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Abstract

In many professional labor markets the number of new workers follows a cyclical time path. This phenomenon is usually explained by means of a cobweb model that is based on the assumptions of myopic wage expectations and occupational immobility. Since both assumptions are questioned by the empirical literature, we develop an alternative model that is based on the assumptions of rational wage expectations and perfect occupational mobility. Depending on the production function, the model can generate cycles in the number of workers who enter a professional labor market.

Contents

1   Introduction

It has long been recognized that the supply of new engineers is cyclical over time. There are periods when engineering schools attract many students, followed by periods when they attract only few. As Freeman and Leonard (1978) have shown, enrollment cycles do not occur in the field of engineering only. They identify them in physics, mathematics, chemistry, and education as well. The cycles observed in these fields are dampened over time. In the absence of exogenous shocks they disappear after a couple of years. This phenomenon can also be observed in countries with educational systems different from the one in the United States. Cycles can be found in Germany, for example, which is illustrated in figure 1.

The first model explaining this phenomenon was proposed by Richard Freeman (1971, 1972, 1975a, 1975b, 1976a, 1976b). His model is based on the assumptions of myopic wage expectations and occupational immobility.[1] We illustrate the model taking the engineering profession as an example. Assume that in an initial period there is an excess demand for engineers. It cannot be satisfied by physicists, mathematicians, or other college graduates who are occupationally immobile. As a consequence, the wage for engineers is high in this period. Because high school graduates are myopic, they believe it will stay high forever. Thus a large number of them enrolls in engineering schools, entering the marketplace in the next period. The excess demand then changes into an excess supply. It cannot be removed by the switching of engineers into other professions. Therefore the wage for engineers is low in this period. Because myopic high school graduates expect it to stay low forever, only few of them enroll in engineering schools this time. The following period is therefore characterized by an excess demand again. Then the process repeats, generating a cobweb pattern of engineering employment.

Freeman's model is not undisputed. One of the controversial points is whether wage expectations are really myopic. Some evidence supports this hypothesis. Le- er and Lindsay (1979), for example, find that students in medicine base their wage forecasts on current wages. Leonard (1982) shows that personnel executives even in large firms tend to badly forecast future salaries. Other evidence rejects the hypothesis of myopic wage expectations, supporting the hypothesis of rational wage expectations instead. Willis and Rosen (1979) show that high school graduates can correctly compare future lifetime earnings associated with a high school diploma and future lifetime earnings associated with a college diploma. Hoffman and Low (1983) find that students in economics can forecast future earnings opportunities very well. Garen (1984) presents evidence implying that young males can correctly forecast lifetime earnings associated with different levels of schooling. Siow (1984) shows that law students are good predictors of lifetime

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[1]In a more sophisticated version of Freeman's model, wage expectations are adaptive. This does not make a big difference, however.
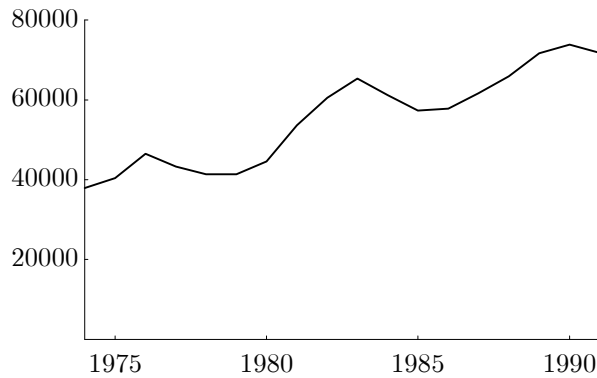
Figure 1: First-Year Students in Engineering, West Germany, 1974 to 1991
Source: Federal Statistical Office, *Statistical Yearbook of the Federal Republic of Germany*

earnings. Berger (1988) presents similar findings for college majors in general. Orazem and Mattila (1991), finally, show that high school graduates are not only able to predict the first moment of the future earnings distribution, but also the second one. So several (though not all) studies suggest that wage expectations are rational and not myopic.[2]

In response to this evidence, Gary Zarkin (1982, 1983, 1985) developed another model aimed at explaining cycles in occupational choice. In his model the assumption of myopic wage expectations is replaced by that of rational wage expectations so that occupations are ranked by future expected lifetime earnings and not by current periodical earnings. The assumption of occupational immobility is not replaced in the model. For an intuitive understanding, return to the example of the engineering profession. Assume that in an initial period the stock of engineers is low and that it cannot be increased by physicists or other professionals switching into the market for engineers. High school graduates then expect to receive lifetime earnings relatively high when entering the engineering profession. So many of them enroll in engineering schools. As a consequence, the number of new engineers is high in the subsequent period, possibly even higher than the number of old engineers retiring from working life. Since it is not possible for new engineers to switch into another occupation, the stock of engineers increases in this case. High school graduates therefore expect to receive lifetime earnings relatively low when entering the engineering profession. Therefore, relatively few of them enroll in engineering schools during this phase. In the following period the number of new engineers thus is low, possibly not even high enough to replace retirees. So the stock of engineers decreases, and expected lifetime

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[2]There is also a study rejecting both the hypothesis of myopic and the hypothesis of rational wage expectations. Betts (1996) finds that only a minority of students is able to accurately rank four different occupations by starting salary.

earnings rise again. At this point the process repeats, generating equilibrium cycles in the market for new engineers.[3]

There is some criticism to Zarkin's model too. The critique refers to the empirical relevance of the assumption of occupational immobility. Many studies find that the degree of occupational mobility is quite high. Börsch-Supan (1990), for example, presents evidence implying that 67 percent of all household heads leave a one digit occupation once or more in 15 years.[4] Velling and Bender (1994) show that 13 percent of all employees leave a two digit occupation once a year. Harper (1995) reports that 11 percent of males in the labor force leave a three digit occupation once a year and that 7 percent do so voluntarily. Mertens (1997) finds occupational turnover rates of 7 percent a year where occupations are defined on the one digit level. Dolton and Kidd (1998), finally, present evidence implying that 24 percent of male college graduates leave a three digit occupation within the first eight years of their working life. This suggests that the assumption of occupational immobility is not consistent with the evidence.

In this paper we present a model which takes this critique seriously. Our model avoids the assumptions of myopic wage expectations and occupational immobility and replaces them by the assumptions of rational wage expectations and perfect occupational mobility. We show that even a model based on these assumptions is able to explain the cyclical behavior of occupational choice observed in many professional labor markets. To simplify the presentation, we proceed in two steps: In the first step, which is found in section 2, we only replace the assumption of myopic wage expectations. In the second step, which is found in section 3, we also abandon the assumption of occupational immobility.

2 A Model with Rational Wage Expectations and Occupational Immobility

Our model is a simple overlapping generations model. In each period a new generation of individuals is born. All generations are equal in size. The size of each generation is normalized to one. Individuals live for two periods. In the first period of life they choose one out of two occupations. In the second period of life they are occupationally immobile and stick to the occupation chosen when young.[5] Occupational choice is made in such a way that lifetime earnings are

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[3]Sometimes the model developed by Zarkin is mentioned in one breath with a model developed by Siow (1984). However, Siow's model can only generate random fluctuations in occupational choice. Cycles cannot be generated by the model.

[4]Studies defining occupations on a one/two/three digit level can distinguish up to ten/hundred/thousand different occupational categories.

[5]We ignore an extra period when individuals attend college. Such a period could easily be introduced into the model, but it would merely increase complexity without yielding any additional insights.

maximized.[6] Individuals are hired by firms. In each period firms thus employ two generations of individuals in two types of occupations. They use the resulting four types of labor to produce a single good. Production is organized so that profits are maximized.

To find the competitive equilibrium of the model, we can either examine the decentralized version of the economy, which is rather tedious and relegated to appendix A, or we can examine the centralized version of the economy, which is much simpler and the way chosen here. The central planner organizes production efficiently. He thus maximizes the present value of production

$$\sum_{t=1}^{\infty} \rho^{t-1} F\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}\right) \tag{1}$$

subject to the complementary constraint, the immobility constraints, and the initial condition

$$
\begin{align}
N_t^B &= 1 - N_t^A, \tag{2a} \\
N_t^{AA} &= N_{t-1}^A, \tag{2b} \\
N_t^{BB} &= N_{t-1}^B, \tag{2c} \\
N_0^A &= \text{given} \tag{2d}
\end{align}
$$

where $t \in \{0, 1, 2, ...\}$ is time, $\rho \in (0, 1)$ is an exogenous discount factor, $F \in [0, \infty)$ is a well-behaved production function, $N_t^A \in [0, 1]$ is the number of young individuals working in occupation A, $N_t^{AA} \in [0, 1]$ is the corresponding number of old individuals, $N_t^B \in [0, 1]$ is the number of young individuals working in occupation B, and $N_t^{BB} \in [0, 1]$ is the corresponding number of old individuals.[7] The complementary constraint says that individuals who do not enter A will enter B. The immobility constraints say that old individuals stick to the occupation chosen when young. The initial condition says that the occupational allocation from period zero is predetermined by the time of period one (the planning period).

To find the solution of the maximization problem, we insert the constraints from (2a), (2b), and (2c) in the objective function from (1). So we get

$$\sum_{t=1}^{\infty} \rho^{t-1} F\left(N_t^A, N_{t-1}^A, 1 - N_t^A, 1 - N_{t-1}^A\right). \tag{3}$$

---

[6] Alternatively we could assume occupational choice to be made such that lifetime utility is maximized. However, in this case individuals might react hesitantly to imbalances in lifetime earnings. In a model with perfect occupational mobility the assumption of maximizing lifetime utility is therefore not appropriate.

[7] For a better understanding of our notation, note that in the next section we will introduce the variables N_t^{AB} and N_t^{BA}. They denote the number of individuals changing the occupation when they enter their second period of life.

The necessary condition for an inner maximum then is

$$F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) = F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right) \tag{4}$$

with

$$\mathbf{X}_t \equiv \left(N_t^A, N_{t-1}^A, 1 - N_t^A, 1 - N_{t-1}^A\right) \tag{5}$$

for $t \in \{1, 2, 3, ...\}$. It says that lifetime marginal productivities and, hence, lifetime earnings must equalize across occupations.

The condition of equal lifetime earnings given in (4) determines the dynamics of N_t^A. Since we are only interested in linear dynamics,[8] we compute the first-order Taylor approximation of the necessary condition around the steady state. As a result we get the linear second-order difference equation

$$G_1\left(N_{t+1}^A - N^A\right) + G_2\left(N_t^A - N^A\right) + G_3\left(N_{t-1}^A - N^A\right) = 0 \tag{6}$$

where

$$
\begin{align}
G_1 &\equiv \rho F_{21} - \rho F_{23} - \rho F_{41} + \rho F_{43}, \tag{7a}\\
G_2 &\equiv F_{11} - F_{13} - F_{31} + F_{33} + \rho F_{22} - \rho F_{24} - \rho F_{42} + \rho F_{44}, \tag{7b}\\
G_3 &\equiv F_{12} - F_{14} - F_{32} + F_{34} \tag{7c}
\end{align}
$$

and where the second partial derivatives of F are evaluated at the steady state. Note that the shorthand variables G_1, G_2, and G_3 may have any numerical value. The only thing we can say about them is that $G_1 = \rho G_3$.

Because we do not know the numerical value of G_1, we must distinguish between two cases. Let us first turn to the case where $G_1 = 0$. As $G_1 = \rho G_3$, we also have $G_3 = 0$ in this case. So the difference equation from (6) reduces to $N_t^A = N^A$. We see that the number of young individuals entering A is on the steady state level in each period. The number is stationary, cycles in occupational choice do not arise. The model cannot explain the evidence in this case. For this reason we ignore the case from now on.

We now turn to the case where $G_1 \neq 0$. In this case it is possible to rewrite the difference equation from (6) as

$$\left(N_{t+1}^A - N^A\right) + H_1\left(N_t^A - N^A\right) + H_2\left(N_{t-1}^A - N^A\right) = 0 \tag{8}$$

where

$$H_1 \equiv \frac{G_2}{G_1} = \frac{F_{11} - F_{13} - F_{31} + F_{33} + \rho F_{22} - \rho F_{24} - \rho F_{42} + \rho F_{44}}{\rho F_{21} - \rho F_{23} - \rho F_{41} + \rho F_{43}}, \tag{9a}$$

$$H_2 \equiv \frac{G_3}{G_1} = \frac{F_{12} - F_{14} - F_{32} + F_{34}}{\rho F_{21} - \rho F_{23} - \rho F_{41} + \rho F_{43}} = \frac{1}{\rho} > 1 \tag{9b}$$

---

[8]We ignore the case of nonlinear dynamics because the empirical findings of Freeman and Leonard (1978) imply that the economy converges to the steady state. In the neighborhood of the steady state it is sufficient to study linear dynamics.

		R_1		R_2	
		sign	abs	sign	abs
$H_1 \in (-\infty, -1 - H_2)$	distinct real roots	> 0	< 1	> 0	> 1
$H_1 = -1 - H_2$	distinct real roots	> 0	$= 1$	> 0	> 1
$H_1 \in \left(-1 - H_2, -2\sqrt{H_2}\right)$	distinct real roots	> 0	> 1	> 0	> 1
$H_1 = -2\sqrt{H_2}$	identical real roots	> 0	> 1	> 0	> 1
$H_1 \in \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$	conjugate complex roots		> 1		> 1
$H_1 = 2\sqrt{H_2}$	identical real roots	< 0	> 1	< 0	> 1
$H_1 \in \left(2\sqrt{H_2}, 1 + H_2\right)$	distinct real roots	< 0	> 1	< 0	> 1
$H_1 = 1 + H_2$	distinct real roots	< 0	> 1	< 0	$= 1$
$H_1 \in (1 + H_2, \infty)$	distinct real roots	< 0	> 1	< 0	< 1

Table 1: The Sign and the Absolute Value of the Characteristic Roots as a Function of the Unknown H_1

are two additional shorthand variables. Because we do not know the numerical value of G_1 and G_2, we do not know the value of H_1. We know the value of H_2, however, which is equal to $1/\rho > 1$.

We can solve the difference equation from (8) with standard methods. The general solution of the difference equation is

$$N_t^A - N^A = \begin{cases} C_1 R_1^t + C_2 R_2^t & \text{if} \quad H_1 \neq -2\sqrt{H_2}, \\ C_1 R_1^t + C_2 R_2^t t & \text{if} \quad H_1 = -2\sqrt{H_2} \end{cases} \tag{10}$$

where C_1 and C_2 are arbitrary constants and R_1 and R_2 are the characteristic roots

$$R_1 = -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2}, \tag{11a}$$

$$R_2 = -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2}. \tag{11b}$$

We analyze the roots in appendix B. The results of the analysis are listed in table 1. As the table shows, the sign and the absolute value of the roots depend on the numerical value of the unknown H_1.

To derive the definite solution of the difference equation, we need two boundary conditions. One is given by the initial condition

$$N_0^A = \text{given}, \tag{12}$$

which is already known from (2d). The other one is assumed to be

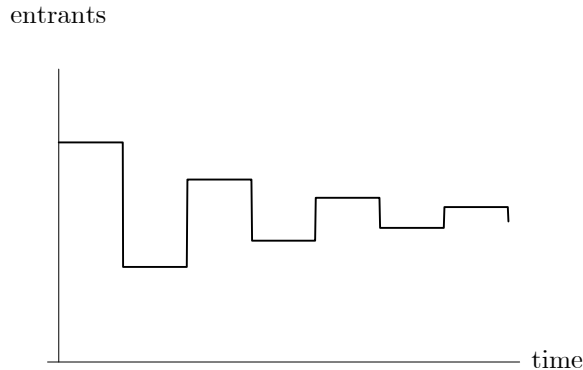$$\lim_{t \to \infty} N_t^A = N^A, \tag{13}$$

entrants

time

Figure 2: The Time Path of the Number of Young Individuals Entering Occupation A if $H_1 \in (1 + H_2, \infty)$

which says that agents expect the economy to converge to the steady state. We could have modelled expectations in any other way, but this way is the only one consistent with the fact that cycles in occupational choice are dampened over time.

As shown in table 1, the economy cannot converge to the steady state if $H_1 \in [-1 - H_2, 1 + H_2]$.[9] In this case both roots are equal to or larger than one in absolute value so that convergence is impossible. A definite solution satisfying the boundary conditions in (12) and (13) does not exist in this case. If $H_1 \notin [-1 - H_2, 1 + H_2]$, one root is smaller than one in absolute value, however, and convergence is possible. If $H_1 \in (-\infty, -1 - H_2)$, the converging root is positive so that the resulting time path of N_t^A is monotonic. If $H_1 \in (1 + H_2, \infty)$, the converging root is negative so that the time path is oscillatory. The definite solution then is

$$N_t^A - N^A = \left(N_0^A - N^A \right) R_2^t. \tag{14}$$

In this case the model generates cycles in occupational choice which are similar to those we can observe in reality. The number of individuals entering A is cyclical, and cycles are dampened over time. An example for this type of dynamic behavior is given in figure 2.

What does the condition $H_1 \in (1 + H_2, \infty)$ mean? As the definitions of H_1 and H_2 in (9) show, it basically is a technological condition. Given the discount factor ρ, it restricts the set of production functions compatible with dampened cycles in occupational choice. A realistic family of production functions satisfying the condition is defined by the following three properties:[10]

---

[9]This statement is perfectly true only for the linear model. In the nonlinear model convergence is impossible if $H_1 \in (-1 - H_2, 1 + H_2)$. The reason for this deviation is that the two versions of the model need not be topologically equivalent at $H_1 \in \{-1 - H_2, 1 + H_2\}$.

[10]In the following definition we use the term substitutes in the sense of Edgeworth or Auspitz-

7

1. Workers who belong to the same occupational group are substitutes, implying that $F_{11} < 0$, $F_{12} < 0$, $F_{21} < 0$, $F_{22} < 0$, $F_{33} < 0$, $F_{34} < 0$, $F_{43} < 0$, $F_{44} < 0$.

2. Workers who belong to different occupational groups are complements, implying that $F_{13} > 0$, $F_{14} > 0$, $F_{23} > 0$, $F_{24} > 0$, $F_{31} > 0$, $F_{32} > 0$, $F_{41} > 0$, $F_{42} > 0$.

3. Workers who belong to the same occupational and generational group are own-substitutes sufficiently close, implying that $F_{11} \ll 0$, $F_{22} \ll 0$, $F_{33} \ll 0$, $F_{44} \ll 0$.

It is easy to see from (9) that for production functions from this family the condition $H_1 \in (1 + H_2, \infty)$ is satisfied.

If we stay with this type of production function for another moment, we can easily explain the intuition of our model. Assume that in an initial period there are many old workers in A and few old workers in B. Because young workers in A are substitutes for old workers in A and complements to old workers in B, this implies that young workers who enter A receive low lifetime earnings. Because young workers in B are complements to old workers in A and substitutes for old workers in B, this also implies that young workers who enter B receive high lifetime earnings. So only few young workers enter A and many young workers enter B. In the next period the number of old workers in A thus is low and the number of old workers in B is high. The scheme of substitutability and complementarity then implies that young workers receive high lifetime earnings when entering A and low lifetime earnings when entering B. So many young workers choose A and few young workers choose B. The following period thus is characterized by a high number of old workers in A and a low number of old workers in B again. At this point the process repeats, generating cycles in occupational choice.

3 A Model with Rational Wage Expectations and Perfect Occupational Mobility

Critical to the model presented in the last section is the assumption that old individuals are occupationally immobile. Now we abandon this assumption and allow for perfect occupational mobility. In each period we thus have young individuals entering A or B (called entrants), old individuals staying in A or B (called stayers), and old individuals switching from A to B or from B to A (called switchers). Firms employ these six types of workers. What is the competitive equilibrium?

Lieben substitutes. Similarly, we use the term complements in the sense of Edgeworth or Auspitz-Lieben complements.

Again the competitive equilibrium can easier be found by employing the concept of the central planner (for the decentralized version of the economy see appendix A). The central planner maximizes

$$\sum_{t=1}^{\infty} \rho^{t-1} F\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA}\right) \tag{15}$$

subject to the constraints

$$\begin{align}
N_t^B &= 1 - N_t^A, \tag{16a}\\
N_t^{AA} &= N_{t-1}^A - N_t^{AB}, \tag{16b}\\
N_t^{BB} &= N_{t-1}^B - N_t^{BA}, \tag{16c}\\
N_0^A &= \text{given} \tag{16d}
\end{align}$$

where $N_t^{AB} \in \left[0, N_{t-1}^A\right]$ and $N_t^{BA} \in \left[0, N_{t-1}^B\right]$ are the numbers of old individuals switching from A to B and from B to A. Note that the production function has six arguments now [compare (15) to (1)]. Note also that we have two switching constraints instead of two immobility constraints [compare (16b) and (16c) to (2b) and (2c)]. The switching constraints say that the number of stayers by definition is equal to the number of former entrants minus the number of switchers.

To solve the maximization problem, we insert the constraints from (16a), (16b), and (16c) into the objective function from (15):

$$\sum_{t=1}^{\infty} \rho^{t-1} F\left(N_t^A, N_{t-1}^A - N_t^{AB}, 1 - N_t^A, 1 - N_{t-1}^A - N_t^{BA}, N_t^{AB}, N_t^{BA}\right). \tag{17}$$

The necessary conditions for an inner maximum then are

$$\begin{align}
F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) &= F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right), \tag{18a}\\
F_2\left(\mathbf{X}_t\right) &= F_5\left(\mathbf{X}_t\right), \tag{18b}\\
F_4\left(\mathbf{X}_t\right) &= F_6\left(\mathbf{X}_t\right) \tag{18c}
\end{align}$$

with

$$\mathbf{X}_t \equiv \left(N_t^A, N_{t-1}^A - N_t^{AB}, 1 - N_t^A, 1 - N_{t-1}^A - N_t^{BA}, N_t^{AB}, N_t^{BA}\right) \tag{19}$$

for $t \in \{1, 2, 3, ...\}$. They imply that lifetime earnings must equalize across occupations, and that periodical earnings must equalize across stayers and switchers.

The necessary conditions determine the dynamics of N_t^A, N_t^{AB}, and N_t^{BA}. Restricting ourselves again to the linear dynamic case, we compute the first-order Taylor approximation of the necessary conditions around the steady state.

This leads to the system of linear second-order difference equations

$$
\begin{pmatrix}
G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} & G_{17} \\
0 & 0 & 0 & G_{24} & G_{25} & G_{26} & G_{27} \\
0 & 0 & 0 & G_{34} & G_{35} & G_{36} & G_{37}
\end{pmatrix}
\begin{pmatrix}
N_{t+1}^A - N^A \\
N_{t+1}^{AB} - N^{AB} \\
N_{t+1}^{BA} - N^{BA} \\
N_t^A - N^A \\
N_t^{AB} - N^{AB} \\
N_t^{BA} - N^{BA} \\
N_{t-1}^A - N^A
\end{pmatrix}
=
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\tag{20}
$$

where

$$
\begin{aligned}
G_{11} &\equiv \rho F_{21} - \rho F_{23} - \rho F_{41} + \rho F_{43}, & \text{(21a)} \\
G_{12} &\equiv -\rho F_{22} + \rho F_{25} + \rho F_{42} - \rho F_{45}, & \text{(21b)} \\
G_{13} &\equiv -\rho F_{24} + \rho F_{26} + \rho F_{44} - \rho F_{46}, & \text{(21c)} \\
G_{14} &\equiv F_{11} - F_{13} + \rho F_{22} - \rho F_{24} - F_{31} + F_{33} - \rho F_{42} + \rho F_{44}, & \text{(21d)} \\
G_{15} &\equiv -F_{12} + F_{15} + F_{32} - F_{35}, & \text{(21e)} \\
G_{16} &\equiv -F_{14} + F_{16} + F_{34} - F_{36}, & \text{(21f)} \\
G_{17} &\equiv F_{12} - F_{14} - F_{32} + F_{34}, & \text{(21g)} \\
G_{24} &\equiv F_{21} - F_{23} - F_{51} + F_{53}, & \text{(21h)} \\
G_{25} &\equiv -F_{22} + F_{25} + F_{52} - F_{55}, & \text{(21i)} \\
G_{26} &\equiv -F_{24} + F_{26} + F_{54} - F_{56}, & \text{(21j)} \\
G_{27} &\equiv F_{22} - F_{24} - F_{52} + F_{54}, & \text{(21k)} \\
G_{34} &\equiv F_{41} - F_{43} - F_{61} + F_{63}, & \text{(21l)} \\
G_{35} &\equiv -F_{42} + F_{45} + F_{62} - F_{65}, & \text{(21m)} \\
G_{36} &\equiv -F_{44} + F_{46} + F_{64} - F_{66}, & \text{(21n)} \\
G_{37} &\equiv F_{42} - F_{44} - F_{62} + F_{64}. & \text{(21o)}
\end{aligned}
$$

Note that some G_{ij}'s are related to each other:

$$
\begin{aligned}
G_{11} &= \rho G_{17}, & \text{(22a)} \\
G_{12} &= -\rho G_{27}, & \text{(22b)} \\
G_{13} &= -\rho G_{37}, & \text{(22c)} \\
G_{15} &= -G_{24}, & \text{(22d)} \\
G_{16} &= -G_{34}, & \text{(22e)} \\
G_{26} &= G_{35}. & \text{(22f)}
\end{aligned}
$$

More information about these variables is not available. In particular, we cannot tell anything about their numerical value.

We now use the system of difference equations to derive a single difference equation that governs the dynamics of N_t^A. For this purpose we rewrite the

system, taking into account that the last two equations, which refer to t and $t-1$, also refer to $t+1$ and t:

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} & G_{17} \\ 0 & 0 & 0 & G_{24} & G_{25} & G_{26} & G_{27} \\ 0 & 0 & 0 & G_{34} & G_{35} & G_{36} & G_{37} \\ G_{24} & G_{25} & G_{26} & G_{27} & 0 & 0 & 0 \\ G_{34} & G_{35} & G_{36} & G_{37} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} N_{t+1}^A - N^A \\ N_{t+1}^{AB} - N^{AB} \\ N_{t+1}^{BA} - N^{BA} \\ N_t^A - N^A \\ N_t^{AB} - N^{AB} \\ N_t^{BA} - N^{BA} \\ N_{t-1}^A - N^A \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (23)$$

Then we rearrange the system as follows:

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{15} & G_{16} \\ 0 & 0 & 0 & G_{25} & G_{26} \\ 0 & 0 & 0 & G_{35} & G_{36} \\ G_{24} & G_{25} & G_{26} & 0 & 0 \\ G_{34} & G_{35} & G_{36} & 0 & 0 \end{pmatrix} \begin{pmatrix} N_{t+1}^A - N^A \\ N_{t+1}^{AB} - N^{AB} \\ N_{t+1}^{BA} - N^{BA} \\ N_t^{AB} - N^{AB} \\ N_t^{BA} - N^{BA} \end{pmatrix} = $$
$$ - \begin{pmatrix} G_{14} & G_{17} \\ G_{24} & G_{27} \\ G_{34} & G_{37} \\ G_{27} & 0 \\ G_{37} & 0 \end{pmatrix} \begin{pmatrix} N_t^A - N^A \\ N_{t-1}^A - N^A \end{pmatrix}. \quad (24)$$

Now we apply Cramer's rule to solve the system for $N_{t+1}^A - N^A$.[11] So we get

$$N_{t+1}^A - N^A = -H_{11}\left(N_t^A - N^A\right) - H_{12}\left(N_{t-1}^A - N^A\right) \quad (25)$$

with

$$H_{11} \equiv \frac{\begin{vmatrix} G_{14} & G_{12} & G_{13} & G_{15} & G_{16} \\ G_{24} & 0 & 0 & G_{25} & G_{26} \\ G_{34} & 0 & 0 & G_{35} & G_{36} \\ G_{27} & G_{25} & G_{26} & 0 & 0 \\ G_{37} & G_{35} & G_{36} & 0 & 0 \end{vmatrix}}{\begin{vmatrix} G_{11} & G_{12} & G_{13} & G_{15} & G_{16} \\ 0 & 0 & 0 & G_{25} & G_{26} \\ 0 & 0 & 0 & G_{35} & G_{36} \\ G_{24} & G_{25} & G_{26} & 0 & 0 \\ G_{34} & G_{35} & G_{36} & 0 & 0 \end{vmatrix}}, \quad (26a)$$

---

[11] We assume that the determinant of the coefficient matrix on the left of (24) is different from zero. This way we exclude special cases similar to that we encountered in the last section. There occupational choice was stationary if $G_1 = 0$.

$$
H_{12} \equiv \frac{\begin{vmatrix} G_{17} & G_{12} & G_{13} & G_{15} & G_{16} \\ G_{27} & 0 & 0 & G_{25} & G_{26} \\ G_{37} & 0 & 0 & G_{35} & G_{36} \\ 0 & G_{25} & G_{26} & 0 & 0 \\ 0 & G_{35} & G_{36} & 0 & 0 \end{vmatrix}}{\begin{vmatrix} G_{11} & G_{12} & G_{13} & G_{15} & G_{16} \\ 0 & 0 & 0 & G_{25} & G_{26} \\ 0 & 0 & 0 & G_{35} & G_{36} \\ G_{24} & G_{25} & G_{26} & 0 & 0 \\ G_{34} & G_{35} & G_{36} & 0 & 0 \end{vmatrix}}. \tag{26b}
$$

Equation (25) is the difference equation we have looked for. Note that it is free from the switching variables N_t^{AB} and N_t^{BA}. Although we allow for occupational switches in this model, the corresponding variables do not appear in the difference equation governing the dynamics of N_t^A. As a result, the difference equation we get in this section is very similar to that obtained in the last section [compare (25) to (8)]. The two difference equations have the same structure, and they share an identical coefficient. By inserting (22) into (26b) and expanding determinants, we can see that H_{12} is equal to $1/\rho$ and, thus, to H_2. Any difference between the model with and the model without occupational mobility thus is reflected in the difference between H_{11} and H_1.

Since the two difference equations are nearly identical, we can solve them in the same way. By following the procedure described in the last section, we find that the time path of N_t^A exhibits dampened cycles if the condition $H_{11} \in (1 + H_{12}, \infty)$ is satisfied. This condition again has a technological character. It restricts the set of production functions compatible with dampened cycles in occupational choice. Let us examine if the family of production functions defined by the following three properties satisfies the condition:

1. Workers who currently belong to the same occupational group are substitutes, implying that $F_{11} < 0$, $F_{12} < 0$, $F_{16} < 0$, $F_{21} < 0$, $F_{22} < 0$, $F_{26} < 0$, $F_{33} < 0$, $F_{34} < 0$, $F_{35} < 0$, $F_{43} < 0$, $F_{44} < 0$, $F_{45} < 0$, $F_{53} < 0$, $F_{54} < 0$, $F_{55} < 0$, $F_{61} < 0$, $F_{62} < 0$, $F_{66} < 0$.

2. Workers who currently belong to different occupational groups are complements, implying that $F_{13} > 0$, $F_{14} > 0$, $F_{15} > 0$, $F_{23} > 0$, $F_{24} > 0$, $F_{25} > 0$, $F_{31} > 0$, $F_{32} > 0$, $F_{36} > 0$, $F_{41} > 0$, $F_{42} > 0$, $F_{46} > 0$, $F_{51} > 0$, $F_{52} > 0$, $F_{56} > 0$, $F_{63} > 0$, $F_{64} > 0$, $F_{65} > 0$.

3. Workers who belong to the same occupational, generational, and transitional group are own-substitutes sufficiently close, implying that $F_{11} \ll 0$, $F_{22} \ll 0$, $F_{33} \ll 0$, $F_{44} \ll 0$, $F_{55} \ll 0$, $F_{66} \ll 0$.

12

In the previous section it was easy to see that the example family of production functions satisfies the condition for dampened cycles in occupational choice. This time it is rather difficult to see this although the families are very similar in both sections. Therefore we approached the problem numerically. We assigned numerical values to $F_{11}, F_{12}, ...$ that conformed to the pattern defined by the three properties. We also assigned a value to ρ. Then we computed H_{11} and H_{12} and checked if the cycle condition was satisfied. We found that for some parameterizations it was, while for others it was not.[12] So the family of production functions defined above does not satisfy the cycle condition in any case. In many cases the condition is met, however. Cycles in occupational choice therefore are possible even if occupational mobility is perfect among workers.

4   Conclusion

Many professional labor markets exhibit systematic ups and downs. There are periods when many new workers enter these markets, followed by periods when there are only few entrants. This phenomenon can be observed on the markets for engineers, physicists, mathematicians, and others. Richard Freeman (1971, 1972, 1975a, 1975b, 1976a, 1976b) has proposed to explain the phenomenon by means of a cobweb model based on the assumptions of myopic wage expectations and occupational immobility. However, some evidence suggests that wage expectations are rational and not myopic. Students in economics and law, for example, have been shown to form their wage expectations rationally. In response to this evidence, Gary Zarkin (1982, 1983, 1985) has proposed to explain the phenomenon by means of a rational expectations model based only on the assumption of occupational immobility. Nevertheless, there is reason to question this assumption as well. Empirical studies report occupational turnover rates between seven and thirteen percent a year. In the long run, about two thirds of all workers seem to change their occupation even if occupations are defined very broadly. Taking this evidence seriously, we have presented a model in this paper which is neither based on the assumption of myopic wage expectations nor on that of occupational immobility. It is based instead on the assumptions of rational wage expectations and perfect occupational mobility. We have shown that even a model based on these assumptions can explain the existence of cycles in occupational choice. There are plausible technologies for which cycles do not

---

[12]We selected the numbers for F_{ij} randomly. If $F_{ij} < 0$ and $i \neq j$, the numbers were uniformly drawn from the interval $[-1, 0]$. If $F_{ij} < 0$ and $i = j$, they were uniformly drawn from $[-7.5, -5]$. If $F_{ij} > 0$, they were uniformly drawn from $[0, 1]$. The number assigned to ρ was 0.5. We performed 100 experiments. The condition for converging cycles was satisfied in 53 cases, in 47 cases it was not. When the model generated cycles, they were weaker than in the model with occupational immobility. When it did not generate cycles, the time path was nevertheless converging. Since the model is quite abstract, one should not overemphasize the numerical results.

disappear if workers are allowed to change their occupation.

Appendix A

Here we examine the decentralized versions of the models presented in sections 2 and 3. We begin with the model from section 2. In this model, individuals behave as follows:

$$
N_t^A \in \begin{cases} \{0\} & \text{if} \quad W_t^A + \rho W_{t+1}^{AA} < W_t^B + \rho W_{t+1}^{BB}, \\ [0,1] & \text{if} \quad W_t^A + \rho W_{t+1}^{AA} = W_t^B + \rho W_{t+1}^{BB}, \\ \{1\} & \text{if} \quad W_t^A + \rho W_{t+1}^{AA} > W_t^B + \rho W_{t+1}^{BB}, \end{cases}
$$
$$
N_t^B = 1 - N_t^A,
$$
$$
N_t^{AA} = N_{t-1}^A,
$$
$$
N_t^{BB} = N_{t-1}^B.
$$

The variables W_t^A, W_t^{AA}, W_t^B, W_t^{BB} denote the wages paid to young workers in A, old workers in A, young workers in B, old workers in B. The firms' behavior is described by

$$
\begin{aligned}
W_t^A &= F_1\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}\right), \\
W_t^{AA} &= F_2\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}\right), \\
W_t^B &= F_3\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}\right), \\
W_t^{BB} &= F_4\left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}\right).
\end{aligned}
$$

In the temporary equilibrium, both systems of equations must hold simultaneously. This implies

$$
N_t^A \in \begin{cases} \{0\} & \text{if} \quad F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) < F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right), \\ [0,1] & \text{if} \quad F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) = F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right), \\ \{1\} & \text{if} \quad F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) > F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right) \end{cases}
$$

where

$$
\mathbf{X}_t \equiv \left(N_t^A, N_{t-1}^A, 1 - N_t^A, 1 - N_{t-1}^A\right).
$$

A temporary equilibrium exists if for each level of N_{t+1}^A and N_{t-1}^A there is a level of N_t^A satisfying this equation. This is the case by Kakutani's fixed point theorem. Given a suitable form of the production function, the equilibrium level of N_t^A must be strictly between zero and one. So the temporary equilibrium is characterized by

$$
F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) = F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right).
$$

This equilibrium condition corresponds to the maximum condition of the central planner in (4). The results of the decentralized economy therefore are identical to the results of the centralized economy.

Now we turn to the model from section 3. Here the individuals' behavior is governed by

$$
N_t^A \ \in \ \begin{cases} \{0\} & \text{if} \quad \begin{aligned} & W_t^A + \rho \max \left\{ W_{t+1}^{AA}, W_{t+1}^{AB} \right\} < \\ & \qquad W_t^B + \rho \max \left\{ W_{t+1}^{BA}, W_{t+1}^{BB} \right\}, \end{aligned} \\[1ex] [0,1] & \text{if} \quad \begin{aligned} & W_t^A + \rho \max \left\{ W_{t+1}^{AA}, W_{t+1}^{AB} \right\} = \\ & \qquad W_t^B + \rho \max \left\{ W_{t+1}^{BA}, W_{t+1}^{BB} \right\}, \end{aligned} \\[1ex] \{1\} & \text{if} \quad \begin{aligned} & W_t^A + \rho \max \left\{ W_{t+1}^{AA}, W_{t+1}^{AB} \right\} > \\ & \qquad W_t^B + \rho \max \left\{ W_{t+1}^{BA}, W_{t+1}^{BB} \right\}, \end{aligned} \end{cases}
$$

$$
N_t^{AB} \ \in \ \begin{cases} \{0\} & \text{if} \quad W_t^{AB} < W_t^{AA}, \\ \left[0, N_{t-1}^A \right] & \text{if} \quad W_t^{AB} = W_t^{AA}, \\ \left\{ N_{t-1}^A \right\} & \text{if} \quad W_t^{AB} > W_t^{AA}, \end{cases}
$$

$$
N_t^{BA} \ \in \ \begin{cases} \{0\} & \text{if} \quad W_t^{BA} < W_t^{BB}, \\ \left[0, N_{t-1}^B \right] & \text{if} \quad W_t^{BA} = W_t^{BB}, \\ \left\{ N_{t-1}^B \right\} & \text{if} \quad W_t^{BA} > W_t^{BB}, \end{cases}
$$

$$
\begin{aligned}
N_t^B &= 1 - N_t^A, \\
N_t^{AA} &= N_{t-1}^A - N_t^{AB}, \\
N_t^{BB} &= N_{t-1}^B - N_t^{BA}
\end{aligned}
$$

where W_t^{AB} and W_t^{BA} are the wages paid to individuals switching from A to B or from B to A. The firms' behavior is defined by

$$
\begin{aligned}
W_t^A &= F_1 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right), \\
W_t^{AA} &= F_2 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right), \\
W_t^B &= F_3 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right), \\
W_t^{BB} &= F_4 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right), \\
W_t^{AB} &= F_5 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right), \\
W_t^{BA} &= F_6 \left(N_t^A, N_t^{AA}, N_t^B, N_t^{BB}, N_t^{AB}, N_t^{BA} \right).
\end{aligned}
$$

In the temporary equilibrium, both systems must hold simultaneously. This implies

$$
N_t^A \in \begin{cases} \{0\} & \text{if} \quad \begin{aligned} & F_1 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_2 \left(\mathbf{X}_{t+1} \right), F_5 \left(\mathbf{X}_{t+1} \right) \right\} < \\ & \qquad F_3 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_6 \left(\mathbf{X}_{t+1} \right), F_4 \left(\mathbf{X}_{t+1} \right) \right\}, \end{aligned} \\[1ex] [0,1] & \text{if} \quad \begin{aligned} & F_1 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_2 \left(\mathbf{X}_{t+1} \right), F_5 \left(\mathbf{X}_{t+1} \right) \right\} = \\ & \qquad F_3 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_6 \left(\mathbf{X}_{t+1} \right), F_4 \left(\mathbf{X}_{t+1} \right) \right\}, \end{aligned} \\[1ex] \{1\} & \text{if} \quad \begin{aligned} & F_1 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_2 \left(\mathbf{X}_{t+1} \right), F_5 \left(\mathbf{X}_{t+1} \right) \right\} > \\ & \qquad F_3 \left(\mathbf{X}_t \right) + \rho \max \left\{ F_6 \left(\mathbf{X}_{t+1} \right), F_4 \left(\mathbf{X}_{t+1} \right) \right\}, \end{aligned} \end{cases}
$$

$$N_t^{AB} \in \begin{cases} \{0\} & \text{if } F_5\left(\mathbf{X}_t\right) < F_2\left(\mathbf{X}_t\right), \\ \left[0, N_{t-1}^A\right] & \text{if } F_5\left(\mathbf{X}_t\right) = F_2\left(\mathbf{X}_t\right), \\ \left\{N_{t-1}^A\right\} & \text{if } F_5\left(\mathbf{X}_t\right) > F_2\left(\mathbf{X}_t\right), \end{cases}$$

$$N_t^{BA} \in \begin{cases} \{0\} & \text{if } F_6\left(\mathbf{X}_t\right) < F_4\left(\mathbf{X}_t\right), \\ \left[0, 1 - N_{t-1}^A\right] & \text{if } F_6\left(\mathbf{X}_t\right) = F_4\left(\mathbf{X}_t\right), \\ \left\{1 - N_{t-1}^A\right\} & \text{if } F_6\left(\mathbf{X}_t\right) > F_4\left(\mathbf{X}_t\right) \end{cases}$$

where

$$\mathbf{X}_t \equiv \left(N_t^A, N_{t-1}^A - N_t^{AB}, 1 - N_t^A, 1 - N_{t-1}^A - N_t^{BA}, N_t^{AB}, N_t^{BA}\right).$$

A temporary equilibrium exists if for any two vectors $\left(N_{t+1}^A, N_{t+1}^{AB}, N_{t+1}^{BA}\right)$ and $\left(N_{t-1}^A, N_{t-1}^{AB}, N_{t-1}^{BA}\right)$ there is a vector $\left(N_t^A, N_t^{AB}, N_t^{BA}\right)$ satisfying this system. This is the case by Kakutani's fixed point theorem. Given a suitable form of the production function, the equilibrium vector $\left(N_t^A, N_t^{AB}, N_t^{BA}\right)$ is strictly between $(0, 0, 0)$ and $\left(1, N_{t-1}^A, 1 - N_{t-1}^A\right)$. So the temporary equilibrium is characterized by

$$F_1\left(\mathbf{X}_t\right) + \rho F_2\left(\mathbf{X}_{t+1}\right) = F_3\left(\mathbf{X}_t\right) + \rho F_4\left(\mathbf{X}_{t+1}\right),$$
$$F_5\left(\mathbf{X}_t\right) = F_2\left(\mathbf{X}_t\right),$$
$$F_6\left(\mathbf{X}_t\right) = F_4\left(\mathbf{X}_t\right).$$

These equilibrium conditions are identical to the maximum conditions of the central planner in (18). The decentralized and the centralized economy therefore lead to the same allocation.

Appendix B

Here we derive the sign and the absolute value of the characteristic roots determining the dynamics of the model presented in section 2. According to (11), the characteristic roots are

$$R_1 = -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2},$$

$$R_2 = -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2}$$

with H_1 being unknown and $H_2 = 1/\rho > 1$. They are real if $H_1 \notin \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$. If $H_1 \in \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$, they are complex.

We first examine the sign of R_1. Since the sign is only defined for real numbers, it is sufficient to study the case where $H_1 \notin \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$. We must

distinguish between two cases. If $H_1 \in \left(-\infty, -2\sqrt{H_2}\right]$, we have

$$R_1 = -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} > -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1}{2} = 0.$$

If $H_1 \in \left[2\sqrt{H_2}, \infty\right)$, we have

$$R_1 = -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} < 0.$$

So $R_1 > 0$ if $H_1 \in \left(-\infty, -2\sqrt{H_2}\right]$, and $R_1 < 0$ if $H_1 \in \left[2\sqrt{H_2}, \infty\right)$. This result is summarized in the third column of table 1.

We now turn to the sign of R_2. It is again sufficient to examine the cases where $H_1 \notin \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$. If $H_1 \in \left(-\infty, -2\sqrt{H_2}\right]$, we have

$$R_2 = -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} > 0.$$

If $H_1 \in \left[2\sqrt{H_2}, \infty\right)$, we have

$$R_2 = -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} < -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1}{2} = 0.$$

Therefore $R_2 > 0$ if $H_1 \in \left(-\infty, -2\sqrt{H_2}\right]$, and $R_2 < 0$ if $H_1 \in \left[2\sqrt{H_2}, \infty\right)$. This result is summarized in the fifth column of table 1.

Next we determine the absolute value of R_1. Since the absolute value is defined for real and complex numbers, we can no longer ignore the case where $H_1 \in \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$. In sum, we must distinguish between five cases. If $H_1 \in (-\infty, -1 - H_2)$, we have $H_2 < -1 - H_1$ and $\frac{H_1 + 2}{2} < 0$ so that

$$
\begin{aligned}
R_1 &= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} < -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 - H_1)} \\
&= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1 + 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 + 2}{2} = 1.
\end{aligned}
$$

If $H_1 = -1 - H_2$, we have $H_2 = -1 - H_1$ and $\frac{H_1 + 2}{2} < 0$ so that

$$
\begin{aligned}
R_1 &= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} = -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 - H_1)} \\
&= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1 + 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 + 2}{2} = 1.
\end{aligned}
$$

18

If $H_1 \in \left(-1 - H_2, -2\sqrt{H_2}\right]$, we have $H_2 > -1 - H_1$ and $\frac{H_1+2}{2} < 0$ so that

$$
\begin{aligned}
R_1 &= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} > -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 - H_1)} \\
&= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1 + 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 + 2}{2} = 1.
\end{aligned}
$$

If $H_1 \in \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$, R_1 is complex so that

$$
\begin{aligned}
|R_1| &= \left| -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} \right| = \left| -\frac{H_1}{2} - i\sqrt{H_2 - \left(\frac{H_1}{2}\right)^2} \right| \\
&= \sqrt{\left(-\frac{H_1}{2}\right)^2 + H_2 - \left(\frac{H_1}{2}\right)^2} = \sqrt{H_2} > 1.
\end{aligned}
$$

If $H_1 \in \left[2\sqrt{H_2}, \infty\right)$, we have

$$
\begin{aligned}
R_1 &= -\frac{H_1}{2} - \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} \leq -\frac{H_1}{2} - \sqrt{\left(\frac{2\sqrt{H_2}}{2}\right)^2 - H_2} \\
&= -\frac{H_1}{2} \leq -\frac{2\sqrt{H_2}}{2} < -1.
\end{aligned}
$$

So $|R_1| < 1$ if $H_1 \in (-\infty, -1 - H_2)$, $|R_1| = 1$ if $H_1 = -1 - H_2$, and $|R_1| > 1$ if $H_1 \in (-1 - H_2, \infty)$. This result is summarized in the fourth column of table 1.

Finally we determine the absolute value of R_2. We must distinguish between five cases again. If $H_1 \in \left(-\infty, -2\sqrt{H_2}\right]$, we have

$$
\begin{aligned}
R_2 &= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} \geq -\frac{H_1}{2} + \sqrt{\left(\frac{-2\sqrt{H_2}}{2}\right)^2 - H_2} \\
&= -\frac{H_1}{2} \geq -\frac{-2\sqrt{H_2}}{2} > 1.
\end{aligned}
$$

If $H_1 \in \left(-2\sqrt{H_2}, 2\sqrt{H_2}\right)$, R_2 is complex so that

$$
\begin{aligned}
|R_2| &= \left| -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} \right| = \left| -\frac{H_1}{2} + i\sqrt{H_2 - \left(\frac{H_1}{2}\right)^2} \right| \\
&= \sqrt{\left(-\frac{H_1}{2}\right)^2 + H_2 - \left(\frac{H_1}{2}\right)^2} = \sqrt{H_2} > 1.
\end{aligned}
$$

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If $H_1 \in \left[2\sqrt{H_2}, 1 + H_2\right)$, we have $H_2 > -1 + H_1$ and $\frac{H_1-2}{2} > 0$ so that

$$
\begin{aligned}
R_2 &= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} < -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 + H_1)} \\
&= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1 - 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 - 2}{2} = -1.
\end{aligned}
$$

If $H_1 = 1 + H_2$, we have $H_2 = -1 + H_1$ and $\frac{H_1-2}{2} > 0$ so that

$$
\begin{aligned}
R_2 &= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} = -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 + H_1)} \\
&= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1 - 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 - 2}{2} = -1.
\end{aligned}
$$

If $H_1 \in (1 + H_2, \infty)$, we have $H_2 < -1 + H_1$ and $\frac{H_1-2}{2} > 0$ so that

$$
\begin{aligned}
R_2 &= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - H_2} > -\frac{H_1}{2} + \sqrt{\left(\frac{H_1}{2}\right)^2 - (-1 + H_1)} \\
&= -\frac{H_1}{2} + \sqrt{\left(\frac{H_1 - 2}{2}\right)^2} = -\frac{H_1}{2} + \frac{H_1 - 2}{2} = -1.
\end{aligned}
$$

So $|R_2| > 1$ if $H_1 \in (-\infty, 1 + H_2)$, $|R_2| = 1$ if $H_1 = 1 + H_2$, and $|R_2| < 1$ if $H_1 \in (1 + H_2, \infty)$. This result is summarized in the last column of table 1.

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