105

Reihe Ökonomie

Economics Series

Elastic Labour Supply and Optimal Taxation in a Model of Sustainable Endogenous Growth

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share "work in progress" in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschafts-wissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

This paper presents an endogenous growth model which features elastic labour supply in order to address the distortions created by labour income and consumption taxation. Introducing elastic labour into an AK model greatly changes the structure of the model and raises problems regarding the existence and the stability of a balanced growth path. Another important feature of this paper is the presence of environmental quality both in the utility and in the production functions and the requirement that the growth path of the economy be environmentally sustainable. Within the framework outlined, the focus of the analysis is on optimal taxation and on the effects of government policy on the growth path of the economy. The basic result is that even in this simple setting the interaction between the economic and the ecological system are complex and the policy outcomes crucially depend on the parameters of the model.

Keywords

Endogenous growth, sustainable growth, environmental externalities, elastic labour supply, optimal taxation

JEL Classifications

O41, D62, Q28, E62

Comments

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1 Introduction

Numerous recent contributions have discussed the key role played by fiscal policy in determining the long-run growth of an economy[1], yet most of these models feature inelastic labour supply. This modelling choice has profound implications for the analysis of fiscal policy since, for example, the taxation of labour (and consumption) doesn't introduce distortions in the economy, affecting the economy in a lump sum fashion. In the simple AK model presented in this paper, an elastic labour supply is introduced in order to be able to address the distortions created by labour income and consumption taxation. The introduction of elastic labour into an AK model greatly changes the structure of the model and raises problems regarding the existence and the stability of a balanced growth path. In this context the equilibrium can be represented using two loci. The first represents the levels of the growth rate and of the share of time devoted to leisure that preserve the equality of the rates of return on consumption and capital. The second represents the combination of the two quantities which guarantees the equilibrium in the goods market. Given the complexity of these two loci, an equilibrium may or may not exist depending upon the structural parameters of the model.

The second important feature of the model presented here is the presence of environmental quality and the requirement that the growth path of the economy be environmentally sustainable. Sustainability in this context requires that the environmental quality level does not decline over time. One can think of the environment as affecting the economy in essentially three ways: through amenity, health and productivity channels. The first two channels will be accounted for here by introducing environmental quality directly into the utility function, while the third channel is represented by allowing the environment to enter the production function. This approach is not uncommon in the literature on environmentally sustainable growth[2]. A new element is introduced by using the environmental sciences literature to specify a plausible dynamic behaviour for the environment. This behaviour describes a resilient system which is damaged by the pollution generated by economic activity and which benefits from efforts made to reduce this impact in the form of pollution abatement expenditure. This formulation also proves to have significant impacts on the properties of the model, since changes in the level of abatement expenditure have now two contrasting effects. On the one hand, they tend to lower the growth rate of the economy due to a reduction of the resources devoted to investment, while on the other hand the increase in environmental quality tends to offset this direct effect by increasing the level of output. The induced changes on the labour-leisure trade-off further complicate matters. It will be shown that these effects

[1] See, for example, Jones *et al.* [3] and Turnovsky [13].

[2] Among authors who have used similar formulations, see, for example, Rubio and Aznar [10], Mohtadi [6], Smulders and Gradus [11]. A large body of literature, mainly published in specialised journals, is available which deals with analyses of the impacts of deteriorating environmental conditions on the productivity of crops, fisheries, forests, on the negative effects on labour productivity due to poorer workers' health or on the increased rate of machinery and building degradation brought about by increasing air pollution.

depend on the parameters of the model.

Within the framework outlined above, the focus of the analysis will be on optimal taxation and on deriving the tax rates which cause the decentralised equilibrium path to mimic the socially optimal path. Doing so necessitates deriving both the socially optimal and market-driven solution paths and characterising the tax rates that make the market equilibrium coincide with the social optimum.

The rest of the paper is organised as follows: the first part of section 2 presents the model, and section 2.1 discusses the social optimum; section 2.2 discusses the property of the decentralised equilibrium and section 2.3 derives the optimal tax rates. Section 3 summarises and concludes.

2 The Model

The economy consists of N identical individuals, each endowed with a unit of time that can be allocated either to leisure l, or to work $1-l$, $l \in (0,1)$. The aggregate output of the economy is given by:

$$Y = \alpha K E^{\beta}(1-l)^{\phi} \tag{1}$$
$$\alpha > 0, 0 \leq \beta \leq 1, 0 < \phi < 1$$

where K is physical capital and E stands for environmental quality, which is assumed to have positive effects on production, and should be viewed as a pure public good. At the aggregate level, the production function exhibits constant returns to scale with respect to capital[3] and positive but diminishing marginal products with respect to labour and to environmental quality.

The representative agent's welfare function[4] is given by:

$$U = \int_0^{\infty} \frac{1}{\gamma}(\frac{C}{N} l^{\theta} E^{\eta})^{\gamma} e^{-\rho t} dt \tag{2}$$
$$\eta > 0,\ \theta > 0;\ \gamma \leq 1,\ \gamma \neq 0$$

Agents derive positive marginal utility from consumption C, leisure l and from an aggregate index of environmental quality E. Strict concavity of the utility function in C, l and E requires that $\gamma(1+\eta) < 1$ and $\gamma(1+\eta+\theta) < 1$. The parameter γ is related to the intertemporal elasticity of substitution, σ, by the relation $\sigma = \frac{1}{1-\gamma}$.

[3] The AK technology at the aggregate level is simply assumed here, and since the paper does not deal with the way the constant returns to scale are brought about, a constant return to scale technology at the individual firms' level will also be assumed. A simple change to this framework could be made following Turnovsky [12] by introducing some form of productive government expenditure as the engine of growth, so that the production function for the individual firm might be: $y = \alpha' G_p^{\kappa}(1-l)^{\phi'} E^{\beta'} k^{1-\kappa}$. Assuming that $G_p = g_p Y$, defining $Y = Ny$ and solving, the aggregate production function is: $Y = \left(\alpha' N^{\beta'} g_p^{\beta'}\right)^{\frac{1}{1-\kappa}} (1-l)^{\frac{\phi'}{1-\kappa}} E^{\frac{\beta'}{1-\kappa}} K$.

[4] The choice of the utility function is such that it satisfies the conditions derived in Ladrón-de-Guevara *et al.* [4] to insure that the introduction of leisure into the utility function is consistent with the existence of a balanced growth path.

E is regarded as a capital good that is depleted over time by pollution but also has its own regenerative capabilities. The flow of pollution, P is an increasing function of the level of output and a decreasing function of the amount of expenditure in pollution abatement[5]: $P = \left(\frac{Y}{A}\right)^{\psi}$.

The level of environmental quality, E, evolves according to the following logistic function[6] with harvesting:

$$\dot{E} = \chi E(1 - \frac{E}{\bar{E}}) - \left(\frac{Y}{A}\right)^{\psi} \tag{3}$$
$$\chi > 0\,, \psi > 0$$

where $\bar{E}$ is the carrying capacity of the environment. Assume for simplicity that the expenditure in pollution abatement is a constant share of output, $A = aY$, so that (3) now becomes:

$$\dot{E} = \chi E(1 - \frac{E}{\bar{E}}) - a^{\omega} \tag{4}$$

where $\omega = -\psi$.

The law of motion is:

$$\dot{K} = Y - C - A = (1 - a)Y - C \tag{5}$$

2.1 The Centralised Problem

The Social Planner[7] maximises the welfare of the representative agent, subject to the production function and to the laws of motion by choosing the control variables C, l and a to solve:

$$\max_{\{C,l,a;E,K\}} \int_0^{\infty} \frac{1}{\gamma}(\frac{C}{N} l^{\theta} E^{\eta})^{\gamma} e^{-\rho t} dt \tag{SPP}$$

$$s.t.\;\; \dot{K} = Y - C - A$$
$$\dot{E} = \chi E(1 - \frac{E}{\bar{E}}) - a^{\omega}$$
$$Y = \alpha K E^{\beta}(1 - l)^{\phi}$$

[5] No specific assumption need to be made on the coefficient ψ for the results of the model to go through, yet it is plausible to imagine that the marginal damage to the environment is increasing with the level of output, given the limited resilience capability of the system, so that $\psi > 1$.

[6] This function was chosen as an approximation of the dynamics of a renewable resource subject to harvesting. It is probably the simplest functional form one can derive from the environmental literature to represent the dynamic behaviour of a resilient system with a fixed carrying capacity, $\bar{E}$. For an introductory discussion on this issue see, for example, Perman *et al.* [7], ch. 9.

[7] I am assuming that the objective of the Social Planner is simply to maximise the welfare of the representative agent. Of course this is not the only possible objective for the Social Planner, yet I will restrict my analysis to this case of a purely (symmetric) utilitarian Social Planner.

After substituting for the production function in $\dot{K}$, the current value Hamiltonian is:

$$\tilde{\mathcal{H}} = \frac{1}{\gamma}(\frac{C}{N}l^{\theta}E^{\eta})^{\gamma} + \lambda\left[(1-a)\alpha K E^{\beta}(1-l)^{\phi} - C\right] + \mu\left[\chi E(1-\frac{E}{\bar{E}}) - a^{\omega}\right] \quad (6)$$

Assuming that a is fixed for the moment[8] and deriving with respect to C, l, K, and E, respectively, the following necessary conditions arise:

$$(\frac{C}{N}l^{\theta}E^{\eta})^{\gamma}\frac{1}{C} = \lambda \quad (7)$$

$$(\frac{C}{N}l^{\theta}E^{\eta})^{\gamma}\frac{\theta}{l} = \lambda\phi(1-a)\frac{Y}{(1-l)} \quad (8)$$

$$\lambda(1-a)\frac{Y}{K} = -\dot{\lambda} + \rho\lambda \quad (9)$$

$$(\frac{C}{N}l^{\theta}E^{\eta})^{\gamma}\frac{\eta}{E} + \lambda(1-a)\beta\frac{Y}{E} + \mu\chi(1-\frac{2E}{\bar{E}}) = -\dot{\mu} + \rho\mu \quad (10)$$

To this are added the derivatives with respect to the co-state variables (which return the constraints) and the transversality conditions:

$$\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$$
$$\lim_{t\to\infty} \mu E e^{-\rho t} = 0$$

The first expression (7) equates the marginal utility of consumption to the marginal value of an additional unit of capital. Equation (8) represents the equality, at the margin, between an additional unit of leisure and the marginal value of the production forgone. Expression (9), which can be rewritten as $\frac{\dot{\lambda}}{\lambda} + (1-a)\frac{Y}{K} = \rho$, states that the return to physical capital (price appreciation plus net marginal product) must equal the social discount rate. Equation (10) can be interpreted as in (9), by including in the return to environmental quality the marginal utility and the marginal product net of the pollution increase caused by the increase in production and of the increase in the regenerative capacity.

The easiest way to characterise the equilibrium in this context is to focus on the so-called *Sustainable Balanced Growth Path*, defined as a solution $\{K, E, \lambda, \mu, l, C, A\}$ to (SPP) for initial conditions $K(0) = K_0$ and $E(0) = E_0$ such that $\dot{l} = 0$, $\dot{E} = 0$ and that the growth rates of K,Y and A are equal and constant. I call this *sustainable*[9] since the environmental quality level is constant along the expansion path of the economy.

In the first part of the Appendix it is shown how the necessary conditions (7)-(10) can be expressed as a differential equation in the share of time devoted to leisure l and that, under the (weak) conditions that assure the existence of

[8] The share of output devoted to abatement expenditure will later be reintroduced as a control variable and computed.

[9] This is the definition of weak sustainability as given in Perman *et al.*[7].

an equilibrium, this equation is locally unstable, implying that the economy will always lie on the balanced growth path just defined.

Following Turnovsky [12], the Macroeconomic Equilibrium can be conveniently written as follows:

$$\frac{C}{Y} = (1-a)\frac{l}{1-l}\frac{\phi}{\theta} \tag{$EQ1$}$$

$$\frac{\dot{C}}{C} = g = \frac{1}{1-\gamma}\left[(1-a)\frac{Y}{K} - \rho\right] \tag{$EQ2$}$$

$$\frac{\dot{K}}{K} = g = \left[(1-a) - \frac{C}{Y}\right]\frac{Y}{K} \tag{$EQ3$}$$

$$\frac{Y}{K} = \alpha E^{\beta}(1-l)^{\phi} \tag{$EQ4$}$$

$$E = \frac{\bar{E}\chi + \sqrt{-4a^{\omega}\bar{E}\chi + \bar{E}^2\chi^2}}{2\chi} \tag{$EQ5$}$$

($EQ1$) describes the familiar intratemporal optimality condition between consumption and leisure and is obtained from the ratio of conditions (7) and (8). Equation ($EQ2$) is the Euler equation which equates the rate of growth of consumption to the difference between the marginal product of capital available for consumption (net of the abatement expenditure, a) and the discount rate, multiplied by the intertemporal elasticity of substitution. This condition equates the marginal return to capital to the rate of return on consumption. Condition ($EQ3$) is just a per-unit-of-capital version of the resource constraint in (5) and provides a condition for the equilibrium in the goods market. ($EQ4$) returns the production function and finally ($EQ5$) is the sustainability condition $\dot{E} = 0$.[10]

Substituting into ($EQ2$) the expression for $\frac{Y}{K}$ from ($EQ4$) and the expression for E from ($EQ5$) one gets the following relation between the growth rate g and the share of time devoted to leisure l, that ensures the equilibrium of the rates of return on capital and on consumption:

$$g = \frac{1}{1-\gamma}\left[\alpha\,(1-a)\,(1-l)^{\phi}\left(\frac{\bar{E}\chi + \sqrt{-4a^{\omega}\bar{E}\chi + \bar{E}^2\chi^2}}{2\chi}\right)^{\beta} - \rho\right] \tag{L_1}$$

Making use of ($EQ3$), ($EQ4$), ($EQ1$)and ($EQ5$) one can express the resource constraint as

$$g = \left[1 - \frac{l}{1-l}\frac{\phi}{\theta}\right]\alpha\,(1-a)\,(1-l)^{\phi}\left(\frac{\bar{E}\chi + \sqrt{-4a^{\omega}\bar{E}\chi + \bar{E}^2\chi^2}}{2\chi}\right)^{\beta} \tag{L_2}$$

[10] Given the logistic structure of the law of motion of E there exist two solutions to $\dot{E} = 0$, here we will concentrate on the stable one. For ($EQ5$) to assume real values the share of abatement expenditure a should meet the restriction $a \geq \left(\frac{\bar{E}\chi}{4}\right)^{\frac{1}{\omega}}$.

Both loci are unambiguously decreasing, with L_2 decreasing to infinity as $l \to 1$ whereas L_1 decreases to $\frac{\rho}{\gamma-1}$. One can show that a sufficient condition for a unique (interior) balanced growth equilibrium to exist is that[11]:

$$\gamma < Min \left\{ \frac{\rho}{\alpha(1-a)\hat{E}^{\beta}}, \frac{1}{1+\theta} \right\} \tag{11}$$

One such equilibrium is shown in Figure1.

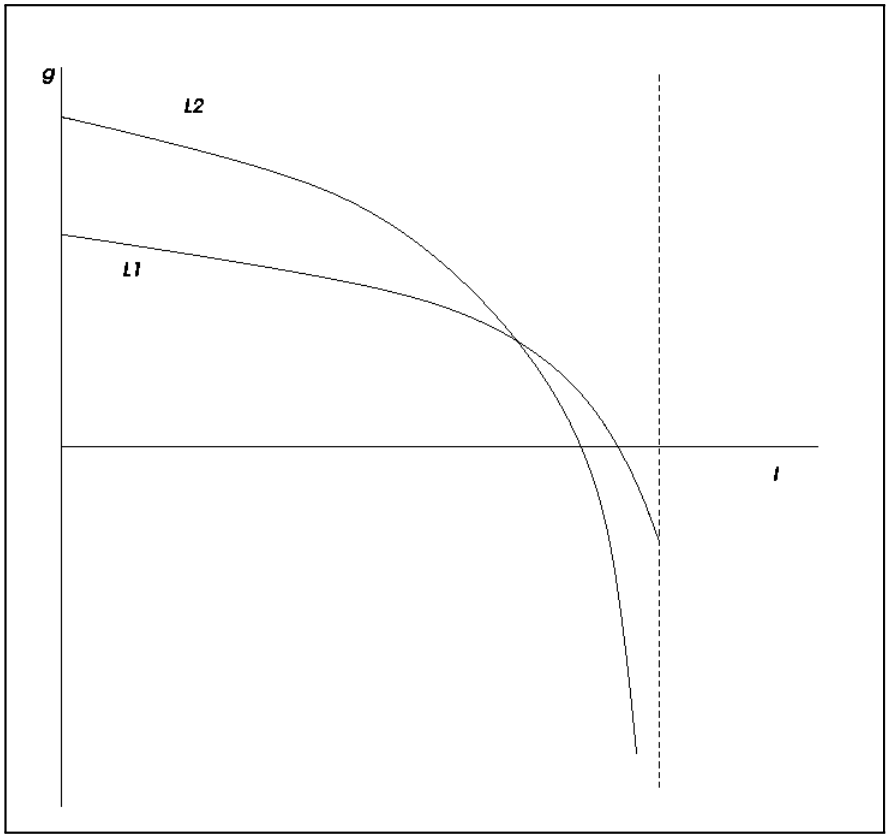

Figure 1: Growth-Leisure Trade-off.

The effects of a change in the share of output devoted to abatement expenditure can be derived from differentiating (7)-(10). Unfortunately the results are not unequiivocal. An increase in the share of output devoted to abatement expenditure directly decreases the growth rate, diverting resources away from investment. Yet it also indirectly increases output via the improvement in environmental quality. The increased environmental quality can also have different effects on the labour leisure choice, depending on the parameters of

[11] This restriction is satisfied by the prevailing empirical evidence on the intertemporal elasticity of substitution which implies a negative value for γ. For a positive growth rate to occur at the equilibrium one also must impose the additional condition $1 - \left(\frac{\rho}{\alpha(1-a)\hat{E}^{\beta}}\right)^{\frac{1}{\phi}} > \frac{\theta}{\theta+\phi}$.

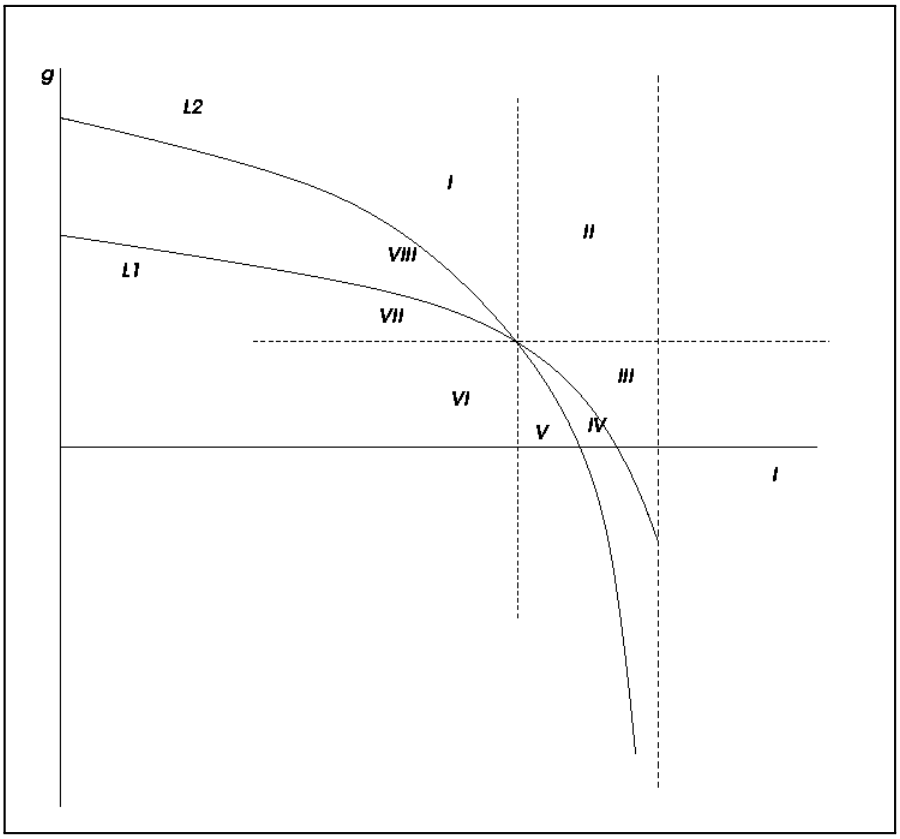

Figure 2: Comparative Statics Regions

the model[12]. An idea of the possible outcome can be obtained with a graphical analysis.

Consider Figure 2. The loci L_1 and L_2 can shift either up or down depending on the predominance of the direct investment effect or of the indirect environmental quality effect and on the direction of the change in l. The locus L_1 always shifts less than L_2. Eventually any outcome can obtain, as each of the eight regions indicated with roman numerals in Figure 2 can be reached. The qualitative results are summarised in Table 1.

	I	II	III	IV	V	VI	VII	VIII
$\frac{\partial \hat{g}}{\partial a}$	+	+	-	-	-	-	+	+
$\frac{\partial \hat{l}}{\partial a}$	-	+	+	+	+	-	-	-

Effects of changes in abatement expenditure on equilibrium values of g and l.

Regions I to III are reached when both loci shift upwards, regions V to VII when they both shift downwards and finally regions IV and VIII are reached

[12]The shifts in the loci L_1 and L_2 as well as the derivatives $\frac{\partial \hat{g}}{\partial a}$ and $\frac{\partial \hat{l}}{\partial a}$, are discussed in the third section of the Appendix. Clearly, the parameters that determine the sign of the derivatives are those relative to the impact of a on E and of the latter on the output level, i.e. β, χ, and ω.

when L_1 shifts downwards (respectively, upwards) and L_2 shifts in the opposite direction.

Suppose now that the social planner also chooses the share of abatement expenditure optimally jointly with the other control variables in the Hamiltonian (6). Given that the law of motion of environmental quality is concave in a, the Hamiltonian remains concave, and the sufficient conditions for the existence of a solution are still satisfied.[13] Maximising (6) with respect to a yields the following additional condition:

$$-\lambda Y - \mu\omega a^{\omega-1} = 0 \tag{12}$$

Solving for a, one gets the following (implicit) optimal level for a (substituting the optimal levels for the other variables).

$$\hat{a} = \left(-\frac{\hat{\lambda}}{\hat{\mu}\omega}\hat{Y}\right)^{\frac{1}{\omega}} \tag{13}$$

2.2 The Decentralised Problem

In the decentralised model the agents maximise their utility choosing their consumption level c and their leisure l subject to the constraint imposed by their income, which they derive from working and holding capital. Agents in this case take the level of environmental quality and the share of abatement expenditure as given[14]. Agents maximise their utility,

$$U = \int_0^\infty \frac{1}{\gamma}(cl^\theta E^\eta)^\gamma e^{-\rho t}dt \tag{14}$$

subject to their individual accumulation constraint,

$$\dot{k} = (1-t_w)w(1-l) + (1-t_k)rk + (1+t_c)c - \frac{T}{N} \tag{15}$$

where w is the real wage, r is the return to capital, $t_{w,k,c}$ are the tax rates on labour, capital and consumption respectively and the ratio $\frac{T}{N}$ represents the lump-sum taxes (or subsidies) necessary to keep the government budget balanced. Assuming perfect competition in both the labour and the capital markets, the equilibrium wage rate and the return to capital are given by the usual marginal product conditions:[15]

$$w = \phi\frac{y}{(1-l)} \quad \text{and} \quad r = \frac{y}{k} \tag{16}$$

[13] The procedure used here is correct due to the particular structure of the relation between environmental quality and the share of output devoted to abatement a. Thich preserves the concavity of the Hamiltonian when a is treated as a control variable.

[14] Alternative formulations are, of course, possible. For an example of firms which take the impact of their productive activities (partially) into account see Rubio and Aznar [10].

[15] The marginal product of capital corresponds to the case of constant returns to scale with respect to capital also at the individual firms' level. Using the alternative formulation based on Turnovsky [12], the marginal return to capital would be $(1-\kappa)\frac{y}{k}$. All the expressions that contain the rate of return to capital in what follows would need to be modified if the alternative specification would be chosen, but this is the only set of modifications required.

Ruling out the possibility of issuing debt, the government must run a balanced budget:

$$Nt_w w(1-l) + Nt_k rk + Nt_c c + T = aY \tag{17}$$

The Hamiltonian for the decentralised problem is then:

$$\tilde{\mathcal{H}} = \frac{1}{\gamma}(cl^\theta E^\eta)^\gamma + \xi\left[(1-t_w)w(1-l) + (1-t_k)rk + (1+t_c)c - \frac{T}{N}\right] \tag{18}$$

and the following necessary conditions arise:

$$(cl^\theta E^\eta)^\gamma \frac{1}{c} = (1+t_c)\xi \tag{19}$$

$$(cl^\theta E^\eta)^\gamma \frac{\theta}{l} = \xi(1-t_w)w \tag{20}$$

$$\xi(1-t_k)r = -\dot{\xi} + \rho\xi \tag{21}$$

With a procedure similar to the Socially Planner problem, the necessary conditions (19)-(21) can be expressed as a differential equation in the share of time devoted to leisure l . As shown in the Appendix, it is also possible to derive conditions that ensure the local instability of the equation and thus that the economy will always lie on the balanced growth path.

Parallel to the description of the equilibrium in the centralised economy, the balanced growth path of the economy (after aggregating over all the agents) can now be written as:

$$\frac{C}{Y} = \frac{(1-t_w)}{(1+t_c)}\frac{l}{1-l}\frac{\phi}{\theta} \tag{$EQ1'$}$$

$$\frac{\dot{C}}{C} = g = \frac{1}{1-\gamma}\left[(1-t_k)r - \rho\right] \tag{$EQ2'$}$$

$$\frac{\dot{K}}{K} = g = \left[(1-a) - \frac{C}{Y}\right]\frac{Y}{K} \tag{$EQ3'$}$$

$$\frac{Y}{K} = \alpha E^\beta (1-l)^\phi \tag{$EQ4'$}$$

The similarity between these conditions and those of the centralised case are evident, the only differences here being that the share a of output devoted to abatement is taken as given and, obviously, that the agents' decisions now depend upon the tax rates. The expressions corresponding to the functions L_1 and L_2 are now given by the following:

$$g = \frac{1}{1-\gamma}\left[\alpha(1-t_k)(1-l)^\phi E^\beta - \rho\right] \tag{L_3}$$

$$g = \left[1 - a - \frac{(1-t_w)}{(1+t_c)}\frac{l}{1-l}\frac{\phi}{\theta}\right]\alpha(1-l)^\phi E^\beta \tag{L_4}$$

The qualitative behaviour of these two loci is similar to that of L_1 and L_2 and, provided that the appropriate restriction on γ holds[16], the equilibrium can

[16] The existence of a unique balanced growth path is guaranteed if $\gamma < Min\left\{\frac{\rho+\alpha E^\beta(t_k-a)}{\alpha(1-a)E^\beta}, \frac{1-a\theta-t_w+\theta t_c a+\theta t_k(1-t_c)}{1-a\theta-t_w+\theta t_c a+\theta(1-t_c)}\right\}$.

also be represented by a graph analogous to that of figure 1. In the absence of taxation, the equilibrium resulting from the decentralised problem would exhibit a higher growth rate and a lower share of time devoted to leisure than is the case in the socially optimal equilibrium. Given the intractability of the analytical solutions to the maximisations, it is necessary to resort to simulations in order to make welfare comparisons between the centralised and the decentralised case. Numerical solutions, making use of plausible parameters values[17] showed that the welfare level as measured by the representative agent's utility is higher under the centralised solution than under the decentralised solution for every value of a chosen in the admissible range.

Given the simple structure of the dependency of L_3 and L_4 on the tax rates, the effects on the equilibrium growth rate and on the equilibrium share of leisure of changes in the tax rates, with tax revenues rebated in lump sum fashion, can be easily demonstrated graphically. Consider Figure 3. A change in the tax rate on capital (t_k) only influences the L_3 locus which rotates downwards to $L_3^{'}$. For an initial given level of l at the initial equilibrium E, the increase causes an immediate decrease in the growth rate and, since the rate of return on consumption, $\rho - \frac{\dot{\lambda}}{\lambda}$, is positively related to the growth rate, this causes the return on consumption to decrease. Agents then substitute leisure for labour and output decreases, which further reduces the growth rate and the final equilibrium is a point like E' in Figure 3. Increases in either the tax on consumption (t_c) or on labour income (t_w) only affect the locus L_4 which rotates upwards to $L_4^{'}$. The initial increase in the growth rate, for given l, increases the return on consumption, so that consumption and leisure increase. Output falls meanwhile, leading to a decrease in the growth rate and of the rates of return on capital and consumption. Eventually the economy reaches a new equilibrium at a point like E''in Figure 3. In conclusion, the effects of tax changes on the equilibrium levels of g and l can be summarised as

$$\frac{\partial \hat{g}}{\partial t_j} < 0 \quad \text{and} \quad \frac{\partial \hat{l}}{\partial t_j} > 0 \qquad j = w, k, c.$$

which is not surprising given the differences between the social optimum and the no-tax market equilibrium noted above.

In this context, in order to analyse an increase in government expenditure one needs an explicit assumption as to the means of financing the expenditure increase. Only the case where the additional expenditures are financed by lump sum taxation so that the distortionary tax rates are not changed will be discussed here. The effects on the equilibrium levels of g and l can once more be determined graphically from the shifts in the loci L_3 and L_4. L_3 shifts up or down depending on whether or not the marginal impact of the change in abatement on leisure is negative or positive[18]. If $\frac{\partial l}{\partial a} < 0$, L_3 shifts unambiguously upwards. If $\frac{\partial l}{\partial a} > 0$, the direction of the shift depends on the relative effect on output of the decrease in labour and of the increase in environmental

[17] The paprameters values used were: $\{\alpha \to .15, \beta \to .01, \gamma \to -1, \phi \to .2, \theta \to .3, \chi \to 5, \bar{E} \to 25, \omega \to -1.5, \eta \to .15, \rho \to .05\}$.

[18] The expressions for the shifts are given in the last section of the Appendix.

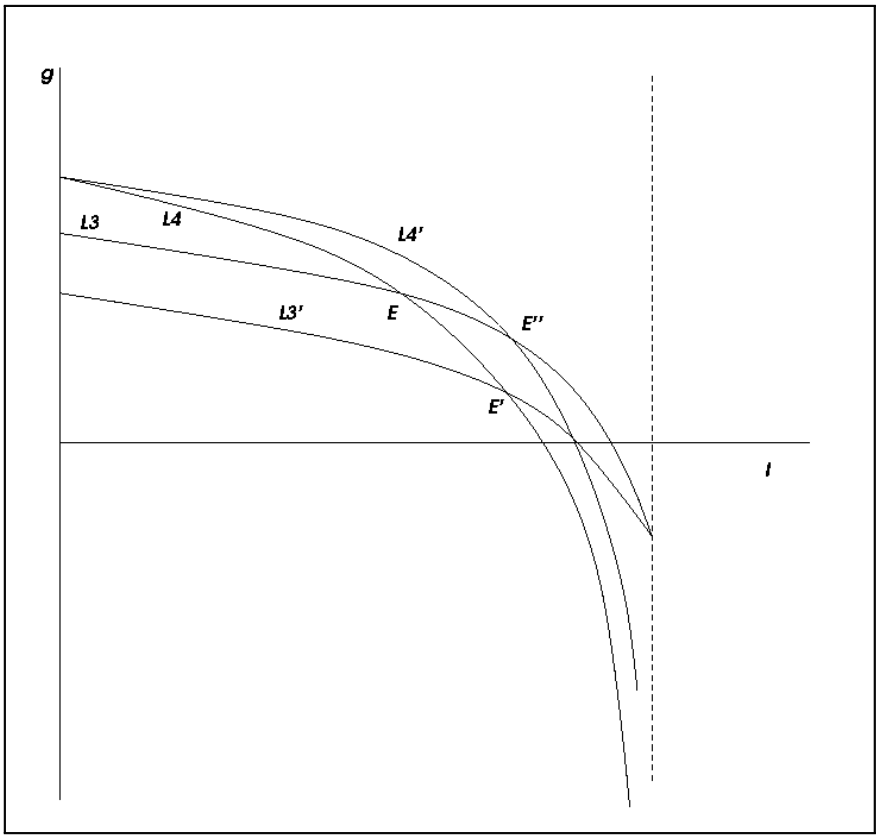

Figure 3: Effects of tax changes on the Equilibrium

quality. When the latter effect is larger, output increases and L_3 moves up. L_4 has a somewhat more complicated structure: $\frac{\partial l}{\partial a} > 0$, it shifts always in the opposite direction relative to L_3. If $\frac{\partial l}{\partial a} < 0$, on the other hand, the shift can be in the same direction (if the leisure decrease is large enough) or in the opposite direction. Just as in the centralised case discussed above, everything can happen with cases equivalent to I-III and V-VII that take place if the shifts are in the same direction (upwards and downwards, respectively) and IV and VIII in the other case. In the latter situation the growth rate decreases, and the level of leisure increases as a consequence of the increase in abatement expenditure whenever the increase in production due to the improved environmental quality is not large enough to offset the decrease brought about by a higher share of time devoted to leisure. It is plausible to assume that such a situation would arise when the level of environmental quality is high and its marginal productivity small. In the case where the level of environmental quality is low, it is possible to envision situations in which the increase in environmental quality could lead to an increase in output and to higher growth rate.

2.3 Optimal Taxation

The central planner sets the optimal tax rates to have the decentralised equilibrium path coincide with the centralised path. By examining the expressions defining the equilibria, it is easy to see that equations $(EQ2)$ and $(EQ2')$ will coincide if and only if:

$$(1-a) = (1-t_k) \tag{22}$$

and that equations $(EQ1)$ and $(EQ1')$ will coincide if and only if:

$$(1-a) = \frac{(1-t_w)}{(1+t_c)}. \tag{23}$$

Condition (22) implies that the growth rate in the decentralised equilibrium will coincide with the growth rate chosen by the social planner provided that the private after-tax marginal return to capital $(1-t_k)r$ coincides with the social rate of return $(1-a)\frac{Y}{K}$. The second condition, equation (23), requires that the consumption-leisure margins be equalised between the two solutions.

From the previous conditions, it emerges that the optimal tax rates must satisfy:

$$t_k = a \quad \text{and} \quad t_w = a - t_c + at_c \tag{24}$$

The optimal taxes and expenditure must also be consistent with the government budget constraint (17). Setting the tax rates according to (24) and evaluating the marginal products of labour and capital at the optimum, the budget constraint becomes:

$$(\hat{a} - \hat{t}_c + \hat{a}\hat{t}_c)\phi\frac{\hat{Y}}{1-\hat{l}} + \hat{a}\hat{Y} + \hat{t}_c\frac{1-\hat{a}+\hat{t}_c-\hat{a}\hat{t}_c}{1+\hat{t}_c}\frac{\phi}{\theta}\frac{\hat{l}\,\hat{Y}}{1-\hat{l}} + T = \hat{a}\hat{Y} \tag{25}$$

so that the social optimum can be enforced by any combination of T and $\hat{t}_c$ that satisfies (25). In particular if $T = 0$, so that no lump sum taxes (or subsidies) are necessary to sustain the social optimum, then the following values for the tax rates emerge:[19]

$$t_k = \hat{a} \qquad \hat{t}_c = \frac{\hat{a}\theta}{(\hat{a}-1)(\hat{l}-\theta)} \qquad \hat{t}_w = \frac{\hat{a}\hat{l}}{(\hat{l}-\theta)}. \tag{26}$$

As is evident by the above expression, the taxes on consumption and labour have opposite signs and, depending on the level of $\hat{l}$, it is optimal either to subsidise labour and tax consumption (if $\hat{l} < \theta$) or to do the opposite and tax labour while subsidising consumption (if $\hat{l} > \theta$).

[19] For the case in which the formulation with decreasing returns to scale with respect to capital were chosen, the appropriate optimal tax rates (always for $T = 0$) would be: $t_k = \frac{a-\kappa}{1-\kappa}$, $\hat{t}_c = \frac{\left((\hat{l}-1)\kappa+\hat{a}\phi\right)\theta}{(\hat{a}-1)(\hat{l}-\theta\phi)}$, and $\hat{t}_w = \frac{\hat{a}\hat{l}+(\hat{l}-1)\theta\kappa}{(\hat{l}-\theta\phi)}$.

3 Conclusions

This paper presents an endogenous growth model in which a distinct role is played by the environment. The environment enters both the utility function and the production functions as a pure public good. It influences both the consumers' and the firms' decisions and determines the socially optimal growth path by means of the sustainability constraint that the level of environmental quality be non-decreasing over time. The environment is modelled from the existing environmental literature and is assumed to exhibit resilience to shocks.

The other distinctive feature of the model is the introduction of an elastic labour supply which not only increases the degree of plausibility of the model but also makes it possible to conduct a richer fiscal policy analysis in which the incentives of the agents are fully taken into account. The introduction of an elastic labour supply could, in principle, lead to problems of non-existence of a balanced growth path. In the model presented, however, under plausible conditions a unique (sustainable) balanced growth path exists. The endogeneity of labour supply has significant consequences for fiscal policy since it causes both the consumption and labour taxes to have a negative effect on the growth rate, in the same way as a tax on capital.

The effects of a lump-sum-financed increase in abatement expenditure has been analysed, showing that the final effect depends on the relative sizes of the changes induced on the output levels by the increased abatement expenditures: 1) directly via the reduced investment and 2) indirectly via the increased environmental quality and the change in the labour supply.

Finally the optimal tax structure has been characterised, which is aimed at correcting the distortions generated by the abatement expenditures on the economy. The tax on capital income is used to correct the distortions in the capital markets and the taxes on consumption and labour income are used to correct the consumption-leisure choice. Naturally enough, they will always have opposite signs.

The work presented in this paper is a first step in the direction of models that try to better characterise the richness of the ecological-economic interactions. In the model some simplifications were made to increase analytical tractability: First, consumers and firms are assumed to care for a single, aggregated index of environmental quality, although it is plausible to imagine that the two types of agents would attach importance to different kinds of environmental services. Second, and along the same line of reasoning, it would be more realistic to imagine different kinds of pollution abatement expenditures, one aiming at maintaining the productivity of the environment, the other more concerned with its contribution to agents' utility; the pollution abatement function would need to be modified accordingly. Finally, the pollution abatement expenditure is assumed throughout the paper to be a constant share of output, thus giving up one degree of freedom in the choice of the optimal path and limiting the feed-backs from abatement expenditure to the production and utility functions.

These limitations notwithstanding, the structure of the model is rich enough to give raise to ambiguous results with regard to the effects of environmental policy in this context. In particular, the contrasting effects of an increase in

abatement expenditure due to the reduction of resources available for capital accumulation and the increase in output brought about by the higher environmental quality stress the complexity of the economic-ecological interactions that this paper is focused on. Moreover the changes in the labour-leisure trade-off contribute to the uncertainty of the sign and the magnitude of the changes in the comparative statics exercises.

The complexity of the dynamics of the model also made it impossible to derive analytically the solutions to the welfare maximisations problems. Accordingly, it was necessary to make use of simulations in order to compare the welfare outcomes in the centralised and decentralised cases. All the simulations showed that the centralised solution invariably lead to higher levels of welfare compared to the unregulated market equilibrium, thus underlining the importance of corrective taxation in this framework.

4 Appendix

4.1 Stability Analysis in the Centralised Problem

The necessary conditions in (7)-(10) can be conveniently expressed as a differential expression in the share of time devoted to leisure l. Indeed it can be shown that from the necessary conditions we can obtain:

$$\frac{dl(t)}{dt} = \frac{\Delta(l)}{\Gamma(l)} \tag{A.1}$$

where

$$\Delta(l) = (a-1)\frac{Y}{K}\left[\gamma - 1 - (\gamma-1)\frac{l}{1-l}\frac{\phi}{\theta} + 1\right] - \tag{A.2}$$
$$- \left[\beta(\gamma-1) + \eta\gamma\right]\left[\chi(1-\frac{E}{\bar{E}}) - \frac{a^{\omega}}{E}\right] + \rho$$

$$\Gamma(l) = \frac{\gamma - 1 + \theta\gamma(1-l) + \phi l(1-\gamma)}{l(1-l)} \tag{A.3}$$

The equilibrium level of $\hat{l}$ is obtained by setting (A.1) equal to zero and solving for l. Since by definition then $\Delta(\hat{l}) = 0$, the linearised dynamics around the steady state can be written as:

$$\frac{dl(t)}{dt} = \frac{\Delta'(\hat{l})}{\Gamma(\hat{l})}(l(t) - \hat{l}) \tag{A.4}$$

The ratio $\frac{\Delta'(\hat{l})}{\Gamma(\hat{l})}$ can be shown to be always positive so that (A.4) is an unstable differential equation. The only solution consistent with a balanced growth equilibrium is therefore $l = \hat{l}$ at all points in time, so that the economy is always on the sustainable balanced path described by equations $(EQ1)$-$(EQ5)$.

4.2 Stability Analysis in the Decentralised Problem

As in the previous case the necessary conditions (19)-(21) can be conveniently expressed as a differential expression in the share of time devoted to leisure l. In particular after taking the appropriate differentials one gets:

$$\frac{dl(t)}{dt} = \frac{\Xi(l)}{\Gamma(l)} \tag{A.5}$$

where

$$\Xi(l) = -\frac{Y}{K}\left[(\gamma-1)(1-a) - (\gamma-1)\frac{(1-t_w)}{(1+t_c)}\frac{l}{1-l}\frac{\phi}{\theta} + 1 - a\right] - \tag{A.6}$$
$$- \left[\beta(\gamma-1) + \eta\gamma\right]\left[\chi(1-\frac{E}{\bar{E}}) - \frac{a^{\omega}}{E}\right] + \rho$$

and $\Gamma(l)$ is the same as before.

Once more the linearised dynamics around the equilibrium level of l, $\check{l}$ say, is given by:

$$\frac{dl(t)}{dt} = \frac{\Xi'(\check{l})}{\Gamma(\check{l})}(l(t) - \check{l}) \tag{A.7}$$

That can be shown to be positive if γ is small enough[20], so that also in this case the instability of the equilibrium is proved and the economy will always lie on the path described by $(EQ1')$-$(EQ4')$.

4.3 Comparative Statics in the Centralised Problem

The shifts in the loci L_1 and L_2 can be obtained by differentiating the expression for the loci with respect to a, they are:

$$L_1 : \frac{1}{1-\gamma}\left[-\alpha(1-l)^{\phi}E^{\beta} - \alpha\,(1-a)\,E^{\beta}\phi(1-l)^{\phi-1}\frac{\partial l}{\partial a} + \alpha\beta\,(1-a)\,(1-l)^{\phi}E^{\beta-1}\frac{\partial E}{\partial a}\right] \tag{A.8}$$

$$L_2 : \left(-\frac{1}{(1-l)^2}\frac{\partial l}{\partial a}\frac{\phi}{\theta}\right) \tag{A.9}$$

$$\left[-\alpha(1-l)^{\phi}E^{\beta} - \alpha\,(1-a)\,E^{\beta}\phi(1-l)^{\phi-1}\frac{\partial l}{\partial a} + \alpha\beta\,(1-a)\,(1-l)^{\phi}E^{\beta-1}\frac{\partial E}{\partial a}\right]$$

The direction of the shift in both cases depend on whether the direct effect on output reduction, the first term in the square bracket in (A.8), is bigger or smaller than the positive effect of the induced change in environmental quality on output. The allocation of time between labour and leisure is also determinant. If $\frac{\partial l}{\partial a} < 0$ then it is much more likely than both loci would shift upwards (indeed if $\frac{\partial l}{\partial a} > 0$ their shifts are discordant and so are also the changes in $\frac{\partial \hat{g}}{\partial a}$ and $\frac{\partial \hat{l}}{\partial a}$). to obtain the analytical expression for $\frac{\partial \hat{g}}{\partial a}$ and $\frac{\partial \hat{l}}{\partial a}$, it is necessary to implicitly differentiate the equations that represent the macroeconomic equilibrium in the centralised case,$(EQ1)$-$(EQ5)$, with respect to a. Solving for the partial derivatives yields, among the others, the expressions:

$$\frac{\partial \hat{g}}{\partial a} = \frac{\alpha(1-l)^{\phi}\hat{E}^{\beta-1}\left[-a\hat{E} - 4(a-1)a^{2\omega}\bar{E}^{2}\beta\chi\omega + (a-1)a^{\omega}\bar{E}^{3}\beta\chi^{2}\omega\right]}{a\,(1+\gamma(-1+(l-1)\theta)+l(\gamma-1)\phi)} \tag{A.10}$$

$$\frac{\partial \hat{l}}{\partial a} = \frac{(1-l)\,[(l-1)\gamma\theta + (\gamma-1)l\phi]\left[-a\hat{E} - 4(a-1)a^{2\omega}\bar{E}^{2}\beta\chi\omega + (a-1)a^{\omega}\bar{E}^{3}\beta\chi^{2}\omega\right]}{(1-a)a\hat{E}\phi\,(1+\gamma(-1+(l-1)\theta)+l(\gamma-1)\phi)}. \tag{A.11}$$

The denominators of both expressions are positive for interior solutions since then $a \in (0,1)$ and $l \in (0,1)$ and $\gamma < \frac{1}{1+\theta}$, which is one of the conditions for the existence of the equilibrium. The first square bracket in the expression for $\frac{\partial \hat{l}}{\partial a}$

[20] A non positivity of γ is a sufficient condition for the instability result, sufficient and necessary conditions are much more complicated in this case in which the taxe rates play an important role in the dynamics. If the taxe rates are set optimally, anyway, a sufficient and necessary condition for the result is that $\gamma < \frac{1-\phi l}{1-\phi l+\theta(1-l)}$.

always negative provided that $l > \frac{\theta}{\theta+\phi}$, and is negative even when the previous condition doesn't hold but $\gamma > \frac{l\phi}{l(\theta+\phi)-\theta}$. In these two cases the signs of the two expressions are discordant, given that the polynomial in a in the square brackets of both expressions is the same. When $l > \frac{\theta}{\theta+\phi}$ and $\gamma < \frac{l\phi}{l(\theta+\phi)-\theta}$, the sign of the two derivatives is the same and depends on the sign of the polynomial $\left[-a\hat{E} - 4(a-1)a^{2\omega}\bar{E}^2\beta\chi\omega + (a-1)a^{\omega}\bar{E}^3\beta\chi^2\omega\right]$ .For plausible parameters values[21] this polynomial, which is defined only in the range $a \in \left(\left(\frac{\bar{E}\chi}{4}\right)^{\frac{1}{\omega}}, 1\right)$, is positive at the values that the simulations assign to the solutions for $\hat{l}$ and $\hat{a}$. Thus, around that equilibrium, an increase in the share of output devoted to pollution abatement is growth increasing and, depending on the equilibrium level of l and on the size of γ, it can prove to be either leisure or labour increasing.

4.4 Comparative Statics in the Decentralised Problem

In order to understand the discussion of section 2.2, the following expressions are necessary, differentiating the expression for the loci L_3 and L_4 with respect to a, the shifts in L_3 and L_4 can be shown to be:

$$L_1 : \frac{1}{1-\gamma}\alpha\left(1-t_k\right)\left[-\phi(1-l)^{\phi-1}E^{\beta}\frac{\partial l}{\partial a} + \beta(1-l)^{\phi}E^{\beta-1}\frac{\partial E}{\partial a}\right] \tag{27}$$

$$L_2 : \alpha\left(-1 - \frac{1}{(1-l)^2}\frac{\partial l}{\partial a}\frac{1-t_w}{1+t_c}\frac{\phi}{\theta}\right)\left[-\phi(1-l)^{\phi-1}E^{\beta}\frac{\partial l}{\partial a} + \beta(1-l)^{\phi}E^{\beta-1}\frac{\partial E}{\partial a}\right]. \tag{28}$$

This is discussed in Section 2.2.

[21] The parameter values I used are: $\{\alpha \to .5, \beta \to .01, \gamma \to -1, \phi \to .3, \theta \to .3, \chi \to 2, \bar{E} \to 10, \omega \to -1.5\}$.

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