Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

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Abstract

This paper studies the impact of income inequality on the level of innovative activities in a model where innovations result in quality improvements. The market for quality goods is characterized by a natural oligopoly with three types of consumers - rich, middle class and poor. In general, we find that for reasons of strategic price setting a more equal distribution of income is favourable for innovation incentives. This is consistent with empirical evidence suggesting that countries with a more equal distribution of income have grown faster.

Keywords

Inequality, income distribution, heterogeneity, innovation, endogenous growth, product quality, vertical product differentiation

JEL-Classifications

o31, L15, L16, D43, H23

Comments

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1. Introduction

Is the existence of a rich class necessary to stimulate innovative activities or is it a high purchasing power of the middle class? According to the former view, high profits accruing from the rich - due to their higher willingness to pay for new goods or better qualities - drive the incentives to conduct R&D. According to the latter, a high purchasing power for the middle class creates large markets, and consequently high innovation incentives.

It is the aim of this paper to study systematically the impact of income inequality on the level of innovative activities for the case that innovations result in quality improvements. While our set-up resembles those of the standard endogenous growth models of vertical product differentiation (Aghion and Howitt, 1992, Grossman and Helpman, 1991), it differs in two important respects. First, previous papers have assumed homothetic preferences for the quality good. This implies that the distribution of income has no impact on the size of the market. As a result income inequality and the rate of innovation are uncorrelated. This is different in our model. Each consumer purchases only one unit of a quality good, while the remaining expenditures are spent on a standardised (composite) commodity. Within this framework the differences in the willingness to pay between consumers determine the price which producers of different qualities can charge. Via this channel income distribution affects profits and the incentives to innovate.[1]

Second, an implication of most previous models is that in equilibrium only the quality leader is on the market. In the present model this turns out to be a special case. The market structure is characterized by a natural oligopoly. The static price equilibrium within such a framework has been studied in various papers (Gabszwicz and Thisse, 1979, 1980, Shaked and Sutton, 1982, 1983) assuming that incomes (or tastes) are uniformly distributed.[2] For our purpose, this has the disadvantage that only one dimension of inequality - the range of the distribution - can be studied. Instead, we concentrate the analysis on a discrete distribution with three types of individuals - the rich, the middle class, and the poor.

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[1] A paper by Glass (1995) uses preferences similar to the Grossman and Helpman (1991) model of quality ladders, but assumes that there are two types of households with different tastes about quality. It is because of this assumption that income distribution plays a role.

[2] O'Donoghue, Scotchmer, and Thisse (1995) extend this framework to study the role of patent policies in a model of cumulative innovations. While using a similar framework to ours they restrict the analysis to study a partial (industry) equilibrium.

The general equilibrium of the model can be characterized by four different regimes. In one equilibrium, the quality leader sells to the rich, whereas the middle class purchases the second-best, and the poor buy the third-best quality. We will call this a "separating equilibrium". In "partially separating equilibria" the top quality producer serves the rich, the second-best supplier sells to the poor, whereas the middle class purchases either the best or the following quality. Finally, in a "pooling equilibrium" the producer of the top quality captures the entire market. It is intuitively clear that the latter situation will arise if the degree of heterogeneity of consumers is rather small. In contrast, when incomes are unevenly distributed, a separating equilibrium will arise.

How income inequality affects the rate of innovation depends crucially on the price regime under consideration. Consider a *pooling* equilibrium. To conquer the whole market, the quality leader has to set a price such that the poor can afford the top quality. Since this price depends exclusively on the poors' willingness to pay for quality, neither the purchasing power of the middle class nor the amount of income concentrated in the hands of the poor have an impact on the incentives to innovate. The results are more subtile in the other regimes. In a *separating* equilibrium the third-best supplier sets the highest possible price to attract the poor, given that worse firms set their price at marginal cost. The leading firm and its immediate follower pursue an analogous policy: given the price of the next lower quality, charge the highest price the own clientel is willing to pay. Under such a situation, both more purchasing power for the poor and for the middle class tends to lead to a more profitable structure of prices for quality goods. The role of the middle class in the *partially separating* regime, however, turns out to be ambiguous.

In general, the results imply that a more equal distribution of income is favourable for innovation incentives. This leads us to two interesting observations. First, while the profits from the rich may be most important in quantitative terms (both because they accrue immediately and because the rich pay the highest prices), it is for strategic reasons why a high purchasing power of the middle class or of the poor is likely to be more favourable for innovation incentives. Second, to the extent that quality improving innovations are the source of economic growth, this relationship between inequality and innovations is consistent with empirical evidence. A number of studies (Persson and Tabellini, 1994, Alesina and Rodrik, 1994, Clarke, 1995) have found that countries with a more equal distribution of income have grown faster. In contrast to many other attempts to explain the evidence, our model stresses the importance of the composition of demand.[3]

The paper is organised as follows. In section 2, we describe the set-up of the model and describe the optimal choice of consumers and firms, both per period and over time. In section 3, we study the properties of the dynamic equilibrium and consider the effects of changes in inequality. Section 4 summarises.

2. The Model

2.1 Consumers

There exist three groups of consumers, P, M and R, distinguished by wealth $A_P < A_M < A_R$. At any instant of time t, consumer i (i = P, M, R) can buy $c_i(t)$ units of a standardised good and one unit of a quality good, where the qualities $q_j(t)$, j = 0, -1, -2, ... are available at prices p_j(t), resp. The price of the standardised good is 1.

Instantaneous utility of consumer i buying quality q_j is described by the utility function $\ln c_i(t) + \ln q_j(t)$, the same for all groups. Thus, at time τ a household maximises life-time utility (θ denotes the rate of time preference)

(2.1) $$U = \int_{\tau}^{\infty} (\ln c_i(t) + \ln q_j(t)) e^{-\theta(t-\tau)} dt$$

s.t.

(2.2) $$A_i(\tau) + \int_{\tau}^{\infty} w e^{-r(t-\tau)} dt \geq \int_{\tau}^{\infty} c_i(t) e^{-r(t-\tau)} dt + \int_{\tau}^{\infty} p_j(t) e^{-r(t-\tau)} dt \,.$$

[3] Exceptions are Murphy, Shleifer and Vishny (1989) who consider the importance of the composition of demand for the adoption of modern technologies, and Falkinger (1994), Zweimüller (1994) who study demand composition effects on the incentives to introduce new goods. Other work has focused either on political issues (see e.g. Alesina and Rodrik, 1994, Persson and Tabellini, 1994, Perotti, 1993, Saint Paul and Verdier, 1993) or have stressed the importance of capital market imperfections (see e. g. Galor and Zeira, 1993, Torvik, 1993, Aghion and Bolton, 1991, Banjeree and Newman, 1993).

where w is wage income, r the interest rate (both constant over time), and $p_j(t)$ is the price of the quality bought in t. The left-hand side of (2.2) can be written as $A_i(t) + w/r$, it is denoted by $K_i(t)$, a person's endowment with human and non-human capital in t.

The following analysis is restricted to steady-states: among the time paths of variables resulting from optimising decisions of the consumers (and of the firms, discussed in 2.2) we concentrate on those where all quantities grow at the same rate.

Equal growth rates of K_i and A_i imply $\dot{K}_i = \dot{A}_i = 0$, as w is constant over time. $\dot{K}_i$ equals current income rK_i minus expenditures $c_i(t) + p_j(t)$, hence the sum $c_i(t) + p_j(t) = rK_i$ must remain constant. Furthermore, in a steady-state the interest rate has to be equal to the rate of time preference. Setting $r = \theta$ in (2.1) and (2.2) implies that the optimum consumption path requires c_i to be constant over time, for any given paths of $q_j(t)$ and $p_j(t)$, hence also constancy of p_j. Thus we can write

(2.3) $c_i = \theta K_i - p_j = w + \theta A_i - p_j, \quad i = P, M, R,$

and instantaneous utility of a consumer i buying quality q_j as $\ln(\theta K_i - p_j) + \ln q_j(t)$. Obviously, a consumer maximises life-time utility by maximising instantaneous utility with respect to quality q_j at any point of time.

2.2 Prices

The market for the c-good is competitive. All firms produce with a unit labour input 1/w. Since this good is the numeraire, w is the wage rate. The market for the quality good is oligopolistic. At any instant of time many different qualities $q_j(t)$, j = 0, -1, -2, ... have been invented, each by a different firm, where $q_0(t) > q_{-1}(t) > q_{-2}(t) > \ldots$. Successive quality levels differ by a factor k > 1: $q_j(t) = k \cdot q_{j-1}(t)$.[4] There are constant marginal costs wa, where a (<1) is the labour coefficient, the same in all firms. The problem is characterized by asymmetric information in the sense that firms cannot distinguish buyers by wealth. The shares of group P, M and R in the population, β_P, β_M and $\beta_R = 1 - \beta_P - \beta_M$, respectively, and the preferences are known.

The life-time of a firm is uncertain. A firm supplies quality q_j until the next innovation takes place. From that event until a further innovation, this firm supplies quality q_{-1}, and so on. It follows that the price setting problem can be viewed as a repeated game in continuous time,

[4] The process of how innovations are created is discussed in Subsection 2.3.

where the firms (the players) want to maximise long-term profits. Since we are interested in steady-states, we look for equilibria where prices are constant over time.

As $q_j = k^m q_{j-m}$, for all j = 0, -1, -2, ..., m = 1, 2, ..., where k > 1, we can compute the maximum price $\overline{p}_j$, given p_{j-m}, such that consumer i prefers quality q_j to q_{j-m}, from the equation ln $(\theta K_i - \overline{p}_j)$ + ln q_j = ln$(\theta K_i - p_{j-m})$ + ln q_{j-m}:[5]

(2.4) $$\overline{p}_j = \theta K_i \left(\frac{k^m - 1}{k^m}\right) + \frac{p_{j-m}}{k^m}.$$

Obviously, (2.4) can also be used to determine the upper bound of p_{j-m} such that consumer i prefers quality q_{j-m}, given $\overline{p}_j$. As $\overline{p}_j$ increases in both K_i and m (note $\theta K_i - p_j = c_i > 0$), we get

Lemma 1: (i) If for prices p_j, p_{j-m} some consumer prefers quality q_j to q_{j-m}, any richer consumer does the same.

(ii) If $p_j \geq wa$, for the price of some quality q_j, j = -1, -2, ..., then for the producer of any quality q_{j+m}, $1 \leq m \leq -j$ there exists a price $p_{j+m} > wa$, such that any consumer prefers quality q_{j+m} to q_j.

Lemma 1 implies that in equilibrium at most the three highest qualities are actually produced and sold.[6] The four possible situations are: (i) q_0 is sold to all three groups of consumers (pooling), (ii) q_0 is sold to R and M, q_{-1} to P (partially separating, case A), (iii) q_0 is sold to R, q_{-1} to M and P (partially separating, case B) and (iv) q_0, q_{-1}, q_{-2} are sold to groups R, M, P, resp. (separating).

Which equilibrium prices correspond to these situations? To study this question, we refer to Figure 1, where p_0, p_{-1}, p_{-2} are shown on the axis. Lines AB and CD, resp., refer to (2.4) with j = -1, m = 1, i = M (i = P, resp.): maximum p_{-1} such that the group-M (group-P) consumers prefer q_{-1} to q_{-2}, given p_{-2}; note $K_P < K_M$. Analogously, lines EF and GH refer to (2.4) with j = 0, m = 1, i = R (i = M, resp.). Finally, line IJ refers to (2.4), with j = 0, m = 1, i = P: maximum p_0 such that the poor prefer q_0 to q_{-1}, given p_{-1}. Points G and I have the same position in the (p_0, p_{-1})

[5] We use the general convention that quality q_j is chosen if, for given p_j, p_{j-m}, buying q_j or q_{j-m} leads to the same utility level.

[6] If q_{-3} was in the market (which means $p_{-3} \geq wa$), one of qualities q_0, q_{-1}, q_{-2} could not be sold, as there are only three groups of consumers. However, by Lemma 1 (ii), higher qualities can always drive out lower qualities.

quadrant as A and C in the (p_{-1},p_{-2}) quadrant. Moreover, the slopes of all lines are equal in the sense that $\partial p_j / \partial p_{j-1} = 1/k$.

Additionally, p^S_{-2} is defined by (2.4) with j = -2, m = 1, i = P and p_{-3} = wa (maximum price of q_{-2} in order to deter q_3 from entry) and p^x_0 is defined by (2.4) with j = 0, m = 2, i = P and p_{-2} = wa (maximum p_0 such that the poor prefer q_0 to q_{-2}, given p_{-2} = wa..

Note that wa is the lowest possible price of any quality and that wa < θK_i, because $\theta K_i = w + \theta A_i$ and a < 1. From Figure 1 and the respective definitions the following facts are immediate:

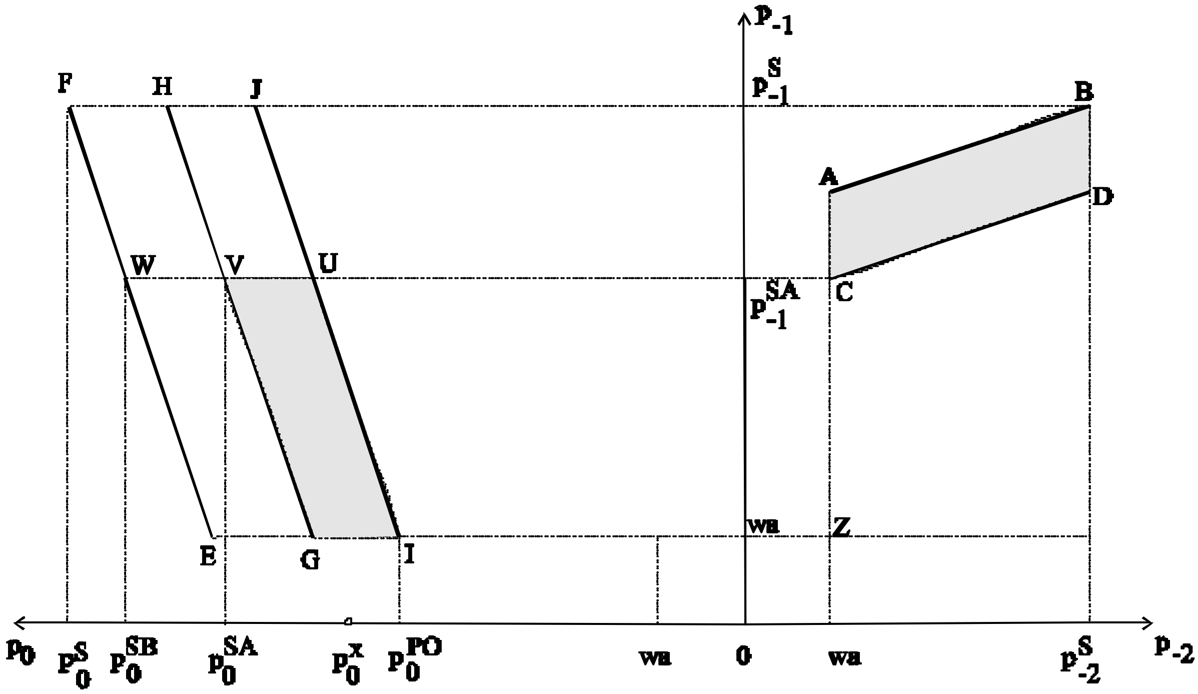

Figure 1

a) wa < p^S_{-2} = $p^{SA}_{-1} = p^{PO}_0 < p^x_0$.

b) In equilibrium $p_{-2} \leq p^S_{-2}$, $p_{-1} \leq p^S_{-1}$, $p_0 \leq p^S_0$ must hold.

c) If and only if $p_1 > p^{SA}_{-1}$, the q_2-producer can choose a price p_{-2} such that group P prefers his quality to q_{-1} and he makes a profit.

d) If and only if $p_0 > p_0^{PO}$, the q_{-1}-producer can choose a price p_{-1} such that group P prefers his quality to q_0 and he makes a profit.

From these facts we can draw the following conclusions:

(i) Pooling occurs only if prices are: $p_0 \leq p_0^{PO}$, p_{-1} = wa, p_{-2} = wa (note d) and $p_0^{PO} < p_0^{x}$).

(ii) Partially separating, case A (R and M buy q_0, P buys q_{-1}) occurs only if prices (p_0, p_{-1}) are in the area GIUV (line UV excluded) and p_{-2} = wa. (Note c) for the upper bound for p_{-1}).

(iii) Partially separating, case B (R buys q_0, M and P buy q_{-1}) occurs only if prices (p_0, p_{-1}) are in the area EGVW (line GV excluded), and p_{-2} = wa.

(iv) Separating occurs only at prices (p_0, p_{-1}, p_{-2}), where (p_0, p_{-1}) is in the area WVHF (VH excluded), and the corresponding pair (p_{-1}, p_{-2}) is in the area ACDB (CD excluded). (Note b) from which the upper bounds for p_{-1} and p_0 follow.)

It is well-known that in an infinitely repeated game a large set of possible solutions exists, each of which can be supported by appropriate punishment strategies. However, in the present case it is possible to restrict this set to four triples of prices, by a simple and plausible general principle: No player is punished if he changes his price without affecting the other players' profits.

After applying this principle the points (I, Z), (V, C), (W, C), (F, B) are left as equilibria, corresponding to the situations (i) - (iv), resp.[7] A pooling equilibrium (I, Z) is decided by the q_0-producer alone, if he receives maximum profit by setting a price sufficiently low such that all groups buy his quality. In this case (I, Z) represents a Nash equilibrium of the stage game.[8]. Concerning a separating equilibrium (F, B) it is straightforward to describe situations where the present value of each firm's profit is higher there than in any other equilibrium. (Clearly, this is always the case for the q_{-2}-firm.) As to the partially separating equilibria, however, there may be a disagreement between the q_0- and q_{-1}-producers, as the latter always prefers (W, C) to (V, C) (because with (W, C) he sells to more consumers at the same price), while the former may prefer (V, C), if it pays for him to attract group M.

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[7] Consider, e.g., the interior of GIUV (partial separation, case A): the q_0 - and q_{-1}-producers can both increase their prices by ε without reducing each other's profit. One the line GV the q_{-1}-producer can increase his price, while on IG and UV the q_0-producer can do the same. A similar reasoning applies for any other possible triple of prices.

[8] The q_{-1} and q_{-2} producers are present as potential competitors, though they do not sell anything in the pooling situation. The q_0-producer looses group P if he increases the price above p_0^{PO}.

Summarising, we have the following triples of equilibrium prices:

(2.5) Pooling: $(p_0, p_{-1}, p_{-2}) = (\theta K_p \frac{k-1}{k} + \frac{wa}{k}, wa, wa)$

(2.6) Partially separating, case A:

$$(p_0, p_{-1}, p_{-2}) = (\theta K_M \frac{k-1}{k} + \theta K_P \frac{k-1}{k^2} + \frac{wa}{k^2}, \theta K_P \frac{k-1}{k} + \frac{wa}{k}, wa)$$

(2.7) Partially separating, case B:

$$(p_0, p_{-1}, p_{-2}) = (\theta K_R \frac{k-1}{k} + \theta K_P \frac{k-1}{k^2} + \frac{wa}{k^2}, \theta K_P \frac{k-1}{k} + \frac{wa}{k}, wa)$$

(2.8) Separating: $(p_0, p_{-1}, p_{-2}) =$

$$(\theta K_R \frac{k-1}{k} + \theta K_M \frac{k-1}{k^2} + \theta K_P \frac{k-1}{k^3} + \frac{wa}{k^3},\ \theta K_M \frac{k-1}{k} + \theta K_P \frac{k-1}{k^2} + \frac{wa}{k^2},$$

$$\theta K_P \frac{k-1}{k} + \frac{wa}{k})$$

2.3 Research and Innovation

Profit-seeking entrepreneurs engage in R&D to improve the quality good. A research success enables a firm to produce a quality k (> 1)-times better than the currently best. Innovations are random and arrive according to a Poisson process with parameter ϕ. For the representative researcher ϕ is a choice variable: employing ϕF workers (where F is a labour coefficient) "produces" R&D intensity ϕ, so the flow of R&D costs is $w\phi F$. The flow of expected profits is ϕB, where B denotes the value of an innovation. B is the present value of profits of a successful researcher. The subsequent life-cycle of such a firm can be divided into several "periods", where a "period" is defined as the (random) interval between two successive innovations in the future. The firm produces the best (second-best, third-best, ...) quality during period t = 0 (1, 2, ...) of its life-cycle. Since the fourth-best producer will never have positive demand, we can confine the discussion to "periods" t = 0, 1, 2. Denoting Π_t as instantaneous profits during period t and ϕ_e as the expected intensity of future research activities, B may be calculated as $V = \sum_{t=0}^{2} \Pi_t \phi_e^t / (\phi_e + \theta)^{t+1}$.[9]

The objective function of the representative research firm may be written as ϕB - ϕwF. Since there is costless access to R&D activities, in equilibrium B $\leq$ wF, with equality for $\phi > 0$. Otherwise entering R&D would still be profitable. Moreover, in a steady-state we must have $\phi = \phi_e$. Then, the innovation equilibrium condition wF = B reads:

$$wF = \sum_{t=0}^{2} \Pi_t \phi^t / (\phi + \theta)^{t+1} \qquad \text{for} \qquad \phi > 0. \tag{2.9}$$

The particular form of the right-hand-side of (2.9) depends on the type of price equilibrium. Denote Π^i as the profit flow from serving the market for group i. Then in a separating equilibrium, we have $\Pi_0 = \Pi^R$, $\Pi_1 = \Pi^M$, $\Pi_2 = \Pi^P$. In a pooling regime, on the other hand, $\Pi_0 = \Pi^R + \Pi^M + \Pi^P$ whereas $\Pi_t = 0$ for t > 0. Finally, in a partially separating equilibrium, $\Pi_2 = 0$ and Π_0 and Π_1 depend on whether the middle class purchases the q_0-or the q_{-1}-quality.

3. Inequality and the rate of innovations

3.1. The distribution of assets and the allocation of resources

It is assumed that profits after an innovation constitute the unique source of aggregate wealth, denoted by v. The distribution of v among households is given by four parameters: d_P, d_M, β_P and β_M. As defined earlier, β_i represents the population share of group i. d_i is the ratio of assets owned by household i relative to per-capita wealth, so $d_i = A_i/(v/L)$, where L is the size of the population. Given values of (d_P, d_M, β_P, β_M) determine the parameters for group R: $\beta_R = 1 - \beta_P - \beta_M$ and $d_R = (1 - \beta_P d_P - \beta_M d_M)/(1- \beta_P - \beta_M)$.[10] With these definitions $\beta_P d_P$, $\beta_M d_M$ and $(1 - \beta_P d_P - d_M \beta_M)$ are the respective shares in aggregate wealth. From inspection of the Lorenz-curve it

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[9] B is $\int_{\tau}^{\infty} e^{-\theta(s-\tau)} E\Pi(s) ds$, where $E\Pi(s)$, expected profit in time s, is the weighted sum of the profits in s in case that zero, one or two innovations will occur until s, i.e., in case that in s the firm will be in period 0, 1 or 2. Hence $E\Pi(s) = \sum_{t=0}^{2} \Pi_t [(\phi_e(s-\tau))^t \exp(-\phi_e(s-\tau)) / t!]$, where the expression in square brackets is the probability of t innovations in the interval [τ ,s]. Evaluating the above integral yields the expression in the text.

[10] The obvious restrictions on d_P and d_M are $0 \leq d_P < \min(1,d_M)$, and $d_P < d_M < (1 - \beta_P d_P)/(1-\beta_P)$, where the last inequality ensures $d_M < d_R$.

follows that increasing d_P or d_M as well as decreasing β_P or β_M unambiguously yields a more even distribution of assets.

By assumption, each household supplies one unit of labour, so total supply is equal to L. How L is allocated among sectors depends (i) on labour demand for research, ϕF, (ii) on employment in the quality sector, aL (note that every individual buys one unit), and (iii) on labour demand for the c-good, $(1/w)[\beta_P c_P L + \beta_M c_M L + (1-\beta_P-\beta_M)c_R L]$. The resource constraint is therefore:

(3.1) $$L = \phi F + aL + (1/w)L[\beta_P c_P + \beta_M c_M + (1-\beta_P-\beta_M)c_R]$$

3.2. The steady-state equilibrium

The general equilibrium is defined by the conditions (2.3) - optimal consumption, one of (2.5) to (2.8) - non-cooperative equilibrium prices for quality goods, (2.9) - innovation equilibrium, and (3.1) - full employment. To solve this system of equations for ϕ and v we first use $A_i = d_i v/L$ and $K_i = A_i + w/\theta$ to express p_j (j = 0, -1, -2) and c_i (i = P, M, R) in terms of the endogenous variable v. Then we can express the innovation equilibrium condition (2.9) and the full employment condition (3.1) in terms of v and ϕ. As a result, the former relation implicitly defines a function $v = \varphi^N(\phi)$, the "N-locus" in Figure 2. The latter relation can be written $v = \varphi^R(\phi)$, the "R-line" in Figure 2. (See the Appendix for details and for the proofs of Lemma 2 and Proposition 1.)

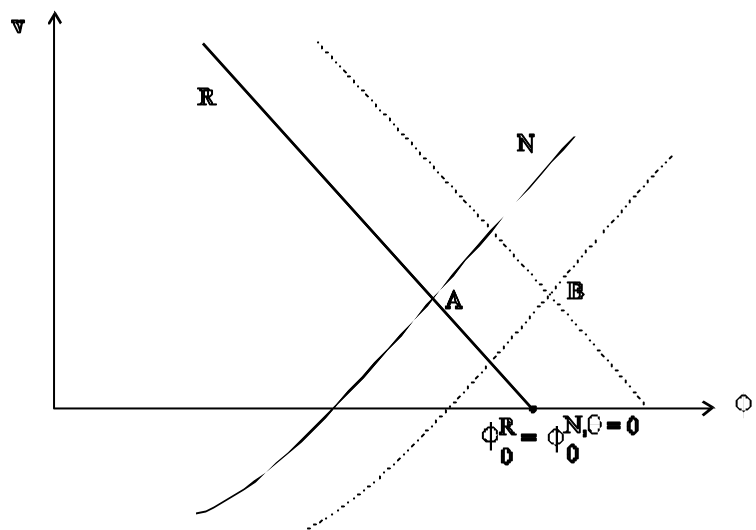

Figure 2

Lemma 2 (i) The N-locus. In a separating and in both types of partially separating price regimes $v = \varphi^N(\phi)$ satisfies $\partial\varphi^N / \partial\phi$ > 0 provided that θ sufficiently small. In a pooling equilibrium $\partial\varphi^N / \partial\phi$ > 0.

(ii) The R-line. The function $v = \varphi^R(\phi)$ is linear in ϕ and has a negative slope.

Lemma 2, part (i) establishes an upward sloping N-locus in Figure 2. To make a higher ϕ profitable, consumers must be willing to pay more for quality which will be the case if assets, v, are higher. According to Lemma 2 (ii), the R-line in Figure 2 has a negative slope. More research activities are only possible if resources are transferred from the production to the R&D sector. This will be the case for a lower v, since then all households become poorer and reduce the level of c_i.

Proposition 1.

> There exists a unique general equilibrium with positive ϕ^* and v*, provided that the rate of time preference is sufficiently small. In a pooling regime, it exists for any (positive) rate of time preference.

The intuition behind Proposition 1 is simple. According to (2.9), ϕ approaches the limit $\Sigma_t \Pi_t / (wF)$ for $\theta \to 0$, thus the R&D intensity ϕ is positive for small θ. Moreover, since marginal utility from consuming the quantity good approaches infinity as c_i goes to zero (see equation (2.1)), it will be consumed in positive amounts, which rules out a situation where the R&D-sector uses the whole resource base. Thus, the existence of an interior solution is guaranteed (point A in Figure 2). By Lemma 2(i), a sufficiently small θ also implies that the N-locus is monotonically increasing under all types of price regimes, whereas the R-locus has a negative slope. Therefore, the equilibrium is unique.

3.3. Inequality and the rate of innovation

We now come to discuss the central question of the paper, namely how inequality in the distribution of assets influences the rate of innovation. It turns out that the answer to this questions depends crucially on the particular price regime. All proofs can be found in the Appendix.

A separating price regime.

Proposition 2. In a separating price regime: (i) $\partial\phi^*/\partial d_P > 0$ if $\beta_P(1-\phi^2/(\phi+\theta)^2) < \beta_R/k^2 + \beta_M\phi/((\phi+\theta)k)$ (sufficient), (ii) $\partial\phi^*/\partial d_M > 0$ if $\beta_M(1-\phi/(\phi+\theta)) < \beta_R/k$ (sufficient), (iii) $\partial\phi^*/\partial\beta_P < 0$ and (iv) $\partial\phi^*/\partial\beta_M < 0$.

Recall that increasing d_i and decreasing β_i (i = P, M) implies a more even distribution of assets. Proposition 2 therefore states that ϕ is unambiguously *increasing* with *less* inequality, if the latter is due to lower group shares β_i of either group M or P (and, consequently a larger share of group R)[11]. Moreover, increasing d_i (i = M, P) at the expense of d_R has a tendency toward a higher ϕ as well, although with respect to this dimension of inequality the effect is ambiguous.

To understand the reason for the results in Proposition 2 it is instructive to consider the impact of the wealth distribution parameters on the price structure (equations (2.5) - (2.8) using $K_i =$ w/θ + $d_i v/L$). If d_P increases, the rich become poorer, whereas the middle class is not concerned. The poor are willing to pay more for quality, so p_{-2} will rise. Therefore also p_{-1} - the maximum price ensuring that the middle class prefers quality q_{-1} to q_{-2} - rises. The impact on p_0, however, is ambiguous. On the one hand, p_0, the maximum price such that the rich prefer q_0 to q_{-1}, can be larger with a larger p_{-1}. On the other hand, a higher d_P reduces the value of assets owned by the rich, and thus depresses their valuation of quality. If the sufficient condition in Proposition 2(i) is satisfied, p_0 will increase. In that case all prices increase and innovation activities are more profitable for any given value of assets. In Figure 2 the N-locus shifts to the right. Moreover, if all p_j's are higher, all consumers will spend less on the standard good c_i, for any given value of v and more resources become available for R&D. The R-locus in Figure 2 shifts also to the right. In sum, the economy is shifted from point A to point B, the latter being characterized by a larger rate of innovation.

An analogous argument can be made for a change in d_M. The only difference is that now p_{-2} remains constant since the change in distribution does not concern the poor. p_{-1} will increase, whereas - just like before - the impact on p_0 is ambiguous. Again, if the condition in Proposition 2 (ii) is met, p_0 increases, resulting in a higher intensity of R&D.

A decrease in β_P implies that the rich become relatively poorer, whereas the wealth positions of groups P and M remain unchanged. Consequently, p_{-2} and p_{-1} stay the same, while p_0

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[11] Recall that $d_R = (1-\beta_P d_P - \beta_M d_M)/(1-\beta_P - \beta_M)$, so decreasing either β_M or β_P decreases d_R as well. Therefore not only becomes the group of rich larger, but also relatively poorer.

decreases. Since group P is now smaller, the market for quality q_2 has shrinked and Π^P has decreased. Group M is unaffected and therefore Π^M stays constant. The market for the q_0-producer has become larger as a result of a higher population share of the rich, whereas p_0 is now smaller, since the rich have become relatively poorer. It is however straightforward to show that (i) Π^R unambiguously increases and (ii) that this increase is larger than the reduction in Π^P. This has two effects: First, it makes innovations more attractive, shifting the N-locus to the right. Second, since profits are higher, a larger part of expenditures is devoted to purchase the quality goods, with a resource releasing effect from the quantity-good sector. Consequently, also R shifts to the right and the economy moves from equilibrium A to B. Analogous arguments can be used to explain result (iv) in Proposition 2.

Partially Separating Price Regime

Case A: Group M purchases quality q_0.

Proposition 3A. In a partially separating equilibrium, where group M purchases quality q_0:

(i) $\partial\phi^*/\partial d_P > 0$, (ii) $\partial\phi^*/\partial d_M > 0$, (iii) $\partial\phi^*/\partial\beta_P < 0$, and (iv) $\partial\phi^*/\partial\beta_M = 0$.

If the middle class purchases the top quality, all distribution parameters have an unambiguous impact on ϕ^*. Increasing d_P yields a larger p_{-1}, hence p_0, which is now the maximum price such that group M prefers q_0 to q_{-1}, will increase. This makes innovation more attractive and shifts the N-locus to the right. For a given value of v, higher prices for quality result in less expenditures for the c-good with a resource-saving effect. As a result, also R shifts to the right and ϕ^* unambiguously increases. A similar argument applies to the effects of an increase in d_M (increase in p_0).

A more even income distribution through a decrease in β_M does not change ϕ^*. The reason is that neither p_1 nor p_0 are affected and the same amount of resources is spent on the c-good (albeit in a changed composition with respect to income classes). So R- and N-curves remain unaffected and ϕ^* stays constant.

Finally, a decrease in β_P leaves prices unchanged, because both the middle class and the poor are as wealthy as before. However, as profits from selling to the rich are higher than those from selling to the poor, total profits increase with a decrease of β_P, implying less expenditures on the c-good. Moreover, innovation becomes more attractive, since the q_0-good has now a larger market share. Consequently, the R- and the N-locus shift to the right and ϕ^* increases.

Case B: Group M purchases q_{-1}.

Proposition 3B. In a partially separating equilibrium, where group M purchases quality q_{-1}:

(i) $\partial\phi^*/\partial d_P > 0$ if $\beta_P(1-\phi/(\phi+\theta)) < \beta_R/k + \beta_M\phi/(\phi+\theta)$, (ii) $\partial\phi^*/\partial d_M < 0$,

(iii) $\partial\phi^*/\partial\beta_P < 0$ and (iv) $\partial\phi^*/\partial\beta_M < 0$.

An increase in d_P raises p_{-1} and has an ambiguous effect on p_0 which now just ensures that the *rich* prefer q_0 to q_{-1}. As long as the sufficient condition in Proposition 3B (i) is satisfied, R- and N-locus both shift to the right leading to an increase in ϕ^*.

Giving more purchasing power to the middle class discourages innovation activities. With a larger d_M (= lower d_R), p_0 decreases whereas p_{-1} is independent of d_M: the N-locus shifts to the left. Also, a lower p_0 induces the rich to spend more on the c-good resulting in higher employment in production and less employment in R&D. So, the R-locus shifts to the left as well.

A smaller group size of the poor increases the R&D intensity ϕ^* by a similar reason than before. p_{-1} stays constant, p_0 is now smaller since a lower β_P decreases d_R. However, the larger group size of the rich offsets this effect, and both R- and N-locus shift to the right. Exactly the same argument applies to a decrease in β_M.

A Pooling Price Regime

Proposition 4. In a pooling price regime (all groups purchase quality q_0): (i) $\partial\phi^*/\partial d_P > 0$, and

(ii) $\partial\phi^*/\partial d_M = \partial\phi^*/\partial\beta_P = \partial\phi^*/\partial\beta_M = 0$.

Increasing d_P increases p_0 which is now the maximum price such that the poor prefer q_0 to q_{-1}. This makes innovations more profitable and reduces the demand for the c-good. Consequently, both R- and N-locus shift to the right, ϕ^* increases.

More purchasing power for the middle class has no effect in this scenario, since it has no impact on prices. With group shares staying the same only the composition (the rich consume less, the middle class consumes more) but not the aggregate demand for the c-good is affected. Consequently, there is no impact on ϕ^*. Since in a pooling equilibrium neither the R- nor the N-curve is affected by the size of the various groups, it immediately follows that neither β_P nor β_M have an impact on ϕ^*.

4. Conclusions

In this paper we have studied the impact of the distribution of income on the rate of innovation. Inequality plays a role because consumers purchase the quality good in a fixed quantity, implying that richer households have a higher willingness to pay for quality. As a result, inequality affects the prices for the quality good and therefore the profitability of conducting R&D. We have confined the analysis to studying the distribution of wealth. It should be clear that exactly the same mechanisms are at work when households have different wage rates.

In equilibrium, more firms in addition to the quality leader may have a positive market share, leading to various possible regimes in equilibrium. This is different from previous endogenous growth models where only the top quality producer is on the market. Within the present framework such a situation can only arise if the degree of heterogeneity between consumers is sufficiently small. With a more skewed distribution up to three firms may have a positive market share, since we have confined the analysis to three types of income classes - the poor, the middle and the rich.

We find that less income inequality tends to improve the profitability of innovations. This is because a more equal distribution allows quality producers to charge higher prices: if only the quality leader is on the market, the price of his product can be larger if, due to more income, the poor are willing to pay more for quality; if more than one firm has a positive market share, not only the price for the worse quality can be higher with more equality, but also better qualities may be selling at higher prices. The role of a high purchasing power of the middle class turns out to be quite ambiguous, depending on the specific price regimes.

Appendix

To proof Lemma 2 and Propositions 1 - 4 we use equations (2.5) - (2.8) together with $\theta K_i = w + \theta d_i v / L$ and the formulas for β_R and d_R to express Π_t, the profit flow of a quality producer during period t (t = 0, 1, 2) in terms of v and the parameters β_P, β_R, d_P, d_R. The Π_t are described at the end of Subsection 2.3 for the four different regimes, where $\Pi^i = \beta_i L(p_j^i - wa)$ with p_j^i as the price of the respective quality purchased by group i, i = P, M, R.

Proof of Lemma 2

(i) <u>The N-locus</u>. φ^N is implicitly defined by writing equation (2.9) as N(ϕ, v; β_P, β_M, d_P, d_M) = 0 with $N(\cdot) = -wF + \sum_{t=0}^{2} \Pi_t \phi^t / (\phi + \theta)^{t+1}$. We have $N_\phi = \sum_{t=0}^{2} \Pi_t \left[(t(\phi + \theta) - (t+1)\phi)\phi^{t-1} / \ (\phi + \theta)^{t+2} \right]$. Clearly, $N_\phi < 0$ for θ small enough, while $N_\phi < 0$ for any θ if $\Pi_1 = \Pi_2 = 0$ (pooling equilibrium). $N_v = \sum_{t=0}^{2} (\partial \Pi_t / \partial v) \phi^t / (\phi + \theta)^{t+1} > 0$ follows from differentiation of the Π_t for the various regimes. $\partial \varphi^N / \partial \phi = -N_\phi / N_v$ completes the proof of Lemma 2 (i).

(ii) <u>The R-line</u>. Equation (3.1) can be rewritten as R(ϕ, v; β_P, β_M, d_p, d_M) = 0 with $R(\cdot) = -wF + (\sum_{t=0}^{2} \Pi_t - \theta v) / \phi$. (Use (2.3), $\theta K_i = w + \theta d_i v / L$, the formulas for β_R and d_R; moreover note that wa can be written as wa(β_P + β_M + (1 - β_P - β_M)) and $\Pi^i = \beta_i L(p_j^i - wa)$). R($\cdot$) = 0 implicitely defines the relation $v = \varphi^R(\phi)$. The Π_t are linear in v but do not depend on ϕ, hence φ^R is a linear relation. Its slope is determined by implicit differentiation: $\partial \varphi^R / \partial \phi = -R_\phi / R_v$, with the partial derivatives $R_\phi = -(\sum_{t=0}^{2} \Pi_t - \theta v) / \phi^2 < 0$ and $R_v = (\sum_{t=0}^{2} \partial \Pi_t / \partial v - \theta) / \phi < 0$ for $\theta > 0$, $R_v = 0$ for $\theta = 0$; where the inequality follows from explicitely taking derivations of the Π_t with respect to v, and using $d_p < d_M$, k > 1 and (1 - β_P)d_M < 1 - $\beta_P d_P$ < 1 (see the footnote in Section 3.1).

Proof of Proposition 1

We compute, for any $\theta \geq 0$, the root ϕ_0^R of φ^R (where $\varphi^R(\phi_0^R) = v = 0$) as $\phi_0^R = (\sum_{t=0}^{2} \Pi_t(0)) / wF$, where $\Pi_t(0)$ denotes profits at v = 0. ϕ_0^R coincides with the root $\phi_0^{N,\theta=0}$ of φ^N, if $\theta = 0$. We know already that $\partial\varphi^N / \partial\phi$ is positive for small θ. Then Proposition 1 follows from continuity of φ^N with respect to θ and the fact that for small θ the root $\phi_0^{N,\theta}$ decreases with θ: $\partial\phi_0^{N,\theta}/\partial\theta = -N_\theta/N_\phi < 0$ (we know $N_\phi < 0$; $N_\theta = -\sum_{t=0}^{2} \Pi_t(t+1)\phi^t / (\phi+\theta)^{t+2} < 0$, as $\partial\Pi_t/\partial\theta = 0$ for v = 0). In case of pooling the respective assertions hold for any $\theta \geq 0$.

Proof of Propositions 2 - 4

The influence of the inequality parameters d_P, d_M, β_P, β_M on the equilibrium value ϕ^* is determined by considering the shifts of the φ^R- and φ^N-curves caused by an increase of each of these parameters in turn. If both curves are shifted into the same direction, then there is an unambiguous effect on ϕ^* (see Figure 2 in Section 3.2). The shifts are determined by implicit differentiation of ϕ with respect to some parameter α, using $R(\cdot) = 0$ and $N(\cdot) = 0$. That is, we compute $\partial\phi^R / \partial\alpha = -R_\alpha / R_\phi$, $\partial\phi^N / \partial\alpha = -N_\alpha / N_\phi$, for $\alpha = d_P, d_M, \beta_P, \beta_M$.

$R_\phi < 0$, $N_\phi < 0$ is known from before. The signs of $R_\alpha = (\sum_{t=0}^{2} \partial\Pi_t / \partial\alpha) / \phi$ and $N_\alpha = \left[\sum_{t=0}^{2} (\partial\Pi_t / \partial\alpha)\phi^t / (\phi+\theta)^t\right] / (\phi+\theta)$ are directly determined by differentiation of the profits with respect to α, for the four equilibrium situations (note $k > 1$, $d_M > d_P$, $1 > \phi/(\phi+\theta) > \phi^2/(\theta+\phi)^2$). Straightforward computation of the bounds on θ in the Propositions 2 (pooling) and 3B (partially separating, B) completes the proof.

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