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An Alternative Estimation to Spurious Regression Model

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Abstract

In sturdy econometrics specification search problems of unit roots and multicollinearity are well documented since the inception of regression analysis. In examining the likely consequences of nonsense relationship Granger and Newbold (1974) make it clear that first differencing is not the universal sure fire solution to problem of spurious regression models. This has prompted the discovery of cointegration regression estimation by Engle and Granger (1987). In recent years applied econometricians are debating with the problem of spurious regression model when the co movements between the variables are different. If the variables of the model are not cointegrated, there is a question whether the background economic or financial theory is plausible with the data that we are analyzing. This paper reviews the debate and proposes an alternative solution to the problem. Our approach uses a suitable data transformation of the variables of the model based on Hendry (1995) and Phillips (1998) approaches to reduce the spurious correlation, stochastic means and variances in standard level. In a non cointegrated USA information processing investment model, we apply our technique and found a meaningful solution.

Key Words: Spurious Regression, Unit Roots, Cointegration
JEL Classifications: C22, C51

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1. Introduction

Zellner (1971), lists three types of inferences namely deductive, inductive and reductive. All these three are important to economic research. Hendry et. al (1984) pointed out that, according to Keynes, all induction is blind as long as the deduction of casual connection is left out of account, and all deduction is barren as long as it does not start from observations. If this is so, then the best decision from a set of economic choices will be obtained by explaining the relationship among economic variables, the direction of the relation and, in some cases, its magnitude. According to Judge et. al (1985) this involves specifying the econometric model by using the economic theory, mathematical economics and statistical inference as analytical foundation stones and economic data as the information base for modifying, refining, or possibly refuting conclusions contained in the economic theory. Through econometric methodology economic data are used to attach signs, numbers and reliability statements to the coefficients of variables in economic relationships, in order that this information can be used as a basis for decision

making and choice. However, during the last few decades, the development in the estimation of spurious regression has changed the direction of classical regression modeling technique.

The classical time series regression model is based on the assumption that the observed data generating process are stationary, i.e. they are time invariant. However, since the economy grows, evolves and changes over time most of the economic data are trending upwards, e.g. Nelson and Plosser (1982), Hendry and Juselius (200). Technological progresses, innovations, legislative and political changes, geographical and environmental changes and globalization and other changes make most of the economic data non stationary. Yule (1926) first suggested that with the trending nonstationary data the problem of “nonsense correlation”, is extremely high and regression based on these data can be spurious. The estimated coefficients in regression are statistically significant when there is no true relationship between the explained and explanatory variables. A new perspective of this problem was further pursued by Granger and Newbold (1974), Hendry (1980), Philips (1986) among others.

If in regression model explanatory variables exhibit non-stationary, it is very likely that the dependent variable will display the similar stochastic trend. Then as sample size increases their coefficient variance will not tend to be constant and the standard consistency property of OLS estimators breaks down. Their estimators’ sampling distribution will take a non-standard form and the usual test statistics based on normal distribution becomes invalid.

Granger and Newbold's (1974) examined some of the likely consequences of spurious regressions in econometrics. They argue that the level of many economic time series are non-stationary and their sample paths are not represented by the Box and Jenkins (1970) ARIMA type process. They showed that this problem arises when independent random walk variables are regressed with one another. They highlighted that a good fit with significant serial correlation in their disturbances is a cause of spurious regression. In other words, regression equations which relate such non-stationary time series frequently encountered high R^2 and very low Durbin-Watson statistics. The sampling experiments they conducted provide strong evidence of biased towards rejection of the null hypothesis of no relationship and hence the acceptance of a spurious regression.

Phillips (1986) develops an asymptotic theory for regressions that relate quite general integrated random processes. This includes Granger-Newbold (1974) spurious type as a special case. Phillips demonstrated that, in the spurious regression with independent random walk the usual t test does not possess a limiting distribution but actually diverges as the sample size increases towards infinity. He also verified that the Durbin-Watson statistics actually converges in probability to zero while the regression R^2 has a non-degenerate limiting distribution as the sample size increases towards infinity.

In the formulation of theoretically meaningful regression, Engle and Granger (1987) pointed out that a linear combination of two or more non stationary series may be stationary and thus are said to be cointegrated. This cointegrated series may be

interpreted as a long run equilibrium relationship among the variables. Usually cointegration relationship exists if the variables are nonstationary and have the same order of integration. The debate focused around the model frame work when the combination of variables based on economic theory were not cointegrated and hence become spurious. Also, when there are mixed integrated variables in the model there is every possibility of having non cointegration relationship among the variables. This paper examines the possibility of alternate solution of the mixed cointegration problem.

When applied econometricians couldn't find any meaningful relationship among mixed integrated variables, the standard practice in time series literature is to look for a combination set of variables that are not spurious. Most of the time, this combination of cointegrated variables may not follow the usual economic theory. The objective of this paper is to examine the cointegration relationship among mixed integrated variables. Following the new tools for spurious regression model of Phillips (1998), and the theoretical frame work of Hendry (1995, Chapter 3), we proposed the transformation of the variables in such a way so that the stochastic trend variance and correlation reduce in a standard level and the variables are cointegrated.

The structure of this paper is as follows. In the following section we will review the existing spurious regression model solutions. Section 3 and 4 are devoted to defining the problem and an alternative solution of the spurious regression model respectively. An empirical illustration will be presented in section 5. Finally, we conclude the paper with an extension of future research.

2. Existing Solutions to the Spurious Regression model

Traditionally, the time series consists of trend, seasonal and cyclical components. The trend and seasonal components were first removed and then the residuals were analyzed in a model. There are two approaches to the removal of trend and seasonal components. The regression method, and the differencing method. If both the variables exhibit strong trends, the high R^2 observed is due to the presence of trend, not due to the true degree of association between the variables. To avoid such spurious relationship, the common practice is to remove the trend effect by regressing the dependent variable with the explanatory variables and the time trend. This practice may be acceptable if the trending variable is deterministic and not stochastic. If the variables contain stochastic trend, the method suggested by Box and Jenkins (1970) is the successive differencing method.

One of the other solutions of the spurious regression models is the formulation of error correction model. The concept of the error correction model dates back to the Sargan(1964). However, the current popularity is due to Hendry and his promotion of the General to Specific modeling approach. For the model

$$y_t = \beta_0 + \beta_1 x_t + u_t \quad (1)$$

Sargan (1964) linked static equilibrium economic theory to dynamic empirical models in an autoregressive distribution set up as:

$$y_t = b_0 + b_1 y_{t-1} + b_2 x_t + b_3 x_{t-1} + \varepsilon_t \quad (2)$$

This can be written in equilibrium correction form as:

$$\Delta y_t = \alpha_0 + \alpha_1 \Delta x_t - \alpha_2 (y_{t-1} - \beta_0 - \beta_1 x_{t-1}) + \varepsilon_t \quad (3)$$

where $\alpha_1 = b_2$, $\alpha_2 = (1 - b_1)$, $\beta_1 = (b_2 + b_3)/(1 - b_1)$, and $\alpha_0 + \alpha_2 \beta_0 = b_0$

The magnitude of the past disequilibrium is measured by $(y_{t-1} - \beta_0 - \beta_1 x_{t-1})$ and the speed of adjustment towards this steady-state by α_2 . Based on (3), Hendry and Anderson (1977) noted that “there are ways to achieve stationarity other than blanket differencing”, and argued that terms like u_{t-1} would often be stationary even when the individual series were not. Later Davidson et.al (1978) introduced a class of models based on (3) and denoted them as error correction models (ECM). With reference to (3), when a genuine relation exists between non-stationary series, Granger (1981) explains them as cointegrated series. Granger uncovers that, if $x_t \sim I(d)$, $y_t \sim I(d)$ and there exists a constant A such that $z_t = y_t - Ax_t \sim I(0)$, then x_t and y_t will be said to be cointegrated. Granger and Weiss (1983), reported that the main purpose of the error correction models is to capture the time series properties of variables through the complex lag-structures allowed, whilst at the same time incorporating an economic theory of an equilibrium type.

Despite being individually nonstationary characteristics, a linear combination of two or more time series can be stationary and cointegrated. Following Granger (1981, 1983) conceptions of cointegration, Engle and Granger (1987) extended the relationship between cointegration and error correction models to develop estimation and test procedures. Engle and Granger proved that ECMs and cointegration were actually two names for the same thing, i.e. cointegration entails a negative feedback involving the lagged levels of the variables, and a lagged feedback entails cointegration. Suppose that the variables y_t and x_t are $I(1)$. Then the variables y_t and x_t are said to be cointegrated of $C(1,1)$, if there exists a β such that $y_t - \beta x_t$ is $I(0)$. More generally, if y_t is $I(d)$ and x_t is $I(d)$, y_t and x_t are $CI(d,b)$ if $u_t = y_t - \beta x_t$ is $I(d-b)$ with $b > 0$. Engle and Granger (1987) pointed out that if two or more variables are cointegrated, they may diverge substantially from equilibrium in the short run but they must obey an equilibrium relationship in the long run.

An alternative solution of the spurious regression model was proposed by Phillips (1998). Phillips proposed that, the deterministic trend functions (or even time path of another trending variable) can be used as a coordinate system for measuring the trend behavior of an observed variable. Much as one set of functions can be used as a coordinate basis for studying another function. As one can write any function $f \in L_2[0,1]$ in terms of an ortho-normal basis $\{\varphi_k\}_k^\infty$ as $f(x) = \sum_{k=1}^{\infty} c_k \varphi_k(x)$. Continuous stochastic processes such as Brownian motion and diffusions also have representations in terms of the functions

φ_k but with coefficients c_k that are random variables rather than constant Fourier coefficients. In a similar way, we can write trending data in terms of coordinates comprised of other trends, like time polynomials, random walks or other observed trends. Such formulations can be given a rigorous function space interpretation in terms of functional representations of the limiting stochastic processes or deterministic functions to which standardized versions of the trending data or trend functions converge. Phillips (2003) reported that, what is particularly interesting about this perspective is that it provides a mechanism for relating variables of different stochastic order (like time polynomials and random walks) so that it can be used to justify relationships between observed variables like interest rates, inflation, money stock and GDP, which have differing memory characteristics, overcoming the problem of stochastically imbalanced relationships. This approach also offers an interpretation of empirical regressions that are deliberately constructed to be spurious such as the celebrated example of prices on cumulative rainfall (Hendry, 1980). Here, cumulative rainfall is a stochastic trend by construction and this trend is simply one possible coordinate (by no means a good one a priori) for measuring trend behavior of prices. Of course, other coordinates, like the aggregate stock of money, may well provide a more economically meaningful coordinate system, but this does not invalidate the rainfall aggregate as potential yardstick for assessing the trend in price levels. Phillips also showed that how we can still perform useful forecasting exercises despite the presence of (inevitably) mis-specified trends. The common theme of these alternative tools is that all the variables share the common feature of a trending mechanism, even though they may otherwise be unrelated and even though the trending mechanisms themselves may be different.

3. The Problem

The economic interpretation of cointegration as reported by Harris and Sollis (2003) is that if two (or more) series linked to form an equilibrium relationship spanning the long run, and even though the series themselves may contain stochastic trends, they will nevertheless move closely together over time and the difference between them is constant i.e. stationary. Thus the concept of cointegration, according to them, mimics the existence of a long-run equilibrium to which an economic system converges over time, and u_t can be interpreted as the disequilibrium error.

As reported before, usually cointegration analysis assumes that variables are integrated of the same order, say I(1). If integration of the variables are mixed, say some are I(2), and some are of order I(1), then cointegration is still possible if the I(2) series cointegrated down to an I(1) variable in order to cointegrated to other I(1) variables. However in the real world this may not be so. When applied econometricians can not find any meaningful relationship among mixed integrated variables, as an alternative they look for a combination set of variables that are not spurious. Most of the time, this combination of cointegrated variables may not follow the usual economic and or financial theory.

Harris (1995) opined that the main reason why relationships are not always in equilibrium centers on the inability of economic agents to adjust to new information instantaneously. There are often substantial costs of adjustment which result in the current value of the

dependent variable being determined by not only by the current value of some explanatory variables but also their past values.

Some good examples of cointegrations are disposable income versus consumption; wages versus prices. Examples to establish explicit links between cointegration and economic theory are Cambell (1987), King et al. (1991), Ogagi (1992), Granger, et. al (1995) among others. Wickens (1993), Bardsen and Fisher (1993) among others discuss the relation between cointegration and structural/reduced form model at a conceptual level, without reference to explicit economic models.

Soderlind and Vredin (1996), observed that cointegration analyses and equilibrium concepts of macroeconomic time series are rarely based on fully specified economic models. They used a theoretical model to scrutinize a common procedure in applied cointegration analysis and suggest that the cointegration analysis without strong links to economic theory as a-theoretical and made the interpretation a dangerous exercise and misleading.

“Problems with modern economics”, Klein (1994) explained that the modern macroeconomics has become vague, subjective, uncertain, and unhelpful in policy formation. He noted that the technique of cointegration to keep differencing data until stationarity is obtained and then relate the stationary series can do damage. He added that the focus of attention now is on co-integration, simplistic causation testing, unit root

extraction, and other things that he does not think are giving any useful information that we do not already have.

As in applied econometric analysis, most of the economic time series exhibit non stationary behavior, there is every possibility that regression of one time series on another is not cointegrated and gives nonsensical or spurious results. Phillips (2003) in “Laws and Limits of Econometrics”, discussed some general weakness and limitations of the econometric approach encounters in explaining and predicting economic phenomena. Phillips explained that the model developed in economic theory are metaphors of reality, sometimes amounting to a very basic set of relations that are easily rejected by the data. Formulating six laws of econometrics, Philips highlighted that no one understands trends in empirical macroeconomic research. Most commonly trend formulations are polynomial time trends, simple trend break polynomials, and stochastic trends which include unit root models, near unit root models and fractional process. Unit roots inevitably cause trouble because of the nonstandard limit distributions (Philips and Xiao (1998)). Unit roots also cause trouble because of the difficulty in discriminating between stochastic trends and deterministic trend alternatives, including models that may have trends break.

4. Proposed Solution

Before examining proposed procedure, it is useful to outline the original concept of regression. By predicting the children heights based on the average height of the parents,

Galton (1885) first proposed “regression towards the mean”. Galton smoothed the plots for a sample of 928 children, and the counts appeared more regular to draw the level curves of the underlying population density. Following Galton, as reported by Koenker (2000), the most likely value of the child’s height given the parent height, that is for any given value of the mid-parent (average height of the parent) height we could ask, what value of the child’s height put us on the highest possible contour of the density. This obviously yields a locus of tangencies of the ellipses with horizontal lines. Stigler (1997) guides this remarkable feature of the conditional densities of jointly Gaussian random variables that the conditioning induces what we may call pure location shift. In Galton’s original example, the height of the mid parent alters only the location of the center of the conditional density of the child’s height; dispersion and shape of the conditional density as invariant to the height of the mid-parent. Which is the essential feature of the classical regression model, i.e. the entire effect of the covariates on the response is captured by the location shift $E(Y|X = x) = x'\beta$ while the remaining randomness of Y given X may be modeled as an additive error independent of X . This attempt to compare random variables in terms of means is most responsible for narrowing the scope of statistical investigations to the comparison of means.

Let us consider a time series regression model of the response variable Y which is explained by a set of explanatory variables $X:(X_1, X_2, \dots, X_k)$ by an unknown functional relationship

$$f(Y|X) = g(X, \theta) + U \tag{4}$$

When the form of f is unknown, θ is not estimable. However, under certain conditions, the sub-space $S(\theta)$ of R^p spanned by θ is estimable. The goal of this paper is to find out a design of experiment so that standard method of estimation may yield useful estimates of $S(\theta)$, when the family $f(Y|\theta'X)$ is unknown. When the functional form of the relationship is unknown, then many standard design methods no longer apply. The choice of an experimental design can depend on many aspects of the phenomena under study, particularly the expected relationship between the response and the design variables. There are several papers that deal with the designs of unknown models, among which are the pioneer paper by Box and Darper (1959), Atkinson (1988).

Most of the economic raw data have strong asymmetry, outliers, and fat tails, and widely different spreads, large and systematic residuals. Hence data needs change of expression in terms of transformation that is much easier to analyze to produce informative display and effective summaries. The primary motives of engaging transformations of variables are to enhance interpretability, stabilize the spread, and enhance symmetry and to deal with non normality, nonlinearity and heterogeneous variances. It is important to know that transformation may destroy the relationship between the dependent and independent variables. Or transformation could solve one problem but give rise to another. The choice of transformation typically driven by the nature of the variables and their relationships.

When the exact functional form of the model is unknown, and have non-normal and nonlinear problems, then various transformations to the original data may be necessary to

retain an efficient parameter of estimations. One of the widely used transformation in econometrics is the Box and Cox (1964) family of transformations which includes logarithmic transformation and no transformation at all in special case. Box-Cox suggests that when their power transformation applied to the dependent variable in a linear regression setting, it might induce normality, error variance homogeneity, and additivity of effects. Carroll and Ruppert (1984) suggested applying this and other transformations to both dependent and independent variables. Successful transformation methodology in regression analysis are Zellner and Revankar (1969), Carrol and Ruppert (1988), Tibshirani (1988), Coulson (1992), Anglin and Gencay (1993), Linton and Hurdle (1996) among others. MacKinnon and Magee (1990) proposed a family of transformation which can sensibly be applied to both dependent and independent variables and scale invariant. One of the popularity of the linear regression model is that even though linearity is an unreasonable assumption for the original data, it often is reasonable for data that have been appropriately transformed.

The role of an Econometrician is to develop simple models for the interpretation of economic data capable of forecasting and hypothesis testing. For a time series econometric model, the methodology is to decompose the series into trend, seasonal, cyclical, and irregular component. We need suitable tests to determine whether a system contains a trend and whether the trend is deterministic or stochastic nature. A series containing a trend will not revert to a long-run level. A serious problem is encountered when the inappropriate method is used to eliminate trend. Based on the nature of trend appropriate transformation is necessary to attain a stationary series. As discussed, the

usual methods for eliminating the trend are differencing and detrending. Detrending entails regressing a variable on time and saving the residuals.

When there are integrated variables in a regression model, applied econometricians are looking for the cointegrated relationship among the variables. When there are no cointegrated relationships among the variables (e.g. cointegration of mixed integrated variables), the standard practice is to look for a combination set of variables that are not spurious. Most of the time, this combination of variables may not follow the usual economic theoretical background. To keep economic theory valid, some times, a unit series can be made stationary by differencing. Due to the differencing, important information with the level variables will be missing. Since trend and seasonal components contain important information, they are to be explained rather than removed. If the original model's disturbance is non-autocorrelated, due to differencing the new disturbance term will exhibit autocorrelation of the moving average type. Thus differencing does not provide any satisfactory solution to the spurious regression model. This does not mean all nonstationary model can be transformed into well behaved models by appropriate differencing.

There are instances in which econometricians advocated transformation of variables to obtain cointegrated relationship e.g. Hallman (1989), Granger and Hallman (1988, 1991), Hendry (1995), Phillips (1989) among others. Granger and Hallman (1988) showed an example where x_t and y_t are not cointegrated but their functions in the form of x_t^2 and y_t^2 are cointegrated.

As we reported in section 2 that when the variables are not cointegrated Phillips (1998) proposed an alternative solution that the deterministic trend functions (or even time path of another trending variable) can be used as a coordinate system for measuring the trend behavior of an observed variable, much as one set of functions can be used as a coordinate basis for studying another function.

As we know the variance of a unit-root process increase over time and successive observations are highly interdependent, we will consider the conceptual framework of Phillips (1998) to propose an alternative solution of the spurious regression model, where the variables of the model are not cointegrated. We will also consider Hendry's (1995, chapter 3) convergence results for normalizing sample moments in conjunction with Philips conceptual frame work.

We propose transformation to the nonstationary variables in such a way, so that the nonstationary variance and inter correlation among the variables reduce in a significant level to have meaningful relation among the variables of the model. We know that the correlation between two deterministic time trends (Z_i , $i=1,2$) is unity and their mean and variance are nonstationary with respect to time. If we transform these deterministic trends by

$$Z_{i,t}^* = (Z_{i,t} + Z_{i,T-t+1})/2, \quad (5)$$

it can be shown that the means of Z_1^* and Z_2^* will be constant, their variance will be zero and correlation between them will also be zero. Variable transformation, applying (5) is

appealing, which could be used to solve the problem of heteroschedasticity, autocorrelation and multicollinearity problem in generalized linear regression models (Rahman, 2004). If the variables have stochastic trend, then we could considering transformation to the original variables (Z_i , $i=1,2$) based on the following form

$$Z_{i,t}^* = (Z_{i,t} + Z_{i,T-t+1})/2d \quad (6)$$

for $t = 1,2,\dots,T$ and $i=1,2$ and d is any divisor based on the nature of sample moment given in Hendry (1995, pp107).

There are two reasons for using this transformation to the original model variables. First in regression analysis we predict the expected value of any dependent variable based on any particular set of values of the independent variables. And estimation of the parameters of the model based on the set of independent variables must be more efficient. If the data suggest that the distribution of error term are not constant across t , one must go beyond standard regression methodology and adopt this transformation to the original variables to get a model like GAM form. If the variables of the original linear regression model are not cointegrated, then the question arise what kind of accepted model we have to fit to predict the variable. If we transform all the variables of the model which will overcome the problem of nonnormality and the nonconstancy of the distribution of error term, then there is a possibility of meaningful regression results. In our research, the proposed solution to the spurious regression model, is restricted to the single equation model.

5. An Empirical Experiment

For an empirical experiment, data used are private fixed investment in information processing and equipment (Y , in billion dollars), sales in total manufacturing and trade (X_1 , in million dollars), and interest rate (X_2 , Moody's triple A corporate bond rate, in percent) [Source, US Economic Report of the President, 2001, Table B-18, B-57, and B-73]. Data for the year 1971-1999 were considered. Based on economic theory the original model considered for this data set is that

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + U_1 \quad (7)$$

where α_0 is the intercept term, α_i ($i=1,2$) are the coefficient parameters and U_1 is the disturbance vector. The first step of the analysis of the model is to examine the order of integration of the individual series. In Table 1, we report the results of unit root tests based on Augmented Dickey-Fuller (1979) test, and Dickey-Fuller Test with GLS Detrending (DFGLS) proposed by Elliot, Rothenberg and Stock (1996). The null hypothesis tested are that the variables have unit root. We apply these tests for level, first and second differences of the series and their cointegration relationships. Each of the unit root tests provide the same results. For the series investment, sales and interest rate, the null hypothesis that the series do not have unit root can not be rejected at 5% significance level. When the data differenced once, the null of nonstationarity can be rejected for the variables X_1 and X_2 by the ADF test. Thus the variables X_1 and X_2 are found to be

integrated of order one. The series Y is found to be integrated of order 2 i.e. I(2). Thus for the mixed integrated variables, we examine the cointegration relationship for the variables Y, X₁ and X₂ and found that they are not cointegrated. The residual term is found to be I(2) under both test procedures. Note that the critical values of the ADF test depend on the number of I(1) and I(2) regressors in the equation and according to Haldrup (1994), his Table 1 must be used for the test.

Given our results for the original model, we now use the transformation to the original model variables to find out whether any meaningful relation could be obtained. Our transformed model is of the type

$$f(Y) = \beta_0 + \beta_1 g(X_1) + \beta_2 h(X_2) + U_2 \quad (8)$$

where, f(Y), g(X₁) and h(X₂) are the transformation functions of the original variables Y, X₁ and X₂. In this experiment, we perform the transformation of the type based on the following form:

$$Z_{it}^* = (Z_{it} + Z_{i,T-t+1})/2 \text{ for } t = 1, 2, \dots, T$$

where the variables Z_i are both dependent and independent variables.

Test for unit roots for the transformed variables showed that except X_1 in DFGLS test, all the transformed variables Y^* , X_1^* and X_2^* have the same order of integrations as the original variables Y , X_1 and X_2 have under both tests. However, the residuals of \hat{U}_2 is found to be $I(0)$, i.e. the transformed variables are cointegrated, though the variables have mixed integrated orders.

OLS regression results for both the original and transformed models are presented in Table 2. The alternative model based on the transformation of its variables clearly fit the data better than its original model. The original model variables are not cointegrated and fail to generate plausible long run relationship. Whereas the transform alternative model variables are cointegrated, and lead to sensible interpretation. A good measure of the relative performance of the two models is the difference between DW statistics. The most interesting feature of the regression output is that for the original model $R^2 > DW$ whereas for the transformed model we obtained $DW > R^2$. Although the original model based on coefficient of determination clearly fit the data well, this result is not surprising. Thus the transformed model represents an improvement over the original model.

6. Conclusions

In regression model when the variables have different unit roots are not cointegrated then the model is concluded as spurious. There are some questions as to whether the background economic or financial theory is plausible with the data that we are analyzing.

If any uncertainty was expressed about the model specification, there was a tendency to acknowledge that the econometric model could not play the role of the real world. This notion could be removed. This paper reviews the debate and attempt to devise an alternative solution to estimate such kind of spurious regression model. Based on the concept of Phillips (1998) alternative solution of the spurious regression model and Hendry's (1995, chapter 3) convergence results for normalizing sample moments, we have introduced a new form of transformation on the variables of the regression model to reduce the changing variance and inter correlations among the variables. We present an empirical example where variables have different unit root orders and are not cointegrated. We transform the variables according to our suggested method and found that our proposed transformation performing significantly better than the original model. We conclude that this kind of transformation benefit is substantial. In view of this, we could suggest that one could consider this kind of transformation to their model to get better meaningful fit of a spurious regression. Discussions of the properties of the transformation with various d and their properties to regression model are kept for future research.

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Table-1: Unit Root Tests

Original Model			Transformed Model		
Variables	ADF	DFGLS	Variables	ADF	DFGLS
Y	2.031 [1] (-2.976)	1.319 [1] (-1.954)	Y*	-1.210 [0] (-2.972)	-0.688 [1] (-1.954)
X ₁	2.062 [0] (-2.972)	0.764 [1] (-1.954)	X ₁ *	-2.877 [0] (-2.972)	-2.167 [0] (-1.954)
X ₂	-1.842 [1] (-2.976)	-1.194 [0] (-1.953)	X ₂ *	-2.027 [1] (-2.976)	-1.730 [1] (-1.954)
D(Y)	1.346 [0] (-2.976)	0.987 [0] (-1.954)	D(Y*)	-1.504 (-1.954)	-1.024 [2] (-1.955)
D(X ₁)	-3.9932 (0.0214)	-3.614 [0] (-1.954)	D(X ₁ *)	-5.731 [0] (-1.954)	-5.386 [0] (-1.954)
D(X ₂)	-3.783 [0] (-1.954)	-3.781 [0] (-1.954)	D(X ₂ *)	-3.270 (-1.954)	-3.226 (-1.954)
D(Y,2)	-4.007 (-1.954)	-4.357 [0] (-1.954)	D(Y*,2)	-5.035 [0] (-1.954)	-5.240 [0] (-3.190)
U1	-0.642 [1] (-2.976)	-1.276 [1] (-1.954)	U2	-3.124 (-1.954)	-2.268 [0] (-1.953)
D(U1)	-0.141 [0] (-1.954)	-0.429 (-1.954)			
D(U1,2)	-5.192 [0] (-1.954)	-5.456 [0] (-1.954)			

Figures in [] and () are the lag length/Band Width and 5% critical values respectively.

Table2: Regression Results of the Original and Transformed Models
(Dependent Variable: Y)

Independent Variable	Original Model	Transformed Model
Intercept	-16.4934 (0.4228)	-154.2234 (0.0295)
X ₁	0.0005 (0.0000)	0.0009 (0.0000)
X ₂	-6.3917 (0.0043)	-9.0828 (0.0000)
R-Squared	0.9649	0.7730
Durbin-Watson Statistics	0.3117	0.9210

Note: p-values are given in the parentheses