

CAN DOMESTIC LIABILITIES EXPLAIN THE HOME BIAS IN UK
INVESTMENT PORTFOLIOS?

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Abstract

It has been suggested that domestic liabilities may be an important factor in explaining the existence of a home bias in international investment portfolios. This paper provides a theoretical justification for this claim in a mean-variance framework. However, an empirical analysis for the UK does not find this effect to be large. Mean-variance efficient portfolios already exhibit significant home bias relative to the world market portfolio. Further, the predicted portfolios differ considerably from the actual portfolios of UK life assurance companies and pension funds. Possible reasons for this include weaknesses in the mean-variance approach and the role of peer pressure.

Key words: mean-variance, liabilities, portfolio allocation, pension funds, insurance companies.

JEL classification: G11, G22, G23.

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1. Introduction

Modern portfolio theory (MPT) suggests that investors choose their portfolio of assets according to the return and risk characteristics of the combined assets. Specifically, investors maximise the expected return on the portfolio for a given level of risk measured by the variance of the expected return. This focus on the first two moments of the returns is known as mean-variance analysis (MVA) and was advocated by Markowitz (1952, 1991). The theory is both a positive explanation of portfolio choice, and a normative one (Markowitz, 1952).

MPT suggests that investors can reduce the level of risk of any portfolio by diversifying across asset types, particularly across countries. Many studies have purported to demonstrate the potential gains from international diversification (Grubel, 1968; Levy and Sarnat, 1970; Solnik, 1974; Eun and Resnick, 1988; Odier and Solnik, 1993). These gains, in terms of higher return and/or lower risk, result more from diversifying across countries than from diversifying across industries (Heston and Rouwenhorst, 1994).

“The portfolios that investors actually hold, however, are markedly different from those predicted by the above studies” (Uppal, 1992, p. 172). Specifically, “the portfolios of investors have a disproportionately high share invested in domestic equities, relative to the market portfolio” (Uppal, 1992, p. 172). This phenomenon, known as home bias or home asset preference, has been documented by French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995). Investors do not even hold the market portfolio of the countries they invest in (Kang and Stulz, 1997). Comparisons with the world market portfolio, however, should be viewed with some caution. Odier and Solnik (1993) show that the world index is not on the frontier of mean-variance efficient portfolios. Hence, holding the

world market portfolio may not be optimal. This is particularly true if there are significant barriers to international investment such as transaction costs, differential taxation, or legal restrictions on investment.

Uppal (1992) reviews several reasons for home bias including a hedge for domestic inflation, institutional barriers to investment, and taxes and other transaction costs. He concludes that “it is unlikely that these three factors are significant enough to explain the degree of the bias in portfolios that is observed empirically” (p. 171). Institutional barriers are not considered to be large enough and are not usually binding. Neither are transactions costs large enough to explain the home bias.

Davis (1995), using interview material from pension fund managers, suggests that the main reason for home bias is a perceived need to match assets with their domestic liabilities. This paper examines the theoretical and empirical basis of this claim in a mean-variance framework. While Elton and Gruber (1992) examine the role of investor liabilities on optimal portfolios, they do not distinguish between domestic and foreign assets. Randall and Satchell (1997) examine the optimal hedge for UK pension fund liabilities but do not include overseas assets. Griffin (1997), using data from 1987, does attempt to show that domestic liabilities substantially explain home bias. However, his use of a ten-year bond as a proxy for liabilities is not ideal as this is also a potential asset. He also restricts the return on domestic and foreign equities to be same.

This paper is organised as follows. Section 2 examines the effect of introducing a domestic liability on an investor’s portfolio choice. It shows that when the domestic asset is more highly correlated with the domestic liability than the foreign asset, then the optimal portfolio will be biased towards the domestic asset. With more than two assets, the theoretical result is not clear cut. Section 3 examines the question empirically using historical data on quarterly and annual asset returns between 1976 and 1997. The results suggest that a domestic liability generally does bias the optimal portfolio towards domestic assets for

UK investors, but the effect is not large. The portfolios already exhibit considerable home bias compared to the world market portfolio. The results also reveal weaknesses in mean-variance analysis as a predictive tool. These issues are discussed in Section 4 where the theory is contrasted with actual investment practice. Section 5 concludes.

2. Theoretical Analysis

Assume that a UK financial institution has a fixed amount W to invest. The financial institution can invest in two assets, the domestic asset (asset 1) and the foreign asset (asset 2). The financial institution chooses a portfolio P defined by the portfolio weights w and $(1-w)$, where w is the proportion of the portfolio invested in the domestic asset.

What is the impact of introducing a domestic liability (asset 3) into the investor's problem? We do this by defining the proportion of the portfolio P invested in the domestic liability as x where x is a parameter such that $x=0$ when the domestic liability is ignored and $x=-1$ when the domestic liability is included in the optimisation problem.

In this framework, W may be viewed as an insurance premium or a pension contribution. The financial institution receives W at the start of the period, invests it in domestic and foreign assets, and must then pay back W with a given but unknown return, at the end of the period. The unknown return may reflect a pension payout related to the retail price index (RPI) or the final salary of the contributor. In essence, the institution borrows W and invests it. However, the interest rate on its loan is unknown.

The vector of expected returns is given by:

$$\mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

And the variance-covariance matrix is given by:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

The investor chooses w in the vector of portfolio weights,

$$\mathbf{w} = \begin{bmatrix} w \\ (1-w) \\ x \end{bmatrix}$$

where $0 \leq w \leq 1$, and x is either 0 or -1 depending upon the scenario considered, to maximise the expected return on the portfolio:

$$\begin{aligned} E(R_p) &= \mathbf{w}' \mathbf{R} \\ &= wR_1 + (1-w)R_2 + xR_3 \end{aligned}$$

for a given variance:

$$\begin{aligned} V(R_p) &= \mathbf{w}' \Sigma \mathbf{w} \\ &= w^2 \sigma_{11} + (1-w)^2 \sigma_{22} + x^2 \sigma_{33} \\ &\quad + 2w(1-w)\sigma_{12} + 2wx\sigma_{13} + 2(1-w)x\sigma_{23} \end{aligned}$$

To simplify the analysis we focus our attention on the minimum variance portfolio (MVP) where the optimal weight on the first asset is given by w^* .

The first order condition for the MVP is given by:

$$\begin{aligned}\frac{\partial V(R_p)}{\partial w} &= 2w^*\sigma_{11} - 2(1-w^*)\sigma_{22} + 2(1-2w^*)\sigma_{12} + 2x\sigma_{13} - 2x\sigma_{23} = 0 \\ \Rightarrow w^*[\sigma_{11} + \sigma_{22} - 2\sigma_{12}] &= \sigma_{22} - \sigma_{12} - x\sigma_{13} + x\sigma_{23} \\ \Rightarrow w^* &= \frac{\sigma_{22} - \sigma_{12} - x(\sigma_{13} - \sigma_{23})}{(\sigma_{11} + \sigma_{22} - 2\sigma_{12})}\end{aligned}$$

Note that the denominator is always non-negative as:

$$V(R_1 - R_2) = \sigma_{11} + \sigma_{22} - 2\sigma_{12} \geq 0$$

and this will generally be strictly positive. This expression is also the second order condition so that w^* does indeed define the minimum variance.

We have two cases to consider:

If there is no liability then $x = 0$ and

$$w^* = w^0 = \frac{\sigma_{22} - \sigma_{12}}{(\sigma_{11} + \sigma_{22} - 2\sigma_{12})}$$

If there is a liability to consider then $x = -1$ and

$$w^* = w^1 = \frac{\sigma_{22} - \sigma_{12} + \sigma_{13} - \sigma_{23}}{(\sigma_{11} + \sigma_{22} - 2\sigma_{12})}$$

The optimal weight on the domestic asset (w) will therefore be greater if a liability is included in the analysis if $w^1 > w^0$, i.e. if

$$\sigma_{13} - \sigma_{23} > 0$$

or

$$\sigma_{13} > \sigma_{23} \tag{1}$$

So, the weight (w) for the domestic asset (asset 1) in the MVP will be greater, when a domestic liability is included in the analysis, if the covariance of the domestic liability (asset 3) with the domestic asset is greater than the covariance of the foreign asset (asset 2) with the domestic asset. This is intuitive. Matching the liability with the asset most correlated to it reduces the overall risk of the portfolio. Note further that the weights in the MVP do not depend upon the variance of the domestic liability (σ_{33})

We have shown that the introduction of a domestic liability alters the optimum portfolio weights in the MVP. The analysis can be extended to that of several assets (see Appendix for details). A condition can be derived under which the liability biases the portfolio towards a single domestic asset, but there is no obvious intuition behind this result for the general case. The introduction of several assets, and the covariance between them, makes it difficult to predict theoretically whether a domestic liability will bias the portfolio choice towards possibly several domestic assets. We must therefore rely on historical data to examine whether any home bias in investors' portfolios is justified, *ex post*.

Several limitations of the above analysis can be noted other than the exclusive focus on the MVP. First, the analysis is a one period analysis. In practice, a pension fund would invest pension contributions over many years before paying out a pension over several years. Second, we assumed that portfolio weights could be

negative. Imposing non-negativity constraints would further complicate the analysis. Third, we assumed that there were no net assets. A pension fund with net assets may use some of its gross assets to hedge its liabilities, and then invest the remainder to maximise its risk-return pay-off irrespective of its liabilities (Elton and Gruber, 1992).

3. Empirical Analysis

Data on the portfolio composition of UK life assurance companies and pension funds (LAPFs) is available since 1963. Their portfolio compositions for the main asset classes, using data from the Office for National Statistics (ONS), are shown graphically in Figures 1 and 2. Equity investments have dominated the portfolios throughout the 1980s, with overseas equities growing in importance, especially since the abolition of exchange controls in 1979. However, UK equities still dominate overseas equities, even though the UK market is only a small fraction of the world stock market by capitalisation. Similarly, UK bond investments dominate overseas bond investments. To what extent can mean-variance analysis explain these portfolio compositions, and does the inclusion of a domestic liability explain the apparent home bias?

3.1. Data

The data on returns used in this study was obtained from Datastream. The time period of the analysis was restricted by the availability of data. Data on UK (FT Actuaries All-Share Index) and world (Morgan Stanley Capital International Index except UK) stock markets is available annually and quarterly from 1970. Data on UK government bonds (FT Actuaries Fixed Interest Index) is only available from 1976 with data on index-linked bonds from 1982. Data on world bonds is only available from 1985 (New Salomon Brothers World Government Bond Index, non-sterling) or 1986 (JP Morgan World Index except UK). These two series were basically the same so the Salomon Brothers series was used as it was available for a longer period. Data

on UK property is available annually from 1970 (Jones Lang Wootton Property Index), but only quarterly from 1977 (Hillier Parker Property Database) or 1987 (Investment Property Databank). The Hillier Parker data was therefore used for the quarterly analyses, and the Jones Lang data for the annual analysis. Data on cash returns is only available quarterly from 1975 (London Interbank Bid Rate). All asset returns are in terms of sterling. The currency risk is assumed to be unhedged.

UK prices (RPI index) and earnings (average earnings index) were used separately as proxies for liabilities in the quarterly analyses; pension payouts (Randall and Satchell, 1997) were also used for the annual analysis.

The empirical analysis was conducted for three periods: quarterly from 1985(1) to 1997(2) with all the major asset classes; quarterly from 1977(2) to 1985(4), excluding index-linked and overseas bonds; and annually from 1976 to 1996, excluding index-linked and overseas bonds.

The historical data was used to compute the sample arithmetic mean vector of returns and the sample variance-covariance matrix. The variance-covariance matrices and vector of returns are presented in Tables 1, 2, and 3. UK equities had the highest return and highest variance in each of the three periods.

The portfolio composition of the minimum variance portfolio (MVP) and the tangency portfolio (TP) were calculated, with and without the domestic liability. The MVP minimises the variance of the portfolio. The TP maximises the Sharpe ratio – the excess of the portfolio return over the risk-free rate (cash) divided by the standard deviation – and is a measure of excess return per unit of risk. The portfolios were calculated using the Solver function on Microsoft Excel. Only the composition of the portfolio containing risky (non-cash) assets was calculated. With a riskless asset (e.g. interest-bearing bank time deposits), fund managers would be expected to hold some proportion

of the risky portfolio and the riskless asset according to their return-risk preferences.

3.2. Results

Tables 4, 5, and 6 present the results of the analysis for the three data periods. They show the portfolio composition of the optimal MVP and TP as well as the actual average portfolio composition of both pension funds (PF) and life assurance companies (LA) over the relevant period. In each period, (UK) property was found to have a large weighting relative to the actual portfolios of pension funds and life assurance companies. This probably reflects its small or negative covariances with other assets (see Tables 1-3). The analyses were therefore repeated excluding property. These results are shown in the bottom half of the respective tables. Note that the portfolio composition reflects the composition of the risky asset portfolio only.

The results show that the MVP is dominated by UK assets in all three periods – about 80-81% of the risky asset portfolio. This is evidence of a considerable home bias compared to the UK's share of the world capital market. Including a liability (such as average earnings or prices) marginally increases the home bias by about 0-5%. However, when actual pension payouts were used as the proxy for liabilities in the annual analysis, the optimal proportion of UK assets in the MVP fell to 67%. Excluding property from the MVP yielded similar results. The inclusion of the liability generally led to a small increase in the home bias, but for the annual data 1976 to 1996, the home bias actually fell.

Actual investors are not likely to be so risk averse as to choose the MVP. The TP was therefore computed to reflect the optimal trade off between return and risk. Investors will then choose a portfolio that is a weighted combination of the riskless asset and the risky (TP) portfolio.

The results show that the TP is also dominated by UK assets – to a greater extent than MVP. The TP consists of nearly 100% in UK assets for each period. The effect of the liability was to increase the home bias to 100% in UK assets, with this exclusively in UK equities. This effect was uniform across the three periods and the three proxies for liabilities.

These results cast some doubt on the usefulness of the traditional measure of home bias – a comparison of actual portfolios with the world market portfolio. Actual UK investment portfolios exhibited a high degree of home bias – 70-92% of the risky portfolio in UK assets, depending upon the sample period. However, the MVP and TP portfolios presented here suggest that this ‘home bias’ was actually optimal – indeed, investors may have diversified overseas too much!

In summary, the impact on ‘home bias’ of including a liability is found to be empirically small and, depending upon the sample period and the proxy, occasionally negative. These results suggest that the role of domestic liabilities in explaining home bias may be overstated by Davies (1995).

However, we should also observe that the actual portfolios of pension funds and life assurance companies differ quite considerably from those predicted by the mean-variance analysis presented here. The TPs are strongly biased towards one or two assets, with 90-100% of risky assets in UK equities when property is excluded. This is much higher than the 45-62% observed by LAPFs. When property is included, the TP predicts over 50% of portfolios should be invested in property for the two quarterly periods 1977 to 1985 and 1985 to 1997. However, actual LAPF portfolios averaged only 1-16% in property. The MVP also assigns a zero weight to UK equities, again in stark contrast to the actual LAPFs portfolios. These results therefore cast strong doubt on the ability of mean-variance analysis to explain actual investment portfolios. Either there are serious weaknesses in the traditional mean-variance framework, or there are other factors that prevent fund managers from pursuing the portfolios predicted by

mean-variance analysis. These issues are discussed in the next section.

4. Portfolio Investment in Practice

Several criticisms have been made about the usefulness of mean-variance analysis (MVA). The first concerns the neglect of domestic liabilities (Gardener, 1989; Wise, 1989; Davis, 1995). This is particularly relevant for pension funds and insurance companies. The theoretical analysis above suggests that liabilities can be incorporated into the mean-variance framework (see also Wise, 1989). The empirical results, however, suggest that the impact on optimal portfolios is not very large, at least in a mean-variance framework.

A second criticism of MVA is that the optimal allocations tend to be very sensitive to the expected returns and variance-covariance matrix, particularly the former. Small changes in expected returns can produce large changes in the optimal portfolio (Jorion, 1985; Koskosidis and Duarte, 1997). This is shown graphically in Figure 3. The graph shows the composition of the risky asset TP over the period 1976 to 1995 (cf. Figures 1 and 2). The variance-covariance matrix was the sample variance-covariance matrix for the whole period, but the return vector was the actual historic return vector for each year. The optimal asset allocations fluctuate widely, with sometimes the whole portfolio in one asset. Such large fluctuations and lack of diversification can only be justified if expected returns can be predicted accurately, and if transactions costs and liquidity effects are very small.

Third, expected returns, variances and covariances may be difficult to forecast accurately. Most empirical studies of international portfolio diversification have used *ex post* mean-variance analysis, as in the empirical analysis here. The expected return vector and the variance-covariance matrix are computed from historical data. But the past may not be a reliable guide for the future. Jorion (1985) shows that such portfolios tend to have poor out-of-sample performance and unstable

portfolio weights. Jorion (1985) suggests a method to overcome the estimation risk by shrinking the sample mean returns to a common mean. Eun and Resnick (1988) use this method as well as the MVP and an equally weighted portfolio to estimate the gains from international diversification taking account of both estimation risk and exchange rate volatility. They show that using one of these three methods, especially with currency hedging, improves over a US-only investment strategy and the traditional certainty equivalence strategy. However, such strategies still do not permit the incorporation of varying degrees of confidence in investor's forecasts, or new information that suggest a divergence from historical returns (Koskosidis and Duarte, 1997).

Fourth, the optimal portfolios in MVA are not necessarily well diversified (Jorion, 1985). In the study by Levy and Sarnat (1970), only 5 to 8 of the 28 countries are included in the optimal portfolio. In the results presented here, the optimal portfolio was usually dominated by one or two assets.

Finally, MVA focuses on only the first two moments of returns. Chunnachinda *et al* (1997) show that stock market returns are skewed and that incorporating skewness affects the optimal (mean-variance efficient) portfolio.

How do investment managers actually make their portfolio allocation decisions? Solnik (1996) discusses several elements of an investor's investment strategy. Investors may choose a passive or active strategy. A passive strategy would attempt to match a particular market index while an active approach may involve strategic and tactical selection of assets and securities. Strategic asset allocation involves selecting a portfolio to "maximise the likelihood of achieving the portfolio objectives given the expected long-term equilibrium relative values of each asset class" (Watsham, 1993, p. 90). Tactical asset allocation, by contrast, "is the practice of deviating from the strategic allocation as asset classes experience short-term deviations from their long-term relative valuations" (Watsham, 1993,

p. 92). This is also known as market timing. Investors may also adopt a top-down or bottom-up approach (Solnik, 1996). In the top-down approach, the portfolio is allocated between various asset classes, and then individual securities are selected to best satisfy that allocation. In the bottom-up approach, individual stocks are chosen on their individual merits, irrespective of nationality or currency.

Davis (1995) distinguished between four main approaches to international asset allocation: international indexation, where asset weights are determined by those in a global index; international portfolio optimisation, using models to trade-off risk and return such as mean-variance analysis; discretionary allocation, a more subjective approach but taking account of economic forecasts, the recent behaviour of markets, and the behaviour of other fund managers; and tactical asset allocation, comparing current asset returns with long-run ratios. Interviews with London pension fund managers indicated that discretion, rather than MVA and portfolio optimisation, was the main strategy adopted (Davis, 1995, p. 276). Several authors have noted a scepticism among UK fund managers of the practical value of modern portfolio theory (Frost and Henderson, 1983; Prodano, 1987. Davis, 1988). This seems to be confirmed by the interviews conducted by Davis (1995) and by this author. The reasons given often relate to the criticisms of mean-variance analysis discussed above. Another reason includes a belief that UK and/or world markets are not efficient implying that there is scope for active portfolio investment. Given a discretionary approach, it is not surprising that approaches and portfolios differ considerably (Jackson, 1987; Davis, 1995).

While such findings suggest reasons why LAPF portfolios do not reflect the predictions of mean-variance analysis, it is not clear what alternative approach would succeed. Relative asset returns are obviously important (Davis, 1988), but so is diversification (Davis, 1995). It is not clear how to combine these two factors to predict investment portfolios. MVA or holding the market portfolio may be simple strategies but they do not seem to reflect actual investment practice.

An important factor usually ignored by MPT is the role of peer pressure. This has been noted by Jackson (1987) and Davis (1995) and confirmed in interviews conducted by this author. Risk may therefore be measured in terms of the likelihood of underperforming the peer group rather than the variance of the actual returns. The implications of such behaviour for overall portfolio allocations are not clear.

It is also important to note that while some researchers suggest that institutional barriers are not important in explaining home bias (French and Poterba, 1991; Uppal, 1992), UK life assurance companies do face an important restriction. Regulation 27 of the Insurance Companies Regulations (51/1994/1516) requires them to hold sufficient assets in sterling to cover at least 80 per cent of their sterling liabilities (if more than 5 per cent of their liabilities are in sterling). Liabilities must also be covered by “assets of appropriate safety, yield and marketability” (Section 35A, Insurance Companies Act, 1987). While overseas assets of insurance companies have only averaged about 11-13% of their portfolios over recent years, these regulations may partly explain why pension funds have been investing a higher proportion of their portfolios overseas (17-21%). Pension funds are not subject to the same regulations (Davis, 1995), but the introduction of a minimum funding requirement (MFR) in the 1995 Pensions Act may have implications for future pension fund asset allocation (Davis, 1995).

5. Conclusion

This paper has argued that, theoretically, the introduction of a domestic liability into a traditional mean-variance framework may bias the optimal portfolio towards domestic assets. However, an empirical analysis for the UK found this effect to be small. Optimal portfolios tended to be heavily weighted towards UK assets suggesting that the observed home bias in investor's portfolios may be optimal. The use of the world market index may not be the most useful benchmark for comparison.

The empirical results also cast doubt on the ability of mean-variance analysis to predict or explain actual investment portfolios of UK life assurance companies and pension funds. Actual portfolios are more diversified, change more slowly, and are less heavily weighted towards property than mean-variance analysis would predict. This may partly reflect weaknesses in the mean-variance framework such as its sensitivity to changes in expected returns or the variance-covariance matrix, and its reliance on historical data. However, it may also reflect institutional features not normally taken into consideration. Investor performance seems strongly motivated by performance relative to peers. Liquidity considerations suggest that investors may not be able to make large changes in asset composition without affecting the prices of securities. Property may not be as liquid as stocks or bonds. Domestic liabilities, and the associated regulation for insurance companies, may also be important, but in ways that the mean-variance framework used here cannot explain.

TABLES AND FIGURES

Table 1: Variance-covariance matrix for quarterly asset returns, 1985(1) to 1997(2)

	UK equity	World equity	UK bonds	World bonds	Index-linked	Property	Earnings	Prices	Pension payouts
UK equity	0.0070								
World equity	0.0049	0.0057							
UK bonds	0.0014	0.0007	0.0013						
World bonds	0.0003	0.0002	0.0004	0.0024					
Index-linked	0.0008	0.0004	0.0009	0.0005	0.0010				
Property	-0.0002	0.0002	-0.0003	-0.0005	-0.0002	0.0017			
Earnings	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001		
Prices	0.0000	0.0000	-0.0001	-0.0001	-0.0001	0.0000	0.0000	0.0001	
Pension payouts									
Mean return	0.0408	0.0327	0.0270	0.0221	0.0197	0.0299	0.0158	0.0110	

Table 2: Variance-covariance matrix for quarterly asset returns, 1977(2) to 1985(4)

	UK equity	World equity	UK bonds	World bonds	Index-linked	Property	Earnings	Prices	Pension payouts
UK equity	0.0061								
World equity	0.0020	0.0030							
UK bonds	0.0030	0.0005	0.0027						
World bonds									
Index-linked									
Property	0.0005	-0.0002	-0.0001			0.0011			
Earnings	-0.0001	-0.0001	0.0000			0.0002	0.0002		
Prices	-0.0002	-0.0002	-0.0001			0.0002	0.0001	0.0001	
Pension payouts									
Mean return	0.0558	0.0347	0.0334			0.0396	0.0276	0.0219	

Table 3: Variance-covariance matrix for annual asset returns, 1976 to 1996

	UK equity	World equity	UK bonds	World bonds	Index-linked	Property	Earnings	Prices	Pension payouts
UK equity	0.0207								
World equity	0.0078	0.0185							
UK bonds	0.0098	0.0000	0.0115						
World bonds									
Index-linked									
Property	0.0018	0.0028	-0.0003			0.0109			
Earnings	0.0002	0.0002	-0.0004			0.0017	0.0017		
Prices	-0.0009	0.0004	-0.0013			0.0020	0.0016	0.0020	
Pension payouts	0.0017	0.0020	-0.0007			-0.0009	0.0013	0.0010	0.0030
Mean return	0.1956	0.1310	0.1338			0.1285	0.0874	0.0704	0.1502

Table 4: Optimal and actual portfolios, 1985(1) to 1997(2)

Portfolio:	Asset							Total UK
	UK equities	World equities	UK bonds	World bonds	Index-linked	Property		
<i>With property:</i>								
MVP	0.00	0.01	0.19	0.18	0.24	0.38	0.81	
MVP(e)	0.00	0.03	0.20	0.15	0.26	0.36	0.82	
MVP(p)	0.00	0.02	0.10	0.17	0.31	0.40	0.81	
TP	0.28	0.00	0.17	0.01	0.00	0.54	0.99	
TP(e)	1.00	0.00	0.00	0.00	0.00	0.00	1.00	
TP(p)	1.00	0.00	0.00	0.00	0.00	0.00	1.00	
PF	0.57	0.18	0.10	0.02	0.04	0.08	0.79	
LA	0.51	0.13	0.29	0.03	0.02	0.01	0.84	
<i>Without property:</i>								
MVP	0.00	0.08	0.19	0.20	0.53	0.72		
MVP(e)	0.00	0.09	0.20	0.17	0.54	0.74		
MVP(p)	0.00	0.09	0.10	0.19	0.62	0.72		
TP	0.88	0.00	0.12	0.00	0.00	1.00		
TP(e)	1.00	0.00	0.00	0.00	0.00	1.00		
TP(p)	1.00	0.00	0.00	0.00	0.00	1.00		
PF	0.62	0.20	0.11	0.03	0.05	0.78		
LA	0.52	0.13	0.30	0.03	0.02	0.84		

Key: MVP = minimum variance portfolio; TP = tangency portfolio;
(e) = with earnings index as liability; (p) = with price index as liability;
PF = actual pension fund portfolio; LA = actual life assurance fund portfolio.

Table 5: Optimal and actual portfolios, 1977(2) to 1985(4)

Portfolio:	Asset							Total UK
	UK equities	World equities	UK bonds	World bonds	Index-linked	Property		
<i>With property:</i>								
MVP	0.00	0.20	0.20			0.60	0.80	
MVP(e)	0.00	0.17	0.17			0.66	0.83	
MVP(p)	0.00	0.14	0.15			0.70	0.85	
TP	0.28	0.02	0.00			0.69	0.97	
TP(e)	1.00	0.00	0.00			0.00	1.00	
TP(p)	1.00	0.00	0.00			0.00	1.00	
PF	0.50	0.11	0.23			0.16	0.89	
LA	0.43	0.08	0.44			0.05	0.92	
<i>Without property:</i>								
MVP	0.00	0.47	0.53				0.53	
MVP(e)	0.00	0.46	0.54				0.54	
MVP(p)	0.00	0.46	0.54				0.54	
TP	1.00	0.00	0.00				1.00	
TP(e)	1.00	0.00	0.00				1.00	
TP(p)	1.00	0.00	0.00				1.00	
PF	0.59	0.13	0.28				0.87	
LA	0.45	0.09	0.47				0.92	

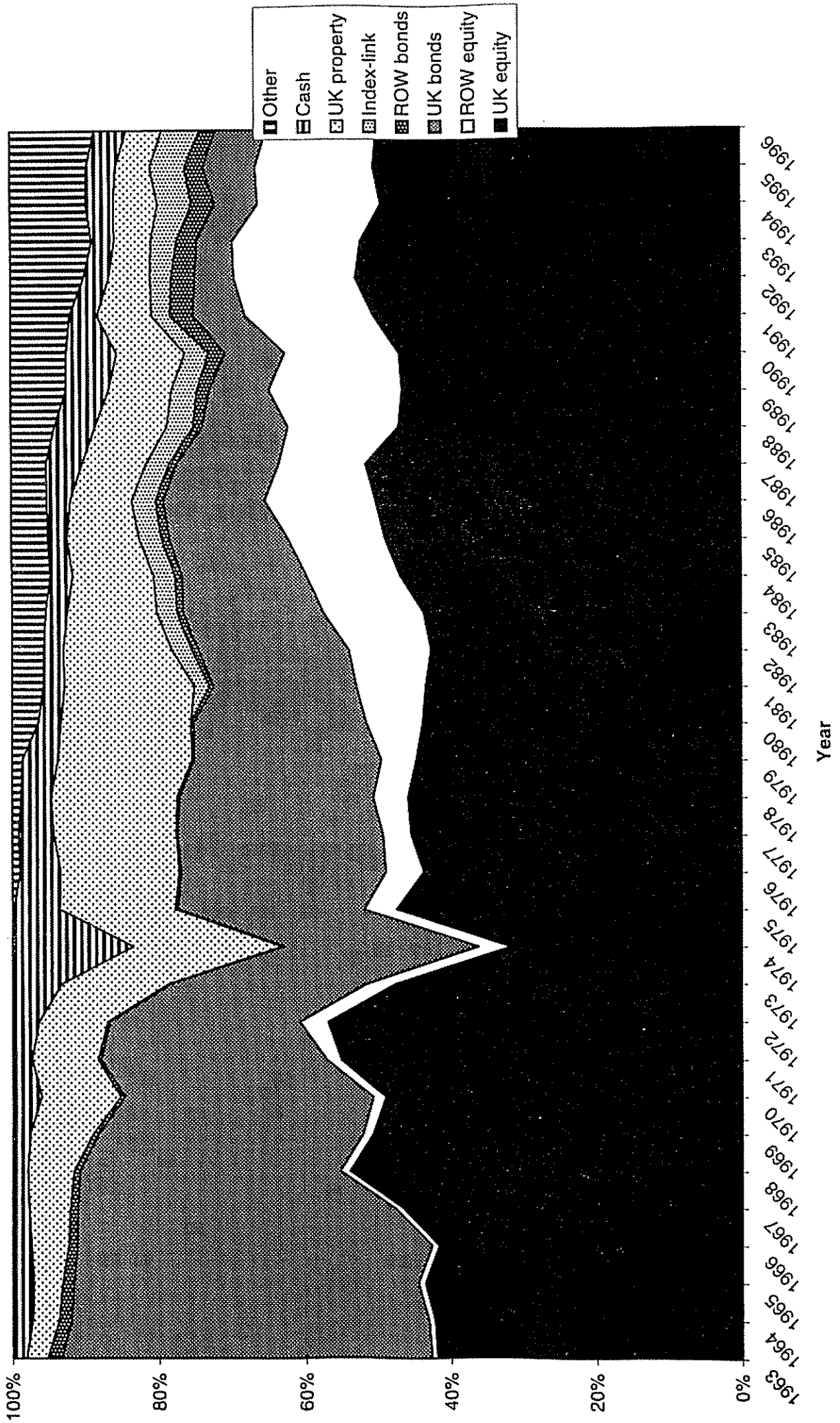
Key: MVP = minimum variance portfolio; TP = tangency portfolio;
(e) = with earnings index as liability; (p) = with price index as liability;
PF = actual pension fund portfolio; LA = actual life assurance fund portfolio.

Table 6: Optimal and actual portfolios, 1976 to 1996

Portfolio:	Asset							Total UK
	UK equities	World equities	UK bonds	World bonds	Index-linked	Property		
<i>With property:</i>								
MVP	0.00	0.19	0.42			0.39		0.81
MVP(e)	0.00	0.16	0.34			0.51		0.84
MVP(p)	0.00	0.17	0.28			0.55		0.83
MVP(pen)	0.00	0.33	0.37			0.30		0.67
TP	0.80	0.00	0.00			0.20		1.00
TP(e)	1.00	0.00	0.00			0.00		1.00
TP(p)	1.00	0.00	0.00			0.00		1.00
TP(pen)	1.00	0.00	0.00			0.00		1.00
PF	0.56	0.15	0.17			0.12		0.70
LA	0.48	0.11	0.37			0.03		0.89
<i>Without property:</i>								
MVP	0.00	0.38	0.62					0.62
MVP(e)	0.00	0.40	0.60					0.60
MVP(p)	0.00	0.44	0.56					0.56
MVP(pen)	0.00	0.47	0.53					0.53
TP	1.00	0.00	0.00					1.00
TP(e)	1.00	0.00	0.00					1.00
TP(p)	1.00	0.00	0.00					1.00
TP(pen)	1.00	0.00	0.00					1.00
PF	0.63	0.17	0.20					0.83
LA	0.45	0.09	0.47					0.91

Key: MVP = minimum variance portfolio; TP = tangency portfolio;
(e) = with earnings index as liability; (p) = with price index as liability;
(pen) = with actual pension payouts as liability
PF = actual pension fund portfolio; LA = actual life assurance fund portfolio.

Figure 1: Portfolio composition of UK pension funds, 1963 to 1996.



Source: ONS

Figure 2: Portfolio composition of UK life assurance companies, 1963 to 1996

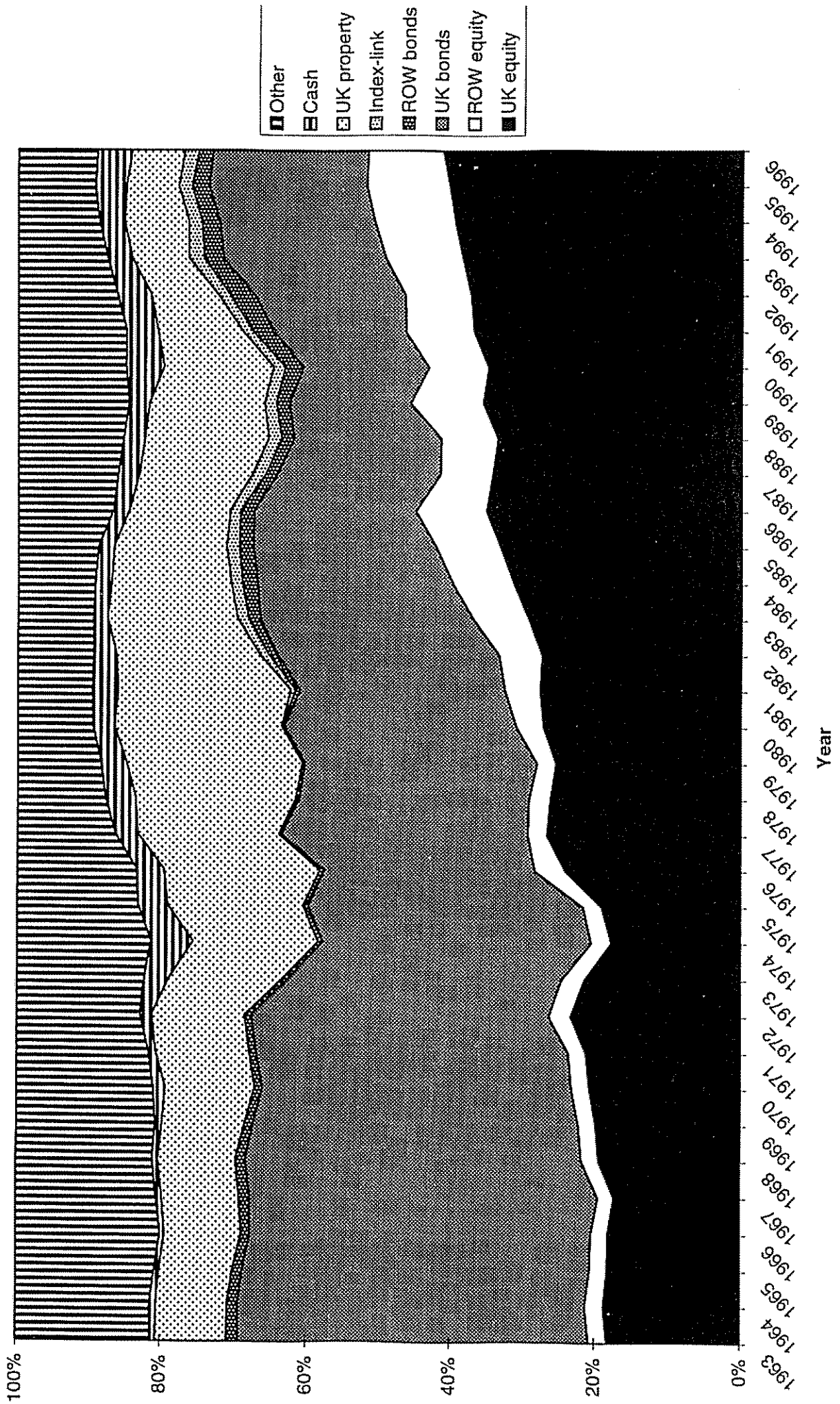
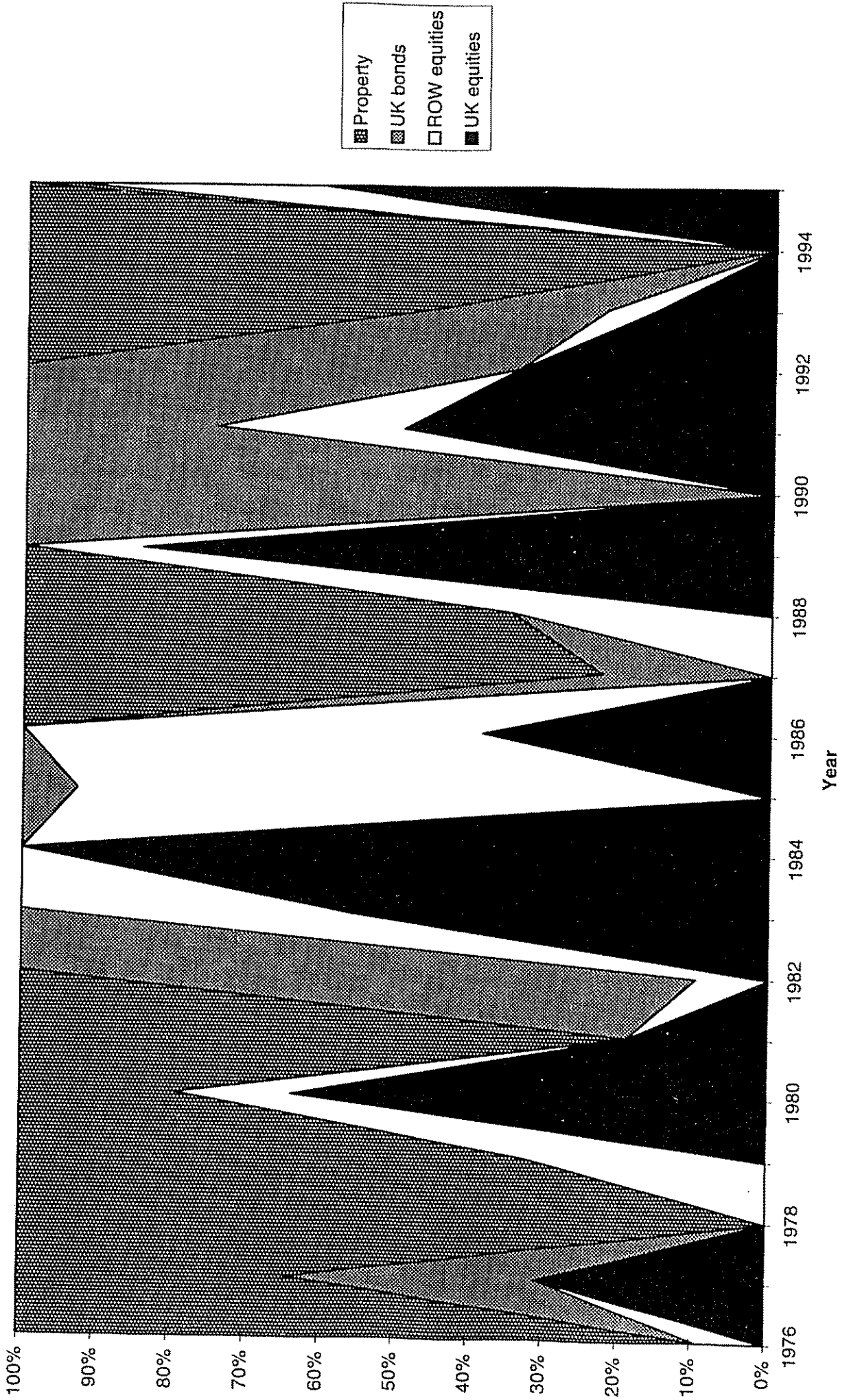


Figure 3: Predicted portfolio composition using historical annual returns, 1976 to 1995



Source: Author's calculations (see text).

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Appendix: The Case of N Assets

The example in the text consisted of just two assets and one liability. In general, investors have the choice of more than two assets. This appendix extends the theoretical analysis to the general case of n assets and one liability. Again we focus on the minimum variance portfolio (MVP). There are n assets and one liability – the $(n+1)$ th asset. The variance-covariance matrix is defined as:

$$\Sigma^* = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} & \sigma_{1n+1} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{2n} & \sigma_{2n+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{1n} & \sigma_{2n} & \cdots & \sigma_{nn} & \sigma_{nn+1} \\ \sigma_{1n+1} & \sigma_{2n+1} & \cdots & \sigma_{nn+1} & \sigma_{n+1n+1} \end{bmatrix} \quad \text{where } \Sigma^* = \Sigma^{*'}$$

The investor chooses portfolio weights:

$$\mathbf{w}^* = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ x \end{bmatrix}$$

where

$$\sum_{i=1}^n w_i = 1 \text{ and } x = 0 \text{ or } -1$$

to minimise the variance of the portfolio:

$$V(R_p) = \mathbf{w}^{*'} \Sigma^* \mathbf{w}^*$$

Note that we cannot simply minimise this function with respect to \mathbf{w}^* as the last element of \mathbf{w}^* is not a choice variable. We remove this problem by partitioning \mathbf{w}^* and Σ^* as follows:

$$\mathbf{w}^* = \begin{bmatrix} \mathbf{w} \\ x \end{bmatrix} \text{ where } \mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

and

$$\Sigma^* = \begin{bmatrix} \Sigma & \sigma_{n+1} \\ \sigma'_{n+1} & \sigma_{n+1n+1} \end{bmatrix}$$

where

$$\Sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{1n} & \cdots & \sigma_{nn} \end{bmatrix} \text{ and } \sigma_{n+1} = \begin{bmatrix} \sigma_{1n+1} \\ \vdots \\ \sigma_{nn+1} \end{bmatrix}$$

Hence

$$\begin{aligned} \mathbf{w}^{*\prime} \Sigma^* \mathbf{w}^* &= [\mathbf{w}' \quad x] \begin{bmatrix} \Sigma & \sigma_{n+1} \\ \sigma'_{n+1} & \sigma_{n+1n+1} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ x \end{bmatrix} \\ &= [\mathbf{w}' \quad x] \begin{bmatrix} \Sigma \mathbf{w} + \sigma_{n+1} x \\ \sigma'_{n+1} \mathbf{w} + \sigma_{n+1n+1} x \end{bmatrix} \\ &= \mathbf{w}' \Sigma \mathbf{w} + \mathbf{w}' \sigma_{n+1} x + x \sigma'_{n+1} \mathbf{w} + x \sigma_{n+1n+1} x \\ &= \mathbf{w}' \Sigma \mathbf{w} + 2x \sigma'_{n+1} \mathbf{w} + x^2 \sigma_{n+1n+1} \end{aligned}$$

So

$$V(R_p) = \mathbf{w}^*{}' \Sigma^* \mathbf{w}^* = \mathbf{w}' \Sigma \mathbf{w} + 2x\sigma'_{n+1} \mathbf{w} + x^2 \sigma_{n+1n+1}$$

The investor's problem is therefore to select w to:

$$\text{minimise } V(R_p) \text{ subject to } \mathbf{e}'\mathbf{w} = 1$$

where

$$\mathbf{e} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

To solve this we set up the Lagrangean:

$$L = \mathbf{w}' \Sigma \mathbf{w} + 2x\sigma'_{n+1} \mathbf{w} + x^2 \sigma_{n+1n+1} + \lambda'(1 - \mathbf{e}'\mathbf{w})$$

Differentiating with respect to w and λ' we obtain the following first order conditions:

$$\frac{\partial L}{\partial \mathbf{w}} = 2\Sigma \mathbf{w} + 2x\sigma_{n+1} - \lambda' \mathbf{e} = \mathbf{0}$$

$$\text{and } \frac{\partial L}{\partial \lambda'} = 1 - \mathbf{e}'\mathbf{w} = 0$$

Defining $\lambda = (\lambda'/2)$, and assuming Σ^{-1} exists, we obtain:

$$\mathbf{w} = \lambda \Sigma^{-1} \mathbf{e} - x \Sigma^{-1} \sigma_{n+1}$$

$$\text{and } \mathbf{e}'\mathbf{w} = 1$$

Hence

$$\mathbf{e}'\mathbf{w} = 1 = \lambda \mathbf{e}'\Sigma^{-1}\mathbf{e} - x \mathbf{e}'\Sigma^{-1}\sigma_{n+1}$$

$$\Rightarrow \lambda = \left(\frac{1 + x \mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} \right)$$

Substituting for λ gives the optimal portfolio weights:

$$\mathbf{w} = \left(\frac{1 + x \mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} \right) \Sigma^{-1}\mathbf{e} - x \Sigma^{-1}\sigma_{n+1}$$

$$\Rightarrow \mathbf{w} = \frac{\Sigma^{-1}\mathbf{e}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} + x \left[\left(\frac{\mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} \right) \Sigma^{-1}\mathbf{e} - \Sigma^{-1}\sigma_{n+1} \right]$$

$$\Rightarrow \mathbf{w} = \frac{\Sigma^{-1}\mathbf{e}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} - x \left[\Sigma^{-1}\sigma_{n+1} - \left(\frac{\mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} \right) \Sigma^{-1}\mathbf{e} \right]$$

Hence, when there are no liabilities to consider, $x=0$ and:

$$\mathbf{w}_0 = \frac{\Sigma^{-1}\mathbf{e}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}}$$

When there is a liability, $x=-1$ and so:

$$\mathbf{w}_{-1} = \frac{\Sigma^{-1}\mathbf{e}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} + \left[\Sigma^{-1}\sigma_{n+1} - \left(\frac{\mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}} \right) \Sigma^{-1}\mathbf{e} \right]$$

$$= \frac{1}{\left(\mathbf{e}'\Sigma^{-1}\mathbf{e}\right)}\Sigma^{-1}\left[\left(\mathbf{e}'\Sigma^{-1}\mathbf{e}\right)\sigma_{n+1} + \left(1 - \mathbf{e}'\Sigma^{-1}\sigma_{n+1}\right)\mathbf{e}\right]$$

Hence,

$$\mathbf{w}_{-1} = \mathbf{w}_0 + \Delta\mathbf{w}$$

where

$$\begin{aligned}\Delta\mathbf{w} &= \left[\Sigma^{-1}\sigma_{n+1} - \left(\frac{\mathbf{e}'\Sigma^{-1}\sigma_{n+1}}{\mathbf{e}'\Sigma^{-1}\mathbf{e}}\right)\Sigma^{-1}\mathbf{e}\right] \\ &= \frac{1}{\left(\mathbf{e}'\Sigma^{-1}\mathbf{e}\right)}\Sigma^{-1}\left[\left(\mathbf{e}'\Sigma^{-1}\mathbf{e}\right)\sigma_{n+1} - \left(1 - \mathbf{e}'\Sigma^{-1}\sigma_{n+1}\right)\mathbf{e}\right]\end{aligned}$$

It is possible to derive the conditions under which the weight of the first (domestic) asset will increase. This depends upon a complex expression involving the cofactors of Σ but without any immediately obvious intuition. For the case of two assets and one liability, the condition simplifies to that obtained above (1).