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Consumer Value-Maximizing Sweepstakes & Contests: A Theoretical and Experimental Investigation

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A Theoretical and Experimental Investigation

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Abstract

Sweepstakes and contests are an extremely common promotional strategy used by firms. The sweepstakes and contests often differ significantly in the design of reward structure. For example, in 1999, Godiva Chocolates conducted a sweepstakes where one box of chocolates contained a diamond jewellery. The chance of winning was 1 in 320,000. In 2000, M&M conducted a contest where the Grand Prize of a $1,000,000 had winning odds of 1 in 380,000,000 and a million second prizes of a coupon redeemable for a M&M packet had the odds of 1 in 380. In a contest conducted by Planters in 2000, the first prize too was a $1 m (odds 1 in 5,000,000) but there were only 100 second prizes of a NFL football jacket with odds of 1 in 50,000. In 1999, Old Navy conducted a sweepstake where there were 4,552 first prize winners who got $100 gift cards with the odds of winning 1 in 1,000, the 9,105 second prize of $ 20 gift certificates had odds of 1 in 500 and the 13,660 third prizes of $10 certificates and 883,476 fourth prizes of $5 had winning odds of 1 in 333 and 1 in 50 respectively. These examples raise the issue of how reward structure would affect consumer valuation and participation. The objective of this paper is to obtain an understanding of how consumers' valuation of sweepstakes varies on the basis of differing consumer segments and the characteristics of the consumers.

Our paper focuses on the decisions pertaining to the reward structure. We examine some commonly used sweepstakes and provide insights on how consumer valuations depend on the number of winners, the number of levels of prizes, and the difference in the awards between the levels (reward spread). We follow the Cumulative Prospect Theory to develop a model for consumer valuations of alternative formats of sweepstakes. The model applies a *S*-shaped probability weighting function and a loss-aversion framework for the consumers who switched to less preferred brands for sweepstakes but eventually did not win any prizes. We analytically derive our theoretical results and experimentally test some of the key implications.

The results of the model show that the sweepstakes reward structure should be based on three factors: the objectives of the firm, the risk aversion of the customers, and the level of sub-additivity of probability weighting. The results of the model prescribes that the firm should begin by setting sweepstake objectives in terms of either attracting switchers or targeting current users. If the objective is to target current users, then the number of prizes awarded should be lower than in the case where the targets are switchers. If the current users are risk neutral, then the consumer value-maximizing award is a single grand prize. If the current users are risk averse, then the award should consist of multiple "large" prizes. When the firm's objective is to draw sales away from competitors, the value-maximizing strategy is to distribute the award money over more prizes. If the non-current user segment is risk neutral with respect to gains but sufficiently risk averse in the domain of losses, then the prescribed reward structure is to have a single grand prize but also include several small prizes which ideally should be close to the opportunity cost of the customers. If the non-loyal customers are risk averse in gain and loss averse, then the best prize allocation is to have both multiple large prizes as well as several small prizes.

Another recommendation from the model analysis is that the firm should minimize the number of prizes at each level. In practice, the costs of implementing and communicating such a prize structure could be high. To trade-off between the logistical and communication costs and the theoretically value-maximizing approach, firms could increase the number of prizes at each level for easier implementation. A trade-off is involved between increasing the attractiveness of the sweepstake and the implementation costs of administering several levels of prizes. Often, when the prizes are products rather than cash, the firm may obtain quantity discounts for the products but the value of the products will be the same for the sweepstake participants.

Key words: Sales promotion, prospect theory, customer loyalty.

Introduction

The use of sweepstakes and contests as a promotional tool is ubiquitous across several categories in durables, non-durables and services. According to the Annual Cox Direct 20th Annual Survey of Promotional Practises, sweepstakes are reported to be used by 73% of the firms surveyed in 1997 as compared to being used by 63% of the firms in 1993. Firms spend considerable amounts of their communication budgets on the rewards as well as advertising the sweepstakes or contests. For example, in 2000, Coke Classic conducted a ‘False Tops’ sweepstakes that included five $1 million cash awards and was promoted using a $15 million advertising campaign. There is considerable anecdotal evidence suggesting that managers consider these a very effective tool in generating sales (e.g. *Marketing News*, 2000). While there is extensive literature on other promotional devices such as coupons (e.g. Dhar, Morrison and Raju 1996; Inman and McAlister 1994; Neslin, 1990), despite its prevalence, very little research has examined as to how consumers respond to sweepstakes promotions.

Consider the following examples of sweepstakes promotions. In 1998, Godiva Chocolates conducted a “Chocolates and Engagement” sweepstakes where three boxes of chocolates contained a diamond engagement ring. The odds of winning the ring were 1 in 320,000. In 1999, the promotion was changed to a single piece of diamond jewelry. In 2000, M&M conducted a contest where the Grand Prize of a $1,000,000 had winning odds of 1 in 380,000,000 and a million second prizes of a coupon redeemable for a M&M packet had the odds of 1 in 380. In a contest conducted by Planters in 2000, the first prize too was a $1 m (odds 1 in 5,000,000) but there were only 100 second prizes of a NFL football jacket with odds of 1 in 50,000. In 1999, Old Navy conducted a sweepstake where there were 4,552 first prize winners who got $100 gift cards with the odds of winning 1 in 1,000, the 9,105 second prize of $ 20 gift certificates had odds of 1 in 500 and the 13,660 third prizes of $10 certificates and 883,476 fourth prizes of $5 had winning odds of 1 in 333 and 1 in 50 respectively.

These examples illustrate the wide variation in how the rewards in sweepstakes and contests are allocated. The number of winners ranges from one in the Godiva (1999) example to a million and one in the M& M’s case. The number of prize levels awarded also varies with two in the case of Planters and four prize levels in the case of Old Navy. Further, the differences in amounts between the levels of prizes are also considerably different. These examples raise the issue of how reward structure would affect consumer valuation and participation. To the best of our knowledge, no research has been conducted to address this issue. The objective of this paper is to obtain an understanding of how consumers valuation of sweepstakes differs on the basis of differing consumer segments and the characteristics of the consumers.

Why are sweepstakes and contests effective in generating consumer response? Literature investigating people's incentive to participate in lotteries offers insights in understanding why sweepstakes may be a useful promotion tool. Friedman and Savage (1948) argued that people's utility function was concave up to a point but later becomes convex. They reasoned that consumers participated in lotteries because winning accorded a possibility of reaching a state of high income that provided disproportionate benefits. Kwang (1965) presents an "indivisibility of expenditures" explanation that states that consumer expenditures cannot be divisible infinitely. If a consumer wished to allocate her resources between a car and a boat, the option of selecting fractions of the product does not exist: the customer has to select one of the products. He demonstrates that rational consumers with limited income wishing to purchase both products will participate in lotteries. Another argument that explains why people participate in lotteries or sweepstakes is the availability bias. Usually, a firm's communication messages emphasize only the winners (e.g. Publishers Sweepstakes) that increases the availability of the positive consequences of participation in the consumers' minds (Tversky and Kahneman, 1974). Finally, there may be a utility of just participating in some contests and sweepstakes (Chandon, Wansink and Laurent 2000).

Sweepstakes and contests are similar to lotteries in that both involve a random drawing process that offers an opportunity to win a prize. The key distinguishing feature of a sweepstake from a lottery is that no consideration is involved. Consideration implies that the customer makes a payment to avail being a participant in the drawing. Conducting a lottery is illegal where a lottery is defined as containing three components: chance, prize and consideration. To be legal, one of these elements has to be eliminated. Therefore, any sweepstakes requires that customers can avail of the opportunity to participate by getting an appropriate form or contest piece free from the firm.

Sweepstakes and contests differ in that while sweepstakes are promotions where only chance is involved, contests also require some level of effort or skill. Very often, the skill required in most contests is minimal and does not differentiate between consumers. For example, in 2000, Folger's Coffee used print media to promote a contest where consumers entered into a drawing if they correctly identified a singer who regularly drank Folger's. Folgers had provided the answer by using the picture and the name of the singer in an ad on the adjoining page of the magazine. We use the terms contests and sweepstakes interchangeably but our results are applicable only to those contests where the skill or effort level does not impact the outcome.

Designing a sweepstake or contest promotion involves a number of interrelated decisions. The elements of a sweepstake design include determining the total reward money and the allocation of the reward money with respect to the total number of winners and the split of the reward between them. Other decisions include determining the theme of contest, kind of prizes (cash or products), the duration and frequency of the sweepstake, whether the rewards are immediate or delayed and the amount of effort the consumer has to expend in participating in the sweepstake. In some cases, contests are designed so that the odds of winning are based on the number of entries received. Therefore, sweepstake contests can be dichotomized as those that have risky prospects (actual winning odds are announced) and those with uncertain prospects (actual winning odds are not known and depend on the number of entries received). In all the motivational examples used earlier, the firm has provided the odds of winning on the packages.

The focus in this paper is only on the decisions pertaining to the reward structure. We examine some commonly used sweepstakes and provide insights on how consumer valuations depend on the number of winners, the number of levels of prizes, and the difference in the awards between the levels (reward spread). We provide a model for consumer valuations of alternative formats of sweepstakes and experimentally test some of the key implications.

Model Development

In this section we set up a model that describes consumer's valuation of sweepstakes. As our focus is limited to the design of the sweepstakes structure, we assume the price of the product to be fixed. We further assume that the firm has allocated a fixed budget of R to be distributed allocated as prize money to the winners. These assumptions allows us focus on alternative sweepstake formats i.e. how many winners there should be in a sweepstakes promotion and how the total reward money should be allocated between them . We let S denote the prize structure of a sweepstakes promotion where

$$S = \{r_1, m_1; r_2, m_2; ...r_n, m_n\} \qquad (1)$$

In equation (1) r_j denotes the j^{th} prize and m_j the number of winners of r_j ($j = 1, 2, \ldots n$), both r_j and m_j are positive. The number of prize levels is denoted by n. Thus, the sweepstake defined by (1) offers m_1 number of first prize at the amount of r_1, m_2 number of second prize at the amount of $r_{2,}$ and so on. Without any loss of generality, we assume that the prizes offered are in the form of cash. Given the promotion budget constraint (R), a feasible prize structure for the sweepstakes should satisfy the following conditions: $\sum_{j=1}^{n} m_j r_j = R$ and $r_i > r_j > 0$ for any $1 \leq i < j \leq n$.

Consumer Valuation

Consumers face a decision on whether or not to purchase the brand conducting the sweepstakes. For expositional simplicity, we assume that each consumer purchases only one unit and therefore has one chance to win a prize.[1] In the rest of the paper, we use the term 'participate' in the contest to reflect that the consumer has decided to both purchase the brand and to avail of the opportunity to win a prize. We do not consider consumers who enter the sweepstakes without purchasing the brand. Discussion with managers indicates that the effort cost to enter sweepstakes without purchasing are usually high enough to induce negligible entry rates.[2] To decide whether or not to participate in the sweepstakes, a consumer (i) has to compare the anticipated value of participation with value of non-participation defined as follows:

$$v_i = \begin{cases} v_i(S) & \text{if consumer i participates in the sweepstakes} \\ v_i^0 & \text{if consumer i does not participate in the sweepstakes} \end{cases} \quad (2)$$

In equation (2), $v_i(S)$ represents the value that consumer i anticipates from the outcome of sweepstake S defined in equation (1); v_i^0 represents the value that consumer i would receive without participating in sweepstakes. Thus, v_i^0 serves as the reference value that consumer i should employ in judging the gain and loss from participating in the sweepstakes. The anticipated gain and loss from the sweepstakes depends on the consumers' preference for the brands as well as from the outcome.

We assume that there is heterogeneity in preferences for the brand. For simplicity, we divide the market into two segments: the first segment is defined as the high-valuation segment (denoted as $i=H$), and the second segment as low-valuation (denoted as $i=L$). The high-valuation segment consists of consumers who, at the time the sweepstake is conducted,

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[1] Usually, consumers can purchase multiple units. We assume that the decision to purchase each unit of the brand is made independently. This is equivalent to analysing the demand by units of product with sweepstakes.

have a relatively higher preference for the brand relative to the competitors. A number of reasons have been suggested as to why a firm will target high-valuation consumers with a promotion even though they are likely to purchase the brand on the occasion the sweepstakes is conducted. These include increasing consumer consumption through purchase of additional units or encouraging purchase acceleration (Blattberg and Neslin 1990). Another reason suggested for promotions is to reward current consumers (Kotler 1997) so as to increase their brand preferences in future periods and prevent defection (Schultz, Robinson and Petrison 1998). For example, in 2000, Patek Philippe conducted a sweepstakes targeted at high valuation customers explicitly advertising that the objective of the sweepstake was to reward their customers. Only customers who had purchased a Patek watch in the past were eligible to enter (by providing evidence of owning a Patek such as the movement or reference number of the watch). U.S Airlines places sweepstakes forms in their in-flight magazines that are available only to flyers who are already current customers. Such promotions impact consumer retention by enhancing the valuation of the brand for future purchases. Low valuation customers prefer competitors' brands at the time the sweepstake is conducted. Thus, the primary objective of the sweepstake or contest is to encourage brand switching.

These two segments of consumers anticipate different levels of post-purchase utility. Since consumers of high-valuation segment will prefer the brand on the particular purchase occasion with or without the sweepstakes promotion, their reference value (v_H^0) is same as the value that they would receive without winning any prizes ($v_H(0)$). These consumers will experience a gain from any prizes that they may win and will not regret their brand choice decision even if they do not win any prize. In contrast, consumers of the low-valuation segment will select other alternatives if the sweepstake is not offered as they value the competing brands more on the particular purchase occasion. Therefore $v_L^0 > v_L(0)$; that is, low-valuation segment's reference value is higher than the value that they would receive without winning any prizes. As a result, a low-valuation consumer will gain from purchasing the firm's brand only if she wins a prize. If the consumer does not win any prizes, then she will experience a loss and regret for her purchase decision. Such a loss is the opportunity cost for the low-valuation consumers. We let $\tau = v_L^0 - v_L(0)$ where τ denotes the opportunity (switching) cost of purchasing the less preferred brand.

[2] For example, in the sweepstakes conducted by M&M in 2000, the firm required that a self-addressed stamped # 10 envelope be sent to obtain the official game form. There was a limit of 1 request per envelope and the game form was available only "while supplies last".

As our focus is on the reward structure, for simplicity, we do not consider important elements like the utility of participation and the disutility of effort involved in a sweepstake. Thus, consumers obtain positive utility from the sweepstakes only through the chances to win prizes. Then, $V_i(S)$, the anticipated values from sweepstake (S) given in equation (1) can be formulated as follows for the high- and low-valuation segments respectively:

$$V_H(S) = V(v_H(S), v_H^0) = \sum_{j=1}^{n} \omega_j g(r_j) + \omega_0 g(0) \tag{3}$$

$$V_L(S) = V(v_L(S), v_L^0) = \sum_{j=1}^{J} \omega_j g(r_j - \tau) - \sum_{j=J+1}^{n} \omega_j l(\tau - r_j) - \omega_0 l(\tau) \tag{4}$$

where $r_j > \tau$ for any $1 \leq j \leq J$ and $r_j \leq \tau$ for any $J < j \leq n$. Note that prizes r_j ($J < j \leq n$) are not large enough to cover the opportunity cost that low-valuation consumers incur. The gain function $g(.)$ represents the consumer's valuation of gain and loss function $l(.)$ represents the consumer's valuation of loss.[3] Decision weights ω_j represents the weight that consumers assign to valuation of prize r_j. Naturally, decision weights ω_j depends on the odds of winning the prize r_j. Both utility formulation (3) and (4) and above assumptions are standard in prospect theory (Kahneman and Tversky 1979 and Tversky and Kahneman 1992). We next discuss the properties of the loss and gain functions as well as the weighting functions in more depth.

The Gain and Loss Function

Prospect Theory proposes that both gain function $g(.)$ and loss function $l(.)$ are positive, monotonically increasing, and concave: $g(x) > 0$, $g'(x) > 0$, $g''(x) \leq 0$, $l(x) > 0$, $l'(x) > 0$, and $l''(x) \leq 0$. Moreover, a loss is more significant than a gain of the same amount due to loss-aversion behaviour; that is, $g(x) < l(x)$ for any positive value of x. The reference-dependence model that we use in equation (3) and (4) was originally proposed by Tversky and Kahneman (1991) and is typical in modelling loss aversion. Loss-aversion behavior has also been demonstrated in the marketing domain (for examples see Loewenstein and Prelec 1993 and Hardie, Johnson, and Fader 1993).

[3] Both gain and loss functions depend on size of prize that consumers receive. We recognize but do not explicitly model the factor that consumers may not always claim the prizes that they win, particularly when the prize is small and the transaction cost to receive the prize is high.

The Weighting Function

In prospect theory, the value of each outcome is multiplied by a decision weight. The decision weight measures the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events. Prospect Theory adopts rank-dependent cumulative decision weighting functions (Quiggin 1982, Tversky and Kahneman 1992, and Tversky and Wakker 1995).

Consider the case of risk prospects sweepstakes where the consumers know the odds of winning. For the sweepstakes defined in (1), we denote the total number of product units with sweepstakes as N [4]. The participant's actual chance to win first prize is m_1/N, chance to win second prize is m_2/N, and so on. The cumulative probability for the event of winning at least the j^{th} prize is $\sum_{k=1}^{j} m_k \big/ N$. The corresponding cumulative decision weights for the event of winning at least the j^{th} prize is $\omega(\sum_{1}^{j} m_k \big/ N)$ where ω is cumulative decision weighting function. The decision weight ω_j, associated with winning the j^{th} prize (j=1,2….n), is the difference between cumulative weights of the events winning at least the j^{th} prize and winning at least (j-1)th prize; that is, $\omega_j = \omega\left(\sum_{k=1}^{j} m_k \big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \big/ N\right)$. For any rank j=1,2….n, we obtain the decision weight ω_j associated with winning the jth prize as follows.

$$\omega_1 = \omega(\frac{m_1}{N}),\ \omega_2 = \omega(\frac{m_1+m_2}{N}) - \omega(\frac{m_1}{N}),\ \ldots\ \omega_j = \omega\left(\sum_{k=1}^{j} m_k \big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \big/ N\right),\ \ldots$$

$$\omega_n = \omega\left(\sum_{k=1}^{n} m_k \big/ N\right) - \omega\left(\sum_{k=1}^{n-1} m_k \big/ N\right),\ \text{and}\ \omega_0 = 1 - \omega\left(\sum_{k=1}^{n} m_k \big/ N\right) \qquad (5)$$

where ω_0 denotes the decision weight for the event of "not winning any prizes". In prospect theory cumulative decision weighting function $\omega(.)$ is a *s*-shaped function as shown in Figure 1, $\omega(0) = 0$ and $\omega(1) = 1$. Such a weighting function overweights small probabilities and underweights moderate and high probabilities. The *s*-shaped weighting function has been empirically verified by a large number of studies, e.g., Kahneman and Tversky (1979), Tversky and Kahneman (1992), Camerer and Ho (1994), and Wu and Gonzalez (1996), etc. Specific *S*-shaped functions have also been suggested by Tversky and Kahneman (1992) and

[4] When consumers do not know the actual number of entries, sweepstakes become uncertain prospects. However our results will remain the same because essentially *N* will be replaced by consumers' expected number of

Prelec (1998) and further estimated using experimental data by Camerer and Ho (1994) and Wu and Gonzalez (1996).

Due to the rank-dependent nature of the weighting function, weights of winning large prizes are evaluated earlier and hence inflated more (relative to actual probabilities). Therefore, the ratio of decision weight and actual probability of winning a particular prize, which measures the degree of overweighting the probability of winning, is larger at higher ranks:

$$\frac{\omega_j}{m_j} > \frac{\omega_k}{m_k} \text{ for any rank } j < k \qquad (6)$$

Consumers' risk attitude depends on both the value functions and the decision weights. First, following the standard approach in Prospect Theory, we conceptualise risk aversion only through the value function. More specifically, a consumer's risk aversion with gain (loss) increases with the concavity of gain (loss) function. Second, in order to compare alternative decision weight functions, we adopt measure of subadditivity (SA) proposed by Tversky and Wakker (1995). Higher SA is interpreted as an ordering by departure from the actual probability of the corresponding outcome. This measure is taken independent of value functions. For example, in Figure 1, $\omega_1(p)$ is said to be more SA than $\omega_2(p)$.

Though not the focus of this paper, factors influencing SA are important to investigate for designing sweepstakes. Several design elements of a sweepstakes can manipulate the level of sub-additivity. For example, increasing the effort level required to participate (eg. completing a form or collecting game pieces) could create an illusion of control (Langer 1975) that may increase sub-additivity.

Utilizing the value functions in (3) and (4) and decision weights in (5), the consumers' anticipated utility of sweepstakes participation can be characterized as follows:

$$V_H(S) = \omega\left(\frac{m_1}{N}\right) g(r_1) + \sum_{j=2}^{n}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) g(r_j) + \left(1 - \omega\left(\sum_{k=1}^{n} m_k \Big/ N\right)\right) g(0) \quad (7)$$

$$\begin{aligned} V_L(S) = {} & \omega\left(\frac{m_1}{N}\right) g(r_1 - \tau) + \sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) g(r_j - \tau) - \\ & \sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) l(\tau - r_j) - \left(1 - \omega\left(\sum_{k=1}^{n} m_k \Big/ N\right)\right) l(\tau) \end{aligned} \quad (8)$$

entries formed according to their beliefs on a probabilistic distribution of *N*. For expositional convenience we will focus on the case with known odds.

Value-Maximizing Sweepstakes

As mentioned earlier, our objective is to study the design of sweepstakes that maximize consumers' valuation. Corresponding to the objective of the sweepstakes promotion, within the promotion budget, we aim to investigate sweepstakes that maximize targeted consumers' anticipated valuation.

$$\text{(P1)} \qquad \max_{s} \delta_H V_H(\text{S}) + (1-\delta_H)V_L(\text{S})$$

$$\text{s.t.} \quad \sum_{j=1}^{n} m_j r_j = R$$

where the anticipated value functions $V_i(\text{S})$ (i =H, L) are given in equation (7) and (8). Parameter $\delta_H \in \{0,1\}$ indicates the targeting segment, where δ_H =1 implies that the sweepstakes is offered to target the high-valuation segment and δ_H =0 implies that the sweepstakes is offered to attract the marginal and switching consumers. As we focus on the design of sweepstakes we also assume that both the price and brand valuation (without the sweepstake) are predetermined.

Model Analysis

We begin our analysis with the case where the sweepstakes promotion targets the high-valuation segment (δ_H =1), followed by the case where the sweepstakes promotion targets the low-valuation segment (δ_H =0). We examine the prize structure of the sweepstakes that will be valued most by each target segment. For each case, we draw conclusions on the reward format in general scenarios as well as scenarios corresponding to specific type of consumers' risk attitudes. We conclude our analysis with a comparison between the value-maximizing reward formats for high- and low-valuation segments.

High-Valuation Segment

Problem (P2) solves the value-maximizing sweepstakes for consumers of high-valuation segment.

$$\text{(P2)}\ \underset{s}{\text{Max}}\, V_H(S) = \omega(\frac{m_1}{N})g(r_1) + \sum_{j=2}^{n}\left(\omega(\frac{\sum_{k=1}^{j} m_k}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j) + \left(1-\omega(\frac{\sum_{j=1}^{n} m_j}{N})\right)g(0)$$

$$\text{s.t.} \sum_{j=1}^{n} m_j r_j = R$$

We solve problem (P2) in Appendix 1 and obtain following the equilibrium condition:

$$\frac{\omega_j}{m_j} g'(r_j^*) = M_H^* \quad (j = 1,2,..n) \tag{9}$$

Equilibrium condition (9) states that, in the value-maximizing sweepstakes design, marginal anticipated value generated from an increase in prize is identical across all ranks that offer positive prizes. The equilibrium ability to raise anticipated-value of high-valuation consumers is denoted by M_H^*, which increases with ω_j / m_j (degree of actual winning probability being overweighed) and $g'(r_j^*)$ (marginal utility from gain). Intuitively, when an extra prize is allocated to a rank that has winning probability more overweighed, consumers should anticipate a larger increase in value of sweepstakes. According to equation (6), the probabilities of winning higher ranks are more overweighed. However, higher ranks are associated with larger prizes and hence lower marginal value $g'(.)$. Next we discuss the implications of equilibrium condition (9) on the design of the sweepstakes.

Risk Neutral in Gain

We begin by analysing the value-maximizing format of sweepstakes with the case where high-valuation consumers are risk neutral in gain. We will later study the implications of risk aversion. When high-valuation consumers are risk neutral in gain, marginal value of gain $g'(r_j)$ becomes constant. Since $\frac{\omega_j}{m_j} > \frac{\omega_k}{m_k}$ for any $j < k$ (equation (6)), the only solution that satisfies equilibrium condition (9) is n=1. Moreover, among the sweepstakes that only offer one level of prize, the value-maximizing solution is to have only one winner because $\frac{\omega(1/N)}{1/N} > \frac{\omega(m_1/N)}{m_1/N}$ for any m_1>1. We summarize the above result in Proposition 1.1.

Proposition 1.1: *When high-valuation consumers are risk neutral in gain, the value-maximizing format of sweepstakes is to offer one grand prize only; that is,* $S_H^* = \{r_1^* = R, m_1^* = 1; r_j^* = 0, j > 1\}$.

The intuition behind the result of Proposition 1.1 lies in the rank-dependent *s*-shaped decision weighting function. Such a decision weighting function overweighs availability of larger prizes (or higher ranks). Therefore, when high-valuation consumers are risk neutral in gain, allocating more prize budget to larger prize increases high-valuation consumers' anticipated value for the sweepstakes. Moreover, for a specific high rank, smaller winning probability is overweighed more. Consequently it is value-maximizing to offer one grand prize only. Next, we consider the scenario when consumers are risk averse in the domain of gains.

Risk Aversion in Gain

When high-valuation consumers are risk averse, they may prefer sweepstakes that offer multiple levels of prizes. On one hand, probabilities of winning higher ranks are still overweighed more. On the other hand, since high-valuation consumers are risk averse in gain, marginal value from gain $g'(r_j) < g'(r_{j+1})$ as $r_j > r_{j+1}$. Thus, equilibrium condition (9) can hold for multiple levels of prizes.

To design a sweepstake that offers multiple levels of prizes, the firm needs to decide the number of winners for each rank of prize $\{m_1, m_2, ..., m_n\}$ and the inter-rank spread (difference in size of prize between consecutive ranks). First, we find that the value-maximizing number of winner for each rank of prize is one. (See Appendix 1 for proof). The intuition behind the results is as follows. Suppose there are two winners for rank *j* with prize r_j. We divide the total prizes of ($2r_j$) into two ranks, rank *j* of prize $r_j+\nabla r$ and rank *j*+1 of prize $r_j-\nabla r$. Given a small ∇r, ranks for all other prizes will not be changed. The difference in the utilities of achieving these ranks, $g(r_j + \nabla r) - g(r_j)$ and $g(r_j) - g(r_j - \nabla r)$, decreases and approaches to zero with smaller ∇r. On the other hand, according to equation (6), the chance to win the j^{th} prize is overweighed *strictly* more than the chance to win the $j+1^{th}$ prize. Moreover, the degree of overweighting is independent of the size of ∇r. Therefore there exists a sufficiently small ∇r so that, by dividing the prizes for two winners of rank *j* into two consecutive ranks, one can increase consumer's anticipated value of sweepstakes. Thus the consumer value-maximizing prize structure should offer only a single prize at every level.

When a sweepstakes consists of a single prize at every level, equilibrium condition (9) becomes

$$\left(\omega\left(j/N\right) - \omega\left(j-1/N\right)\right) g'(r_j^*) = M_H^* \tag{10}$$

for $j = 1,2,..n$. Condition (10) implies that, given a decision weighting function (ω), when high-valuation consumers are more risk averse, they should prefer a sweepstake of smaller inter-rank ($d_j^* = r_j^* - r_{j+1}^*$) and more winners. To see this, note that the ratio of marginal gains from consecutive ranks ($g'(r_j^*)/g'(r_{j+1}^*)$) is a constant. When high-valuation consumers are more risk averse, their marginal utilities functions become steeper. For a given value of r^*_{j+1}, a smaller r^*_j is required to satisfy condition (10), leading to a smaller inter-rank spread d^*_j. From condition (10), we can also see that when the decision-weighting function is more sub-additive (SA), the ratio ($g'(r^*_j)/g'(r^*_{j+1})$) is required to be smaller. Following a similar line of reasoning as above, we can conclude that given a fixed level of risk aversion, high-valuation consumers prefer a sweepstake that has larger inter-rank spread and smaller number of winners.

Proposition 1.2: *If high-valuation consumers are sufficiently risk averse, the value-maximizing sweepstakes should consist of multiple big prizes. The number of prizes increases but the inter-rank spread decreases with (a) the increase in consumer risk aversion and (b) decrease in sub-additivity.*

Low-valuation Segment

We now discuss the reward structure of value-maximizing sweepstake when the firm targets the low valuation segment. A key distinction from the high valuation segment in this case is that consumers who do not win a prize will experience a loss as they incur an opportunity cost of not purchasing the brand they prefer on the specific purchase occasion. Problem (P3) defines the value-maximizing sweepstakes for the low-valuation segment.

(P3)
$$\underset{s}{Max}\, V_L(S) = \omega\left(\frac{m_1}{N}\right) g(r_1 - \tau) + \sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) g(r_j - \tau)$$
$$- \sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) l(\tau - r_j) - \left(1 - \omega\left(\sum_{j=1}^{n} m_j \Big/ N\right)\right) l(\tau)$$

s.t. $\sum_{j=1}^{n} m_j r_j = R$

<u>No Loss if Winning a Prize</u>

Our first finding is that value-maximizing prize structure of the sweepstakes targeting low-valuation segment should only offer prizes at least as large as opportunity cost τ. If a consumer does win a prize, she should anticipate either a gain or at least indifference (no gain and no loss). We state the result in Lemma 1 and provide a formal proof in Appendix 2.

Lemma 1: *If the sweepstakes is targeted to the low-valuation segment, the lowest prize should be at least as large as opportunity cost τ.*

Lemma 1 implies that firms should not offer any prize that is smaller than τ. Consumers of low-valuation segment will experience no loss (and no gain) if they win the smallest prize. To see the intuition behind this result, assume that a sweepstakes contains a bottom prize $r_n < \tau$. We find that the low-valuation consumers' anticipated value from this sweepstakes increases if we raise the prize r_n by a small amount. First, since the loss function is concave, the marginal utility becomes larger when the size of prize goes increasingly closer to τ. Second, given the same budget for the sweepstakes contest, an increase in r_n leads to a decrease in the number of winners (m_n). A smaller number of last prize winners, hence lower probability of winning the last prize (while the chance to win all other prizes remain the same), implies a greater marginal decision weight for the last prize. This is the result of *s*-shaped decision weighting function. Combining these two effects, we can conclude that a marginal increase in the amount of the last prize towards opportunity switching cost τ will always increase the low-valuation consumers' anticipated value for the sweepstakes. In the rest of analysis on Problem (P3) we will apply the result of Lemma 1.

Equilibrium Condition

According to Lemma 1, in the value-maximizing sales contests offered to low-valuation segment, all the prizes other than the last prize are larger than the opportunity cost τ. Similar to our analysis of the high-valuation segment, we now derive the equilibrium conditions for the value-maximizing sales contests for the low-valuation segment, $S_L^* = \{r_1^*, m_1^*; r_2^*, m_2^*; ... r_n^* = \tau, m_n^*\}$. (Please see Appendix 2 for the derivation.)

$$\frac{\omega_j}{m_j} g'(r_j^*) = M_L^* \quad (j = 1,2,..n\text{-}1) \tag{11}$$

$$\frac{\omega_n}{m_n} l'(0) \geq M_L^* \geq \frac{\omega_n}{m_n} g'(0) \tag{12}$$

Similar to equation (9), equation (11) requires identical anticipated value-generating ability M_L^* from the top (n-1) prizes. Consumers will experience gain if they win any of these prizes. Equilibrium sweepstakes will also contain a bottom prize equal to τ if (12) holds; that is, anticipated marginal value of loss aversion shall be sufficiently high at the bottom prize. Since $l'(0) / g'(r_j)$ measure the degree of loss aversion, condition (12) indicates that the value-maximizing sweepstakes for the low-valuation consumers should consist of a small prize when the low-valuation consumers are sufficiently loss averse. Intuitively, since low-valuation consumers are loss averse, they anticipate a much higher value of participation if the chance to experience loss is reduced.

Risk Neutral with Gain Function

Similar to the procedure adopted in the analysis for high-valuation segment, we first examine the case where the low-valuation segment is risk neutral with respect to gain; that is, $g'(.)$ is a constant (while loss function can be either risk averse or neutral). When low-valuation consumers are risk neutral in gain, we find that the prize structure of value-maximizing sweepstake consists of only two levels of prize: one larger than opportunity cost (τ) and one equal to τ. The intuition for the result of one large prize follows the same logic behind Proposition 1.1. Consider a sweepstakes as defined by equation (1) for the low-valuation segment. According to Lemma 1, we infer that the first (n-1) prizes must be larger than τ. Among these top (n-1) prizes, since $\frac{\omega_j}{m_j} > \frac{\omega_k}{m_k}$ for any $j < k$ according to equation (6) and $g'(r-\tau)$ is constant, the only solution to satisfy equilibrium condition (11) is $n = 2$. That is, the value-maximizing format of sweepstakes for low-valuation segment is to offer only one level of prize that is larger than the opportunity cost τ. Moreover, following the same argument as given in the high-valuation segment case, the value of sweepstakes is higher by offering one grand price than offering multiple top prizes of the same level; that is, $m_1^*=1$. Therefore, when low valuation consumers are risk neutral in gain, the value-maximizing format of sweepstakes should not offer more than one big prize.

In addition to the grand prize, the value-maximizing sweepstakes for the low-valuation segment also contains many small prizes equal to τ. Such a prize structure becomes an equilibrium outcome if the marginal anticipated value from prize level τ is sufficiently high for the low-valuation segment. According to (12), the sufficient condition for such a prize structure to be value-maximizing is

$$l'(0)\frac{\omega_2}{m_2} \geq g'(r_1-\tau)\frac{w_1}{1} \qquad (13)$$

This condition holds when the low-valuation consumers are sufficiently loss averse.

Now we analyse what the size of the first prize (r_1) and number of small-prize winners (m_2) should be for the type of sweepstakes proposed above for low-valuation segment. In Appendix 2 we derive the equilibrium condition for the value-maximizing number of small-prize winners:

$$\omega'\left(1+m_2^*\big/N\right)=\frac{\omega\left(1\big/N\right)}{1/N}\frac{g'(r_1^*-\tau)}{l(\tau)\big/\tau} \tag{14}$$

In equation (14), the marginal decision weight $\omega'\left(\frac{1+m_2^*}{N}\right)$ decreases with m_2, slope $\frac{\omega(1/N)}{1/N}$ increases with the SA of the decision weighting function, and ratio $\frac{g'(r_1^*)}{l(\tau)/\tau}$ decreases with loss aversion and increases with opportunity cost τ. Therefore, the number of small-prize winners (m_2) should increase with loss aversion but decrease with the size of opportunity cost (τ) and SA of the decision weighting function. Since the size of first prize decreases with number of small-prize winners, we can infer that the spread between the first prize and small prize should decrease with loss aversion but increase with SA of the decision weighting function.

The results of the above analysis on the value-maximizing structure of sweepstakes for low-valuation consumers lead to the following proposition.

Proposition 2.1:

(a) *If the low-valuation segment is risk neutral with respect to gain and sufficiently loss averse, then the value-maximizing prize structure for the sweepstakes should consist of a grand prize and many small prizes equal to* τ. *That is,* $s_L^*=\{r_1^*, m_1^*=1; r_2^*=\tau, m_2^*\}$.

(b) *In a value-maximizing sweepstakes, the number of small-prize winners (* m_2^**) increases with loss aversion and decrease with Sub-Additivity of the decision weighting functions.*

<u>*Impact of Risk Aversion in Gain Function*</u>

We now analyse the implications of risk aversion in gain on the design of sweepstakes for the low-valuation segment. The degree of risk aversion in gain is measured by the concavity of gain function with respect to the amount of gain that a low-valuation consumer receives from winning the sweepstakes. Similar to the analysis for the high-valuation consumers who are risk averse, low-valuation consumers who are risk averse may also prefer sweepstakes that offer multiple levels of prizes above switching cost τ. That is, condition (11) may hold for multiple levels of prizes above switching cost τ. On the one hand, $\frac{\omega_j}{m_j}>\frac{\omega_{j+1}}{m_{j+1}}$ still holds for any *j* due to the *s*-shaped decision weighting function. This is tempered by the risk averseness in gain with the marginal value from gain $g'(r_j-\tau)<g'(r_{j+1}-\tau)$ for $r_j>r_{j+1}$. Thus, necessary condition (11) can hold for multiple levels of rewards.

To design a sweepstake that offers multiple levels of prizes, the firm needs to decide the number of winners for each rank of prize $\{m_1, m_2, ..., m_n\}$ and inter-rank spread. As in the analysis for the high-valuation consumers, we find that value-maximizing number of winner for each rank of prize (above τ) is one. The intuition behind this result is same as discussed for high-valuation segment. In essence, the concavity of the value function and the *s*-shape of the decision weighting function determine that two different prizes are more effective than one big prize of the same amount. In summary, the value-maximizing format for the top (*n*-1) prizes is to offer a single prize at many levels.

When a sweepstakes offers many single prizes, equilibrium condition (11) becomes

$$\left(\omega(\frac{j}{N}) - \omega(\frac{j-1}{N})\right) g'(r_j^* - \tau) = M_L^* \quad (j = 1,2,..n\text{-}1) \qquad (15)$$

for $j = 1,2,..n\text{-}1$. Condition (15) implies that, given the decision weighting function, when low-valuation consumers are more risk averse, they prefer a sweepstake of smaller inter-rank spread (r_j - r_{j+1}) and larger number of winners. Moreover, given a fixed level of risk aversion, when decision-weighting function is more sub-additive (SA), a sweepstake that has larger inter-rank spread for the top prizes and fewer number of winners provides maximum value.

The number of last-prize winners also changes with consumer risk aversion. We derive the equilibrium condition for value-maximizing number of winners in Appendix 2 as follows:

$$\omega'\left(\frac{n-1+m_n^*}{N}\right) = \frac{\omega\left(\frac{j}{N}\right) - \omega\left(\frac{j-1}{N}\right)}{1/N} \frac{g'(r_j - \tau)}{l(\tau)/\tau} \qquad (j=1,2,\ldots n\text{-}1) \qquad (16)$$

We can infer from condition (16) that the number of last-prize winners increase with loss aversion but decrease with the size of switching cost. Moreover, the number of last-prize winners increases with the low-valuation consumers' risk aversion.

Proposition 2.2:

(a) *If low-valuation consumers are risk averse, the value-maximizing sweepstakes should consist of multiple big prizes and many small prizes equal to* τ.
(b) *The number of prizes that are larger than opportunity cost* τ *should increase but the inter-rank spread decrease with degree of consumer risk aversion.*
(c) *The number of winners of last prize should increase with loss aversion and risk aversion, but decrease with the size of switching cost and SA of decision weighting function.*

The major distinction between the designs of sweepstakes targeted towards the high-valuation vs. low-valuation segment is that the reward structure should include more and smaller amount prizes for the latter. A sweepstakes that offers many small prizes to the low-valuation consumers efficiently reduces the anticipated loss resulting from switching from her preferred brands. This is unnecessary for a sweepstakes that targets the high-valuation consumers.

The implication of risk aversion is that consumers prefer sweepstakes that offer a number of large prizes rather than just one grand prize. This pattern should be the same for both high-valuation segment and low-valuation segment. However, for a sweepstakes that targets low-valuation consumers, when the level of loss aversion is high, the number of small prizes should be increased to maximize consumers' valuation. Thus, in addition to offering multiple large prizes rather than just one grand prize, increased number of small prizes also leads to a smaller chance for low-valuation consumers to experience a loss from participating a sweepstakes.

Experimental Test

Testing the model based on behavior rather than stated preferences posed a challenge. As conducting a field test to judge the effectiveness of alternative sweepstakes was not feasible, an experimental method is employed to examine whether high valuation and low valuation customers prefer the sweepstakes as suggested by the theoretical model. In addition, the impact of risk aversion is also tested.

Subjects and Procedure

The subjects were 128 undergraduate business majors who participated to get extra course credit. The task given to subject was to select between alternative sweepstakes. Subjects were asked to imagine that they were at a grocery store where they could select between two brands in four different product categories. Based on pre tests, two product categories were selected that met two criteria (a) students usage rates of the product categories were high and (b) there were two brands that constituted a major market share in the subject population. The categories and brands selected were Toothpaste (Crest/Colgate) and Colas (Diet Pepsi/Diet Coke). We use two product categories to check for consistency of results. As the stimuli for the two focal product categories was quite similar, subjects were also responded to sweepstakes for two other categories: Peanut Butter (Jif /Skippy) and Canned Tuna (Bumble

Bee/Chicken of the Sea) where the sweepstakes stimuli were different from the designs that are the focus of this study.

Subjects were given a booklet (Questionnaire 1) where their relative preference for the two brands in the product category was elicited using a 15-point scale anchored between Strongly Prefer Brand X-Will Never Consume Brand Y to Strongly Prefer Brand Y-Will Never Consume Brand X where the mid point (8) indicated indifference between the two brands. Thus, responses from 1-7 reflected preference for Brand X with 1 showing a strong preference for Brand X. Responses from 9-15 reflected preference for Brand Y with 15 an indicator of strong preference for Brand Y. To illustrate the experimental procedure, we use the example of the toothpaste category. Subjects were asked to imagine that they had a choice of either purchasing Crest or of purchasing Colgate. They were told that Colgate was running a sweepstake promotion where different packages had different sweepstakes in that the odds of winning a prize varied. The subjects were asked to look at the (five) options available to them that included buying Crest that had no sweepstake or one of the four Colgate packages. Each Colgate package had a different sweepstake promotion. The stimuli of the alternative sweepstakes is provided in Table 1. The sweepstakes in Tube 1 is designed as a single grand prize, Tube 2 multiple large prizes, Tube 3 is multiple large prizes and several small prizes while Tube 4 is a grand prize with several small prizes.

Subjects were asked to rank order the five alternatives in order of preference[5]. However, we use only data of the most preferred option. It is important to note that the task was entirely incentive compatible. There is no a priori theoretical reason to suggest that the rank order task causes any preference reversal as compared to a simple choice decision.

After subjects completed their responses for the sweepstakes in the four product categories, they were given a filler task. The filler task consisted of evaluating advertisements and took approximately 25 minutes. After completing the filler task, subjects were handed another booklet (Questionnaire 2) where their sweepstakes preferences were elicited again in a manner identical to the first task except that the brand with the sweepstakes promotion was switched (e.g. Crest had the identical sweepstakes to that in Questionnaire 1 whereas Colgate had no sweepstake). The questionnaires were counterbalanced to eliminate order effects. At the end of the task, measures of risk aversion were obtained using certainty equivalence questions. Risk aversion was assessed by responses to four questions (see, for example, Johnson & Schkade 1989). Two of the questions were framed in terms of gains and two in terms of losses. In the first two questions, the subjects were asked to indicate how much they were willing to receive a sure payment for a gamble that had a 0.5 (0.3) chance of winning $200 ($160) and 0.5 (0.7) chance off getting $ 0($75). In the domain of losses, subjects were asked to indicate what they were willing to pay to give away a gamble that offered a 0.3 (0.5) chance of losing $200 ($90) and a 0.7 (0.5) chance of losing $415 ($210).

To ensure that the subjects took the task seriously, they were told that they would play one of the alternative sweepstakes for real based on their decisions and were also given the product. For each product category, bags containing cards with winning odds as in the stimuli were placed in view of the students. After completing both questionnaires, subjects approached the experimenter where they first drew a card that contained the name of the product category and questionnaire (first or second) that they would play for real money. After the category and questionnaire number were determined, the experimenter tossed a coin. If the subject called correctly, subjects played got their first choice in the product category and questionnaire they had drawn. If the subject called incorrectly, the coin was tossed gain until the subject called correctly. Recall that the subjects rank-ordered five options. If the subject called incorrectly 4 times, they were asked to play their last choice. Based on the drawing obtained by the subject, they were given the corresponding sweepstake to play. Finally, the subjects were paid, given the brand of the option they played and debriefed. Thus, for the toothpaste category, if subjects played their first choice indicated in Questionnaire 1, they were given a tube of Colgate toothpaste and played the Colgate sweepstakes (or were given Crest without playing if that was their first choice).

Results

Eight subjects did not provide completed surveys and were dropped from the analysis. Two adjustments were made in the sample that are included in the analysis. First, it was observed that the sweepstakes choices of subjects who were indifferent between the two brands varied systematically. This was due to the fact that subjects who were indifferent between the two brands either were loyal users of a third brand or were switchers between the two major brands. We therefore dropped the subjects who were loyal to alternative brands. Second, the first choice of some subjects was to get their preferred brand and not participate in the sweepstakes. These subjects are also not included in the results presented.

[5] Due to the financial and logistical costs involved in the experiment, a rank-ordered task was used to gather as much information as possible for additional insights.

The results of the preferred sweepstakes are provided in Table 2a. The percentages reflect the most preferred sweepstake selected by the subjects. The subjects were categorized as high valuation customers if the responses on the loyalty scale exceeded 10 or was below 6. Responses between 6 and 10 on the 15-point scale were categorized as low-valuation customers. Risk aversion was determined by aggregating the responses to the four certainty equivalence questions. The top and bottom one-third of the subjects were categorized as the low and high-risk averse subjects.

We first test whether there are any differences in the distribution of choices between the four alternative sweepstakes that the subjects had to select from across the four conditions. We begin with the analysis of the stimuli used for the toothpaste category and examine whether the choices made by the high-valuation segment differ from those made by the low-brand valuation segment. Within the subjects who are categorized as low risk averse, the choices by the high-valuation segment are significantly different than the choice made by the low-valuation segment ($\chi^2_{1,3} = 45.36, p < .001$). Similarly, when the subjects were high risk averse, the choices made by the high-valuation segment differ significantly from the low-valuation segment ($\chi^2_{1,3} = 24.90, p < .001$). According to Proposition 1.1, the most effective sweepstakes for the high valuation consumers who have low risk aversion is a single grand prize. Consistent with the proposition, the grand prize only sweepstake is the most preferred option with 52.10 % of the subjects selecting this option. Proposition 2.1 states that the Grand Prize and multiple small prizes is the best sweepstake if the target are consumers with low-brand valuation and high risk aversion. Also, consistent with the prediction, 59.02 % of the subjects selected this sweepstake.

Now we examine the results when consumers differ in terms of their risk aversion. Within the high-brand valuation condition, there is a significant difference in choices between the low and high risk averse subjects ($\chi^2_{1,3} = 50.46, p < .001$). Similarly, there is also a significant difference between the low and high-risk averse subjects when the valuation of brands is low ($\chi^2_{1,3} = 23.87, p < .001$). For consumers who have high brand valuation but are risk averse, the most effective sweepstake according to Proposition 1.2 is to have multiple prizes. However, this option was selected by 36.13 % of the subjects in contrast to 36.97 % of the subjects who selected Grand Prize+Small prize option. Similarly, in the high-risk averse condition where subjects had low-brand valuations, the predicted Multiple Large

Prize+Small prize sweepstake was selected by 39.13 % of the subjects as compared to 40.00 % who selected the Grand Prize+Small option.

The results for the Cola category provide stronger support for the model. As in the toothpaste category, comparison of the choices made by the high-valuation segment and the low-valuation segment reveals differences. The distribution of choices made by the high-valuation and low-valuation are significantly different in the low risk averse condition ($\chi^2_{1,3} = 70.19, p < .001$) as well as in the high risk averse condition ($\chi^2_{1,3} = 15.58, p < .01$). As predicted by Proposition 1.1, the Grand Prize only sweepstake is most preferred (62.86 %) by the high-brand valuation segment that has low risk aversion. Also, as predicted by Proposition 2.1, the Grand Prize+Small Prize sweepstake is the modal pick (58.65 %) for the high-brand valuation high-risk averse segment.

The results reveal that the level of risk aversion also has an impact on choices. The choices made by the high-brand valuation segment are significantly different between the low-risk averse and high risk-averse segment ($\chi^2_{1,3} = 75.20,\ p < .001$) as are the choices made by the low-brand valuation segment between the risk aversion conditions ($\chi^2_{1,3} = 23.68,\ p < .001$). As predicted by Proposition 1.2, the Multiple Prize sweepstake is most preferred (35.71 %) as predicted when the subjects have high-brand valuations but are risk averse. In the high-risk averse condition where the brand valuation is low, the most preferred sweepstake is Multiple Prize+Small Prizes (43.12 %) which is consistent with the prediction of Proposition 2.2. The difference in results between the toothpaste and the cola categories can be explained by the difference in the stimuli. Note that the stimuli for the Multiple Prize+Small Prizes in the cola category has a small spread between the prizes than it does in the toothpaste category.

The experiment finds strong support for Propositions 1.1 and 2.1. The results are directionally correct in support of Propositions 1.2 and 2.2 particularly for sweepstakes conducted for the cola category. A limitation of the experiment may account for the lack of similarly strong results for Propositions 1.2 and 2.2. Note that an infinite number of prize structures can be created for the 'multiple prize' category of sweepstakes. It is likely that the reward structure created was not the 'value-maximizing' one for the subject population. In the cola stimuli, there are more prizes in the 'multiple' prizes sweepstakes suggesting that the toothpaste stimuli was further away from the 'value-maximizing' reward structure. Therefore, even directional results can be viewed as quite encouraging. Another limitation of the experiment is that subjects did not pay for the products. Finally, subjects selected between several sweepstakes whereas brands offer only one option [6].

Overall, the experiment finds strong support for the sweepstake preferences predicted by the model when consumers vary in their valuations of the product and when they are not risk averse. In the case of high-risk aversion, the experiment finds directional support. We view this result not as a limitation of the model but rather in the implementation of the 'best' sweepstake design for the high-risk averse subject population.

Conclusion

Sweepstake and contests are commonly used as a promotional tool and the incidence of usage has been growing. Firms also devote large budgets to communicate the promotion to consumers. The purpose of this paper is to provide guideline on designing sweepstakes that will increase consumers' motivation to participate and thus generate additional sales. The results of the model show that the sweepstakes reward structure should be based on three factors: the objectives of the firm, the risk aversion of the customers and the level of sub-additivity. The results of the model prescribes that the firm should begin by setting sweepstake objectives in terms of either attracting switchers or targeting current users. If the objective is to target current users, then the number of prizes awarded should be lower than in the case where the targets are switchers. If the current users are risk neutral, then the consumer value-maximizing award is a single grand prize. If the current users are risk averse, then the award should consist of multiple "large" prizes. When the firm's objective is to draw sales away from competitors, the value-maximizing strategy is to distribute the award money over more prizes. If the non-current user segment is risk neutral with respect to gains but sufficiently risk averse in the domain of losses, then the prescribed reward structure is to have a single grand prize but also include several small prizes which ideally should be close to the opportunity cost of the customers. If the non-loyal customers are risk averse in gain and loss averse, then the best prize allocation is to have both multiple large prizes as well as several small prizes.

Another recommendation from the model analysis is that the firm should minimize the number of prizes at each level. In practice, the costs of implementing and communicating such a prize structure could be high. To trade-off between the logistical and communication

[6] Note that a field test will be required to gather data where subjects view only one sweepstakes.

costs and the theoretically value-maximizing approach, firms could increase the number of prizes at each level for easier implementation. A trade-off is involved between increasing the attractiveness of the sweepstake and the implementation costs of administering several levels of prizes. Often, when the prizes are products rather than cash, the firm may obtain quantity discounts for the products but the value of the products will be the same for the sweepstake participants.

An implicit assumption in the model is that the opportunity cost of switching brands also is homogenous. This is a major assumption but the implications when the assumption does not hold are also clear from the analysis. The firm needs to segment the low valuation consumers in terms of the amount of opportunity cost and then calculate the relative size of each segment. If the segment size is skewed towards high opportunity costs, the size of the smaller prizes needs to be increased. However, if the segment is skewed towards smaller opportunity costs, the smaller prizes should be of a lower amount. Another implicit assumption made in the analysis is that the level of customer risk aversion is homogenous. If the customers are heterogenous, then the firm should measure the extent of risk aversion and the degree of heterogeneity. Under certain conditions such as when there is a large segment of risk averse consumers, it is profitable to increase the level of prizes.

A factor that impacts the value-maximizing reward structure is the degree of sub-additivity. It is important that future research investigate the sweepstake design factors that affect subadditivity. As mentioned earlier, creating illusion of control by increasing the effort required to participate may be a fruitful avenue for future research. The effort levels can be thought of as either being incorporated in the game itself or in terms of effort required to submit the entry forms. Other potential factors likely to influence subadditivity that require further research are the degree of brand loyalty and whether the product is immediately consumed or if the consumption is delayed.

Another avenue for future research is to examine which of these two formats is more effective in generating additional sales and the conditions under which one approach will be superior to the other. Under certain conditions (e.g. products positioned as niche brands), consumers may under or over-estimate sweepstake participation rates thereby changing the perceived attractiveness of the contest. Several other formats of sweepstakes also deserve attention in future research. One format is a combination of risk prospects and uncertainty where the odds of winning *a* prize is provided but the specific odds of winning specific prizes are not. For example, in 2000, Folgers Coffee conducted a sweepstake where they announced what the prizes were but only specified that the odds of winning a prize was 1 in

7. Another format are sequential contests where the initial prize is a small fixed amount but the winners have the opportunity to play for larger amounts in subsequent rounds either as a sweepstake or based on skill (e.g. Beck's 2 million Deutsche Marks Golf Putt Sweepstake).

Though sweepstakes are used very frequently, there is no evidence how this promotional tool is more effective than others. It is important to identify the conditions under which sweepstakes are better than other options available to managers. Clearly, sweepstakes do not possess the disadvantage of reducing brand equity through price discounts. Also, it is likely that advertising the sweepstakes may attract more consumer attention than other forms of promotions. However, little is known regarding the relative benefits of sweepstakes and the conditions and/or segments they are likely to have more impact. Given the large expenditures incurred in sweepstakes, these issues merit future research.

Table 1

Sweepstake Choices (Toothpaste Category)

Buy Colgate:

_____ Tube 1 $ 200 Grand Prize(1)	1 out of 200 tubes contains the winning number for $ 200
_____ Tube 2: $ 10 Prize (20)	20 out of 200 tubes contain the winning number for $ 10
_____ Tube 3: $100 Grand Prize(1)	1 out of 200 tubes contains the winning number for $ 100
$ 2 First Prize (50)	50 out of 200 tubes contain the winning number for $ 2
_____ Tube 4: $ 5 First Prize (20)	20 out of 200 tubes contain the winning number for $ 5
$ 2 Second Prize (50)	50 out of 200 tubes contain the winning number for $ 2
_____ Buy Crest	

Sweepstake Choices (Cola)

Buy Diet Pepsi:

_____ Can 1: $ 175 Grand Prize (1)	1 out of 300 tubes contains the winning number for $ 175
_____ Can 2: $ 25 Grand Prize (1)	1 out of 300 tubes contains the winning number for $ 25
$ 5 First Prize (30)	30 out of 300 cans contain the winning number for $ 5
_____ Can 3 $ 75 Grand Prize(1)	1 out of 300 cans contains the winning number for $ 75
$ 1 Prize (100)	100 out of 300 cans contain the winning number for $ 1
_____ Can 4: $ 2.00 (50)	50 out of 300 cans contain the winning number for $ 2.00
$ 0.75 (100)	100 out of 300 cans contain the winning number for $ 0.75

_____ Buy Diet Coke

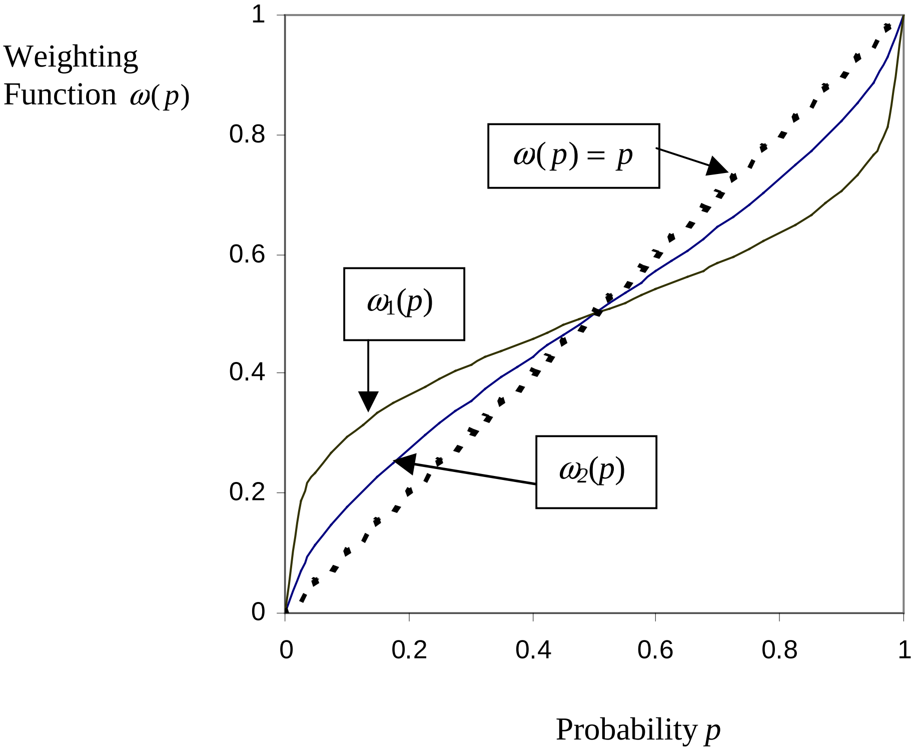
Table 2(a). Choice of Sweepstakes in Toothpaste Category

	Brand Valuation			
	High		Low	
Low Risk Aversion	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n = 119)	52.10 % 14.29 % 27.73 % 5.88 %	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n=122)	18.85 % 4.10 % 59.02 % 18.03 %
High Risk Aversion	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n = 119)	10.08 % 36.13 % 36.97 % 16.81 %	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n=115)	8.70 % 12.17 % 40.00 % 39.13 %

Table 2(b). Choice of Sweepstakes in Cola Category

	Brand Valuation			
	High		Low	
Low Risk Aversion	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n=105)	62.86 % 5.71 % 23.81 % 8.40 %	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple & Small (n=104)	7.69 % 19.23 % 58.65 % 14.42 %
High Risk Aversion	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple+Small (n=112)	8.93 % 35.71 % 32.14 % 23.21 %	Only Grand Prize Multiple Prizes Grand Prize & Small Multiple+Small (n=109)	2.75 % 19.27 % 34.86 % 43.12 %

Figure 1: Decision Weighting Functions



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Appendix 1. Sweepstakes for High-valuation Segment

Equilibrium condition

(P2) $$\operatorname*{Max}_{s} V_H(S) = \omega(\frac{m_1}{N})g(r_1) + \sum_{j=2}^{n}\left(\omega(\frac{\sum_{k=1}^{j} m_k}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j) + \left(1-\omega(\frac{\sum_{j=1}^{n} m_j}{N})\right)g(0)$$

$$\text{s.t. } \sum_{j=1}^{n} m_j r_j = R$$

We characterize value-maximizing sweepstakes based on equilibrium conditions. Let the value-maximizing sweepstakes be $S^* = \{r_1^*, m_1^*; r_2^*, m_2^*; ...r_n^*, m_n^*\}$. Consider a very small amount of prize (∇r) reduced from r_k^* and allocated to r_j^* ($1\leq j, k\leq n, j\neq k$), while prizes of all other ranks remaining the same. With total budget R fixed, r_k^* should decrease by $m_j \nabla r / m_k$.

Therefore, $dr_k / dr_j = -m_j / m_k$. Such an allocation should not change anticipated value from sweepstakes, that is,

$$\frac{dV_H(S^*)}{dr_j} = \omega_j g'(r_j^*) + \omega_k g'(r_k^*)\frac{dr_k}{dr_j} = m_j\left[\frac{\omega_j}{m_j}g'(r_j^*) - \frac{\omega_k}{m_k}g'(r_k^*)\right] = 0 \qquad \text{(A1)}$$

Equation (A1) leads to equilibrium condition for value-maximizing sweepstakes of (P2):

$$\frac{\omega_j}{m_j}g'(r_j^*) = M_H^* \quad (j = 1,2,..n) \qquad \text{(A2)}$$

Proof there is only one winner at each rank when salespeople are risk averse in gain

Suppose there are two winners at j^{th} rank for prize r_j. Consumer's anticipated value from this rank of prize is equal to

$$\left(\omega\left(\left(\sum_{k=1}^{j-1} m_k + 2\right)\Big/N\right) - \omega\left(\left(\sum_{k=1}^{j-1} m_k\right)\Big/N\right)\right)g(r_j) =$$

$$\left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j) + \left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N})\right)g(r_j)$$

For a positive and sufficiently small σ, we can reallocate prize ($2r_j$) into (r_j -σ) and (r_j +σ), keeping other prizes and their associated decision weights unchanged. With such a change in prize structure, the anticipated value from these two prizes become

$$\left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k}{N})\right)g(r_j+\sigma) + \left(\omega(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}) - \omega(\frac{\sum_{k=1}^{j-1} m_k + 1}{N})\right)g(r_j-\sigma)$$

Then the change in anticipated value of sweepstakes resulting from prize reallocation is

$$\left(\omega\left(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}\right) - \omega\left(\frac{\sum_{k=1}^{j-1} m_k}{N}\right)\right)\left(g(r_j+\sigma)-g(r_j)\right) - \left(\omega\left(\frac{\sum_{k=1}^{j-1} m_k + 2}{N}\right) - \omega\left(\frac{\sum_{k=1}^{j-1} m_k + 1}{N}\right)\right)\times$$
$$\left(g(r_j)-g(r_j-\sigma)\right) \qquad \text{(A3)}$$

In (A3), according to equation (6), $\omega\left(\left(\sum_{k=1}^{j-1} m_k + 1\right)\Big/N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)$ is larger than $\omega\left(\left(\sum_{k=1}^{j-1} m_k + 2\right)\Big/N\right) - \omega\left(\left(\sum_{k=1}^{j-1} m_k + 1\right)\Big/N\right)$. Moreover, *the difference is strictly positive and independent of σ*. On the other hand, when σ becomes smaller, the difference between $\left(g(r_j)-g(r_j-\sigma)\right)$ and $\left(g(r_j+\sigma)-g(r_j)\right)$ decreases and eventually approaches to zero. Therefore, there exists a positive and sufficiently small σ^* so that (A3) is positive for any $0<\sigma<\sigma^*$. In other words, the firm can increase the anticipated value of sweepstakes promotion by reallocating ($2r_j$) into (r_j-σ) and (r_j+σ) as long as σ is small enough. Thus, it is not value-maximizing to have a rank with more than one winner. Instead, only a single winner should be awarded for every level of prize.

Appendix 2: Sweepstakes for Low-valuation Segment

(P3)
$$\underset{s}{Max}\, V_L(S) = \omega\left(\frac{m_1}{N}\right) g(r_1-\tau) + \sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) g(r_j-\tau)$$
$$- \sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_k \Big/ N\right) - \omega\left(\sum_{k=1}^{j-1} m_k \Big/ N\right)\right) l(\tau-r_j) - \left(1-\omega\left(\sum_{j=1}^{n} m_j \Big/ N\right)\right) l(\tau)$$
$$\text{s.t.} \quad \sum_{j=1}^{n} m_j r_j = R$$

<u>Proof that the lowest prize should be at least as large as opportunity cost τ.</u>

Suppose a sweepstake *S* includes a prize smaller than opportunity cost τ. Without loss of generality, we let $S=\{r_1,m_1;r_2,m_2;...;r_n,m_n\}$ where $r_n m_n = R_n$, $\sum_{j=1}^{n} m_j r_j = R$, and $r_n<\tau$.

Now we show that under the same budget (*R*), we can enhance the valuation of sweepstakes with an increase in lowest reward r_n. Keeping $\{r_1,m_1;r_2,m_2;...;r_{n-1},m_{n-1}\}$ the same, we let r_n increase by a very small amount while keeping R_n (hence total expense *R*) unchanged. With a constant R_n, since $m_n = R_n \big/ r_n$, an increase in r_n implies a decrease in m_n. An increase in r_n will lead to following changes in the low-valuation consumers' anticipated value from sweepstakes participation:

$$\frac{\partial V_L(S)}{\partial r_n} = \frac{\partial \omega_n}{\partial r_n}\left[-l(\tau-r_n)\right] + \omega_n l'(\tau-r_n) + \frac{\partial \omega_n}{\partial r_n}\left[l(\tau)\right]$$

$$= \frac{m_n}{N}\left[\frac{\omega(\sum_{j=1}^{n} m_j \big/ N) - \omega(\sum_{j=1}^{n-1} m_j \big/ N)}{m_n/N} l'(\tau - r_n) - \omega'\left(\sum_{j=1}^{n} m_j \Big/ N\right)\frac{l(\tau) - l(\tau - r_n)}{r_n}\right] > 0 \quad \text{(A4)}$$

The above inequality always holds because a) $\frac{\omega(\sum_{j=1}^{n} m_j / N) - \omega(\sum_{j=1}^{n-1} m_j / N)}{m_n/N} > \omega'\left(\sum_{j=1}^{n} m_j / N\right)$ due to the *s*-shaped decision weighting function, and b) $l'(\tau - r_n) > \frac{l(\tau) - l(\tau - r_n)}{r_n}$ due to the concavity of the loss function. Since $\frac{\partial V_L(S)}{\partial r_n} > 0$, sweepstakes valuation can be enhanced with an increase in r_n. Therefore a sweepstakes $S = \{r_1, m_1; r_2, m_2; ...; r_n, m_n\}$ with $r_n < \tau$ is not value-maximizing. The lowest prize in value-maximizing sweepstakes should be at least as large as opportunity cost τ.

Equilibrium Condition

As in Appendix 1, we characterize value-maximizing sweepstakes based on equilibrium conditions. Let the value-maximizing sweepstakes be $S^* = \{r_1^*, m_1^*; r_2^*, m_2^*; ...r_n^*, m_n^*\}$. Since the smallest prize should be as large as τ, we let $r_n^* = \tau$. We now consider a very small amount of prize (∇r) *deducted from* r_k^* and allocated to r_j^* ($1 \le j, k \le n, j \ne k$), while prizes of all other ranks remain the same. With total budget *R* fixed, r_k^* should increase by $m_j \nabla r / m_k$. Therefore, $dr_k / dr_j = -m_j / m_k$. Anticipated value from sweepstakes would then change by:

$$\frac{dV_L(S^*)}{dr_j} = \omega_j g'(r_j^* - \tau) + \omega_k g'(r_k^* - \tau)\frac{dr_k}{dr_j} = m_j\left[\frac{\omega_j}{m_j} g'(r_j^* - \tau) - \frac{\omega_k}{m_k} g'(r_k^* - \tau)\right] = 0 \quad (j,\ k \ne n) \quad \text{(A5)}$$

$$\frac{dV_L(S^*)}{dr_n} = \omega_n g'(r_n^* - \tau) + \omega_k g'(r_k^* - \tau)\frac{dr_k}{dr_n} = m_n\left(\frac{\omega_n}{m_n} g'(0) - \frac{\omega_k}{m_k} g'(r_k^* - \tau)\right) \quad (j=n) \quad \text{(A6)}$$

$$\frac{dV_L(S^*)}{dr_j} = \omega_j g'(r_j^* - \tau) + \omega_n l'(r_k^* - \tau)\frac{dr_n}{dr_j} = m_j\left(\frac{\omega_j}{m_j} g'(r_j^* - \tau) - \frac{\omega_n}{m_n} l'(0)\right) \quad (k=n) \quad \text{(A7)}$$

Equation (A5) characterizes equilibrium condition for first (*n*-1) prizes,

$$\frac{\omega_j}{m_j} g'(r_j^* - \tau) = M_L^* \quad (j = 1,2,..n\text{-}1) \quad \text{(A8)}$$

Similar to (A2), (A8) requires identical anticipated value generating ability M_L^* from the top (*n*-1) prizes. At the bottom prize that equals to switching cost τ, anticipated value is not differentiable because of loss aversion. The value-maximizing sweepstakes contain a bottom prize equal to τ if

$$\frac{\omega_n}{m_n} l'(0) \ge M_L^* \ge \frac{\omega_n}{m_n} g'(0) \quad \text{(A9)}$$

Condition (A9) ensures that reducing the lowest prize will reduce the anticipated value.

When low-valuation consumers are risk-averse in gain, (A8) and (A9) indicate that the value-maximizing sweepstakes is to have multiple big prizes in addition to the bottom prize equal to τ. Following exactly the same logic as given in second part of Appendix 1, we can show that number of winners for each rank of big prizes should be equal to one. Then the equilibrium condition (A8) becomes:

$$\left(\omega(\frac{j}{N})-\omega(\frac{j-1}{N})\right)g'(r_j^*-\tau)=M_L^* \ (j = 1,2,..n\text{-}1) \qquad \text{(A10)}$$

Value-maximizing number of lowest-prize winners

We now analyze the value-maximizing number of winners for the lowest prize (τ). Consider a very small increase (∇m) in number of last-prize winners (m_n^*) and a decrease in r_j^* ($1\le j\le n\text{-}1$), while keeping prizes of all other ranks the same. To maintain the same total budget R, r_j^* should decrease by $\tau\nabla m$. Therefore, $\partial r_j/\partial m_n=-\tau$. Such a reallocation should not change the low-valuation consumers' anticipated value of sweepstakes:

$$\frac{\partial V_L(S^*)}{\partial m_n}=\omega_j g'(r_j^*-\tau)\frac{\partial r_j}{\partial m_n}+\frac{\partial\omega\left(n-1+m_n^*/N\right)}{\partial m_n}\left(l(\tau)-l(0)\right)$$
$$=\tau\left(\frac{l(\tau)}{\tau}\frac{\omega'\left(n-1+m_n^*/N\right)}{N}-\omega_j g'(r_j^*-\tau)\right)=0 \qquad \text{(A11)}$$

Combining condition (A11) with (A10), we have the following equilibrium condition:

$$\left(\omega\left(j/N\right)-\omega\left(j-1/N\right)\right)g'(r_j^*-\tau)=\frac{l(\tau)}{\tau}\frac{\omega'\left(n-1+m_n^*/N\right)}{N} \quad (j=1,2,\ldots n\text{-}1)$$

which can be rewritten as

$$\omega'\left(n-1+m_n^*/N\right)=\frac{\omega\left(j/N\right)-\omega\left(j-1/N\right)}{1/N}\frac{g'(r_j-\tau)}{l(\tau)/\tau} \qquad (j=1,2,\ldots n\text{-}1) \qquad \text{(A12)}$$

In the special case of low-valuation consumers being risk-neutral in gain, the value-maximizing sweepstakes only offers one big prize; that is, n=2. Then condition (A12) simplifies to:

$$\omega'\left(1+m_2^*/N\right)=\frac{\omega\left(1/N\right)}{1/N}\frac{g'(r_1^*-\tau)}{l(\tau)/\tau} \qquad \text{(A13)}$$