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A Comparative Simulation Study of Fund Performance Measures

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Abstract

This study critically reviews current fund performance measures. The performance measure derived from the return-based style analysis by Sharpe (1992) is introduced and compared with other regression-based measures. A comparative simulation is set up to test the robustness, accuracy, and efficiency of the measures. The evidence shows that the RBSA measure is superior to other measures. The performance of the simple Jensen measures is sensitive to fund types. More complicated measures, like market-timing measures and multifactor measures show spurious market timing and wrong fund type information.

JEL Classification: G0, G1, C1, D4
Key words: Mutual Fund, Performance Measure, Market-timing, Return-Based Style Analysis

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A Comparative Simulation Study of Fund Performance Measures

1. Introduction

Most of current fund performance measures are estimated by the regression method and are actually an application of the Capital Asset Pricing Model (CAPM). Depending on their assumptions about the measure of fund performance, the measure of fund risk, and the behavior of fund managers, we classify the measures into three general categories: (i) unconditional measures, where it is assumed that there is no market-timing activity, for example, the Jensen (1968) measure. It was later extended to Fama-French's (1993) three-factor measure and Carhart's (1997) four-factor measure; (ii) market-timing measures, where they control the measurement bias caused by the fund manager's market timing behavior. There are two popular ways to control the market-timing behavior: Treynor-Mazuy model (Treynor and Mazuy, 1966) and Henriksson-Merton model (Henriksson and Merton, 1981), which were later refined by Bhattacharya & Pfleiderer (1983). They assumed that the risk level of the portfolio varies when managers adopt market-timing strategies; (iii) conditional measures that control the investment strategies using publicly available macroeconomic information, most typically is the study by Ferson and Schadt (1996).

Traditional measures suffer a number of limitations. Firstly, it is difficult to find a proxy for market portfolio (it is called benchmark inefficiency). This difficulty poses a serious problem when evaluating fund performance, because if the market portfolio used is not a perfect market portfolio the covariance of the return of the fund and the return of the market portfolio can not correctly measure the risk born by the fund. Thus the alpha derived from the measure is biased. Later efforts, like the Fama-French three-factor measure and the Carhart four-factor measure, attempted to solve this problem by adding more risk factors into the Jensen measure. Although they could reduce the inefficiency problem to some extent, the inefficiency is still material as noted by Grinblatt and Titman (1994). In addition, the complex multi-factor measures brought two other problems along: it consumes more degrees of freedom, making statistical inference of coefficients unreliable. And it is difficult to interpret the beta coefficients. They provide no quantified information about the fund's asset allocations to each asset category, which is valuable for the in-depth analysis of fund risk level.

Secondly, although market timing and conditional measures are theoretically attractive, it is practically impossible to implement them. When managers invest in options or option-like securities, spurious market-timing ability and selectivity ability may be observed as noted by Jagannathan and Korajczyk (1986). In addition, when managers trade securities in less than one month, which is common in practice, we could also observe spurious market-timing ability (Ferson and Schadt, 1996). The correct separation of market-timing ability from selection ability, denoted by alpha, depends on some impractical constrictions. Regarding conditional measures, the measures are complicated in multi-factor models, making the inference about beta coefficients and alpha unreliable within a three-year evaluation period. And we can not increase the sample size to deal with this problem, because the fund may significantly shift its investment strategy or change fund managers in the longer sample period. But, it is a common practice to use three-year data to evaluate fund performance, see, for example, Cai et al. (1997), Carhart (1997), Elton et al. (1996), and Kosowski et al. (2001).

Thirdly, all these measures are estimated by the regression method. An underlying assumption is that ε_{it} is normally distributed in order to make hypothesis tests on betas and alpha. But many empirical studies have shown this assumption is not likely true, for example, a recent study by Kosowski et al. (2001), where they used bootstrap analysis to assess the p value of alphas.

Sharpe (1992) proposed to measure fund performance based on the return-based style analysis, which overcomes some limitations of traditional measures because of its diffrrent rationale and estimate techniques. It attracted a lot of attentions since this pioneering work, please see, for example, Buetow, et al. (2000), Christopherson (1995, 1999), Cummisford, et al. (1996), Lieberman (1996), and Mayes, et al. (2000). The model is,

$$r_t = \beta_1 f_{1t} + \beta_2 f_{2t} + \ldots + \beta_k f_{kt} + \varepsilon_t \qquad (1)$$

where r_t is the fund return from period 1 to T. T is the number of observations in the sample period. f_{kt} is the kth index return in period t. f_{1t} to f_{kt} are called style indexes.

Return-based style analysis can be naturally extended to measure fund performance. We term it the RBSA measure. It decomposes the return in (1) into two parts. One is, $\beta_1 f_{1t} + \beta_2 f_{2t} + \ldots + \beta_k f_{kt}$, attributable to fund styles; the other is attributable to ε_t, due to the active management like securities selection and asset allocation. It is defined it as the tracking error at period t. The expected value of the tracking error, E(ε_t), is defined as the performance of the fund, Alpha. It is the difference between the realized fund return and the return of passive style indexes.

The measure has several advantages compared to traditional measures estimated by the regression method. Firstly, we do not require that ε_{it} should be normally distributed. ε_{it} can be distributed differently. In addition, the expected value of ε_{it} is not even required to be zero. We interpret the non-zero value of ε_{it} as the management effect, caused by securities selection or asset rotations. The expected value of ε_{it} is the measure of fund performance, a counterpart of the alpha of traditional measures in this chapter. Secondly, we circumvent the benchmark inefficiency problem by including all the investable style indexes in the RBSA measure. The only requirements about the style indexes are that they are exhaustive and exclusive of each other. These requirements are easily accommodated by a large amount of indexes publicly available in the market. Thirdly, the betas estimated in the RBSA measure provide useful information about fund styles. Fund styles are essential for the decomposed-analysis of the fund's risk level by institutional investors.

In this paper, we intend to test the robustness, accuracy, and efficiency of the measure, and compare the RBSA measure with traditional measures by a comparative simulation experiment. We present the setup of the experiment in section 2, then we show the simulation results of alpha, betas and R^2 in section 3, finally we summarize our findings in section 4.

2. Setup of Simulation Experiment

The fund returns are simulated from,

$$r_t = a + \beta_1 R_{1t} + \beta_2 R_{2t} + \beta_3 R_{3t} + \beta_4 R_{4t} + \beta_5 R_{5t} + \varepsilon_t \tag{2}$$

where a is set at 5% annually. It is possible to change the value of a in the simulation, but the results (not reported) show that the selection of a does not change our conclusions about the accuracy and efficiency of the measures. In (2) R_{1t}, R_{2t}, R_{3t}, R_{4t} and R_{5t} stand for three-month Treasury bill rates, Russell Top 200 Growth Index, Russell Top 200 Value Index, Russell 2000 Growth Index, and Russell 2000 Value Index [1] respectively. These five indexes represent the fund's asset allocation to currency asset, large-cap growth stocks, large-cap value stocks, small-cap growth stocks, and small-cap value stocks. ε_t is a randomly generated residual with a mean of zero and standard deviation calculated from the actual style analysis of more than 1000 US domestic well-diversified equity mutual funds, following normal distribution.

To test the measures' ability to measure fund performance and its styles in different situations, we use four sets of beta coefficients below,

$$\begin{bmatrix} 0.05 & 0.48 & 0.47 & 0 & 0 \\ 0.05 & 0 & 0 & 0.48 & 0.47 \\ 0.05 & 0.35 & 0.35 & 0.13 & 0.12 \\ 0.05 & 0.13 & 0.12 & 0.35 & 0.35 \end{bmatrix} \tag{3}$$

The four sets of beta coefficients are to mimic the fund return behavior of four general types of funds: large-cap funds, small-cap funds, well-diversified funds with a preference

[1] The definitions of the indexes are available at http://www.russell.com/US/Indexes/US/Definitions.asp.

to large-cap stocks, and well-diversified funds with a preference to small-cap stocks. For example, the first set of beta coefficients, [0.05 0.48 0.47 0 0], means that the simulated funds put 5% of assets in treasury bills, 48% of assets in well-diversified large-cap growth stocks, 47% of assets in well-diversified value stocks, and no assets in small-cap stocks.

With the simulated return series of the fund, we are testing the power of the following performance measures that we reviewed and proposed in section 1:

1. RBSA measure that is formulated under the framework of a convex quadratic programming problem (RBSA):

$$r_t = \beta_1 f_{1t} + \beta_2 f_{2t} + \ldots + \beta_k f_{kt} + \varepsilon_t$$

subject to $\beta' e = 1$ and $\beta \geq 0$

In the simulation setup, the alpha of RBSA measure is simplified as the expected value of the in-sample ε_t .

2. Jensen measure (JS):

$$r_t - r_{ft} = \alpha_t + \beta_m (r_{mt} - r_{ft}) + \varepsilon_t$$

3. Jensen measure with Treynor-Mazuy market-timing adjustment (JS-TM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \gamma^{TM} (r_{mt} - r_{ft})^2 + \varepsilon_t$$

4. Jensen measure with Henriksson-Merton market-timing adjustment (JS-HM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \gamma^{HM} MAX(0, r_{mt} - r_{ft}) + \varepsilon_t$$

5. Fama-French three-factor measure (FF3):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \varepsilon_t$$

6. Fama-French three-factor measure with Treynor-Mazuy market-timing adjustment (FF3-TM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \gamma^{TM} (r_{mt} - r_{ft})^2 + \varepsilon_t$$

7. Fama-French three-factor measure with Henriksson-Merton market-timing adjustment (FF3-HM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \gamma^{HM} MAX(0, r_{mt} - r_{ft}) + \varepsilon_t$$

where r_t is fund retunrn. Betas are risk exposures. And in the RBSA measure, beats are style coefficients. The risk-free rate r_{ft} is three-month Treasury Bill Rates. The market portfolio r_{mt} is S&P 500, the most frequently used proxy for market portfolio. γ^{TM} and γ^{HM} are market-timing coefficients measured by Treynor-Mazuy method and Henriksson-Merton method respectively. In Fama-French three-factor models, $r_{SMB,t}$ and $r_{HML,t}$ are used to control investment strategies due to size effect and B/M ratio respectively, where $r_{SMB,t}$ is the difference of returns between the monthly return of Russell 1000 index and Russell 2000 index, and $r_{HML,t}$ is the difference of returns between the monthly return of Russell 3000 Value Index and Russell Growth Index.

3. Simulation Results and Analysis

3.1 Simulation Results and Analysis of Alpha and R^2

Table 1 shows simulation results of alpha and R^2 from seven measures, based on 1000 simulations of randomly generated fund return series under four sets of style coefficients in (3). They are presented in table 1 from panel 1 to panel 4. The alpha and R^2 are the average values of the estimation from 1000 simulations. The bias is reported as the difference between the estimated alphas from the measures and the true alpha, which is fixed at 5% in the simulation. To show the efficiency of the performance measurements, we also report the empirical confidence interval at 95% from the simulations. The lower bound is the 5th percentile of the 1000 estimated alphas and the upper bound is the 95th percentile of the 1000 estimated alphas. Because the index return series is possibly not normal due to the cross correlations among stocks in the index portfolios (Kosowski et al., 2001), we construct the confidence intervals from simulation instead of constructing them from t values.

Table 1: Simulation I(Alpha and R^2)

Panel I (β1=0.05, β2=0.48, β3=0.47, β4=0, β5=0)

Measures	Alpha	Bias	C.I.	Size	R^2
RBSA	4.76	-0.24	[2.05 7.59]	5.54	0.96
JS	1.13	-3.87	[-1.46 3.75]	5.21	0.94
JS-TM	0.51	-4.49	[-2.83 3.63]	6.46	0.94
JS-HM	-0.32	-5.32	[-4.43 3.68]	8.11	0.94
FF3	2.68	-2.32	[-4.03 9.13]	13.16	0.95
FF3-TM	3.01	-1.99	[-4.20 10.16]	14.36	0.95
FF3-HM	2.62	-2.38	[-5.15 10.41]	15.56	0.95

Panel II (β1=0.05, β2=0, β3=0, β4=0.48, β5=0.47)

Measures	Alpha	Bias	C.I.	Size	R^2
RBSA	5.16	0.16	[2.45 7.82]	5.37	0.97
JS	13.95	8.95	[11.51 16.39]	4.88	0.52
JS-TM	24.03	19.03	[20.89 27.23]	6.34	0.55
JS-HM	27.48	22.48	[23.32 31.87]	8.55	0.54
FF3	1.89	-3.11	[-4.71 9.02]	13.73	0.97
FF3-TM	2.19	-2.81	[-4.87 8.91]	13.78	0.97
FF3-HM	2.21	-2.79	[-5.44 9.54]	14.98	0.97

Panel III (β1=0.05, β2=0.35, β3=0.35, β4=0.13, β5=0.12)

Measures	Alpha	Bias	C.I.	Size	R^2
RBSA	5.1	0.1	[2.31 8.03]	5.72	0.96
JS	4.71	-0.29	[2.04 7.29]	5.25	0.92
JS-TM	6.75	1.75	[3.42 9.98]	6.56	0.92
JS-HM	7.03	2.03	[2.72 11.14]	8.42	0.92
FF3	2.53	-2.47	[-4.17 9.01]	13.18	0.95
FF3-TM	2.75	-2.25	[-4.33 9.75]	14.08	0.95
FF3-HM	2.49	-2.51	[-4.99 10.22]	15.21	0.95

Panel IV (β1=0.05, β2=0.13, β3=0.12, β4=0.35, β5=0.35)

Measures	Alpha	Bias	C.I.	Size	R^2
RBSA	5.02	0.02	[2.31 7.81]	5.5	0.97
JS	10.64	5.64	[8.15 13.13]	4.98	0.67
JS-TM	17.86	12.86	[14.63 21.27]	6.64	0.69
JS-HM	20.09	15.09	[15.67 24.44]	8.77	0.68
FF3	1.87	-3.13	[-4.53 9.06]	13.59	0.96
FF3-TM	2.25	-2.75	[-4.74 9.06]	13.8	0.96
FF3-HM	2.14	-2.86	[-5.21 9.62]	14.81	0.96

Table 1: Simulation I (Alpha and R^2)(continue)

Panel V Summary

	RBSA	JS	JS-TM	JS-HM	FF3	FF3-TM	FF3-HM
Alpha							
Panel 1	4.76	1.13	0.51	-0.32	2.68	3.01	2.62
Panel 2	5.16	13.95	24.03	27.48	1.89	2.19	2.21
Panel 3	5.10	4.71	6.75	7.03	2.53	2.75	2.49
Panel 4	5.02	10.64	17.86	20.09	1.87	2.25	2.14
Average	5.01	7.61	12.29	13.57	2.25	2.55	2.36
Bias	0.01	2.61	7.29	8.57	-2.75	-2.45	-2.64
Std.	0.15	4.99	9.20	10.86	0.37	0.34	0.20
R^2							
Panel 1	0.96	0.94	0.94	0.94	0.95	0.95	0.95
Panel 2	0.97	0.52	0.55	0.54	0.97	0.97	0.97
Panel 3	0.96	0.92	0.92	0.92	0.95	0.95	0.95
Panel 4	0.97	0.67	0.69	0.68	0.96	0.96	0.96
Average	0.97	0.76	0.78	0.77	0.96	0.96	0.96
Size of C.I.							
Panel 1	5.54	5.21	6.46	8.11	13.16	14.36	15.56
Panel 2	5.37	4.88	6.34	8.55	13.73	13.78	14.98
Panel 3	5.72	5.25	6.56	8.42	13.18	14.08	15.21
Panel 4	5.50	4.98	6.64	8.77	13.59	13.80	14.81
Average	5.53	5.08	6.50	8.46	13.42	14.01	15.14

The table provides simulation results of alpha and R^2 under four sets of beta coefficients presented in (4.2). Fund returns are simulated from an alpha,fixed at 5%, a random error, and five style indexes, that is, three-month Treasury bill rates, Russell Top 200 Growth Index, Russell Top 200 Value Index, Russell 2000 Growth Index,and Russell 2000 Value Index. Beta coefficients correspond to the proportions of assets allocated to Treasury bill and four style indexes. We simulate four types of funds, that is, large-cap funds, small-cap funds, well-diversified funds with a preference of large-cap stocks, and well-diversified funds with a preference of small-cap stocks. RBSA stands for RBSA measure by quadratic programming; JS stands for Jensen measure; JS-TM stands for Jensen measure with Treynor-Mazuy market-timing adjustment; JS-HM stands for Jensen measure with Henriksson-Merton market-timing adjustment; FF3 stands for Fama-French three-factor measure; FF3-TM stands for Fama-French three-factor measure with Treynor-Mazuy market-timing adjustment; FF3-HM stands for Fama-French three-factor measure with Henriksson-Merton market-timing adjustment.

C.I. is the empirical confidence interval of alpha estimator based on simulations. Size is the length of C.I.

Panel I of table 1 shows alpha estimates of the simulated fund with style coefficients $\begin{bmatrix} 0.05 & 0.48 & 0.47 & 0 & 0 \end{bmatrix}$, meaning 5% of fund asset is allocated to currency asset, 48% to well-diversified large-cap growth stocks, 47% to large-cap value stocks, and no asset is allocated to small stocks. We find that RBSA is the most accurate measure with the bias only -0.24% annually. The other measures' accuracy is not comparable to that of the RBSA measure. The biases are larger than 1% as shown in the panel. Using the first set of betas, the three Jensen-based measures, that is, JS, JS-TM, and JS-HM, are less accurate than three FF3-based measures, that is, FF3, FF3-TM, and FF3-HM. The average bias of three JS-based measures is about two times larger than the average bias of three FF3-based measures.

After adjusting market-timing behavior, which actually does not exist in our simulation, with methods suggested by Treynor-Mazuy and Henrikksson-Merton, the biases are even larger, except for FF3-TM. Since there is no market-timing in the simulation, we should not observe any change of biases after adding a market-timing term if the market-timing models are solid. We observed spurious market-timing in the simulation. The spurious market timing is also found empirically by Cai, et al. (1997), Glosten and Jagannathan (1994), and Jagannathan and Korajczyk (1986).

The size of confidence interval indicates the efficiency of measures. JS measure has the smallest size, however since the alphas are severely biased, the efficiency gain has no meaning. The size of RBSA is similar to JS but less biased. The size of confidence interval is largest for FF3-based measures, which are around two times of the size of JS-based measures. This wider confidence interval of FF3 measures is mainly caused by using more variables at the right side of the regression. This kind of correlation may cause inaccurate estimation of alphas in FF3 measures.

We also notice that the R^2 is highest for RBSA measure whose average is 96%. FF3-based measures show a little higher R^2 than JS-based measures. Therefore, using the first set of betas that mimics a large-cap fund we find RBSA measure is less biased and has the largest explanatory power and efficiency.

Panel II shows results using another set of betas. The simulated fund behaves like a small-cap fund according to style coefficients that we set in simulation. The magnitude of the bias of RBSA is similar to what we observed in panel I, but now is upwardly biased. And again RBSA has the smallest bias. But now we observe that bias of JS-based measures is much larger and R^2 is quite low, ranging from 52% to 55%. This is because we are using S&P 500 as the market benchmark, in which most of the stocks are large-cap stocks. This bias clearly illustrates the incapability of JS-based measures in measuring performance when funds invest small-cap securities. FF3-based measures are using the same market benchmark as JS-based measures, but the biases are much smaller, which is due to the explicit incorporation of two risk factors related to size effect and the B/M ratio. We also observe the explanation power of FF3 is comparable to that of the RBSA measure. Therefore, when a fund is a small-cap fund, JS-based measures are not capable of estimating the true alpha. FF3-based measures are more robust than JS-based measures, because they explicitly consider the size effect in the model. RBSA is still the best measure in this case with high R^2, small bias and efficient estimation.

In panel III, we randomly generate a fund that widely invests in all the stocks in the market, but leans to large-cap stocks. We notice that the bias of RBSA is 0.1, but JS-based measures also have small biases when evaluating this kind of fund. The average is -0.29. The bias, efficiency and R^2 of FF3-based measures are similar to what we observed in panel I and panel II. In this set of style coefficients, JS-based measures are comparable to RBSA in terms of bias and efficiency but RBSA is more powerful to explain the fund's return behavior with the highest R^2, 0.96.

In panel IV we generate a fund that widely invests in all the stocks in US market, but leans to small stocks with 70% of assets allocated to small stocks. We find RBSA is very accurate with only a 0.02% bias. The magnitude of bias and R^2 for FF3 measures is stable through the four situations. Regarding JS measures, in panel IV we again observe large bias and low explanation power ranging from 66% to 68%, as we observed in panel II.

From the summary panel of table 1, we find that RBSA unanimously has small biases with an average bias of 0.01% annually, high R^2 accounting for 97% of return variation, and small size of confidence intervals, through the four situations in table 1. FF3-based measures have high R^2, stable biases, and stable size of confidence intervals, but the average bias is around 2.5%, which is much larger than the average bias of RBSA. JS-based measures have the largest biases and the biases are volatile depending on the type of the simulated fund. Although the size of the confidence intervals of JS-based measures is relatively small, the biases and variation of estimated alphas make the efficiency not meaningful. Adjusting market timing for JS and FF3 only makes the estimation less efficient, and causes biases larger in JS-based measures Therefore, from simulation results we may say the RBSA is a better measure in measuring fund performance and explaining the fund return variation compared to other traditional measures.

3.2 Simulation Results and Analysis of Style Coefficients (betas)

Table 2 presents simulation results of style coefficients (betas) in four situations. To test the robustness, accuracy, and efficiency of the seven measures in estimating style coefficients, we simulate four types of funds, that is, large-cap funds, small-cap funds, well-diversified funds with a preference of large-cap stocks, and well-diversified funds with a preference of small-cap stocks. The estimates of betas in the table are average betas of 1000 simulations, and the empirical confidence interval is obtained by setting the 5^{th} percentile of the estimates as the lower bound and 95^{th} percentile as the upper bound.

Panel I shows the estimation results when the simulated fund behaves like a large-cap fund. Our estimates of betas using RBSA are very close to the actual betas. The non-negativity constraints of betas may cause a small upward bias when betas are actually zeros and a small downward bias for other positive betas with the same magnitude. When we use traditional measures: JS-based measures and FF3-based measures, we find that the betas of the market benchmark are uniformly above 0.9. Considering the actual asset

Table 2: Simulation II(Style Coefficients)

Panel I (β1=0.05, β2=0.48, β3=0.47, β4=0, β5=0)

Measures	β_1=0.05	β_2=0.48	β_3=0.47	β_4=0	β_5=0
RBSA	0.05	0.47	0.47	0.01	0.01

	β_m	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS	0.96				
JS-TM	0.96			0.28	
				[-0.47 1.07]	
JS-HM	0.92				0.07
					[-0.08 0.21]
FF3	0.96	-0.09	-0.01		
		[-0.14 -0.04]	[-0.11 0.1]		
FF3-TM	0.96	-0.09	-0.01	-0.05	
		[-0.15 -0.04]	[-0.11 0.09]	[-0.83 0.75]	
FF3-HM	0.96	-0.09	-0.01		0.01
		[-0.14 -0.04]	[-0.11 0.09]		[-0.15 0.16368]

Panel II (β1=0.05, β2=0, β3=0, β4=0.48, β5=0.47)

Measures	β1=0.05	β2=0	β3=0	β4=0.48	β5=0.47
RBSA	0.04	0.01	0.01	0.48	0.46

	β_m	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS	0.92				
JS-TM	0.87			-3.8	
				[-4.57 -3.02]	
JS-HM	1.19				-0.6
					[-0.75 -0.46]
FF3	0.94	0.95	0.04		
		[0.9 1]	[-0.07 0.14]		
FF3-TM	0.94	0.95	0.04	-0.1	
		[0.9 1]	[-0.07 0.14]	[-0.9 0.7]	
FF3-HM	0.95	0.95	0.03		-0.01
		[0.9 1.01]	[-0.07 0.13]		[-0.17 0.15]

Table 2: Simulation II (Style Coefficient) (continue)

Panel III (β1=0.05, β2=0.35, β3=0.35, β4=0.13, β5=0.12)

Measures	β1=0.05	β2=0.35	β3=0.35	β4=0.13	β5=0.12
RBSA	0.05	0.34	0.36	0.13	0.11
	β_m	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS	0.95				
JS-TM	0.94			-0.79	
				[-1.57 0.01]	
JS-HM	1				-0.11
					[-0.25 0.04]
FF3	0.95	0.18	0		
		[0.13 0.24]	[-0.1 0.1]		
FF3-TM	0.96	0.18	0	-0.07	
		[0.13 0.24]	[-0.11 0.11]	[-0.82 0.66]	
FF3-HM	0.95	0.18	0		0
		[0.13 0.24]	[-0.1 0.1]		[-0.15 0.15]

Panel IV (β1=0.05, β2=0.13, β3=0.12, β4=0.35, β5=0.35)

Measures	β1=0.05	β2=0.13	β3=0.12	β4=0.35	β5=0.35
RBSA	0.05	0.13	0.12	0.35	0.35
	β_m	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS	0.93				
JS-TM	0.9			-2.7	
				[-3.44 -1.92]	
JS-HM	1.12				-0.42
					[-0.58 -0.28]
FF3	0.94	0.68	0.03		
		[0.63 0.73]	[-0.08 0.13]		
FF3-TM	0.94	0.68	0.03	-0.1	
		[0.62 0.73]	[-0.08 0.13]	[-0.87 0.68]	
FF3-HM	0.95	0.67	0.02		0
		[0.62 0.72]	[-0.08 0.12]		[-0.15 0.14]

The table provides simulation results of alpha and R^2 under four sets of beta coefficients presented in (4.2). Fund returns are simulated from an alpha, fixed at 5%, a random error, and five style indexes, that is, three-month Treasury bill rates, Russell Top 200 Growth Index, Russell Top 200 Value Index, Russell 2000 Growth Index, and Russell 2000 Value Index. Beta coefficients correspond to the proportions of assets allocated to Treasury bill and four style indexes. We simulate four types of funds, that is, large-cap funds, small-cap funds, well-diversified funds with a preference of large-cap stocks, and well-diversified funds with a preference of small-cap stocks. RBSA stands for RBSA measure by quadratic programming; JS stands for Jensen measure; JS-TM stands for Jensen measure with Treynor-Mazuy market-timing adjustment; JS-HM stands for Jensen measure with Henriksson-Merton market-timing adjustment; FF3 stands for Fama-French three-factor measure; FF3-TM stands for Fama-French three-factor measure with Treynor-Mazuy market-timing adjustment; FF3-HM stands for Fama-French three-factor measure with Henriksson-Merton market-timing adjustment.
C.I. is the empirical confidence interval of alpha estimator based on simulations.
Size is the length of C.I.

allocation where 95% of assets are invested in large-cap stocks, this beta estimation is acceptable. FF3 measures are capable to capture the style of the fund. We find that β_{smb} is significant in all three cases, indicating a large-cap fund.

When we study the performance measurement of a fund that behaves like a small-cap fund, which is shown in panel II, we have different results. The estimates based on RBSA are similar to the first panel, but we observe spurious market timing, when using JS-based measures. In both JS-TM and JS-HM, we observe significant negative market timing. This may be caused by different return behavior of small-cap stocks from large-cap stocks, because after we control the size effect in FF3-based measures we don't observe market timing behavior of the fund. Again we find that FF3-based measures are capable of capturing the fund style, since β_{smb} is positive and significant, meaning that the fund generally moves in the same direction as the small stocks.

In panel III we investigate the measures' accuracy in measuring a well-diversified equity fund that leans to large-cap stocks. The accuracy in estimation of RBSA is stable as we observed before. But we find that FF3 measures show that the fund is a small-cap fund, which gives a significant positive β_{smb}. The result contradicts the actual asset allocation of the simulated fund, which invests 70% of its assets in large-cap stocks. Therefore, FF3-based measures don't correctly estimate the coefficients in this situation.

Panel 4 gives the estimation results of a well-diversified equity fund that leans to small-cap stocks. The estimates of RBSA are unbiased in this situation. In RBSA, all five estimates of betas are precisely the true values. We again observe the spurious negative market timing in JS-based measures, but no market timing in FF3 measures. The styles from FF3 measures are accurate, which indicates that it is a small-cap fund.

From simulation results of beta estimation, RBSA is quite successful in identifying the true asset allocation no matter whether it is a large-cap fund, a small-cap fund, or a well-diversified fund. FF3-based measures are capable of capturing the true fund style when the fund is exclusively investing in large-cap or small-cap stocks; however, when the fund is a well-diversified fund, FF3-based measures seem difficult to identify the true styles. Another finding is that FF3-based measures may avoid the spurious market timing that we observed in JS-based measures.

4. Conclusion

From our simulation results of the performance measurement and style identification, we find that the RBSA measure seems to be the best measure among the seven measures. The RBSA measure is accurate, efficient and robust, and its performance does not depend on the type of the fund in the study. The average bias of alphas is around 0.01% annually, whereas the average biases of other measures range from 2.45% to 8.57% in absolute value. The beta coefficients estimation is also satisfactory, very close to the true betas as shown in table 2. However, the beta estimation may be upwardly biased when the beta is actually zero. Since we observed that the bias is quite small around 0.01, it does not pose any difficulty in implementation.

The estimates of JS-based measures are unstable, depending on the fund type. When the fund is a large-cap fund, the results are acceptable. However, when funds invest in small-cap stocks, there are some problems. Firstly, it can not identify fund styles, secondly, it shows spurious negative market-timing, and thirdly it captures only a relatively small part of return variations, where R^2 is quite small compared with other measures with R^2 well above 90%.

FF3-based measures have stable estimates, not depending on the fund type. We find that using FF3-based measures we may avoid spurious market-timing that we observed in JS-based measures. However, they are unable to identify the true fund style of a well-diversified equity fund, thus the alpha estimates derived from the measures are also questionable. In addition, the accuracy and efficiency of the measures are not comparable with the RBSA measure.

Therefore, based on the criterion of accuracy, efficiency and robustness of the estimation of alpha and betas, RBSA comes to be superior to other measures.

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