85

Reihe Ökonomie

Economics Series

Relative Consumption and Endogenous Labour Supply in the Ramsey Model: Do Status-Conscious People Work Too Much?

Walter H. Fisher, Franz X. Hof

85

Reihe Ökonomie

Economics Series

Relative Consumption and Endogenous Labour Supply in the Ramsey Model: Do Status-Conscious People Work Too Much?

Walter H. Fisher, Franz X. Hof

September 2000

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Contact:

Walter H. Fisher
☎: +43/1/599 91-253
email: fisher@ihs.ac.at

Franz X. Hof
Institute of Economics
University of Technology Vienna
Karlsplatz 13, A-1040 Vienna, Austria
☎: +43/1/58801-17566
Fax: +43/1/58801-17599
email: fxhof@pop.tuwien.ac.at

Abstract

This paper introduces consumption externalities into a Ramsey-type model with endogenous labour supply and homogeneous agents. The instantaneous utility of any consumer is assumed to depend on work effort, own consumption and relative consumption, where the latter determines the individual's status in the society. Appropriate normality conditions with respect to consumption and leisure ensure that at least in the long run status-conscious individuals consume and work too much, compared to the social optimum, and that the capital stock is too high. Public policy can, however, induce the private sector to attain the social optimum by designing an optimal consumption tax policy.

Keywords

Status, relative consumption, work effort

JEL Classifications

D62, D91, E21

Contents

1 Introduction

The goal of this paper is to investigate the influence of consumption externalities in a standard version of the Ramsey model with an endogenous labour supply decision. Specifically, we introduce consumption externalities into the representative agent's instantaneous utility function, which depends on own consumption, work effort, and relative consumption. This specification expresses two ideas: i) economic agents care about their relative position in society, and ii) social status is determined by relative consumption.[1] The major innovation in our paper is to model employment as a choice variable. The alternative approach is to treat work effort as a fixed variable and has been used by authors such as Rauscher (1997b), Hof (1999a), and Fisher and Hof (2000).[2] Our extension allows us to consider whether status-conscious people work "too much" or "too little", compared to some hypothetical social optimum.

The rationale for studying the influence of status preference in a dynamic framework is to investigate the divergent short and long-run effects that this preference has on the economy's development. For instance, a preference for relative consumption may, depending on the specific economic setting, stimulate short-run consumption at the expense of long-run consumption, or vice-versa. We will show in this paper that the impact of status on the economy's macroeconomic dynamics depends crucially on the endogenous response of employment. In particular, we will prove that if work effort is endogenously determined, then not only the dynamic behavior of the economy is affected by a preference for status, but also the properties of its stationary equilibrium. Appropriate normality conditions with respect to consumption and leisure are sufficient to demonstrate that status-conscious individuals, at least in the long run, consume and work too much and accumulate an excessive stock of physical capital, compared to a hypothetical social optimum.[3] The government, nevertheless, can induce the private sector to attain the social optimum by designing an optimal consumption tax policy.[4] We will calculate an optimal consumption tax in which the tax rate is a function of the average level of consumption and average hours worked. We can further illustrate that there exist quite standard specifications of the instanta-

[1] An alternative branch of this literature models status as determined by relative wealth rather than by relative consumption. This approach is employed by Corneo and Jeanne (1997), Rauscher (1997a), and Futagami and Shibata (1998).

[2] In Persson (1995) labour supply is also treated as endogenously determined. In contrast to our work in which attention will be restricted to the case of homogeneous agents, Persson considers heterogeneous agents, but neglects any intertemporal considerations.

[3] In models with fixed employment, such as Rauscher (1997b), Hof (1999a), and Fisher and Hof (2000), the equilibrium values of consumption and capital in the stationary steady state are independent of consumption externalities, although these authors do show that these externalities may influence the economy's speed of transitional adjustment.

[4] Hof (1999a) and Fisher and Hof (2000) have shown in the fixed employment case that there exist several quite general types of instantaneous utility functions in which the decentralized solution equals its corresponding socially optimal counterpart in spite of the existence of consumption externalities. This situation eliminates, of course, the need for optimal government intervention.

neous utility function in which the optimal tax rate is constant over time and depends positively on the degree of agents' status-consciousness.

The rest of the paper is organized as follows: section 2 describes the basic model, while section 3 derives the intertemporal equilibrium of the planned and the decentralized macroeconomy. Using two different utility functions, we then illustrate in section 4 the results of the preceding section. Section 5 derives the optimal consumption tax policy, while section 6 contains some brief concluding remarks. The paper closes with an appendix that contains some mathematical results and proofs.

2 The Model

The economy is populated by a large number of identical, infinitely-lived individuals. For simplicity, we assume that the population size remains constant over time. As is usual in the Ramsey framework, we restrict attention to the case in which agents possess perfect foresight. The representative individual chooses paths of consumption and leisure in order to maximize discounted intertemporal utility, which is given by

$$\int_0^\infty e^{-\rho t} u\left(c\left(t\right), l\left(t\right), z\left(t\right)\right) dt, \qquad z\left(t\right) \equiv c\left(t\right) / C\left(t\right),$$

where ρ is the constant rate of time preference and u denotes the instantaneous utility function. We assume that the instantaneous utility of the representative individual depends on his absolute consumption, c, work effort (measured by hours worked), l, and relative consumption, $z \equiv c/C$, where C denotes the average (or per capita) consumption in the economy.[5] This specification of u implies that in our model an individual's status is solely determined by his relative consumption. In other words, consumption is treated as a positional good, while leisure is assumed to be a non-positional good.[6] We further assume that u possesses continuous first-order and second-order partial derivatives that have the following signs

$$u_c > 0, \quad u_{cc} < 0, \quad u_l < 0, \quad u_{ll} < 0, \quad u_z > 0, \quad u_{zz} < 0, \tag{1}$$

where we define u over the following inequalities: $c > 0, 0 < l < l^{\max}$ and $c/C > 0$, where $l^{\max}$ is exogenously given. According to (1), the representative individual derives positive and diminishing marginal utility from both own and relative consumption, in addition to positive and increasing marginal disutility from working, (i.e., positive, but diminishing, marginal utility from leisure). In order to obtain a well-behaved optimization problem we will assume that the function $U\left(c, l, C\right) \equiv u\left(c, l, c/C\right)$ is strictly concave in (c, l). Observe that the expression $U_c = u_c + C^{-1} u_z > 0$ measures the total marginal utility of own consumption.

[5] The variable z is introduced for notational convenience. The partial derivative $u_{(c/C)(c/C)}$, for instance, is represented by u_{zz}.

[6] See Frank (1985) for a discussion of positional goods.

In other words, a rise in c, for given values of l and C, leads to an increase in both own and relative consumption. In order to ensure that $U(c,l,C)$ is strictly concave in (c,l), we introduce the following additional assumptions:

$$U_{cc} = u_{cc} + 2C^{-1}u_{cz} + C^{-2}u_{zz} < 0,$$

$$U_{cc}U_{ll} - U_{cl}^2 = \left(u_{cc} + 2C^{-1}u_{cz} + C^{-2}u_{zz}\right)u_{ll} - \left(u_{cl} + C^{-1}u_{zl}\right)^2 > 0.$$

Finally, we let

$$(MRS)^d(c,l,C) \equiv -\frac{U_c(c,l,C)}{U_l(c,l,C)} = -\frac{u_c(c,l,c/C) + C^{-1}u_z(c,l,c/C)}{u_l(c,l,c/C)}$$

(in the paper the superscript d stands for "decentralized") denote the marginal rate of substitution of consumption for leisure as perceived by the representative consumer who treats the average level of consumption in the economy, C, as given. We will assume that $(MRS)^d$ depends negatively on own consumption c and positively on leisure for any given level of average consumption in the economy, C, i.e., $(MRS)_c^d(c,l,C) < 0$ and $(MRS)_l^d(c,l,C) < 0$. In other words, we assume that both own consumption and leisure are normal goods. Further assumptions with respect to u and U, respectively, will be introduced below.

In this simple framework we specify that individuals own the economy's physical capital, the services of which are rented to firms in a perfectly competitive capital market that yields a real return of r. In addition, the representative individual supplies l units of labour services per unit of time and receives the real wage w, which is determined in a perfectly competitive labour market. Individuals can lend to and borrow from other individuals. Since physical capital and loans are assumed to be perfect substitutes as stores of value, they must pay the same real return of r. The flow budget constraint of the representative agent is then given by

$$\dot{a} = ra + wl - c, \tag{2}$$

where a denotes the representative agent's stock of net assets, consisting of physical capital k and net loans b. We will assume that $k(0) = k_0 > 0$ and $b(0) = b_0 = 0$ so that $a(0) = k_0$, where k_0 is exogenously given and positive. We further assume that the credit market imposes the following no-Ponzi-game condition on the agent's borrowing:

$$\lim_{t\to\infty}\left\{a(t)\exp\left[-\int_0^t r(v)\,dv\right]\right\} \geq 0. \tag{3}$$

Since agents in our model are identical in every respect, the representative agent will end up holding zero net loans in any symmetric macroeconomic equilibrium, which implies that $a = k \geq 0$.

With respect to the production sector, we assume that there is large number of perfectly competitive firms that rent the services of physical capital (which does not depreciate) and labour to produce output. Each firm has access to the same production possibilities, although technology itself does not improve. The production function $y = F(k, l)$ (using y to denote output) is assumed to have the usual neoclassical properties of positive and diminishing marginal productivity with respect to capital and labour and is homogeneous of degree one. We have now prepared all prerequisites for analysing the decentralized solution. To consider whether the introduction of relative consumption into the instantaneous utility function leads to Pareto nonoptimality, we will compare the decentralized solution with the solution from a hypothetical social planner's problem. Since the decentralized economy is more complicated than the command economy, we will begin in the next section with the social planner's problem.

3 The Social Planner's Problem and the Decentralized Solution

Suppose that there exists a benevolent social planner who dictates the choices of consumption and hours worked over time and seeks to maximize the welfare of the representative individual. Since individuals are identical, we assume that the social planner assigns to each the same consumption level and the same level of work effort. Consequently, $c(t) = C(t)$ holds for all t, so that the instantaneous utility of each individual becomes $U^s(c, l) \equiv u(c, l, 1)$, where the superscript s stands for "symmetric" case. It is clear that $U_c^s = u_c > 0$, $U_{cc}^s = u_{cc} < 0$, $U_l^s = u_l < 0$, and $U_{ll}^s = u_{ll} < 0$, where the signs hold due to the assumptions made in (1). In order to ensure that $U^s(c, l)$ is jointly strictly concave in c and l, we impose

$$u_{cc}(c, l, 1)\, u_{ll}(c, l, 1) - [u_{cl}(c, l, 1)]^2 > 0,$$

which implies that $U_{cc}^s U_{ll}^s - (U_{cl}^s)^2 > 0$. Finally, let $(MRS)^p$ (in the paper the superscript p stands for social "planner") denote the marginal rate of substitution of leisure for consumption as perceived by the social planner, who ensures that $c = C$ for each individual. It is further assumed that the centralized marginal rate of substitution, equal to

$$(MRS)^p(c, l) \equiv -\frac{U_c^s(c, l)}{U_l^s(c, l)} = -\frac{u_c(c, l, 1)}{u_l(c, l, 1)}$$

depends negatively on consumption and positively on leisure, i.e., $(MRS)_c^p(c, l) < 0$ and $(MRS)_l^p(c, l) < 0$. In other words, we assume that both consumption and leisure are normal goods with respect to the instantaneous utility function $U^s(c, l) \equiv u(c, l, 1)$.

The social planner then chooses the time paths of c and l to maximize

$$\int_0^\infty e^{-\rho t} U^s(c, l)\, dt = \int_0^\infty e^{-\rho t} u(c, l, 1)\, dt$$

subject to the economy's resource constraint

$$\dot{k} = F(k, l) - c \tag{4}$$

and the initial condition $k(0) = k_0 > 0$, where k_0 is exogenously given. The current-value Hamiltonian is given by

$$H = U^s(c,l) + \lambda[F(k,l) - c] = u(c,l,1) + \lambda[F(k,l) - c],$$

where the costate variable λ denotes the shadow price of capital. The necessary conditions for (c,l,k) to be an optimal path are equal to

$$U_c^s(c,l) = u_c(c,l,1) = \lambda, \tag{5}$$

$$U_l^s(c,l) = u_l(c,l,1) = -\lambda F_l(k,l), \tag{6}$$

$$\dot{\lambda} = -[F_k(k,l) - \rho]\lambda. \tag{7}$$

The transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda k = 0. \tag{8}$$

The assumptions made so far ensure that if (c,l,k) satisfies (4), (5), (6), (7), (8) and the initial condition $k(0) = k_0$, then it is an optimal path.

In the decentralized economy the representative individual chooses the time paths c and l to maximize

$$\int_0^\infty e^{-\rho t}U(c,l,C)\,dt = \int_0^\infty e^{-\rho t}u(c,l,c/C)\,dt$$

subject to the flow budget constraint (2), the no-Ponzi-game condition (3) and the initial condition $a(0) = k_0$. A crucial feature of this optimization problem is that the representative agent takes not only the time paths of r and w, but also the time path of C as given. In other words, each individual is small enough to neglect his own contribution to the average consumption level in the economy. The current-value Hamiltonian of this optimization problem is written as

$$H = U(c,l,C) + \lambda(ra + wl - c) = u(c,l,c/C) + \lambda(ra + wl - c),$$

where, as before, λ denotes the shadow price of wealth. The necessary conditions for (c,l,a) to follow an optimal path are equal to

$$U_c(c,l,C) = u_c(c,l,c/C) + C^{-1}u_z(c,l,c/C) = \lambda, \tag{9}$$

$$U_l(c,l,C) = u_l(c,l,c/C) = -\lambda w, \tag{10}$$

$$\dot{\lambda} = -(r - \rho)\lambda. \tag{11}$$

The transversality condition given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda a = 0 \tag{12}$$

ensures that the no-Ponzi-game condition (3) holds with equality. If (c,l,a) satisfies (9), (10), (11), (12), (2), and the initial condition $a(0) = k_0$, then it is an optimal path.

The next step in our analysis is to derive the symmetric macroeconomic equilibria for the decentralized economy. Since all individuals are identical, this means that i) identical individuals make identical choices so that $c = C$, or $z \equiv c/C = 1$ for all t, ii) each individual holds zero net loans so that net wealth is simply equal to the capital stock, i.e., $a = k$, iii) the real rental rate and the real wage are determined by the profit-maximizing conditions $r = F_k(k, l)$ and $w = F_l(k, l)$, and iv) the constant returns to scale assumption implies that $F(k, l) = rk + wl$. Substituting these relationships into (9), (10), (11), (12), (2), and the initial condition $a(0) = k_0$, the dynamic evolution of the decentralized economy is determined by the optimality condition

$$u_c(c, l, 1) + c^{-1}u_z(c, l, 1) = \lambda \tag{13}$$

along with the optimality condition (6), the differential equations (7) and (4), the transversality condition (8) and the initial condition $k(0) = k_0$. Obviously, the only difference between this system and the system which determines the evolution of (c, l, k) in the socially planned economy is that (5) is replaced by (13).

Note that the differential equations $\dot{\lambda} = -[F_k(k, l) - \rho]\lambda$ and $\dot{k} = F(k, l) - c$ are common to both the decentralized and the socially planned economies. Since, in addition, the production function exhibits constant returns to scale, it is clear that in steady state equilibrium, in which $\dot{k} = \dot{\lambda} = 0$, the capital-labour ratio, k/l, and the consumption-labour ratio, c/l, are equal in both economies and determined by the following equations:

$$F_k(k/l, 1) = \rho, \qquad c/l = F(k/l, 1).$$

It follows that the decentralized steady state ratios, denoted by $\tilde{k}^d/\tilde{l}^d$ and $\tilde{c}^d/\tilde{l}^d$, are identical to their socially optimal counterparts, equal to $\tilde{k}^p/\tilde{l}^p$ and $\tilde{c}^p/\tilde{l}^p$, where the symbol $\sim$ denotes a steady state value. In the following analysis we will, however, show that while $\tilde{k}^d/\tilde{l}^d = \tilde{k}^p/\tilde{l}^p$ and $\tilde{c}^d/\tilde{l}^d = \tilde{c}^p/\tilde{l}^p$, it is also the case that $\tilde{l}^d > \tilde{l}^p$, $\tilde{c}^d > \tilde{c}^p$ and $\tilde{k}^d > \tilde{k}^p$ under plausible assumptions with respect to preferences. This means that (at least) in the long run people work and consume too much, and accumulate to much physical capital.

We will next study the dynamic behavior of both the decentralized and the socially planned economy.[7] The first step is to solve the necessary optimality conditions, given by (5) and (6) in the centralized case and by (13) and (6) in the decentralized case, for c and l. This yields

$$c = \hat{c}^j(\lambda, k), \qquad l = \hat{l}^j(\lambda, k), \qquad j = p, d, \tag{14}$$

with the partial derivatives for the centralized economy, $\hat{c}^p$ and $\hat{l}^p$, equal to

$$\begin{aligned} \hat{c}^p_\lambda &= \frac{u_{ll} + u_{cl}F_l + \lambda F_{ll}}{D^p}, & \hat{l}^p_\lambda &= -\frac{u_{lc} + u_{cc}F_l}{D^p}, \\ \hat{c}^p_k &= \frac{u_{cl}\lambda F_{lk}}{D^p}, & \hat{l}^p_k &= -\frac{u_{cc}\lambda F_{lk}}{D^p}, \end{aligned} \tag{15}$$

[7]For an account of this type of analysis, see Turnovsky (1995), Chapter 9, after p. 234.

where $D^p = u_{cc}(u_{ll} + \lambda F_{ll}) - u_{cl}^2 > 0$.[8] Our assumptions concerning the properties of the instantaneous utility function u and the production function F imply that $D^p > 0$ and that the partial derivatives have the following signs:

$$\hat{c}^p_\lambda < 0, \qquad \hat{l}^p_\lambda > 0, \qquad \operatorname{sgn}\left(\hat{c}^p_k\right) = \operatorname{sgn}\left(u_{cl}\right), \qquad \hat{l}^p_k > 0. \tag{16}$$

Due to normality, both own consumption and leisure depend negatively on the shadow price of capital λ. Equally, work effort depends positively on λ. Since $F_{kl} > 0$ by assumption, a rise in capital k raises the marginal product of labour. This effect causes labour (resp. leisure) to depend positively (resp. negatively) on the capital stock. Own consumption depends negatively on the capital stock if $U^s_{cl}(c,l) = u_{cl}(c,l,1) < 0$, i.e., if a rise in leisure increases the marginal utility of own consumption $U^s_c(c,l) = u_c(c,l,1)$. While Barro and Sala-i-Martin (1995) and Turnovsky (1995) (in models without consumption externalities) consider the case in which $u_{cl} < 0$ to be intuitively more plausible, we will not rule out the alternative that $U^s_{cl}(c,l) = u_{cl}(c,l,1) > 0$.[9]

In the decentralized economy the partial derivatives of $\hat{c}^d$ and $\hat{l}^d$ are given by

$$\hat{c}^d_\lambda = \frac{u_{ll} + \left(u_{cl} + c^{-1}u_{zl}\right)F_l + \lambda F_{ll}}{D^d},$$

$$\hat{l}^d_\lambda = -\frac{u_{cl} + \left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right)F_l}{D^d}, \tag{17}$$

$$\hat{c}^d_k = \frac{\left(u_{cl} + c^{-1}u_{zl}\right)\lambda F_{kl}}{D^d}, \qquad \hat{l}^d_k = -\frac{\left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right)\lambda F_{kl}}{D^d},$$

where $D^d = \left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right)(u_{ll} + \lambda F_{ll}) - u_{cl}\left(u_{cl} + c^{-1}u_{zl}\right)$.[10] Observe that the signs of D^d, $\hat{c}^d_\lambda$, $\hat{c}^d_k$, $\hat{l}^d_\lambda$, and $\hat{l}^d_k$ cannot be determined without introducing further assumptions. For this reason, we assume first that

$$\left(u_{cc} + c^{-1}u_{cz} - c^{-2}u_z\right)u_{ll} - \left(u_{cl} + c^{-1}u_{zl}\right)u_{cl} > 0. \tag{18}$$

Loosely speaking, this condition ensures that individuals do not overreact to changes in average consumption.[11] We impose next an additional normality condition that the decentralized symmetric marginal rate of substitution

[8] In these expressions the second-order partial derivatives of u and F are evaluated at $(c,l,1) = \left(\hat{c}^p(\lambda,k), \hat{l}^p(\lambda,k), 1\right)$ and $(k,l) = \left(k, \hat{l}^p(\lambda,k)\right)$, respectively.

[9] The analysis becomes, nevertheless, much simpler under the assumption that $u_{cl}(c,l,1) \leq 0$. One reason is that $u_{cl}(c,l,1) \leq 0$ is a sufficient (but not necessary) condition for normality.

[10] In these expressions $c = \hat{c}^d(\lambda,k)$, and the partial derivatives of u and F are evaluated at $(c,l,1) = \left(\hat{c}^d(\lambda,k), \hat{l}^d(\lambda,k), 1\right)$ and $(k,l) = \left(k, \hat{l}^d(\lambda,k)\right)$, respectively.

[11] The initial necessary optimality conditions of the representative individual's optimization problem (9) and (10) can be solved for c and l in the form $c = \hat{c}^{d1}(\lambda,w,C)$ and $l = \hat{l}^{d1}(\lambda,w,C)$. The inequality (18) ensures that $\hat{c}^{d1}_C(\lambda,w,c) < 1$, i.e., that the partial derivative of own consumption c with respect to average consumption in the economy C is less than unity where $c = C$ holds. For a detailed discussion, see Hof (1999b).

$(MRS)^d(c,l,c)$ depends negatively on consumption c and positively on leisure, i.e., $(MRS)_c^d(c,l,c)+(MRS)_C^d(c,l,c)<0$ and $(MRS)_l^d(c,l,c)<0$.[12] Finally, we assume that the labour market exhibits Walras stability. Under these conditions it follows that

$$D^d>0, \qquad \hat{c}_\lambda^d<0, \qquad \hat{l}_\lambda^d>0, \tag{19}$$

while the signs of $\hat{c}_k^d$ and $\hat{l}_k^d$ remain ambiguous.

Substituting (14) into (4) and (7), we obtain the dynamic system(s) that jointly determines the evolution of k and λ. It is equal to

$$\begin{aligned} \dot{k} &= F\left(k,\hat{l}^j(\lambda,k)\right)-\hat{c}^j(\lambda,k) \\ \dot{\lambda} &= -\left[F_k\left(k,\hat{l}^j(\lambda,k)\right)-\rho\right]\lambda \end{aligned} \qquad j=p,d. \tag{20}$$

Linearizing (20) about the steady states $\left(\tilde{k}^j,\tilde{\lambda}^j\right)$, $j=p,d$, the local dynamics can be approximated by the following system of linear differential equations:

$$\begin{pmatrix} \dot{k} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} \omega_{11}^j & \omega_{12}^j \\ -\tilde{\lambda}^j\omega_{21}^j & -\tilde{\lambda}^j\omega_{22}^j \end{pmatrix} \begin{pmatrix} k-\tilde{k}^j \\ \lambda-\tilde{\lambda}^j \end{pmatrix}, \quad j=p,d, \tag{21}$$

where

$$\begin{aligned} \omega_{11}^j &= \left[F_k+F_l\hat{l}_k^j-\hat{c}_k^j\right]_{SSj}, \qquad & \omega_{12}^j &= \left[F_l\hat{l}_\lambda^j-\hat{c}_\lambda^j\right]_{SSj}, \\ \omega_{21}^j &= \left[F_{kk}+F_{kl}\hat{l}_k^j\right]_{SSj}, & \omega_{22}^j &= \left[F_{kl}\hat{l}_\lambda^j\right]_{SSj}, \end{aligned} \tag{22}$$

and where $[\bullet]_{SSj}$ indicates that the expressions between the brackets are evaluated at the appropriate steady state. Substitution of (15) and (17), respectively, into (22) yields the complete solutions of the ω_{im}^j elements. These expressions are stated in the appendix (see subsection 7.1). To examine the stability properties of the two economies, we calculate the characteristic polynomial of (21), which is equal to

$$P^j(\xi) \equiv \xi^2-\left(\omega_{11}^j-\tilde{\lambda}^j\omega_{22}^j\right)\xi-\tilde{\lambda}^j\left(\omega_{11}^j\omega_{22}^j-\omega_{12}^j\omega_{21}^j\right)=0. \tag{23}$$

Using the partial derivatives (16) and the expressions for the ω_{im}^p elements given in the appendix (see subsection 7.1), we can show that our assumptions made with respect to preferences and technology are sufficient for

$$\omega_{11}^p>0, \qquad \omega_{12}^p>0, \qquad \omega_{21}^p<0, \qquad \omega_{22}^p>0. \tag{24}$$

From (24) follows that the term $-\tilde{\lambda}^p\left(\omega_{11}^p\omega_{22}^p-\omega_{12}^p\omega_{21}^p\right)$ is unambiguously negative, which implies that the roots, or eigenvalues, of the characteristic polynomial

[12] The first inequality means that if the economy moves from one symmetric situation to another with a higher common level of consumption, then the marginal rate of substitution as perceived by the representative consumer decreases.

of the socially planned solution are of opposite sign, i.e., $\xi_1^p < 0$ and $\xi_2^p > 0$, and, thus, that the centralized steady state equilibrium is a saddlepoint. Using the expression given in the appendix it can be further shown that

$$\omega_{12}^d > 0, \quad \omega_{21}^d < 0, \quad \omega_{22}^d > 0, \quad -\tilde{\lambda}^d \left(\omega_{11}^d \omega_{22}^d - \omega_{12}^d \omega_{21}^d\right) < 0. \tag{25}$$

Hence, the roots of the characteristic polynomial of the decentralized solution are as well of opposite sign, i.e., $\xi_1^d < 0$ and $\xi_2^d > 0$. The stationary equilibrium of the decentralized economy is, consequently, also a saddlepoint. Employing standard methods and imposing the transversality condition, we can show that the solution to (20) can be approximated by the following equations for $j = p, d$:

$$k^j = \tilde{k}^j + \left(k_0 - \tilde{k}^j\right) \exp\left(\xi_1^j t\right),$$

$$\lambda^j - \tilde{\lambda}^j = -\frac{\omega_{11}^j - \xi_1^j}{\omega_{12}^j}\left(k^j - \tilde{k}^j\right) = -\frac{\tilde{\lambda}^j \omega_{21}^j}{\tilde{\lambda}^j \omega_{22}^j + \xi_1^j}\left(k^j - \tilde{k}^j\right). \tag{26}$$

Given $\omega_{11}^p > 0$, $\omega_{12}^p > 0$, and $\xi_1^p < 0$, it is obvious from (26) that the stable arm corresponding to the socially planned solution is negatively sloped in the (k, λ) plane. In other words, a rise in the capital stock is accompanied by a decline in the its shadow value and that both adjust monotonically. What can we infer concerning the slope of the stable arm of the decentralized economy? In the appendix we demonstrate that $\omega_{11}^d - \xi_1^d > 0$. Since, in addition, $\omega_{12}^d > 0$ (see (25)), it is clear from (26) that this stable arm is also negatively sloped in the (k, λ) plane.

What can be said about the optimal paths of own consumption and work effort, c^j and l^j? Using (14), we can write the expressions for consumption and leisure as $c^j = \hat{c}^j\left(\lambda^j, k^j\right)$ and $l^j = \hat{l}^j\left(\lambda^j, k^j\right)$, respectively. Consider, for instance, the socially planned economy and assume that $k_0 < \tilde{k}^p$. In this case the transition to the steady state is characterized by a rising capital stock k and a falling shadow price of capital λ. Using (16), we can infer that the fall in λ leads to an increase in the optimal levels of consumption and leisure. In contrast, the rise in k causes the optimal level of leisure to fall, while its effect on consumption is ambiguous, since it depends on the sign of u_{cl}. The direction of the net effects can be determined, nevertheless, as follows: linearizing $c^j = \hat{c}^j\left(\lambda^j, k^j\right)$ about steady state and substituting (26), we obtain the (k, c) locus given by

$$c^j - \tilde{c}^j = -\left(\frac{\eta_{11}^j - \xi_1^j}{\eta_{12}^j}\right)\left(k^j - \tilde{k}^j\right), \quad j = p, d,$$

where η_{1m}^j, $m = 1, 2$ coefficients are defined in the appendix (see subsection 7.2). Since we can show that $\eta_{11}^j > 0$ and $\eta_{12}^j < 0$ for $j = p, d$, the stable locus in the (k, c) plane is positively sloped in both economies, which implies that a rise in the capital stock is accompanied by a rise in consumption if $k_0 < \tilde{k}^p$. Clearly, both the capital stock and consumption converge monotonically to their stationary values. Following the same procedure for labour, where $l^j = \hat{l}^j\left(\lambda^j, k^j\right)$, we cannot, however, determine whether l^j rises or falls monotonically toward its steady-state value.

The following proposition summarizes the results derived to this point and states the key result of the paper that is proved in the appendix.

Proposition 1 *Under the assumptions made above, the following results hold:*

A) The steady-state values of the capital-labour ratio and the consumption-labour ratio are identical in the two economies:

$$\tilde{k}^d/\tilde{l}^d = \tilde{k}^p/\tilde{l}^p, \quad \tilde{c}^d/\tilde{l}^d = \tilde{c}^p/\tilde{l}^p.$$

B) Both the centralized and the decentralized steady states are saddlepoints. Furthermore, in both cases the stable arm exhibits the property that the capital stock and consumption converge monotonically to their steady-state levels such that the two variables always move in the same direction.

C) The optimal plan chosen by the benevolent social planner leads to a steady state in which private individuals work and consume less and the capital stock is lower than in decentralized economy:

$$\tilde{c}^p < \tilde{c}^d, \quad \tilde{l}^p < \tilde{l}^d, \quad \tilde{k}^p < \tilde{k}^d.$$

Proposition 1 shows that at least in the long run status-conscious people work and consume too much, and the capital stock is too high, compared to the social optimum. We have shown both A) and B) above. The proof of C) is given in the appendix (see subsection 7.3) and will be illustrated with two examples in the next section.

4 Illustrations

4.1 Additively Separable Utility Function

In this subsection we analyze the case in which the instantaneous utility function u takes the following additively separable form

$$u(c, l, z) = v(c, l) + \delta \cdot s(z), \tag{27}$$

where $\delta > 0$ is a fixed preference parameter. The partial derivatives of the utility functions v and s are assumed to have the following properties:

$$v_c > 0, \quad v_{cc} < 0, \quad v_l < 0, \quad v_{ll} < 0, \quad v_{cc}v_{ll} - v_{cl}^2 > 0, \quad s' > 0, \quad s'' < 0.$$

To simplify the analysis, we will assume in this case that an increase in leisure raises the marginal utility of own consumption, so that $v_{cl} < 0$.[13] The decentralized optimality conditions (13) and (6) become

$$v_c(c,l) + c^{-1}\delta s'(1) = \lambda, \qquad v_l(c,l) = -\lambda F_l(k,l),$$

while their socially planned counterparts (5) and (6) are equal to

$$v_c(c,l) = \lambda, \qquad v_l(c,l) = -\lambda F_l(k,l).$$

It is clear that the decentralized economy coincides with the socially planned solution if status does not play any role, i.e., if $\delta = 0$.

We can show that the steady-state values of the decentralized economy's endogenous variables depend crucially on the preference parameter for status, which we specify as $\vartheta \equiv \delta s'(1)$. We can calculate the following comparative static expressions:

$$\frac{\partial\left(\tilde{k}^d/\tilde{l}^d\right)}{\partial\vartheta} = \frac{\partial\tilde{r}^d}{\partial\vartheta} = \frac{\partial\tilde{w}^d}{\partial\vartheta} = 0, \qquad \frac{\partial\tilde{c}^d}{\partial\vartheta} = \frac{\partial\tilde{y}^d}{\partial\vartheta} = \left[\frac{(F/l)F_l}{G}\right]_{SSd} > 0$$

$$\frac{\partial\tilde{k}^d}{\partial\vartheta} = -\left[\frac{F_l F_{kl}}{F_{kk}G}\right]_{SSd} > 0, \qquad \frac{\partial\tilde{l}^d}{\partial\vartheta} = \left[\frac{F_l}{G}\right]_{SSd} > 0$$

where

$$G = -\left\{(v_{ll} + v_{cl}F_l) + (F/l)\left[v_{lc} + \left(v_{cc} - c^{-2}\vartheta\right)F_l\right]\right\}c > 0.$$

According to these results the steady state has the following properties: i) the capital-labour ratio, the real interest rate, and the real wage are independent of ϑ and ii) $\tilde{c}^d$, $\tilde{y}^d$, $\tilde{l}^d$, and $\tilde{k}^d$ depend positively on ϑ. Observe that the solution to the social planner's problem coincides with the decentralized solution if $\delta = 0$. Since $\delta = 0$ implies that $\vartheta = 0$, this is equivalent to $\tilde{x}^p = \tilde{x}^d\big|_{\vartheta=0}$. Finally, since $\partial\tilde{c}^d/\partial\vartheta > 0$, $\partial\tilde{y}^d/\partial\vartheta > 0$, $\partial\tilde{k}^d/\partial\vartheta > 0$, and $\partial\left(\tilde{k}^d/\tilde{l}^d\right)/\partial\vartheta = 0$, it is clear that:

$$\tilde{c}^p < \tilde{c}^d, \quad \tilde{y}^p < \tilde{y}^d, \quad \tilde{l}^p < \tilde{l}^d, \quad \tilde{k}^p < \tilde{k}^d, \qquad \tilde{k}^p/\tilde{l}^p = \tilde{k}^d/\tilde{l}^d.$$

In other words, private households in the stationary decentralized equilibrium work and consume more, (and produce more output) than they do in the centralized case, as is maintained in part C of proposition 1.

4.2 An Isoelastic Utility Function

The utility function studied in the preceding subsection exhibits the property that $u_{lz} = u_{cz} = 0$. In this subsection we consider a specification that permits

[13] If we, instead, allow $v_{cl} > 0$, then further conditions must be imposed in order for (27) to satisfy the assumptions made in our paper. See Hof (1999b) for further details.

$u_{cz} \neq 0$. Specifically, we consider the utility function that is isoelastic in own and relative consumption and separable in work effort

$$u = \frac{1}{1-\theta}\left[\left(c^{1-\beta}z^{\beta}\right)^{1-\theta} - 1\right] - \mu l^{1+\sigma}, \quad \theta > 0,\, \mu > 0,\, \sigma > 0,\, 0 < \beta < 1 \quad (28)$$

where (28) satisfies our curvature assumptions and the parameter β represents the agent's preference for status. If, in addition, the production function takes the Cobb-Douglas form $F(k,l) = Bk^{\alpha}l^{1-a}$, with $B > 0$ and $0 < \alpha < 1$, then the steady-state values of c, l, and k are easily calculated. In particular, we can show that ratios of decentralized to centralized consumption, work effort, physical capital are equal and exceed unity according to the following relationship

$$\frac{\tilde{c}^d}{\tilde{c}^p} = \frac{\tilde{l}^d}{\tilde{l}^p} = \frac{\tilde{k}^d}{\tilde{k}^p} = \left(\frac{1}{1-\beta}\right)^{1/(\sigma+\beta+(1-\beta)\theta)} \equiv \Psi(\beta,\theta,\sigma) > 1,$$

where the expressions for $\tilde{c}^p$, $\tilde{l}^p$, and $\tilde{k}^p$ are given in the appendix (see subsection 7.4) and $\Psi(\beta,\theta,\sigma)$ represents the scale factor according to which economic activity in the status-conscious economy rises relative to its socially planned counterpart. It can be shown that $\Psi(\beta,\theta,\sigma)$ is greater, the larger is the status parameter β, the stronger is the willingness to shift consumption toward the future, which is inversely related to θ, and weaker is disutility of work effort, which depends positively on σ.

5 Optimal Taxation

We will analyze in this section whether the social optimum can be attained in the decentralized economy by means of optimal taxation. The key distortion is that the willingness to substitute consumption for leisure is too high in the decentralized economy, i.e.,

$$(MRS)^d(c,l,c) = \left(\frac{u_c(c,l,1) + c^{-1}u_z(c,l,1)}{u_c(c,l,1)}\right)(MRS)^p(c,l) > (MRS)^p(c,l).$$

An obvious policy response is to impose a tax on consumption, since it raises the price of present consumption in terms of present as well as future leisure. If the government imposes a consumption tax and returns lump-sum transfers, then the flow budget constraint of the representative household equals

$$\dot{a} = ra + wl - c(1+\tau) + T,$$

where τ and T denote the consumption tax rate and lump-sum transfers, respectively. We assume that each individual takes not only the time path of C, but also the time paths of τ and T as given. In the following, we will restrict attention to symmetric macroeconomic equilibria in which the government runs a balanced budget for all t. In such a situation the conditions $c = C$ and $\tau c = T$ hold. In this case the dynamic evolution of $(c,\, l,\, k,\, \lambda)$ is governed by the first-order condition

$$u_c(c,l,1) + c^{-1} u_z(c,l,1) = \lambda(1+\tau) \tag{29}$$

along with the optimality condition (6), the differential equations (7) and (4), the transversality condition (8) and the initial condition $k(0) = k_0$. By introducing the consumption tax, the only difference is that (13) is replaced by (29). The crucial question is whether the social optimum can be attained in the decentralized economy by choosing the consumption tax rate appropriately.

Proposition 2 *If the government sets the consumption tax rate τ according to*

$$\tau = \Phi(C,L) \equiv \frac{u_z(C,L,1)}{C u_c(C,L,1)}, \tag{30}$$

where L denotes the average hours worked in the economy, then the social optimum is attained by the decentralized economy.

The idea of the proof is as follows: Since the structures of the two economies are identical, except with respect to optimality conditions for own consumption, a tax policy which ensures that (29) is equivalent to (5), ensures that the decentralized economy reproduces the social optimum. Assume that the government sets τ according to (30). Since the representative individual takes the time paths of C and L as given, he also takes the time path of $\tau = \Phi(C,L)$ as given. In a symmetric equilibrium in which identical individuals make identical choices, $\tau = \Phi(c,l)$ holds. Substituting this relationship into (29) and rearranging we obtain:

$$\left(1 + \frac{u_z(c,l,1)}{c u_c(c,l,1)}\right)\left[u_c(c,l,1) - \lambda\right] = 0. \tag{31}$$

Since $u_c > 0$ and $u_z > 0$, it is obvious that (31) is equivalent to (5), so that $u_c(c,l,1) = \lambda$. Hence, the tax policy given by (30) will ensure that the social optimum is attained by the decentralized economy.

We will give two specifications of the instantaneous utility function in which the optimal tax rate is constant over time. First, if u takes the form

$$u(c,l,z) = \alpha \ln c + g(l) + \delta s(z), \qquad g' < 0, \quad g'' < 0, \quad s' > 0, \quad s'' < 0,$$

which is a special case of (27), then $u_c(C,L,1) = \alpha C^{-1}$ and $u_z(C,L,1) = \vartheta$ where $\vartheta \equiv \delta s'(1)$. Substitution of this result into (30) yields $\tau = \Phi(C,L) = \vartheta/\alpha$. This constant tax rate, which ensures that the social optimum is attained by the decentralized economy, depends positively on both δ and $s'(1)$ and negatively on α.

Next, if u takes the form given by (28), then

$$u_c(C,L,1) = (1-\beta)\, C^{-[\beta+(1-\beta)\theta]}, \quad u_z(C,L,1) = \beta C^{-[\beta+(1-\beta)\theta]}.$$

Substitution of these results into (30) yields $\tau = \Phi(C,L) = \beta/(1-\beta)$. Obviously, the optimal tax rate is constant over time and depends positively on the parameter β. It is zero if $\beta = 0$, i.e., if relative consumption does not matter at all, and tends to infinity as $\beta \to 1$.

6 Concluding Remarks

We have examined the effects of competition for status within a Ramsey-type model in which labour supply is endogenously determined. In this model the status of any individual is solely determined by his relative consumption. Appropriate normality conditions with respect to consumption and leisure ensure that at least in the long run status-conscious individuals consume and work too much and the capital stock is too high compared to the socially optimal solution. The government can induce, however, the private sector to attain the social optimum by designing an optimal consumption tax policy.

7 Appendix

7.1 The ω_{im}^{j} elements

In this section of the appendix we give the expressions for the ω_{im}^{j} elements of the coefficient matrix of the system (21), where $i, m = 1, 2$, $j = p, d$. For the planned, or centralized economy, these are given by:

$$\omega_{11}^{p} = \rho - \tilde{\lambda}^{p} \left[\frac{(u_{cc}F_l + u_{cl})\, F_{lk}}{D^p} \right]_{SSp} > 0,$$

$$\omega_{12}^{p} = - \left[\frac{(u_{lc} + u_{cc}F_l)\, F_l + (u_{ll} + u_{cl}F_l) + \lambda F_{ll}}{D^p} \right]_{SSp} > 0,$$

$$\omega_{21}^{p} = \left[\frac{F_{kk}\left(u_{cc}u_{ll} - u_{cl}^{2}\right)}{D^p} \right]_{SSp} < 0,$$

$$\omega_{22}^{p} = - \left[\frac{F_{kl}\left(u_{lc} + u_{cc}F_l\right)}{D^p} \right]_{SSp} > 0.$$

For the decentralized economy, these elements equal:

$$\omega_{11}^{d} = \rho - \tilde{\lambda}^{d} \left[\frac{F_{kl}\left[\left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right) F_l + \left(u_{cl} + c^{-1}u_{zl}\right)\right]}{D^d} \right]_{SSd}$$

$$\omega_{12}^{d} = - \left[\frac{F_l\left[u_{cl} + \left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right) F_l\right] + u_{ll} + \left(u_{cl} + c^{-1}u_{zl}\right) F_l + \lambda F_{ll}}{D^d} \right]_{SSd}$$

$$\omega_{21}^{d} = \left[\frac{F_{kk}\left[\left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right) u_{ll} - u_{lc}\left(u_{cl} + c^{-1}u_{zl}\right)\right]}{D^d} \right]_{SSd}$$

$$\omega_{22}^{d} = - \left[\frac{F_{kl}\left[u_{cl} + \left(u_{cc} - c^{-2}u_z + c^{-1}u_{zc}\right) F_l\right]}{D^d} \right]_{SSd}$$

We can show that the determinant of (21), $-\tilde{\lambda}^d\left(\omega_{11}^d\omega_{22}^d-\omega_{12}^d\omega_{21}^d\right)$, is equal to

$$-\tilde{\lambda}^d\left\{\left[\frac{F_{kk}\,(F/l)\left[u_{lc}+\left(u_{cc}-c^{-2}u_z+c^{-1}u_{zc}\right)F_l\right]}{D^d}\right]_{SSd}\right.$$
$$\left.+\left[\frac{F_{kk}\left[u_{ll}+\left(u_{cl}+c^{-1}u_{zl}\right)F_l\right]}{D^d}\right]_{SSd}\right\}<0$$

Next, we will show that $\omega_{11}^d-\xi_1^d>0$. Since the steady state is a saddlepoint, the characteristic polynomial (23) has the following properties:

$$P^d\,(\xi)>0\Leftrightarrow\xi<\xi_1^d<0\text{ or }\xi>\xi_2^d>0;\quad P^d\,(\xi)<0\Leftrightarrow\xi_1^d<\xi<\xi_2^d \tag{32}$$

From (25) and $P^d\left(\omega_{11}^d\right)=\tilde{\lambda}^d\omega_{12}^d\omega_{21}^d$ follows that $P^d\left(\omega_{11}^d\right)<0$. Hence, according to (32) we have shown that $\xi_1^d<\omega_{11}^d<\xi_2^d$, which implies that $\omega_{11}^d-\xi_1^d>0$.

7.2 The η_{1m}^j elements

Using the expressions for the partial derivatives $\hat{c}_\lambda^p$ and $\hat{c}_k^p$, (see (15)), and the elements ω_{11}^p and ω_{12}^p, given in subsection 7.1, we can calculate that η_{1m}^P, $m=1$, 2, are equal to:

$$\eta_{11}^p=\rho-\left[\frac{\lambda F_lF_{kl}}{u_{ll}+u_{cl}F_l+\lambda F_{ll}}\right]_{SSp},$$
$$\eta_{12}^p=-\left[\frac{\left(u_{lc}+u_{cc}F_l\right)F_l+u_{ll}+u_{cl}F_l+\lambda F_{ll}}{u_{ll}+u_{cl}F_l+\lambda F_{ll}}\right]_{SSp}.$$

The conditions for normality and the properties of the production function ensure that $\eta_{11}^p>0$ and $\eta_{12}^p<0$.

Substituting for the partial derivatives $\hat{c}_\lambda^d$ and $\hat{c}_k^d$, (see (17)), and the elements ω_{11}^d and ω_{12}^d stated above in subsection 7.1, we can show that:

$$\eta_{11}^d=\rho-\left[\frac{\lambda F_lF_{kl}}{u_{ll}+\left(u_{cl}+c^{-1}u_{zl}\right)F_l+\lambda F_{ll}}\right]_{SSd},$$
$$\eta_{12}^d=-\left[\frac{\left[u_{cl}+\left(u_{cc}-c^{-2}u_z+c^{-1}u_{zc}\right)F_l\right]F_l+u_{ll}+\left(u_{cl}+c^{-1}u_{zl}\right)F_l+\lambda F_{ll}}{u_{ll}+\left(u_{cl}+c^{-1}u_{zl}\right)F_l+\lambda F_{ll}}\right]_{SSd}.$$

From normality and the properties of the production function, it follows that $\eta_{11}^d>0$ and $\eta_{12}^d<0$.

7.3 Proof of proposition 1 part C

In the socially planned economy the steady-state values of c, l, k, and λ are obtained from the following equations:

$$u_c\left(\tilde{c}^p,\tilde{l}^p,1\right)-\tilde{\lambda}^p=0, \tag{33}$$

$$u_l\left(\tilde{c}^p,\tilde{l}^p,1\right)+\tilde{\lambda}^p F_l\left(\tilde{k}^p,\tilde{l}^p\right)=0, \tag{34}$$

$$\rho=F_k\left(\tilde{k}^p,\tilde{l}^p\right), \tag{35}$$

$$\tilde{c}^p=F\left(\tilde{k}^p,\tilde{l}^p\right). \tag{36}$$

In the decentralized economy the corresponding steady-state values are determined by

$$u_c\left(\tilde{c}^d,\tilde{l}^d,1\right)+u_z\left(\tilde{c}^d,\tilde{l}^d,1\right)\left(1/\tilde{c}^d\right)-\tilde{\lambda}^d=0, \tag{37}$$

$$u_l\left(\tilde{c}^d,\tilde{l}^d,1\right)+\tilde{\lambda}^d F_l\left(\tilde{k}^d,\tilde{l}^d\right)=0, \tag{38}$$

$$\rho=F_k\left(\tilde{k}^d,\tilde{l}^d\right), \tag{39}$$

$$\tilde{c}^d=F\left(\tilde{k}^d,\tilde{l}^d\right). \tag{40}$$

Since by assumption the production exhibits constant returns we have

$$F(k,l)=l\cdot f(k/l), \tag{41}$$

$$F_k(k,l)=f'(k/l), \tag{42}$$

$$F_l(k,l)=f(k/l)-(k/l)\cdot f'(k/l), \tag{43}$$

where $f(k/l)\equiv F(k/l,1)$. From (35), (39), and (42), it follows that

$$\tilde{k}^d/\tilde{l}^d=\tilde{k}^p/\tilde{l}^p=\kappa, \quad \kappa\equiv\left(f'\right)^{-1}(\rho), \tag{44}$$

which implies that the steady-state values of the capital-labour ratios are identical in the two economies. Using (36), (40), (41), and (44), we obtain

$$\tilde{c}^d/\tilde{l}^d=\tilde{c}^p/\tilde{l}^p=f(\kappa). \tag{45}$$

Using (43) and (44), we obtain

$$F_l\left(\tilde{k}^d,\tilde{l}^d\right)=F_l\left(\tilde{k}^p,\tilde{l}^p\right)=\psi, \quad \psi\equiv f(\kappa)-\kappa f'(\kappa). \tag{46}$$

From (33), (34), (45) and (46), it follows that $\tilde{l}^p$ is a solution to $\Theta_1(l)-\Theta_2(l)=0$, where the functions Θ_1 and Θ_2 are defined as

$$\Theta_1(l)\equiv-(1/\psi)\,u_l(l\cdot f(\kappa),l,1)>0, \qquad \Theta_2(l)\equiv u_c(l\cdot f(\kappa),l,1)>0.$$

The first-order derivatives of Θ_1 and Θ_2 are given by

$$\Theta_1'(l)=-(1/\psi)\left[u_{lc}(l\cdot f(\kappa),l,1)\,f(\kappa)+u_{ll}(l\cdot f(\kappa),l,1)\right],$$

$$\Theta_2'(l)=\left[u_{cc}(l\cdot f(\kappa),l,1)\,f(\kappa)+u_{cl}(l\cdot f(\kappa),l,1)\right].$$

Hence,

$$\begin{aligned}\Theta_1'\left(\tilde{l}^p\right)-\Theta_2'\left(\tilde{l}^p\right) &= -\frac{1}{\psi}\left[u_{lc}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)f(\kappa)+u_{ll}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)\right]\\ &\quad -\left[u_{cc}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)f(\kappa)+u_{cl}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)\right].\end{aligned}$$

Using $\psi\equiv f(\kappa)-\kappa f'(\kappa)$, this result can be rewritten as

$$\begin{aligned}&\Theta_1'\left(\tilde{l}^p\right)-\Theta_2'\left(\tilde{l}^p\right) \qquad (47)\\ =\ &-\frac{1}{\psi}\left[u_{ll}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)+\psi u_{cl}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)\right]\\ &-\frac{f(\kappa)}{\psi}\left[u_{cl}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)+\psi u_{cc}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)\right].\end{aligned}$$

The normality assumption that the centralized marginal rate of substitution of consumption for leisure $(MRS)^p(c,l)$ depends negatively on consumption and positively on leisure implies that

$$u_{lc}(c,l,1)\,u_c(c,l,1)-u_l(c,l,1)\,u_{cc}(c,l,1)<0, \qquad (48)$$

$$u_{ll}(c,l,1)\,u_c(c,l,1)-u_l(c,l,1)\,u_{cl}(c,l,1)<0. \qquad (49)$$

From (33), (34), and (46), it follows that

$$u_c\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)=\tilde{\lambda}^p,\qquad -u_l\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)=\psi\tilde{\lambda}^p.$$

Substitution of these results into (48) and (49) yields

$$u_{cl}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)+\psi u_{cc}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)<0, \qquad (50)$$

$$u_{ll}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)+\psi u_{cl}\left(\tilde{l}^p\cdot f(\kappa),\tilde{l}^p,1\right)<0. \qquad (51)$$

From (47), (50), and (51), it follows that $\Theta_1'\left(\tilde{l}^p\right)-\Theta_2'\left(\tilde{l}^p\right)>0$. Hence, if $\tilde{l}^p$ is a solution to $\Theta_1(l)-\Theta_2(l)=0$, then $\Theta_1'\left(\tilde{l}^p\right)-\Theta_2'\left(\tilde{l}^p\right)>0$. In other words, at any point of intersection with the horizontal axis, the function $\Theta_1(l)-\Theta_2(l)$ is positively sloped. Taking into account that the utility function as well as its first-order and second-order partial derivatives are continuous functions by assumption, the following result are obvious: If $\tilde{l}^p$ is a solution to $\Theta_1(l)-\Theta_2(l)=0$, then (a) it is the unique solution, and (b)

$$\Theta_1(l)-\Theta_2(l)>0\Leftrightarrow l>\tilde{l}^p. \qquad (52)$$

From (37), (38), (45) and (46), it follows that $\tilde{l}^d$ is a solution to $\Theta_1(l)-[\Theta_2(l)+\Theta_3(l)]=0$, where the function Θ_3 is defined as

$$\Theta_3(l)\equiv u_z(l\cdot f(\kappa),l,1)\,\frac{1}{l\cdot f(\kappa)}>0.$$

Hence, we have

$$\Theta_1\left(\tilde{l}^d\right)-\Theta_2\left(\tilde{l}^d\right)=\Theta_3\left(\tilde{l}^d\right)>0. \qquad (53)$$

From (52) and (53) it is obvious that $\tilde{l}^d>\tilde{l}^p$. From (44) and (45) then follows that $\tilde{k}^p<\tilde{k}^d$ and $\tilde{c}^p<\tilde{c}^d$.

7.4 Utility function (28) – The solutions for $\tilde{c}^p$, $\tilde{l}^p$, and $\tilde{k}^p$

If u takes the form given by (28) and F takes the Cobb-Douglas specification $F(k,l) = Bk^{\alpha}l^{1-\alpha}$, we can then calculate the following stationary equilibrium values of consumption, work effort, and the stock of capital for the socially planned economy:

$$\tilde{c}^p = \left(\Gamma\left[B\left(\frac{\alpha}{\rho}\right)^{\alpha}\right]^{\frac{1+\sigma}{1-\alpha}}\right)^{\frac{1}{\sigma+\beta+(1-\beta)\theta}}, \quad \tilde{l}^p = \left(\Gamma\left[B\left(\frac{\alpha}{\rho}\right)^{\alpha}\right]^{\frac{(1-\beta)(1-\theta)}{1-\alpha}}\right)^{\frac{1}{\sigma+\beta+(1-\beta)\theta}}$$

$$\tilde{k}^p = \left(\Gamma\left[B^{1+\sigma}\left(\frac{\alpha}{\rho}\right)^{\sigma+\beta+(1-\beta)[\alpha+\theta(1-\alpha)]}\right]^{\frac{1}{1-\alpha}}\right)^{\frac{1}{\sigma+\beta+(1-\beta)\theta}}, \quad \Gamma = \frac{(1-\alpha)(1-\beta)}{\mu(1+\sigma)}.$$

References

[1] Barro, R.J., and X. Sala-i-Martin, 1995, *Economic Growth*, McGraw-Hill Advanced Series in Economics.

[2] Corneo, C. and O. Jeanne, 1997, *On Relative Wealth Effects and the Optimality of Growth*, Economics Letters 54 (1), pp. 87–92.

[3] Fisher, W.H. and F.X. Hof, 2000, *Relative Consumption, Economic Growth, and Taxation*, Journal of Economics, forthcoming.

[4] Frank, R.H., 1985, *The Demand for Unobservable and Other Nonpositional Goods*, The American Economic Review, March, 101–116.

[5] Futagami, Koichi, and Akihisa Shibata, 1998, *Keeping one step ahead of the Joneses: status, the distribution of wealth, and long run growth*, Journal of Economic Behavior and Organization (36) 1, pp. 93–111.

[6] Gali, J., 1994, *Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices*, Journal of Money, Credit, and Banking, vol. 26, no. 1, February, 1–8.

[7] Harbaugh, R., 1996, *Falling behind the Joneses: relative consumption and the growth-savings paradox*, Economics Letters 53, 297–304.

[8] Hof, F.X., 1999a, *Consumption Externalities, Economic Growth, and Optimal Taxation: A General Approach*, working paper 99/01, Institute of Economics, University of Technology, Vienna.

[9] Hof, F.X., 1999b, *Relative Consumption and Endogenous Labor Supply in the Ramsey Model: Do Status Conscious People Work Too Much*?, working paper 99/02, Institute of Economics, University of Technology, Vienna.

[10] Persson, M., 1995, *Why are Taxes so High in Egalitarian Societies?*, Scandinavian Journal of Economics 97(4), 569–580.

[11] Rauscher, M., 1997a, *Protestant Ethic, Status Seeking, and Economic Growth*, Discussion paper, University of Rostock, Germany.

[12] Rauscher, M., 1997b, *Conspicuous Consumption, Economic Growth, and Taxation*, Journal of Economics, Vol. 66, No. 1, 35–42.

[13] Turnovsky, St.J., 1995, *Methods of Macroeconomic Dynamics*, The MIT Press.

Authors: Walter H. Fisher, Franz X. Hof

Title: Relative Consumption and Endogenous Labour Supply in the Ramsey Model:
Do Status-Conscious People Work Too Much?

Reihe Ökonomie / Economics Series 85

ISSN: 1605-7996

ISSN: 1605-7996