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A Panel-CADF Test for Unit Roots

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Abstract

In this paper we propose the extension of the covariate-augmented Dickey Fuller (CADF) test for unit roots developed by Hansen (1995) to the panel case. We show that the extension is viable and gives power gains with respect to the time series approach. Particular attention is paid to cross-unit dependence.

JEL classification: C12, C22, C23
Key words: Unit root, Panel data, Cross-unit dependence

1 Introduction

It is well known that standard unit root tests suffer from low power (see e.g. Campbell and Perron, 1991). Starting from the mid-nineties, it has been proposed that a viable way to increase power in unit root testing is to exploit cross section variation together with univariate dynamics (see Quah, 1994; Levin et al., 2002, among others). Panel unit root tests have become increasingly popular since then. Of course, potential power gains are not the only reason for using panel data. In fact, some specific cross-country macroeconomic analyses, such as the PPP hypothesis or the study of convergence, fit naturally in a macro-panel framework. However, the power gain argument has been forcefully put forward in a number of papers and it has been questioned only recently (see e.g. Banerjee et al., 2005).

In order to obtain more powerful unit root tests, Hansen (1995) followed instead a different route. Rather than using panel data on a single variable, Hansen (1995) suggested using stationary covariates in an otherwise standard Dickey-Fuller framework, therefore proposing his covariate augmeted Dickey-Fuller test (CADF). Hansen (1995) and Caporale and Pittis (1999) show that substantial power gains can be achieved using the CADF test. Interestingly, they also show that power increases can be obtained without incurring in severe size distortions.

In this paper we couple the two approaches,[1] extending the CADF test to panels, with the aim of obtaining even more powerful tests. This extension is carried out using an approach advocated in Choi (2001). This method essentially combines p-values from a unit root test

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[1] A panel unit root test with covariates has been developed independently by Chang and Song (2005) using a rather different approach from the one proposed here.

applied to each group in the panel data. There are several reasons for this choice. First, as far as we can compute the p-values for the CADF distribution, the extension to panels is straightforward. Second, the asymptotics carries through for $T \to \infty$, without requiring also $N \to \infty$ as other approaches do: in our view, given that allowing $N \to \infty$ in typical macro-panel applications is an implausible hypothesis, this is an extremely important feature of the tests based on Choi (2001). Third, we don't need balanced panel data sets (individual time series may be of different lengths and span different sample periods). Fourth, the stochastic as well as the non stochastic components can be different across individual time series. Last, the alternative hypothesis is not necessarily that all the individual time series are stationary: the alternative where some individual time series have a unit root and others do not can be dealt with by the approach developed in Choi (2001).

The methodology advocated in Choi (2001) is based on the hypothesis that the individual time series are cross sectionally independent. Indeed, this is a common assumption in many papers dealing with panel unit roots and panel cointegration (see Banerjee, 1999; Baltagi and Kao, 2000, for comprehensive surveys). However, it is well known (see e.g. O'Connell, 1998; Banerjee et al., 2005) that both short-term and long-run cross section dependence adversely affects the performance of these panel unit roots tests. Therefore, we extend the approach to cross sectionally dependent panel units by using the correction suggested in Hartung (1999) and Demetrescu et al. (2006). This correction is derived under rather strict conditions, but Demetrescu et al. (2006) show that it is robust under more general conditions than those assumed in Hartung (1999); in particular they show that Hartung's correction is applicable in the case of Dickey-Fuller based panel unit root tests.

The paper is organized as follows. Section 2 is devoted to a brief discussion of the test proposed in Hansen (1995). We also illustrate the method we use to obtain the necessary p-values. In Section 3 an extensive Monte Carlo analysis is carried out with both independent and cross-dependent time series. Both short-run and long-run cross-unit dependence are considered. The last section concludes.

2 The CADF test and the p-values approximation

The CADF test proposed in Hansen (1995) starts from the idea that real economic phenomena are not univariate in general. Therefore, using extra information in unit root testing can make test regressions more efficient, allowing more precise inferences.

Formally, Hansen (1995) assumes that the series we want to test for a unit root can be written as

$$\begin{aligned} y_t &= d_t + s_t && (1) \\ a(L)\Delta s_t &= \delta s_{t-1} + v_t && (2) \\ v_t &= \boldsymbol{b}(L)' \left(\Delta \boldsymbol{x}_t - \boldsymbol{\mu}_x\right) + e_t && (3) \end{aligned}$$

where d_t is a deterministic term (usually a constant or a constant and a linear trend), $a(L) := (1 - a_1 L - a_2 L^2 - \ldots - a_p L^p)$ is a polynomial in the lag operator L, $\boldsymbol{x}_t$ is an m-vector, $\boldsymbol{\mu}_x = \mathrm{E}(\boldsymbol{x})$, $\boldsymbol{b}(L) := (\boldsymbol{b}_{q_2} L^{-q_2} + \ldots + \boldsymbol{b}_{q_1} L^{q_1})$ is a polynomial where both leads and lags are allowed. Furthermore, consider the long-run covariance matrix

$$\boldsymbol{\Omega} := \sum_{k=-\infty}^{\infty} \mathrm{E}\left[\begin{pmatrix} v_t \\ e_t \end{pmatrix} \begin{pmatrix} v_{t-k} & e_{t-k} \end{pmatrix} \right] = \begin{pmatrix} \sigma_v^2 & \sigma_{ve} \\ \sigma_{ve} & \sigma_e^2 \end{pmatrix} \qquad (4)$$

and define the long-run squared correlation between v_t and e_t as

$$\rho^2 := \frac{\sigma_{ve}^2}{\sigma_v^2\, \sigma_e^2}\,. \qquad (5)$$

When $\Delta \boldsymbol{x}_t$ explains nearly all the zero-frequency variability of v_t, then $\rho^2 \approx 0$. On the contrary, when $\Delta \boldsymbol{x}_t$ has no explicative power on the long-run movement of v_t, then $\rho^2 \approx 1$.

Similarly to the conventional ADF test, the CADF test is based on three different models representing the "no-constant", "with constant", and "with constant and trend" case, respectively

$$a(L)\Delta y_t = \delta y_{t-1} + \boldsymbol{b}(L)'\Delta \boldsymbol{x}_t + e_t \tag{6}$$
$$a(L)\Delta y_t = \mu + \delta_\mu y_{t-1} + \boldsymbol{b}(L)'\Delta \boldsymbol{x}_t + e_t \tag{7}$$
$$a(L)\Delta y_t = \mu^* + \theta\, t + \delta_\tau y_{t-1} + \boldsymbol{b}(L)'\Delta \boldsymbol{x}_t + e_t \tag{8}$$

and is computed as the t-statistic for δ, $\widehat{t(\delta)}$. Hansen (1995, p. 1154) proves that under the unit-root null, the asymptotic distribution of $\widehat{t(\delta)}$ in (6) is

$$\widehat{t(\delta)} \xrightarrow{w} \rho \frac{\int_0^1 W\, dW}{\left(\int_0^1 W^2\right)^{1/2}} + \left(1-\rho^2\right)^{1/2} \mathrm{N}(0,1) \tag{9}$$

where W is a standard Wiener process and $\mathrm{N}(0,1)$ is a standard normal independent of W. Therefore, the asymptotic distribution is a mixture of a Dickey-Fuller and a standard normal distribution. It can be noticed that, if $\rho^2 \neq 1$, the conventional ADF test is conservative (and therefore has low power). The distribution depends on the nuisance parameter ρ^2 but, provided ρ^2 is given, it can be simulated using standard techniques. The mathematical expression remains unchanged if a model with constant ($\widehat{t(\delta_\mu)}$) or a model with constant and trend ($\widehat{t(\delta_\tau)}$) are considered, except that demeaned and detrended Wiener processes are used instead of the standard Wiener process W.

In order to extend Hansen's CADF unit root test to the panel case using the approach outlined in Choi (2001), we need to compute the p-values of the CADF unit root distribution.

We derive the quantiles of the asymptotic distribution for different values of ρ^2. Given that our goal is the computation of p-values, we simulate the distributions for 40 values of ρ^2 ($\rho^2 = 0.025, 0.05, 0.0725, \ldots, 1$) using 100,000 replications for each value of ρ^2 and $T = 5000$ as far as the Wiener functionals are concerned.[2] From the simulated values we derive 1005 estimated asymptotic quantiles, (0.00025, 0.00050, 0.00075, 0.001, 0.002, ..., 0.998, 0.999, 0.99925, 0.99950, 0.99975).

Figure 1 reports the estimated asymptotic quantiles for the model with constant, without any smoothing. The surface is extremely regular.[3] Similar considerations carry over for the "no constant" and the "constant plus trend" cases. Therefore we expect that the simulated values can be successfully used to derive asymptotic p-values along lines similar to MacKinnon (1996).

In order to derive p-values from tabulated quantiles of a given distribution, MacKinnon (1996, p. 610) proposed using a local approximation of the kind

$$\Phi^{-1}(p) = \gamma_0 + \gamma_1 \widehat{q(p)} + \gamma_2 \widehat{q(p)}^2 + \gamma_3 \widehat{q(p)}^3 + \nu_p \tag{10}$$

where $\Phi^{-1}(p)$ is the inverse of the cumulative standard normal distribution function evaluated at p and $\widehat{q(p)}$ is the estimated quantile.[4] Equation (10) is not estimated globally (as one would do with a standard response surface). Rather, it is estimated only over a relatively small number of points, in order to obtain a *local* approximation (see MacKinnon, 1996, p. 610, for details).

With respect to MacKinnon (1996), we have the extra difficulty that we have to deal with the nuisance parameter ρ^2, so that the local approximation must be obtained along two dimensions. However, given that quantiles change fairly smoothly by varying ρ^2, and given that we can at best have only a consistent estimate of ρ^2 (which is unknown in real situations), we decided to adopt a rather straightforward two-step procedure. In a first step we interpolate the

[2] Simulations have been carried out using `R` (see R Development Core Team, 2006).

[3] Figure 1 reports the estimated asymptotic quantiles using a coarser resolution than the one used in the computations.

[4] In MacKinnon (1996) approximate finite sample quantiles are used, instead of the asymptotic ones.

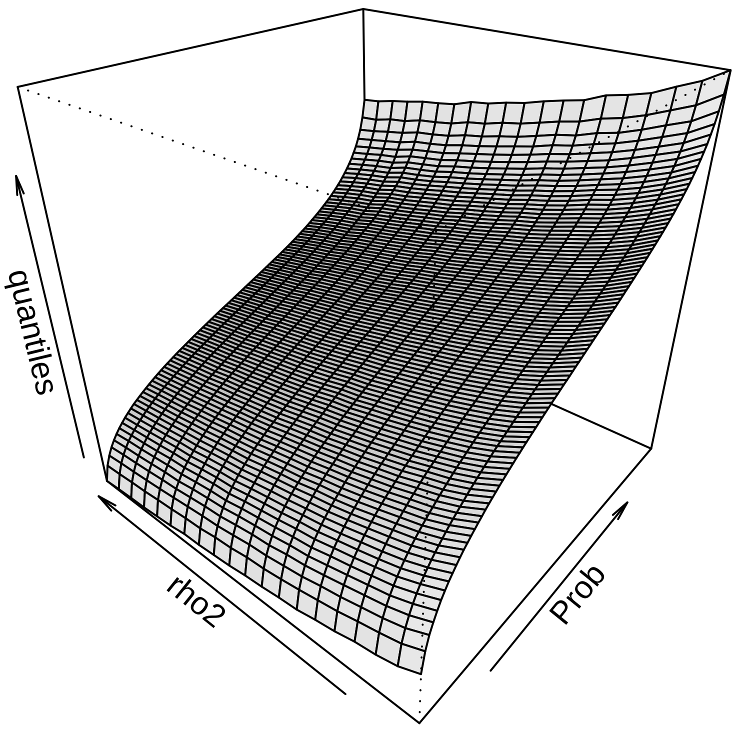

Figure 1: Estimated asymptotic quantiles for $t(\hat{\delta}^{\mu})$.

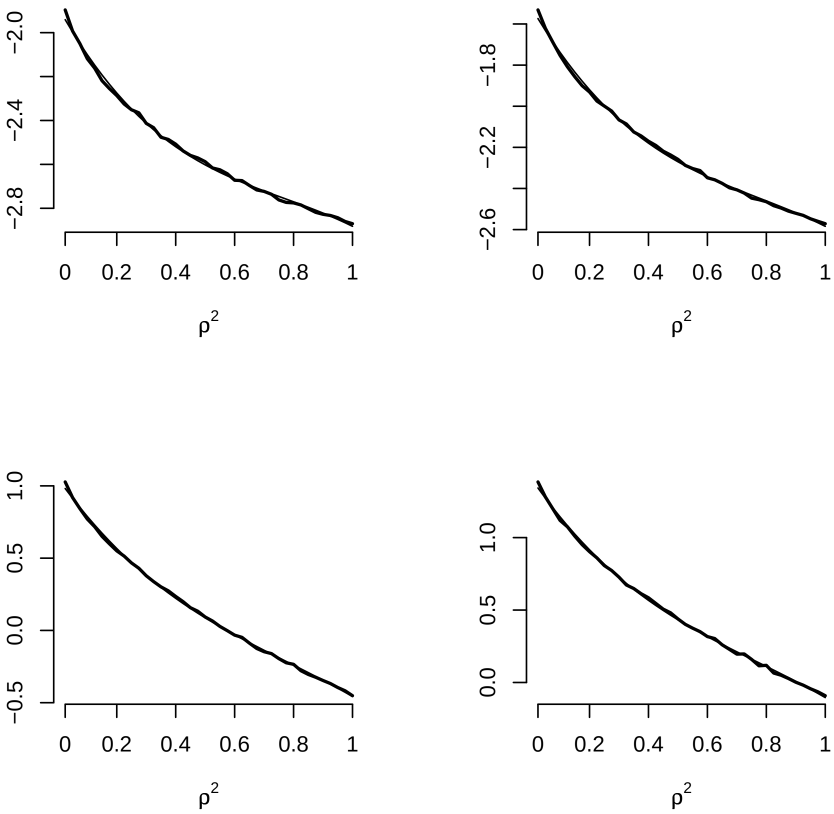

Figure 2: Interpolation of the quantiles $\widehat{q_\rho(p)}$. From upper-left clockwise: $\alpha = 5\%$, $\alpha = 10\%$, $\alpha = 95\%$, $\alpha = 90\%$. The thick solid lines are the simulated quantiles. The thin lines are the interpolated values.

quantiles $\widehat{q(p)}$ to obtain an approximation for the relevant estimated value of ρ^2. In practice we use

$$\widehat{q_\rho(p)} = \beta_0 + \beta_1 \rho^2 + \beta_2 \rho^4 + \beta_3 \rho^6 + \varepsilon_\rho \tag{11}$$

where we have used the subscript ρ in $\widehat{q_\rho(p)}$ to indicate the dependence of the quantiles on ρ^2. Interpolation is always very good, as can be gathered from Figure 2.

As a by-product of our analysis, we compute a detailed table of asymptotic critical values of the CADF test using equation (11) (see Table 1).

Then we apply the procedure advocated in MacKinnon (1996) on the interpolated quantiles to obtain the p-values.

Once we can obtain asymptotic p-values from the CADF distribution, the extension to the panel case is immediate. In fact, Choi (2001) shows that under some regularity conditions

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\widehat{p}_i) \xrightarrow{w} \mathrm{N}(0,1) \tag{12}$$

as $T \to \infty$, where $N < \infty$ is the number of individual time series, $\Phi(\cdot)$ is the standard normal cumulative distribution function, and $\widehat{p}_i$ are the estimated individual p-values for $i = 1, \dots, N$.[5] The null hypothesis is $\mathrm{H}_0 : \delta = 0 \,\forall i$, while the alternative is $\mathrm{H}_1 : \delta < 0$ for at least one i, with $i = 1, 2, \dots, N$. This alternative allows more flexibility with respect to those tests whose alternative is $\mathrm{H}_1 : \delta < 0 \,\forall i$ (see e.g. Levin and Lin, 1993; Quah, 1994).

3 Monte Carlo analysis

In this Section we evaluate the performance of the CADF test both in the pure time series framework ($N = 1$), as well as in the panel context ($N > 1$). As far as the panel case is considered, both the cross-sectionally independent and the cross-sectionally dependent case are examined. The latter is investigated both in the presence of short-run and long-run dependence. All experiments are based on 5,000 replications.

3.1 Independent time series units and short-run cross-dependence

We start by generalizing the Data Generating Process (DGP) used in Hansen (1995, p. 1161) and Caporale and Pittis (1999, p. 586) in order to cope with the panel dimension. For each unit $i = 1, \dots, N$ we have

$$\Delta y_{i,t} = -\frac{c}{T} y_{i,t-1} + u_{i,t} \tag{13}$$

$$\begin{pmatrix} u_{i,t} \\ \Delta x_{i,t} \end{pmatrix} = \begin{pmatrix} a_{i,11} & a_{i,12} \\ a_{i,21} & a_{i,22} \end{pmatrix} \begin{pmatrix} u_{i,t-1} \\ \Delta x_{i,t-1} \end{pmatrix} + \begin{pmatrix} e_{i,1t} \\ e_{i,2t} \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} e_{i,1t} \\ e_{i,2t} \end{pmatrix} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_i) \tag{15}$$

with

$$\boldsymbol{\Sigma}_i := \begin{pmatrix} \sigma_{i,11} & \sigma_{i,21} \\ \sigma_{i,21} & \sigma_{i,22} \end{pmatrix}. \tag{16}$$

Note that (14) can be written compactly as

$$\boldsymbol{w}_{i,t} = \boldsymbol{A}_i \boldsymbol{w}_{i,t-1} + \boldsymbol{\varepsilon}_{i,t} \tag{17}$$

with $\boldsymbol{w}_{i,t} := (u_{i,t}, \Delta x_{i,t})'$, $\boldsymbol{\varepsilon}_{i,t} := (e_{i,1t}, e_{i,2t})'$ and

$$\boldsymbol{A}_i := \begin{pmatrix} a_{i,11} & a_{i,12} \\ a_{i,21} & a_{i,22} \end{pmatrix}. \tag{18}$$

[5] Choi (2001) proposes also other tests based on the combination of the p-values. However, the Z test seems to have better properties. Choi's combination test is based on Lipták (1958). In Choi (2001) $T \to \infty$ is required for the relevant statistics to converge to a proper continuous distribution, under some regularity conditions. Maddala and Wu (1999) also use Fisher's combination test (Fisher, 1932) to derive a panel unit root test and show that Bonferroni-like methods have low power.

Therefore it is possible to write (14) $\forall i$ as

$$\boldsymbol{w}_t = \boldsymbol{A}\boldsymbol{w}_{t-1} + \boldsymbol{\varepsilon}_t \,. \tag{19}$$

with

$$\boldsymbol{A} := \begin{pmatrix} \boldsymbol{A}_1 & \mathbf{O} & \dots & \mathbf{O} \\ \mathbf{O} & \boldsymbol{A}_2 & \dots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \dots & \boldsymbol{A}_n \end{pmatrix} \tag{20}$$

and

$$\boldsymbol{\varepsilon} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{V}) \tag{21}$$

with $\boldsymbol{V}$ a $(2N \times 2N)$ covariance matrix. The $\boldsymbol{A}_i$'s are such that the resulting VARs are stationary. Short-run cross-unit dependence here is introduced when $\boldsymbol{V}$ is not block-diagonal.

3.1.1 Independent time series units

Consider first the case when $\boldsymbol{V} := \mathrm{diag}(\boldsymbol{\Sigma}_1, \dots, \boldsymbol{\Sigma}_N)$, that is

$$\boldsymbol{V} := \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{O} & \dots & \mathbf{O} \\ \mathbf{O} & \boldsymbol{\Sigma}_2 & \dots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \dots & \boldsymbol{\Sigma}_N \end{pmatrix} . \tag{22}$$

Denoting by $\boldsymbol{Z}_{t-1}$ the past of the variables and letting $\delta := -c/T$, we can write

$$\begin{aligned} \mathrm{E}(\Delta y_{i,t} | \Delta x_{i,t}, \boldsymbol{Z}_{t-1}) &= \delta y_{i,t-1} + \mathrm{E}(u_{i,t} | \Delta x_{i,t}, \boldsymbol{Z}_{t-1}) \\ &= \delta \left(1 - a_{i,11} + \frac{\sigma_{i,21}}{\sigma_{i,22}} a_{i,21} \right) y_{i,t-1} + (1+\delta) \left(a_{i,11} - \frac{\sigma_{i,21}}{\sigma_{i,22}} a_{i,21} \right) \Delta y_{i,t-1} + \\ &\quad + \frac{\sigma_{i,21}}{\sigma_{i,22}} \Delta x_{i,t} + \left(a_{i,12} - \frac{\sigma_{i,21}}{\sigma_{i,22}} a_{i,22} \right) \Delta x_{i,t-1} \end{aligned} \tag{23}$$

which is analogous to the expression derived in Caporale and Pittis (1999, p. 586). Therefore, depending on the values of the design parameters, the correct model is CADF(1,1,0). Testing for a unit root using an ADF(1) model would ignore $\Delta x_{i,t}$ and $\Delta x_{i,t-1}$, resulting in incorrect inference, unless $\sigma_{i,21} = a_{i,12} = 0$. An ADF($n$), with $n >> 1$ should otherwise be used if $0 < |a_{i,22}| < 1$.[6]

We consider $T \in \{50, 100, 150\}$ and $N \in \{1, 5, 10, 25\}$, where T is the time dimension and N is the number of individual time series. As in Hansen (1995), we set $\sigma_{i,11} = \sigma_{i,22} = 1 \; \forall i$. Furthermore, we set $a_{i,11} = a_{i,22} = 0 \; \forall i$, given that these parameters do not affect the nuisance parameter ρ^2. The experiments we consider are a subset of those examined in Hansen (1995) and in Caporale and Pittis (1999), where $\sigma_{i,21} \in \{0, 0.4\}$, $a_{i,21}$ and $a_{i,12} \in \{-0.3, 0, 0.3, 0.6\}$. We set $c = 0$ to investigate size properties, and $c = 4$ and $c = 8$ to verify the power of the test.

In practice, when $N > 1$ we generate N independent series using the same DGP (13)-(15) for each i. We investigate the size and power properties of the panel-CADF test both in the Choi (2001) version, as well as using the correction for dependent p-values proposed by Hartung (1999).[7] The reason is that, although we expect some form of dependence to be present across units in macro panels, nevertheless, in practice we can't rule out the possibility that we are correcting in the presence of genuinely cross-sectionally independent time series.

Each experiment is carried out over 5,000 replications using the demeaned version of the ADF(2) and CADF(1,1,0) tests. Consistently with (23), when $\sigma_{21} = a_{12} = 0$ an ADF(1) is used.[8] As far as the case for $N = 1$ is concerned, our results are nicely consistent with those reported

[6] An infinite number of lags should theoretically be used. If $a_{i,22} = 0$, then an ADF(2) would be sufficient. However, the residuals of the ADF(2) would be MA(1).

[7] In all instances, when the correction for cross-dependence is used, we refer to equation (2.4) in Hartung (1999), with $\lambda_i = 1$ and $\kappa = 0.2$, as suggested in Hartung (1999).

[8] Simulations have been carried out using `Gauss 6.0`.

in Hansen (1995). Furthermore, the simulations results reported in Table 2 indicate that the both the ADF and the CADF do not suffer from large size distortions. Both tests are oversized only when $\sigma_{21} = 0.4$, $a_{12} = a_{21} = 0.6$ with higher size distortions for the ADF test. Finally, notice also that correcting for dependence when this is absent makes the test undersized, with the size distortion growing with N.

Power results are reported in Tables 3 and 4 for $c = 4$ and $c = 8$, respectively. Consistently with Hansen (1995) and Caporale and Pittis (1999), we find considerable power gains under many parameters configurations.

Note that erroneously correcting for cross-dependence when the time series units are in fact cross-independent leads to sizeable power losses. However, these losses are larger for the ADF test. Especially as far as the CADF test is considered, under some parameters configurations power losses are absent or negligible.

3.1.2 Short-run dependence

Short-run cross-unit dependence is considered by allowing $\boldsymbol{V}$ to be a generic $(2N \times 2N)$ covariance matrix. The diagonal of $\boldsymbol{V}$ is a vector of ones, so that $\boldsymbol{V}$ is indeed a correlation matrix.

In our simulations we consider a "strong-correlation" and a "moderate-correlation" settings. In the "strong-correlation" scenario, the off-diagonal elements of $\boldsymbol{V}$ are such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.8)$. In the "moderate-correlation" setting we use $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.4)$. No special correlation structure is imposed on $\boldsymbol{V}$. Of course, in any instance $\boldsymbol{V}$ is generated in order to be symmetric and strictly positive definite.

As expected, if no correction is operated the tests are generally oversized (see Tables 5 and 6). However, the correction suggested in Hartung (1999) seems to work properly, especially so for the CADF test, and only minor problems remain.

As far as power is concerned (see Tables 7-10), correcting for cross-dependence reduces the power somewhat, as expected. However, correction in this DGP is essential in order not to incur in severe size distortions. The CADF test performs much better than the ADF, showing high power also in circumstances where the corrected ADF has no power at all.

3.2 Long-run dependence

It has already been emphasized that the assumption of cross-section independence is rather unrealistic, especially for macro-panels. Therefore we continue our investigation of the panel-CADF unit root test by running a number of Monte Carlo experiments based on a DGP originally proposed by Banerjee et al. (2005). This DGP, apart being empirically sound, can be very interesting to study the properties of our panel-CADF test. Again, the results are based on 5,000 replications with $N \in \{1, 5, 10, 25\}$ and $T \in \{50, 100, 150\}$. The DGP is discussed in detail in Banerjee et al. (2005) and is reported here for convenience.

Let y_{it} be the variable for which we want to test the unit root hypothesis, and x_{it} a possibly related (in a sense to be defined below) variable, where $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Furthermore, consider $\boldsymbol{Y}_t = (y_{1t}, \ldots, y_{Nt})'$ and $\boldsymbol{X}_t = (x_{1t}, \ldots, x_{Nt})'$. Data are generated using

$$\begin{pmatrix} \Delta \boldsymbol{Y}_t \\ \Delta \boldsymbol{X}_t \end{pmatrix} = \boldsymbol{\alpha}\boldsymbol{\beta}' \begin{pmatrix} \boldsymbol{Y}_{t-1} \\ \boldsymbol{X}_{t-1} \end{pmatrix} + \varepsilon_t \tag{24}$$

$$\varepsilon_t \sim \text{i.i.d. } \mathrm{N}(\boldsymbol{0}, \boldsymbol{\Sigma}) \tag{25}$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_{2N}^2 \end{pmatrix} \tag{26}$$

$$\sigma_j^2 \sim \mathrm{U}(0.5, 1.5) \quad j = 1, \ldots, 2N\,. \tag{27}$$

This setting allows us to test the size and power properties of the panel-CADF test with both cross-sectionally independent time series as well as with cross-sectionally dependent time series. The stationary covariates to be used in the unit root test are given by Δx_{it}. Given that an analysis under cross-section independence has been already discussed with reference to

Hansen (1995), here we will focus on the case under cross-section dependence. Dependence in this DGP is in the form of $N-b$ (with $0<b<N$) cointegrating relations between x_{it} and $x_{i+1,t}$, that Banerjee et al. (2005) label "cross-unit cointegration". Furthermore N within-unit cointegrating relations of the form $y_{it}-x_{it}$ are allowed for.

As in Banerjee et al. (2005), size properties are investigated by setting

$$\boldsymbol{\alpha} = 0.1 \begin{pmatrix} -\boldsymbol{I}_N & \mathbf{O}_N \\ \mathbf{O}_N & -\boldsymbol{I}_N \end{pmatrix} \tag{28}$$

$$\boldsymbol{\beta}' = \begin{pmatrix} \boldsymbol{I}_N & \boldsymbol{I}_N \\ \mathbf{O}_N & \boldsymbol{B}_N \end{pmatrix} \tag{29}$$

$$\boldsymbol{B}_N = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix} \tag{30}$$

so that all the $2N$ individual series are I(1), but there are less than $2N$ independent random walks. The number of independent random walks is given by b, the number of zero rows in $\boldsymbol{B}_N$. The degree of cross-unit cointegration is measured by $q := (N-b)/N$.

Our results (listed in Table 11) are not strictly comparable with those reported in Banerjee et al. (2005), given that they do not consider Choi-based unit root tests.[9] However, we want to stress that our results are broadly consistent with theirs, in that we find large size distorsions when no correction is carried out. When the correction suggested in Hartung (1999) is carried out, then the size of the tests is fairly well behaved, irrespective of CADF or ADF being used. Only in a few instances we find moderately undersized outcomes.

4 Concluding remarks

A Covariate-Augmented Dickey Fuller (CADF) test for unbalanced heterogeneous panels is proposed. The test is a generalization of Hansen (1995) and is developed along the lines suggested in Choi (2001). This allows us to be very general in the specification of the individual unit root tests. Furthermore, since the asymptotics used in Choi (2001) does not require $N \to \infty$, our test is especially well suited to deal with macroeconomic panels.

The size and power properties of the panel-CADF test are investigated. It is shown that the panel-CADF test does not suffer from important size distortions and is more powerful than the standard panel-ADF test.

Furthermore, it is shown that correcting for cross-dependence is important: we find that Hartung's correction gives good results in the presence of either short-run or long-run cross-unit dependence.

[9] Banerjee et al. (2005) examine the properties of the tests proposed by Levin and Lin (1992, 1993), Im et al. (2003), and Maddala and Wu (1999). When dealing with the ADF-based tests, consistently with Banerjee et al. (2005), we use an ADF(2) parameterization.

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Table 1: Asymptotic critical values of the CADF test

	Standard			Demeaned			Detrended		
ρ^2	1%	5%	10%	1%	5%	10%	1%	5%	10%
0.05	-2.426	-1.740	-1.380	-2.661	-1.987	-1.626	-2.794	-2.125	-1.767
0.10	-2.450	-1.770	-1.410	-2.760	-2.091	-1.733	-2.937	-2.274	-1.921
0.15	-2.470	-1.795	-1.436	-2.847	-2.183	-1.829	-3.063	-2.408	-2.058
0.20	-2.488	-1.818	-1.460	-2.924	-2.266	-1.915	-3.175	-2.527	-2.181
0.25	-2.503	-1.837	-1.481	-2.990	-2.339	-1.991	-3.274	-2.633	-2.291
0.30	-2.515	-1.854	-1.500	-3.049	-2.403	-2.060	-3.360	-2.727	-2.389
0.35	-2.525	-1.868	-1.517	-3.099	-2.460	-2.121	-3.436	-2.810	-2.476
0.40	-2.534	-1.880	-1.531	-3.142	-2.510	-2.174	-3.502	-2.883	-2.553
0.45	-2.540	-1.890	-1.544	-3.179	-2.554	-2.222	-3.560	-2.947	-2.622
0.50	-2.545	-1.898	-1.555	-3.211	-2.593	-2.265	-3.610	-3.005	-2.683
0.55	-2.550	-1.905	-1.565	-3.239	-2.628	-2.303	-3.654	-3.055	-2.738
0.60	-2.553	-1.911	-1.573	-3.264	-2.658	-2.338	-3.693	-3.101	-2.788
0.65	-2.555	-1.916	-1.581	-3.286	-2.686	-2.369	-3.729	-3.143	-2.834
0.70	-2.558	-1.921	-1.587	-3.307	-2.712	-2.399	-3.762	-3.183	-2.877
0.75	-2.560	-1.925	-1.593	-3.327	-2.737	-2.428	-3.794	-3.220	-2.919
0.80	-2.563	-1.929	-1.598	-3.348	-2.762	-2.456	-3.825	-3.258	-2.960
0.85	-2.566	-1.933	-1.603	-3.371	-2.787	-2.484	-3.858	-3.296	-3.002
0.90	-2.569	-1.938	-1.608	-3.395	-2.814	-2.514	-3.893	-3.336	-3.046
0.95	-2.574	-1.944	-1.613	-3.423	-2.843	-2.545	-3.932	-3.379	-3.093
1.00	-2.580	-1.950	-1.618	-3.455	-2.874	-2.580	-3.975	-3.427	-3.144

Table 2: Size of the ADF and CADF tests: independent time series units

				N=1		N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	CADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.049	0.058	0.054	0.039	0.058	0.052	0.050	0.028	0.057	0.036	0.049	0.012	0.056	0.017
50	0.0	0.6	0.3	0.055	0.060	0.049	0.040	0.067	0.066	0.051	0.028	0.075	0.053	0.049	0.013	0.082	0.028
50	0.0	0.6	-0.3	0.049	0.056	0.048	0.042	0.055	0.050	0.047	0.028	0.054	0.038	0.044	0.009	0.054	0.021
50	0.4	0.0	0.0	0.047	0.060	0.053	0.041	0.064	0.052	0.046	0.022	0.060	0.048	0.047	0.012	0.061	0.023
50	0.4	0.0	-0.3	0.050	0.054	0.053	0.042	0.054	0.059	0.048	0.029	0.061	0.042	0.049	0.011	0.056	0.021
50	0.4	0.0	0.3	0.045	0.063	0.052	0.042	0.064	0.060	0.054	0.030	0.072	0.043	0.045	0.011	0.079	0.028
50	0.4	0.0	0.6	0.045	0.061	0.052	0.040	0.058	0.058	0.052	0.029	0.066	0.042	0.057	0.015	0.076	0.026
50	0.4	0.3	0.0	0.054	0.055	0.044	0.036	0.052	0.047	0.050	0.024	0.037	0.029	0.052	0.013	0.038	0.012
50	0.4	0.6	0.0	0.049	0.058	0.049	0.042	0.050	0.043	0.050	0.028	0.045	0.029	0.045	0.012	0.034	0.013
50	0.4	0.6	0.6	0.061	0.068	0.083	0.060	0.079	0.069	0.088	0.051	0.071	0.055	0.115	0.039	0.079	0.042
100	0.0	0.0	0.0	0.052	0.059	0.051	0.042	0.052	0.046	0.051	0.025	0.048	0.031	0.051	0.012	0.044	0.016
100	0.0	0.6	0.3	0.051	0.051	0.050	0.040	0.067	0.050	0.053	0.025	0.067	0.040	0.045	0.012	0.074	0.021
100	0.0	0.6	-0.3	0.054	0.058	0.043	0.033	0.055	0.040	0.052	0.025	0.058	0.037	0.051	0.009	0.069	0.016
100	0.4	0.0	0.0	0.057	0.052	0.053	0.039	0.055	0.048	0.050	0.024	0.056	0.028	0.049	0.012	0.056	0.017
100	0.4	0.0	-0.3	0.050	0.054	0.052	0.039	0.058	0.046	0.054	0.027	0.059	0.034	0.045	0.011	0.059	0.013
100	0.4	0.0	0.3	0.054	0.055	0.045	0.035	0.059	0.052	0.050	0.028	0.064	0.032	0.054	0.012	0.068	0.018
100	0.4	0.0	0.6	0.055	0.056	0.049	0.042	0.058	0.048	0.050	0.026	0.057	0.035	0.052	0.013	0.065	0.017
100	0.4	0.3	0.0	0.045	0.052	0.052	0.040	0.044	0.042	0.046	0.025	0.047	0.023	0.048	0.011	0.034	0.013
100	0.4	0.6	0.0	0.055	0.057	0.050	0.039	0.045	0.037	0.048	0.026	0.040	0.025	0.046	0.010	0.035	0.009
100	0.4	0.6	0.6	0.059	0.061	0.076	0.058	0.067	0.058	0.101	0.055	0.075	0.043	0.114	0.037	0.086	0.032
150	0.0	0.0	0.0	0.047	0.058	0.051	0.043	0.049	0.041	0.050	0.026	0.051	0.041	0.053	0.014	0.044	0.001
150	0.0	0.6	0.3	0.051	0.055	0.049	0.038	0.058	0.042	0.053	0.026	0.060	0.042	0.053	0.012	0.073	0.002
150	0.0	0.6	-0.3	0.048	0.055	0.050	0.042	0.063	0.040	0.051	0.024	0.053	0.040	0.050	0.014	0.063	0.002
150	0.4	0.0	0.0	0.054	0.057	0.055	0.039	0.054	0.045	0.054	0.027	0.061	0.045	0.052	0.010	0.056	0.002
150	0.4	0.0	-0.3	0.047	0.052	0.055	0.041	0.053	0.041	0.054	0.025	0.054	0.041	0.049	0.012	0.054	0.002
150	0.4	0.0	0.3	0.049	0.052	0.047	0.036	0.059	0.056	0.049	0.024	0.061	0.056	0.047	0.012	0.061	0.003
150	0.4	0.0	0.6	0.050	0.061	0.051	0.038	0.054	0.039	0.051	0.030	0.056	0.039	0.049	0.013	0.065	0.002
150	0.4	0.3	0.0	0.050	0.049	0.053	0.044	0.041	0.046	0.048	0.025	0.040	0.046	0.049	0.010	0.036	0.000
150	0.4	0.6	0.0	0.053	0.050	0.047	0.038	0.043	0.041	0.047	0.022	0.042	0.041	0.042	0.009	0.037	0.000
150	0.4	0.6	0.6	0.068	0.054	0.076	0.057	0.060	0.056	0.084	0.048	0.074	0.056	0.121	0.037	0.087	0.003

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$).

Table 3: Power of the ADF and CADF tests: $c = 4$, independent time series units

				N=1		N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	CADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.088	0.106	0.254	0.114	0.320	0.136	0.435	0.123	0.556	0.151	0.799	0.106	0.916	0.128
50	0.0	0.6	0.3	0.096	0.080	0.235	0.108	0.371	0.139	0.422	0.119	0.507	0.154	0.775	0.117	0.810	0.212
50	0.0	0.6	-0.3	0.089	0.225	0.255	0.105	0.736	0.407	0.454	0.115	0.952	0.515	0.829	0.108	1.000	0.641
50	0.4	0.0	0.0	0.096	0.100	0.262	0.123	0.321	0.135	0.440	0.117	0.559	0.146	0.813	0.108	0.917	0.134
50	0.4	0.0	-0.3	0.088	0.108	0.246	0.111	0.324	0.138	0.445	0.121	0.559	0.146	0.816	0.109	0.919	0.139
50	0.4	0.0	0.3	0.090	0.100	0.238	0.107	0.320	0.144	0.441	0.111	0.555	0.148	0.815	0.115	0.925	0.131
50	0.4	0.0	0.6	0.086	0.105	0.241	0.111	0.325	0.141	0.447	0.121	0.558	0.151	0.812	0.108	0.921	0.128
50	0.4	0.3	0.0	0.085	0.106	0.242	0.119	0.340	0.149	0.427	0.116	0.596	0.160	0.795	0.102	0.940	0.157
50	0.4	0.6	0.0	0.095	0.119	0.244	0.111	0.360	0.165	0.420	0.111	0.642	0.191	0.780	0.102	0.950	0.186
50	0.4	0.6	0.6	0.110	0.126	0.309	0.158	0.320	0.114	0.532	0.179	0.555	0.159	0.874	0.228	0.823	0.251
100	0.0	0.0	0.0	0.097	0.100	0.295	0.124	0.340	0.143	0.513	0.136	0.566	0.150	0.876	0.119	0.926	0.127
100	0.0	0.6	0.3	0.092	0.111	0.292	0.131	0.466	0.133	0.493	0.127	0.584	0.176	0.860	0.126	0.881	0.236
100	0.0	0.6	-0.3	0.099	0.251	0.295	0.129	0.777	0.450	0.529	0.131	0.966	0.535	0.887	0.104	1.000	0.694
100	0.4	0.0	0.0	0.101	0.106	0.285	0.132	0.320	0.139	0.504	0.129	0.582	0.142	0.875	0.119	0.924	0.123
100	0.4	0.0	-0.3	0.093	0.097	0.283	0.125	0.319	0.137	0.504	0.124	0.573	0.148	0.887	0.117	0.928	0.122
100	0.4	0.0	0.3	0.089	0.105	0.281	0.121	0.333	0.143	0.506	0.130	0.569	0.131	0.880	0.114	0.916	0.128
100	0.4	0.0	0.6	0.101	0.104	0.280	0.120	0.331	0.143	0.519	0.133	0.553	0.145	0.887	0.117	0.926	0.134
100	0.4	0.3	0.0	0.094	0.112	0.282	0.126	0.338	0.147	0.493	0.127	0.598	0.160	0.876	0.125	0.939	0.149
100	0.4	0.6	0.0	0.094	0.121	0.277	0.114	0.382	0.165	0.482	0.124	0.653	0.193	0.860	0.108	0.964	0.184
100	0.4	0.6	0.6	0.125	0.125	0.389	0.187	0.416	0.011	0.637	0.221	0.659	0.184	0.948	0.244	0.965	0.291
150	0.0	0.0	0.0	0.099	0.103	0.302	0.124	0.325	0.140	0.520	0.130	0.579	0.145	0.902	0.116	0.926	0.124
150	0.0	0.6	0.3	0.088	0.119	0.307	0.125	0.541	0.124	0.514	0.130	0.614	0.015	0.882	0.119	0.896	0.256
150	0.0	0.6	-0.3	0.097	0.249	0.300	0.130	0.777	0.444	0.54	0.127	0.972	0.561	0.911	0.113	1.000	0.691
150	0.4	0.0	0.0	0.097	0.100	0.305	0.127	0.324	0.141	0.526	0.136	0.568	0.142	0.896	0.108	0.931	0.124
150	0.4	0.0	-0.3	0.099	0.109	0.294	0.125	0.319	0.132	0.516	0.134	0.576	0.141	0.896	0.110	0.924	0.120
150	0.4	0.0	0.3	0.090	0.101	0.298	0.124	0.335	0.141	0.544	0.138	0.576	0.146	0.898	0.108	0.927	0.118
150	0.4	0.0	0.6	0.096	0.104	0.296	0.124	0.319	0.131	0.542	0.139	0.566	0.139	0.890	0.114	0.928	0.118
150	0.4	0.3	0.0	0.099	0.115	0.290	0.121	0.342	0.150	0.515	0.129	0.605	0.155	0.892	0.115	0.941	0.145
150	0.4	0.6	0.0	0.093	0.121	0.282	0.117	0.381	0.174	0.502	0.124	0.659	0.193	0.884	0.106	0.968	0.175
150	0.4	0.6	0.6	0.119	0.122	0.408	0.192	0.457	0.152	0.683	0.224	0.715	0.214	0.964	0.222	1.000	0.289

5% nominal size. 5,000 replications. $c = 4$ in equation (13). ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$).

Table 4: Power of the ADF and CADF tests: $c = 8$, independent time series units

				N=1		N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	CADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.172	0.212	0.667	0.259	0.767	0.216	0.930	0.269	0.965	0.254	1.000	0.358	1.000	0.281
50	0.0	0.6	0.3	0.161	0.560	0.614	0.251	0.995	0.612	0.903	0.254	1.000	0.767	0.999	0.251	1.000	0.918
50	0.0	0.6	-0.3	0.186	0.256	0.709	0.263	0.811	0.252	0.954	0.285	0.977	0.308	1.000	0.281	1.000	0.392
50	0.4	0.0	0.0	0.178	0.307	0.670	0.246	0.908	0.320	0.934	0.269	0.996	0.387	1.000	0.281	1.000	0.466
50	0.4	0.0	-0.3	0.171	0.225	0.673	0.253	0.718	0.220	0.930	0.276	0.946	0.288	1.000	0.271	1.000	0.365
50	0.4	0.0	0.3	0.171	0.378	0.668	0.244	0.969	0.397	0.932	0.269	1.000	0.486	1.000	0.267	1.000	0.610
50	0.4	0.0	0.6	0.169	0.441	0.669	0.257	0.983	0.462	0.926	0.267	1.000	0.599	1.000	0.265	1.000	0.746
50	0.4	0.3	0.0	0.167	0.433	0.654	0.237	0.979	0.486	0.911	0.254	1.000	0.604	1.000	0.268	1.000	0.780
50	0.4	0.6	0.0	0.162	0.622	0.630	0.242	0.998	0.717	0.917	0.257	1.000	0.854	1.000	0.245	1.000	0.978
50	0.4	0.6	0.6	0.185	0.951	0.632	0.293	1.000	0.997	0.900	0.343	1.000	1.000	0.999	0.385	1.000	1.000
100	0.0	0.0	0.0	0.200	0.215	0.775	0.299	0.817	0.515	0.976	0.317	0.984	0.612	1.000	0.342	1.000	0.763
100	0.0	0.6	0.3	0.194	0.623	0.745	0.303	0.999	0.986	0.962	0.315	1.000	0.999	1.000	0.326	1.000	1.000
100	0.0	0.6	-0.3	0.200	0.272	0.794	0.306	0.846	0.618	0.978	0.329	0.991	0.748	1.000	0.336	1.000	0.910
100	0.4	0.0	0.0	0.196	0.328	0.776	0.300	0.935	0.736	0.976	0.326	0.998	0.854	1.000	0.337	1.000	0.976
100	0.4	0.0	-0.3	0.209	0.227	0.776	0.305	0.777	0.528	0.973	0.319	0.969	0.646	1.000	0.341	1.000	0.812
100	0.4	0.0	0.3	0.197	0.419	0.770	0.293	0.986	0.876	0.977	0.321	1.000	0.970	1.000	0.342	1.000	0.990
100	0.4	0.0	0.6	0.192	0.525	0.766	0.292	0.996	0.951	0.975	0.323	1.000	0.995	1.000	0.336	1.000	1.000
100	0.4	0.3	0.0	0.194	0.491	0.763	0.297	0.989	0.941	0.971	0.312	1.000	0.989	1.000	0.335	1.000	1.000
100	0.4	0.6	0.0	0.184	0.670	0.761	0.282	0.999	0.996	0.968	0.296	1.000	0.999	1.000	0.308	1.000	1.000
100	0.4	0.6	0.6	0.227	0.991	0.797	0.369	1.000	1.000	0.976	0.434	1.000	1.000	1.000	0.523	1.000	1.000
150	0.0	0.0	0.0	0.218	0.227	0.807	0.323	0.816	0.885	0.984	0.342	0.985	0.970	1.000	0.357	1.000	0.998
150	0.0	0.6	0.3	0.203	0.640	0.780	0.305	1.000	1.000	0.980	0.344	1.000	1.000	1.000	0.340	1.000	1.000
150	0.0	0.6	-0.3	0.206	0.273	0.815	0.323	0.859	0.923	0.984	0.336	0.992	0.984	1.000	0.372	1.000	1.000
150	0.4	0.0	0.0	0.216	0.321	0.815	0.325	0.945	0.983	0.982	0.342	1.000	0.999	1.000	0.358	1.000	1.000
150	0.4	0.0	-0.3	0.207	0.233	0.814	0.321	0.785	0.861	0.982	0.338	0.978	0.959	1.000	0.360	1.000	0.998
150	0.4	0.0	0.3	0.211	0.431	0.795	0.307	0.993	0.999	0.983	0.336	1.000	1.000	1.000	0.354	1.000	1.000
150	0.4	0.0	0.6	0.208	0.560	0.803	0.305	0.999	1.000	0.983	0.338	1.000	1.000	1.000	0.368	1.000	1.000
150	0.4	0.3	0.0	0.199	0.496	0.800	0.297	0.993	0.999	0.983	0.342	1.000	1.000	1.000	0.350	1.000	1.000
150	0.4	0.6	0.0	0.210	0.689	0.792	0.290	1.000	1.000	0.985	0.321	1.000	1.000	1.000	0.331	1.000	1.000
150	0.4	0.6	0.6	0.248	0.996	0.841	0.391	1.000	1.000	0.991	0.486	1.000	1.000	1.000	0.575	1.000	1.000

5% nominal size. 5,000 replications. $c = 8$ in equation (13). ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$).

Table 5: Size of the ADF and CADF tests: moderate cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.057	0.050	0.073	0.061	0.061	0.054	0.071	0.053	0.070	0.052	0.093	0.054
50	0.0	0.6	0.3	0.060	0.054	0.078	0.064	0.050	0.053	0.085	0.054	0.060	0.051	0.120	0.049
50	0.0	0.6	-0.3	0.052	0.053	0.053	0.044	0.038	0.051	0.068	0.053	0.057	0.050	0.085	0.050
50	0.4	0.0	0.0	0.058	0.048	0.066	0.056	0.062	0.055	0.078	0.055	0.069	0.054	0.096	0.552
50	0.4	0.0	-0.3	0.058	0.053	0.068	0.061	0.068	0.051	0.075	0.053	0.069	0.054	0.105	0.058
50	0.4	0.0	0.3	0.058	0.054	0.069	0.060	0.063	0.053	0.076	0.048	0.075	0.049	0.109	0.053
50	0.4	0.0	0.6	0.057	0.047	0.065	0.057	0.066	0.051	0.080	0.050	0.074	0.054	0.102	0.056
50	0.4	0.3	0.0	0.064	0.054	0.064	0.054	0.039	0.053	0.071	0.053	0.057	0.049	0.086	0.051
50	0.4	0.6	0.0	0.058	0.051	0.062	0.052	0.041	0.055	0.071	0.054	0.060	0.053	0.096	0.057
50	0.4	0.6	0.6	0.062	0.057	0.100	0.065	0.050	0.052	0.122	0.061	0.071	0.058	0.182	0.063
100	0.0	0.0	0.0	0.057	0.043	0.065	0.052	0.065	0.054	0.071	0.058	0.077	0.056	0.091	0.055
100	0.0	0.6	0.3	0.055	0.052	0.066	0.053	0.045	0.052	0.087	0.054	0.057	0.055	0.118	0.048
100	0.0	0.6	-0.3	0.056	0.052	0.056	0.055	0.034	0.052	0.066	0.056	0.058	0.052	0.083	0.055
100	0.4	0.0	0.0	0.065	0.042	0.067	0.050	0.061	0.053	0.074	0.057	0.080	0.056	0.094	0.051
100	0.4	0.0	-0.3	0.056	0.046	0.057	0.042	0.069	0.054	0.071	0.056	0.079	0.053	0.095	0.050
100	0.4	0.0	0.3	0.054	0.052	0.066	0.058	0.068	0.052	0.078	0.049	0.086	0.054	0.093	0.055
100	0.4	0.0	0.6	0.059	0.047	0.065	0.056	0.065	0.052	0.075	0.051	0.081	0.054	0.095	0.049
100	0.4	0.3	0.0	0.055	0.049	0.063	0.051	0.043	0.052	0.072	0.049	0.064	0.053	0.094	0.057
100	0.4	0.6	0.0	0.058	0.051	0.061	0.048	0.042	0.052	0.070	0.055	0.060	0.055	0.090	0.059
100	0.4	0.6	0.6	0.063	0.052	0.082	0.061	0.049	0.055	0.116	0.055	0.065	0.050	0.177	0.055
150	0.0	0.0	0.0	0.058	0.054	0.064	0.065	0.070	0.054	0.064	0.053	0.082	0.056	0.089	0.057
150	0.0	0.6	0.3	0.062	0.052	0.069	0.051	0.052	0.051	0.089	0.054	0.053	0.053	0.122	0.057
150	0.0	0.6	-0.3	0.053	0.051	0.055	0.061	0.036	0.051	0.065	0.051	0.061	0.049	0.080	0.052
150	0.4	0.0	0.0	0.061	0.050	0.066	0.047	0.064	0.051	0.072	0.056	0.085	0.055	0.097	0.051
150	0.4	0.0	-0.3	0.059	0.046	0.058	0.044	0.062	0.052	0.065	0.054	0.084	0.053	0.089	0.057
150	0.4	0.0	0.3	0.055	0.051	0.062	0.053	0.066	0.053	0.071	0.057	0.087	0.056	0.094	0.054
150	0.4	0.0	0.6	0.058	0.050	0.064	0.049	0.066	0.053	0.072	0.056	0.089	0.058	0.098	0.056
150	0.4	0.3	0.0	0.037	0.053	0.064	0.049	0.056	0.052	0.075	0.051	0.064	0.052	0.089	0.057
150	0.4	0.6	0.0	0.053	0.050	0.054	0.053	0.057	0.051	0.070	0.056	0.048	0.050	0.097	0.050
150	0.4	0.6	0.6	0.040	0.053	0.088	0.063	0.045	0.052	0.112	0.053	0.065	0.054	0.169	0.051

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Moderate cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.4)$ $(i \neq j)$.

Table 6: Size of the ADF and CADF tests: strong cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.074	0.058	0.088	0.064	0.085	0.044	0.109	0.055	0.100	0.033	0.149	0.052
50	0.0	0.6	0.3	0.047	0.030	0.093	0.067	0.053	0.024	0.121	0.061	0.066	0.019	0.157	0.065
50	0.0	0.6	-0.3	0.037	0.021	0.073	0.056	0.046	0.021	0.076	0.042	0.050	0.017	0.098	0.039
50	0.4	0.0	0.0	0.079	0.057	0.090	0.065	0.092	0.051	0.110	0.058	0.104	0.035	0.134	0.052
50	0.4	0.0	-0.3	0.083	0.060	0.085	0.059	0.091	0.052	0.108	0.060	0.112	0.039	0.135	0.050
50	0.4	0.0	0.3	0.073	0.052	0.090	0.066	0.089	0.047	0.104	0.060	0.107	0.035	0.150	0.061
50	0.4	0.0	0.6	0.075	0.056	0.096	0.071	0.088	0.048	0.123	0.066	0.111	0.040	0.143	0.052
50	0.4	0.3	0.0	0.040	0.031	0.084	0.062	0.050	0.020	0.097	0.052	0.062	0.017	0.118	0.047
50	0.4	0.6	0.0	0.037	0.022	0.081	0.063	0.046	0.021	0.096	0.054	0.064	0.019	0.122	0.046
50	0.4	0.6	0.6	0.053	0.028	0.114	0.079	0.070	0.028	0.149	0.084	0.094	0.027	0.209	0.084
100	0.0	0.0	0.0	0.079	0.051	0.083	0.056	0.097	0.040	0.101	0.048	0.121	0.034	0.133	0.040
100	0.0	0.6	0.3	0.034	0.017	0.084	0.053	0.047	0.013	0.118	0.050	0.071	0.012	0.156	0.048
100	0.0	0.6	-0.3	0.031	0.014	0.070	0.051	0.044	0.013	0.080	0.037	0.051	0.012	0.098	0.030
100	0.4	0.0	0.0	0.084	0.048	0.082	0.055	0.092	0.044	0.095	0.044	0.122	0.038	0.141	0.042
100	0.4	0.0	-0.3	0.072	0.047	0.081	0.053	0.098	0.045	0.095	0.046	0.119	0.036	0.126	0.039
100	0.4	0.0	0.3	0.076	0.051	0.086	0.059	0.103	0.044	0.102	0.051	0.117	0.037	0.133	0.046
100	0.4	0.0	0.6	0.079	0.053	0.086	0.058	0.102	0.048	0.100	0.047	0.126	0.035	0.140	0.043
100	0.4	0.3	0.0	0.040	0.020	0.071	0.047	0.054	0.018	0.089	0.047	0.070	0.012	0.123	0.040
100	0.4	0.6	0.0	0.036	0.016	0.080	0.050	0.045	0.015	0.090	0.045	0.056	0.011	0.128	0.044
100	0.4	0.6	0.6	0.047	0.018	0.111	0.065	0.059	0.019	0.142	0.068	0.082	0.016	0.203	0.068
150	0.0	0.0	0.0	0.074	0.048	0.078	0.046	0.095	0.042	0.096	0.048	0.121	0.034	0.132	0.037
150	0.0	0.6	0.3	0.039	0.016	0.085	0.061	0.046	0.012	0.117	0.053	0.064	0.010	0.151	0.046
150	0.0	0.6	-0.3	0.034	0.014	0.073	0.040	0.040	0.012	0.084	0.040	0.050	0.008	0.107	0.032
150	0.4	0.0	0.0	0.075	0.046	0.081	0.053	0.098	0.039	0.098	0.044	0.121	0.032	0.121	0.033
150	0.4	0.0	-0.3	0.081	0.051	0.079	0.050	0.095	0.035	0.093	0.040	0.129	0.035	0.132	0.033
150	0.4	0.0	0.3	0.076	0.051	0.082	0.054	0.094	0.041	0.099	0.044	0.124	0.032	0.131	0.037
150	0.4	0.0	0.6	0.082	0.050	0.088	0.057	0.104	0.043	0.106	0.043	0.127	0.032	0.140	0.038
150	0.4	0.3	0.0	0.037	0.022	0.075	0.048	0.055	0.018	0.094	0.042	0.070	0.012	0.129	0.037
150	0.4	0.6	0.0	0.028	0.012	0.079	0.048	0.044	0.013	0.087	0.041	0.056	0.009	0.121	0.038
150	0.4	0.6	0.6	0.043	0.020	0.097	0.061	0.062	0.017	0.132	0.057	0.084	0.015	0.198	0.062

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Strong cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.8)$ $(i \neq j)$.

Table 7: Power of the ADF and CADF tests: $c = 4$, moderate cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.290	0.159	0.359	0.193	0.471	0.183	0.580	0.218	0.758	0.192	0.855	0.257
50	0.0	0.6	0.3	0.286	0.114	0.893	0.664	0.474	0.122	0.987	0.769	0.754	0.129	1.000	0.897
50	0.0	0.6	-0.3	0.160	0.079	0.385	0.231	0.247	0.077	0.622	0.293	0.470	0.075	0.893	0.369
50	0.4	0.0	0.0	0.283	0.157	0.343	0.192	0.474	0.180	0.571	0.218	0.765	0.206	0.856	0.253
50	0.4	0.0	-0.3	0.229	0.137	0.285	0.157	0.368	0.143	0.451	0.187	0.675	0.173	0.767	0.223
50	0.4	0.0	0.3	0.357	0.194	0.414	0.229	0.563	0.206	0.646	0.262	0.822	0.240	0.895	0.290
50	0.4	0.0	0.6	0.416	0.221	0.460	0.246	0.638	0.253	0.681	0.289	0.865	0.289	0.908	0.332
50	0.4	0.3	0.0	0.237	0.105	0.467	0.256	0.383	0.109	0.705	0.329	0.661	0.108	0.933	0.388
50	0.4	0.6	0.0	0.194	0.095	0.668	0.410	0.340	0.087	0.896	0.518	0.607	0.088	0.990	0.636
50	0.4	0.6	0.6	0.414	0.174	0.977	0.880	0.618	0.202	0.999	0.951	0.821	0.241	1.000	0.991
100	0.0	0.0	0.0	0.319	0.153	0.369	0.178	0.546	0.164	0.590	0.179	0.840	0.167	0.879	0.180
100	0.0	0.6	0.3	0.345	0.093	0.934	0.675	0.586	0.096	0.995	0.793	0.825	0.078	1.000	0.887
100	0.0	0.6	-0.3	0.166	0.065	0.418	0.223	0.303	0.062	0.655	0.271	0.554	0.044	0.924	0.310
100	0.4	0.0	0.0	0.332	0.153	0.360	0.163	0.542	0.160	0.595	0.187	0.841	0.161	0.887	0.179
100	0.4	0.0	-0.3	0.254	0.128	0.285	0.143	0.464	0.146	0.484	0.151	0.761	0.131	0.803	0.163
100	0.4	0.0	0.3	0.402	0.184	0.439	0.192	0.637	0.201	0.679	0.215	0.892	0.194	0.919	0.217
100	0.4	0.0	0.6	0.476	0.198	0.489	0.222	0.726	0.232	0.741	0.259	0.929	0.236	0.939	0.259
100	0.4	0.3	0.0	0.271	0.101	0.502	0.237	0.479	0.095	0.764	0.286	0.758	0.072	0.948	0.308
100	0.4	0.6	0.0	0.234	0.077	0.718	0.384	0.412	0.073	0.916	0.478	0.697	0.055	0.994	0.579
100	0.4	0.6	0.6	0.512	0.162	0.993	0.925	0.748	0.187	1.000	0.975	0.896	0.155	1.000	0.997
150	0.0	0.0	0.0	0.355	0.147	0.365	0.158	0.581	0.164	0.605	0.171	0.863	0.154	0.891	0.158
150	0.0	0.6	0.3	0.375	0.091	0.946	0.671	0.607	0.084	0.994	0.785	0.852	0.063	1.000	0.898
150	0.0	0.6	-0.3	0.178	0.060	0.432	0.220	0.319	0.057	0.678	0.271	0.576	0.038	0.922	0.314
150	0.4	0.0	0.0	0.358	0.151	0.384	0.164	0.574	0.158	0.625	0.175	0.868	0.142	0.886	0.155
150	0.4	0.0	-0.3	0.282	0.130	0.291	0.138	0.481	0.149	0.499	0.153	0.783	0.130	0.817	0.141
150	0.4	0.0	0.3	0.431	0.187	0.441	0.179	0.673	0.190	0.713	0.203	0.913	0.183	0.928	0.184
150	0.4	0.0	0.6	0.500	0.194	0.509	0.213	0.754	0.233	0.761	0.239	0.941	0.213	0.947	0.224
150	0.4	0.3	0.0	0.295	0.099	0.521	0.245	0.501	0.085	0.777	0.268	0.782	0.067	0.960	0.281
150	0.4	0.6	0.0	0.251	0.076	0.718	0.396	0.424	0.061	0.924	0.466	0.715	0.041	0.995	0.556
150	0.4	0.6	0.6	0.563	0.154	0.998	0.941	0.770	0.157	1.000	0.979	0.919	0.126	1.000	0.999

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Moderate cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.4)$ $(i \neq j)$.

Table 8: Power of the ADF and CADF tests: $c = 8$, moderate cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.686	0.337	0.798	0.430	0.915	0.423	0.971	0.512	0.996	0.477	1.000	0.621
50	0.0	0.6	0.3	0.706	0.310	0.998	0.958	0.904	0.333	1.000	0.988	0.979	0.356	1.000	0.999
50	0.0	0.6	-0.3	0.475	0.185	0.845	0.528	0.741	0.192	0.975	0.663	0.948	0.169	1.000	0.800
50	0.4	0.0	0.0	0.699	0.354	0.807	0.435	0.915	0.414	0.964	0.515	0.996	0.468	0.999	0.617
50	0.4	0.0	-0.3	0.575	0.293	0.692	0.363	0.836	0.334	0.911	0.430	0.984	0.389	0.997	0.519
50	0.4	0.0	0.3	0.764	0.421	0.836	0.506	0.950	0.508	0.973	0.581	0.997	0.578	0.999	0.685
50	0.4	0.0	0.6	0.820	0.485	0.834	0.513	0.955	0.578	0.964	0.629	0.998	0.674	0.999	0.747
50	0.4	0.3	0.0	0.638	0.266	0.903	0.593	0.877	0.295	0.989	0.715	0.983	0.304	1.000	0.830
50	0.4	0.6	0.0	0.610	0.210	0.978	0.824	0.854	0.233	1.000	0.911	0.975	0.232	1.000	0.981
50	0.4	0.6	0.6	0.756	0.440	0.999	0.993	0.897	0.516	1.000	0.999	0.978	0.601	1.000	1.000
100	0.0	0.0	0.0	0.799	0.369	0.838	0.399	0.961	0.415	0.973	0.473	0.999	0.473	1.000	0.541
100	0.0	0.6	0.3	0.851	0.336	0.999	0.978	0.961	0.362	1.000	0.996	0.995	0.382	1.000	1.000
100	0.0	0.6	-0.3	0.573	0.171	0.886	0.550	0.843	0.161	0.986	0.639	0.974	0.140	1.000	0.784
100	0.4	0.0	0.0	0.793	0.359	0.850	0.414	0.967	0.420	0.982	0.488	0.999	0.463	0.999	0.545
100	0.4	0.0	-0.3	0.685	0.290	0.747	0.337	0.923	0.336	0.945	0.406	0.997	0.373	0.999	0.447
100	0.4	0.0	0.3	0.864	0.444	0.893	0.484	0.983	0.498	0.984	0.569	1.000	0.582	1.000	0.647
100	0.4	0.0	0.6	0.901	0.512	0.901	0.552	0.987	0.598	0.985	0.631	1.000	0.683	0.999	0.723
100	0.4	0.3	0.0	0.760	0.272	0.933	0.596	0.942	0.285	0.998	0.704	0.997	0.296	1.000	0.817
100	0.4	0.6	0.0	0.719	0.220	0.985	0.838	0.926	0.215	1.000	0.925	0.991	0.205	1.000	0.984
100	0.4	0.6	0.6	0.877	0.541	1.000	1.000	0.965	0.610	1.000	1.000	0.993	0.665	1.000	1.000
150	0.0	0.0	0.0	0.836	0.390	0.853	0.406	0.974	0.424	0.984	0.449	1.000	0.479	1.000	0.536
150	0.0	0.6	0.3	0.875	0.334	1.000	0.985	0.974	0.380	1.000	0.998	0.996	0.373	1.000	1.000
150	0.0	0.6	-0.3	0.615	0.161	0.889	0.559	0.873	0.153	0.989	0.642	0.984	0.135	1.000	0.758
150	0.4	0.0	0.0	0.808	0.363	0.866	0.404	0.976	0.439	0.984	0.461	0.999	0.472	0.999	0.520
150	0.4	0.0	-0.3	0.730	0.300	0.762	0.329	0.936	0.344	0.955	0.376	0.999	0.358	0.998	0.422
150	0.4	0.0	0.3	0.899	0.457	0.910	0.475	0.985	0.509	0.993	0.555	0.999	0.584	1.000	0.638
150	0.4	0.0	0.6	0.921	0.539	0.925	0.552	0.990	0.606	0.991	0.639	1.000	0.702	1.000	0.722
150	0.4	0.3	0.0	0.808	0.272	0.945	0.604	0.958	0.288	0.997	0.703	0.998	0.286	1.000	0.822
150	0.4	0.6	0.0	0.772	0.214	0.990	0.841	0.945	0.211	1.000	0.931	0.993	0.191	1.000	0.984
150	0.4	0.6	0.6	0.913	0.579	1.000	1.000	0.977	0.639	1.000	1.000	0.997	0.692	1.000	1.000

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Moderate cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.4)$ $(i \neq j)$.

Table 9: Power of the ADF and CADF tests: $c = 4$, strong cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.686	0.337	0.798	0.430	0.915	0.423	0.971	0.512	0.996	0.477	1.000	0.621
50	0.0	0.6	0.3	0.706	0.310	0.998	0.958	0.904	0.333	1.000	0.988	0.979	0.356	1.000	0.999
50	0.0	0.6	-0.3	0.475	0.185	0.845	0.528	0.741	0.192	0.975	0.663	0.948	0.169	1.000	0.800
50	0.4	0.0	0.0	0.699	0.354	0.807	0.435	0.915	0.414	0.964	0.515	0.996	0.468	0.999	0.617
50	0.4	0.0	-0.3	0.575	0.293	0.692	0.363	0.836	0.334	0.911	0.430	0.984	0.389	0.997	0.519
50	0.4	0.0	0.3	0.764	0.421	0.836	0.506	0.950	0.508	0.973	0.581	0.997	0.578	0.999	0.685
50	0.4	0.0	0.6	0.820	0.485	0.834	0.513	0.955	0.578	0.964	0.629	0.998	0.674	0.999	0.747
50	0.4	0.3	0.0	0.638	0.266	0.903	0.593	0.877	0.295	0.989	0.715	0.983	0.304	1.000	0.830
50	0.4	0.6	0.0	0.610	0.210	0.978	0.824	0.854	0.233	1.000	0.911	0.975	0.232	1.000	0.981
50	0.4	0.6	0.6	0.756	0.440	0.999	0.993	0.897	0.516	1.000	0.999	0.978	0.601	1.000	1.000
100	0.0	0.0	0.0	0.799	0.369	0.838	0.399	0.961	0.415	0.973	0.473	0.999	0.473	1.000	0.541
100	0.0	0.6	0.3	0.851	0.336	0.999	0.978	0.961	0.362	1.000	0.996	0.995	0.382	1.000	1.000
100	0.0	0.6	-0.3	0.573	0.171	0.886	0.550	0.843	0.161	0.986	0.639	0.974	0.140	1.000	0.784
100	0.4	0.0	0.0	0.793	0.359	0.850	0.414	0.967	0.420	0.982	0.488	0.999	0.463	0.999	0.545
100	0.4	0.0	-0.3	0.685	0.290	0.747	0.337	0.923	0.336	0.945	0.406	0.997	0.373	0.999	0.447
100	0.4	0.0	0.3	0.864	0.444	0.893	0.484	0.983	0.498	0.984	0.569	1.000	0.582	1.000	0.647
100	0.4	0.0	0.6	0.901	0.512	0.901	0.552	0.987	0.598	0.985	0.631	1.000	0.683	0.999	0.723
100	0.4	0.3	0.0	0.760	0.272	0.933	0.596	0.942	0.285	0.998	0.704	0.997	0.296	1.000	0.817
100	0.4	0.6	0.0	0.719	0.220	0.985	0.838	0.926	0.215	1.000	0.925	0.991	0.205	1.000	0.984
100	0.4	0.6	0.6	0.877	0.541	1.000	1.000	0.965	0.610	1.000	1.000	0.993	0.665	1.000	1.000
150	0.0	0.0	0.0	0.836	0.390	0.853	0.406	0.974	0.424	0.984	0.449	1.000	0.479	1.000	0.536
150	0.0	0.6	0.3	0.875	0.334	1.000	0.985	0.974	0.380	1.000	0.998	0.996	0.373	1.000	1.000
150	0.0	0.6	-0.3	0.615	0.161	0.889	0.559	0.873	0.153	0.989	0.642	0.984	0.135	1.000	0.758
150	0.4	0.0	0.0	0.808	0.363	0.866	0.404	0.976	0.439	0.984	0.461	0.999	0.472	0.999	0.520
150	0.4	0.0	-0.3	0.730	0.300	0.762	0.329	0.936	0.344	0.955	0.376	0.999	0.358	0.998	0.422
150	0.4	0.0	0.3	0.899	0.457	0.910	0.475	0.985	0.509	0.993	0.555	0.999	0.584	1.000	0.638
150	0.4	0.0	0.6	0.921	0.539	0.925	0.552	0.990	0.606	0.991	0.639	1.000	0.702	1.000	0.722
150	0.4	0.3	0.0	0.808	0.272	0.945	0.604	0.958	0.288	0.997	0.703	0.998	0.286	1.000	0.822
150	0.4	0.6	0.0	0.772	0.214	0.990	0.841	0.945	0.211	1.000	0.931	0.993	0.191	1.000	0.984
150	0.4	0.6	0.6	0.913	0.579	1.000	1.000	0.977	0.639	1.000	1.000	0.997	0.692	1.000	1.000

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Strong cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.8)$ $(i \neq j)$.

Table 10: Power of the ADF and CADF tests: $c = 8$, strong cross-correlation

				N=5				N=10				N=25			
T	σ_{21}	a_{12}	a_{21}	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF	ADF	HADF	CADF	HCADF
50	0.0	0.0	0.0	0.467	0.249	0.864	0.535	0.658	0.266	0.967	0.593	0.873	0.297	0.998	0.666
50	0.0	0.6	0.3	0.932	0.777	0.997	0.968	0.983	0.840	1.000	0.990	0.998	0.930	1.000	0.999
50	0.0	0.6	-0.3	0.498	0.300	0.886	0.649	0.679	0.332	0.976	0.723	0.893	0.406	0.999	0.840
50	0.4	0.0	0.0	0.485	0.254	0.861	0.534	0.663	0.274	0.968	0.598	0.879	0.295	0.999	0.681
50	0.4	0.0	-0.3	0.353	0.187	0.729	0.411	0.505	0.206	0.904	0.456	0.755	0.245	0.991	0.547
50	0.4	0.0	0.3	0.575	0.316	0.893	0.619	0.745	0.344	0.979	0.682	0.914	0.343	0.999	0.759
50	0.4	0.0	0.6	0.642	0.364	0.903	0.678	0.799	0.407	0.976	0.739	0.932	0.427	0.997	0.829
50	0.4	0.3	0.0	0.618	0.369	0.929	0.713	0.779	0.406	0.990	0.799	0.945	0.463	1.000	0.875
50	0.4	0.6	0.0	0.773	0.554	0.981	0.878	0.907	0.616	0.999	0.939	0.988	0.707	1.000	0.979
50	0.4	0.6	0.6	0.984	0.920	0.999	0.993	0.998	0.966	1.000	0.999	1.000	0.992	1.000	1.000
100	0.0	0.0	0.0	0.500	0.230	0.887	0.536	0.687	0.236	0.978	0.583	0.892	0.219	0.998	0.620
100	0.0	0.6	0.3	0.959	0.798	0.999	0.987	0.994	0.869	1.000	0.997	1.000	0.932	1.000	1.000
100	0.0	0.6	-0.3	0.530	0.290	0.912	0.645	0.725	0.332	0.985	0.722	0.918	0.370	1.000	0.829
100	0.4	0.0	0.0	0.499	0.226	0.891	0.548	0.682	0.229	0.981	0.589	0.886	0.215	0.999	0.635
100	0.4	0.0	-0.3	0.362	0.172	0.770	0.379	0.529	0.174	0.938	0.440	0.786	0.178	0.996	0.471
100	0.4	0.0	0.3	0.612	0.308	0.926	0.632	0.791	0.301	0.990	0.705	0.933	0.272	1.000	0.745
100	0.4	0.0	0.6	0.685	0.378	0.947	0.724	0.840	0.382	0.989	0.781	0.954	0.340	1.000	0.817
100	0.4	0.3	0.0	0.651	0.361	0.956	0.731	0.829	0.403	0.996	0.795	0.948	0.394	1.000	0.876
100	0.4	0.6	0.0	0.815	0.541	0.990	0.899	0.940	0.600	0.999	0.948	0.992	0.669	1.000	0.986
100	0.4	0.6	0.6	0.995	0.950	1.000	0.999	1.000	0.980	1.000	1.000	1.000	0.997	1.000	1.000
150	0.0	0.0	0.0	0.495	0.227	0.900	0.534	0.690	0.224	0.983	0.575	0.888	0.201	0.999	0.621
150	0.0	0.6	0.3	0.968	0.807	1.000	0.985	0.995	0.870	1.000	0.999	1.000	0.939	1.000	1.000
150	0.0	0.6	-0.3	0.546	0.293	0.913	0.665	0.735	0.320	0.987	0.733	0.926	0.341	1.000	0.830
150	0.4	0.0	0.0	0.496	0.225	0.896	0.537	0.699	0.216	0.978	0.573	0.894	0.196	1.000	0.618
150	0.4	0.0	-0.3	0.370	0.166	0.786	0.394	0.558	0.169	0.942	0.412	0.803	0.151	0.998	0.457
150	0.4	0.0	0.3	0.625	0.284	0.940	0.667	0.799	0.281	0.989	0.699	0.943	0.255	0.999	0.738
150	0.4	0.0	0.6	0.692	0.354	0.956	0.743	0.848	0.380	0.992	0.778	0.961	0.323	1.000	0.826
150	0.4	0.3	0.0	0.669	0.369	0.960	0.746	0.830	0.368	0.996	0.810	0.955	0.371	1.000	0.873
150	0.4	0.6	0.0	0.827	0.548	0.994	0.904	0.939	0.590	0.999	0.957	0.992	0.661	1.000	0.987
150	0.4	0.6	0.6	0.996	0.963	1.000	1.000	1.000	0.989	1.000	1.000	1.000	0.998	1.000	1.000

5% nominal size. 5,000 replications. ADF(2) and CADF(1,1,0) with constant. "HADF" and "HCADF" indicate ADF and CADF tests corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). Strong cross-unit dependence is present. The correlation matrix $\boldsymbol{V}$ is such that $v_{ij} = v_{ji} \sim \mathrm{U}(0, 0.8)$ $(i \neq j)$.

Table 11: Size of the ADF and CADF tests in the presence of long-run cross-dependence

N	T	q	CADF(1,1,0)	HCADF	ADF(2)	HADF
5	50	0.2	0.064	0.061	0.078	0.056
5	50	0.4	0.071	0.062	0.084	0.054
5	50	0.6	0.071	0.066	0.082	0.056
5	50	0.8	0.073	0.058	0.084	0.049
5	100	0.2	0.058	0.047	0.086	0.052
5	100	0.4	0.074	0.052	0.099	0.058
5	100	0.6	0.090	0.057	0.122	0.064
5	100	0.8	0.113	0.063	0.145	0.064
5	150	0.2	0.059	0.050	0.094	0.058
5	150	0.4	0.082	0.059	0.121	0.070
5	150	0.6	0.120	0.072	0.177	0.080
5	150	0.8	0.147	0.066	0.199	0.070
10	50	0.2	0.058	0.042	0.080	0.040
10	50	0.4	0.063	0.042	0.086	0.040
10	50	0.6	0.074	0.046	0.093	0.038
10	50	0.8	0.067	0.040	0.090	0.039
10	100	0.2	0.060	0.043	0.103	0.051
10	100	0.4	0.077	0.048	0.120	0.054
10	100	0.6	0.094	0.049	0.145	0.052
10	100	0.8	0.108	0.048	0.156	0.047
10	150	0.2	0.059	0.041	0.119	0.063
10	150	0.4	0.092	0.051	0.152	0.069
10	150	0.6	0.130	0.056	0.196	0.074
10	150	0.8	0.139	0.060	0.208	0.068
25	50	0.2	0.058	0.022	0.103	0.025
25	50	0.4	0.058	0.020	0.108	0.025
25	50	0.6	0.080	0.032	0.138	0.032
25	50	0.8	0.098	0.041	0.170	0.041
25	100	0.2	0.057	0.018	0.133	0.037
25	100	0.4	0.071	0.026	0.156	0.040
25	100	0.6	0.104	0.035	0.194	0.045
25	100	0.8	0.137	0.039	0.249	0.047
25	150	0.2	0.058	0.021	0.155	0.051
25	150	0.4	0.091	0.033	0.187	0.056
25	150	0.6	0.127	0.041	0.232	0.057
25	150	0.8	0.180	0.046	0.291	0.063

5% nominal size. 5,000 replications. The DGP is given by (24)-(30). "q" represents the degree of cross-unit cointegration. "HADF" and "HCADF" indicate ADF(2) and CADF(1,1,0) corrected for cross-dependence using the procedure proposed by Hartung (1999, eq. 2.4 with $\lambda_i = 1$ and $\kappa = 0.2$). All models include a constant term.