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# **Intertemporal Budget Policies and Macroeconomic Adjustment in a Small Open Economy**

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

## **Abstract**

This paper analyzes the role of nominal assets in ranking intertemporal budget policies in a growing open economy. The budget policies are ranked in terms of the public's intertemporal stock of tax liabilities. Our main result is that, in a small open economy, the valuation of private and public assets is in terms of the exogenous foreign price level under purchasing power parity. This constraint limits the scope of government to influence the real value of assets using fiscal and monetary policy shocks.

## **Keywords**

Government budget, taxation, nominal assets, current account

## **JEL Classifications**

E5, E6, F4

**Comments**

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## 1. Introduction

An enduring topic of economic policy is the study of the effects of changes in fiscal and monetary instruments on the financial position of the public sector. Indeed, discussions in the political arena often revolve around the question of the response of policy to current fiscal deficits or surpluses. For example, a major focus of the recent presidential campaign in the United States was the question of the allocation of the current budget surplus between tax cuts, new expenditure, and debt reduction. The major political parties had strikingly different tax and spending proposals, with the Republicans offering significantly lower taxes and the Democrats advocating a “paying-down” of the Federal debt within a specified period of time. An oft-cited justification of tax cuts is that they pay—at least partially—for themselves, since they also increase the level of economic activity and, consequently, the tax base. Early discussions of the supply-side impact of tax cuts focused on whether a reduction in the marginal tax rate on labor income would lead to an increase in tax revenues through greater work-effort. The empirical consensus that emerged subsequently was that the response of labor supply to changes in the after-tax real wage, at least in the United States, was too small to generate such Laffer-curve effects.<sup>1</sup>

This issue has been revisited recently as researchers have applied the insights of endogenous growth theory to the relationship between fiscal policy decisions and the dynamic evolution of the government budget.<sup>2</sup> The earlier Laffer argument was static in nature, since it relied on a one-time increase in labor supply and output to lead to a rise in the level of tax revenues and, thus, to a corresponding decrease in the size of the government budget deficit. The newer research, as exemplified by Ireland (1994) and Bruce and Turnovsky (1999), considers the effect of government expenditure and tax policy not

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<sup>1</sup>See Laffer (1979) for an early statement of the potential supply-side effects of tax reductions. More recently, Slemrod (1994) found evidence that a Laffer curve effect holds for high-income earners.

<sup>2</sup>Authors who analyzed the influence of government expenditure and tax policy on the equilibrium rate of growth include Barro (1990), Jones and Manuelli (1990), Rebelo (1991), and Jones, Manuelli, and Rossi (1993).

only on the growth rate of the economy, but also on the growth rate of the tax base, the path of government debt, and the value of *future* tax payments required to maintain the intertemporal solvency of the public sector. As discussed by Bruce and Turnovsky, this effect on the growth of the tax base opens-up the possibility of intertemporal Laffer-curve effects, which they call *dynamic scoring*. In other words, fiscal policies can be “scored” in terms of their impact on long-run value of future tax payments. For instance, if the growth in the tax base is sufficiently great subsequent to a cut in taxes, it is then possible that a cut in current taxes *lowers* the present discounted value of future tax liabilities. This can be the case even if the cut in taxes results in a decline in *current* tax revenues. In their closed economy model Bruce and Turnovsky derive the conditions under which dynamic scoring takes place subsequent to a cut in income taxes and show that it can occur if the intertemporal elasticity of substitution exceeds unity.<sup>3</sup>

Bianconi (1999) extends the work of Bruce and Turnovsky (1999) by introducing nominal assets—and hence an inflation tax—into his analysis. He finds that the existence of nominal assets introduces another channel through which changes in fiscal policy can affect the long-term tax liability of the private sector. Through the mechanisms of greater inflation tax revenue and price level effects that lower the burden of the public sector real debt, he shows that changes in both government expenditure and tax policy can reduce the long-run tax liability. Bianconi supports these analytical results with numerical simulations that suggest the role of nominal assets in determining future tax liabilities may be of empirical relevance.

In this paper we will extend this analysis to a small open economy that includes nominal assets. We think this is an useful extension in light of the increasing integration of the world economy and because rules enforcing public sector financial stability are becoming a more important part of multi-lateral economic agreements, such as the Maastricht criterion for European monetary integration. We will develop a single-good, small open economy

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<sup>3</sup>Bruce and Turnovsky (1999) also derive the conditions for the implementation of welfare-maximizing fiscal policy. This will not be our concern here, although our model can be employed to address this issue. See also Agell and Persson (2000).



model in which physical capital accumulation, as in Turnovsky (1996), is the engine of economic growth. In addition to spending real resources, the government in our model levies lump-sum and income taxes and issues internationally traded bonds and domestic money balances.

After deriving for the economy's equilibrium growth path, we will consider the effects of the following policy shocks on the value of future tax liabilities: an increase in the share of government expenditure in output; a cut in the capital tax rate, holding the share of government expenditure constant; a balanced-budget cut in the capital tax rate in which share of government expenditure in output falls with the tax rate; and a change in the rate of growth of nominal balances. We examine these questions by analyzing the solution to the theoretical model and by conducting a numerical simulation exercise. In the case in which the proportion of lump-sum taxes in output is held constant, we show for an increase in the share of government expenditure that dynamic scoring—in contrast to Bianconi (1999)—cannot take place. Indeed, the existence of nominal assets in the small open economy case tends to magnify the increase in the private sector's future tax liabilities subsequent to this government expenditure shock. We show that this is due to the fact that small open economy is constrained to deflate its public sector debt by the exogenous foreign price level, which implies that the value of government assets cannot be eroded through a higher domestic price level. In this context, we also derive the conditions in which dynamic scoring can take place subsequent to a reduction in capital taxes, both holding the share of government expenditure constant and in the balanced budget case. We show that a crucial factor in determining the response of future tax liabilities is the share of government expenditure in output compared to the (initial) rate of capital taxation. As in the case of the government expenditure shock, the response of inflation tax revenues is important in scaling the change in the future tax liability. If the response of inflation tax revenues is sufficiently strong, it can even qualitatively determine the change in the future tax liability. We show in our simulation exercise that while dynamic scoring does not occur subsequent to a cut in capital taxes, given our choice of parameters, it does take place in the case of a balanced-budget tax cut. In addition,

we examine the impact of increasing the rate of growth of nominal balances. This policy shock does reduce, through greater inflation tax revenues, the future tax liabilities of the public, although less than in the closed economy due the lack of price level effects just described.

The paper is organized as follows. Section 2 describes the private sector and derives its optimal intertemporal choices. The asset constraints of the public sector and the current account balance are also introduced in this section. In section 3 we calculate the growth equilibrium of the open economy and derive the expressions for the equilibrium current account balance, real money balances, aggregate stock of wealth, and consumer welfare. We next determine in section 4 the effect of fiscal and monetary policy variables on the economy's equilibrium growth rate, the initial levels of consumption and real money balances, and overall welfare. In section 5, which contains the major results of the paper, we calculate the public sector's intertemporal budget constraint and consider how changes in the government expenditure, tax, and monetary policy affect the private sector's the intertemporal tax burden, paying particular attention to the role of nominal assets. We then simulate these results numerically in section 6. Section 7 briefly concludes.

## 2. The Model

In this section we will describe the components of the small open economy model. The economy produces, consumes, and trades a single good with a fixed terms of trade equal to unity. As such, purchasing power parity holds. This implies—in percentage terms—the relationship  $p = p^* + e$ , where  $p$  is the domestic rate of inflation,  $p^*$  is the exogenous foreign rate of inflation, and  $e$  is the rate of depreciation of the domestic in terms of the foreign currency.<sup>4</sup> The economy is also “small” in terms of international financial markets, since it takes as given the world nominal interest rate, which is linked to the domestic nominal rate according to uncovered interest parity,  $i = i^* + e$ , where  $i$  is the domestic and  $i^*$  is the

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<sup>4</sup>In levels this corresponds to  $EP^*/P = 1$  where  $E$  is the level of the nominal exchange rate and  $P$ ,  $P^*$  are, respectively, the domestic and foreign price levels.

world nominal interest rate. Let us next specify the optimization problem of the private sector.

## 2.1. The Private Sector

We model the private sector as a representative consumer-producer, who solves the following maximization problem

$$Z = \max \int_0^{\infty} U(c, m) e^{-\delta t} dt \quad (1)$$

subject to

$$\dot{m} + \dot{b} + \Phi(I, K) = (1 - \tau)\alpha K + (i^* - p^*)b - c - (p^* + e)m - T, \quad (2a)$$

$$\dot{K} = I \quad (2b)$$

and the following exogenous initial stocks of assets

$$K(0) = K_0 > 0, \quad M(0) = M_0 > 0, \quad b(0) = b_0 = B_0/P_0^* > 0 \quad (2c)$$

where  $c$  = consumption,  $m = M/P$  = the real stock of domestic money balances,  $b = B/P^*$  = the real stock of international bonds,  $P$  = the domestic price level,  $P^*$  = the exogenous foreign price level,  $M$  = the nominal money supply in terms of domestic currency,  $B$  = the nominal stock of international bonds in terms of foreign currency,  $K$  = the stock of domestic physical capital,  $I$  = the level of investment expenditure,  $\delta$  = the exogenous consumer rate of time preference,  $\tau$  = the tax rate on physical capital, and  $T$  = the level of lump-sum taxes imposed by the domestic government, (the flow budget constraint for the domestic government will be introduced below). According to equation (1), the agent receives utility services, as in Sidrausky (1967), from real consumption and real money balances. Observe in equation (2a) that the representative agent accumulates domestic money, foreign bonds, and gross-of-cost additions to the physical capital

stock, the latter represented by the function  $\Phi(K, I)$ , which we will specify below. This is the residual after consumption expenditure, the inflation tax on real money balances, and lump-sum taxes have been subtracted from after-tax output and real interest income from foreign bonds, where  $(i^* - p^*)$  is world real interest rate. The agent uses a linear production function  $Y = \alpha K$ ,  $\alpha > 0$ , where  $Y$  represents domestic output. As in Rebelo (1991), Bruce and Turnovsky (1999) and Bianconi (1999), the level of employment is exogenous. This permits us to concentrate on the intertemporal growth effects of government policy, rather on the static effects, which depend largely on changes in the level of work effort. The flow constraint for the accumulation of domestic capital is given by equation (2b) and abstracts from physical depreciation.

To simplify to the solution of the agent's optimization problem, we follow Bianconi (1999) and specify that  $U(c, m)$  and  $\Phi(K, I)$  assume the following functional forms:

$$U(c, m) = \log c + \gamma \log m, \quad \gamma > 0, \quad (3a)$$

$$\Phi(K, I) = I + (h/2)(I^2/K) = I[1 + (h/2)(I/K)], \quad h > 0. \quad (3b)$$

Equation (3a) implies that both consumption and real balances have an intertemporal elasticity equal to unity and that the parameter  $\gamma$  weighs the utility services of money. Note that our specification means that the domestic economy does not accumulate foreign money balances and, further, that domestic money does not “leak-out” abroad.<sup>5</sup> Equation (3b) is a standard quadratic representation, following Hayashi (1982), of the convex installation costs of physical capital, where the parameter  $h$  measures the “slope” of the marginal cost of investing an additional unit of output. This formulation prevents the physical capital stock of the small open economy from taking discrete jumps.<sup>6</sup>

To obtain the solution to the representative agent's maximization problem, we form

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<sup>5</sup>In other words, the issue of currency substitution does not arise in our model, although it could be easily extended to do so.

<sup>6</sup>See Turnovsky (1997), Chapters 3 and 5, for an extensive discussion of the use of functions such as (3b) in an open economy context.

the following current value Hamiltonian

$$H = \log c + \gamma \log m + \lambda \{ (1 - \tau) \alpha K + (i^* - p^*) b - I [1 + (h/2)(I/K)], \\ -c - (p + e)m - T \} + q' I \quad (4)$$

where  $\lambda$  is the costate variable associated with the constraint (2a) and represents the shadow value of financial wealth, while  $q' \equiv q\lambda$  is the shadow value, in terms of financial wealth, of the domestic capital stock. The necessary conditions for consumption, real balances, investment, foreign bonds, and domestic capital are given, respectively, by the following expressions

$$\frac{1}{c} = \lambda, \quad (5a)$$

$$\dot{\lambda} = (\delta + p^* + e) \lambda - \frac{1}{\gamma m}, \quad (5b)$$

$$1 + h \frac{I}{K} = q, \quad (5c)$$

$$\dot{\lambda} = [\delta - (i^* - p^*)] \lambda, \quad (5d)$$

$$\frac{(1 - \tau) \alpha}{q} + \frac{\dot{q}}{q} + \frac{(q - 1)^2}{2hq} = (i^* - p^*) \quad (5e)$$

and include the following transversality conditions for foreign bonds, real money balances and physical capital:

$$\lim_{t \rightarrow \infty} \lambda b e^{-\delta t} = \lim_{t \rightarrow \infty} \lambda m e^{-\delta t} = \lim_{t \rightarrow \infty} q \lambda K e^{-\delta t} = 0. \quad (5f)$$

Equation (5a) states that the marginal utility of consumption equals the shadow value of wealth,  $\lambda$ , while (5c) equates the marginal cost of investment to its shadow value,  $q$ . Equations (5b, d) are the marginal conditions for money balances and bonds in terms of the rate of return of consumption. This can be seen by rewriting these two conditions as

$$\delta - \frac{\dot{\lambda}}{\lambda} = \frac{\gamma}{\lambda m} - (p^* + e) = (i^* - p^*) \quad (6a)$$

where the left-hand-side represents the rate of return of consumption and the next two equalities represent the real rates of return, respectively, of real money balances and bonds. Noting equation (5e), these are equal to the after-tax rate of return of physical capital. Observe that equation (5c) can be rewritten as the rate of accumulation of physical capital

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{q-1}{h} \equiv \phi \quad (6b)$$

where  $\phi$  denotes the growth rate of the capital stock (and output). This implies the path of the capital stock, starting from the initial value  $K_0$ , equals:

$$K(t) = K_0 e^{\int_0^t \phi(s) ds}. \quad (6c)$$

## 2.2. The Public Sector and the Current Account Balance

We now introduce a public sector that issues internationally traded bonds and domestic money balances to cover the flow difference between real expenditures, interest service, and aggregate tax revenues, where the latter consists of lump-sum taxes, the revenues from the capital income tax and real money balances.<sup>7</sup> In this framework government expenditure does not add to the agent's utility or to the productivity of output. As such, it merely withdraws resources from the private sector. This implies the following public sector flow budget constraint

$$\dot{a} + \dot{m} = G + (i^* - p^*)a - T - \tau\alpha K - (p^* + e)m \quad (7a)$$

where  $G$  is real government expenditure and  $a$  is the real stock of internationally traded domestic government bonds, where  $(i^* - p^*)a$  represents real interest service.<sup>8</sup> We also assume that government bonds evolve from a given initial value such that  $a(0) = a_0 = A_0/P_0^* > 0$ ,

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<sup>7</sup>In order to enhance the transparency of our subsequent results, we restrict ourselves to just two distortionary taxes, the income tax and the inflation tax. It is, nevertheless, straightforward to incorporate into our model a consumption tax and a tax on interest income.

<sup>8</sup>Domestic government bonds are then perfect substitutes for internally traded assets.

where  $A$  = the nominal stock of government bonds in terms of foreign currency, and that the evolution of government debt is subject to the following transversality condition:

$$\lim_{t \rightarrow \infty} \lambda a e^{-\delta t} = 0. \quad (7b)$$

Regarding monetary policy, we assume that the public sector sets a constant growth rate of nominal balances. This implies that the accumulation of real money balances is equal to

$$\dot{m} = (\sigma - p)m = (\sigma - p^* - e)m \quad (7c)$$

where  $\sigma = \dot{M}/M$  is the growth rate of the nominal money supply. Following Bianconi (1999), we will specify that both government expenditure and lump-sum taxes are set proportional to output. In the case of government expenditure, this relationship corresponds to  $G(t) = g^* \alpha K$ , where  $g^*$  is a constant policy parameter, while in the case of lump-sum taxes, the proportion  $T^*(t)$  varies with time according to the relationship  $T^*(t) = T(t)/\alpha K$ .<sup>9</sup>

To derive the flow equation for the current account balance, we next substitute the public sector constraint (7a) into private sector constraint (2a). This yields:

$$\dot{b} - \dot{a} = (1 - g^*)\alpha K - c - I [1 + (h/2)(I/K)] + (i^* - p^*)(b - a). \quad (8a)$$

Letting  $n \equiv b - a$  denote the real net credit position of the small open economy, we rewrite (8a) as:

$$\dot{n} = (1 - g^*)\alpha K - c - I [1 + (h/2)(I/K)] + (i^* - p^*)n. \quad (8b)$$

This equation corresponds to the current account balance, which equals output net of government expenditure, plus net interest income, less private expenditures on consump-

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<sup>9</sup>If government expenditure generates utility services, then a policy that fixes the level of government expenditure in proportion to the capital stock is infeasible in the small open economy framework, (see Turnovsky 1996, 1997). Government expenditure must then be set in proportion to either the level of consumption expenditure or to the level of wealth.

tion and capital formation. For expositional purposes, we will assume that the economy inherits a positive stock of initial credit, i.e.,  $n(0) = n_0 = b_0 - a_0 > 0$ . Finally, the open economy is subject to the following intertemporal solvency condition:

$$\lim_{t \rightarrow \infty} \lambda n e^{-\delta t} = 0. \quad (8c)$$

### 3. The Open Economy Growth Equilibrium

In this section we will derive the growth equilibrium of the open economy and the solutions for the current account balance and overall level of welfare. First, to solve for the growth rate of consumption, we take the time differential of (5a) and combine the resulting expression with equation (6a) to obtain

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda} = (i^* - p^*) - \delta = \psi \quad (9a)$$

where  $\psi$  denotes the constant growth rate of consumption. According to this standard Euler relationship, the growth rate of consumption equals the difference between the world real interest rate and the domestic rate of time preference. Since the growth rate of consumption is a function only of exogenous variables, it does not have transitional dynamics. Solving (9a) yields the following expressions for the paths of consumption and its shadow value

$$c(t) = c(0)e^{\psi t}, \quad \lambda(t) = \lambda(0)e^{-\psi t} \quad (9b)$$

where the initial values  $c(0)$  and  $\lambda(0)$  will be determined below.

To determine the equilibrium growth rate of the capital stock, we must determine the behavior of the shadow price of physical capital,  $q$ . Rewriting (5e), the arbitrage condition for capital, we obtain the following differential equation for  $q$ :

$$\dot{q} = (i^* - p^*)q - \alpha(1 - \tau) - \frac{(q - 1)^2}{2h}. \quad (10)$$



In order to obtain an equilibrium with a constant growth rate of physical capital, the expression for  $\dot{q} = 0$  must have at least one real root. This requires that the steady state solution satisfy the following quadratic equation,

$$\alpha(1 - \tau) + \frac{(q - 1)^2}{2h} = (i^* - p^*)q, \quad (11)$$

where the necessary condition for equation (11) to possess real roots is given by:

$$\alpha(1 - \tau) \square (i^* - p^*) \left[ 1 + \frac{h(i^* - p^*)}{2} \right]. \quad (12)$$

Using equation (11), we can solve for the two roots of  $\dot{q} = 0$ . By constructing a phase diagram and using the solutions for physical capital (6c), the shadow value of wealth (9b), and the transversality condition (5f), we can show that the steady state solution of  $\dot{q} = 0$  is equal to the smaller and unstable root, which implies that physical capital and its shadow value do not display transitional dynamics.<sup>10</sup> The equilibrium growth rate of capital is then equal to

$$\phi = \frac{q - 1}{h} \quad (13)$$

where  $q$  is stated in footnote 10.

The next step in our analysis is to calculate the equilibrium path of the real stock of international credit. To do so, we substitute, using equation (6b), the expressions for investment and physical capital, the path of consumption (9b), and  $i^* - p^* = \psi + \delta$  into the current account balance (8b). This yields

$$\dot{n} = (\psi + \delta)n + \zeta K_0 e^{\phi t} - c(0)e^{\psi t}, \quad (14a)$$

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<sup>10</sup>The expression for the equilibrium value of  $q$  is given by  $q = [1 + h(i^* - p^*)] - \sqrt{\Delta}$ , where  $\Delta = 2h[(i^* - p^*) - \alpha(1 - \tau)] + h^2(i^* - p^*)^2$ . In addition, the cases  $q > 1$ ,  $q < 1$  can be identified. The first obtains if  $(i^* - p^*) < \alpha(1 - \tau)$ , while the second occurs if  $(i^* - p^*) > \alpha(1 - \tau)$ . We will assume the former holds so that equilibrium output growth is positive. See Turnovsky (1996, 1997) for a detailed explanation of this type of analysis.

where

$$\zeta = (1 - g^*)\alpha - \frac{(q^2 - 1)}{2h} = q(\psi + \delta - \phi) - (g^* - \tau)\alpha. \quad (14b)$$

and where the second equality in (14b) is calculated using steady state arbitrage condition (11). Following Turnovsky (1996), we can identify the quantity  $\zeta/(\psi + \delta - \phi)$  as the fiscal policy-adjusted price of capital, since it depends on both the capital tax rate and the proportion of government spending in output. Integrating (14a), we can obtain the following expression for the path of net foreign assets:

$$n(t) = \left( n_0 + \frac{\zeta K_0}{\psi + \delta - \phi} - \frac{c(0)}{\delta} \right) e^{(\psi + \delta)t} - \frac{\zeta K_0}{\psi + \delta - \phi} e^{\phi t} + \frac{c(0)}{\delta} e^{\psi t}. \quad (15)$$

Applying the intertemporal solvency condition (8c) for the small open economy and substituting for  $\lambda(t) = \lambda(0)e^{-\psi t}$ , we can show that the following relationships must hold:

$$\psi + \delta - \phi > 0, \quad (16a)$$

$$c(0) = \lambda^{-1}(0) = \delta \left( n_0 + \frac{\zeta K_0}{\psi + \delta - \phi} \right). \quad (16b)$$

Using the solution to  $q$ , (see footnote 10), we can confirm (16a). Equation (16b), in turn, pins-down the initial level of consumption. Substituting the expression for  $c(0)$  into (15), the equilibrium path of net credit then becomes:

$$n(t) = \left( n_0 + \frac{\zeta K_0}{\psi + \delta - \phi} \right) e^{\psi t} - \frac{\zeta K_0}{\psi + \delta - \phi} e^{\phi t}. \quad (17)$$

Observe that the path of net credit—unlike that of consumption and physical capital—displays transitional dynamics, since it is a function of both  $\psi$  and  $\phi$ . Nevertheless, its growth rate of  $\dot{n}/n$  will converge in the asymptotic limit to  $\max(\psi, \phi)$ .<sup>11</sup>

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<sup>11</sup>Whether  $\psi$  is greater or less than  $\phi$  depends on  $\text{sgn}(\psi - \phi)$ . We can show that the difference  $(\psi - \phi)$  is equal to

$$h^{-1}\sqrt{\Delta} - \delta$$

where  $\Delta$  is defined in footnote 10. Thus, for a “patient” consumer with a relatively “small”  $\delta$ ,  $(\psi - \phi) > 0$ , and the small open economy accumulates net assets. The opposite holds if  $(\psi - \phi) < 0$ , which implies that the economy accumulates international debts.

We next derive the equilibrium path of real money balances,  $m(t)$ . To calculate this expression, we combine the equation for the accumulation of real balances (7c) with the arbitrage condition (5b) and the first-order condition for consumption. After substituting for  $c(t) = c(0)e^{\psi t}$  and using the fact that  $\dot{\lambda}/\lambda = -\psi$ , we derive the following equation describing the evolution of real money balances:

$$\dot{m} = (\psi + \delta + \sigma) m - \gamma c(0)e^{\psi t}. \quad (18a)$$

Integrating this expression, we obtain the path of real money balances  $m(t)$ , starting from  $m(0)$ :

$$m(t) = e^{(\psi + \delta + \sigma)t} \left( m(0) - \frac{\gamma c(0)}{\sigma + \delta} (1 - e^{-(\sigma + \delta)t}) \right). \quad (18b)$$

The value of  $m(0)$  is then determined by imposing the transversality condition (5f) on equation (18b). It implies

$$m(0) = \frac{\gamma c(0)}{\sigma + \delta} \quad (18c)$$

and yields the following solution for real money balances:

$$m(t) = m(0)e^{\psi t} = \frac{\gamma \delta}{\sigma + \delta} \left( n_0 + \frac{\zeta K_0}{\psi + \delta - \phi} \right) e^{\psi t}. \quad (18d)$$

Equations (18c, d) imply that the initial level of real balances is proportional to the initial level of consumption and that both grow at the common rate of  $\psi$ . Since  $m(0) = M_0/P(0)$ , the expression for initial real balances also determines the initial domestic price level and nominal exchange rate. The initial domestic price level and nominal exchange rate are equal to, respectively,  $P(0) = M_0/m(0) = (\sigma + \delta) M_0/\gamma c(0)$ ,  $E(0) = P(0)/P_0^*$ . Thus, any variation in the domestic price level leads to an identical variation in the nominal exchange rate. This, in turn, implies that the real values of nominal assets are insulated from variations in the price level and nominal exchange rate due to the fixed real exchange rate.

Using equation (7c), the equilibrium path of real balances also determines the equi-

librium rate of depreciation  $e$ , since:

$$\frac{\dot{m}}{m} = \psi = \sigma - p^* - e. \quad (19a)$$

This relationship also fixes the equilibrium rate of domestic inflation, which is equal to:

$$p = p^* + e = \sigma - \psi. \quad (19b)$$

In other words, the domestic rate of inflation equals the difference between the rates of growth of nominal money balances and consumption.

We close this section by deriving the expression for economy's level of welfare, which we identify with equation (1), the present discounted utility of the representative consumer-producer. Employing our logarithmic parameterization and substituting equations (9b), (16b), and (18d) into (1), we obtain the following expression for  $Z$ :

$$Z = (1 + \gamma)\delta^{-1} \log \delta \left[ n_0 + qK_0 - \frac{(g^* - \tau)\alpha K_0}{\psi + \delta - \phi} \right] + \gamma\delta^{-1} \log \frac{\gamma}{\sigma + \delta} + (1 + \gamma)\delta^{-2}\psi. \quad (20)$$

This expression reveals that consumer welfare depends on the government's fiscal and monetary policy variables,  $g^*$ ,  $\tau$ ,  $\sigma$ ; the two equilibrium growth rates,  $\psi$ ,  $\phi$ ; the inherited stocks of net credit and physical capital,  $n_0$ ,  $K_0$ ; and such "fundamental" parameters as the rate of time preference, the utility weight on real money balances, and the marginal physical product of capital,  $\delta$ ,  $\gamma$ ,  $\alpha$ . In the section 6 of the paper we will calculate the effect of changes in the fiscal and monetary policy instruments on consumer welfare.

#### 4. The Effects of Policy on the Growth Equilibrium

In this section of the paper we will analyze the impact of government policy on the small open economy equilibrium. Considering first the impact of a change in the proportion of output devoted to government spending,  $g^*$ , we calculate the following comparative static

expressions, using equations (6b), (9a), (13a), (16b), and (18c):

$$\frac{\partial \psi}{\partial g^*} = \frac{\partial q}{\partial g^*} = h \frac{\partial \phi}{\partial g^*} = 0, \quad (21a)$$

$$\frac{\partial c(0)}{\partial g^*} = \frac{-\delta \alpha K_0}{\psi + \delta - \phi} < 0, \quad \frac{\partial m(0)}{\partial g^*} = \frac{\gamma}{\sigma + \delta} \frac{\partial c(0)}{\partial g^*} < 0. \quad (21b)$$

Examining these expressions, we can conclude that while an *increase* in  $g^*$  leaves the two open economy growth rates unchanged, it lowers, through the resource-withdrawal effect, the initial levels of consumption and real money balances.<sup>12</sup>

We next consider the effects of a change in the capital or income tax rate,  $\tau$ . The comparative static expressions are given by:

$$\frac{\partial \psi}{\partial \tau} = 0, \quad \frac{\partial q}{\partial \tau} = h \frac{\partial \phi}{\partial \tau} = \frac{-\alpha}{\psi + \delta - \phi} < 0, \quad (22a)$$

$$\frac{\partial c(0)}{\partial \tau} = \frac{\delta(g^* - \tau)h^{-1}\alpha^2 K_0}{(\psi + \delta - \phi)^3}, \quad \frac{\partial m(0)}{\partial \tau} = \frac{\gamma}{\sigma + \delta} \frac{\partial c(0)}{\partial \tau}. \quad (22b)$$

From equations (22) it is clear that while a *cut* in the tax rate  $\tau$  does not affect  $\psi$  and, thus, does not influence the growth rate of consumption, real money balances, it does increase the shadow value of domestic capital and, consequently, raise the growth rate of output.<sup>13</sup> In addition, whether a decrease in the capital tax raises or lowers consumption and real money balances depends on  $\text{sgn}(g^* - \tau)$ . If  $(g^* - \tau) > 0$ , then a cut in  $\tau$  lowers initial consumption and real money demand, while the opposite is the case if  $(g^* - \tau) < 0$ . Because government spending is tied to output, a tax cut that raises  $\phi$  will also increase the growth rate of government spending. If  $(g^* - \tau) > 0$ , then this effect crowds-out consumption through the resource-withdrawal effect. The opposite is true if  $(g^* - \tau) < 0$ , since the tax cut in this case results in more resources for consumption.

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<sup>12</sup>Observe that the decline in  $m(0)$  depends positively on the utility weight  $\gamma$  and negatively on the rate of growth of nominal balances  $\sigma$ .

<sup>13</sup>To derive the expression for  $\partial q/\partial \tau$  in (22a), we employed the expression for  $q$  in footnote 10 to calculate  $\partial q/\partial \tau = -h\alpha\Delta^{-1/2}$ . We then used the fact that  $\psi + \delta - \phi = h^{-1}\Delta^{1/2}$  (see footnote 11) to obtain  $\partial q/\partial \tau$ .

Turning the question of the impact of monetary policy on the small open economy, we can calculate that an increase  $\sigma$  in leads to the following equilibrium effects:

$$\frac{\partial \psi}{\partial \sigma} = \frac{\partial q}{\partial \sigma} = \frac{\partial \phi}{\partial \sigma} = \frac{\partial c(0)}{\partial \sigma} = 0, \quad \frac{\partial m(0)}{\partial \sigma} = \frac{-\gamma c(0)}{(\sigma + \delta)^2} < 0, \quad (23a)$$

$$\frac{\partial e}{\partial \sigma} = \frac{\partial p}{\partial \sigma} = 1. \quad (23b)$$

Consistent with the classical dichotomy, an *increase* in the growth rate of nominal balances lowers the demand for real money balances, but does not affect the equilibrium real growth rates  $\psi$  and  $\phi$  and the initial level of consumption. Given with the economy's interest rate and purchasing power parity relationships, equation (23b) shows that a rise in  $\sigma$  leads to a one-for-one increase in the rates of depreciation and domestic inflation.

We complete this section by considering the impact of the these policy changes on overall welfare,  $Z$ . Using equation (20), we can show that

$$\frac{\partial Z}{\partial g^*} = \frac{1 + \gamma}{\delta c(0)} \frac{\partial c(0)}{\partial g^*} < 0, \quad \frac{\partial Z}{\partial \tau} = \frac{1 + \gamma}{\delta c(0)} \frac{\partial c(0)}{\partial \tau}, \quad \frac{\partial Z}{\partial \sigma} = -\frac{\gamma}{\delta(\sigma + \delta)} < 0, \quad (24)$$

where the expressions for  $\partial c(0)/\partial g^*$  and  $\partial c(0)/\partial \tau$  are given, respectively, by equations (21b) and (22b). Whether  $Z$  rises or falls in response to changes in  $g^*$  and  $\tau$ , depends on whether initial consumption rises or falls. Thus, an *increase* in  $g^*$  lowers overall welfare, since it also lowers initial consumption. In contrast, a *cut* in  $\tau$  raises overall welfare if it increases initial consumption, which is the case if  $(g^* - \tau) < 0$ . Finally, since an *increase* in the rate growth of nominal balances lowers real money demand, a rise in  $\sigma$  directly lowers  $Z$ . Observe that the *size* of the response of  $Z$  for all three policy shifts is scaled by parameter  $\gamma$ , which reflects the role of real money balances in generating utility and overall welfare. In section 6 below we will simulate numerically the impact of these policies on  $Z$ .

## 5. Intertemporal Government Budget Constraint

Preparatory to analyzing the effects of alternative fiscal and monetary policies, we will determine in this section the public sector's intertemporal budget constraint. This is derived by first substituting  $[G(t) - T(t)] = [g^* - T^*(t)]\alpha K(t)$ ,  $K(t) = K_0 e^{\phi t}$ ,  $\dot{m} = \psi m$  and the equilibrium conditions (9a) and (18d) into the flow government budget constraint (7a). We then obtain:

$$\dot{a} - (\psi + \delta)a = (g^* - (\tau + T^*))\alpha K_0 e^{\phi t} - \frac{\gamma\sigma c(0)}{\sigma + \delta} e^{\psi t} \quad (25a)$$

where  $c(0)$  is given by equation (16b). Assuming that the real stock of government debt evolves from the initial value  $a_0$ , integration of (25a) yields:

$$a(t) = a_0 e^{(\psi + \delta)t} + \frac{(g^* - \tau)\alpha K_0}{\psi + \delta - \phi} (e^{\phi t} - e^{(\psi + \delta)t}) + \frac{\gamma\sigma c(0)}{(\sigma + \delta)\delta} (e^{\psi t} - e^{(\psi + \delta)t}) - \alpha K_0 e^{(\psi + \delta)t} \int_0^t T^*(s) e^{(\phi - (\psi + \delta))s} ds. \quad (25b)$$

We then apply the transversality condition (7b) of the public sector to (25b) to obtain, using  $\lambda(t) = \lambda(0)e^{-\psi t}$ , the following expression for the constraint on the initial stock of public debt:

$$\int_0^\infty T^*(t) e^{(\phi - (\psi + \delta))t} dt = \frac{a_0}{\alpha K_0} + \frac{g^* - \tau}{\psi + \delta - \phi} - \frac{\gamma\sigma c(0)}{\delta(\sigma + \delta)\alpha K_0}. \quad (25c)$$

Observe that this expression determines the path of  $T^*(t)$  required to maintain the intertemporal solvency of the public sector. Substitution of this stock constraint into (25b) then determines the equilibrium path of government debt:

$$a(t) = \alpha K_0 e^{(\psi + \delta)t} \int_t^\infty T^*(s) e^{(\phi - (\psi + \delta))s} ds - \frac{(g^* - \tau)\alpha K_0}{\psi + \delta - \phi} e^{\phi t} + \frac{\gamma\sigma c(0)}{\delta(\sigma + \delta)} e^{\psi t}. \quad (25d)$$

Equation (25d) reveals that the path of government debt, like that of net credit, is a “mixed” process that depends on both open economy growth rates,  $\psi$  and  $\phi$ . Next, using the fact that  $T^*(t) = [T(t)/\alpha K_0] e^{-\phi t}$ , the stock constraint (25c) can be rewritten in terms of the path of lump-sum taxes:

$$V(T) = \int_0^\infty T(t) e^{-(\psi+\delta)t} dt = a_0 + \frac{(g^* - \tau) \alpha K_0}{\psi + \delta - \phi} - \frac{\gamma \sigma c(0)}{\delta(\sigma + \delta)}. \quad (25e)$$

We define  $V(T)$  as the present discounted value of future lump-sum taxes that is required to maintain public sector solvency. Following Bruce and Turnovsky (1999), we interpret  $V(T)$  as a measure of the “sustainability” of any combination of fiscal and monetary policies described by  $\{g^*, \tau, \sigma\}$ . This means a shift in  $\{g^*, \tau, \sigma\}$  must be accompanied by a shift in  $V(T)$  in order to sustain public sector solvency. Observe, in addition, that we can identify the last two terms on the right-hand-side of (25e) with the primary deficit of the public sector. We next consider how changes in fiscal and monetary policy affect the value of  $V(T)$ . For convenience we will assume in subsection 5.1 that the proportion  $T^*$  is fixed. Subsequently, we analyze in 5.2 how our results differ if the government maintains a balanced budget. We finally solve in 5.3 for the fiscal and monetary policies that insure long-run government solvency.

### 5.1. A Constant $T^*$ Policy

We begin by considering a budgetary stance that keeps the fraction  $T^*$  constant. Such a policy implies that lump-sum taxes grow at the same rate as the capital stock and output, so that  $T(t) = T_0 e^{\phi t}$ , where  $T_0$  is the implied initial level of lump-sum taxes. Substituting  $T(t) = T_0 e^{\phi t}$  into (25e), we solve for the initial level of lump-sum taxes. It is given by:

$$T_0 = (\psi + \delta - \phi) \left[ a_0 - \frac{\gamma \sigma c(0)}{\delta(\sigma + \delta)} \right] + (g^* - \tau_k) \alpha K_0. \quad (26a)$$

We can also calculate the path of government debt when the constant  $T^*$  policy is pursued. Substituting  $T(t) = T_0 e^{\phi t}$  and the solution for  $T_0$  into the expression for  $a(t)$  given in (25d),



we can show that  $a(t)$  is equal to:

$$a(t) = a_0 e^{\phi t} + \frac{\gamma \sigma c(0)}{\delta(\sigma + \delta)} (e^{\psi t} - e^{\phi t}). \quad (26b)$$

This implies that the rate of growth of government debt, as in the case of net credit, depends on  $\max(\psi, \phi)$ .

Using our expression for  $V(T)$ , we next calculate the impact of changes in the fraction output absorbed by the government,  $g^*$ , the tax rate on capital income,  $\tau$ , and the growth rate of nominal money balances,  $\sigma$ , on the aggregate tax liability of the private sector in the constant  $T^*$  case.<sup>14</sup> For a shift in  $g^*$  the change in the liability is given by:

$$\frac{\partial V(T)}{\partial g^*} \Big|_{T^*} = \frac{\alpha K_0}{\psi + \delta - \phi} - \frac{\gamma \sigma}{\delta(\sigma + \delta)} \frac{\partial c(0)}{\partial g^*} = \left(1 + \frac{\gamma \sigma}{\sigma + \delta}\right) \frac{\alpha K_0}{\psi + \delta - \phi} > 0. \quad (27)$$

This expression reveals that an *increase* in  $g^*$  unambiguously raises the future tax burden of the private sector. This is due to the direct effect of an increase in  $g^*$  on the primary fiscal deficit and because the rise in  $g^*$  causes, through the resource-withdrawal effect, a decline in consumption and real money demand, which, in turn, lowers the inflation tax base. In this context, observe that the rise in  $V(T)|_{T^*}$  depends positively on the size of  $\gamma$ . Clearly then, the larger are the utility services of money, the more a rise in  $g^*$  increases the future tax burden. The existence of nominal assets serves to magnify the impact of a fiscal expansion on the private sector's future tax liabilities in the small open economy. This is in contrast to the closed economy result of Bianconi (1999) in which a dynamic scoring result is possible, due to a sufficiently large fall in the burden of real public debt. The reason why dynamic scoring does not occur in this small open economy model is because the nominal value government debt is deflated by the exogenous foreign price level and not by its domestic counterpart. Consequently, the increase in the domestic price level that occurs to maintain money market equilibrium does *not* affect the real value of government debt.<sup>15</sup> The level of future private sector tax liabilities is, thus, unaffected through this

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<sup>14</sup>These expressions will hold, however, even if the fraction  $T^*$  is not constant.

<sup>15</sup>Using the definitions of  $P(0)$  and  $E(0)$  given above, the increases in the initial domestic price level

channel. This will also be the case in our subsequent examples and will have a dramatic impact on the influence of changes in  $g^*$ ,  $\tau$ , and  $\sigma$  if a balanced budget policy is pursued.

A marginal change in the tax rate on capital  $\tau$  has the following impact on  $V(T)|_{T^*}$

$$\begin{aligned} \frac{\partial V(T)}{\partial \tau} \Big|_{T^*} &= \frac{-\alpha K_0}{\psi + \delta - \phi} + \frac{(g^* - \tau)\alpha K_0}{(\psi + \delta - \phi)^2} \frac{\partial \phi}{\partial \tau} - \frac{\gamma \sigma}{\delta(\sigma + \delta)} \frac{\partial c(0)}{\partial \tau} \\ &= \frac{-\alpha K_0}{\psi + \delta - \phi} - \left[ 1 + \frac{\gamma \sigma}{\sigma + \delta} \right] \frac{(g^* - \tau) h^{-1} \alpha^2 K_0}{(\psi + \delta - \phi)^3} \end{aligned} \quad (28a)$$

where we have substituted for the expressions for  $[\partial \phi / \partial \tau]$  and  $[\partial c(0) / \partial \tau]$  given in equations (22a, b) to derive the second equality in (28a). Examination of the first equality in equation (28a) shows that the impact of a *decrease* in the capital tax can be broken-down into three parts. The first term in this expression describes the direct positive effect of a cut in  $\tau$  on the primary deficit, which acts to raise the tax liability  $V(T)|_{T^*}$ . The next term in this equality describes the effects on  $V(T)|_{T^*}$  that arise from a higher growth rate  $\phi$ . We observe that it has ambiguous effect on future liabilities, since it depends on *sgn* ( $g^* - \tau$ ). If  $(g^* - \tau) > 0$ , then the tax liability rises, because the accompanying increase in government expenditure—recall that it is tied to the growth rate of physical capital and output—swamps the increase in the tax base due to the higher growth rate  $\phi$ . The opposite is true if  $(g^* - \tau) < 0$ . In this case the increase in the tax base overwhelms the rise in government expenditure and tends to lower  $V(T)|_{T^*}$ . Indeed, if this latter effect is sufficiently strong, then dynamic scoring is possible.<sup>16</sup> The third term in the first equality of (28a) describes the influence of changes in the inflation tax on the tax liability. Its sign depends on whether initial consumption rises or falls subsequent to the cut in  $\tau$ . If  $c(0)$  rises, the case  $(g^* - \tau) < 0$ , then real money demand also increases, which, in turn, increases inflation tax revenue and tends to reduce the tax liability. The opposite holds

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and exchange rate equal:

$$\partial P(0) / \partial g^* = \partial E(0) / \partial g^* = -[P(0) / \gamma c(0)] \cdot \partial c(0) / \partial g^* > 0.$$

As such, purchasing power parity insulates the small open economy terms of trade from the shock to  $g^*$ .

<sup>16</sup>This means that dynamic scoring can take place in our model, unlike in Bruce and Turnovsky (1999), even though the elasticity of substitution is unity.

if  $c(0)$  falls, which is true if  $(g^* - \tau) > 0$ . Here, real money demand declines and, consequently, so does inflation tax revenue. If the former increase in inflation tax revenues is sufficiently strong, then a cut in  $\tau$  can lower the future tax liability of the private sector through this channel as well. These considerations lead to the following proposition.

**Proposition 1: Dynamic Scoring and Reductions in the Capital Tax**

(a) *A sufficient condition for a cut  $\tau$  to increase future tax liabilities is  $(g^* - \tau) > 0$ .*

That is:

$$\frac{\partial V(T)}{\partial \tau} \Big|_{T^*} < 0 \Leftrightarrow (g^* - \tau) > 0. \quad (28b)$$

(b) *In the case  $(g^* - \tau) < 0$ , a sufficient condition for a cut in  $\tau$  to reduce future tax liabilities, i.e., to cause dynamic scoring is:*

$$\frac{\partial V(T)}{\partial \tau} \Big|_{T^*} > 0 \Leftrightarrow \left[ 1 + \frac{\gamma \sigma}{\sigma + \delta} \right] \frac{(g^* - \tau)}{(\psi + \delta - \phi)} \frac{\partial \phi}{\partial \tau} > 1. \quad (28c)$$

The proof of part (a) is obvious from our discussion above, since (28a) is unambiguously negative if  $(g^* - \tau) > 0$ . Part (b) is derived using the second equality of (28a), after substituting for  $[\partial \phi / \partial \tau]$  and finding the condition for  $[\partial V(T) / \partial \tau]_{|T^*} > 0$  if  $(g^* - \tau) < 0$ . In section 6 below, we simulate the model numerically to determine whether condition (28c) is satisfied for a plausible set of parameter values.

We next calculate the impact of a *balanced-budget tax cut* on the value of future tax liabilities. The expression is given by

$$\begin{aligned} \frac{\partial V(T)}{\partial \tau} \Big|_{T^*, d\tau=dg^*} &= \frac{(g^* - \tau) \alpha K_0}{(\psi + \delta - \phi)^2} \frac{\partial \phi}{\partial \tau} \Big|_{d\tau=dg^*} - \frac{\gamma \sigma}{\delta (\sigma + \delta)} \frac{\partial c(0)}{\partial g^*} \Big|_{d\tau=dg^*} \\ &= \frac{\gamma \sigma \alpha K_0}{(\sigma + \delta) (\psi + \delta - \phi)} - \left[ 1 + \frac{\gamma \sigma}{(\sigma + \delta)} \right] \frac{(g^* - \tau) h^{-1} \alpha^2 K_0}{(\psi + \delta - \phi)^3}, \end{aligned} \quad (29a)$$

where

$$\frac{\partial \phi}{\partial \tau} \Big|_{d\tau=dg^*} = \frac{\partial \phi}{\partial \tau}, \quad \frac{\partial c(0)}{\partial \tau} \Big|_{d\tau=dg^*} = -\delta \left[ 1 - \frac{(g^* - \tau) h^{-1} \alpha}{(\psi + \delta - \phi)^2} \right] \frac{\alpha K_0}{(\psi + \delta - \phi)} \quad (29b)$$

and where we have substituted for (29b) to obtain the second equality of (29a). Comparing the equations (29a) to (28a), we observe that since fraction  $g^*$  falls with  $\tau$  in the balanced budget case, the direct positive effect of a tax cut on the primary deficit washes-out. This implies that the balanced budget tax cut influences future tax liabilities only through its effect on the growth rate and inflation tax revenues. Because, however, the growth rate  $\phi$ , according to (21a), is independent of  $g^*$ , this term has the same (ambiguous) impact on future tax liabilities as in the previous case in which  $g^*$  is held constant. On the other hand, a balanced budget tax cut has a distinct impact on consumption and real money demand, since the reduction in  $g^*$  “crowds-in” consumption and real money balances and, hence, acts to increase inflation tax revenue.<sup>17</sup> If the latter effect is sufficiently strong, then dynamic scoring can take place even if  $(g^* - \tau) > 0$ . Given these considerations, we state the next proposition.

**Proposition 2: Dynamic Scoring and Balanced-Budget Reductions in the Capital Tax**

(a) *The case  $(g^* - \tau) > 0$  is not a sufficient condition for a balanced-budget tax cut to increase future tax liabilities. If  $(g^* - \tau) > 0$ , a sufficient condition for dynamic scoring is:*

$$\frac{\partial V(T)}{\partial \tau} \Big|_{T^*, d\tau=dg^*} > 0 \Leftrightarrow \left[ 1 + \frac{\sigma + \delta}{\gamma \delta} \right] \frac{(g^* - \tau)}{(\psi + \delta - \phi)} \frac{\partial \phi}{\partial \tau} > -1. \quad (30a)$$

(b) *A sufficient condition for a balanced-budget tax cut to reduce future tax liabilities is  $(g^* - \tau) < 0$ . That is:*

$$\frac{\partial V(T)}{\partial \tau} \Big|_{T^*, d\tau=dg^*} > 0 \Leftrightarrow (g^* - \tau) < 0. \quad (30b)$$

The proof of part (a) is determined using the second equality of (29a), after substituting

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<sup>17</sup>A sufficient condition for a balanced-budget tax cut to increase  $c(0)$  if  $(g^* - \tau) > 0$  is:

$$\frac{(g^* - \tau)}{(\psi + \delta - \phi)} \frac{\partial \phi}{\partial \tau} > -1.$$

for  $[\partial\phi/\partial\tau]$  and solving for  $[\partial V(T)/\partial\tau]_{|T^*, d\tau=dg^*} > 0$  if  $(g^* - \tau) > 0$ . The proof of part (b) is obvious, since the expression (29a) is unambiguously positive if  $(g^* - \tau) < 0$ .

Turning to monetary policy, a change in  $\sigma$  results in the following adjustment in the private sector tax liability:

$$\frac{\partial V(T)}{\partial\sigma} \Big|_{T^*} = \frac{-\gamma c(0)}{(\sigma + \delta)^2} < 0. \quad (31)$$

This implies that an *increase* in the rate of growth of nominal balances, as in the closed economy model of Bianconi (1999), increases the inflation tax revenues and reduces the tax liability  $V(T)_{|T^*}$ . This, of course, also means that dynamic scoring *cannot* take place after a *cut* in  $\sigma$ . The impact on future tax liabilities in (31) is precisely one-half of that calculated by Bianconi (1999). This reflects, as before, the fact that the rise in the domestic price level does not lower the value of public sector liabilities. Due to the classical dichotomy, there is no dynamic feedback on the “real-side” of the economy and, thus, on capital tax revenues.

## 5.2. A Balanced-Budget Policy

In this subsection we will consider the implications of a budgetary stance that maintains a continuously balanced budget, i.e., a stance in which the primary deficit is always zero. Such a policy choice implies the stock of government debt remains constant at its initial level, i.e.,  $a(t) = a_0, \forall t \geq 0$ . Using our solution (25d) for the path of government debt, this can be attained if fiscal and monetary policy authorities take the following actions:

(i) set one of  $\{g^*, \tau, \sigma\}$  so that

$$\frac{(g^* - \tau) \alpha K_0}{\psi + \delta - \phi} e^{\phi t} - \frac{\gamma \sigma c(0)}{\delta(\sigma + \delta)} e^{\psi t} = 0, \quad (32a)$$

(ii) set

$$T^*(t) = T_0^* e^{-\phi t}, \quad (32b)$$

(iii) set

$$T_0 = (\psi + \delta) a_0. \quad (32c)$$

Note that step (i) eliminates the primary deficit through the choice of one of the policy instruments. To satisfy (32a), the other two instruments are then left to freely adjust. Observe also that due to the exponent terms in (32a), the choice one of  $\{g^*, \tau, \sigma\}$  varies through time. This is another consequence of the small open economy equilibrium, which can maintain different aggregate growth rates. Step (ii) implies that the fraction  $T^*(t)$  declines at the rate  $\phi$ , while step (iii) fixes the initial level of lump-sum taxes  $T_0$  that maintains the intertemporal solvency of the public sector. Taken together, the balanced-budget policies (32a-c) imply that the private sector bears the following future tax liability

$$V(T)|_{BB} = a_0 \quad (33)$$

which simply corresponds to the initial stock of government debt. Since, as we indicated above, the nominal stock of government debt is deflated by the exogenous foreign price level, the tax liability in balanced budget equilibrium is independent of small changes in  $g^*$ ,  $\tau$ , or  $\sigma$ . This is contrast to the closed economy results of Bianconi (1999) and is a further example of the limited scope of action possessed by the fiscal and monetary authorities in the small open economy.

### 5.3. Long-Run Fiscal Solvency

We indicated above that the intertemporal solvency of the public sector is a function of the present discounted value of the tax liability  $V(T)$ . Another criterion for intertemporal solvency—one that can be considered more stringent than the balanced-budget criterion—is for the private sector’s future tax liability to be equal to zero,  $V(T) = 0$ .<sup>18</sup> In terms of

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<sup>18</sup>According to Bruce and Turnovsky (1999),  $V(T) = 0$  is “sustainable” in the sense that no further policy shifts need be taken to maintain public sector intertemporal solvency.

equation (25e) for the constant  $T^*$  case, this criterion implies

$$\begin{aligned}
& a_0 + \frac{(g^* - \tau) \alpha K_0}{\psi + \delta - \phi} - \frac{\gamma \sigma c(0)}{\delta(\sigma + \delta)} \\
& = a_0 + \left(1 + \frac{\gamma \sigma}{\sigma + \delta}\right) \left(\frac{(g^* - \tau) \alpha K_0}{\psi + \delta - \phi}\right) - \frac{\gamma \sigma (n_0 + q K_0)}{\sigma + \delta} = 0
\end{aligned} \tag{34}$$

where we have substituted for  $c(0)$  to derive the second equality of (34). To maintain long-run fiscal solvency, one of the three policy tools  $\{g^*, \tau, \sigma\}$  is chosen to satisfy (34). Solving (34), we obtain the following expressions for  $\{g^*, \tau, \sigma\}$  under this constraint

$$g^* = \tau - (\psi + \delta - \phi) \left(1 + \frac{\gamma \sigma}{\sigma + \delta}\right)^{-1} \cdot \left\{ \frac{a_0}{\alpha K_0} - \frac{\gamma \sigma}{\sigma + \delta} \left(\frac{n_0}{\alpha K_0} + \frac{q}{\alpha}\right) \right\}, \tag{35a}$$

$$\tau = g^* + (\psi + \delta - \phi) \left(1 + \frac{\gamma \sigma}{\sigma + \delta}\right)^{-1} \cdot \left\{ \frac{a_0}{\alpha K_0} - \frac{\gamma \sigma}{\sigma + \delta} \left(\frac{n_0}{\alpha K_0} + \frac{q}{\alpha}\right) \right\}, \tag{35b}$$

$$\sigma = \frac{-\delta [(\psi + \delta - \phi) a_0 + (g^* - \tau) \alpha K_0]}{(\psi + \delta - \phi) [a_0 - \gamma (n_0 + q K_0)] + (1 + \gamma) (g^* - \tau) \alpha K_0} \tag{35c}$$

where the other two policy tools are chosen freely. We will use these expressions in the next part of the paper to simulate the welfare implications of maintaining of the public sector solvency using the policy instruments (35a-c).<sup>19</sup>

## 6. Numerical Simulations

In order to assess the impact of the alternative policies on the liabilities of the private sector that guarantee intertemporal solvency and to examine the welfare effects of the alternative policies, we resort to a simple numerical simulation of the model. The benchmark set of parameter values is given at the bottom of Table 1 and is a plausible one, since it leads to a positive endogenous growth rate and because the tax rate exceeds the fraction of government spending in output,  $(g^* - \tau) < 0$ . In the context of the model, the

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<sup>19</sup>In calculating equations (35a-c), we have treated  $q$  and  $\phi$  parametrically, although they are, of course, functions of  $\tau$ . This discrepancy, nevertheless, will not materially affect our simulation results.

parameters imply an equilibrium endogenous growth rate equal to 2% such that  $\psi = \phi$ . Additionally, the consumption share is about 53% of output, the initial stock of government debt is 50% of output with the net foreign asset position is positive and equal to 5% of output. We further assume that foreign nominal interest rate equals 10%, the foreign inflation is 4%, and, thus, that the foreign real interest rate is 6%. This parameterization implies lump-sum tax credits, or transfers, on the order of 97% of output to guarantee long-run intertemporal solvency.

Table 1 summarizes the effects of the alternative intertemporal budget policies as well as effects of arbitrary marginal cuts in each of the policy instruments,  $\{g^*, \tau, \sigma\}$ . The first column of Table 1 denotes the change in the tax liability  $V(T)$  relative to the benchmark of the constant  $T^*$  policy. The second and third columns illustrate, respectively, the change in welfare,  $Z$ , in the constant  $T^*$  case and change in welfare,  $Z|_{LC}$ , in the case in which the long-run constraint (34) binds. The expressions for the changes in welfare are evaluated using equation (20).

The result of a cut in  $g^*$  is a 58.8% welfare gain in the constant  $T^*$  case. In contrast, welfare falls by 47.6% if, instead, government spending is endogenously increased to achieve long-run fiscal solvency. Long-run solvency is satisfied here by increasing government spending, because the initial equilibrium is one in which private sector receives positive transfers. A cut in government spending decreases the tax liability of the private sector by 130.7% in the constant  $T^*$  policy. A reduction in the capital income tax yields a much smaller welfare gain, about 5.2%, and results in an increase in the lump sum tax liability of 88.2%. Consequently, a policy of simultaneously cutting government spending and the tax rate yields a welfare gain of 64% and a decrease in the tax liability of 42.5%, as we should expect from Proposition 2, since  $(g^* - \tau) < 0$ . Again, the long-run solvency is achieved with a cut in the tax rate, because the initial equilibrium is one in which there are lump-sum tax credits. If  $\tau$  is cut to endogenously balance the budget, there is a welfare gain equal to 13.3%. The deflationary policy—reducing the rate of growth of  $\sigma$ —leads to very small welfare gains and changes in the private sector’s tax liability



when compared to the other two policy instruments in the constant  $T^*$  case. However, a deflationary policy that satisfies the long-run constraint turns the inflation tax into a subsidy, which, as in Bianconi (1999), yields more significant welfare gains.

The key issue is the absence of dynamic scoring in the case of fall in the capital income tax rate, holding  $g^*$  constant. In order for condition (28c) of Proposition 1 to be satisfied, we would have to choose an implausible parameterization of the model, in particular, in terms of the difference in the growth rates  $(\psi - \phi)$ . This suggests that the opportunities for dynamic scoring in the case of the small open economy are limited. Indeed, even the closed economy, Bianconi (1999) showed that a relatively “large” rate of time preference compared to the rate of nominal money growth is required for dynamic scoring—brought about by the inflation tax and price level effects—to occur. But, as we have seen, the price level effect is fully absorbed by movements in the nominal exchange rate in the small open economy case, making this channel less effective and these parameters less important. In the small open economy, the adjustment cost parameter,  $h$ , and the foreign interest and inflation rates,  $i^*$  and  $p^*$ , play key roles in determining the discrepancy between the growth rates  $\psi$  and  $\phi$ . Due, however, to the nonlinearities in the equilibrium, we were unable to find a reasonable combination of parameters that resulted simultaneously in dynamic scoring and a plausible difference in the growth rates. Nevertheless, if the alternative policy of simultaneous cuts in  $g^*$  and  $\tau$  is enacted, a sufficient condition for dynamic scoring is  $(g^* - \tau) < 0$  and we observe in Table 1 that it does take place.

To sum-up, in the small open economy framework, a simultaneous cut in government spending and the capital tax rate provides a welfare gain and reduction in tax liabilities. This is the most attractive of our policy options, since attains both objectives—greater welfare and lower tax liabilities—simultaneously. Cutting government spending alone has a similar effect, but cutting tax rates (both on capital and on money balances) cannot yield both objectives at the same time. The inflation tax effects are quantitatively small because of the denomination of domestic assets in foreign currency.

## 7. Concluding Remarks

In this paper we have analyzed the effects of fiscal and monetary policies on the long-run tax liability of the private sector in a small open economy model with nominal assets. Among our major results, we found that a rise in the fraction output devoted to government expenditure unambiguously increased the future tax liabilities of the private sector, without any possibility “dynamic scoring.” In addition, we analyzed the conditions in which a tax cut results in dynamic scoring, i.e., a reduction in the long-run tax burden. A key factor in the determination of our theoretical findings was the response of the inflation tax base to the shift in fiscal policy. The existence of nominal assets can either magnify the effect of the change in fiscal policy, as in the case of a government expenditure shock, or, as in the case of a tax cut, it can offset the positive impact of the tax cut on the primary deficit and lead to lower intertemporal tax burdens. Our simulation results suggest that while dynamic scoring does not take place if the capital tax alone is reduced, it can occur in the balanced-budget case. The one component of the tax burden that the policy authorities cannot alter, however, is the real value of public sector debt, which is determined by the exogenous foreign price level under purchasing power parity. This factor limits the ability of the government to manipulate intertemporal tax burdens in small open economies.

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TABLE 1

TAX LIABILITIES AND THE WELFARE GAINS/COSTS OF BUDGET POLICIES IN SMALL OPEN ECONOMY

	$V(T)$	$Z$	$Z _{LC}$
<b>A. Constant <math>T^*</math> Policy</b>			
$g _{T^* \text{ constant}}^* < 0, g^* = 0.20$	-130.7	58.8	-
$\tau _{T^* \text{ constant}} < 0, \tau = 0.25$	88.2	5.2	-
$\sigma _{T^* \text{ constant}} < 0, \sigma = 0.04$	0.3	0.2	-
$g _{T^* \text{ constant}}^* < 0, g^* = 0.20, \tau = 0.25$	-42.5	64.0	-
<b>B. Long-run Constraint</b>			
$g _{LC}^* > 0$	-	-	-47.6 <sup>a</sup>
$\tau _{LC} < 0$	-	-	13.3 <sup>a</sup>
$\sigma _{LC} < 0$	-	-	29.5 <sup>a</sup>

Notes: The first two columns represent the percentage changes in  $V(T)$  and  $Z$  in the benchmark Constant  $T^*$  Policy case.

<sup>a</sup>These refer to the endogenous choices of  $\{g^*, \tau, \sigma\}$  required to satisfy (35a,b,c), and indicate percentage changes in welfare,  $Z|_{LC}$ , if the long-run fiscal constraint is imposed.

The benchmark set of parameter values are:  $h = 10$ ;  $\delta = 0.04$ ;  $\alpha = 0.1$ ;  $\tau = 0.30$ ;  $g^* = 0.25$ ;  $\gamma = 0.025$ ;  $\sigma = 0.04125$ ;  $K_0 = 10$  (so that  $\alpha K_0 = 1$ );  $M_0 = 0.16$ ;  $b_0 = 0.50$ ;  $i^* = 0.10$ ;  $p^* = 0.04$ ;  $a_0 = 0.45$ ;  $n_0 = 0.05$ . Also, the implied value of  $q$  is 1.2.

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