Economic Growth Centre
Working Paper Series

Markov-switching Unit Root Test: A study of the Property Price Bubbles in Hong Kong and Seoul

by

XIAO Qin and TAN Gee Kwang, Randolph

Economic Growth Centre
Division of Economics
School of Humanities and Social Sciences
Nanyang Technological University
Nanyang Avenue
SINGAPORE 639798

Website: http://www.hss.ntu.edu.sg/egc/

Working Paper No: 2006/02

Copies of the working papers are available from the World Wide Web at:

Website: http://www.hss.ntu.edu.sg/egc/

Markov-switching Unit Root Test: A study of the property price bubbles in Hong Kong and Seoul

Version 3(January 2006)

Xiao Qin

G. K. Randolph Tan

Abstract:

Evans (1991) demonstrates that the unit root tests recommended by Hamilton and Whiteman (1985) and Diba and Grossman (1988) will fail to detect periodically collapsing rational bubbles. Hall et al. (1999) however show that the power of this test procedure can be improved by incorporating a Markov-switching state variable. In this paper, we apply both procedures to selected data from Hong Kong and Seoul. Both point to the possible existence of a periodically-collapsing bubble in each price series investigated, with the second procedure more precise on timing the bubble. Our Markov-switching model is validated using a symmetry test and a Wald test.

JEL classification: G12, C12, C52

Keywords: Markov-switching, unit root test, periodically-collapsing bubble, real-estate

1. Introduction

Hamilton and Whiteman (1985) and Diba and Grossman (1988) suggest that stationarity tests may be used to detect an explosive rational speculative bubble. use of such a test does not preclude the possible influences of unobservable market fundamentals. The rationale of this procedure is as following. If the first-difference of a dividend and those of unobservable fundamentals are stationary in mean, and if no rational bubble exists, then the first difference of the associated stock price must be stationary. Differencing a stock price a finite number of times would not render it stationary, however, if it contains a rational bubble. Due to the possible presence of unobserved variables, finding that the first-difference of a stock price is not stationary does not automatically establish the existence of a rational bubble. However, the converse inference is possible. That is, evidence that the first-difference of a stock price has a stationary mean would be evidence against the existence of a rational bubble in that price.

Evans (1991) shows that stationarity tests, suggested by Diba and Grossman (1988) and Hamilton and Whiteman (1985), are incapable of detecting periodically collapsing rational bubbles. Hall et al. (1999) show that the power of these tests can be improved significantly by incorporating a state variable that follows a Markov process. They argue that testing for a periodically-collapsing bubble essentially involves distinguishing the expanding phase from the collapsing phase of the bubble. The two phases can be modeled by a two-state Markov chain. In this case, the data generating process (DGP) would take different parameters in different states. If we model this DGP with a Markov-switching AR(p) process, then a generalized ADF unit root test would detect the bubble, if it exists, quite effectively.

In this study, we will apply the ADF test to both a non-switching AR(p) model (henceforth, the linear model) and a Markov-switching AR(p) model (the MS model, henceforth). The linear model is qualified by the usual F test, while the MS model by a symmetry test and a Wald test. The results of the two tests are then compared. Our data are drawn from the property market of Hong Kong and Seoul.

2. A Review of the Real-estate Price Bubble Literature

In the language of economists, a bubble exists in a price if the price is other than what is warranted by its fundamentals. The issue of speculative bubble arises because of uncertainties surrounding the "fundamentals". In the real estate market, for instance, a buyer of a property is willing to pay a price which he perceives to be equal to the "fundamental" value. In assessing this "fundamental" value, he will make use of information on rental flows and future price changes. Unavoidably, his assessment must rely on expectations about some relevant future events. The expectations in turn are based on *subjective*, instead of the actual, probabilities of such events. Therefore, the assessment of market fundamentals is inherently subjective (Shiller 2001). As a result, the actual price will reflect the true "fundamental value" only by chance! That is a price will contain a bubble element as a matter of course.

The real-estate market has the longest and the most reliable history of boom and bust cycles stretching back to the early 1800s (Carrigan 2004). Researchers point to speculation as a prime force behind these cycles (Malpezzi and Wachter 2002). Such speculation could initiate the formation of speculative bubbles. Evidence of speculative bubbles in the real estate market has been found in countries around the world.

Abraham and Hendershott (1994) found that, as of the end of 1992, there was a 30% “above market” premium in prices in the Northeast of the United States, a 15%–20% premium in prices on the West Coast of the country, and probably a significantly negative premium in Texas. In their model, Abraham and Hendershott incorporate a proxy for the tendency of a bubble to burst and a proxy for the tendency of a bubble to swell.[i] They found that the proxy does indeed work and is especially useful in explaining the large cyclical swings in real house prices in the West. The lagged appreciation term that represents speculative pressures in the market performs admirably in soaking up the volatility.

Bjorklund and Soderberg (1999) examine in their paper the 1985–1994 cycle in the Swedish property market. Their study shows that the ratio of the property value to the rent increased too much during this sample period, indicating that a bubble might exist. Scott (1990) and Brooks et al. (2001) apply variance-bound tests in their studies to test the rationality of real-estate share prices. Scott analyzes price indices of 13 REITs. His sample stretches from the late 1960s or early 1970s to 1985. Brooks et al. examine the prices of U.K. property stocks. Both studies indicate the existence of irrational, speculative bubbles.

The large swings of property prices in Japan in the late 1980s and early 1990s have intrigued many researchers. Ito and Iwaisako (1995) attempt to measure how much of the asset price variation observed in Japan in the late 1980s and the early 1990s can be attributed to changes in the "fundamentals" As represented by the standard present value model. They conclude that "it seems impossible to offer a rational explanation of the asset price inflation in the second half of the 1980s by changes in fundamentals (p. 10)." This conclusion is reinforced by a variance decomposition analysis (ref. their p. 20). Basile and Joyce (2001) use the method by Fortune (1991) to measure the size of the asset price bubble, which is the difference between the ex post returns of an asset and the required return. By this measure, they find that the land market bubble grows evenly through mid-1990s before declining.

There have been quite a number of studies on the real-estate price bubbles in South Korea. Lee (1997) conducted a test of bubble in the land price of Korea between 1964 and 1994. Using a structural model with GNP, interest rate and money supply as fundamentals, he found the hypothesis that only market fundamental drove the land price in Korea can be rejected. Kim and Lee (2000) adapt the idea that the existence of an equilibrium relationship excludes the possibility of a price bubble. They found that, in the long run, nominal and real land prices are cointegrated with market fundamentals (approximated by nominal and real GDP respectively). However, in the short run, such cointegration relationship does not exist. This is consistent with the notion that speculative bubbles are periodically collapsing (Blanchard and Watson 1982). In the short run, speculative forces could drive prices away from market fundamentals. In the long run, fundamental forces will eventual reassert themselves. Lim (2003) conducted two bubble tests based on the present value relation on the housing price of Korea, one is a modified volatility test (MRS test) suggested by Mankiw et al (1985), another combine unit-root test suggested by Diba and Grossman (1988), and cointegration test by Campbell and Shiller (1987). Their MRS test show that the null hypothesis of market efficiency is rejected, indicating the existence of an irrational bubble. Their unit-root test and cointegration test however suggest that bubbles do not exist! This is in contrast with our findings in this paper, which employ a Markov-switching ADF approach. However, the data series employed in the current paper are not identical to those used by Lim (2003). Park et al (1998) suggests that the price bubble in South Korea in housing and land stood at 58% and 40% respectively in 1991 at its peak, disappeared almost completely by 1997.

Hong Kong underwent extraordinary swings in the 1980s and 1990s. In size, the price swings have been as dramatic as those happened elsewhere in the world. In frequency, they have been more dramatic. This fact makes Hong Kong property market one of the most interesting for the studying of speculative bubbles. However, there have been relatively few papers devoted to this subject. Chan et al.’s (2001) is one of them. The study uses a standard present value model, but allows for a specification error. The data they use are monthly averaged rentals and quarterly averaged prices of the private domestic properties within the class A, which is defined as those apartments with sizes less than 39.9 m^2. The sample period runs from the first quarter of 1985 to the third quarter of 1997. Their results show that there exists misspecification error in the model noises as well as a bubble. The path of the bubble shows that the bubble exploded most sharply between 1990 and 1992, and between 1995 and 1997. Xiao and Tan (2006) use a similar approach, but a different technique[ii], to study the property market of Hong Kong. They use monthly observations between December 1980 and January 2003 from four different sectors: the office, the domestic premises, the flatted factories and the retail premises sectors. They conclude that a periodically-collapsing rational speculative bubble could be responsible for the observed volatilities in each sector. The peak of the bubble occurred in the mid-1994 and/or the mid-1997 in all cases.

3. Methodology

3.1. Conventional Unit-root Tests

Dickey and Fuller (1979) suggested a battery of tests based on a regression of the form $y_t = \rho y_{t-1} + u_t$ or $y_t = \mu + \rho y_{t-1} + u_t$ or $y_t = \mu + \delta t + \rho y_{t-1} + u_t$, and the true process being $y_t = y_{t-1} + u_t$ or $y_t = \mu + y_{t-1} + u_t$. In these tests, the disturbance term is assumed to be i.i.d. and normal with zero mean and constant variance (Hamilton 1994, p. 502).

Phillips (1987) and Phillips and Perron (1988) suggest some modifications to the DF test statistics to take care of serially correlated and heteroscedastic disturbance terms. The test suggested by Phillips and Perron are referred to as the PP test.

Dickey and Fuller (1979) provide an alternative approach that controls for serial correlation by including higher-order autoregressive terms in the regression. That is the model $y_t = \mu + \delta t + \rho y_{t-1} + \sum_{i=1}^{p-1} \varsigma_i \Delta y_{t-i} + u_t$ is to be estimated with possibly zero coefficients on the constant and the trend terms. This modified DF test is referred to as the ADF test.

Various suggestions have been proposed regarding how to proceed when the process is deemed as $AR(p)$ with p unknown but finite. Hamilton (1994) suggests a simple approach that takes p to be some pre-specified upper bound $\bar{p}$ (We set $\bar{p} = \sqrt{T}$ in this paper. T is the sample size).The OLS t-ratio of $\varsigma_{\bar{p}-1}$ can then be compared with the usual critical value for a t statistic. If the null hypothesis is accepted, then the OLS F test of the joint null hypothesis (that both $\varsigma_{\bar{p}-1} = 0$ and $\varsigma_{\bar{p}-2} = 0$) can be compared with the usual F(2,T-K) distribution. The procedure continues sequentially until the joint null hypothesis (that $\varsigma_{\bar{p}-1} = 0$, $\varsigma_{\bar{p}-2} = 0$, …, $\varsigma_{\bar{p}-l} = 0$) is rejected for some l. Greene (1997, p. 787) discusses the merits and flaws of this procedure. As our purpose is to remove serial correlations among residuals, this procedure will suffice when combined with the Durbin Watson test.

3.2. A Markov-switching AR(p) Model and its Estimation Procedure

A MS model assumes that time series data may display periodic changes in their observed behavior, and it accounts for such changes through switches in states. The average duration of each state is allowed to differ. Furthermore, the statistical features and identification of the states are not imposed exogenously on the data, but determined endogenously by the estimation procedure.

Consider

$$\Delta y_t = \mu^{st} + \phi^{st} y_{t-1} + \sum_{k=1}^{K} \psi_k^{st} \Delta y_{t-k} + \varepsilon_t \quad \varepsilon_t \xrightarrow{d} iid, N(0, \sigma^2), \qquad (1)$$

where $s_t \in \{1,2,..., N\}$, a state variable following the first order Markov chain:

$$\begin{aligned} &\Pr\left(s_{t+1} = j \middle| s_t = i, s_{t-1} = i_1,..., \zeta_t\right) \\ &= \Pr\left(s_{t+1} = j \middle| s_t = i\right) \\ &\equiv p_{ij} \end{aligned} \qquad (2)$$

where $\zeta_t = (y_t, y_{t-1},..., y_1)$, representing the information set available at time t, and p_{ij} is the probability that state i will be followed by state j given $s_t = i, s_{t-1} = i_1,..., and \zeta_t$. Equation (2) says that the probability distribution of s_{t+1} depends on past events only through the value of s_t [iii].

The state variable s_t is not observable, but its probability for a given sample of size T, $\Pr\left(s_t = i \middle| \zeta_T\right)$, can be inferred using the discrete Kalman filter (refer to Hamilton 1989, 1994 a&b for detail).

In calculating the smoothed inference of the state variable, $\Pr\left(s_t = i \middle| \zeta_T\right)$, we assume that the DGP parameters, $\beta = (\ \mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}, \sigma)'$, are known to us. In fact, these parameters need to be estimated. We can estimate them by maximizing the log likelihood function of the observed data using the EM algorithm, since the EM algorithm is efficient, simple, and stable.[iv] The log-likelihood function to be maximized is $LL = \sum_{t=1}^{T} \log f\left(y_t \middle| x_t, \zeta_{t-1}\right)$, with $f\left(y_t \middle| x_t, \zeta_{t-1}\right)$ the density of y_t conditional on x_t and ζ_{t-1}. The estimation steps are given below.

1. Make an arbitrary guess about the values of $\mu^{st}, \phi^{st}, \psi_k^{st}, p_{ij}$ and σ .
2. Calculate the smoothed probabilities of s_t using the discrete Kalman filter.
3. OLS regress $y_t \sqrt{\Pr\left(s_t = i \middle| \zeta_T\right)}$ on $x_t \sqrt{\Pr\left(s_t = i \middle| \zeta_T\right)}$, $i = 1,2$, which gives the ML estimates $\tilde{\mu}^{st}, \tilde{\phi}^{st}, \tilde{\psi}_k^{st}$, $\left(k = 1,2,...K\right)$. Notice that $y_t \equiv \Delta y_t$, $x_t \equiv (1, y_{t-1}, \Delta y_{t-1}, ..., \Delta y_{t-k})'$, $\left(k = 1,2,...K\right)$.
4. *U*pdate σ^2 using the OLS residuals.

$$\tilde{\sigma}_{st}^{\ 2} = \frac{\left(y_t - x_t' \tilde{\beta}_{st}\right)'\left(y_t - x_t' \tilde{\beta}_{st}\right)}{\left(T - J\right)}, \tag{3}$$

   where J = the number of parameters estimated in each state.

5. Update p_{ij} .

$$p_{ij} = \frac{\sum_{t=2}^{T} \Pr\left(s_t = j, s_{t-1} = i \middle| \zeta_T\right)}{\sum_{t=2}^{T} \Pr\left(s_{t-1} = i \middle| \zeta_T\right)} \tag{4}$$

6. Update π.

$$\pi_i = \Pr\left(s_1 = i \middle| \zeta_T\right) \qquad (5)$$

Repeat steps 2 through 6 until the parameters and the likelihood converge.

3.3. Specification Tests of the Markov-switching Model

An important issue pertaining to an MS model is the number of states characterizing the data. The standard distributional theory is not applicable for evaluating the Markov-switching model against some popular alternatives, such as a linear time-series model. Hamilton (1989) shows that conventional tests of a Markov-switching model would render the Markov transition matrix unidentified and the information matrix singular, under the null hypothesis of a single state.

Several authors have proposed alternative testing procedures that attempt to overcome these problems. However, the application of these procedures can be problematic. The problems arise because that we have limited knowledge of the respective powers of these tests, and these tests are, in general, computationally demanding (Raymond and Rich 1997). For these reasons, perhaps, the previous studies seldom validate their Markov-switching specifications[v].

Breunig et al. (2003) argue that a Markov-switching model should be put to specification tests, like any other model. Among the four types of tests suggested, they highly recommend a Wald-type test, which they call “encompassing test”. They show that this encompassing test is the most powerful way of examining the ability of the model to match the data.

We will put our MS model to two types of specification tests in this paper: a symmetry test following Cecchetti et al. (1990) and an encompassing test suggested by Breunig et al (2003). Under the null $p_{11} = p_{22}$, the symmetry test statistic has a standard normal distribution. The encompassing test procedure is described below.

Let $\hat{\gamma}$ be a quantity that has been estimated from the data. This $\hat{\gamma}$ can be the mean, the variance, or something else. We denote a comparable quantity *implied* by the MS model by $\gamma_M\left(\hat{\theta}\right)$, where $\hat{\theta}$ is the MLE of the parameter θ associated with the MS model. In particular, we simulate data with the estimated MS model, and estimate $\gamma_M\left(\hat{\theta}\right)$ from the simulated series. A scaling factor of $\left(1+\frac{T}{M}\right)$ is applied to the variance of any test statistic to make an allowance for the effect of the simulation error upon the variance of an estimator, where M is the number of replication and T the number of observations in the sample.

Consider the statistic

$$\hat{\tau} = \hat{\gamma} - \gamma_M\left(\hat{\theta}\right).$$

Under the null

$$\tau_0 = \gamma_0 - \gamma_M\left(\theta_0\right),$$

where θ_0 is the true value θ and γ_0 the true value of γ, we have

$$T^{1/2}(\hat{\tau}-\tau_0) \xrightarrow{d} N(0, V_\tau).$$

Consider the test statistic

$$R^* = \hat{\tau}'[Var(\hat{\tau})]^{-1}\hat{\tau},$$

which has a χ^2 distribution with degree of freedom equals to the dimension of τ, with

$$Var(\hat{\tau}) = Var(\hat{\gamma}) - Var(\gamma_M(\hat{\theta})).$$

This statistic can be replaced by

$$R = \hat{\tau}'[Var(\hat{\gamma})]^{-1}\hat{\tau}.$$

As $R < R^*$, if R exceeds the critical value, we would reject the null even more strongly with R*.

Under the null, that MS model is correct and characterized by parameter $\hat{\theta}$, we can simulate data from the model and find out $Var(\hat{\gamma})$ from the simulated series. Alternatively, we may use asymptotic theory and compute a robust estimator of $Var(\hat{\gamma})$.

In the current study $\hat{\gamma}$ corresponds to SSE[vi] from the MS model. $\gamma_M(\hat{\theta})$ is the sample mean of SSE from the simulation with 10,000 replications, and var($\hat{\gamma}$) the sample variance of SSE. Under the null $\hat{\gamma} = \gamma_M(\hat{\theta})$, the test statistic R has a $\chi^2(1)$ distribution. A scaling factor $\left(1+\frac{T}{M}\right)$ is applied to $Var(\hat{\gamma})$, as discussed before.

3.4. Bootstrapping

The null distributions of the statistics for the linear unit-root tests are tabulated in Hamilton (1994). Those for the Markov-switching ADF tests are unknown but can be generated by bootstrapping.[vii]

Bootstrapping is a method for estimating the distribution of an estimator or test statistic by resampling the data. It amounts to treating the data as if they were the population for the purpose of evaluating the distribution of interest. Under mild regularity conditions, Bootstrapping yields an approximation to the distribution of an estimator or test statistic that is at least as accurate as the approximation obtained from first-order asymptotic theory. Thus, bootstrapping provides a way to substitute computation for mathematical analysis if calculating the asymptotic distribution of an estimator or statistic is difficult.

In fact, bootstrapping is more accurate in finite samples than first-order asymptotic approximations and does not entail the algebraic complexity of higher-order expansions. The first-order asymptotic theory often gives poor approximations to the distributions of test statistics with the sample sizes available in applications. As a result, the nominal probability that a test based on an asymptotic critical value rejects a true null hypothesis can be very different from the true rejection probability (RP). Bootstrapping often provides a tractable way to reduce or eliminate finite-sample errors in the RPs of statistical tests.

The method nevertheless has its own limitations and should not be used blindly, but it works well in general. The readers are referred to the *Handbook of Econometrics*, vol. 5, chapter 52 for details on the sampling procedure and the consistency of bootstrapping.

The steps of bootstrapping are described below.

1. Save the ML parameter estimates $\tilde{\theta}$ and residuals $\{\tilde{\varepsilon}_t\}_{t=1}^{T}$ from the MS model.
2. Construct the random disturbance term e (to be explained later in this section).
3. Take a random draw of e, denote as $e_1^{(1)}$, and set

$$\Delta y_1^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{-k} + e_1^{(1)} ,$$

$$\Delta y_2^{(1)} = \tilde{\mu} + \tilde{\psi}_1 \Delta y_1^{(1)} + \sum_{k=2}^{K} \tilde{\psi}_k \Delta y_{-k} + e_1^{(1)} ,$$

…

$$\Delta y_T^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{T-k}^{(1)} + e_1^{(1)} ,$$

where $\Delta y_t^{(1)}$ = simulated values of Δy_t,

Δy_{-k} = actual observed values of Δy_t, and

$\tilde{\mu}, \tilde{\psi}_k$ = ML estimates.

This gives a full sample $\{y_t^{(1)}\}_{t=1}^{T}$, where T is the number of observations in the sample.

4. Fit the artificial sample to equation (1), producing estimates of model parameters, $\tilde{\theta}^{(1)}$, and their associated τ and rho statistics.

5. Repeat steps 3 and 4 10,000 times,[viii] giving $\{\tilde{\theta}^{(i)}\}_{i=1}^{10000}$ and the 10,000 associated τ and rho values. The 95% confidence interval for the ML estimates of $\tilde{\theta}$, and the τ and rho statistics constructed under the null hypothesis include 95% of the values of $\tilde{\theta}^{(i)}$ and the associated values of τ and rho, respectively.

The random disturbance term, e, is not constructed as an $i.i.d.$ process. This is because that our MS model residuals exhibit clusters when plotted against time (figures 5 through 8). We model the MS residuals, e_t, with an ARCH(q) process of the form

$$\varepsilon_t = u_t \left[\alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \right]^{\frac{1}{2}},$$

where

$$u_t \xrightarrow{d} iid(0,1)$$

and

$$\begin{aligned} &E[\varepsilon_t | \varepsilon_{t-i}] = 0 \\ &Var[\varepsilon_t | \varepsilon_{t-i}] = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \qquad (6) \\ &Cov[\varepsilon_t, \varepsilon_{t-i}] = 0, \qquad for \quad i \geq 1 \end{aligned}$$

where q is selected by the usual F test. This model has the feature that disturbances are heteroscedastic but serially uncorrelated, as the covariance is zero between ε_t and ε_{t-i} for all $i \geq 1$ in this model. We can estimate this ARCH(q) model using the following procedure (Greene 1997; Bollerslev 1986)

1. Regress the squared ML residuals on their corresponding four lagged values to give the first estimates of α_i, denoted by a_i, $i = 0,1,...,4$.

2. Compute the conditional variances using $\hat{\sigma}_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + a_2\varepsilon_{t-2}^2 + a_3\varepsilon_{t-3}^2 + a_4\varepsilon_{t-4}^2$.

   Run the regression $\left(\frac{e_t^2}{\sigma_t^2} - 1\right) = d_0 \frac{1}{\sigma_t^2} + d_1 \frac{e_{t-1}^2}{\sigma_t^2} + d_2 \frac{e_{t-1}^2}{\sigma_t^2} + d_3 \frac{e_{t-1}^2}{\sigma_t^2} + d_4 \frac{e_{t-1}^2}{\sigma_t^2}$.

3. The asymptotically efficient estimator of α is given by $\hat{\alpha} = a + d$, where $\hat{\alpha}, a, d$ are all 5×1 vectors and the $Asy.Var(\alpha) = 2(Z'Z)$, where $Z : T \times 5$ and $z_t = [1 \quad e_{t-1}^2 \quad e_{t-2}^2 \quad e_{t-3}^2 \quad e_{t-4}^2]$.

The estimated parameters of the ARCH(q) model are then used to generate the disturbance term in the bootstrapping procedure.

4. Data

The data we use in this study are price and rent indices for building structures, as opposed to raw land, of Hong Kong and Seoul. There is no convincing reason to believe that a bubble is more likely to exist in the price of a building than in that of a plot of land. Empirical studies on speculative bubbles use data on both. In fact, following the argument of Homer Hoyt (1933), if a bubble exists in the land price, it is likely to be transmitted to the housing price, and vice versa.

We do have a few reasons for choosing data from Hong Kong and Seoul. During the 1980s and 1990s, the two cities experienced dramatic property-price swings (figure 1).

These swings were suspected by practitioners and academics to be the results of speculative bubbles. Although quality data on the property sector are available, there have been relatively few research papers devoted to the study of the speculative bubble in the property market of Hong Kong. More studies are available on property-price bubbles in South Korea, the methodologies used in these studies are, however, crude in general.

In Hong Kong, the data available for building structures are divided into four categories: domestic premises, office, retail premises, and flatted factories. Xiao and Tan (2006) show that each of these four sectors was plagued by a periodically collapsing speculative bubble during the period of interest, and the patterns of the bubble movements in these four sectors are very similar. Thus, we select arbitrarily Hong Kong office price index and its associated rent series in this study. We will use in this study the Seoul housing-price index and its associated rent series, as it is the only data available for building structures in Seoul. All data series come from the CEIC database, a comprehensive source of economic statistics for Asian economies[ix].

The series are monthly data deflated by their respective CPI[x]. Each series of Seoul has 210 observations running between January 1986 and June 2003. Each series of Hong Kong makes use of two data sets of different frequencies: the first set is a quarterly data between January 1984 and September 2000, and the second a monthly data from January 1993 to April 2003. In order to combine them, we have converted the first set into monthly data by means of cubic splining. Thus, the first half of each series of Hong Kong, running from January 1984 to December 1992, consists of the splined output from the first data set; the remaining half of the series is drawn from the second data set. The raw data have 232 observations for both price and rent series.

A plot of the data in figure 1 shows that the Hong Kong office price is highly volatile., The price index more than doubled in a mere 15 months between Dec. 1987 and March 1989. Another sharp increase of the price occurred in the first half of 1994, with an average value of 6.1% per month. The price crashed after July 1997, following the Asian financial crisis. By April 2003, the price index stood only at about 22% of its peak value (occurred in May 1994). Xiao and Tan (2006) show that a periodically-collapsing speculative bubble is responsible for this observed behavior, with the bubble peaked in May 1994 and again in June 1997. The Seoul housing price is less volatile (compared with the Hong Kong office price). After rising for four years between 1988 and 1992, it declined gradually throughout the remaining part of the 1990s. The price started clime up again in 2001.

Theories suggest that, in the absence of a bubble, a price and its associated rent should move more or less hand in hand. But this seems not the case in our data (figure 2). The price-rent ratio of Hong Kong office behaved very much like the price series. It increased continuously between 1990 and 1997, with a few temporary reversals. This ratio crashed to its historical low after the late 1997. On the other hand, the price-rent ratio of Seoul housing was on a declining trend throughout the sample period, with only a few brief episodes of reversals. Does the behavior of the price-rent ratio imply the existence of a speculative bubble in the prices of interests? This is the question the current paper is interested in.

5. Empirical Results

The empirical literature suggests that bubbles are short-lived phenomena, and estimations using a long sample may fail to capture a bubble if it does not last long enough (Rappoport and White 1994; Kim and Lee 2000). We take two approaches to resolve this problem in this study. One is to apply the linear unit-root tests to shorter samples (referred to as the shorter-sample approach), the other is to model the data generating process as Markov-switching AR(p) (referred to as the Markov-switching approach). The Markov-switching approach has the advantage that the change of a state is determined endogenously, rather than pre-imposed by the researchers, as is the case in the shorter-sample approach.

In taking the shorter-sample approach, for purpose of comparison we investigate the entire sample (the *long sample*), as well as sub-periods of each sample (the *short samples*), of each data series. The relevant sample periods used for Seoul are 86:1–03:6 (the long sample), 86:1–91:6, 91:7–98:6, and 98:7–03:6 (the short samples); those for Hong Kong are 84:1–03:4 (the long sample), 84:1–94:5, and 94:6–03:4 (the short samples). The selection of sub-samples is based on graphical and empirical evidences. Empirical studies show that property price bubbles may have occurred in Seoul during the late 1980s and since the late 1990s, and in Hong Kong during the early and mid-

1990s (Kim and Sub 1993; Park et al. 1998; Chung and Kim 2004; Chan, Lee, and Woo 2001, Xiao and Tan 2006). A breaking point is chosen at where a price shows a dramatic change in trend[xi].

An AR(p) model is fitted for each sample for the purpose of linear and Markov-switching ADF test. The AR(p) model we estimate for each sample has a constant term but no trend, that is, $y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p-1} \varsigma_i \Delta y_{t-i} + u_t$, as the samples show no signs of trending. The number of lags, p, is selected using method described in section 3.1. The selected AR(p) model for each sample is then subjected to a Durbin-Watson test. If a model fails the DW test at the 5% level, we add one more lag and, in all cases except one, the autocorrelation found among residuals is adequately removed. In the exceptional case, that is Hong Kong office prices between 84:1 and 94:5, adding one or more lags to the F-test selected model does not allow the residuals to pass the DW test at the 5% level. Hence we stick to the F-test selected model, as the null hypothesis of no autocorrelation can be accepted at the 1% level. The number of lags selected for each sample is listed in table 1.

In the shorter-sample approach, we conduct the linear ADF test as well as the linear PP test for each sample. When the long sample of Seoul (86:1–03:6) is investigated, all test statistics accept the null hypothesis of “no bubble” (a 5% significance level is used for all tests unless otherwise specified). Recall that a “no bubble” conclusion is reached if price and rent behave in a similar manner, that is, both are stationary, or have one unit root, or are explosive (table 2).

The same conclusion of "no bubble" is drawn for Seoul using the ADF τ and rho statistics in the sample period 86:1–91:6. However, the ADF F statistic for the price series rejects the joint hypothesis of zero constant and unit root in the left tail, while that for the rent series accepts the null hypothesis. Looking at the plot of the price and the rent in this sample period (figure 1) we realize that there was a wide gap between the price and the rent at the beginning of the sample. That gap narrowed down in the first half of this sample period, and stabilized in the remaining half. The PP test strongly suggests a positive price bubble might have occurred in this period[xii], with an explosive price path accompanied by a unit-root rent path. We adopt the conclusion drawn from the PP test, as this test takes care of the possible heteroscedasticity (table 2).

For the period between 91:7 and 98:6, both the ADF τ and rho statistics show a negative bubble might have occurred in the Seoul housing price, as both statistics tell us price had one unit root while rent was explosive. These results are reconfirmed by the PP test. But the ADF F statistic accepts the joint hypothesis of zero constant term and unit root for both price and rent (table 2). For the 98:7-03:6 sub-sample, both ADF τ and rho statistics show that a positive bubble possibly existed in the Seoul housing price, as price was explosive while rent had one unit root. Again, the PP test confirms these results. The ADF F statistic accepted the null in the price but rejected it in the rent (table 2). As the F statistic is difficult to interpret, we again follow the conclusion drawn by the PP test.

When the battery of linear ADF and PP tests is applied to Hong Kong office prices and rent indices between 84:1 and 03:4 (the long sample), all statistics, except PP-rho, accepted the null hypothesis of no bubble (table 3). The PP-rho suggests a negative bubble possibly occurred in this sample period. The PP-τ statistic only accepts the null marginally at 5% with a P-value equal to 0.948. All statistics (except the ADF F statistic, which accepted the null for both price and rent), rejected the null of no bubble for the sample period 84:1 to 94:5. One possible alternative is that a positive bubble existed in this period. For the sample period 94:6 to 03:4, both τ and rho statistics accepted the null. The F statistic, while accepting the joint hypothesis of “zero constant and unit root” for the price series, rejected it for the rent series in the right region. A plotting of the series shows that prices in this period moved from about 20% above the rent to about 10% below it (figure 1). We suspect that both positive and negative bubble might have occurred in this sample period, and the conclusion from τ and rho statistics could bear the effect of averaging.

In short, the linear ADF and PP tests accept the null hypothesis of “no bubble” for the Seoul housing price, when the whole sample period 86:1–03:6 is investigated. PP statistics strongly suggest that a positive bubble might exist in the period 86:1–91:6. Both ADF and PP tests suggest that there might be a negative bubble existing during 91:7–98:6 and a positive bubble during 98:7–03:6 (table 2).

For the Hong Kong office prices series, when the whole sample period 84:1–03:4 is investigated, the linear ADF test accepts the null of “no bubble”, and the PP statistics suggest the possible existence of a negative bubble. Both ADF and PP tests rejected the null of no bubble for the period 84:1–94:5, indicating a positive bubble might exist then, but accepted the null for the period 94:6–03:4 (table 3).

The linear unit-root tests suggest structural changes in our data sets (Seoul housing prices and rent 86:1 to 03:6, Hong Kong office prices and rent 84:1 to 03:4). These tests show that the Seoul housing price series followed a unit-root process in the period 91:7–98:6, an explosive process in the periods 86:1–91:6 and 98:7–03:6; and the rent series followed a unit-root process in the periods 86:1–91:6 and 98:7–03:6, but an explosive process in the period 91:7–98:6. The tests indicate that Hong Kong office price series followed an explosive process between 84:1 and 94:5, but a unit-root process between 94:6 and 03:4. For the rent, though both rho and τ statistics of ADF and PP tests suggest the series has one unit root in the first (84:1-94:5) and the second (94:6-03:4) sub-sample periods, the F statistic accepted the joint hypothesis of "zero constant and unit root" for first sub-sample, but rejected the null in the second sub-sample. These conclusions imply that a two-state Markov-switching AR(p) model may better represent the data generating processes.

In fitting a two-state Markov-switching model to the Seoul housing prices and rent, we scanned for a proper starting point in the data set, so that the resulting smoothed probabilities show reasonable fluctuations.[xiii] The starting point finally chosen for Seoul housing was 90:7. Therefore, the selected data set (90:7–03:6) roughly correspond to the second and third sub-sample period (91:7–98:6 and 98:7–03:6). The linear ADF and PP tests accept the null of no bubble for this new data set (table 2). The long sample for

Hong Kong office sector is adopted, for reasonable movements in smoothed probabilities are obtained with this sample[xiv].

The actual observed and the fitted values using the MS model are shown in figures 3 and 4 (all values are in first differences). We list the maximum likelihood estimates of the MS model parameters in tables 4 to 7, alongside their Hessian and White's t-ratios[xv]. Notice that the White's t-ratios are much less significant than their Hessian counterparts.

Both symmetry and Wald (encompassing) specification tests show that the two-state Markov-switching model is accepted for all data sets under consideration (table 9). The alternative is a linear model. Table 8 records the state transition probabilities. These transition probabilities show that whenever the Seoul housing price reaches the state 2, it will switch back to state 1 with certainty, as p21=1; state 1 is persistent in Seoul housing rent, because p11=0.99; both states are likely to appear in Hong Kong office price, given the values of the switching probabilities p12 and p21; the rent series of Hong Kong office is more likely to switch to and stay in state 2, as p12 is as high as 0.826, and p22=0.853. These observations are more or less confirmed by the unconditional state probabilities $\pi_i, i = 1,2$ and the second eigenvalues of the state transition matrix, λ_2. λ_2 shows that the states of the two prices have negatively serial correlation. That is state 1 is likely to be followed by state 2, and vise versa, while the states of the other two time series are positively serially correlated. With positive autocorrelation, a state is likely to persist once the data generating process enters that state.

A plot of the smoothed probabilities of the states in figure 9 echoes the predictions of state transition probabilities and those of the eigenvalues of the transition matrix. There are frequent switches of states in Seoul housing price. On the other hand, state one of its corresponding rent series is highly persistent. There are also frequent switches of states in Hong Kong office price, but state two is more persistent in its associated rent series.

Combine the above observations with the result of the Markov-switching ADF test (table 14), we can conclude that a bubble, if exists in the price of interests, is periodically collapsing. Furthermore, the Markov chain we estimated is ergodic, because one eigenvalue of the state transition matrix is unity and the other is inside the unit circle. Hence, the long term forecast of the Markov chain is given by the unconditional state probabilities (Hamilton 1994, p 682).

The parameter estimates of the ARCH model for the MS model residuals (along with their t-ratios) are summarized in table 10 and 11. Not all t-ratios of the estimated parameters are significant, but the F test shows that the selected models are jointly significant at the 5% level (table 12). The test statistics in table 13 for ARCH effect, TR^2, are highly significant in all cases (R^2 is the goodness of fit measure of the regression, T the sample size). We plot the squared ML residuals and their corresponding predicted values using the estimates of ARCH model in figures 5 through 8. These plots demonstrate that the estimated models capture quite well the patterns of the MS residuals.

Thus, we incorporate ARCH disturbances into the bootstrapping procedure in generating the distribution of τ and rho statistics for the Markov-switching ADF test. For each series, 10,000 replications are used. We then compare the τ and the rho statistics of the MS models with these simulated distributions, and test the null hypothesis of "unit root" (table 14).

A problem arises when we try to draw conclusions. The two statistics, τ and rho, suggest different behavior of the series on three occasions: Seoul housing prices and rent in state two, and Hong Kong Office price in state two (table 14). Therefore, if τ is used, the conclusion would be that, for most of the period between July 1990 and the end of 1992, Seoul housing prices might contained a positive speculative bubble. This bubble disappeared but reappeared in late 2001 (table 15, figure 9). This conclusion is consistent with the observations from the plots in figures 1, and also partly compatible with the results from the linear unit-root tests (table 2). The τ statistic identified four possible bubble episodes in Hong Kong office prices: late 1987 to late 1989, early 1994, late 1997 to late 1998, and early 2001 (table 15, figure 9). These are largely consistent with the facts shown in the plots of figures 1, and with the conclusions of Chan et al. (2001) and Xiao and Tan (2006).

If rho is used, then the conclusion is that Seoul housing prices might contained a positive bubble throughout most of the period between July 1990 and the end of 1992. This possible-bubble disappeared thereafter until late 1997, when it reasserted itself for a brief period (about a year), and revived again in late 2001. These accounts are mostly compatible with the conclusion of τ , except for the period involving late 1997 and early 1998. But the rho side of the story about Hong Kong is less interesting. It says that the city might have experienced a positive bubble throughout the entire sample period (table 15).

6. Conclusions

A Speculative bubble is likely to occur in an asset, such as a real estate, whose fundamental value is difficult to assess. The real-estate market is more prone to speculative bubbles than other types of asset market for reasons such as restrictions in supply and institutional arrangements (Xiao and Tan 2006).

Identifying a speculative bubble is nevertheless a thorny issue, mainly because of the unobservable market fundamentals. Hamilton and Whiteman (1985) and Diba and Grossman (1988) suggest that stationarity tests may be used to detect an explosive rational speculative bubble. These tests need not preclude the influences of unobserved market fundamentals. If the first-difference of a dividend and those of unobservable fundamentals are stationary in mean, and if no rational bubble exists, then the first difference of the associated stock price must be stationary. Differencing a stock price a finite number of times would not render it stationary, however, if it contains a rational bubble. Due to the possible presence of unobserved variables, finding that the first-difference of a stock price is not stationary does not automatically establish the existence of a rational bubble. However, the converse inference is possible. That is, evidence that the first-difference of a stock price has a stationary mean would be evidence against the existence of a rational bubble in that price.

The empirical literature suggests that bubbles are short-lived phenomena, and statistical tests using a long sample may fail to capture a bubble if it does not last long enough (Evans 1991; Rappoport and White 1994; Kim and Lee 2000). In this paper, we take two approached to deal with this problem. One is to divide a long sample into shorter samples, and apply the linear Phillips-Perron and the augmented Dickey-Fuller tests to these samples (referred to as the shorter sample approach); the other is to deploy a Markov-switching AR(p) model in the ADF test (referred to as the Markov-switching approach).

The Markov-switching approach is more appealing than the shorter sample approach ex ante. This is because that the switching (breaking) points are determined endogenously, rather than pre-imposed by the researchers. Nevertheless, the two approaches in our study give compatible results ex post, with the Markov-switching approach more precise in timing a bubble, if it exists. By breaking Seoul data set (1986:1-2003:6) into three sub-samples, we identified with the linear ADF and PP tests that a positive bubble possibly existed between 1986:1 and 1991:6, and between 1998:7 and 2003:6, and a negative bubble possibly occurred between 1991:7 and 1998:6[xvi]. The Markov-switching ADF test does not point to the existence of a negative bubble in Seoul housing price. It nevertheless identified three episodes in which a positive bubble might be active: from July 1990 to the end of 1992, between the late 1997 and the late 1998, and between the late 2001 and the early 2003.

For the purpose of linear unit-root tests, we break the Hong Kong data set into two sub-samples. The linear unit-root tests rejected the null of no bubble. Therefore, a positive bubble might have occurred for the first sub-sample (1984:1-1994:5). We accepted the null in the second sub-sample (1994:6-2003:4). The Markov-switching ADF test identified four possible bubble incidences between 1984:1 and 2003:4. They occurred between the late 1987 and the late 1989, in the early 1994, between the late 1997 and the late 1998, and again in the early 2001. We suspect, however, the last one might be a false alarm, given the behavior of the price and the rent in this period (figure 1). Our findings are consistent with those in Chan et al. (2001) and in Xiao and Tan (2006).

Table 1. **Number of Lags Selected in an AR(p) Model**

	Seoul Housing Sector					Hong Kong Office Sector		
Sample periods	**86:1 to 03:6**	**90:7 to 03:6**	**86:1 to 91:6**	**91:7 to 98:6**	**98:7 to 03:6**	**84:1 to 03:4**	**84:1 to 94:5**	**94:6 to 03:4**
Price	5	5	1	1	1	8	15	0
Rent	5	5	1	2	2	3	1	2

Table 2. Linear ADF and PP Unit Root Tests for Seoul Housing Price and Rent

1. Phillips Perron Test is adopted in drawing conclusions when conflicting results arise between PP and ADF tests and PP strongly suggest H0 or H1, as PP takes care of the Heteroscedasticity existing in our data. 2. "*"means significant in the left tail. 3. "**" means significant in the right tail.4: in all cases, the Durbin-Watson statistic accept H0: no serial correlation among residuals.

		1986:1 to 2003:6		1990:7 to 2003:6	
		Price	**Rent**	**price**	**rent**
ADF Tests	**DW**	2.05	2.02	1.86	1.96
H0: unit root	**Rho**	-0.34	-2.05	-6.134	-11.902
H1:not unit root	**(Pr<Rho)**	(0.614)	(0.078)	(0.331)	(0.080)
(5% level)					
	τ	-1.30	-2.35	-2.83	-2.16
	(Pr< τ **)**	(0.633)	(0.157)	(0.057)	(0.220)
	F	0.86	2.91	4.31	2.38
	(Pr>F)	(0.851)	(0.330)	(0.070)	(0.465)
PP Tests	**Rho**	-1.25	-4.82	-2.35	-2.31
H0: unit root	**(Pr<Rho)**	(0.861)	(0.450)	(0.734)	(0.740)
H1:not unit root	τ	-0.97	-1.73	-2.42	-1.07
(5% level)	**(Pr<** τ **)**	(0.765)	(0.414)	(0.137)	(0.727)

		1986:1 to1991:6		1991:7 to 1998:6		1998:7 to 2003:6	
		price	**rent**	**price**	**rent**	**price**	**rent**
ADF Tests	**DW**	2.04	1.83	1.95	1.88	1.79	2.01
H0: unit root	**Rho**	-0.59	-3.50	-1.98	7.27**	0.68**	-1.36
H1: not unit root	**(Pr<Rho)**	(0.918)	(0.587)	(0.777)	(0.999)	(0.980)	(0.847)
(5% level)							
	τ	-0.29	-1.46	-1.43	1.66**	0.41**	-1.65
	(Pr< τ **)**	(0.920)	(0.549)	(0.565)	(0.999)	(0.982)	(0.452)
	F	0.47*	1.52	4.22	2.69	1.57	4.84*
	(Pr>F)	(0.957)	(0.687)	(0.079)	(0.395)	(0.675)	(0.046)
PP Tests	**Rho**	0.23**	-2.34	-1.75	8.87**	1.30**	-1.22
H0: unit root	**(Pr<Rho)**	(0.966)	(0.732)	(0.805)	(0.999)	(0.992)	(0.861)
H1:not unit root	τ	0.14**	-1.25	-2.13	3.86 **	1.37**	-1.62
(5% level)	**(Pr<** τ **)**	(0.967)	(0.647)	(0.234)	(0.999)	(0.999)	(0.468)

Table 3. Linear ADF and PP Unit Root Tests for Hong Kong Office Price and Rent

1. Phillips Perron Test is adopted in drawing conclusions when conflicting results arise between PP and ADF tests and PP strongly suggest H0 or H1, as PP takes care of the Heteroscedasticity existing in our data. 2. "*"means significant in the left tail. 3. "**" means significant in the right tail.4: in all cases, the Durbin-Watson statistic accept H0: no serial correlation among residuals.

		1984:1 to 2003:4		1984:1 to1994:5		1994:6 to 2003:4	
		Price	**Rent**	**price**	**rent**	**price**	**rent**
ADF Tests	**DW**	1.96	2.00	1.97	2.36	2.29	2.07
H0: unit root	**Rho**	-5.67	-6.79	1.37**	-5.53	-2.19	-3.29
H1:not unit	**(Pr<Rho)**	(0.371)	(0.285)	(0.993)	(0.380)	(0.752)	(0.616)
root	τ	-1.60	-1.67	0.61**	-1.89	-1.70	-2.52
(5% level)	**(Pr<** τ **)**	(0.480)	(0.443)	(0.990)	(0.336)	(0.430)	(0.115)
	F	1.29	1.41	1.58	2.19	4.45	5.03**
	(Pr>F)	(0.743)	(0.711)	(0.669)	(0.513)	(0.061)	(0.038)
PP Tests	**Rho**	-1.61	-0.12**	2.49 **	-0.99	-2.06	-2.04
H0: unit root	**(Pr<Rho)**	(0.823)	(0.951)	(0.999)	(0.885)	(0.768)	(0.771)
H1:not unit	τ	-0.87	-0.09	2.06 **	-0.88	-1.74	-3.35
root	**(Pr<** τ **)**	(0.797)	(0.948)	(0.999)	(0.791)	(0.408)	(0.015)
(5% level)							

Table 4. Markov Switching Model Parameter Estimates for *Seoul Housing Price*

1. inter j: ML estimate of the constant term in state j; 2. lpj: ML estimate of coefficient on y_{t-1} for state j; 3. ldpij: ML estimate of coefficient on Δy_{t-i} in state j. 4. t (H): t ratio obtained from Hessian; 5. t (W): t ratio obtained from White covariance; 6. sig j: variance estimate in state j; 7. Pi j: unconditional state probability in state j. These notations are applicable throughout this chapter.

	State one			State two			
	Parameter	t(H)	t(W)		Parameter	t(H)	t(W)
inter 1	1.155	11.326	0.006	inter 2	0.020	0.100	0.000
lp1	-0.012	-12.350	-0.006	lp2	0.000	0.082	0.000
ldp11	0.588	7.952	0.052	ldp12	0.917	5.340	0.025
ldp21	-0.164	-1.896	-0.009	ldp22	-0.147	-1.072	-0.004
ldp31	-0.074	-1.035	-0.006	ldp32	0.176	0.976	0.004
ldp41	0.072	0.898	0.006	ldp42	-0.255	-1.669	-0.010
ldp51	0.142	2.020	0.015	ldp52	0.462	3.260	0.030
Sig1	0.898	1.944		Sig2	1.384	1.754	
Pi1	0.760	0.010		Pi2	0.240	0.015	

Table 5. Markov Switching Model Parameter Estimates for *Seoul Housing Rent*

	State one				State two		
	Parameter	t (H)	t (W)		Parameter	t (H)	t (W)
inter 1	2.520	24.578	0.004	inter 2	-8.254	-5.131	0.000
lp1	-0.024	-23.802	-0.004	lp2	0.070	4.039	0.000
ldp11	0.620	8.916	0.061	ldp12	0.419	0.899	0.001
ldp21	-0.121	-1.913	-0.008	ldp22	0.242	0.398	0.001
ldp31	-0.167	-2.721	-0.015	ldp32	-0.279	-0.390	-0.001
ldp41	0.009	0.154	0.001	ldp42	0.480	0.492	0.001
ldp51	0.210	4.292	0.026	ldp52	0.208	0.073	0.000
Sig1	1.519	4454.727		Sig2	13.489	1.538	
Pi1	0.996	99.629		Pi2	0.004	0.004	

Table 6. Markov Switching Model Parameter Estimates for *Hong Kong Office Price*

	State one				State two		
	PARM	**t(H)**	**t(W)**		**PARM**	**t(H)**	**t(W)**
inter 1	0.405	0.636	0.004	**inter 2**	0.612	1.692	0.002
lp1	0.000	0.001	0.000	**lp2**	-0.008	-1.525	-0.002
ldp11	0.059	0.245	0.033	**ldp12**	0.112	0.771	0.034
ldp21	-0.173	-0.813	-0.064	**ldp22**	0.390	3.769	0.124
ldp31	0.124	0.568	0.043	**ldp32**	-0.013	-0.118	-0.004
ldp41	0.410	1.835	0.169	**ldp42**	0.040	0.354	0.012
ldp51	0.187	1.063	0.059	**ldp52**	-0.055	-0.496	-0.016
ldp61	0.148	0.715	0.046	**ldp62**	0.015	0.146	0.004
ldp71	0.120	0.546	0.048	**ldp72**	0.072	0.515	0.020
ldp81	0.134	0.699	0.050	**ldp82**	-0.253	-2.273	-0.040
Sig1	25.441	35096.060		**Sig2**	14.263	39909.788	
Pi1	0.722	6698.769		**Pi2**	0.278	3301.492	

Table 7. Markov Switching Model Parameter Estimates for *Hong Kong Office Rent*

	State one				State two		
	Parameter	**t (H)**	**t (W)**		**Parameter**	**t (H)**	**t (W)**
inter 1	0.85	3.35	0.01	**inter 2**	0.14	1.87	0.00
lp1	-0.01	-4.43	-0.01	**lp2**	0.00	-1.09	0.00
ldp11	-0.04	-0.34	0.00	**ldp12**	0.77	16.08	0.10
ldp21	0.55	4.65	0.03	**ldp22**	0.32	6.62	0.03
ldp31	0.07	0.65	0.00	**ldp32**	-0.10	-2.02	-0.01
sig1	1.45	2.30		**sig2**	0.90	3.95	
pi1	0.10	0.09		**pi2**	0.90	0.96	

Table 8. ***State Transition Probabilities***

1. pij is the probability of state j in time t+1, given the time t state is i. 2. $\pi_i, i = 1,2$ is the unconditional probability of state i. 3. $\lambda_i, i = 1,2$ is the eigenvalue of the transition matrix. 4.the average duration of state i (i=1,2) is given by $(1 - p_{ii})^{-1}$ (Raymond and Rich, 1997, page 202).

	Seoul Housing Sector				Hong Kong Office Sector			
	Price		Rent		Price		Rent	
	Parameter	t (H)	Parameter	t (H)	Parameter	t (H)	Parameter	t (H)
p11	0.510	2.719	0.990	1190.391	0.264	139.900	0.174	0.890
p12	0.490	6.250	0.010	171.662	0.736	695.086	0.826	3.610
p21	1.000	6.231	0.267	104.334	0.500	692.805	0.147	3.610
p22	0.000	0.000	0.733	122.948	0.500	201.223	0.853	9.409
π_1	0.671		0.964		0.405		0.151	
π_2	0.329		0.036		0.595		0.849	
λ_1	1		1		1		1	
λ_2	-0.49		0.723		-0.236		0.027	
ave. duration state 1 (months)	2.040		96.421		1.358		1.210	
ave. duration state 2 (months)	1.000		3.740		2.001		6.783	

Testing Significance of Markov Switching Model

Table 9. Testing for the Significance of the Markov Switching Model

(i) The symmetry test statistic has standard normal distribution. (ii) The Wald statistic is generated using bootstrapping with 10,000 replications. Details are given in section II. E.

		Seoul Housing Sector		Hong Kong Office Sector	
		Price	**Rent**	**Price**	**Rent**
Symmetry test: H0:p11=p22 (linear model) H1:p11<p22 (MS model)	**Statistic**	2.719	309.155	-125.435	-3.476
	(Prob(Z $\leq$ z))	(0.996)	(1.000)	(0.000)	(0.000)
	Conclusion	reject H0 in favor of H1: MS model	reject H0 in favor of H1: MS model	reject H0 in favor of H1: MS model	reject H0 in favor of H1: MS model
Wald Test H0: MS model H1: linear model	**Wald statistic**	1.098	0.361	0.004	0.637
	(Prob $\left(\chi_n^2(1)\right) \leq c$)	(0.750)	(0.500)	(0.050)	(0.750)
	Conclusion	accept H0 MS model	accept H0 MS model	accept H0 MS model	accept H0 MS model

Table 10. Parameter Estimates of ARCH Model for MS Residuals of Seoul Housing Price and Rent

		State one		State two	
		Parameter	t ratio	Parameter	t ratio
Price	a0	0.387	2.528	0.458	2.792
	a1	0.198	2.444	-0.044	-0.729
	a2	0.003	0.038	0.092	1.527
	a3	0.216	2.961	0.023	0.377
	a4	0.072	0.985	0.117	1.914
	a5	-0.040	-0.544	0.290	5.047
Rent	a0	0.493	3.809	1.032	7.585
	a1	0.420	13.277	0.605	39.319
	a2	0.097	2.913	0.057	3.260
	a3	0.094	2.818	0.228	13.128
	a4	0.030	0.970	-0.025	-1.644

Table 11. Parameter Estimates of ARCH Model for MS Residuals of Hong Kong Office Price and Rent

		State one		State two	
		Parameter	t ratio	Parameter	t ratio
Price	a0	6.120	54.945	2.338	20.714
	a1	0.227	110.743	0.302	95.732
	a2	0.099	50.328	0.252	81.858
	a3	0.409	208.776	0.407	132.345
	a4	-0.128	-62.532	-0.005	-1.601
Rent	a0	0.337	3.212	0.461	3.836
	a1	0.449	11.369	0.174	3.349
	a2	0.199	4.154	0.138	2.642
	a3	-0.089	-1.850	-0.070	-1.347
	a4	0.119	3.004	0.230	4.432

Table 12. F Test for Joint Significance of ARCH Model of MS Residuals

All test statistics are significant at 5% level, showing the models are accepted.

		Seoul Housing Sector		Hong Kong Office Sector	
		Price	Rent	Price	Rent
State one	F	3.264	7.947	8.793	40.327
	df	(5, 150)	(4, 151)	(4, 218)	(4, 223)
State two	F	4.508	36.267	15.048	4.626
	df	(5, 150)	(4, 151)	(4, 218)	(4, 223)

Table 13. TR^2 Test For ARCH Effect in MS Residuals

The test statistic TR^2 has $\chi^2(q)$ distribution. All statistics are significant at 5% level, indicating the ARCH effect is indeed present in the MS residuals.

	Seoul Housing Sector		Hong Kong Office Sector	
	Price $(q=5)$	Rent $(q=4)$	Price $(q=4)$	Rent $(q=4)$
State one	10.688	20.611	29.000	120.480
State two	16.085	67.476	25.809	12.998

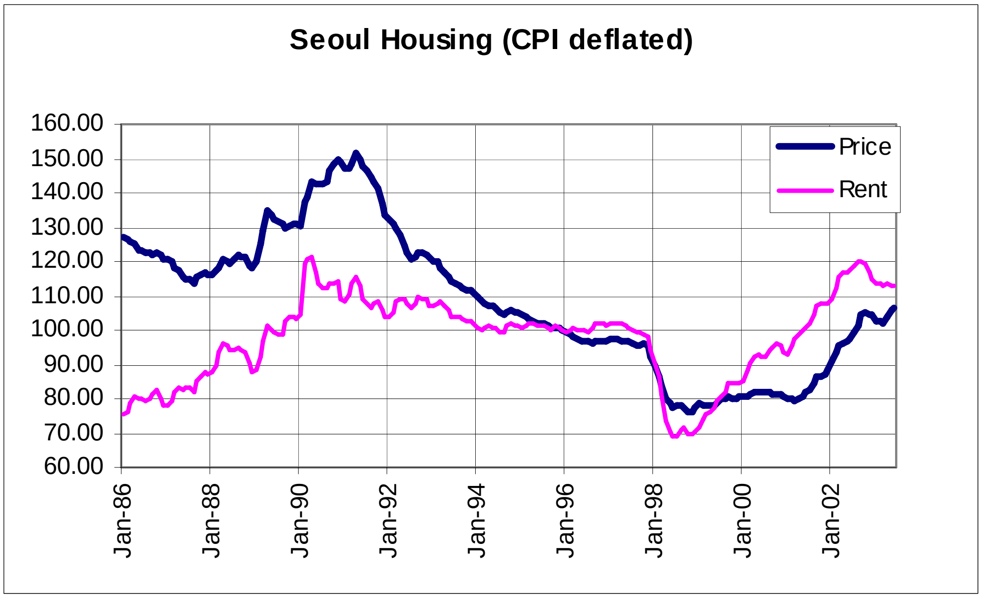
Table 14. ADF Unit Root Tests with Markov Switching

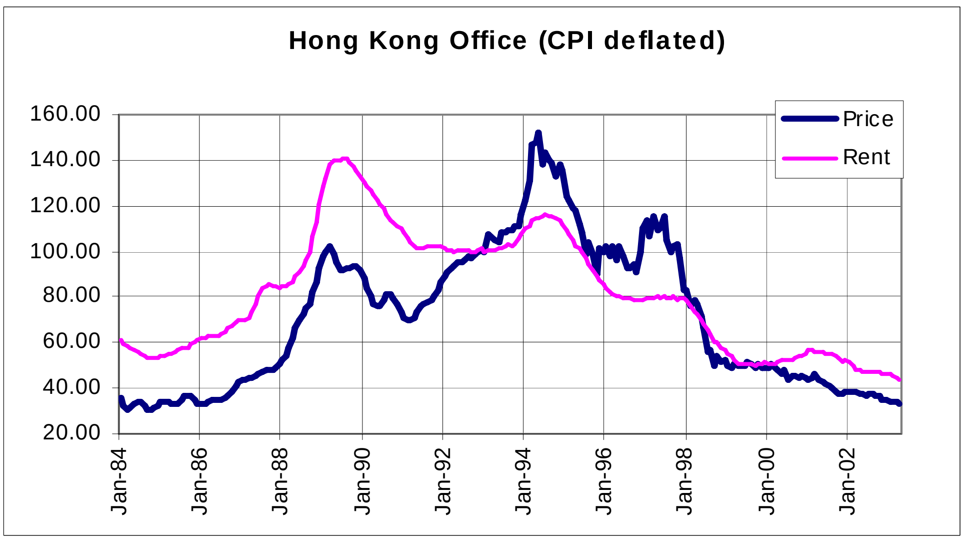
1. Probabilities are generated using bootstrapping with 10,000 replications. 2.”*” means significant in the left tail; “**” means significant in the right tail. 5% levels are used for all.

		Seoul Housing Sector				Hong Kong Office Sector			
		state one		state two		state one		state two	
		Price	Rent	Price	Rent	Price	Rent	Price	Rent
H0: unit root H1: H0 not true	Rho (Pr<rho)	-4.405* (0.000)	-8.428* (0.000)	-0.160 (0.723)	-155.166* (0.000)	-0.274 (0.184)	-7.285* (0.000)	-2.507 (0.257)	-18.928* (0.021)
	conclusion	reject H0 in favor of stationarity	reject H0 in favor of stationarity	accept H0	reject H0 in favor of stationarity	accept H0	reject H0 in favor of stationarity	accept H0	reject H0 in favor of stationarity
H0: unit root H1: H0 not true	τ (Pr< τ)	-0.006* (0.000)	-0.004* (0.000)	0.000** (0.998)	0.000** (1.000)	0.000 (0.333)	-0.008* (0.000)	-0.002 (0.510)	-0.002 (0.194)
	conclusion	reject H0 in favor of stationarity	reject H0 in favor of stationarity	reject H0 in favor of explosive	reject H0 in favor of explosive	accept H0	reject H0 in favor of stationarity	accept H0	accept H0

Table 15. MS ADF Tests Identified Periods Possibly with Positive Bubbles (Sorted by Statistics)

	Rho	τ
Seoul Housing Price	1. July 90 to end 92 2. late 97 to late 98 3. since late 01	1. July 90 to end 92 2. since late 01
Hong Kong Office Price	Through out the sample period	1. late 87 to late 89 2. early 94 3. late 97 to late 98 4. early 01





Source: CEIC database

Figure 1. CPI Deflated Price and Rent Indices

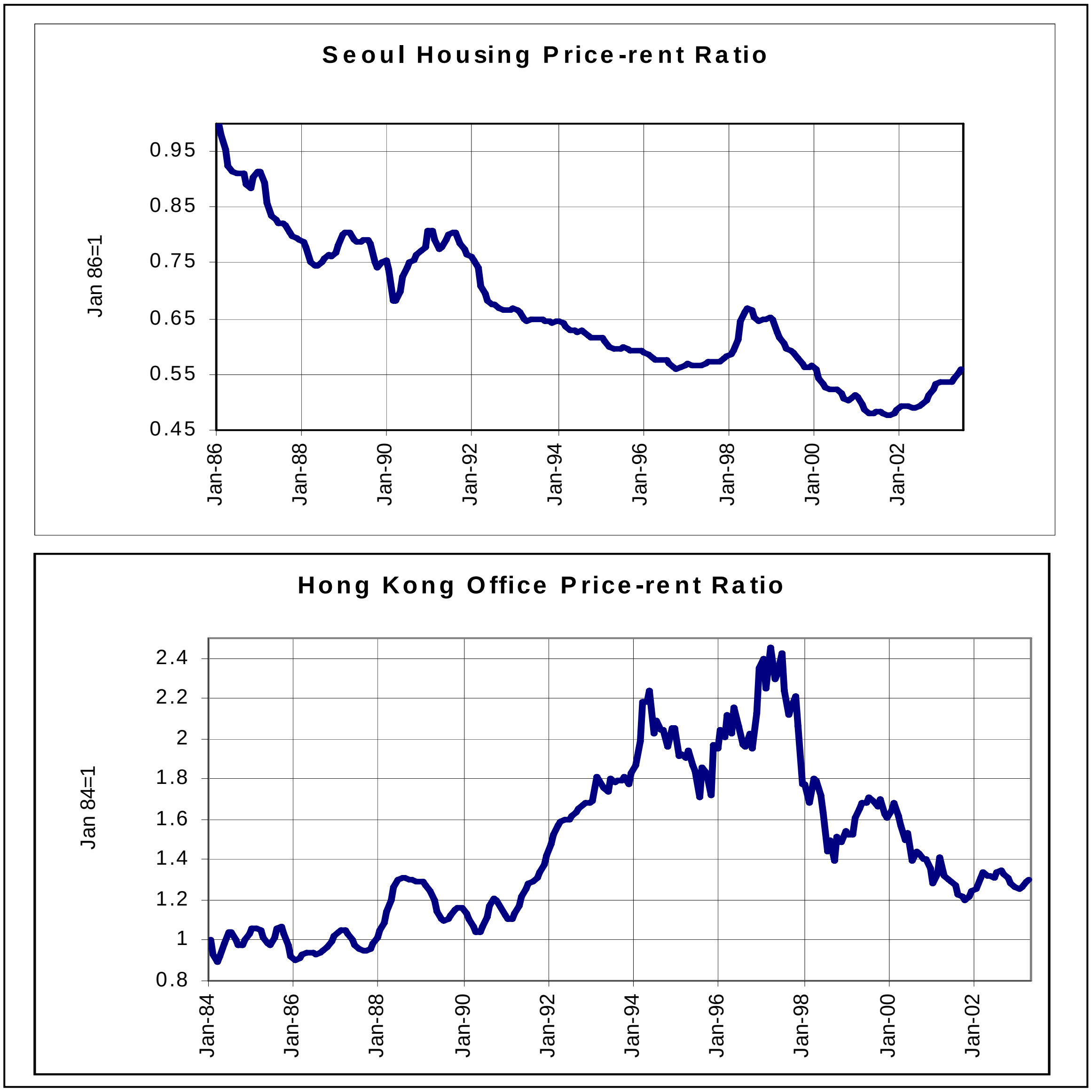


Figure 2. Price-rent Ratio

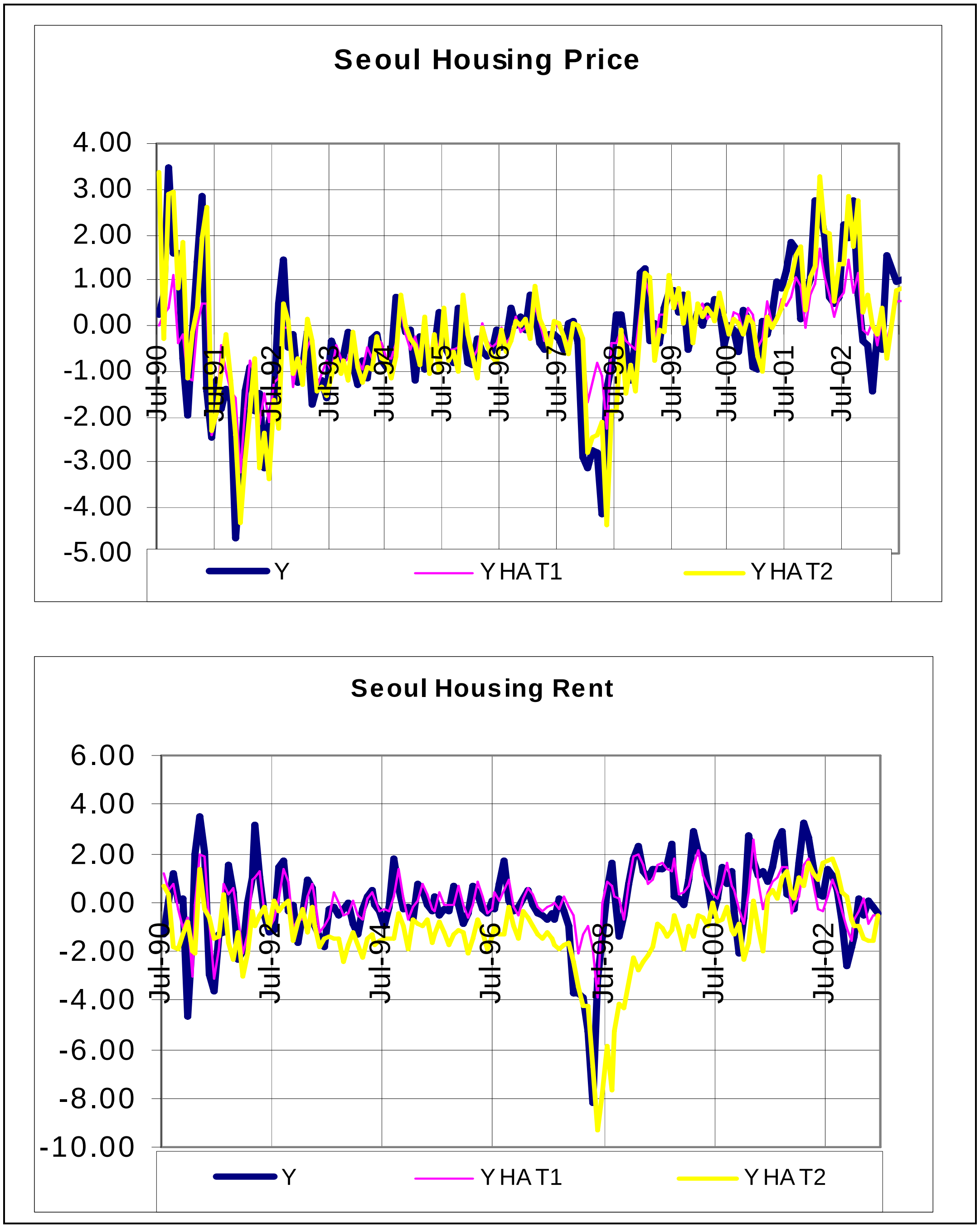


Figure 3 Fitted Values with Markov Switching Model (Seoul Housing Price and Rent)

Y: actual observed value; Yhat1: fitted value using estimates of state one parameters of MS model; Yhat2: fitted value using estimates of state two parameters of MS model. All values are in their first difference.

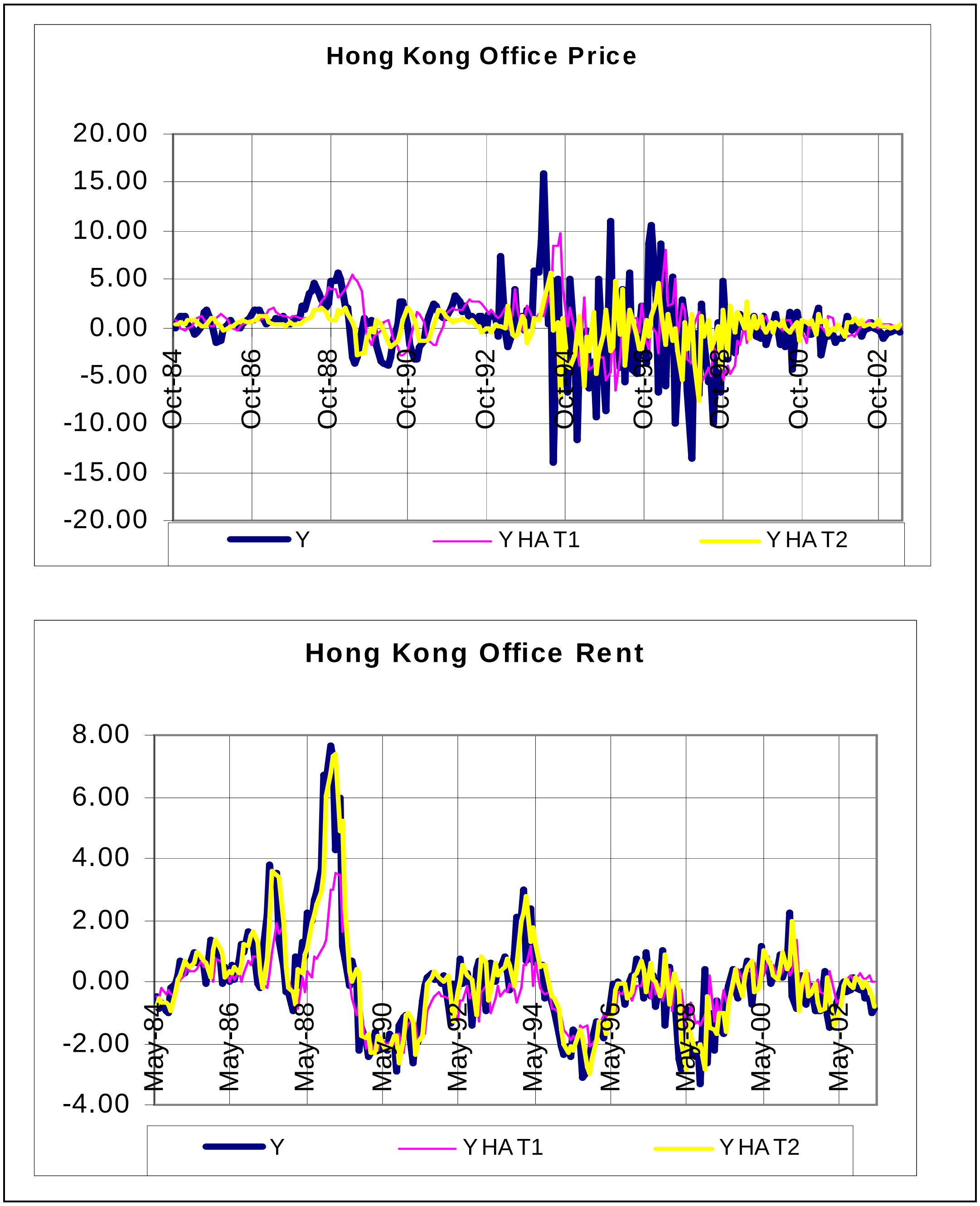


Figure 4 Fitted Values with Markov Switching Model (Hong Kong Office Price and Rent)

Y: actual observed value; Yhat1: fitted value using estimates of state one parameters of MS model; Yhat2: fitted value using estimates of state two parameters of MS model. All values are in their first difference.

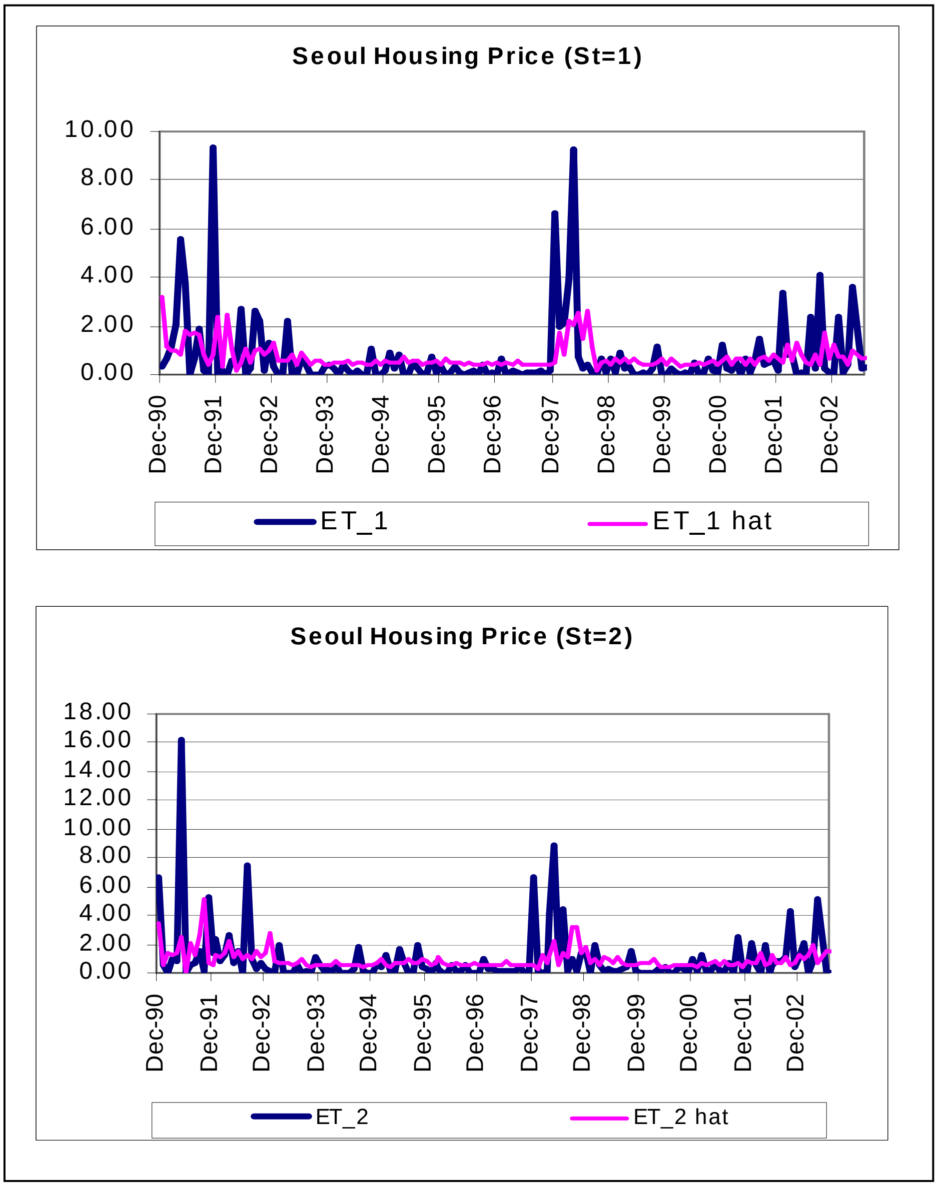


Figure 5. ARCH Modeling of MS Residuals (Seoul Housing Price)

St=j: state j (j=1,2); Et_j: square of state j (j=1,2) MS residuals; Et_j hat: fitted value of square of MS residuals using ARCH model. These notations are applicable for Figure 4 through 7. Plots in Figure 4 through 7 show that the ARCH model is a reasonable description of the actual process of the MS residuals.

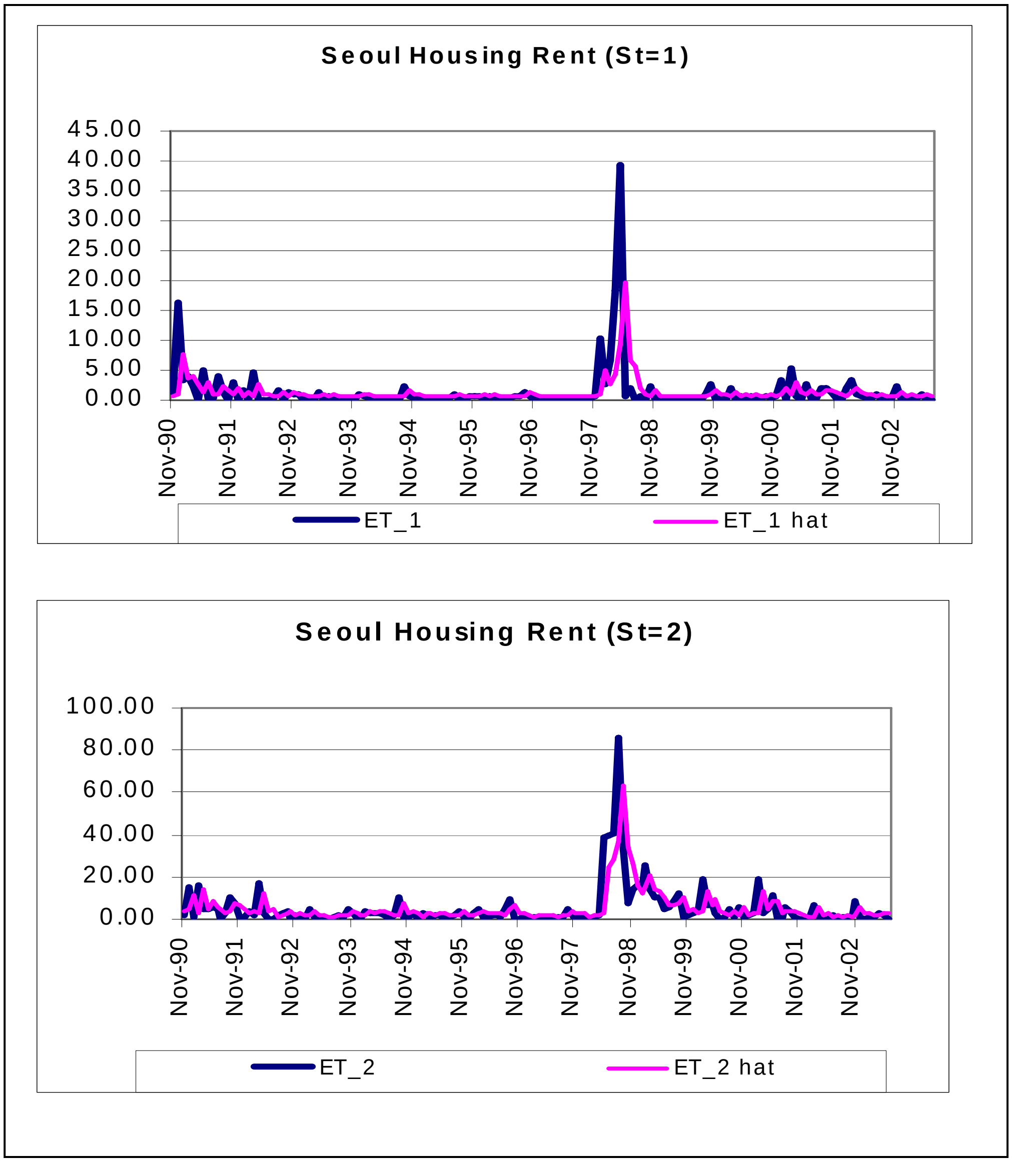


Figure 6. ARCH Modeling of MS Residuals *(Seoul Housing Rent)*

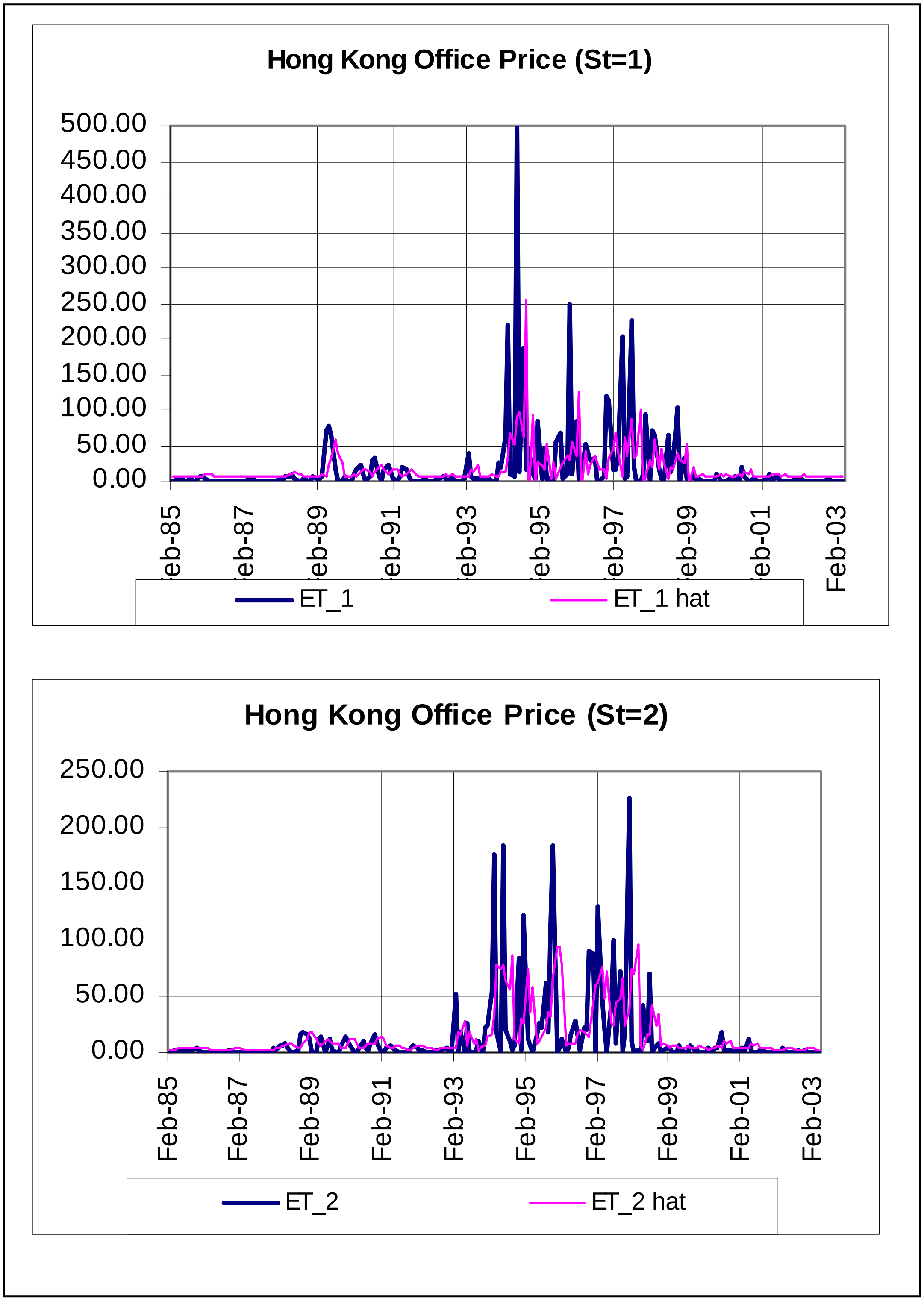


Figure 7. ARCH Modeling of MS Residuals (Hong Kong Office Price)

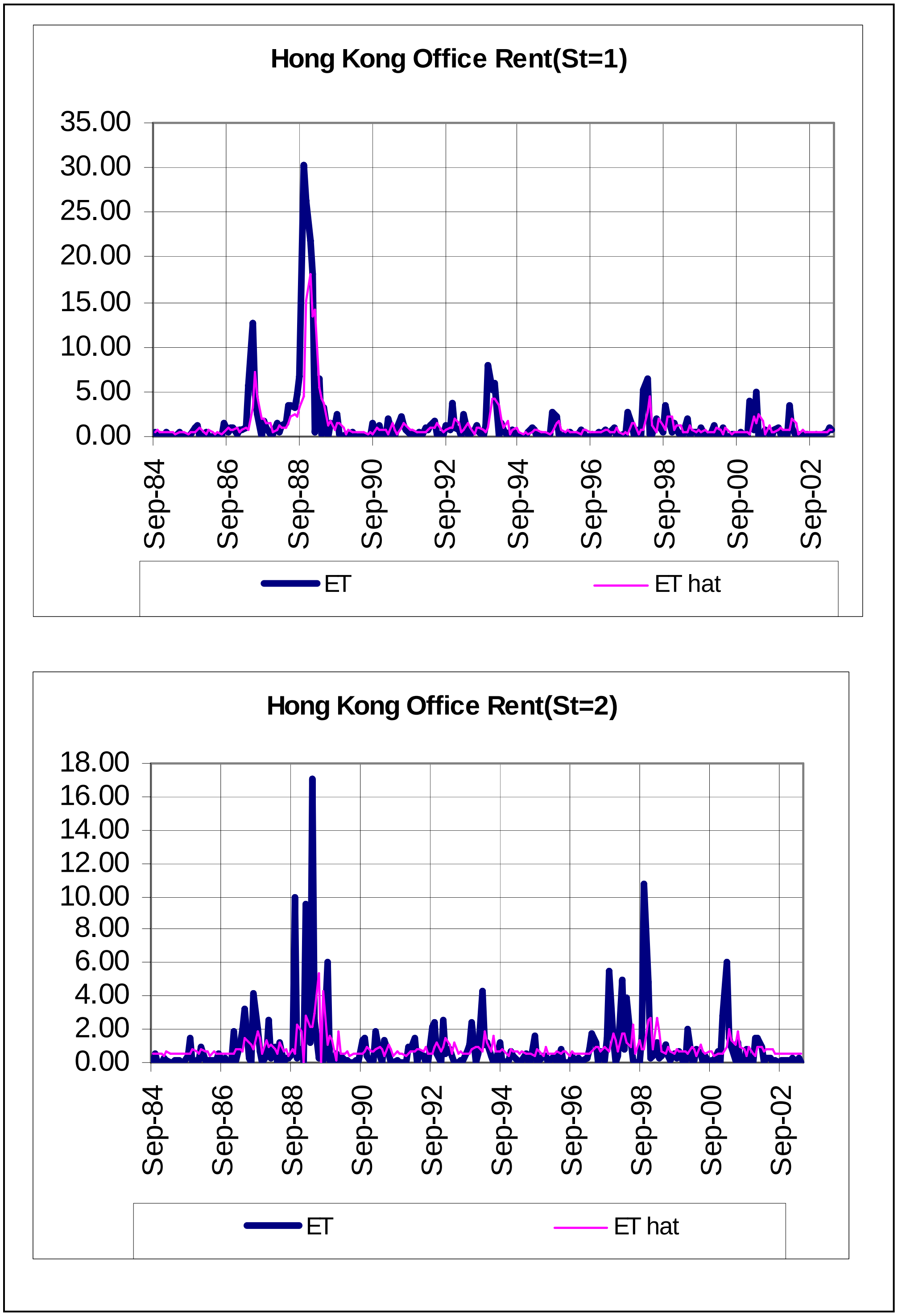


Figure 8. ARCH Modeling of MS Residuals (Hong Office Rent)

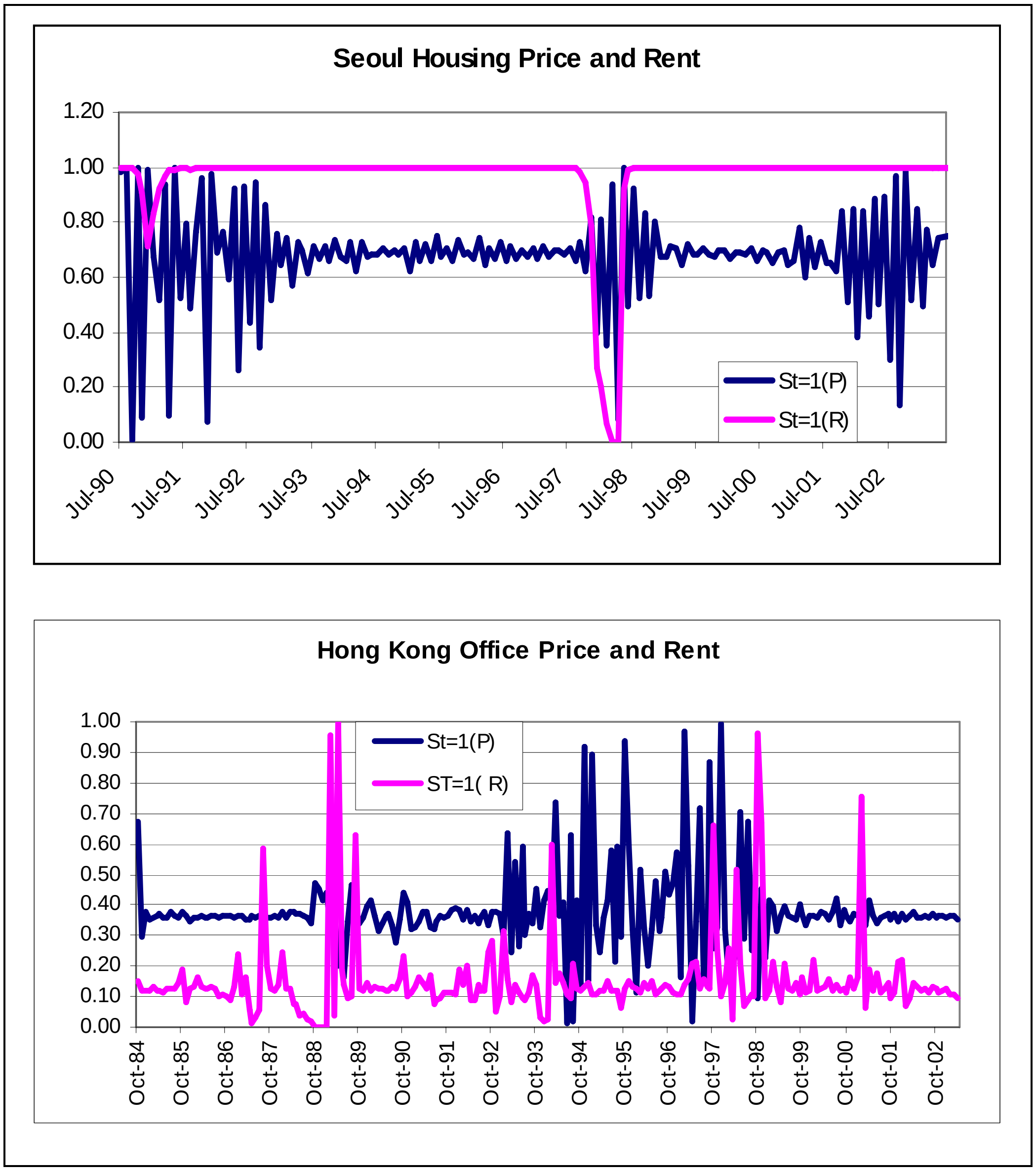


Figure 9 ADF Unit Root Tests for Rational Speculative Bubble with Markov Switching

St=j(P) denote smoothed probabilities of price in state j, j=1,2. The plots in this figure show that Seoul Housing Price contained positive rational speculative bubble between July 1990 and the end of 1992, between late 1997 and late 1998, and since late 2001; Hong Kong Office Price contained positive rational speculative bubble between late 1987 and late 1989, in early 1994, between late 1997 and late 1998, and in early 2001.

References:

1. Bhargava, A. (1986). "On the theory of testing for unit roots in observed time series." The Review of Economic Studie **53**(3): 369-384.

2. Bollerslev, T. (1986). "Generalized autoregressive conditional heteroskedasticity." Journal of Econometrics **31**: 307-327.

3. Breunig, R., S. Najarian, et al. (2003). "Specification testing of Markov switching models." Oxford Bulletin of Economics and Statistics **65**(Supplement 0305-9049).

4. Caines, P. E. (1988). Linear stochastic systems,. New York, Willey.

5. Campbell, J. Y. and S. R. J. (1987). "Cointegration and tests of present value models,." The Journal of Political economy **95**(5): 1062-1088.

6. Cavaliere, G. (2003). "Asymptotics for unit root tests under Markov regime-switching." Econometrics Journal **6**: 193-216.

7. Cecchetti, S., P. S. Lam, et al. (1990). "Mean reversion in equilibrium asset prices." The American Economic Review **80**(3): 398-418.

8. Chan, H. L., S. K. Lee, et al. (2001). "Detecting rational bubbles in the residential housing markets in Hong Kong." Economic Modeling **18**: 61-73.

9. Chung, H. S. and J. H. Kim (2004). Housing speculation and housing price bubble in Korea. KDI School of Public Policy and Management Working Paper.

10. Diba, B. T. and H. I. Grossman (1988). "Explosive rational bubbles in stock prices?" The American Economic Review **78**(3): 520-530.

11. Dickey, D. A. and W. A. Fuller (1979). "Distribution of the estimators for autoregressive time series with a unit root." Journal of the American statistical association **74**(366): 427-431.

12. Enders, W. (1995). Applied econometric time series, Wiley.

13. Evans, G. W. (1991). "Pitfalls in testing for explosive bubbles in asset prices." The American Economic Review **81**(4): 922-930.

14. Flood, R. P., R. J. Hodrick, et al. (1986). An evaluation of recent evidence on stock market bubbles. NBER working paper series.

15. Granger, C. and R. Engle (1987). "Dynamic model specification with equilibrium constraints: Conintegration and error-correction." Econometrica **55**: 251-76.

16. Greene, W. H. (1997). Econometric analysis, Prentice Hall.

17. Hall, S. G., Z. Psaradakis, et al. (1999). "Detecting periodically collapsing bubbles: A Markov-switching unit root test." Journal of Applied Econometrics **14**: 143-154.

18. Hamilton, J. D. (1989). "A new approach to the economic analyusis of nonstationary time series and the business cycle." Econometrica **57**(2): 357-384.

19. Hamilton, J. D. (1994a). Time series analysis, Princeton University Press.

20. Hamilton, J. D. (1994b). State-space models. Handbook of Econometrics 2. **4:** 3039-3080.

21. Horowitz, J. L. (2001). The bootstrap, in James J. Heckman and Edward Leamer. Handbook of Econometric 2, Elsevier Science B. V. **5**.

22. Ito, T. and T. Iwaisako (1995). Explaining asset bubbles in Japan. NBER working paper.

23. Kim, K.-H. and H. S. Lee (2000). Real estate price bubble and price forecasts in Korea. Deparment of Eocnomics, Sogang University working paper.

24. Kim, K.-H. and S.-H. Suh (1993). "Speculation and price bubbles in the Korean and Japanese real estate markets." Journal of Real Estate Finance and economics **6**: 73-87.

25. Lee, J. S. (1997). "An Ordo-liberal perspective on land problems in Korea." Urban Studies **34**(7): 1071-84.

26. Lim, H. Y. (2003). Asset price movements and monetary policy in South Korea,. Monetary policy in a changing environment,, BIS Papers no. 19.

27. Newey, W. K. and K. D. West (1987). "A simple positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix." Econometrica **55**: 703-708.

28. Phillips, P. C. B. (1987). "Time series regression with a unit root,." Econometrica **55**: 277-301.

29. Phillips, P. C. B. and P. Perron (1988). "Testing for a unit root in time series regression." Biometrika **75**(2): 335-346.

30. Rappoport, P. and E. N. White (1994). The New York Stock market in the 1920s and 1930s: Did stock prices move together too much? NBER working paper.

31. Rappoport, P. and E. N. White (1994). The New York Stock market in the 1920s and 1930s: Did stock prices move together too much? NBER working paper.

32. Raymond, J. E. and R. W. Rich (1997). "Oil and the macroeconomy: a Markov state-switching approach." Journal of Money, Credit and Banking **29**(2): 193-213.

33. Shiller (2001). Bubbles, human judgment, and expert opinion. Cowles Foundation discussion paper.

34. Wheatley, G. (2004). Applied numerical analysis, Pearson Education, Inc.

35. White, H. (1980). "A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity." Econometrica **48**: 817-838.

36. Xiao, Q. and R. G. K. Tan (2006). Signal Extraction with Kalman Filter: A study of the Hong Kong property price bubbles. Nanyang Technological University EGC Working Paper Series.

---

[i] The proxy for a bubble to swell is the price appreciation lagged one period; that for a bubble to burst is the deviation of accrual price from the equilibrium price in the previous period (Abraham and Hendershott 1994, 2, 4).
[ii] Chan et al. use signal extraction method of Durlauf and Hall (1989a,b), whereas Xiao and Tan use the Kalman filter method to estimate the specification error.
[iii] Refer to Hamilton (1994) page 691 for an argument of this specification.
[iv] For an introduction to EM, refer to Frank Dellaeret (2002), which also is a good source of references.
[v] For example, Hall et al. (1999).
[vi] SSE is the sum of the squared weighted average of residuals from both states, with weights being the smoothed probabilities associated with each state.
[vii] Cavaliere (2003) derives asymptotics for unit-root tests with Markov-switching. But we will adopt the bootstrapping approach for reasons to be explained below.
[viii] The theory does not suggest a specific number of replications. The guideline is to stop when changes in the distribution are negligible (*Handbook of Econometrics*, vol. 5, chapter 52). We nevertheless conducted 10,000 replications to be on the safe side.
[ix] The CEIC Economic Databases have been established since 1992. Its core economic database is the CEIC Asia Economic Database with over 190,000 data series. Its prime sources of data include over 150 major

government statistical agencies, over 80 recognized non-government issuing agencies, and over 300 reference statistical publications. Please visit http://www.ceicdata.com/ for more information on the profile of CEIC Data Company Ltd ("CEIC")

[x] The rationale for using CPI as a deflator is that buying a property is an investment decision, which very much depends on one's consumption choices.
[xi] Breunig, Najarian, and Pagan [2003, 705] argue that graphical evidence is often the most effective procedure for model evaluation. It is nevertheless less appealing than the Markov-switching approach ex ante, as the latter determines the breaking points endogenously.
[xii] We use the term "might" or "possible" throughout the paper because that the result could be due to some unobserved market fundamentals, rather than a rational speculative bubble.
[xiii] Reasonable in the sense that it give results consistent with the earlier results from the linear ADF test and the conclusion from the visual inspection of the plots (figure 1).
[xiv] These exercises show that the results of estimation and testing are indeed data sensitive, as noted by previous researchers (Lim 2003).

[xv] As the autocorrelations among residuals have been removed in the first place, we will use the White heteroscedasticity consistent estimator, instead of the Newey-West autocorrelation consistent covariance estimator.