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A Suggestion

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Abstract

This paper presents a characterization of a welfare index for the evaluation of *primary goods* (to be understood as those goods that all agents *should* enjoy equally). The welfare associated with a given distribution of n primary goods among m agents is measured as the sum of n real-valued functions, one for each good, which are increasing in the aggregate consumption and decreasing in its dispersion (measured by Theil's first inequality index).

Keywords

Welfare evaluation, inequality, primary goods

JEL Classifications

D30, D63

Comments

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Contents

1 Introduction

Rawls (1971) introduces the notion of *primary goods* as a reference for his theory of justice. These goods are deemed essential for the survival and self-respect of agents, so that no trade-off between these and other goods can be admitted. Rights, liberties, income and opportunities are the main categories of this type of goods. The chief idea in Rawls formulation is that all agents have the right to enjoy these goods equally, so that the key principle for social evaluation is the maximin criterion. Similar ideas have been pursued by some economists, trying to disentangle variables such as opportunities, rights or needs from utilities. Among them let us mention the works of Kolm (1972) and Sen (1985) [the reader is referred to the discussion in Fleurbeay (1996) and Roemer (1996, ch. 5); see also the approach in Herrero (1996)].

Let us narrow the scope of the analysis and concentrate on distribution problems. Following the idea behind Rawls' difference principle, let us apply the term "primary goods" to *those commodities that should be distributed equally among the agents*. We can think of goods such as basic schooling, primary health services, social security benefits, etc. That is, goods which have to do with the equality of opportunity of economic agents. These goods typically represent the material counterpart of some basic rights that define the entitlements of the citizens in a given society. Note that in some cases the consumption of primary goods is to be interpreted as *potential* consumption rather than real consumption (meaning that, as in the case of public goods, what is important is the availability of these goods).

The purpose of this paper is to provide an index that can be considered as *a cardinal welfare measure* of the allocation of these primary goods. For that we start by restricting the choice of welfare indices to the family of homogeneous functions. This allows to evaluate allocations as a weighted sum of individual consumption vectors. These weights describe the individuals' social marginal worth in the evaluation function. Therefore, our value judgements on social welfare can be naturally expressed in terms of the weighting system.

The key equity principle in this analysis will be that of *progressivity*. By this we mean that the ith agent's social marginal worth, with respect to a given commodity, is negatively correlated and inversely proportional to her share in total consumption. That is, we are going to give progressively more weight in social welfare to those agents with smaller shares in total consumption, and viceversa. Combining the progressivity principle with a suitable scaling system permits one to measure social welfare as the sum of n partial indices (one for each primary good). Each of these partial indices is a function increasing in the amount of the good consumed and decreasing in its dispersion, measured by Theil's first inequality index.

The analysis is carried out according to two methodological principles:

(i) *Social welfare is defined directly in terms of the availability of primary goods and not as a function of agents' utilities.* This is an explicit departure from welfarism because agents' utilities are not the leading variables in the welfare evaluation of primary goods.

(ii) *The social marginal worth attached to the consumption of a given primary good depends on the good considered and the agent who consumes it.* This principle is related to Sen's (1976) "personalized goods approach" and allows us to evaluate allocations by means of a system of shadow prices which weight commodities differently, depending on the agent who consumes them.

This methodological approach has been successfully applied to the welfare analysis of one-dimensional distribution problems, following the ideas of Sen (1976), (1979) [see for instance Osmani (1982), Chakravarty & Dutta (1990), Herrero &Villar (1992)]. Our contribution here extends the works in Herrero & Villar (1989) and Tomás & Villar (1993) on the evaluation of income distributions, allowing for a multidimensional variable and refining the axiomatization of the welfare measure.

It will be implicitly assumed through the paper that: (a) The class of primary goods is already given —that is, we shall not discuss here which commodities ought to be considered as primary goods; (b) Primary goods are measurable by some real numbers that describe their availability (physical units); (c) Each primary good has associated with it a *market price* (or a well defined *unitary cost*); and (d) Agents are taken to be homogeneous; namely, we shall ignore the scaling problem that arises when agents are different in size and characteristics [there are standard procedures to deal with this problem by means of equivalence scales; see for instance Deaton & Muellbauer (1980), Ruiz-Castillo (1995)].

Let us point out that these results are applicable to a number of practical problems. One of particular interest refers to the social evaluation of *local* *public goods*. This is an important problem when we consider an economy consisting of m different regions which are responsible for the provision of a number of public services (health, education, unemployment benefits, etc.). One can think of the European Union or of a Federal State that gives their citizens the right to enjoy these basic services, no matter where they choose to live.[1] The welfare index proposed here provides a social evaluation of the overall allocation of public services, depending upon the quantities provided by each State and its dispersion.

The work is organized as follows. Section 2 contains the model and the characterization result. Section 3 elaborates on the application of this formula to the analysis of the many-groups many-goods case.

2 The model and the results

Consider an economy consisting of m homogeneous agents and n primary goods, measured in some given units. A point $\mathbf{x}_i \in \mathbb{R}^n_{++}$ denotes a **consumption vector** for the ith agent, $i = 1, 2, ..., m$ (that is, x_{ij} describes the amount of the jth primary good available for this agent, that we take to be strictly positive). A point $\mathbf{x} \in \mathbb{R}^{mn}_{++}$ describes an **allocation** for the economy. For every $j = 1, 2, ..., n$, call $X_j = \sum_{i=1}^{m} x_{ij}$ —that is, X_j is the aggregate amount of commodity j in the distribution $\mathbf{x}$.

The key point of our analysis is the identification of a welfare criterion that enables the evaluation of allocations $\mathbf{x} \in \mathbb{R}^{mn}_{++}$. That is, we look for a **social evaluation function** $V : \mathbb{R}^{mn}_{++} \to \mathbb{R}$ which permits one to perform welfare assessments of the overall allocation of primary goods. The properties of this evaluation function will reflect the value judgments involved. We shall restrict the search of this evaluation function to the family of cardinal (and smooth) measures. To be precise, we denote by $\mathbb{V}$ the family of *evaluation functions* $V : \mathbb{R}^{mn}_{++} \to \mathbb{R}$ *that are homogeneous of degree one and twice differentiable*.

The homogeneity property is equivalent to the existence of a complete, continuous and homothetic social preference preordering on the set of allocations $\mathbb{R}^{mn}_{++}$. It introduces a cardinal element in the evaluation, as $V(\lambda \mathbf{x}) =$

[1] Needless to say that in this case agents may be widely different witht respect to their size and characteristics, and that the scaling problem comes to the forefront.

$\lambda V(\mathbf{x})$ for every $\lambda > 0$. The differentiability property is an operational requirement that will facilitate our reasoning.

For any given $V \in \mathbb{V}$ Euler's theorem implies that the evaluation of a given allocation $\mathbf{x} \in \mathbb{R}^{mn}_{++}$ can be expressed as $V(\mathbf{x}) = \nabla V(\mathbf{x})\mathbf{x}$, where $\nabla V(\mathbf{x})$ stands for the vector of partial derivatives of V. Hence, calling $f_{ij}(\mathbf{x}) = \partial V(\mathbf{x})/\partial x_{ij}$ for each $\mathbf{x}$ in $\mathbb{R}^{mn}_{++}$ we have:

$$V(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}(\mathbf{x})x_{ij} \qquad [1]$$

This expression says that the social welfare of allocation $\mathbf{x}$ can be measured as a weighted sum of individual consumption levels, where the coefficient $f_{ij}(\mathbf{x})$ describes the **social marginal worth** of individual i as a consumer of the jth primary good. Note that taking V in $\mathbb{V}$ implies that these weights are homogeneous of degree zero. That is, each agent's social marginal worth depends on the distribution of primary goods but not their levels.

Now we shall establish some assumptions on this evaluation function $V \in \mathbb{V}$. These axioms express our value judgments, in terms of properties of the weighting system $f_{ij}(\mathbf{x})$.

Axiom 1 (Independence) *For every $i = 1, 2, ..., m$ and every $j = 1, 2, ..., n$, $f_{ij}(\mathbf{x}) = f_j(x_{ij}, X_j)$.*

Axiom 2 (Minimal Equity) *Let $\mathbf{x}$, $\mathbf{x}'$ be such that $\sum_{i=1}^{m} x_{ik} = \sum_{i=1}^{m} x'_{ik}$ for all k, and let $x_{ij} > x'_{ij}$. Then, $f_{ij}(\mathbf{x}) < f_{ij}(\mathbf{x}')$.*

Axiom 3 (All for Nothing) $\lim_{x_{ij} \to X_j} f_{ij}(\mathbf{x}) = 0$.

Axiom 4 (Progressivity) *Let $\varepsilon_{ij}(\mathbf{x})$ denote the elasticity of the weighting function with respect to x_{ij}. For all $i = 1, 2, ..., m$, all $j = 1, 2, ..., n$,*

$$\varepsilon_{ij}(\mathbf{x}) = \frac{\alpha_j}{f_{ij}(\mathbf{x})}$$

Axiom 1 translates into this context the notion of *decentralizability* commonly used in the literature on cost/surplus sharing problems [e.g. Moulin (1988, ch. 6)]. It may be seen as combining informational efficiency (as this is the only relevant information) and anonymity (the social marginal worth does not depend on the names of the agents). When $n = 1$ it simply says that the social marginal worth of each agent depends only on its own consumption and on the aggregate (actually on her *share*). When $n > 1$ this also expresses the idea that the marginal worth of an agent, as a consumer of a given good, is evaluated independently of her consumption of other goods. Namely, there is no substitutability in agents' marginal worth, as the very notion of primary goods suggests.

Axiom 2 says that the change in the social marginal worth of the ith agent due to a change in her own consumption, while keeping constant the total amount available, is negatively correlated to her consumption level. The axiom of Minimal Equity, introduced by Sen (1973, p. 18), constitutes a basic value judgment: we are going to give more weight in social welfare to those agents with smaller relative consumption of primary goods.

Axiom 3 says, roughly speaking, that when a single agent is the only consumer of a given primary good her social marginal worth is taken to be zero. So getting all means contributing nothing to social welfare. Note that axioms 2 and 3 together imply that $f_{ij}(\mathbf{x}) \geq 0$, that is, social welfare grows with the amounts of goods available.

Finally, axiom 4 postulates that the elasticity of f_{ij} with respect to x_{ij} is inversely proportional to f_{ij}. That is, the ith agent's social marginal worth as a consumer of the jth primary good changes more the smaller her weight. It follows from axioms 2 and 3 that $\alpha_j < 0$. Therefore, axioms 2, 3 and 4 together establish that the poorer an agent is, the smaller the reduction of her weight in social welfare associated with a given increase in her share.

Remark 1 *Progressivity makes this welfare index compatible with the "principle of Dalton", which postulates that a transfer from rich to poor that does not change their ranking increases social welfare. It can also be regarded as an instance of "second order Minimal Equity" (as it applies this principle to the change in the social marginal worth).*

Let us recall here the definition of Theil's first inequality index.[2] Let $\mathbf{y} \in \mathbb{R}^m_{++}$ stand for a distribution of a given one-dimensional variable, and call $z_i = y_i/\mu$ (where μ stands for the average). Theil's first inequality index is given by:

$$T(\mathbf{y}) = \frac{1}{m}\sum_{i=1}^{m} z_i \ln z_i$$

When $\mathbf{y}$ is a vector of personal incomes $T(\mathbf{y})$ can be interpreted as a measure of the distance between population shares and income shares [see Theil (1967)]. It is easy to see that $0 \leq T(\mathbf{y}) \leq \ln m$. This suggests that we can define the **normalized Theil's inequality index** as $\widetilde{T}(\mathbf{y}) = \frac{1}{\ln m}T(\mathbf{y})$, so that $0 \leq \widetilde{T}(\mathbf{y}) \leq 1$.

For $\mathbf{x} \in \mathbb{R}^{mn}_{++}$, let $T_j(\mathbf{x})$ denote the value of Theil's inequality index relative to the distribution of the jth variable, and $\widetilde{T}_j(\mathbf{x})$ the associated normalized index. The following result is obtained:

Theorem 1 *A social evaluation function $V \in \mathbb{V}$ satisfies axioms 1 to 4 if and only if, for every $\mathbf{x} \in \mathbb{R}^{mn}_{++}$ we have*

$$V(\mathbf{x}) = \sum_{j=1}^{n} \beta_j X_j \left[1 - \widetilde{T}_j(\mathbf{x})\right]$$

where $\beta_j > 0$ for all j.

Proof.

We know that the evaluation function $V \in \mathbb{V}$ can be written as $V(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}(\mathbf{x})x_{ij}$. It follows from axioms 1 to 4 that:

$$\frac{\partial f_{ij}(\mathbf{x})}{\partial x_{ij}} = \frac{\partial f_j(x_{ij}, X_j)}{\partial x_{ij}} = \frac{\alpha_j}{x_{ij}} \qquad [2]$$

with $\alpha_j < 0$. Moreover, as f_{ij} is homogeneous of degree zero in $\mathbf{x}$, it follows that $f_j(\lambda x_{ij}, \lambda X_j) = f_j(x_{ij}, X_j)$. Therefore, letting $\lambda_j = m/X_j$ we can define an auxiliary function $\gamma_j : \mathbb{R}_{++} \to \mathbb{R}$ as follows:

[2] For a discussion of this inequality index the reader is referred to Blackorby & Donaldson (1978), Bourguignon (1979), Cowell & Kuga (1981) and Foster (1983).

$$f_{ij}(\frac{1}{\mu_j}\mathbf{x}) = f_j(\frac{x_{ij}}{\mu_j}, m) = \gamma_j(s_{ij})$$

where $\mu_j = X_j/m$ is the average consumption of the jth primary good and $s_{ij} = x_{ij}/\mu_j$ represents the ith agent's share in this average. By construction, and in view of [2] above, we can write:

$$\frac{\partial f_j}{\partial x_{ij}} = \frac{d\gamma_j}{ds_{ij}}\frac{1}{\mu_j} = \frac{\alpha_j}{x_{ij}}$$

which gives us:

$$\frac{d\gamma_j(s_{ij})}{ds_{ij}} = \frac{\alpha_j}{s_{ij}}$$

Solving this differential equation we obtain:

$$\gamma_j(s_{ij}) = C_j + \alpha_j \ln s_{ij} \qquad [3]$$

Axiom 4 establishes that $f_{ij}(\mathbf{x}) \rightarrow 0$ when $x_{ij} \rightarrow X_j$. Hence, taking the limit case we find that $\gamma_j(m) = 0$, which in view of [3] implies that $C_j = -\ln m\alpha_j$. Therefore,

$$\begin{aligned} V(\mathbf{x}) &= \sum_{j=1}^{n}\left[\sum_{i=1}^{m}(-\ln m\alpha_j + \alpha_j \ln s_{ij})\, x_{ij}\right] = \sum_{j=1}^{n}\left[-\ln m\alpha_j X_j + \alpha_j \sum_{i=1}^{m} x_{ij}\ln s_{ij}\right] \\ &= \sum_{j=1}^{n}\left[-\ln m\alpha_j X_j + \alpha_j\mu_j \sum_{i=1}^{m} s_{ij}\ln s_{ij}\right] = \sum_{j=1}^{n} X_j \left[-\ln m\alpha_j + \alpha_j \frac{1}{m}\sum_{i=1}^{m} s_{ij}\ln s_{ij}\right] \\ &= \sum_{j=1}^{n} X_j\left[-\ln m\alpha_j + \alpha_j T_j(\mathbf{x})\right] = \sum_{j=1}^{n}(-\ln m\alpha_j)X_j\left[1-\widetilde{T}_j(\mathbf{x})\right] \end{aligned}$$

Finally, letting $\beta_j = -\ln m\alpha_j > 0$, we get:

$$\sum_{j=1}^{n}\beta_j X_j\left[1-\widetilde{T}_j(\mathbf{x})\right]$$

■

Theorem 1 says that choosing an evaluation function in the set $\mathbb{V}$ that satisfies axioms 1 to 4 is equivalent to measuring social welfare as a weighted sum of the amounts of primary goods available, each deflated by a term that expresses the distance with respect to the egalitarian distribution, measured by Theil's normalized inequality index.

Note that each term $X_j\left[1-\widetilde{T}_j(\mathbf{x})\right]$ corresponds to the **egalitarian equivalent** amount of commodity j, in the sense of Atkinson-Kolm-Sen. That is, the amount of commodity j that equally distributed would make the society as well-off as with the real amount available, when inequality is measured by Theil's normalized inequality index. To see this notice that the egalitarian equivalent amount of the jth commodity, whose distribution is described by a vector $\mathbf{x}^j = (x_{1j}, x_{2j}, ..., x_{mj}) \in \mathbb{R}^m_{++}$, with $X_j = \sum_{i=1}^m x_{ij}$ is given by mx_j^*, where x_j^* is the quantity that satisfies:

$$\widetilde{T}_j(\mathbf{x}) = 1 - \frac{x_j^*}{\mu_j}$$

Therefore, $mx_j^* = X_j[1-\widetilde{T}_j(\mathbf{x})]$. Consequently, by letting X_j^* to denote the egalitarian equivalent amount of j, the Theorem boils down to:

$$V(\mathbf{x}) = \sum_{j=1}^n \beta_j X_j^*$$

Remark 2 *Observe that the homogeneity of V ensures a one to one correspondence between welfare measures and inequality measures [see Blackorby & Donaldson (1978)].*

The coefficients $\beta_1, ..., \beta_n$ scale the contribution of the egalitarian equivalent amounts of primary goods to social welfare. The determination of these parameters requires imposing further restrictions on the evaluation function. In so doing one has to take into account that our evaluation function V might be sensitive to the units in which different commodities are measured. That is, it might be that $V(\mathbf{x}) > V(\mathbf{y})$ for some $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{mn}_{++}$, whereas $V(\mathbf{x}') < V(\mathbf{y}')$, when $\mathbf{x}', \mathbf{y}'$ are the same commodity vectors measured in different units.[3]

[3]Needless to say that this problem does not appear when $n = 1$, or when commodities are expressed in value terms (i.e. $x_{ij} = p_j y_{ij}$, where y_{ij} denotes a given amount of good j in physical units and p_j its corresponding price).

There is a number of alternative scales that permits one to close the evaluation formula. Each one amounts to assign relative weights to primary goods as components of social welfare. The procedure we adopt here is to find these parameters by fixing the values that individual weights f_{ij} take on at the egalitarian distribution, $x_{ij} = \mu_j$ for all i, j. This is the polar case considered in the "All for nothing" axiom, and provides a natural way to define an additional reference point.

Let $\mathbf{p} \in \mathbb{R}^n_{++}$ stand for the vector of market prices (or unit costs) of primary goods. By taking this vector as given, we postulate:

Axiom 5 (Price scale) *$x_{ij} = X_j/m$ for all $i = 1, 2, ..., m$, all $j = 1, 2, ..., n$, implies $f_{ij}(\mathbf{x}) = p_j$.*

This axiom establishes that when *all* agents consume identical amounts of a given commodity, a perfectly egalitarian allocation, we take commodity prices as a measure of social welfare. One may interpret this as a way of measuring social welfare which is respectful with agents' *unanimous* judgments. Namely, all agents agree on the marginal worth of that commodity (as market prices correspond precisely to the ratio of agents' marginal utilities), and all agents enjoy the same consumption.

Scaling the weighting system by means of a reference price vector has three major advantages: (1) It makes V independent of changes in the units of measurement (because changing the unit of commodity j by a factor λ_j implies dividing its price by λ_j); (2) It has a well defined economic meaning: it fixes a scale that reflects the relative weight that markets give to these goods (and that we would admit as welfare weights *only* in the hypothetical situation in which all agents consume identical amounts); and (3) It permits one to choose units so that $p_j = 1$ for all j. In this case x_{ij} represents both the physical amount of the j commodity available for the ith agent, and the ith agent's expenditure in the jth good.

The following result is obtained:

Corollary 2 *A social evaluation function $V \in \mathbb{V}$ satisfies axioms 1 to 5 if and only if, for every $\mathbf{x} \in \mathbb{R}^{mn}_{++}$ we have*

$$V(\mathbf{x}) = \sum_{j=1}^{n} p_j X_j \left[1 - \widetilde{T}_j(\mathbf{x})\right]$$

Proof.

Axiom 5 says that $f_{ij}(\mathbf{x}) = p_j$ when $x_{ij} = \mu_j$, that is, $\gamma_j(1) = p_j$. Substituting this value in equation [3] and bearing in mind that axiom 4 implies that $C_j = \beta_j$, it follows that $\beta_j = p_j$. Substituting in the last equation we obtain the desired result. ■

This result says that the welfare evaluation associated to axioms 1 to 5 is given by the sum of n terms, one for each primary good. Each term measures the contribution of a particular good to the social welfare as its market worth deflated by the normalized Theil's inequality index. Therefore, one can take the aggregate worth of primary goods as a suitable measure of social welfare if and only if all goods are equally distributed (that is, when $T_j(\mathbf{x}) = 0$ for all j so that $V(\mathbf{x}) = \sum_{j=1}^{n} p_j X_j$).

Remark 3 *Observe that when our variables are expressed in value terms (as it will typically be the case in many empirical applications) this price scale is automatically incorporated by letting $\beta_j = 1$ for all j.*

Remark 4 *The price scale can be criticized because market prices partly reflect the initial distribution of endowments (via the income and substitution effects between primary goods and other commodities). To avoid this problem one might take a different reference price vector, such as the equilibrium price vector associated to the egalitarian distribution of all resources.*

3 A final comment: The many-goods many-groups case

Let us conclude by briefly commenting on the application of this welfare measure to the analysis of a society made of different sub-societies (think of the regions in a country, the states in the U.S.A., or the countries of the European Union). This is a classical topic in the study of inequality measurement, that is usually addressed under the heading of "decomposability". Note however that here we cannot apply the standard decomposability procedure to our welfare index, because the normalized Theil index is not decomposable. Moreover, normalizing the index is necessary to ensure that our welfare measure is well defined (more precisely, that V is increasing in the amounts of goods and decreasing in their dispersion). There is nevertheless a simple way of assessing the overall welfare loss that is due to the distribution of the primary goods between and within the groups of the society under consideration.

Consider a society made of k sub-societies. To fix ideas let us consider the case of a state consisting of k regions. Let m_s denote the population of region s $(s = 1, 2, ..., k)$. The distribution of the jth primary good *within* region s is described by a vector $\mathbf{y}(j, s) \in \mathbb{R}^{m_s}_{++}$, whereas $\widetilde{\mathbf{y}}(s) = [\mathbf{y}(1, s), ..., \mathbf{y}(n, s)] \in \mathbb{R}^{nm_s}_{++}$ stands for the vector that describes the distribution of the n primary goods within region s.

Under axioms 1 to 5 we can measure the welfare of region s associated with a distribution $\widetilde{\mathbf{y}}(s)$ as follows:

$$V[\widetilde{\mathbf{y}}(s)] = \sum_{j=1}^{n} Y_j^s \left[1 - \widetilde{T}[\mathbf{y}(j, s)]\right] \qquad s = 1, 2, ..., k$$

where Y_j^s is the total worth of good j that is available in region s (evaluated at the appropriate market prices) and $\widetilde{T}[\mathbf{y}(j, s)]$ is the normalized Theil's measure of the inequality in region s with respect to the jth good.

Similarly, let $\widetilde{\mathbf{y}} = [\widetilde{\mathbf{y}}(1), \widetilde{\mathbf{y}}(2), ..., \widetilde{\mathbf{y}}(k)] \in \mathbb{R}^{nm}_{++}$, with $m = \sum_{s=1}^{k} m_s$, denote the whole country's distribution of the n goods. Now for each $j = 1, 2, ..., n$, let $Y_j = \sum_{s=1}^{k} Y_j^s$ and let $\widetilde{T}_j(\widetilde{\mathbf{y}})$ denote the normalized inequality index in the whole population, relative to the jth primary good. Applying our welfare measure country-wise, we obtain:

$$V(\widetilde{\mathbf{y}}) = \sum_{j=1}^{n} Y_j \left[1 - \widetilde{T}_j(\widetilde{\mathbf{y}})\right]$$

Now observe that, for all $s = 1, 2, ..., k$, all $j = 1, 2, ..., n$, the number $Y_j^s \widetilde{T}[\mathbf{y}(j, s)]$ measures the welfare loss that occurs *within* region s due to the unequal distribution of the jth primary good. Therefore,

$$\sum_{s=1}^{k} Y_j^s \widetilde{T}[\mathbf{y}(j, s)]$$

is the aggregate welfare loss within the k regions that derives from the allocation of the jth primary good. Therefore,

$$W = \sum_{j=1}^{n} \sum_{s=1}^{k} Y_j^s \widetilde{T}[\mathbf{y}(j, s)]$$

is *the aggregate welfare loss within the k regions derived from the unequal distribution of the whole bundle of primary goods.*

Similarly, $Y_j \widetilde{T}_j(\widetilde{\mathbf{y}})$ measures the welfare loss of the country due to the overall inequality between individuals associated with the jth primary good, whereas $\sum_{j=1}^{n} Y_j \widetilde{T}_j(\widetilde{\mathbf{y}})$ measures the country's total welfare loss due to the overall inequality. Therefore, for all $j = 1, 2, ..., n$, the difference between these two numbers,

$$Y_j \widetilde{T}_j(\widetilde{\mathbf{y}}) - \sum_{s=1}^{k} Y_j^s \widetilde{T}[\mathbf{y}(j, s)] = B_j$$

gives us the welfare loss that can be attributed to the differences *between* the regions in the allocation of good j, whereas:

$$\begin{aligned} B &= \sum_{j=1}^{n} B_j = \sum_{j=1}^{n} \left(Y_j \widetilde{T}_j(\widetilde{\mathbf{y}}) - \sum_{s=1}^{k} Y_j^s \widetilde{T}[\mathbf{y}(j, s)] \right) \\ &= V(\widetilde{\mathbf{y}}) - \sum_{s=1}^{k} V[\widetilde{\mathbf{y}}(s)] \end{aligned}$$

gives us the total welfare loss that can be attributed to the overall inequality between the k regions.

Since B is precisely the difference between the welfare measure defined on the distribution of the primary goods among whole population and the sum of all regions' welfare measures, we can write:

$$V(\widetilde{\mathbf{y}}) = Y - W - B$$

This expression says that the aggregate welfare of a country, associated with a given distribution of n primary goods, can expressed as the aggregate worth of these goods $Y = \sum_{j=1}^{n} Y_j$, deflated by two components, W and B, that describe the welfare loss due to the inequality within and between the regions, respectively.

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