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Economics Series

Non-implementation of Rational Expectations as a Perfect Bayesian Equilibrium

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Abstract

We show that a rational expectations equilibrium need not be incentive compatible, need not be implementable as a perfect Bayesian equilibrium and may not be fully Pareto optimal, unless the utility functions are state independent. A comparison of rational expectations equilibria with core concepts is also provided.

Keywords

Differential information economy, rational expectations equilibrium, coalitional Bayesian incentive compatibility, implementation, game trees, private core, weak fine core, interim weak fine core

JEL Classification

C71, C72, D5, D82

Comments
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Contents

1 Introduction

A differential information economy (DIE) consists of a finite set of agents each of which is characterized by a random utility function, a random consumption set, random initial endowments, a private information set and a prior probability distribution on a finite set of states of nature. A DIE is an extension of the Arrow - Debreu - McKenzie economy which enables us to model the idea of trade under asymmetric information.

The rational expectations equilibrium (REE) concept (see for example Radner (1979) and Allen (1981) among others), is an extension of the deterministic Walrasian equilibrium. Unlike the Walrasian equilibrium, the REE may not exist in well behaved economies (for example Kreps (1978)) and, moreover, may not satisfy certain efficiency criteria (see Allen (1981), Laffont (1985) and Grossman (1981)).

Despite the above non-attractive properties of REE, this concept is widely used because it is an extension of the classical Walrasian equilibrium idea. Given the central role of REE in economic theory it is of interest to investigate this concept further, to find out if it has any other attractive or non-attractive features. Moreover, if it does have non-attractive features, the question arises whether there is any other concept which behaves better.

The purpose of this paper is four-fold:

First, we show that the REE, whenever it exists, can be fully interim Pareto optimal and we provide a positive result to that effect. In particular we demonstrate that, under certain conditions, the REE is always in the interim weak fine core (IWFC), defined in Yannelis (1991), and therefore it is "full information" Pareto optimal. This result depends crucially on the fact that the utility function of each agent is state independent. Indeed we show that if the utility function depends on the state of nature then the REE need not be in the IWFC. Furthermore, we show that the REE may not be ex ante Pareto optimal. We demonstrate this by examining the relationship between REE and the weak fine core (WFC).

Second, we examine the incentive compatibility of the REE. Since the REE is capturing the idea of contracts under asymmetric information one would like to know if contracts (trades) are incentive compatible. It turns out that if there is one good per state of nature then the REE is always Bayesian incentive compatible. However this result ceases to be true if there is more than one good per state of nature. [1] It should be noted that the Bayesian incentive compatibility criterion used here is coalitional and it implies individual Bayesian incentive compatibility.

Third, following the ideas of Glycopantis - Muir - Yannelis (2001), we investigate whether or not the REE can be implemented as a perfect Bayesian equilibrium (PBE) of an extensive form game. A PBE consists of optimal behavioral strategies of the players and the consistent with these decisions, beliefs attaching a probability distribution to the nodes of each information set. It is a variant of the Kreps - Wilson (1982) concept of sequential equilibrium.

The attempt to implement the REE as a PBE of an extensive form game is interesting, because we see the dynamics of the agents' decisions, i.e. how agents move sequentially (or simultaneously) to reach a final outcome. We found that the REE need not be implementable as a PBE. Thus, if one believes that the PBE is a reasonable rationality criterion

[1] A related example can be found in Palfrey - Srivastava (1986) and Hahn - Yannelis (2001).

that most equilibrium notions must satisfy, then the REE should also satisfy this, but we show that this is not the case.

Fourth, in view of the unsatisfactory features of the REE noted above, one would like to know if there are better alternatives. To this end, we compare the REE with the private core (Yannelis (1991). Analyzing Example 3.1, below, we see that, in general, the REE is not Bayesian incentive compatible and it cannot be implemented as a PBE. On the other hand, the private core exists under the standard continuity and concavity assumptions on the utility functions, (Yannelis (1991) and Glycopantis - Muir - Yannelis (2001)), and has desirable properties. [2] Moreover, we examine the example of Kreps (1978), where, although fully revealing or non-revealing REE do not exist, the private core does exist. Hence the private core provides a more sensible outcome than the REE.

The paper is organized as follows. Section 2 defines a differential information exchange economy and the concept of REE. Section 3 explains the idea of incentive compatibility and Section 4 discusses the non-implementation of REE as a PBE, in terms of a particular example. Section 5 looks at the relationship between REE and weak core concepts, Section 6 comments on the relationship between REE and the private core and Section 7 concludes the discussion with some remarks. Appendix I discusses the relation between Nash equilibria and PBE, in terms of the same particular example.

2 Differential information economy and REE

For simplicity we confine ourselves to the case where the set of states of nature, Ω, is finite and there is a finite number, ℓ, of goods per state. $I = \{1, 2, \ldots, n\}$ is a set of agents, or players, and $\mathbb{R}_+$ will denote the set of positive real numbers and $\mathbb{R}^l_+$ its l-fold Cartesian product.

A differential information exchange economy $\mathcal{E}$ is a set

$$\{((\Omega, \mathcal{F}), X_i, \mathcal{F}_i, u_i, e_i, q_i) : i = 1, \ldots, n\}$$

where

1. $\mathcal{F}$ is a σ-algebra generated by the singletons of Ω;

2. $X_i : \Omega \rightarrow 2^{\mathbb{R}^l_+}$ is the set-valued function giving the *random consumption set* of Agent (Player) i, who is denoted by Pi;

3. $\mathcal{F}_i$ is a partition of Ω generating a sub-σ-algebra of $\mathcal{F}$, denoting the *private information*[3] of Pi. We assume that[4] $\mathcal{F} = \bigvee_{i \in I} \mathcal{F}_i$;

4. $u_i : \Omega \times \mathbb{R}^l_+ \rightarrow \mathbb{R}$ is the *random utility* function of Pi; for each $\omega \in \Omega$, $u_i(\omega, .)$ is continuous, concave and monotone;

5. $e_i : \Omega \rightarrow \mathbb{R}^l_+$ is the *random initial endowment* of Pi, assumed to be $\mathcal{F}_i$-measurable, with $e_i(\omega) \in X_i(\omega)$ for all $\omega \in \Omega$;

[2]The private core is always Bayesian incentive compatible (Koutsougeras - Yannelis (1993) and Hahn - Yannelis (2001)) and can be implemented as a PBE of an extensive form game (Glycopantis - Muir - Yannelis (2001)).

[3]Sometimes $\mathcal{F}_i$ will denote the σ-algebra generated by the partition, as will be clear from the context.

[4]The "join" $\bigvee_{i \in S} \mathcal{F}_i$ denotes the smallest σ-algebra containing all $\mathcal{F}_i$, for $i \in S \subseteq I$.

6. q_i is an $\mathcal{F}$-measurable probability function on Ω giving the *prior* of Pi. It is assumed that on all elements of $\mathcal{F}_i$ the aggregate q_i is strictly positive. If a common prior is assumed on $\mathcal{F}$, it will be denoted by μ.

We refer to a function with domain Ω, constant on elements of $\mathcal{F}_i$, as $\mathcal{F}_i$-*measurable*, although, strictly speaking, measurability is with respect to the σ-algebra generated by the partition. It is assumed that the players' information partitions are common knowledge.

Agents make contracts in the ex ante stage. In the interim stage, i.e. after they have received a signal as to what is the event containing the realized state of nature, one considers the incentive compatibility of the contract (allocation).

For any $x_i : \Omega \to \mathbb{R}^l_+$ we define

$$v_i(x_i) = \sum_{\Omega} u_i(\omega, x_i(\omega))q_i(\omega). \tag{1}$$

Equation (1) gives the *ex ante expected utility* of Pi.

Let $\mathcal{G}$ be a partition of (or σ-algebra on) Ω, belonging to Pi. For $\omega \in \Omega$ denote by $E_i^{\mathcal{G}}(\omega)$ the element of $\mathcal{G}$ containing ω; in the particular case where $\mathcal{G} = \mathcal{F}_i$ denote this just by $E_i(\omega)$. Pi's conditional probability for the state of nature being ω', given that it is actually ω, is then

$$q_i\big(\omega'|E_i^{\mathcal{G}}(\omega)\big) = \begin{cases} 0 & : \quad \omega' \notin E_i^{\mathcal{G}}(\omega) \\ \frac{q_i(\omega')}{q_i\left(E_i^{\mathcal{G}}(\omega)\right)} & : \quad \omega' \in E_i^{\mathcal{G}}(\omega). \end{cases}$$

The *interim expected utility* function of Pi, $v_i(x_i|\mathcal{G})$, is given by

$$v_i(x_i|\mathcal{G})(\omega) = \sum_{\omega'} u_i(\omega', x_i(\omega'))q_i\big(\omega'|E_i^{\mathcal{G}}(\omega)\big). \tag{2}$$

Equation 2 defines a $\mathcal{G}$-measurable random variable.

We denote by $L_1(q_i, \mathbb{R}^l)$ the space of equivalence classes of $\mathcal{F}$-measurable functions $f_i : \Omega \to \mathbb{R}^l$; when a common prior μ is assumed $L_1(q_i, \mathbb{R}^l)$ will be replaced by $L_1(\mu, \mathbb{R}^l)$. L_{X_i} is the set of all $\mathcal{F}_i$-measurable selections from the random consumption set of Agent i, i.e.,

$$L_{X_i} = \{x_i \in L_1(q_i, \mathbb{R}^l) : \ x_i : \Omega \to \mathbb{R}^l \text{ is } \mathcal{F}_i\text{-measurable and } x_i(\omega) \in X_i(\omega) \ q_i\text{-}a.e.\}$$

and let $L_X = \prod_{i=1}^{n} L_{X_i}$.

Also let

$$\bar{L}_{X_i} = \{x_i \in L_1(q_i, \mathbb{R}^l) : \ x_i(\omega) \in X_i(\omega) \ q_i\text{-}a.e.\}$$

and let $\bar{L}_X = \prod_{i=1}^{n} \bar{L}_{X_i}$.

An element $x = (x_1, \ldots, x_n) \in \bar{L}_X$ will be called an *allocation*. For any subset of players S, an element $(y_i)_{i \in S} \in \prod_{i \in S} \bar{L}_{X_i}$ will also be called an allocation, although strictly speaking it is an allocation to S.

We now discuss the notion of REE. We shall need the following. Let $\sigma(p)$ be the smallest sub-σ-algebra of $\mathcal{F}$ for which a price system $p : \Omega \rightarrow \mathbb{R}^l_+$ is measurable and let $\mathcal{G}_i = \sigma(p) \vee \mathcal{F}_i$ denote the smallest σ-algebra containing both $\sigma(p)$ and $\mathcal{F}_i$.

Definition 2.1. A pair (p, x), where p is a price system and $x = (x_1, \ldots, x_n) \in \bar{L}_X$ is an allocation, is a *REE* if

(i) for all i the consumption function $x_i(\omega)$ is $\mathcal{G}_i$-measurable;

(ii) for all i and for all ω the consumption function maximizes $v_i(x_i|\mathcal{G}_i)(\omega)$ subject to the budget constraint at state ω,

$$p(\omega)x_i(\omega) \leq p(\omega)e_i(\omega);$$

(iii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$ for all $\omega \in \Omega$.

REE is an interim concept which allows us to condition also on information from prices. A REE is said to be *fully revealing* if $\mathcal{G}_i = \mathcal{F}$ for all $i = 1, 2, \ldots, n$.

In the next section we shall discuss the general idea of whether allocations have certain desirable properties from the point of view of incentive compatibility. Following this, we shall turn our attention in particular to REE allocations and their possible implementation as PBE in the context of contracts described in terms of a game tree.

3 On the incentive compatibility of REE

Since we are concerned with multilateral contracts, we think that the appropriate incentive compatibility concept should be coalitional rather than individual. In particular, contracts which are individually Bayesian incentive compatible may not be coalitional Bayesian incentive compatible and therefore not stable, i.e. viable and self-enforceable (see also Allen - Yannelis (2001)).

The basic idea is that an allocation is incentive compatible if no coalition can misreport the realized state of nature and have a distinct possibility of making its members better off.

Suppose we have a coalition S, with members denoted by i. Their pooled information $\bigvee_{i \in S} \mathcal{F}_i$ will be denoted by $\mathcal{F}_S$. We have assumed that $\mathcal{F}_I = \mathcal{F}$. Let the realized state of nature be ω^*. Each member $i \in S$ sees $E_i(\omega^*)$. Obviously not all $E_i(\omega^*)$ need be the same, however all Agents i know that the actual state of nature could be ω^*.

Consider a state ω' such that for all $j \in I \setminus S$ we have $\omega' \in E_j(\omega^*)$ and for at least one $i \in S$ we have $\omega' \notin E_i(\omega^*)$. Now the coalition S decides that each member i will announce that she has seen her own set $E_i(\omega')$ which, of course, contains a lie. On the other hand we have that $\omega' \in \bigcap_{j \notin S} E_j(\omega^*)$.

The idea is that if all members of $I \setminus S$ believe the statements of the members of S then each $i \in S$ expects to gain. For *coalitional Bayesian incentive compatibility* (CBIC) of an allocation we require that this is not possible. This is the incentive compatibility condition used in Glycopantis - Muir - Yannelis (2001).[5]

[5]See Krasa - Yannelis (1994) and Hahn - Yannelis (1997) for related concepts.

CBIC coincides in the case of a two-agent economy with the concept of *Individually Bayesian Incentive Compatibility* (IBIC), which refers to the case when S is a singleton.

We consider here a strengthening of the concept of Coalitionally Bayesian Incentive Compatibility (CBIC)which allows for transfers between the members of a coalition.

Definition 3.1. An allocation $x = (x_1, \dots, x_n) \in \bar{L}_X$, with or without free disposal, is said to be *Transfer Coalitionally Bayesian Incentive Compatible* (TCBIC) if it is not true that there exists a coalition S, states ω^* and ω', with ω^* different from ω' and $\omega' \in \bigcap_{j \notin S} E_j(\omega^*)$ and a net-trade vector among the members of S: $z = (z_i)_{i \in S}$, with z_i being $\mathcal{F}_i$-measurable such that $\sum_{i \in S} z_i = 0$, such that for all $i \in S$ there exists $\bar{E}_i(\omega^*) \subseteq Z_i(\omega^*) = E_i(\omega^*) \cap (\bigcap_{j \notin S} E_j(\omega^*))$, for which

$$\sum_{\omega \in \bar{E}_i(\omega^*)} u_i(\omega, e_i(\omega) + x_i(\omega') - e_i(\omega') + z_i) q_i\big(\omega | \bar{E}_i(\omega^*)\big) > \sum_{\omega \in \bar{E}_i(\omega^*)} u_i(\omega, x_i(\omega)) q_i\big(\omega | \bar{E}_i(\omega^*)\big). \tag{3}$$

The definition in (3) implies that no coalition can hope that by misreporting a state, every member could become better off if they are believed by the members of the complementary set.

We condition the interim expected utility on $\bar{E}_i(\omega^*)$ which implies that we require consistency between the declaration of the members of S and the observations of the agents in the complementary set.

Returning to Definition 3.1, one can define CBIC to correspond to $z = 0$ and then IBIC to the case when S is a singleton. It follows that TCBIC $\Rightarrow$ CBIC $\Rightarrow$ IBIC.

In terms of game trees, an allocation will be IBIC if there is a profile of optimal behavioral strategies and equilibrium paths along which no player misreports the state of nature he has observed. Players might lie from information sets which are not visited by an optimal play.

The definition of CBIC and its variants is about situations where a lie might be beneficial. On the other hand the extensive form forces us to consider earlier decisions by other players to lie or tell the truth and what payoffs will occur whenever a lie is detected, through observations or incompatibility of declarations. Only in this fuller description can players make a decision whether to risk a lie. Such considerations probably open the way to an incentive compatibility definition based on expected gains from lying.

Observation 3.1. In a differential information economy with one good per state and monotonic utility functions any REE is TCBIC.

Proof. With one good per state, the measurability of the allocations implies that the only REE, fully revealing or not, is the initial allocation which is incentive compatible.

In oder to see that the REE and initial allocation coincide we argue briefly as follows. A price function, $p(\omega)$, known to all agents, is defined on Ω. Each agent apart from his own private $E_i \subseteq \mathcal{F}_i$, receives also a price signal. Combining the two types of signals he deduces the event $G_i \in \mathcal{G}_i$ that he is observing. If the price function is fully revealing then G_i will consist of just one state. On the other hand if prices provide no extra information to the agent then $G_i = E_i$.

Then he acts under the constraint of the measurability condition on his consumption with respect to his new, possibly more refined information. This means, given also the measurability of his initial allocation, that he chooses a constant quantity which maximizes his interim expected utility subject to the budget set at state ω in the event he has observed.

More formally Agent i maximizes $v_i(x_i|\mathcal{G}_i)(\omega)$ subject to the budget constraint at state ω, i.e. $p(\omega)x_i(\omega) \leq p(\omega)e_i(\omega)$, and given that there is only one good and the utility function is monotone he chooses $x_i(\omega) = e_i(\omega)$. Hence the REE implies no trade.

The following example shows that for more than one good REE is not necessarily CBIC.

Example 3.1 $I = \{1, 2\}$ with two commodities, i.e. $X_i = \mathbb{R}^2_+$ for each agent i, and three states of nature $\Omega = \{a, b, c\}$.

We assume that the endowments, per state a, b, and c respectively, and information partitions of the agents are given by

$$e_1 = ((7,1),(7,1),(4,1)), \qquad \mathcal{F}_1 = \{\{a,b\},\{c\}\};$$

$$e_2 = ((1,10),(1,7),(1,7)), \qquad \mathcal{F}_2 = \{\{a\},\{b,c\}\}.$$

We shall denote $A_1 = \{a, b\},\ c_1 = \{c\},\ a_2 = \{a\},\ A_2 = \{b, c\}$.

It is also assumed that $u_i(\omega, x_{i1}, x_{i2}) = x_{i1}^{\frac{1}{2}} x_{i2}^{\frac{1}{2}}$, where the second index refers to the good, which is a strictly quasi-concave, and monotone function in x_{ij}, and that every player expects that each state of nature occurs with the same probability, i.e. $\mu(\{\omega\}) = \frac{1}{3}$, for $\omega \in \Omega$. Some calculations are

$u_1(7,1) = 2.65,\ u_1(4,1) = 2,\ u_2(1,10) = 3.16,\ u_2(1,7) = 2.65.$

The expected utilities, multiplied by 3, are given by $\mathcal{U}_1 = 7.3$ and $\mathcal{U}_2 = 8.46$.

Straightforward calculations show that there is only one, fully revealing REE. The prices, the allocations and the corresponding utilities are:

In state a, $(p_1, p_2) = (1, \frac{8}{11})$; $x^*_{11} = \frac{85}{22}$, $x^*_{12} = \frac{85}{16}$, $x^*_{21} = \frac{91}{22}$, $x^*_{22} = \frac{91}{16}$; $u^*_1 = 4.53$, $u^*_2 = 4.85$.

In state b, $(p_1, p_2) = (1, 1)$; $x^*_{11} = 4,\ x^*_{12} = 4,\ x^*_{21} = 4,\ x^*_{22} = 4$; $u^*_1 = 4, u^*_2 = 4$.

In state c, $(p_1, p_2) = (1, \frac{5}{8})$; $x^*_{11} = \frac{37}{16}$, $x^*_{12} = \frac{37}{10}$, $x^*_{21} = \frac{43}{16}$, $x^*_{22} = \frac{43}{10}$; $u^*_1 = 2.93, u^*_2 = 3.40$.

The normalized expected utilities of the REE are $\mathcal{U}_1 = 11.46,\ \mathcal{U}_2 = 12.25$.

The quantities obtained are different in each state of nature and therefore the REE does not belong to the private core because this concept require $\mathcal{F}_i$-measurability of the allocations. On the other hand a REE is in the WFC under certain conditions which are satisfied here. However this relation is not stable. We explain this below.

In order to show that the REE redistribution is not CBIC we argue as follows. Suppose that the realized state of nature is $\{a\}$ so that P1 sees $\{a, b\}$, and P2 sees $\{a\}$ but misreports $\{b, c\}$. If P1 believes the lie then state b is believed. So P1 agrees to get the allocation (4, 4). P2 receives the allocation $e_2(a) + x_2(b) - e_2(b) = (1, 10) + (4, 4) - (1, 7) = (4, 7)$ with $u_2(4, 7) = 5.29 > u_2\left(\frac{91}{22}, \frac{91}{16}\right) = 4.85$. Hence P2 has a possibility of gaining by misreporting and therefore the REE is not CBIC (IBIC).

We can also explain the allocation that P2 receives by arguing in an alternative manner. P1 agrees to get (4, 4), and P2 receives the rest of the total initial endowments in state a, i.e.

$e_1(a)+e_2(a)-x_1(b) = (7,1)+(1,10)-(4,4) = (4,7)$ as above. The expression on the left-hand-side matches up with $e_2(a)+x_2(b)-e_2(b)$ by taking into account that by measurability of allocations we have $e_1(a) = e_1(b)$ and by feasibility $e_1(b) + e_2(b) = x_1(b) + x_2(b)$.

On the other hand if P2 sees $\{b,c\}$ and P1 sees $\{c\}$, the latter cannot misreport $\{a,b\}$ and hope to gain if P2 believes it is b because the calculations now give $e_1(c) + x_1(b) - e_1(b) = (4,1) + (4,4) - (7,1) = (1,4)$ with $u_1(1,4) = 2 < u_1\left(\frac{37}{16}, \frac{37}{10}\right) = 2.93$.

4 Non-implementation of REE allocations as a PBE.

In this section we use Example 3.1 to demonstrate that a fully revealing REE, which is not incentive compatible, is also not implementable as a PBE.[6] Therefore, in general, REE allocations are not implementable as a PBE.

First we look at the REE and show that it is not CBIC, by considering which agent, and under which circumstances, can misreport what he has observed. Then we consider sequential and also simultaneous play to show that REE is not implementable. Initially, we assume that P1 acts first and that when P2 is to act he has heard the declaration of P1. Then we reverse the roles of the two agents. Finally we consider a version with simultaneous declarations.

4.1 Sequential decisions

Next we show using the sequential decisions approach that the REE is not implementable as a PBE. Comparisons will be made with $\mathcal{U}_1$ and $\mathcal{U}_2$ of initial endowments and of REE. Throughout, payoffs are given in terms of utility.

First we consider the non-simultaneous, sequential decisions case with P1 acting first. We specify the *rules* for calculating payoffs, i.e. the terms of the contract:

(i) If the declarations of the two players are incompatible, that is (c_1, a_2), then this implies that no trade takes place.
(ii) If the declarations of the two players are (A_1, A_2) then this implies that state b is really declared. The player who believes it (because he has no reason to disbelieve it) gets his REE allocation (4, 4) and the other player gets the rest. So aA_1A_2 means that P2 has lied but P1 believes it is state b and gets (4, 4). P2 gets the rest under state a, that is (4, 7); bA_1A_2 means that both believe that it is the (actual) state b and each gets (4, 4); cA_1A_2 means that P2 believes it is state b and gets (4, 4) and P1 gains nothing from his lie as he gets (1, 4).
(iii) $aA_1a_2, bA_1A_2, cc_1A_2$ imply that everybody tells the truth and the contract implements the REE allocation under state a, b, and c respectively. (bA_1A_2 in (ii) and (iii) give, of course, an identical result).
(iv) ac_1A_2 implies that both lie but their declarations are not incompatible. Each gets his REE under c and there is free disposal of (3, 3) which the difference between the total endowments under state a and the allocation the agents receive.
(v) cA_1a_2 means that both lie and stay with their initial endowments as they cannot get

[6] A related example with three agents has been given in Hahn - Yannelis (2001). The extensive form analysis of the section goes beyond the discussion in Hahn - Yannelis. See also Dubey - Geanakoplos - Shubik (1987) for related ideas.

the REE allocations under state a which is the intersection of A_1 and a_2.
(vi) bA_1a_2 implies that P2 misreports and P1 believes and gets his REE under a; P2 gets the rest under b, that is $\left(\frac{91}{22}, \frac{43}{16}\right)$. Then $u_2 = 3.33 < 4$ and $u_1 = 4.53 > 4$ and the lie of P2 really benefits P1.
(vii) bc_1A_2 means that P1 lies and P2 believes that it is state c. P2 gets his REE allocation under c and P1 gets the rest under b, that is the allocation $\left(\frac{85}{16}, \frac{37}{10}\right)$. Then $u_1 = 4.43 > 4$ and $u_2 = 3.4 < 4$ and P1 benefits from lying.

The analysis is in Figures 1, 2 and the complete optimal paths are shown in Figure 3. Probabilities next to the nodes of the information sets denote the players' beliefs. We assume that each player chooses optimally from his information set. Optimal decisions and equilibrium paths are shown through heavy lines.

In Figure 1 we show the optimal decisions of P2. It is clear that from all information sets he will choose to play A_2 as it means for him a better payoff than a_2. Hence the tree in Figure 1 folds back to the one in Figure 2, in which the optimal decisions of P1 are shown. Given the prior probabilities on nature's choices, P1 expects to find himself with probability $\frac{1}{2}$ on each node of the information set I_1^1 and therefore he chooses A_1 from this set. From the singleton he chooses c_1 as it dominates A_1.

In Figure 3 we show through heavy lines, plays of the game corresponding to choices by nature and optimal behavior strategies of the players. Their beliefs have been obtained through Bayesian updating. Strategies and beliefs satisfy the condition of a PBE.

The probabilities are calculated as follows. We label the nodes of the information sets: From left to right, in I_1^1 we denote them by j_1 and j_2, in I_2^1 by n_1 and n_2, and in I_2^2 by n_3 and n_4. The probabilities of the nodes in I_1^1 follow from the fact that the states of nature are equally probable. The rest of the conditional probabilities are calculated given the choices of nature and the strategies of the players by using the Bayesian formula for updating beliefs.

$$Pr(n_1/A_1) = \frac{Pr(A_1/n_1) \times Pr(n_1)}{Pr(A_1/n_1) \times Pr(n_1) + Pr(A_1/n_2) \times Pr(n_2)} = \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 1 \times 0} = 1 \qquad (4)$$

and

$$Pr(n_3/c_1) = \frac{Pr(c_1/n_3) \times Pr(n_3)}{Pr(c_1/n_3) \times Pr(n_3) + Pr(c_1/n_4) \times Pr(n_4)} = \frac{1 \times 0}{1 \times 0 + 1 \times \frac{1}{3}} = 0. \qquad (5)$$

It follows from (4)and (5) that $Pr(n_2/A_1) = 0$ and $Pr(n_4/c_1) = 1$

Our analysis shows that there is a unique PBE. The corresponding normalized expected payoffs of the players are $\mathcal{U}_1 = 10.93$ and $\mathcal{U}_2 = 12.69$.

The equilibrium paths imply that REE is not implementable. This matches up with the fact that it is not CBIC. However comparing the normalized expected utility of the PBE with those corresponding to the initial allocation we conclude that the proposed contract will be signed. This follows from the fact that both agents gain from this proposed contract.

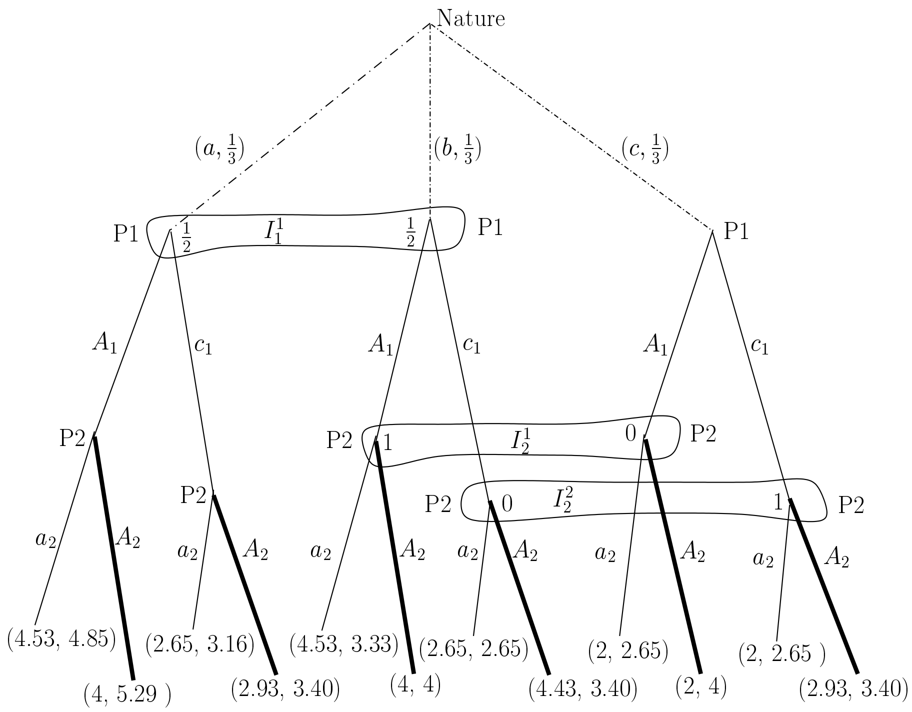


Figure 1

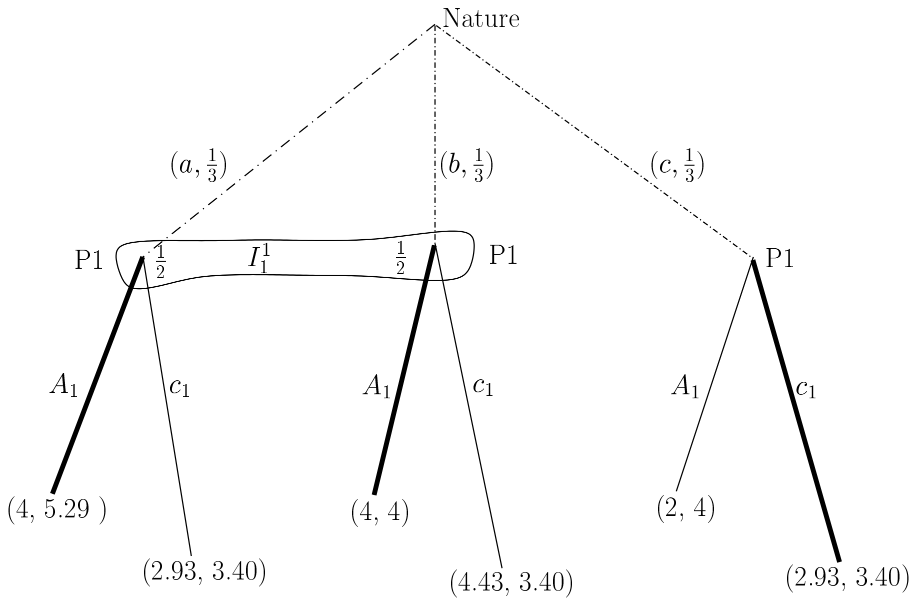


Figure 2

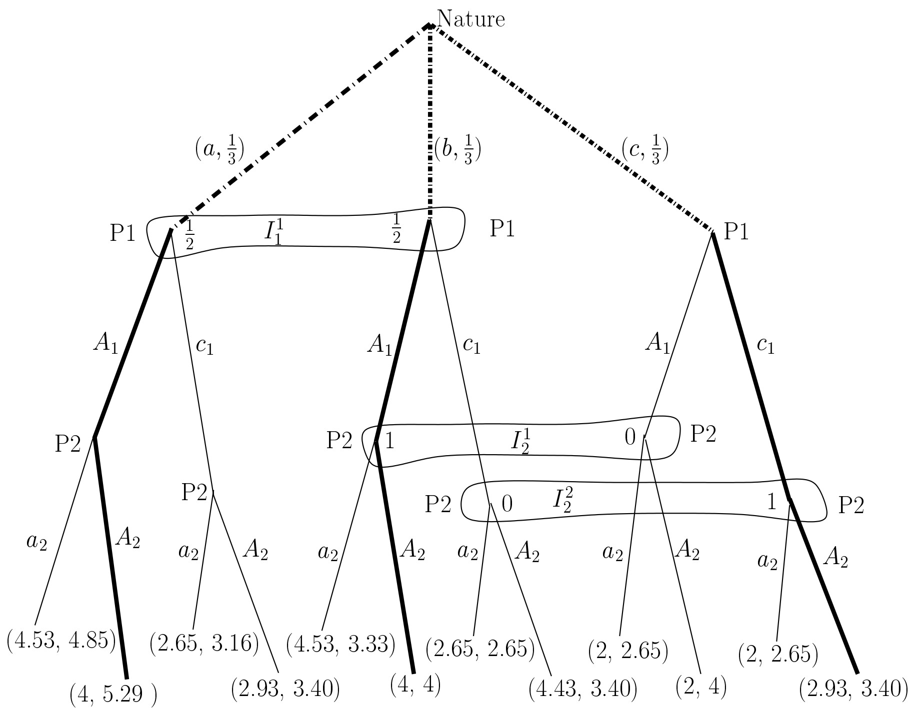


Figure 3

On the other hand P2, because it is advantageous to him to do so, stops P1 from realizing his normalized REE utility. P1 ends up with $\mathcal{U}_1 = 10.93$ rather than $\mathcal{U}_1 = 11.46$.

We now consider the case when P2 plays first and when P1 is to act he has heard the declaration of P2. The terms of the contract are the same as in the previous case. They are repeated here, adjusted for the order of play. Explicitly, the *rules* are now:

(i) If the declarations of the two players are incompatible, that is (a_2, c_1), then this implies that no trade takes place.
(ii) If the declarations of the two players are (A_2, A_1) then this implies that state b is believed. The player who believes it gets his REE allocation (4, 4) and the other player gets the rest. So aA_2A_1 means P2 has lied but P1 believes it is state b and gets (4, 4). P2 gets the rest under state a, that is (4, 7); bA_2A_1 means that both believe that it is the (actual) state b and each gets (4, 4); cA_2A_1 means that P2 believes it is state b and gets (4, 4) and P1 gains nothing from his lie as he gets (1, 4).
(iii) $aa_2A_1, bA_2A_1, cA_2c_1$ imply that everybody tells the truth and the contract implements the REE allocation under state a, b, and c respectively. (bA_2A_1 gives an identical result in both (ii) and (iii)).
(iv) aA_2c_1 implies that both lie but their declarations are not incompatible. Each gets his REE under c and there is free disposal.
(v) ca_2A_1 means that both lie and stay with their e_i's.
(vi) ba_2A_1 implies that P2 misreports and P1 believes and gets his REE under a; P2 gets the rest under b.
(vii) bA_2c_1 means that P1 lies and P2 believes that it is state c. P2 gets his REE allocation c and P1 the rest under b.

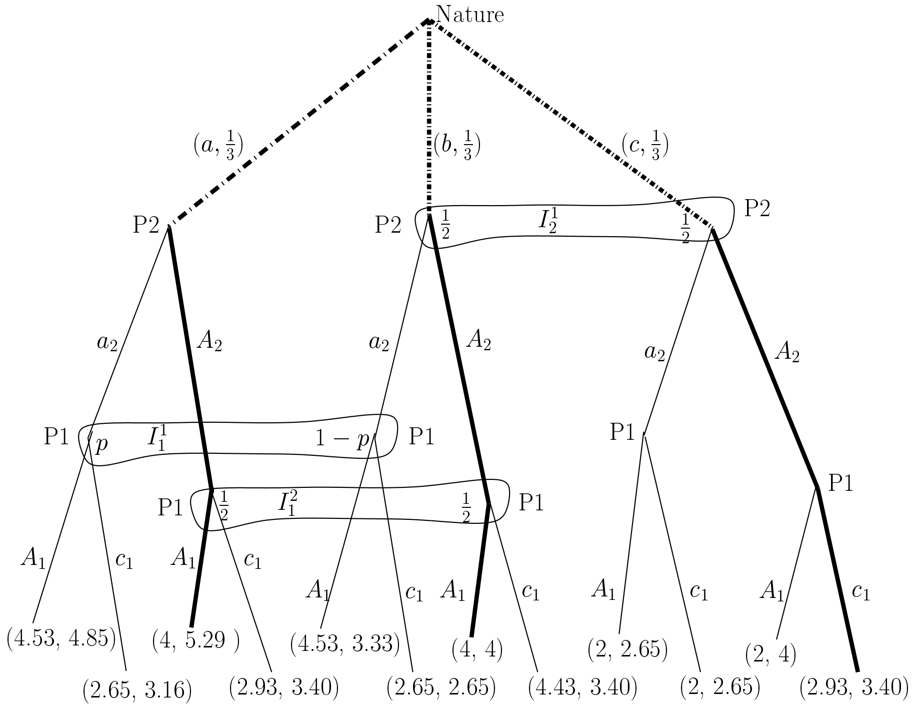


Figure 4

We can see now that the PBE depend on the sequence of play, i.e. they vary with a change in the player who moves first. First it is straightforward to show that one of them is the same as in the previous case, i.e. we obtain $\mathcal{U}_1 = 10.93$ and $\mathcal{U}_2 = 12.69$. We only show in Figure 4 the optimal paths. The implied decisions along these paths coincide with those in the previous case already considered.

A brief argument to justify this conclusion is as follows. From I_1^1 player P1 will always play A_1, and P2 will always play A_2 from I_2^1. The only issue is what P2 will play from the singleton. This will determine also what P1 does from I_1^2. The optimal paths shown above have P2 play A_2 from the singleton and P1 play A_1 from I_1^2. Strategies and beliefs form a PBE. Information set I_1^1 is not visited and therefore the probability, p, determining the beliefs attached to its nodes by P1, is arbitrary.

However in this case of sequential decisions there exist other PBEs as well. A further PBE is shown in detail in Figures 5, 6 and 7. When P1 is to act from I_1^2 he believes now that he is at the right-hand-side node and chooses c_1. For consistency P2 must have played a_2 from the singleton and this is shown in Figure 6. Optimal decisions following choices by nature, and the new PBE, including beliefs consistent with strategies, are shown on Figure 7. The normalized expected utilities are $\mathcal{U}_1 = 11.89$ and $\mathcal{U}_2 = 11.65$.

The equilibrium paths imply that REE is not implementable but comparing the normalized expected utility of the PBE with those corresponding to the initial allocation we conclude again that the proposed contract will be signed. On the other hand P1, because it is advantageous to him, stops P2 from realizing his normalized REE utility. He ends up with $\mathcal{U}_2 = 11.65$ rather than $\mathcal{U}_2 = 12.69$.

It is possible but more involved to show that there is one more PBE which contains

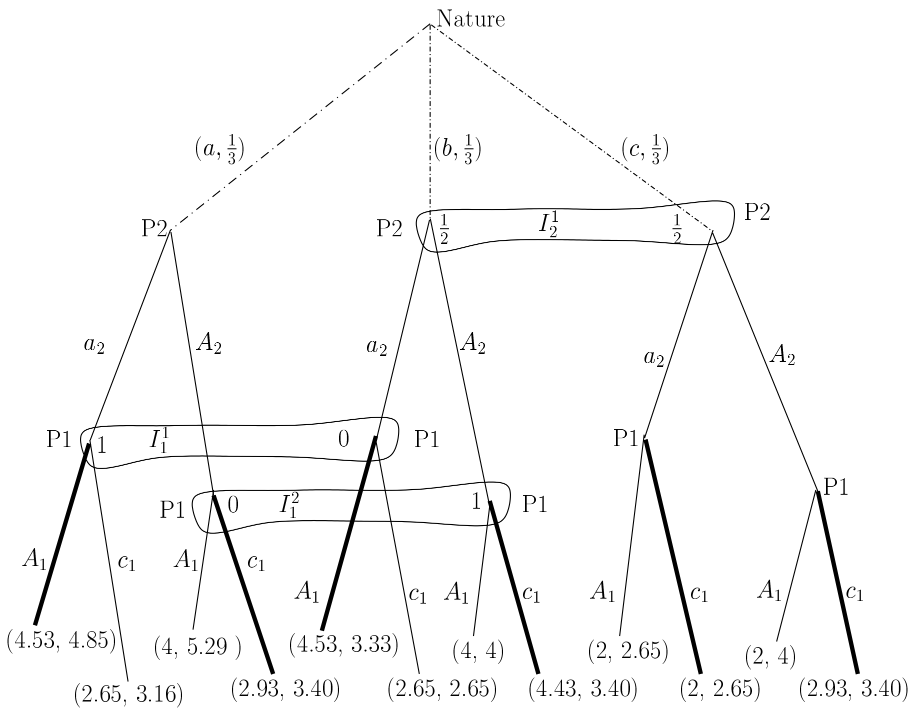


Figure 5

completely stochastic behavioral strategies. P1 plays A_1 from I_1^1; from I_1^2 he plays c_1 with probability 0.233; from the first singleton he has an arbitrary choice; and from the second singleton he plays c_1. P2 plays from the singleton A_2 with probability 0.402 and from I_2^1 he plays A_2. Beliefs consistent with these behavioral strategies are $\left(\frac{1}{2}, \frac{1}{2}\right)$ for the nodes of I_2^1, from left to right, (1, 0) for I_1^1 and (0.287, 0.713) for I_1^2. The normalized expected utilities are $\mathcal{U}_1 = 11.25$ and $\mathcal{U}_2 = 12.10$.

The three PBEs just described are the only ones and there is a presumption that P2 might be able to force the one with $\mathcal{U}_1 = 10.93$ and $\mathcal{U}_2 = 12.69$ because he is playing first. Finally mixing the behavioral strategies of the PBEs obtained above will not achieve another potential equilibrium. This follows from the well known Kuhn's theorem for games with perfect recall which implies that there will be equivalent behavioral strategies, and all these have been considered.

4.2 Simultaneous decisions

Next we consider the simultaneous decisions case. We look at it in terms of trees with enlarged information sets of the players. We produce two different sets of three tree graphs, each corresponding to one of the two cases as to who is placed first on the graph. Then we shall construct normal form type games.

The rules for calculating payoffs are the same as in the corresponding earlier cases when the player to act second hears the choice made by the player acting before him. The third graph in each case, i.e. Figures 10 and 13 describe the unique PBE, identical to each

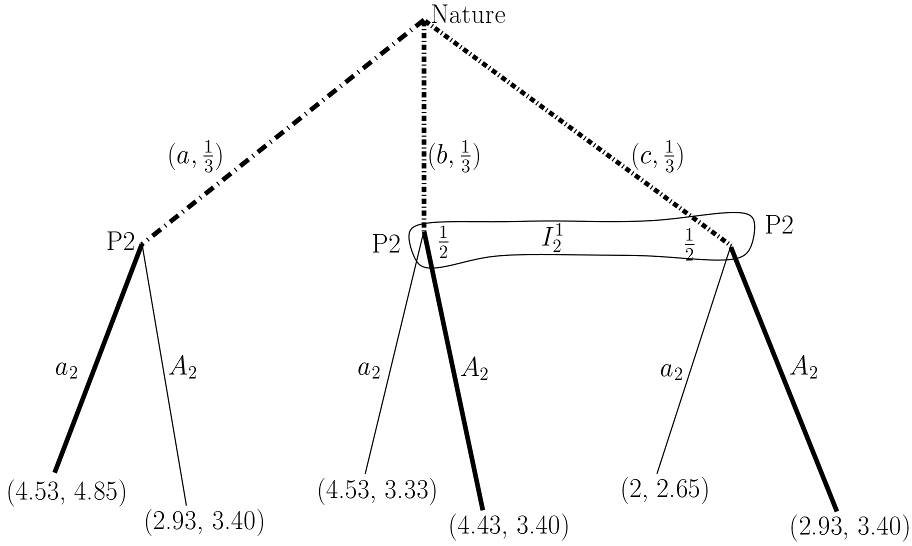


Figure 6

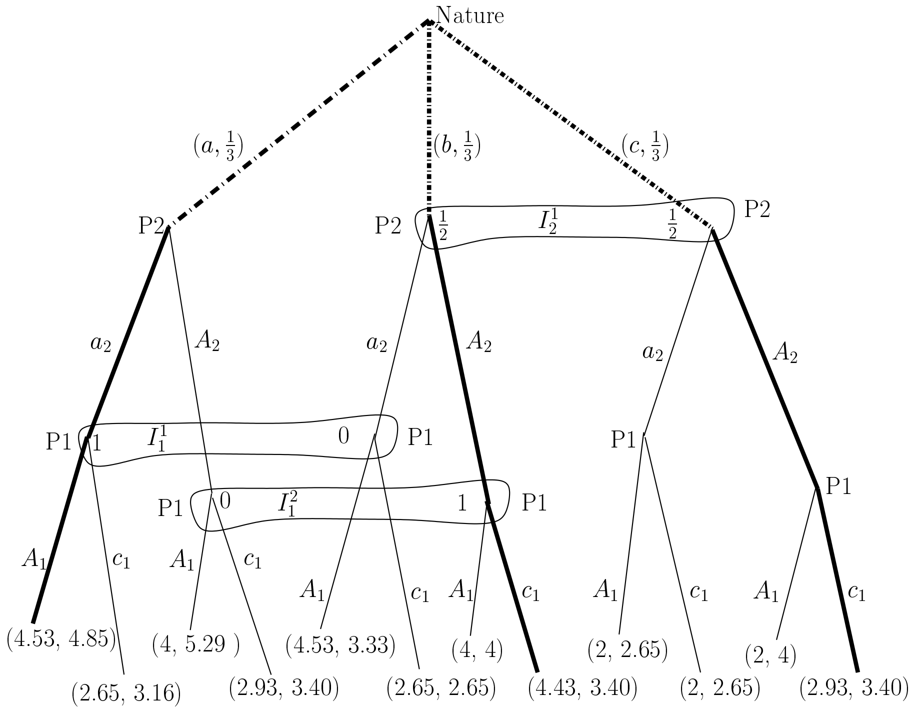


Figure 7

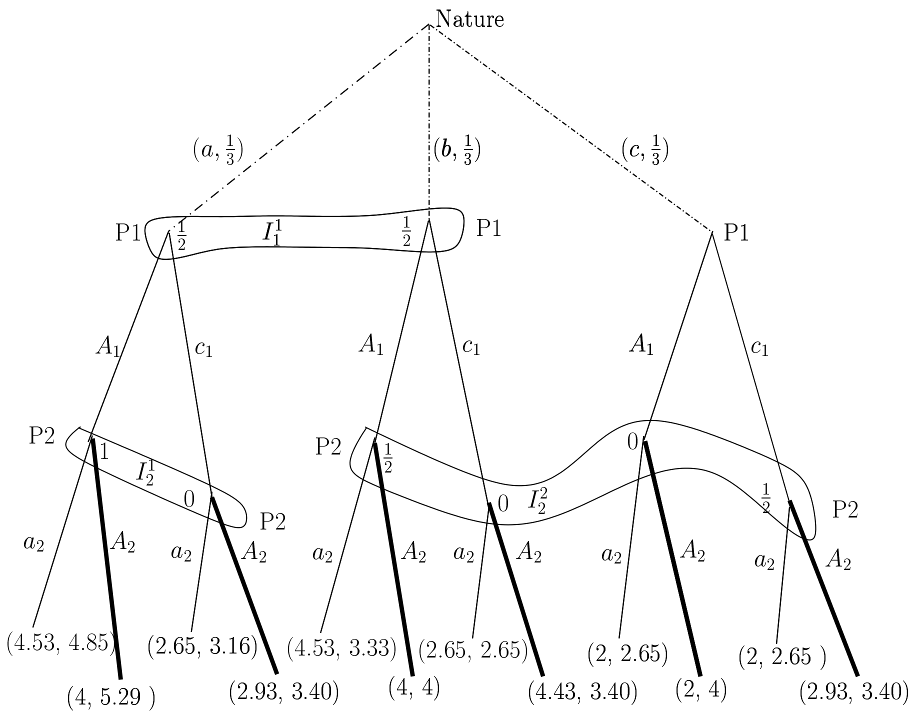


Figure 8

other. From the analysis of the graphs we obtain that P1 plays A_1 from $\{a, b\}$ and c_1 from $\{c\}$. On the other hand P2 plays A_2 from both $\{b, c\}$ and a.

Consider the analysis of their decisions through Figure 8. P2 plays A_2 from both I_2^1 and I_2^2 because it dominates a_2. Then Figure 9 shows that P1 plays A_1 from I_1^1 and c_1 from the singleton.

Next consider the analysis starting with Figure 11. P1 plays A_1 from I_1^1 and c_1 from I_1^2. The choice c_1 from I_1^2 is obvious. In order to justify the choice A_1 from I_1^1 we argue as follows:
A_1 will imply $4.53\pi_1 + 4\pi_2 + 4.53\pi_3 + 4\pi_4$,
c_1 will imply $2.65\pi_1 + 2.93\pi_2 + 2.65\pi_3 + 4.43\pi_4$,
where $\pi_1, \pi_2, \pi_3, \pi_4$ are his beliefs attached to the nodes, from left to right, with $\pi_1 + \pi_2 = \dfrac{1}{2}$ and $\pi_3 + \pi_4 = \dfrac{1}{2}$. Now the most favourable probabilities for choosing c_1 are $p_2, p_4 = \dfrac{1}{2}$. However A_1 does better even under these conditions. As we move away from this vector of probabilities A_1 does even better. Turning next to the optimal choices of P2, Figure 12 shows that he will always play A_2.

It follows that with respect to their optimal decisions, it does not matter whom we place first in the tree form representation of the simultaneous game. In effect, in one case we do backward induction and in the other case we cut through the tree from above. The outcome is different from the one in the sequential Section 4.2 in which case the sequence in which the players act matters.

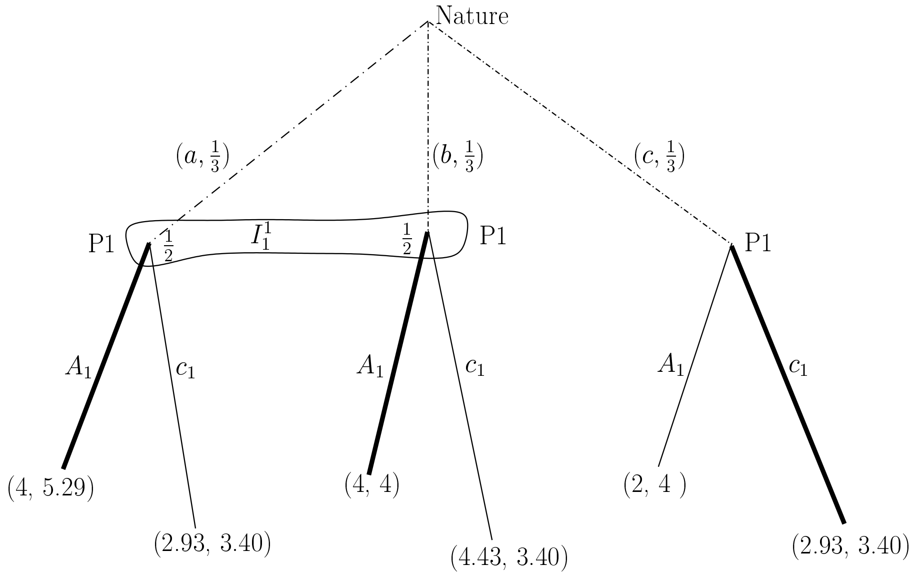


Figure 9

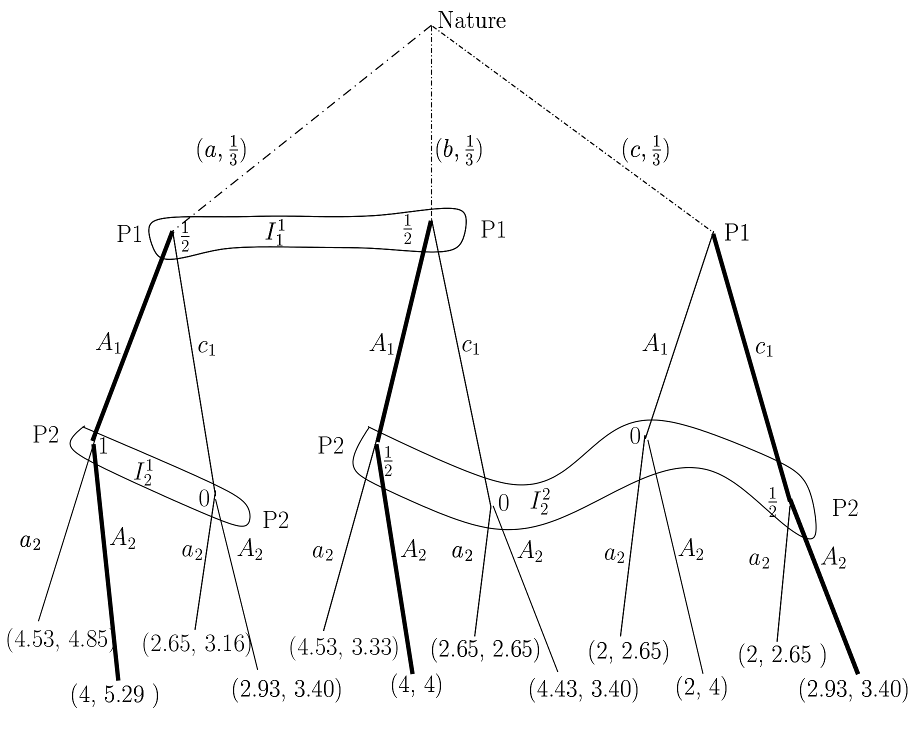


Figure 10

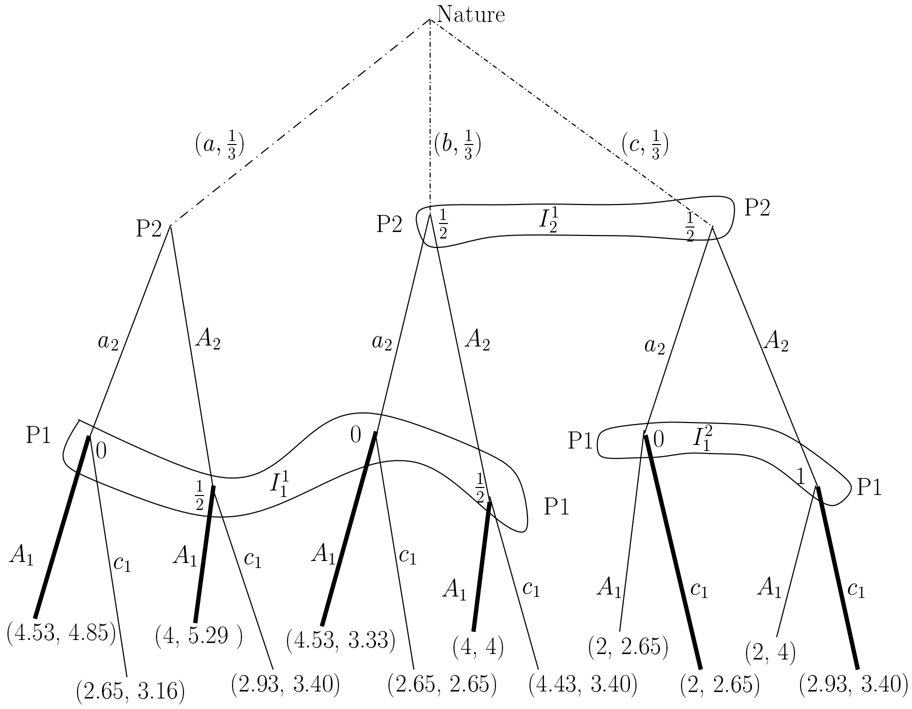


Figure 11

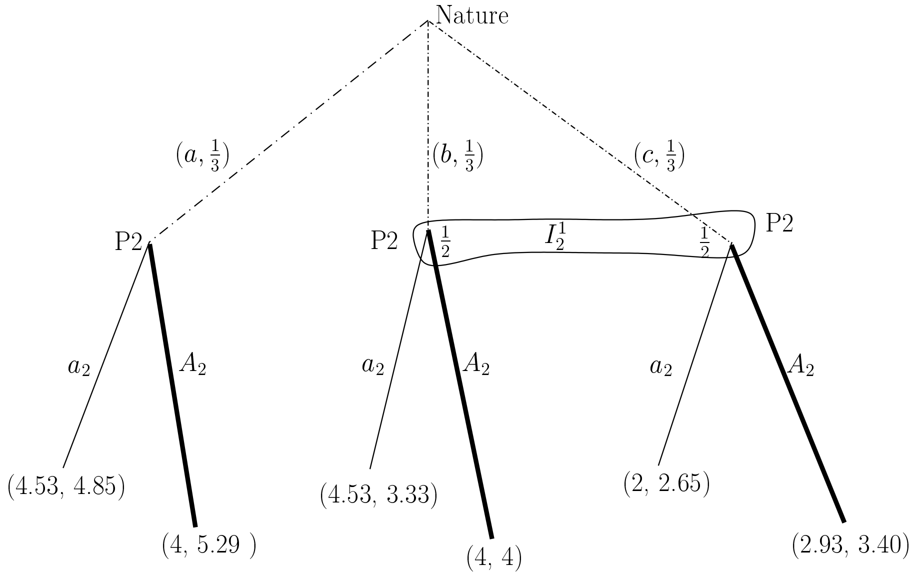


Figure 12

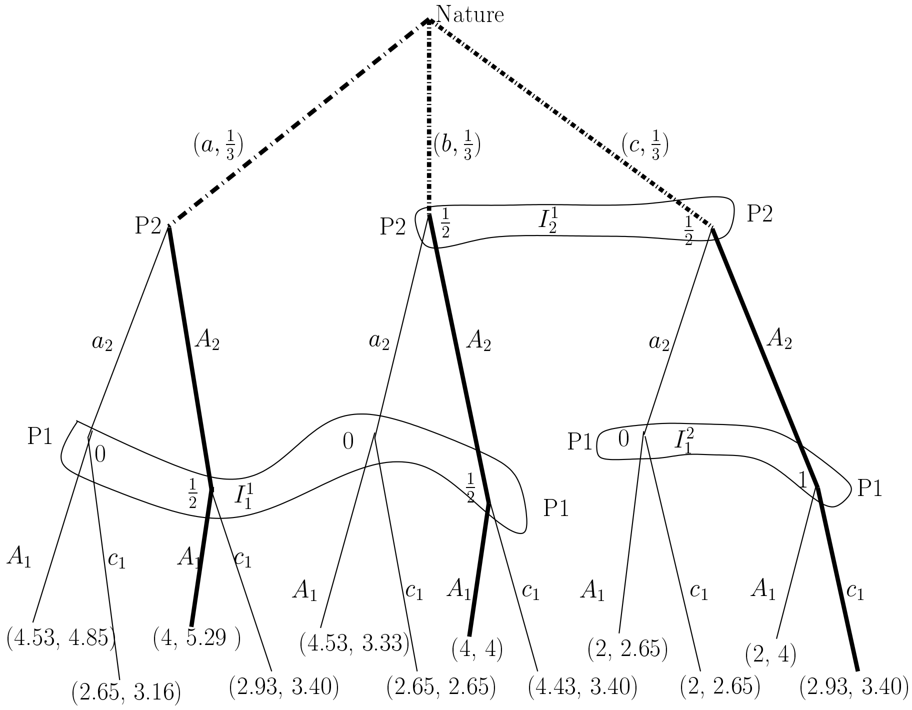


Figure 13

4.3 Normal form games interpretations

Next we cast the problem in Example 3.1 for the case with the simultaneous decisions in a normal form type game. The purpose of this section is to provide with the opportunity to compare the explicit, sequential decisions, game tree formulation with the static normal form approach.

In general we get a much clearer picture from a game tree of how an equilibrium is reached, and hence why a particular contract is accepted or rejected. This is especially so when backward induction is possible.

This section shows that in a normal formulation, of which we offer two types, the construction is really based on having the extensive form game in mind and the interpretation is more complicated. These features are even more pronounced in the discussion in Section 5.3 in which the model is partly based on the analysis of example 3.1, under the assumption that P1 plays first.

In summary, a normal game does not allow us to capture fully the dynamics of a sequence of decisions, while the extensive form approach does. In terms of outcome the normal form here leads to the same outcome as in the case in the previous section when decisions were simultaneous.

The interpretation of the decisions (strategies) is as follows. $A_1\{a,b\}$ means that P1 has seen $\{a,b\}$ and declares A_1; similarly $a_2\{b,c\}$ means that P2 has seen $\{b,c\}$ and declares a_2, etc. In all cases the sign X means that, given their information partition, it is impossible, i.e. not compatible, for P1 to see $\{c\}$ and for P2 to see $\{a\}$.

Table 1: Observations, Strategies and Payoffs

	P2:	$a_2\{a\}$	$a_2\{b,c\}$	$A_2\{a\}$	$A_2\{b,c\}$
P1:					
$A_1\{a,b\}$		(4.53, 4.85)	(4.53, 3.33)	(4, 5.29)	(4, 4)
$A_1\{c\}$		X	(2, 2.65)	X	(2, 4)
$c_1\{a,b\}$		(2.65, 3.16)	(2.65, 2.65)	(2.93, 3.4)	(4.43, 3.4)
$c_1\{c\}$		X	(2, 2.65)	X	(2.93, 3.40)

Basically, each player is interested in what his opponent might declare. He is not interested, as in any case it is not possible to confirm it precisely, in what his opponent has seen. On the other hand he is interested in what he has seen, himself. In order to establish the Nash equilibria of this game we argue as follows. The first and second columns of payoffs are eliminated because they are dominated, from the point of view of P2, by the third and fourth column respectively. Then in the reduced table the second row is eliminated because it is overtaken, from the point of view of P1 by the fourth row.

Thus we are left with a reduced table with six entries:

Table 2: Remaining Observations , Strategies and Payoffs

	P2:	$A_2\{a\}$	$A_2\{b,c\}$
P1:			
$A_1\{a,b\}$		(4, 5.29)	(4, 4)
$c_1\{a,b\}$		(2.93, 3.4)	(4.43, 3.4)
$c_1\{c\}$		X	(2.93, 3.40)

However, this is not an ordinary normal form game. The table separates according to what the players have seen. The first and second row correspond to P1 seeing $\{a,b\}$ and being unable to distinguish between them. Given the prior probability distribution on the choices of nature, P1 attaches probability $\frac{1}{2}$ to each of a and b and this implies that the second row is dominated by the first one. This means that we get the same answer as from the graphs.

We are in effect crossing the product of the players' strategies, $S_1 = \{A_1, c_1\}$ and $S_2 = \{a_2, A_2\}$, with their observations $O_1 = \{\{a,b\},\{c\}\}$ and $O_2 = \{\{a\},\{b,c\}\}$, and obtained $\{S_1 \times O_1\} \times \{S_2 \times O_2\}$, where the observations of each player are taken with a probability distribution on their elements. In this way the idea of PBE, which is defined in terms of game trees, is approached.

Next we cast the problem with the simultaneous decisions in a normal form game of the usual type. We do this employing Figure 10. For simplicity we have labeled the terminal nodes from 1 to 12, left to right.

In describing the strategies of a player the first letter refers to his decision from the first information set, from left to right, and the letter which follows to the one from his second information set. Given a pair of strategies by the two players the game reaches three terminal nodes and we calculate the normalized utility payoffs by adding the appropriate payoff vectors.

Table 3: Strategies and Payoffs

P1: \ P2:	a_2a_2	a_2A_2	A_2a_2	A_2A_2
A_1A_1	(11.06, 10.83)	(10.53, 12.85)	(10.53, 9.14)	(10.53, 13.29)
$(Nodes)$	(1+5+9)	(1+6+10)	(2+5+9)	(2+6+10)
A_1c_1	(11.06, 10.83)	(11.46, 12.25)	(10.53, 11.27)	(10.93,12.69) **(N)**
$(Nodes)$	(1+5+11)	(1+6+12)	(2+5+11)	(2+6+12)
c_1A_1	(7.3, 8.46)	(9.08, 10.56)	(7.58, 8.7)	(9.36, 10.8)
$(Nodes)$	(3+7+9)	(3+8+10)	(4+7+9)	(4+8+10)
c_1c_1	(7.3, 8.46)	(10.01, 9.96)	(7.58, 8.7)	(10.29, 10.2)
$(Nodes)$	(3+7+11)	(3+8+12)	(4+7+11)	(4+8+12)

We are now looking for a Nash equilibrium and it is possible to argue in terms of dominant strategies. From the point of view of P1, the row A_1c_1 dominates all other available strategies. Then P2 will play A_2A_2 and we have obtained a Nash equilibrium, indicated also on the table. We could have also argued that A_2A_2 dominates all other strategies of P2.

Notice that the same table of payoffs can also be obtained by combining the strategies of the players as they appear in Figure 8. Obviously, although the payoffs in the entries of the table would be identical, the numbering of the terminal nodes from which they are obtained would be different.

The Nash equilibrium, or PBE, obtained in the simultaneous decisions case coincides with the one obtained in Figures 1 to 3. It is only in the case of Figures 4 to 7 that more equilibria appear. On the other hand this implies that this type of sequential decision, in which P2 plays first and his declaration is heard, offers more information than the simultaneous decisions problem.

5 REE and weak fine cores

In this section we look at the relation between REE and weak core concepts. The IWFC is conditional on some information already obtained and shared by coalitions of agents. We show that only for state independent utilities, no coalition of agents can block a fully revealing REE and therefore in this case the REE is interim "fully" Pareto optimal. We also show that in general a REE does not belong to the WFC. If it so happens that REE does belong to this set then a slight modification of the utility functions imply that the two sets do not overlap anymore.

5.1 REE and IWFC

Definition 5.1.1. An allocation $x = (x_1, \ldots, x_n) \in \bar{L}_X$ is said to be a IWFC allocation if

(i) each $x_i(\cdot)$ is $\mathcal{F}_I$-measurable;[7]

[7]Recall that for $S \subseteq I$, $\mathcal{F}_S$ denotes the "join" of coalition S, i.e. $\bigvee_{i\in S} \mathcal{F}_i$.

(ii) $\sum_{i=1}^{n} x_i(\omega) = \sum_{i=1}^{n} e_i(\omega)$ for all $\omega \in \Omega$;

(iii) there do not exist state of nature $\omega^* \in \Omega$, coalition S and allocation $(y_i)_{i\in S} \in \prod_{i\in S} \bar{L}_{X_i}$ such that $y_i(\cdot) - e_i(\cdot)$ is $\mathcal{F}_S$-measurable for all $i \in S$, $\sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega)$, for all ω and $v_i(y_i|\mathcal{F}_S)(\omega^*) > v_i(x_i|\mathcal{F}_S)(\omega^*)$ for all $i \in S$.

The definition, (see Yannelis (1991)), implies that no coalitions of agents can pool their own information and make each of its members better off.

Proposition 5.1.1. For state independent utility functions, a fully revealing REE allocation belongs to the IWFC.

Proof. Let (x, p) be a fully revealing REE, so that the state of nature that has occurred is known to everybody and x be feasible and measurable with respect to $\mathcal{F}_I$. Suppose now that x is not an element of IWFC. Then there exists $\omega^* \in \Omega$, a coalition S and feasible $(y_i)_{\in S} \in \prod_{i\in S} \bar{L}_{X_i}$ which is $\mathcal{F}_S$-measurable $\forall\, i \in S$, such that $\sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega)\ \forall\, \omega \in \Omega$ and

$$v_i(y_i|\mathcal{F}_S)(\omega^*) > v_i(x_i|\mathcal{G}_i)(\omega^*). \tag{6}$$

On the right-hand-side of (6) we have that $\mathcal{G}_i = \mathcal{F}$ which in this case is generated by singletons.

We consider the two terms in relation to the Definition 5.1.1. The right-hand-side is $v_i(x_i|\mathcal{G}_i)(\omega^*) = u_i(x_i(\omega^*))$, i.e. one single term with probability one. This follows from the fact that x is fully revealing and therefore $E_i^{\mathcal{G}_i}(\omega^*) = \{\omega^*\}$.

On the other hand the left-hand-side is

$$v_i(\omega^*, y_i(\omega^*)) = \sum_{\omega'} u_i(y_i(\omega'))q_i\big(\omega'|E_i^{\mathcal{F}_S}(\omega^*)\big), \tag{7}$$

where in (7)

$$q_i\big(\omega'|E_i^{\mathcal{F}_S}(\omega^*)\big) = \begin{cases} 0 & : \quad \omega' \notin E_i^{\mathcal{F}_S}(\omega^*) \\ \frac{q_i(\omega')}{q_i\left(E_i^{\mathcal{F}_S}(\omega^*)\right)} & : \quad \omega' \in E_i^{\mathcal{F}_S}(\omega^*). \end{cases}$$

and $E_i^{\mathcal{F}_S}(\omega^*)$ is a subset of $\mathcal{F}_S$ on which y_i is constant.

This allows us to take the utility term out of the sum[8] and deduce that $u_i(y_i(\omega^*)) > u_i(x_i(\omega^*))$. This implies that when x_i was chosen y_i was too expensive and therefore $p(\omega^*)y_i(\omega^*) > p(\omega^*)x_i(\omega^*) = p(\omega^*)e_i(\omega^*)\quad \forall\, i \in S$. Then summing up with respect to $i \in S$ we obtain

$$p(\omega^*)\sum_{i\in S} y_i(\omega^*) = \sum_{i\in S} p(\omega^*)y(\omega^*) > \sum_{i\in S} p_i(\omega^*)e_i(\omega^*) = p(\omega^*)\sum_{i\in S} e_i(\omega^*). \tag{8}$$

[8]Notice that if $u_i(\omega', x_i(\omega'))$ depended separately on ω' then, in general, it would not have been possible to take $u_i(\omega', y_i(\omega'))$ out of the sum. On the other hand measurability of u_i with respect to its first argument would rescue the proof.

Relation (8) is a contradiction to $\sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega)$ because in order to obtain the inequality $p(\omega^*) \sum_{i \in S} y_i(\omega^*) > p(\omega^*) \sum_{i \in S} e_i(\omega^*)$ at least one element of the vector $\sum_{i \in S} y_i(\omega)$ must be larger than the corresponding element of $\sum_{i \in S} e_i(\omega)$.

Remark 5.1.1. With state independent utilities, Proposition 5.1.1 can be proven even if x is a partially revealing or non-revealing, REE. It does not matter whether the information of the coalition is finer or not than the one of the REE. Also with state dependent utilities the proposition can be proven for general REE and an appropriately defined WFC concept if coalitions are only allowed to form which have the same information as REE. Then there is no need to take the utility expressions out of the relation $v_i(y_i|\mathcal{F}_S)(\omega^*) > v_i(x_i|\mathcal{G}_i)(\omega^*)$ An interpretation of what the proposition implies is that, under certain conditions, allowing all possible coalitions to share their information will not block the REE allocations.

Kwasnica (1998) has discussed a related result for a different core concept which is not interim fully Pareto optimal.

The conditions under which Proposition 5.1.1 holds are limited. We now construct examples to show that it does not necessary hold when we have state dependent utilities. The introduction of Agent 3 is done so that the REE satisfy (i) in the definition of the IWFC. Alternatively, without introducing a third agent we can argue that given a REE there exists an IWFC allocation which improves the conditional utility of an agent given some particular state.

Example 5.1.1 There are only two, equally probable, states of nature, (one can add more states to make the model richer but this is not important), and two goods. Players 1 and 2 cannot distinguish between states a and b. On the other hand their utility functions differ per state. Player 3 can distinguish between all states of nature, has no initial endowments and has some utility function. His role is to ensure that the vector x described below satisfy condition (i) of IWFC. We turn our attention to the other players.

We are assuming the following. In state a: $u_1 = \min\{\epsilon x_{11}, x_{12}\}$, where $\epsilon > 1$, and $e_1 = (2,0); u_2 = \min\{x_{21}, x_{22}\}$, and $e_2 = (0,2)$. In state b: $u_1 = \min\{x_{11}, x_{12}\}$, and $e_1 = (2,0)$ $u_2 = (x_{21}x_{22})^c$, where $c > 0$ will be determined later, and $e_2 = (0,2)$.

We construct two Edgeworth boxes and find the fully revealing REE, and hence our vector x, to be as follows. In state a: $p_1 = 0, p_2 = 1$; Agent 1 gets zero quantities and Agent 2 gets everything; $u_1 = 0$ and $u_2 = 2$. In state b: $p_1 = 1, p_2 = 1$; every agent gets 1 unit if each good; $u_1 = 1$ and $u_2 = 1$. In both states, Players 3 receives no quantities.

We will now show that this REE is not in the IWFC. Since, when the two players share their information, they still cannot distinguish between the two states we still require measurability of the feasible allocation to satisfy condition (iii) of IWFC.

The proposed allocation is that Agent 1 gets $y_1(a) = y_1(b) = (0.75, 0.75)$ and Agent 2 gets $y_2(a) = y_2(b) = (1.25, 1.25)$. The utility levels are as follows. In state a: $u_1 = 0.75$, and $u_2 = 1.25$ and in state b: $u^1 = 0.75$, $u^2 = (1.25 \times 1.25)^c$.

We choose state a for the condition (iii) of IWFC. For agent 1 we have that $v_1(y_1)(a)$ is larger than his REE utility which is zero. Also, for suffiently large c, we have for agent 2 that $v_2(y_2)(a) = \left(\frac{1}{2}\right)1.25 + \left(\frac{1}{2}\right)(1.25 \times 1.25)^c > u_2 = 2$ (REE utility under a).

As for the alternative approach, without introducing a third agent we can argue that, given a REE, there exists an IWFC allocation which does better for some agent. First we use the above y_i allocation to show that it does better under a. Then we can argue that there exists an IWFC allocation which for some agent does even better in terms of utility conditioned on state a.

Example 5.1.2 There are two, equally probable states $\Omega = \{a, b\}$ and three players $I = 1, 2, 3$. Player 3 can detect all states, but he has no initial endowments; his only role is to ensure that the x_i calculated below satisfy condition (i) of IWFC. Players 1 and 2 cannot distinguish between the states.

We are assuming that in state a: $u_1 = x_{11}^2 x_{12},\ u_2 = x_{21}^2 x_{22}^2\ e_1 = \left(\frac{9}{13}, \frac{9}{13}\right),\ e_2 = \left(\frac{4}{13}, \frac{4}{13}\right)$, and in state b: $u_1 = x_{11}^{0.5} x_{12},\ u_2 = x_{21} x_{22},\ e_1 = \left(\frac{9}{13}, \frac{9}{13}\right),\ e_2 = \left(\frac{4}{13}, \frac{4}{13}\right)$.

The REE is given by $p(a) = (8, 5),\ x_1(a) = (0.75, 0.6),\ x_2(a) = (0.25, 0.4)$, and $p(b) = (5, 8),\ x_1(b) = (0.6, 0.75),\ x_2(b) = (0.4, 0.25)$.

In the IWFC definition choose $\omega^* = a,\ S = \{1, 2\},\ y_1(a) = y_1(b) = (0.6, 0.8)$, and $y_2(a) = y_2(b) = (0.4, 0.2)$.

Then $v_1(y_1)(a) = 0.454,\ u_1(a, x_1(a)) = 0.337,\ v_2(y_2)(a) = 0.043,\ u_2(a, x_2(a)) = 0.01$.

5.2 REE and WFC

Next we define the WFC (Yannelis (1991) and Koutsougeras - Yannelis (1993)). This is a refinement of the fine core concept of Wilson (1978). The fine core notion of Wilson as well as that in Koutsougeras and Yannelis may be empty in well behaved economies. However, WFC allocations always exist, provided the utility functions are concave and continuous.

Definition 5.2.1. An allocation $x = (x_1, \ldots, x_n) \in \bar{L}_X$ is said to be a *WFC allocation* if

(i) each $x_i(\omega)$ is F_I-measurable;

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$, for all $\omega \in \Omega$;

(iii) there do not exist coalition S and allocation $(y_i)_{i \in S} \in \prod_{i \in S} \bar{L}_{X_i}$ such that $y_i(\cdot) - e_i(\cdot)$ is $\mathcal{F}_S$-measurable for all $i \in S$, $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $v_i(y_i) > v_i(x_i)$ for all $i \in S$.

As comparisons are made on the basis of expected utility, the weak fine core is also an ex ante concept. It captures the idea of an allocation which is ex ante "full information" Pareto optimal. It can easily be shown that the WFC and the IWFC are different concepts. The purpose of this section is to see whether or not REE is in the WFC. The examples below show that the REE need not be in the WFC. This is not surprising because the REE is an interim concept while the WFC is an ex ante one. Thus, one should not expect interim contracts to be ex ante fully Pareto efficient, as the examples below demonstrate.

Example 5.2.1 Consider the following two agents economy, $I = \{1, 2\}$ with one commodity, i.e. $X_i = \mathbb{R}_+$ for each i, and three states of nature $\Omega = \{a, b, c\}$.

The endowments and information partitions of the agents are given by

$$e_1 = (5, 5, 0), \qquad \mathcal{F}_1 = \{\{a, b\}, \{c\}\};$$
$$e_2 = (5, 0, 5), \qquad \mathcal{F}_2 = \{\{a, c\}, \{b\}\}.$$

The utility functions of the two agents are state independent, given by $u_i(x_i) = x_i^{\frac{1}{2}}$ and every player has the same prior distribution $\mu(\{\omega\}) = \frac{1}{3}$, for $\omega \in \Omega$.

As explained in Observation 3.1, the REE coincides with the initial endowments. On the other hand, for the WFC the agents pool their information and therefore any feasible consumption vector to either agent will be measurable.

There are uncountably many WFC allocations, as for example

$$\begin{pmatrix} 5 & 2.5 & 2.5 \\ 5 & 2.5 & 2.5 \end{pmatrix}.$$

This allocation is $\bigvee_{i=1}^{2} \mathcal{F}_i$-measurable and cannot be dominated by any coalition of agents using their pooled information. Furthermore it is Pareto superior to REE allocation.

Next we turn our attention to a more general model.

Example 5.2.2 For simplicity, we treat originally a case with two players, two goods and two states. We also assume, in the beginning, that the players are, in all states, endowed with strictly positive endowments of both goods and that for both players all states are equally probable. We assume that all states, $j \in \Omega$, are distinguishable by the two players when they pool their information.

The normalized expected utility functions of the two players are $\mathcal{U}_1 = \sum_j (x_{11}^j)^\alpha (x_{12}^j)^\beta$ and $\mathcal{U}_2 = \sum_j (x_{21}^j)^\alpha (x_{22}^j)^\beta$. Namely we assume that they have identical, state independent utility functions. These assumptions can be relaxed. In summary, the result of the analysis is that in general the REE does not belong to the WFC.

The WFC allocations are characterized through the following problem:

Maximize $\sum_j (x_{11}^j)^\alpha (x_{12}^j)^\beta$

Subject to

$$\sum_j (S_1^j - x_{11}^j)^\alpha (S_2^j - x_{12}^j)^\beta = \bar{\mathcal{U}}_2 \text{ (fixed)},$$
$$0 \le x_{11}^j \le S_1^j, \;\; 0 \le x_{12}^j \le S_2^j \;\; \forall j,$$

where S_i^j denotes the total quantity of Good i in state j. Note that $0 < \mathcal{U}_2 < \sum_j (S_1^j)^\alpha (S_2^j)^\beta$.

Because of the feasibility constraints on quantities, the Lagrange theory cannot be applied in general in order to obtain the solution. However we can comment on the relation between REE and WFC allocations by arguing through another route.

We apply a Gorman-type separation argument (see Gorman (1955)). We consider the contract curve per state. First we consider the following problem.

Maximize $(x_{11}^j)^\alpha (x_{12}^j)^\beta$

Subject to

$$(S_1^j - x_{11}^j)^\alpha (S_2 - x_{12}^j)^\beta = u_2{}^j \text{ (fixed)},$$
$$0 \le x_{11}^j \le S_1^j, \;\; 0 \le x_{12}^j \le S_2^j.$$

The solution implies $S_2^j x_{11}^j = S_1^j x_{12}^j$, which is the diagonal of the Edgeworth box. All WFC allocations are on contract curve in each state, for otherwise we can move to a Pareto superior point on the contract curve. It is also true that a REE, fully revealing or not, will be on the diagonal with every agent receiving positive quantities from both goods. This follows from the fact that otherwise, in at least one state, the markets will not clear.

The actual solution is

$$x_{11}^j = \left(\frac{S_1^j}{S_2^j}\right)^{\frac{\beta}{\alpha+\beta}} \left[(S_1^j)^{\frac{\alpha}{\alpha+\beta}}(S_2^j)^{\frac{\beta}{\alpha+\beta}} - (u_2^j)^{\frac{1}{\alpha+\beta}}\right],$$

$$x_{12}^j = \left(\frac{S_2^j}{S_1^j}\right)^{\frac{\alpha}{\alpha+\beta}} \left[(S_1^j)^{\frac{\alpha}{\alpha+\beta}}(S_2^j)^{\frac{\beta}{\alpha+\beta}} - (u_2^j)^{\frac{1}{\alpha+\beta}}\right].$$

We write $(S_1^j)^{\frac{\alpha}{\alpha+\beta}}(S_2^j)^{\frac{\beta}{\alpha+\beta}} = T^j$ and $(u_2^j)^{\frac{1}{\alpha+\beta}} = W^j$, and substitute into the objective function to get $\sum_j [T^j - W^j]^{(\alpha+\beta)}$ which is to be maximized subject to the constraints $\sum_j u_2^j = \bar{\mathcal{U}}_2$. and $u_2^j \geq 0$ which are equivalent to $\sum_j (W^j)^{(\alpha+\beta)} = \bar{\mathcal{U}}_2$. and $W^j \geq 0$ and considering the solution for the $x's$ we also have $0 \leq W^j \leq T^j$. So in summary we are solving:

Maximize $\sum_j [T^j - W^j]^\gamma$

Subject to

$\sum_j (W^j)^\gamma = \bar{\mathcal{U}}_2$ (fixed), and

$0 \leq W^j \leq T^j$.

where $\gamma = \alpha + \beta$.

We now look at the form of the functions. Consider $\sum_j (W^j)^\gamma = 1$ for any $\gamma > 0$.

For $\gamma = 1$ this is a hyperplane. In the positive orthant, $\gamma > 1$ causes the surface to bulge away from the hyperplane so as to enclose a convex set including the origin ($\gamma = 2$ is the exemplary case, which is a hypersphere). Conversely for $\gamma < 1$ produces a surface which bulges in towards the origin. $\sum_j (W^j)^\gamma = \bar{\mathcal{U}}_2$ is similar in shape but scaled by a factor $\bar{\mathcal{U}}_2^{\frac{1}{\gamma}}$.

Finally the shape of $\sum_j [T^j - W^j]^\gamma = K$ (fixed) can be derived from the above. The origin has been shifted to the point with coordinates (T^j) after the surface has been reflected along each coordinate axis.

Now we look at the solution of the overall Gorman problem. We distinguish between:

(i) $\gamma > 1$; the constraint is concave, in the nonnegative area, with perpendicular intersections with the axes. The indifference curves of the objective function are convex, with nonnegative coordinates, (see Figure 14), and increase in value as we move in the direction of the origin. It follows that the maximum will be at one or both of the corner points. This means that the REE is not in the WFC.

(ii) $\gamma < 1$; in this case the constraint is convex and the indifference curves are concave, (see Figure 14), and increase in value as we move in the direction of the origin. The solution is away from the corner points at a point of tangency. Even under symmetric conditions there is no reason why the REE should be in the WFC.

(iii) $\gamma = 1$; inspection of the objective function and the constraint shows that the WFC

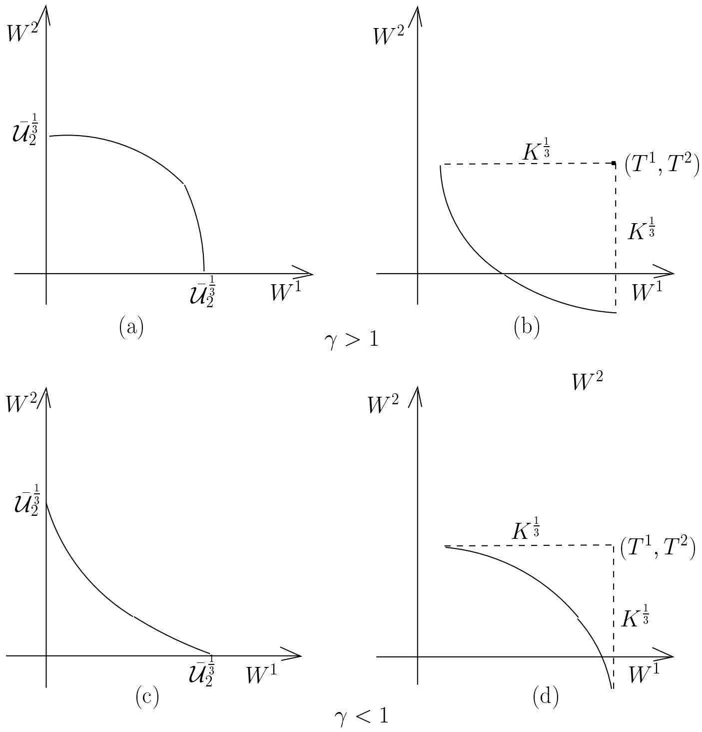


Figure 14

coincides with the linear constraint. It follows that the REE allocation is in the WFC and this is the case in Example 3.1. However, attaching a weight to the utility of player 1 in one state implies a corner solution and therefore the REE is not in the WFC.

5.3 A decomposable model; non-revealing REE and WFC

The idea is to construct a model such that we have a non-revealing REE which does not belong to the WFC. At the same time we want to show how a model decomposes into two independent ones. The model we are considering uses in part Example 3.1. We also use this model for further comparisons between the dynamic, extensive form formulation of a game and the static, normal form approach.

Example 5.3.1 We assume that there two agents, $I = \{1, 2\}$, two commodities, i.e. $X_i = \mathbb{R}^2_+$ for each $i \in I$, and five states of nature $\Omega = \{a, b, c, d, e\}$.

We further assume that the endowments, per state a, b, c, d and e respectively, and information partitions of the agents are given by

$$e_1 = ((7,1),(7,1),(4,1),(4,0),(4,0)), \qquad \mathcal{F}_1 = \{\{a,b\},\{c\},\{d,e\}\};$$
$$e_2 = ((1,10),(1,7),(1,7)(0,4),(0,4)), \qquad \mathcal{F}_2 = \{\{a\},\{b,c\}\{d\},\{e\}\}.$$

We shall denote $A_1 = \{a, b\}$, $c_1 = \{c\}$, $D_1 = \{d, e\}$, $a_2 = \{a\}$, $A_2 = \{b, c\}$, $d_2 = \{d\}$, $e_2 = \{e\}$. It is also assumed that $u_i(\omega, x_{i1}, x_{i2}) = x_{i1}^{\frac{1}{2}} x_{i2}^{\frac{1}{2}}$ and that every player expects that all states are equi-probable.

The REE for states a, b and c are described in Example 3.1 and for states d and e the prices are equal and the allocations are (2,2), per agent, per state. Since no information is provided to Player 1 concerning states d and e the REE is non-revealing.

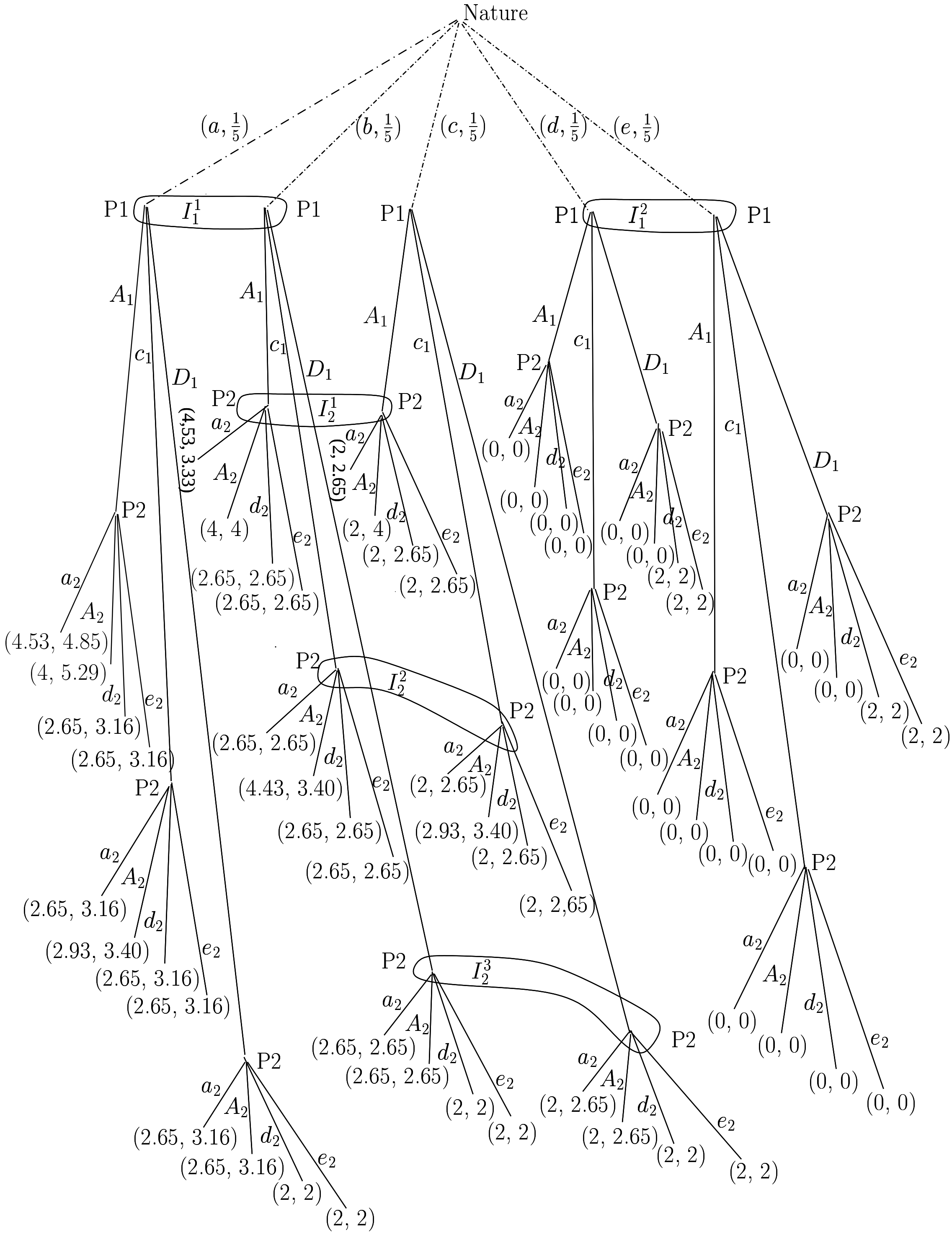
Now the REE will belong to the WFC because it does so separately in the two examples which have been put together. However attaching a higher weight to the utility function of Player 1 in one of the periods will imply that the REE will not lie in the WFC.

We now wish to see how the overall model decomposes into two separate ones. We assume that P1 plays first. We specify the *rules* for calculating payoffs, i.e. the terms of the contract. Payoffs are in utility terms:

(i) If the declarations of the two players are incompatible, that is one of
(A_1, d_2), (A_1, e_2), (c_1, a_2), (c_1, d_2), (c_1, e_2), (D_1, a_2), (D_1, A_2),
then this implies that no trade takes place.
(ii) If the state is a, b, or c and the declarations of the two players are (A_1, A_2) then this implies that state b is really declared. The player who believes it (because he has no reason to disbelieve it) gets his REE allocation (4, 4) and the other player gets the rest. So aA_1A_2 means P2 has lied but P1 believes it is state b and gets (4, 4). P2 gets the rest under state a that is (4, 7); bA_1A_2 means that both believe that it is the (actual) state b and each gets (4, 4); cA_1A_2 means that P2 believes it is state b and gets (4, 4) and P1 gains nothing from his lie as he gets (1, 4). If the state is d or e and the declarations of the two players are (D_1, e_2) or (D_1, d_2), respectively, then the state in the overlap is believed and the two players get their REE allocation under this state and end up with allocation (2, 2) each, which implies $u_i = 2$.
(iii) aA_1a_2, bA_1A_2, cc_1A_2, dD_1d_2, eD_1e_2, imply that everybody tells the truth and the contract implements the REE allocation under state a, b, c, d and e respectively. (bA_1A_2 in (ii) and (iii) give of course an identical result).
(iv) ac_1A_2, $,aD_1d_2$, aD_1e_2, bD_1d_2, bD_1e_2, cD_1d_2, cD_1e_2,
imply that both lie but their declarations are not incompatible. Each gets his REE under the overlapping state of the difference between the total endowments under the true state and the allocation which the agents receive.
(v) cA_1a_2, dA_1a_2, dA_1A_2, dc_1A_2, eA_1a_2, eA_1A_2, ec_1A_2
means that both lie and stay with their initial endowments as they cannot get the REE allocations under the state in the overlap of their declarations.
(vi) bA_1a_2 implies that P2 misreports and P1 believes and gets his REE under a; P2 gets the rest under b, that is $\left(\frac{91}{22}, \frac{43}{16}\right)$. Then $u_2 = 3.33 < 4$ and $u_1 = 4.53 > 4$ and the lie of P2 really benefits P1.
(vii) bc_1A_2 means that P1 lies and P2 believes that it is state c. P2 gets his REE allocation under c and P1 gets the rest under b, that is the allocation $\left(\frac{85}{16}, \frac{37}{10}\right)$. Then $u_1 = 4.43 > 4$ and $u_2 = 3.4 < 4$ and P1 benefits from lying.

We are looking for a PBE and we analyse Figure 15 by considering first optimal decisions of P2. From information sets corresponding to states a, b, c, player P2 will never play d_2 or e_2, since these are dominated by strategy A_2. In states d or e, P2 never gains anything by lying and optimal decisions are to play truthfully, d_2 or e_2. Figure 16 contains only these optimal decisions of P2.

We now consider optimal decisions of P1 on the base of Figure 16. In states a, b, c, player P1 does not need to play D_1 since this is weakly dominated by c_1, and in states d or e he


Nature
(a, 1/5)
(b, 1/5)
(c, 1/5)
(d, 1/5)
(e, 1/5)
P1
I11
P1
P1
P1
I12
P1
A1
c1
D1
P2
I21
P2
a2
A2
d2
e2
(4.53, 3.33)
(2, 2.65)
(4, 4)
(2.65, 2.65)
(2.65, 2.65)
(2, 4)
(2, 2.65)
(2, 2.65)
(4.53, 4.85)
(4, 5.29)
(2.65, 3.16)
(2.65, 3.16)
P2
I22
P2
(2.65, 2.65)
(4.43, 3.40)
(2.65, 2.65)
(2.65, 2.65)
(2, 2.65)
(2.93, 3.40)
(2, 2.65)
(2, 2,65)
(2.65, 3.16)
(2.93, 3.40)
(2.65, 3.16)
(2.65, 3.16)
P2
I23
P2
(2.65, 2.65)
(2.65, 2.65)
(2, 2)
(2, 2)
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(2, 2.65)
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(0, 0)

Figure 15

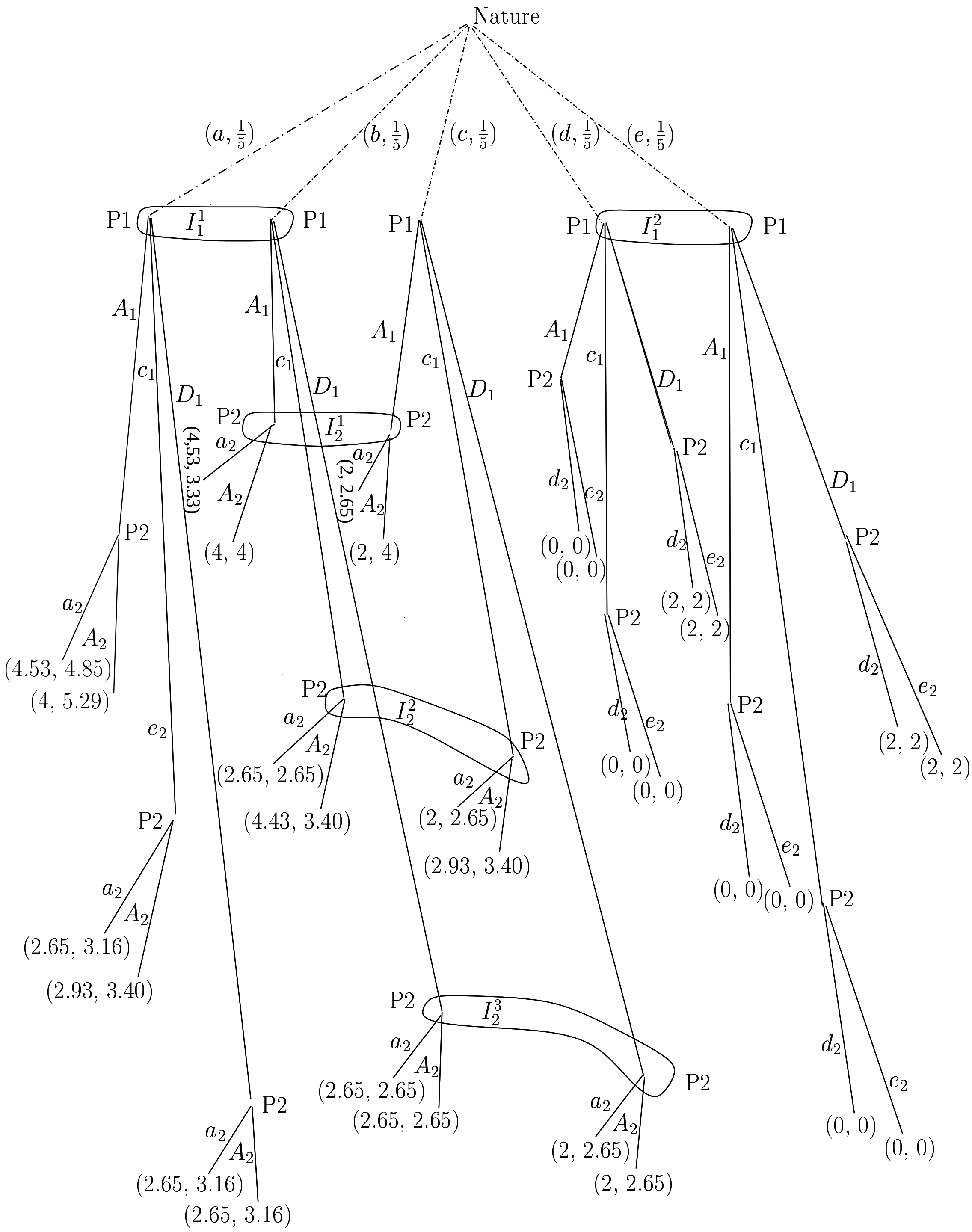
Nature
$(a, \frac{1}{5})$
$(b, \frac{1}{5})$
$(c, \frac{1}{5})$
$(d, \frac{1}{5})$
$(e, \frac{1}{5})$
P1
I_1^1
P1
P1
P1
I_1^2
P1
A_1
c_1
D_1
P2
I_2^1
P2
a_2
A_2
(4.53, 3.33)
(2, 2.65)
(4, 4)
(2, 4)
(4.53, 4.85)
(4, 5.29)
e_2
P2
I_2^2
(2.65, 2.65)
(4.43, 3.40)
(2, 2.65)
(2.93, 3.40)
(2.65, 3.16)
(2.93, 3.40)
I_2^3
(2.65, 2.65)
(2.65, 2.65)
(2, 2.65)
(2, 2.65)
(2.65, 3.16)
(2.65, 3.16)
d_2
(0, 0)
(2, 2)

Figure 16

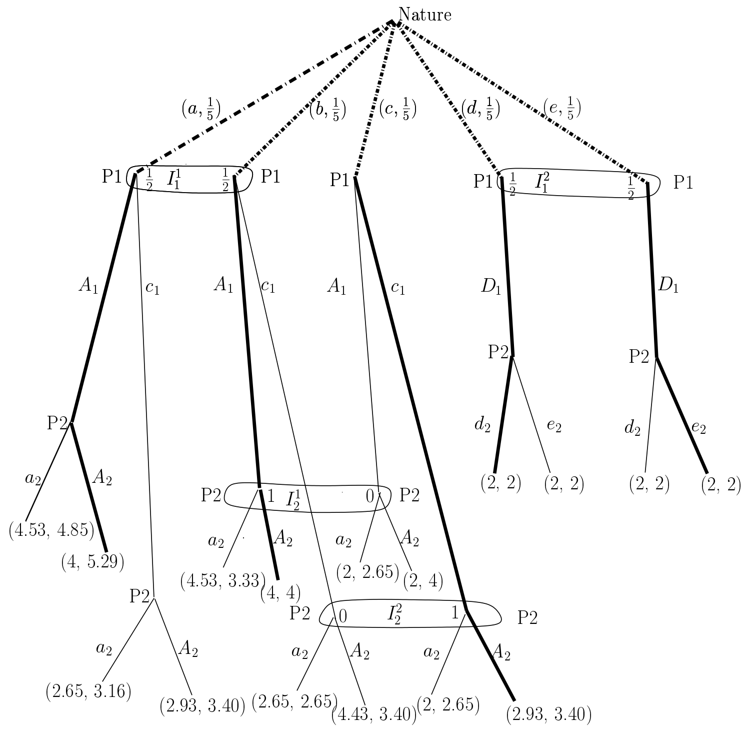

Figure 17

will always play D_1. Hence we obtain Figure 17, in which heavy lines show plays of the game corresponding to choices by nature and optimal behavior strategies of the players. Their beliefs are also indicated and the conditions for a PBE are satisfied.

Figure 17 and the conclusions reached shows that the game splits into two independent games. One concerns a, b and c, already analysed in Example 3.1, and the other states d and e. We have the REE for both models and so we have the REE for the combined model which will not be fully revealing because the common prices for state d and e will not allow P1 to deduce the state of nature.

Note that instead of analyzing the example through a game tree which is slightly larger than usual, we can tabulate the payoffs for strategy pairs in each state, as it is shown below in Tables 4 to 7. However, these tables have to be interpreted with care as they are not independent. This follows from the fact that there exist information sets with more than one node.

We read the tables assuming P2 plays second having heard the declaration of P1. The first observation is that in cases a, b, c player P2 will never play d_2 or e_2, since these are dominated by strategy A_2. This does not contradict the fact that P2 cannot distinguish between states b and c. This means that the last two columns in Tables 4, 5 and 6 are eliminated from consideration by P1 who, by taking into account this optimal response by P2, can discard strategy D_1 without any loss. In effect P1 chooses as optimal strategies A_1 from the equi-probable, to him, states a and b and c_1 from state c.

In states d and e, player P2 can play either d_2 or e_2, and therefore it is also optimal to play truthfully. Hence in Table 7 there is only one column for P1 to consider and he plays

D_1. It does not matter that he cannot distinguish between d and e.

Hence the game has been fully analyzed through the tables. In effect it is seen that it splits into two independent games. This enables us to conclude that a non-revealing REE may not be implementable as a PBE of an extensive form game.

The tables are given below:

Table 4: Strategies and Payoffs; State of Nature a

	P2:	a_2	A_2	d_2	e_2
P1:					
A_1		(4.53, 4.85)	(4.5, 5.29)	(2.65, 3.16)	(2.65, 3.16)
c_1		(2.65, 3.16)	(2.93, 3.40)	(2.65, 3.16)	(2.65, 3.16)
D_1		(2.65, 3.16)	(2.65, 3.16)	(2, 2)	(2, 2)

Table 5: Strategies and Payoffs; State of Nature b

	P2:	a_2	A_2	d_2	e_2
P1:					
A_1		(4.53, 3.33)	(4, 4)	(2.65, 2.65)	(2.65, 2.65)
c_1		(2.65, 2.65)	(4.43, 3.40)	(2.65, 2.65)	(2.65, 2.65)
D_1		(2.65, 2.65)	(2.65, 2.65)	(2, 2)	(2, 2)

Table 6: Strategies and Payoffs; State of Nature c

	P2:	a_2	A_2	d_2	e_2
P1:					
A_1		(2, 2.65)	(2, 4)	(2, 2.65)	(2, 2.65)
c_1		(2, 2.65)	(2.93, 3.40)	(2, 2.65)	(2, 2.65)
D_1		(2, 2.65)	(2, 2.65)	(2, 2)	(2, 2)

Table 7: Strategies and Payoffs; State of Nature d or e

	P2:	a_2	A_2	d_2	e_2
P1:					
A_1		(0, 0)	(0, 0)	(0, 0)	(0, 0)
c_1		(0, 0)	(0, 0)	(0, 0)	(0, 0)
D_1		(0, 0)	(0, 0)	(2, 2)	(2, 2)

6 REE versus the private core

We comment in this, and the following two subsections, on the relation between the REE and the private core. First we define the notion of the private core (Yannelis (1991)).

Definition 6.1. An allocation $x \in L_X$ is said to be a *private core allocation* if
(i) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$ and
(ii) there do not exist coalition S and allocation $(y_i)_{i \in S} \in \prod_{i \in S} L_{X_i}$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $v_i(y_i) > v_i(x_i)$ for all $i \in S$.

The private core is an ex ante concept and under mild conditions it is not empty.

It has been shown in Yannelis (1991) and in Glycopantis - Muir - Yannelis (2001) that the private core exists under the standard concavity and continuity assumptions on the utility functions. Moreover, the private core is always TCBIC as shown in Koutsougeras - Yannelis (1993) and Hahn - Yannelis (2001).

We make our first point by using Example 3.1 above. We have seen that the normalized expected utilities corresponding to the REE allocations are $\mathcal{U}_1 = 11.46$ and $\mathcal{U}_2 = 12.25$. On the other hand the following allocation $x_1 = ((5.5, 5.5),\ (5.5, 5.5),\ (2.5, 5.5))$ and $x_2 = ((2.5, 5.5),\ (2.5, 2.5),\ (2.5, 2.5))$ is in the private core yielding normalized expected utilities $\mathcal{U}_1 = 14.70$ and $\mathcal{U}_2 = 8.10$. This example shows that one agent's utility function is improved by going from REE to this private core allocation. Therefore there is no reason why this agent should agree to the REE distributive scheme. Most importantly the REE allocation is not, in general, incentive compatible, (contrary to the private core which is), and therefore the contracts may not viable.

Further, it should noted that Glycopantis - Muir - Yannelis (2001, 2003) show that the private core can be implemented as a PBE, and as a sequential equilibrium of an extensive form game, contrary to the REE. Therefore it appears the private core is more acceptable than the REE. Below we show that the private core exists in situations in which the REE does not. To this end we present the Kreps (1977) example.

6.1 The Kreps example

Kreps (1977), in a general example of a differential information economy, showed the non-existence of a REE, revealing or not. Here we present a specific example which satisfies all his assumptions. On the other hand the private core exists which, again suggests that the latter concept has an advantage over that of REE.

Example 6.1.1 There are two agents $I = \{1, 2\}$, two commodities, i.e. $X_i = \mathbb{R}^2_+$ for each agent, i, and two states of nature $\Omega = \{\omega_1, \omega_2\}$, considered by the agents as equally probable. In x_{ij} the first index will refer to Agent i and the second to Good j. If it is necessary we shall also write $x_{ij}(\omega_1)$ and $x_{ij}(\omega_2)$.

We assume that the endowments, per state $\omega_1 = 1$ and $\omega_2 = 2$ respectively, and information partitions of the agents are given by

$$e_1 = ((1.5, 1.5), (1.5, 1.5), \qquad \mathcal{F}_1 = \{\{\omega_1\}, \{\omega_2\}\};$$

$$e_2 = ((1.5, 1.5), (1.5, 1.5), \qquad \mathcal{F}_2 = \{\{\omega_1, \omega_2\}\}.$$

The utility functions, of Agents 1 and 2 respectively, are for ω_1 given by $u_1 = \ log\ x_{11} + x_{12}$ and $u_2 = 2\ log\ x_{21} + x_{22}$ and for state ω_2 by $u_1 = 2\ log\ x_{11} + x_{12}$ and $u_2 = \ log\ x_{21} + x_{22}$.

We consider first the possibility of REE.

Case 1. Fully revealing REE.

Suppose that there exist, after normalization, prices $(p_1(1), p_2(1)) \neq (p_1(2), p_2(2))$, where $p_i(j)$ denotes the price of good i in state j. In this case every agent would know the state of nature. We now check whether this is possible. The problems of two agents would be as follows.

State ω_1:

Agent 1:

Maximize $u_1 = \ log\ x_{11} + x_{12}$
Subject to

$$p_1(1)x_{11} + p_2(1)x_{12} = 1.5(p_1(1) + p_2(1))$$

and

Agent 2:

Maximize $u_2 = 2\ log\ x_{21} + x_{22}$
Subject to

$$p_1(1)x_{21} + p_2(1)x_{22} = 1.5(p_1(1) + p_2(1)).$$

The agents solve analogous problems in state ω_2. However it is not possible to find, after normalization, $(p_1(1), p_2(1)) \neq (p_1(2), p_2(2))$. The reason is that in the two problems the demands of the agents are interchanged so that the total demand stays the same while the total supply is fixed. It is also straightforward to check that there is no multiplicity of equilibria per state.

Case 2. Non-revealing REE.

Now we consider the possibility of having prices $p_1(1) = p_1(2) = p_1$ and $p_2(1) = p_2(2) = p_2$. The two agents would act as follows.

Agent 1:

He can tell the states of nature and obtains the demand functions for ω_1, $x_{11} = \frac{p_2}{p_1}$ and $x_{12} = \frac{1.5p_1}{p_2} + 0.5$ and for ω_2, $x_{11} = \frac{2p_2}{p_1}$ and $x_{12} = \frac{1.5p_1}{p_2} - 0.5$ for $3p_1 \geq p_2$.

It is clear that the demands differ per state of nature.

Agent 2:

He sets $x_{21}(\omega_1) = x_{21}(\omega_2) = x_{21}$ and $x_{22}(\omega_1) = x_{22}(\omega_2) = x_{22}$ and solves the problem

Maximize $u_2 = \frac{1}{2}(2\ log\ x_{21} + x_{22}) + \frac{1}{2}(\ log\ x_{21} + x_{22}) = 1.5\ log\ x_{21} + x_{22}$
Subject to

$$p_1 x_{21} + p_2 x_{22} = 1.5(p_1 + p_2).$$

So the highest indifference curve touches the budget constraint only once. On the other hand the demands of Agent 1 differ per ω. It follows that the markets cannot be cleared with common prices in both states of nature. It follows that there is no REE, revealing or non-revealing, in this model.

Next we consider the existence of private core allocations. These are obtained as solutions of the problem:

Maximize $E_2 = 1.5 \, log \, x_{21} + x_{22}$
Subject to

$$\frac{1}{2}(\, log \, x_{11}(\omega_1) + x_{12}(\omega_1)) + \frac{1}{2}(\, log \, x_{11}(\omega_2) + x_{12}(\omega_2)) \geq E_1 \text{ (fixed)},$$
$$x_{1j}(\omega_1), x_{1j}(\omega_2) \geq 0, \; E_1, \; E_2 \geq 1.5 \, log \, 1.5 + 1.5$$
$$x_{21} + x_{11}(\omega_1) \leq 3, \quad x_{21} + x_{11}(\omega_2) \leq 3,$$
$$x_{22} + x_{12}(\omega_1) \leq 3, \quad x_{22} + x_{12}(\omega_2) \leq 3.$$

This problem always has a solution because of the continuity of the objective function and the compactness of the feasible set. If we set the quantity constraints equal to 3 and $1.5 \, log \, 1.5 + 1.5 = E_1$ then the initial allocation is in the private core.

Also this analysis indicates that the REE may not be an appropriate concept to capture trades under asymmetric information. The agents here receive no instructions as to what they should be doing.

6.2 REE and informational asymmetries

We consider the following three agents economy.

Example 6.2.1 We assume that there are two agents $I = \{1, 2\}$, one commodity, i.e. $X_i = \mathbb{R}_+$ for each $i \in I$, and three states of nature $\Omega = \{a, b, c\}$, considered be the agents as equally probable.

We further assume that the endowments, per state a, b, and c respectively, and information partitions of the agents are given by

$$e_1 = ((8), (8), (0)), \qquad \mathcal{F}_1 = \{\{a, b\}, \{c\}\};$$
$$e_2 = ((8), (0), (8)), \qquad \mathcal{F}_2 = \{\{a\}, \{b, c\}\};$$
$$e_3 = ((0), (0), (0)), \qquad \mathcal{F}_3 = \{\{a\}, \{b\}, \{c\}\}.$$

The utility functions of the agents are $u_i(\omega, x_i) = x_i^{\frac{1}{2}}$.

A private core allocation in this economy is given by the feasible and $\mathcal{F}_i$-measurable allocation $x_1 = \left(\frac{32}{5}, \frac{32}{5}, \frac{18}{5}\right)$, $x_2 = \left(\frac{32}{5}, \frac{18}{5}, \frac{32}{5}\right)$ and $x_3 = \left(\frac{16}{5}, 0, 0\right)$. For all three agents, this allocation gives higher expected utility than the initial endowment.

Although Agent 3 has zero initial endowment in each state, she brings about a Pareto improvement to the whole economy and she is rewarded for this. In effect by revealing her superior information she makes trade possible between the other two agents in states b and c and she receives a positive quantity under state a. Of course the outcome depends on the third agent having more information than the other traders. If the private information set of Agent 3 were to change to the trivial one $\mathcal{F}_3 = \{a, b, c\}$ then she would get zero quantities in each state.

On the other hand the REE cannot capture this phenomenon. It will give zero quantities to an agent who has zero initial allocation in all states, irrespective of his private information, i.e. whether it is the full information partition or the trivial one.

7 Concluding remarks

In this paper we showed that under certain conditions the REE is in the IWFC and therefore it is interim fully Pareto optimal. This result depends crucially on the fact that each agent's utility function is state-independent. In addition we showed that the REE need not be in the WFC and therefore the REE may not be ex ante "fully" Pareto optimal.

Furthermore, it was shown that the REE can result in allocations (contracts) that are not incentive compatible and cannot be supported or implemented as a PBE or a sequential equilibrium. The latter is quite striking because it means that the REE does not fulfil a widely accepted Bayesian rationality criterion. This casts doubts on the sustainability of the REE as an appropriate solution concept which can be used to examine contracts in a differential information economy.

In order to present alternatives to the REE we looked at the example used to show that the REE is not incentive compatible and cannot be supported as a PBE. In the same example a private core allocations exists, it is coalitionally Bayesian incentive compatible, (thus the contract is stable), and can also be supported as a PBE. Moreover we reconsidered the well known example of Kreps (1978) of a non-existant REE and showed that the private core exists.

Finally, we looked at examples where the private core provides to agents superior outcomes in terms of expected utility than the REE. Another advantage of the private core is that it is sensitive to the private information of the agents, i.e. a change in the private information of an agent changes the equilibrium outcome.

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Appendix I: On Nash equilibria and PBEs

In this Appendix we are concerned with the relation between Nash equilibria and PBE's in the model of the Example 3.1. The purpose of this analysis is to compare the PBE with the Nash equilibria. A PBE is one of the Nash equilibria, satisfying further conditions. It is now possible that a pair of players might choose to play simply a Nash equilibrium. This could be explained on the basis of a bounded rationality argument, i.e. that the calculation of a PBE is much more involved.

An efficient way of identifying all the Nash equilibria is by the use of the players' reaction functions. We shall cast the analysis in terms of reaction functions for the first case, shown in Figures 1 to 3, and also for the second and more difficult one of Figures 4 to 7.

P1 plays first.

P1 plays from I_1^1, his pure strategies, from left to right, with probabilities $1-x$ and x respectively and from the singleton with $1-y$ and y. P2 plays his pure strategies as follows. From the singleton at the end of aA_1 he plays with probabilities $1-z$ and z, and from the one at the end of ac_1 with probabilities $1-w$ and w. From the information set I_2^1 he plays with probabilities $1-k$ and k and from I_2^2 with probabilities $1-\ell$ and ℓ.

The normalized expected utilities from which the reaction functions will be

obtained can be seen, through routine calculations, to be

$E_1 = 4.53(1-x)(1-z) + 4(1-x)z + 2.65x(1-w) + 2.93xw + 4.53(1-x)(1-k) + 4(1-x)k + 2.65x(1-\ell) + 4.43x\ell + 2(1-y)(1-k) + 2(1-y)k + 2y(1-\ell) + 2.93y\ell$ and

$E_2 = 4.85(1-x)(1-z) + 5.29(1-x)z + 3.16x(1-w) + 3.40xw + 3.33(1-x)(1-k) + 4(1-x)k + 2.65x(1-\ell) + 3.40x\ell + 2.65(1-y)(1-k) + 4(1-y)k + 2.65y(1-\ell) + 3.40y\ell$.

We need to consider the structure of the reaction functions. In E_1 we need only consider terms which depend on x, y and in E_2 only those which depend on z, w, k, ℓ.

In E_1 the coefficient of x is $0.53z + 0.28w + 0.53k + 1.78\ell - 3.76 < 0$ which implies $x = 0$. On the other hand the coefficient of y is 0.93ℓ, which implies $y = 1$ if $\ell > 0$ and $y =$arb (arbitrary) if $\ell = 0$.

Next we substitute $x = 0$ into E_2 and obtain the reaction of P2 in this case.
The coefficient of z is 0.44 so $z = 1$. The coefficient of w is zero and therefore w is arbitrary; of k is $0.67 + 1.35(1-y)$ so $k = 1$; of ℓ is $0.75y$ so $\ell = 1$ if $y > 0$ and $\ell =$arb if $y = 0$.

The relations connecting y, ℓ admit two solutions which correspond to Nash equilibria.
<u>Case 1</u>: $y = 0, \ell = 0$
<u>Case 2</u>: $y = 1, \ell = 1$

Turning to PBE conditions we see immediately that the arbitrariness of w is untenable. That is $w = 1$ if the strategy of $P2$ is to be optimal from the corresponding singleton.

For Case 1 the beliefs in I_2^1 are $\frac{1}{2}$ for each node and are arbitrary in I_2^2. In effect any beliefs in I_2^1 are compatible with P2's strategy from there, because P2 is always going to play to the right from all his information sets, but the beliefs $(\frac{1}{2}, \frac{1}{2})$ are obtained from P1's strategy. However $\ell = 0$ is not optimal from I_2^2 and therefore Case 1 is not a PBE.

This leaves Case 2 with $(x, y, z, w, k, \ell) = (0, 1, 1, 1, 1, 1)$ as the only PBE.

P2 plays first.

We now use the following notation. P2 plays from the singleton, his pure strategies, left and right, with probabilities $1-x_2$ and x_2 respectively and from I_2^1 with $1-y_2$ and y_2. P1 plays his pure strategies as follows. From I_1^1 he plays with probabilities $1-z_1$ and z_1, from I_1^2 with probabilities $1-w_1$ and w_1, from the singleton at the end of ca_2 with probabilities $1-k_1$ and k_1 and from the one at the end of cA_2 with probabilities $1-\ell_1$ and ℓ_1.

Routine calculations imply now

$E_1 = -1.88(2-x_2-y_2)z_1+(0.43y_2-1.07x_2)w_1+0.93y_2\ell_1+$terms independent of $z_1,\ w_1,\ k_1,\ \ell_1$.

$E_2 = (0.44+1.69z_1-1.89w_1)x_2+(2.02+0.68z_1-0.60w_1-0.60\ell_1)y_2$ + terms independent of x_2, y_2.

Of course k_1 is arbitrary throughout because it is not present in E_1. Immediately from E_2 we get $y_2 = 1$ and inserting this into E_1 we obtain $\ell_1 = 1$.

The reaction function for P1 then depends only on x_2. We have

Case 1 If $x_2 < 0.402$ then $z_1 = 0,\ w_1 = 1$

Case 2 If $x_2 = 0.402$ then $z_1 = 0,\ w_1 =$arb

Case 3 If $0.402 < x_2 < 1$ then $z_1 = w_1 = 0$

Case 4 If $x_2 = 1$ then $z_1 =$arb, $w_1 = 0$.

Substituting each of the above in turn into E_2 we obtain that

Case 1 implies $x_2 = 0 < 0.402$ which gives $(z_1, w_1, \ell_1, x_2, y_2) = (0, 1, 1, 0, 1)$

Case 2 implies $x_2 = 0$ *or* 1 which gives a contradiction unless $w_1 = 0.233$ which implies $(z_1, w_1, \ell_1, x_2, y_2) = (0, 0.233, 1, 0.402, 1)$

Case 3 implies $x_2 = 1$ which gives a contradiction. It drops out, exactly in the same way as when P1 plays first.

Case 4 implies $x_2 = 1$ which gives $(z_1, w_1, \ell_1, x_2, y_2) =$(arb,0,1,1,1).
The above Cases 1, 2 and 4 describe the Nash equilibria.

Next we consider the PBEs. Case 1 above is already a PBE since all non-singular information sets are visited with non-zero probabilities, so no beliefs are arbitrary. For I_2^1 we always have $\left(\frac{1}{2}, \frac{1}{2}\right)$, for I_1^1 we have (1, 0) and for I_1^2 the beliefs are (0, 1). Starting from these with the given beliefs the strategies are optimal.

In Case 4 the beliefs are $\left(\frac{1}{2}, \frac{1}{2}\right)$ for I_2^1, for I_1^1 they are arbitrary (1-p, p) and for I_1^2 we have $\left(\frac{1}{2}, \frac{1}{2}\right)$. Starting from I_1^2 the criterion to be optimal with the given beliefs is $-0.32w_1$ which implies $w_1 = 0$. So the strategy is optimal from I_1^2. ¿From I_1^1 only $z_1 = 0$ is optimal giving $(z_1, w_1, \ell_1, x_2, y_2) = (0, 0, 1, 1, 1)$. Thus for a PBE the arbitrariness of z_1 seen above is removed.

Case 2 gives another possibility in which the beliefs are $\left(\frac{1}{2}, \frac{1}{2}\right)$ for I_2^1, for I_1^1 they are (1, 0) and for I_1^2 we have (0.287, 0.713). This is consistent with the non-extreme value for w_1 because its coefficient, starting from I_1^2, is zero.

It is interesting that in Case 4 if we look at the strategies of the two players even though

P1 is playing 2nd he will tell the truth whatever he hears P2 say; but P2, although he plays first, lies when he sees state a and that is essentially the same thing as what happens when P1 plays first. This confirms that these strategies are feasible no matter who plays first and we obtain the same payoffs. On the other hand, Case 1 and the one with completely stochastic behavioral strategies are new.

Cases 1, 2 and 4 do not implement REE. We can see this by calculating normalized expectations. Also conceptually REE assumes that people tell the truth while each of the above cases requires a player to lie.

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