardinality of X itself:

Theorem 4 *If a choice function defined on a domain of finite sets maximizes a utility function then it has a standard checklist.*

The following example shows that a domain restriction is indeed required in Theorem 4.

Example 5 Let X be the interval $[0,1]$, let the domain of c be the closed sets in X, let the utility function $u : X \longrightarrow \mathbb{R}$ that c maximizes be defined by $u(x) = x$, and suppose P is a standard checklist for c. We will sketch a proof in *Canonical checklists* below that we may assume that the checklist consists only of properties $P(i)$ that are weak or strict upper contour sets, i.e., sets of the form $\{x \in X : x \geq q\}$ or $\{x \in X : x > q\}$ for some $q \in X$. In other words, if $\widehat{P}$ is a standard checklist for c then there is also a standard checklist P for c that consists solely of upper contour sets.

Assume then that there is a P that is a standard checklist for c that consists of upper contours. If we call glb(i) the greatest lower bound of $P(i)$, then there will be at most countably many glb(i) for the properties in P. Pick some $y \in X$ that is not one of these glb(i), and set $S = \{x \in X : x \leq y\}$. Then, for any i, $M_i(S)$ will equal the nonempty interval whose lower boundary equals $\max\{\text{glb}(k) : \text{glb}(k) < y \text{ and } k \leq i\}$ and whose upper boundary equals y. (This interval contains y but may or may not contain its lower boundary.) Since $M_i(S) \neq \{y\} = c(S)$ for all i, P could not in fact be a checklist for c. ■

Canonical checklists. That we may take a checklist in Example 5 to consist solely of upper contours illustrates a wider principle. Fix some X and suppose that c is defined on a domain that includes the two-element subsets of X and that c has a standard checklist $\widehat{P}$. Then c maximizes some preference relation $\succsim$ whose strict part we label $\succ$. We may assume, without loss of generality, that $\succsim$ is in fact a linear order. Call $U \subset X$ an *upper cut* if ($x \in U$ and $y \succsim x$) $\Longrightarrow y \in U$,[11] and call $I \subset X$ *convex* if ($x, y \in I$ and $x \succsim z \succsim y$) $\Longrightarrow z \in I$. Then c also has a checklist consisting solely of upper cuts. To see why, observe that $\widehat{P}(1)$ must be an upper cut since if $x \in \widehat{P}(1)$, $y \succ x$, and $y \notin \widehat{P}(1)$, then $c(\{x, y\}) = \{x\}$, and so c would not maximize $\succsim$. So set $P(1) = \widehat{P}(1)$. The argument then proceeds by induction. To illustrate how the induction works, observe that while $\widehat{P}(2)$ need not be an upper cut, we can conclude that if $\widehat{P}(2)$ is not an upper cut then it must equal the union of an upper cut U and a convex L such that $P(1) \cup L$ is an upper cut and $P(1) \cap L = \varnothing$. If this conclusion were false, then there would be a $w \in \widehat{P}(2)$ and a $z \notin \widehat{P}(2)$ such that $z \succ w$ and either $\{w, z\} \subset P(1)$ or $\{w, z\} \cap P(1) \neq \varnothing$; hence $c(\{w, z\}) = \{w\}$, again violating the assumption that c maximizes $\succsim$. So in the case where $\widehat{P}(2)$ is an upper cut, set $P(2) = \widehat{P}(2)$ and in the case where $\widehat{P}(2) = U \cup L$, set $P(2) = U$ and $P(3) = \widehat{P}(1) \cup L$. An explicit induction argument would show that each $\widehat{P}(i)$ must be the union of an upper cut and sets which can form upper cuts when joined with the $P(j)$, $j < i$, specified in the previous steps. It is easy to confirm that the P constructed in this way is a checklist for c. ■

While Example 5 shows that some domain limitation is needed in Theorem 4, the restriction can be weakened. For instance, the conclusion of the theorem still holds on any domain that includes at most countably many infinite sets. But we do not have an attractive characterization of the maximum permissible domain. So, while the converse result, Theorem 2, is clearcut, the ideal way to fill the gap in 'A choice function ... if and only if it has a standard checklist' remains an open question.

For readers familiar with cardinal numbers, we can summarize the 'shortness' Theorems 3 and 4 concisely: if an agent has a utility function u with n indifference classes, where n is a cardinal number, then a choice function that maximizes u on a domain of finite sets has a checklist of cardinality k if k satisfies $2^k \geq n$.[12]

[11] For the preference relation $\geq$ on $\mathbb{R}$, an upper cut must be a weak or strict upper contour set, but an upper cut of an arbitrary preference relation $\succsim$ need not have a $\succsim$-greatest lower bound.

[12] Two sets A and B have the same cardinality if they can be put into a one-to-one correspondence, and A has larger cardinality if there is a one-to-one function from B to A but not vice versa.

6 Utility maximizers always have short approximate checklists

As we have seen, the choice behavior of utility maximizers does not coincide exactly with that of agents who use a standard checklist (a domain restriction is necessary), nor of agents who use an extended checklist (since then we go beyond utility maximization to preference maximization). Nevertheless, standard checklists can closely *approximate* utility maximization regardless of the domain.

To capture the idea that a checklist could approximate the decision $c(S)$ we consider the limit of the set of survivors selected by a standard checklist: although the procedure never yields exactly the decision $c(S)$ at any finite step, it approximates $c(S)$ more and more accurately as the number of steps increases. In fact, in the limit, we get exact equivalence between the choices of standard checklist users and utility maximizers.

As no notion of distance is present in our set-up, we use a set-theoretic definition of the convergence of the $M_i(S)$. A choice function $c : \Sigma \to X$ has an **approximate checklist** if and only if it has a standard checklist P and (defining the $M_i(S)$ as in section 2.1)

$$c(S) = \bigcap_{i \in I} M_i(S)$$

for all $S \in \Sigma$. Thus, although after any finite number of steps the set of surviving alternatives may still contain other alternatives beside the chosen ones, it is only the chosen alternatives that survive all steps of elimination: for any alternative rejected by the choice function, there exists a property that it does not have.

Theorem 5 *A choice function maximizes a utility function if and only if it has an approximate checklist.*

Approximate checklist help explain how a countable checklist would work practically. A countable checklist can raise a termination problem: even if no further eliminations occur after some property $P(j)$, the agent may not know this fact. The agent will know it for choice functions that always select singletons or subsets of a single indifference class (see footnote 2). But in all other cases, the practical distinction between standard and approximate checklists is not sharp. For both of these checklist models, the agent would have to declare at some point that the set of alternatives had been winnowed down adequately.

7 Remarks on preference representation

Checklists shed light on the question of how to represent preferences that cannot be summarized by a real-valued utility function. Since Birkhoff [2], the theorem that a preference relation $\succsim$ on X can be represented by a real-valued utility function if and only if X has a countable $\succsim$-order-dense subset has become widely known. Yet this theorem stands as an isolated fact in lattice theory; as Birkhoff himself pointed out, it is not useful in abstract versions of the subject. Checklists can give order density a broader role as a representation tool. If X has a $\succsim$-order-dense subset D of cardinality k and the choice function c, defined on a domain of finite sets, maximizes $\succsim$, then c has a checklist of cardinality k: as in the proof of Theorem 4, augment D as necessary to a larger set D^+ of the same cardinality, then define a property $P(d) = \{x \in X : x \succsim d\}$ for each $d \in D^+$ and finally well-order these properties to create a checklist. For example, in a set theory that admits sets of a cardinality m strictly between $\aleph_0$ (the cardinality of the natural numbers) and $2^{\aleph_0}$ (the cardinality of the real numbers), there can be preference relations $\succsim$ on $\mathbb{R}^n$ where there is no countable $\succsim$-order-dense subset of $\mathbb{R}^n$ but where there is a $\succsim$-order-dense subset of cardinality m. To build an example, take a subset $Y \subset X$ of cardinality m, let $\succsim$ well-order Y, set $x \sim z$ for all $x, z \in X \backslash Y$, and set $y \succ x$ for all $y \in Y$ and $x \in X \backslash Y$. In cases like this, $\succsim$ has no real-valued utility function. But $\succsim$ can still be 'represented' concisely since there is a checklist of length m for any c defined on a domain of finite sets: we do not have to go to the extreme of specifying $2^{\aleph_0}$ properties.[13]

[13]This example but not the general point turns on the continuum hypothesis (that there is no set of cardinality between $\aleph_0$ and $2^{\aleph_0}$). The continuum hypothesis in fact can be posed as a conjecture solely about checklists. Suppose $\succsim$ is a preference relation on R^n and that c, defined on the finite subsets of R^n, maximizes $\succsim$. If c fails to have a standard checklist, then does the shortest extended checklist for c have to have the same cardinality as R^n? In standard set theory, the question is undecidable: one cannot prove or disprove the claim that the answer is 'yes.'

Checklists as a representation tool are related to Chipman's [4] classical work on utility theory.[14] Among other topics, Chipman considered how to represent a preference relation $\succsim$ on X when $\succsim$ has no real-valued utility function. His proposal was to use a well-ordered and perhaps uncountable sequence of utility functions with each utility defined on a domain of cardinal numbers. The theory is much simpler, however, if utilities are defined on X. With this change, the Chipman proposal uses a sequence $(u_i)_{i \in I}$ where each u_i maps X to $\mathbb{R}$ and where the set I has a well-ordering $\leq$. Then we say $(u_i)_{i \in I}$ *Chipman represents* $\succsim$ if $x \succsim y \iff$ (for any i with $u_i(y) > u_i(x)$ there exists $j \leq i$ with $u_j(x) > u_j(y)$). Any preference relation $\succsim$ can be Chipman represented: as in the proof of Theorem 4, let $\leq$ be a well-ordering of X, define $P(x) = \{y \in X : y \succsim x\}$ for any x, and then set $u_x(z) = 1$ if $z \in P(x)$ and $u_x(z) = 0$ if $z \notin P(x)$. In Chipman's proof of this result, his specification of $(u_i)_{i \in I}$ was more complicated, but it shares the feature that the range of each u_i consists of only two points. Since a utility with a two-point range defines a partition of X, Chipman's proof implicitly specifies a checklist. In Chipman's general framework, on the other hand, utilities map to $\mathbb{R}$, and so his model does not normally define a set of properties or a sequential decision-making procedure. Chipman did not consider the possibility, mentioned in the previous paragraph, of a $(u_i)_{i \in I}$ with fewer than $2^{\aleph_0}$ functions for a $\succsim$ on $\mathbb{R}^n$ that does not have a classical utility function.[15] The possibility shows, in either Chipman's framework or ours, that all is not lost when countable order-density fails: concise representation is still feasible.

8 Concluding remarks

Although we believe that the checklist model is new, we should mention Rubinstein [16] which (to the best of our knowledge) is the first mention in the economic literature of the potential importance of *unary* relations (what we call *properties*) in decision making. Although distantly related, that work was the initial stimulus for this project.

There are ways to choose by checklist that do not fit the model of this paper. Check-

[14] We thank Chris Tyson for stressing the connection between ours and Chipman's work.

[15] Not surprisingly since Chipman's work precedes that of Cohen [5] [6] showing that sets of cardinality between $\aleph_0$ and $2^{\aleph_0}$ are consistent with the standard axioms of set theory.

lists as we have defined them are fixed across choice sets. Consequently an agent shopping for a camera, who looks for cameras on the top shelf, then for those priced between \$225 and \$250, and then for those with black finish could choose different cameras from stores that stocked the same set of cameras but put them on different shelves. The properties (sets of cameras) in this list differ by store; if we think of a store as a choice set, our model rules this out. Rubinstein and Salant [17] better fits this situation: the alternatives in each choice problem are presented to the decision maker in an exogenously specified order (e.g., the element on the top shelf is seen before the element on the next shelf). A choice problem is then an ordered list of alternatives $(a_1, ..., a_k)$, and a choice function associates each such list with one of its elements.

Manzini and Mariotti [15] characterize the choice behavior that arises when an agent sequentially eliminates alternatives using an ordered sequence of *binary* relations (keeping at each step only the maximal alternatives). In that case behavior may be very far from utility maximization.

With unary relations, the gulf between extended checklists and standard checklists underlines how the latter form fast and efficient choice procedures. That such models turn out to be so close to utility maximization came as a surprise to us.

9 Appendix: Proofs

Proof of Theorem 1: Let the choice function c have the extended checklist P. We identify each $x \in X$ with the vector $p_x \in \{0,1\}^I$ given by $p_x(i) = 1$ if $x \in P(i)$ and $p_x(i) = 0$ if $x \notin P(i)$ (of course each p_x can be associated with many alternatives). We order $\{0,1\}^I$ lexicographically: for $p, q \in \{0,1\}^I$, $p \succsim q \iff (q(i) > p(i) \implies \exists k < i$ with $p(k) > q(k))$. The asymmetric and symmetric parts of $\succsim$ are labeled $\succ$ and $\sim$ respectively. To conclude that $\succsim$ is a linear order, we could appeal to the fact that if $\succsim$ is the lexicographic order of any family of linear orders with well-ordered indices then $\succsim$ is also a linear order. But to argue directly, completeness follows from the fact that (1) if $p = q$ then $(q(i) > p(i) \implies \exists k < i$ with $p(k) > q(k))$ obtains vacuously, while (2) if $p \neq q$ then the well-ordering of I implies that $j = \min\{i : p(i) \neq q(i)\}$ is well-defined and hence

$p \succ q$ if $p(j) > q(j)$ and $q \succ p$ if $q(j) > p(j)$. Case (2) also yields antisymmetry. For transitivity, if $p \sim q \sim r$ then $p = q = r$ and hence $p \sim r$. If on the other hand $p \succsim q \succ r$ or $p \succ q \succsim r$ set $j = \min\{i : p(i) \neq q(i) \text{ or } q(i) \neq r(i)\}$. Then $p(j) \geq q(j) \geq r(j)$ with at least one strict inequality. Hence $p(j) > r(j)$ and $p(i) = r(i)$ for $i < j$, i.e., $p \succ r$.

Let $\succsim$ also now denote the relation on X given by $x \succsim y \iff p_x \succsim p_y$: since $\succsim$ on $\{0,1\}^I$ is a linear order, $\succsim$ on X is a preference relation. To see that for any $S \in \Sigma$, $c(S) = \{x \in S : x \succsim y \text{ for all } y \in S\}$, suppose first that $x \in c(S)$. If $y \succ x$ for some $y \in S$ and we set $j = \min\{i : p_x(i) \neq p_y(i)\}$ then the fact that $x \in M_i(S)$ for all $i < j$ implies that $y \in M_i(S)$ for all $i < j$. But since $y \in P(j)$ and $x \notin P(j)$, $x \notin M_j(S)$, contradicting $x \in c(S)$. Conversely suppose $x \in S$ and $x \succsim y$ for all $y \in S$. Then, since $c(S)$ is nonempty, $x \succsim z$ for some $z \in c(S)$. Since $z \in M_i(S)$ for all i, $x \succsim z$ implies $\{i : p_x(i) \neq p_z(i)\} = \varnothing$ (otherwise z would be eliminated at $\min\{i : p_x(i) \neq p_z(i)\}$). So $x \in M_i(S)$ for all i, i.e., $x \in c(S)$.

Now suppose that c maximizes some preference relation $\succsim$. To construct a checklist, let $I = X \cup \{0\}$ and let $\leq$ be a well-ordering of I with $0 < x$ for any $x \in X$. (This is the nonconstructive step mentioned in the text: the principle that any set can be well-ordered relies on the axiom of choice.) For each $x \in X$ define $P(x) = \{y \in X : y \succsim x\}$. Fix $S \in \Sigma$ and some $x \in c(S)$. Then, for any $z \in X$ with $x \notin P(z)$, the fact that $x \succsim y$ for $y \in S$ and the transitivity of $\succsim$ imply $y \notin P(z)$ for $y \in S$. So, for any $z \in X$, if $x \in \cap_{w<z} M_w(S)$ then $x \in M_z(S)$. Since $x \in M_0(S)$, transfinite induction implies that $x \in M_z(S)$ for all $z \in X$. Moreover, for all $y \notin c(S)$, $y \notin P(x)$ and so $y \notin M_x(S)$. Finally observe that $M_z(S) = M_x(S)$ for all z such that $x \leq z$, so that the terminal step j in Definition 1 is well defined. ■

Proof of Theorem 2: Let c have a standard checklist $P : I \to 2^X$. As in Theorem 1, given P, each $x \in X$ can be associated with a unique $p_x \in \{0,1\}^I$, where the i^{th} component is defined by $p_x(i) = 1$ if $x \in P(i)$ and $p_x(i) = 0$ if $x \notin P(i)$. Define $u : X \to \mathbb{R}$ by

$$u(x) = \sum_{i \in I} \frac{p_x(i)}{3^i}.$$

Since $\sum_{j>i} \frac{1}{3^j} < \frac{1}{3^i}$ for any $i \in I$, this u represents the lexicographic order $\succsim$ on $\{0,1\}^I$ defined in the proof of Theorem 1 (that is, $x \succsim y \Leftrightarrow u(x) \geq u(y)$). That proof also shows that

$c(S) = \{x \in X : x \succsim y \text{ for all } y \in X\}$ for all $S \in \Sigma$. Hence $c(S) = \{x \in X : u(x) \geq u(y)$ for all $y \in X\}$. ■

Proof of Theorem 3: For any n, let $1, ..., n$ denote the indifference classes of the preference relation $\succsim$ and let the linear order over $\{1, ..., n\}$ that c induces be $\geq$ (the standard order on the integers). That is, $g \geq h$ for $g, h \in \{1, ..., n\}$ if and only if, for all $x \in g$ and $y \in h$, $x \succsim y$. It is sufficient to consider a choice function c defined on subsets of $\{1, ..., n\}$ that always selects the $\geq$-maximal element. Specifically, if $\widehat{c}$ is the choice function that maximizes $\succsim$, then let S be in the domain of c if and only if there is a $\widehat{S}$ in the domain of $\widehat{c}$ such that $\Big((x \in \widehat{S} \text{ and } x \in g) \Longrightarrow g \in S\Big)$ and $\Big(g \in S \Longrightarrow (\exists x \in \widehat{S} \text{ such that } x \in g)\Big)$.

Both conclusions of the theorem hold for $n = 1$ since the empty set of properties is minimal. So assume henceforth that $n > 1$.

Regarding minimality, suppose c has a checklist P with s properties. As in the proof of Theorem 1, identify each $x \in \{1, ..., n\}$ with the $p_x \in \{0, 1\}^s$ given by $p_x(i) = 1$ if $x \in P(i)$ and $p_x(i) = 0$ if $x \notin P(i)$. Since there are 2^s elements in $\{0, 1\}^s$ and given that $n > 1$, $2^s < n$ would imply that $p_x = p_y$ for some distinct pair $x, y \in \{1, ..., n\}$. Since the domain of c contains the two-element sets, then $\{x, y\} \in \Sigma$ and thus $c(\{x, y\}) = \{x, y\}$, contradicting the assumption that c maximizes $\geq$. So for this domain we cannot have $2^s < n$.

Regarding 'there exists a checklist with k properties, where k is the smallest integer such that $2^k \geq n$,' suppose this claim holds for $1, ..., n-1$. Partition $\{1, ..., n\}$ into $Z_l = \{1, ..., m\}$ and $Z_u = \{m+1, ..., n\}$, where $m = n/2$ if n is even and $m = (n+1)/2$ if n is odd. Then, since $n > 1$, we have $2^{k-1} \geq |Z_r|$ for both $r = l$ and $r = u$. The induction hypothesis implies that $c|Z_u$ (the choice function defined by restricting c to subsets of Z_u) has a checklist $P = (P(1), ..., P(k-1))$ and that $c|Z_l$ has a checklist $P' = (P'(1), ..., P'(k-1))$. Define the checklist Q by $Q(1) = Z_u$ and $Q(i+1) = P(i) \cup P'(i)$ for $i = 1, ..., k-1$.

For any checklist R, let $M_i^R(S)$ denote the ith set of survivors when R is applied to the choice set S.

To see that Q is a checklist for c, notice first that if $S \in Z_u$ then $M_k^Q(S) = M_{k-1}^Q(S) =$

$c|Z_u(S \cap Z_u) = c(S)$, and similarly if $S \in Z_l$ then $M_k^Q(S) = c(S)$. For all S that contain both elements of Z_l and elements of Z_u, application of $Q(1)$ yields $M_1^Q(S) = S \cap Q(1) = S \cap Z_u$. Since $Q(i+1) \cap Z_u = P(i)$, for $i = 1, ..., k-1$, application of properties $Q(2)$ through $Q(k)$ yields $M_k^Q(S) = M_{k-1}^P(S \cap Z_u) = c|Z_u(S \cap Z_u) = c(S)$. ■

Proof of Theorem 4: Let c be a choice function with domain Σ that maximizes $u : X \to \mathbb{R}$, let $\succsim$ be the preference relation on X that u represents, and let $\succ$ and $\sim$ be, respectively, the asymmetric and symmetric part of $\succsim$. Since $\succsim$ has a utility, there exists a finite or countable order dense subset $D \subset X$, that is a D such that for all $x, y \in X$ with $x \succ y$ there exists $d \in D$ with $x \succsim d \succsim y$ (see, e.g., Fishburn [8], chapter 7). For $\succsim$ where no indifference class has an immediate successor, as in Examples 4 and 5, the upper contour sets defined by the $d \in D$ will discriminate between any two strictly ranked options. For the general case, we augment D as follows. For each $d \in D$, define the (possibly empty) set of immediate successors to d, $\sigma(d) = \{x \in X : x \succ d$ and there does not exist $y \in X$ such that $x \succ y \succ d\}$. If $\sigma(d) \neq \varnothing$, let $s(d)$ be an arbitrary element of $\sigma(d)$. Then set $D^+ = D \cup \{z \in X : z = s(d)$ for some $d \in D$ with $\sigma(d) \neq \varnothing\}$. Since D is at most countable, so is D^+. (For the purposes of section 7, we record that for a D with infinite cardinality, D^+ will have the same cardinality as D.)

Enumerate D^+: for either $I = \{1, ..., n\}$ or $I = \mathbb{N}$ let f be a bijection from I to D^+. Define the checklist P by $P(i) = \{x \in X : x \succsim f(i)\}$ for $i \in I$.

Fix some $S \in \Sigma$ and let $x \in c(S)$. So $x \succsim y$ for all $y \in S$. Since for all $y \in S$ and all $i \in I$, $(y \in P(i) \Rightarrow x \in P(i))$, $x \in M_i(S)$ for all $i \in I$. For each $y \in S \backslash c(S)$, $x \succ y$ and so there exists a $d \in D$ with $x \succsim d \succsim y$. If $d \succ y$ for one such d then there is a $\widehat{i} \in I$ with $x \in P(\widehat{i})$ and $y \notin P(\widehat{i})$. If on the other hand $d \sim y$ for all $d \in D$ with $x \succsim d \succsim y$ then $x \in \sigma(y)$ (if not then there would be a z with $x \succ z \succ y$ and hence a $d' \in D$ with $x \succsim d' \succsim z \succ y$). So there exists $e \in D^+$ with $e \sim x$ and again there exists $\widehat{i} \in I$ with $x \in P(\widehat{i})$ and $y \notin P(\widehat{i})$. We can define a $\widehat{i}$, say $\widehat{i}(y)$, for any $y \in S \backslash c(S)$. Since S is finite, there is a $j \in I$ such that $j \geq \widehat{i}(y)$ for all $y \in S \backslash c(S)$. Hence $M_i(S) = M_j(S)$ for all $i \geq j$ and thus $M_j(S) = c(S)$. ■

Proof of Theorem 5: The part of the proof of Theorem 1 that shows that a c with a checklist $P : I \longrightarrow 2^X$ maximizes the lexicographic order on $\{0,1\}^I$ never uses the fact that P finitely terminates. The proof of Theorem 2 therefore also does not use finite termination, and so that proof establishes the 'if' part of the present Theorem. For the 'only if' part, where we are given a utility u that represents some $\succsim$ and a c that maximizes u, we can follow the construction of the $\succsim$-dense set D^+, associated indices I, and checklist P in the proof of Theorem 4. Once again for all $x \in c(S)$ and $i \in I$, we have $(y \in P(i) \Rightarrow x \in P(i))$. So if $x \in \cap_{i<j} M_i(S)$ for some j, it must be $x \in M_j(S)$, and therefore $x \in M_i(S)$ for all $i \in I$. And for all $y \in S \setminus \{c(S)\}$, where therefore $x \succ y$, there must exist $i \in D^+$ such that $x \in P(i)$ and $y \notin P(i)$ (again, by the same argument given in the previous proof). So it must be that $y \notin \cap_{i \in D^+} M_i(S)$, and thus $c(S) = \cap_{i \in D^+} M_i(S)$. ■

References

[1] Bereby-Meyer, Y., A. Assor and I. Katz (2004) "Children's choice strategies: the effects of age and task demands", *Cognitive Development*, 19: 127-146.

[2] Birkhoff, G. (1940) *Lattice Theory*, American Mathematical Society: New York.

[3] Brandstätter, B., G. Gigerenzer, and R. Hertwig (2006) "The priority heuristic: Making choices without trade-offs", *Psychological Review*, 413: 409-432.

[4] Chipman, J. (1960), "The foundations of utility," *Econometrica* 28:193-224.

[5] Cohen, P. (1963), "The independence of the continuum hypothesis," *Proceedings of the National Academy of Sciences* 50: 1143–1148.

[6] Cohen, P. (1964), "The independence of the continuum hypothesis, II," *Proceedings of the National Academy of Sciences* 51: 105–110.

[7] Fischer, J. E., F. Steiner, F. Zucol, C. Berger, L. Martignon, W. Bossart, M. Altwegg, and D. Nadal (2002) "Using simple heuristics to target macrolide prescription in children with community–acquired pneumonia", *Archives of Pediatrics*, 156, 1005–1008.

[8] Fishburn, P. C. (1972) *Mathematics of Decision Theory*, Mouton: The Hague, Paris.

[9] E. N. Gilbert, E. (1962), "Games of Identification or Convergence," *SIAM Review* 4: 16-24.

[10] Gigerenzer, G., P. M. Todd and the ABC Research Group (1999), *Simple Heuristics That Make Us Smart*. Oxford University Press, New York.

[11] Halmos, P.R. (1970) *Naive Set Theory*. Springer Verlag, New York.

[12] Huffman, D. (1952), "A method for the construction of minimum-redundancy codes," *Proceedings of the Institute of Radio Engineers* 40: 1098-1101.

[13] Katsikopoulos, K. V. and L. Martignon (2006) "Naive heuristics for paired comparisons: Some results on their relative accuracy", *Journal of Mathematical Psychology*, 50: 488-494.

[14] Knuth, D. (1973) *The Art of Computer Programming, Volume 3: Sorting and Searching*. Addison-Wesley: Reading MA.

[15] Manzini, P. and M. Mariotti (2007) "Sequentially Rationalizable Choice", *American Economic Review*, 97: 1824-1839.

[16] Rubinstein, A. (1996) "Why Are Certain Properties of Binary Relations Relatively More Common In Natural Language?", *Econometrica* 64: 343-355.

[17] Rubinstein, A. and Y. Salant (2006) "A Model of Choice from Lists" Theoretical Economics, 1: 3-17.

[18] Tversky, A. (1969) "Intransitivity of preferences", *Psychological Review*, 76: 31–48.

[19] Yee, M., E. Dahan, J. R. Hauser and J. B. Orlin (2007) "Greedoid-Based Non-Compensatory Inference", *Marketing Science*, 26: 532-549.

[20] Zimmerman, S. (1959) "An optimal search procedure," *American Mathematical Monthly* 66: 690-693.