SIGNIFICANT FEEDBACKS IN FIRM GROWTH AND MARKET STRUCTURE

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by

Paul A. Kattuman
Judge Institute of Management
University of Cambridge
Trumpington Street
Cambridge CB2 1AG
United Kingdom
Tel: 01223 764136
e-mail: p.kattuman@jims.cam.ac.uk

Alexandru Chirmiciu
King's College
University of Cambridge
Cambridge CB2 1ST
United Kingdom
e-mail: ac296@cam.ac.uk

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Abstract

There are some markets where the growth of firms are held to be subject to diminishing returns, or negative feedbacks; and there are other markets where firm growth is believed to be subject to increasing returns, or positive feedbacks. A long run tendency towards monopoly might be expected in this latter market type, as opposed to a tendency towards relative equality of size shares in the former. It would be useful to draw inferences about the nature of the feedback process from observed market shares and concentration. We motivate and develop a test for feedbacks in firm growth under the null hypothesis that there are none. We use the equivalence between an urn model of the no-feedback process and the asymptotic distribution of sums of ordered intervals in the random division of the unit interval. In the empirical application for the United States, we find that most markets are subject to significant positive feedbacks.

JEL Codes: C16, L11, L60

Key words: Firm growth, urn models, feedback process, size distribution, concentration ratio

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Corresponding Author: Paul Kattuman

1. Introduction

There are a number of celebrated instances where, from among firms in a market, one is seen to have grown swiftly to near-monopoly on the basis of having won some degree of market share advantage. The dominance of the Microsoft operating system, among other examples, has been portrayed in these terms. At the other end, negative feedbacks are also conceivable, where a large initial market share makes it more difficult to keep the lead in the market.

The implication of the growth process for market structure has been an important concern in industrial economics, though the precise notion of feedback has remained largely implicit in this literature. Gibrat's law (Gibrat, 1911), which models the growth of firms in terms of dependence on current size, has been subjected to direct empirical testing in a very large and continuing literature (for a detailed review, see Audretsch *et. al.,* 2002).[1] Though there are a variety of formal models of feedback built upon urn scheme formalisms (Johnson and Kotz, 1977) and their applications have grown prominent in many contexts (Arthur, 1994; Shapiro and Varian, 1999), there has been almost no work explicitly interpreting market structures with the help of probabilistic models of feedback in firm growth. Our primary objective is to formulate a simple, model based statistical test for positive and negative feedbacks in firm growth.

Our second motivation follows directly from this. A variety of structural measures are used in industrial economics to describe the extent to which any market is dominated by large firms, and the corresponding potential for anticompetitive outcomes. None of these measures map, in any precise way, onto the nature or degree of competition in models of markets. In turn, models of growth or competition do not suggest *robust* benchmarks for evaluating observed concentration. In empirical work, concentration has been assessed *ad hoc*, against the benchmark of no concentration. The "bounds" approach of Sutton (1991,1998) marks a departure. Based on a robust model of competition, Sutton derives the expected, conditional, limiting value for a simple structural measure of market concentration, providing a benchmark that could be used to assess observed market structures.[2] Our second objective in this paper is to motivate a method of assessing departures, using an appropriate metric, from a similar model-based benchmark. This requires the limiting probability distribution of the concentration measure, explicitly derived from the model of competition.

We proceed by making explicit the connection between an urn scheme representation of the no-feedback growth process and the asymptotic distribution of the concentration ratio, C_k. The latter arises from the direct application of a result in the distribution theory of the “random division of the unit interval” (Mauldon, 1951). The result is a well defined probability metric to assess departures from the no-feedback process in either direction: positive, or negative feedbacks in growth. Under the null hypothesis that there is no feedback in the growth of firms, observed concentration ratios can be assessed against critical values from the probability distribution of C_k. We follow up with an empirical illustration for United States manufacturing.

The precedent to our work is that of Parker (1991) who employed Mauldon's result in a model-free way to determine “significantly concentrated” industries. He compared observed concentration ratios with the distribution of the C_k ratio under a null that he characterised, as arising in a “totally unconcentrated market”, drawing on the analogy of *random* division of the unit interval. This has been critiqued for not being based on any behavioural model of firm growth or competition (Hviid and Villadsen, 1995). We provide the accurate model-based interpretation of the null hypothesis.

2. Models of firm growth

Sutton's bounds approach model has been stated as follows: a sequence of discrete, equal sized (in terms of revenue / profit) and independent investment opportunities arise over time. Here the market can be interpreted as comprising of a number of independent submarkets - in terms of product attributes, taste niches or geographic locations. A firm's size is measured by the number of opportunities it has taken up. If opportunities are labelled by $t=1,2,...,T$, then n_{it} can denote the number of firms of size i at stage t , and N_t can denote the number of active firms at time t .

$$N_t = \sum_{i=1}^{t} n_{it} \tag{1}$$

The process begins at stage $t=1$, when the first opportunity is taken up by some firm. Thereafter, each opportunity may be taken up either by some active firm or by a new one. The question of interest is: how does the number of firms N_t and their size distribution, the vector n_{it} evolve? The evolution of market concentration will depend on the pattern of entry by firms into the submarkets: on whether there is any systematic bias in favour of large firms or small - is the next opportunity taken by an incumbent, more likely to be taken by a larger, or by a smaller firm.[3]

The bounds approach develops a model of competition based on a principle of symmetry, or equality of opportunity, which excludes the possibility that any firm might be privileged by age, size or experience. The stipulation that the next market opportunity is filled by **any** currently active firm amounts to the proviso of no feedbacks. [4] [5] In the bounds approach, this probabilistic process defines the “expected” size distribution for any given total number of firms. Note that the bounds approach rules out the possibility of smaller firms being systematically advantaged in entry: so observed concentration cannot lie below the bound.

Stochastic processes akin to the above have been modelled as urn processes in mathematics. The classical Polya urn model (Johnson and Kotz, 1977) considers an urn with two kinds of balls, black and white. The replacement policy is defined in terms of drawing a ball from the urn, observing its colour and putting it back in the urn along with $a>0$ balls of the same color. Generalisations of the replacement policy in the basic urn model have become the main methodology in modelling the effect of feedback in growth processes.

Consider a non-linear generalisation of the classical urn model where the probability of drawing a ball of a specific colour from an urn is proportional to a non-linear function (x^p) of the number of balls (x) of that colour in the urn. The case where the number of colours are fixed are analytically tractable – if colour represent firms, this corresponds to the situation where the number of firms is fixed.[6] The case when $p=1$ is equivalent to the Polya-Eggenberger model (Johnson and Kotz, 1977) – and corresponds to Gibrat's Law - each firm has a probability of taking the next opportunity that is proportional to its size; growth rates are independent of size. If $p=0$, the model specifies throwing balls independently and uniformly at random - this corresponds to the case where each active firm has the same probability of taking the next opportunity: the lower bound model. This is the no-feedback case - with the number of firms fixed at the start, and no entry. It is known (Athreya, 1969) that in this case, if we start with k colours, with one ball of each colour, the distribution of the asymptotic proportion of colours in the urn will be uniform. We draw together this result with a result on the distribution of sums of ordered intervals under the random division of the unit interval (Mauldon, 1951) to derive the asymptotic distribution of the concentration ratio under Gibrat's law. In the rest of this section, for completeness, we recount the definition of Generalized Polya's urn schemes and the result of Athreya (1969).

The no-feedback rule enshrines a probabilistic growth process, and this model leads us to a limiting probability distribution of the market concentration index. This distribution can be used to assess point estimates of observed concentration, to test the hypothesis that the growth model operating in any market is the 'no-feedbacks' rule. This will be a more sound assessment of the market structure than merely considering where observed market concentration lies in its statistical range.

2.1 Generalized Polya's urn scheme

A Generalized Polya's urn scheme is defined as follows: At date 0 , an urn has $s_0=(s_{01}, s_{02}, \ldots , s_{0k})$ balls $(s_{0i} \geq 1)$ respectively of colors indexed $1, 2, \ldots , k$. The tth draw consists of the following operations:

1. Pick a ball at random from the urn, notice its color C and return it to the urn.
2. If $C=i$, add N_t balls of color i to the urn, where N_t is a random variable with probability generating function f_i .

Let $s_n = (s_{n1}, s_{n2}, \ldots, s_{nk})$ denote the composition of the urn after n successive draws, where s_{ni} is the number of balls of colour i in the urn.

In this analysis of the evolution of the size distribution $\{ n_{it} \}$, the competitive benchmark is the equal opportunities case when each active firm has the same probability of taking the next opportunity.[7]

The stochastic process $(s\_n;\ n=0,\ 1,\ 2,\ \ldots)$ has been defined a generalized Polya's urn process, denoted by GP$\{k;\ s_0, f_1, \ldots , f_k \}$ to indicate the parameters involved. Define $\boldsymbol{p_n} = (p_{n1}, p_{n2}, \ldots , p_{nk})$ as the share partition, where

$$p_{ni} = \frac{s_{ni}}{\sum_{i=1}^{k} s_{ni}}$$

Using a technique of embedding urn processes in continuous time multitype Markov branching processes, Athreya (1969) proves that:

1. $\lim_{n\to\infty} \boldsymbol{p_n} = \boldsymbol{p} = (p_1, \ldots, p_k)$ exists with probability 1, if the probability generating function f_i is defined in the following way: $f_i(z) = \sum_{j=1}^{\infty} q_{ij} \cdot z^j$ and for all i, $f_i(0) = 0$, $0 < f'(1) = \lambda_i < \infty$; $\sum_{j=1}^{\infty} q_{ij} \cdot j \cdot \log j < \infty$, where $f_i(z) = \sum_{j=1}^{\infty} q_{ij} z^j$ and λ_i is a set of parameters such that $\lambda_1 = \lambda_2 = \ldots = \lambda_r > \lambda_{r+1} \geq \lambda_{r+2} \geq \ldots \lambda_k$.

For all $i > r$, $p_i = 0$ with probability 1, i.e. asymptotically only the first r colours have non-zero proportions in the urn.

2. If $f_i(z) = z^\lambda$, $i = 1, 2, \ldots , r$ and $f'(1) < \lambda$ for $i > r$ then $(p_1, p_2, \ldots , p_r)$ has a generalised $\beta(\frac{s_{01}}{\lambda}, \frac{s_{02}}{\lambda}, \ldots, \frac{s_{0r}}{\lambda})$ distribution over the simplex $\Delta = \{\boldsymbol{x} = (x_1, \ldots , x_r): x_i \geq 0, \sum_{i=1}^{r} x_i = 1\}$.

3. Consider a GP where $f_i(z)$ is independent of i and satisfies the conditions in (1). Then $p = \lim_{n\to\infty} = (p_1, p_2, \ldots , p_k)$ has a $\beta(\frac{s_{01}}{\lambda}, \frac{s_{02}}{\lambda}, \ldots, \frac{s_{0r}}{\lambda})$ distribution for all initial set up $s_0 \neq 0$ if and only if $f(z) = z^\lambda$

2.2 The no-feedback growth process and random division of the unit interval

The generalized Polya process defined above may be characterised simply in the case where the limiting joint distribution is uniform on the unit simplex. Then, the limiting distribution is $\beta(1, \ldots , 1)$ and the necessary and sufficient conditions are: $s_{01} = s_{02} = \ldots = s_{0k} = s_0$, and $\lambda = 1/s_0$. One obvious solution is when the urn contains one ball of each colour, and $\lambda = 1$. In this case, conditioned on N , the total number of firms (colours), if at each date t , each firm (colour) has the same probability of taking the next market opportunity (ball), then as t passes to infinity, each firm (colour) has equal probability of being of any market share in (0,1). As t goes to infinity, each firm's market share is drawn independently from the uniform distribution (0,1). In other words, if a market has a fixed number of firms, and a firm's probability of taking a new opportunity that arises in the market is independent of its size, then asymptotically the joint market share distribution is uniform.

Formally, for a given N_t, as t tends to infinity: Consider $n-1$ points x_j, $(j = 1, \ldots , n-1)$ selected independently at random from interval (0,1), with the distribution of any x_j, the rectangular distribution $dF = dx_j$ $(0 < x_j \leq 1)$. The $n-1$ points selected independently, at random from interval (0,1), divide the unit interval into n sub-intervals, such that each one is drawn independently, at random, from (0,1). Consider each of these sub-intervals to be the size share of a firm. As t tends to infinity, for each firm its size share has the rectangular distribution, i.e. there is equal probability for any firm that its share is any value in (0,1). Correspondingly, each possible size share distribution is equally possible.

Fisher (1929) characterised the above process and characterised the distribution of the largest share. Mauldon (1951) extended this to provide the sampling distribution for the total share C_k of the largest k segments as:

$$F(C_k) = \begin{cases} 0 & \text{for} \quad C_k \le k/n \\ 1 & \quad\quad\; C_k \le 1 \\ \sum_p (-1)^{n-p} p^{-1} \Lambda_p (pC_k - k)^{n-1} & \quad\quad\; k/n \le C_k \le 1 \end{cases} \tag{2}$$

where sum is over values of p for which $k/n < p \le n$ and

$$\Lambda_p = \frac{1}{k^{n-k-1}(p-k)^{k-1}} \cdot \frac{n!}{(n-p)!(p-k)!k!}$$

In Sutton's bounds model, the expected asymptotic firm size distribution under equal opportunities is approximated by an exponential distribution, and under this approximation, the median concentration ratio is well defined.

$$C_{k,median} = \frac{k}{n}\left(1 + \sum_{i=k+1}^{n} \frac{1}{i}\right)$$

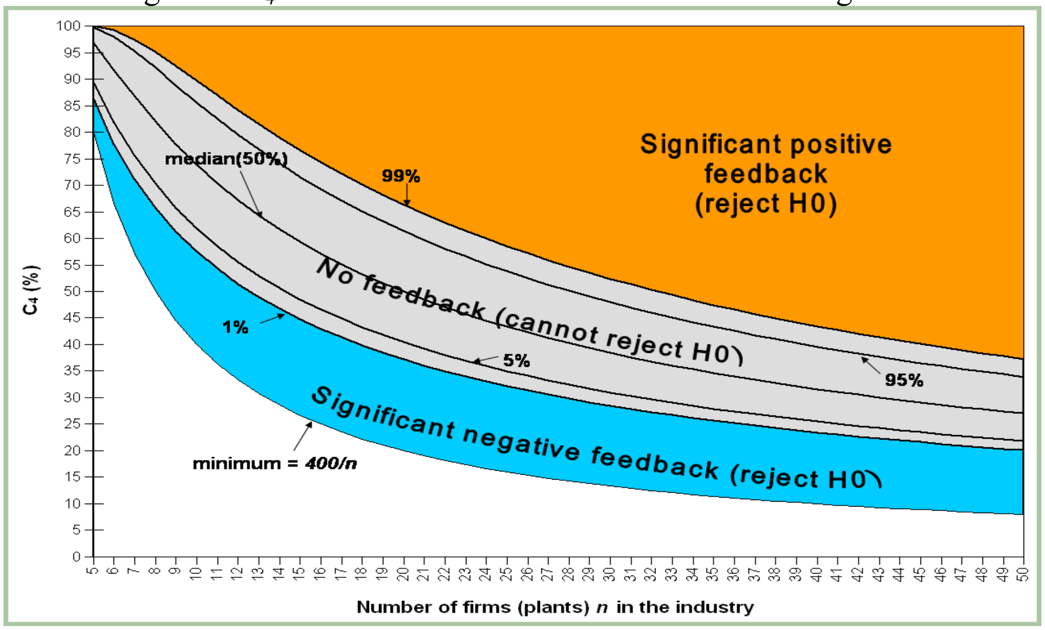
This is indeed the expected value for Mauldon's (1951) distribution above.

2.3 Assessing significance of feedbacks

Our objective is to motivate a statistical test for positive or negative feedbacks, based on a null hypothesis of no feedbacks. We can use Mauldon's (1951) sampling distribution in (2) to compute the confidence bands for the share of the largest k firms, C_k in an industry. Conditioning on the number of firms (industry size) if the underlying competition and growth process follows a no-feedback process, the observed concentration ratios must lie within the confidence bands defined according to the above distribution. Figure 1 plots the confidence bands for C_4 under a no-feedback process, showing how they depart significantly from the theoretical minimum, *400/n* and the maximum, *100* %.

Given the limiting distribution of the concentration ratio, observed values in the upper tail would suggest that larger firms enjoy size advantages over smaller firms; the larger firms are able to pre-empt market opportunities and gain higher market shares than smaller firms. This would be characteristic of economies of scale and scope, and also of intensity in endogenous sunk costs, for example, R&D. Reputation effects, for instance linking entry deterrence activities across submarkets could also be a reason.

Figure 1: C_4 confidence bands under no-feedback firm growth



An observed concentration ratio in the lower tail of the limiting distribution would suggest that larger firms are systematically disadvantaged in competing with small firms. The limiting case is when firms are of equal size.

The usefulness of the probability metric derived from the model of equal opportunity competition is that one can identify the cases where observed concentration is significantly far away from the no-feedbacks benchmark. In Appendix C we examine the nature of the size distribution that is consistent with the Mauldon's distribution. We compare (2) above with two popularly estimated firm size distributions: Pareto and exponential.

3. An application to the US manufacturing sector

Where do observed concentration ratios lay in comparison with the theoretically derived confidence bands? If the market structure is driven by an equal opportunities growth process, observed concentration ratios must lie within the confidence bands of the limiting concentration ratio distribution. If an observed ratio lies in the lower critical region (below the 5% critical value in the no-feedback process), we may conclude that growth in that market favours firms with smaller market shares. Conversely, if the observed concentration ratio lies in the upper critical region (above the 99% critical value), the market would appear to favour firms with larger market shares.

In Section 2 we established that a no-feedback, equal opportunity, growth process would lead to the limiting distribution of concentration ratio described by Mauldon (1951). We use the 1997 US Census of Manufacturing to compare the 6-digit NAICS product market concentration ratios against the bounds derived using the cdf in (2). We detail the critical values in Appendix A. The census covers 473 product markets at the 6-digit level, of which one has only 4 firms (C_4=100). Concentration ratio (C_4) data is supplied for both sales and value added. The computational complexity of Mauldon's formula (eq. 2) impedes the computation of confidence bands for C_4 in industries with more than 500 firms. We directly compare observed concentration ratios against critical values for product markets with not more than 500 competing firms (Table 1).

Table 1: *Number of US Manufacturing industries with negative, no- and positive feedback in growth (1) – industries with up to 500 firms*

Type of growth feedback	Sales	Value Added
Negative feedback	0	0
No-feedback (cannot reject H_0)	5	6
Positive feedback (at 95%)	4	4
Positive feedback (at 99%)	312	311
Total	**321**	**321**

Notes: The null hypothesis is the no-feedback process. If the observed concentration C_4 is less than the corresponding 5% bound of the no-feedback concentration, then the industry must have negative feedback characteristics. Similarly, above the 95% or 99% bounds, industries are positive feedback. No-feedback industries are those for which observed concentration is within the bounds.
Source: US Census Bureau and authors' computation

If the number of firms in a product market exceeds 500, we cannot directly compare the observed concentration ratio against the corresponding critical values. We therefore employ an indirect way of assessing the feedback process. Given that critical concentration ratios are decreasing in the number of firms, all critical values for industries with more than 500 firms must be lower than the critical values for n=500. If the observed concentration in an industry with more than 500 firms is in the upper critical region of an industry with 500 firms, then it must lie in the upper critical region for its own number of firms. This procedure allows us to conclude on most industries of the census (Table 2).

Table 2: *Number of US Manufacturing industries with negative, no- and positive feedback in growth (2) – industries with more than 500 firms*

Type of growth feedback	Sales	Value Added
Positive feedback (at 99%)	142	142
Positive feedback (at 95%)	2	2
Other (no assessment)	7	7
Total	**151**	**151**

Notes: The assessments are based on the critical values for industries with 500 ¯rms. If the observed $C4$ in an industry with n firms is above the 99% bounds for 500 firms, it is also above the critical values for n.
Source: US Census Bureau and authors' computation

The results in Tables 1 and 2 suggest that market structure is mostly driven by positive feedback in firm growth. We find evidence of positive feedback in 460 product markets out of the total of 472. There is no evidence of negative feedbacks and only 5 industries (6 for VA)[8] may have been driven by no-feedbacks in firm growth. The majority of US manufacturing industries are positive feedback industries, where concentration significantly diverges from theoretical minima.

4. Conclusion

Positive or negative feedbacks in the firm growth process have fundamental implications for the evolution of market structure. But while the estimation of firm growth models has always been concerned with assessing positive or negative dependence of firm growth rates on size, there has been little work relating market concentration and feedbacks in firm growth in a model that is amenable to statistical inference procedures. While a method to determine "significantly concentrated" markets was suggested (Parker, 1991), there has been little progress in deriving probability distributions of market concentration drawing from general models of firm growth and competition.

Our objective has been to determine a probability metric for the assessment of feedbacks in the growth process of firms. We showed how a no-feedback growth process can be represented as an urn scheme. This behavioural model asymptotically leads to the concentration ratio distribution identical to the distribution of sums of ordered intervals from the random division of the unit interval. Thus under a null hypothesis that there is no feedback in the growth of firms, observed concentration ratios can be assessed against critical values from the limiting probability distribution. Observed concentration in the upper critical region would suggest that larger firms are able to pre-empt smaller firms in taking new market opportunities - possible evidence of economies of scale and scope. Observed concentration ratio in the lower critical region would suggest that larger firms are systematically disadvantaged in competing with small firms. The probability metric derived from the no-feedback model can be used to identify cases where there are significant positive or negative feedbacks. In the empirical application for the United States, we found that almost all markets showed evidence of significant positive feedback.

Notes

1 Growth rates of surviving firms appear to decline systematically with size, at least in manufacturing.

2 The measure used by Sutton, the concentration ratio C_k is defined as the sum of the market shares of the largest k firms in a market of n firms, where k is a specified small number, usually between 3 and 8. The concentration ratio has well-known shortcomings (Sleuwaegen and Dehandschutter, 1986), but is popular in empirical work. Its ease of computation and general availability from government statistical sources make it suitable for comparing $C_k \in [k/n, 1]$ across industries, countries and over time. The program of empirical work that followed the bounds approach has gone on to estimate the lower envelope of the cross sectional relationship between C_k and market size (or n), for different classes of industries.

3 It depends to a lesser extent on de-novo entry. The extreme case of minimal concentration will result if each of the submarkets were equal in size and these were taken up in succession by new firms, leading to perfect equality between firms. McCloughan (1995) used simulations to determine the role of different processes in shaping concentration. He found that the most important determinant of concentration was systematic firm-level growth; entry and exit were much less significant.

4 In contrast, Gibrat's law postulates that the “probability that the next opportunity is taken up by any particular active firm is proportional to the current size of the firm” (Sutton, 1997, p. 43).

5 Sutton argues that in the light of the difficulty in choosing among the many model specifics of unobservable firm decisions, robustness requires the dropping of the standard game theoretic principle of the legitimacy of all perfect Nash equilibria. Then the definition of rationality involves only a viability condition, interpreted as avoidance of loss-making strategies; and a stability condition, which is a no-arbitrage principle. The competition to enter any submarket can be modelled as a generic two stage game; firms first making various sunk cost investments, setting up plants, designing and developing product attributes, etc., leading to a configuration in the space of plant locations, or product characteristics. This is followed in the second stage by market competition which may take any form: Cournot, Bertrand, or otherwise, but in which choices made in the first stage are parameters into the firms' payoff functions.

Sutton shows that in modelling the evolution of market structure, the above described firm growth process nests into a robust characterisation of the set of outcomes that can be supported as an equilibrium of any candidate competition model.

6 While this appears to depart from Sutton's model that allows entry, it must be noted Sutton's results are conditioned on chosen limiting values of N_t.

7 This corresponds to the case when the resulting size distribution will be of minimum inequality, among all the cases where the probability that the next market opportunity is taken by an active firm is non-decreasing in the size of the firm.

8 The industries are beet sugar (NAICS 311313), tire cord and tire fabric mills (314992), newsprint mills (322122), electrometallurgical ferroalloy products (331112) and primary aluminium production (331312) for sales and flat glass (327211) additionally for value added. Notice that the metallurgical industries (aluminium and ferroalloy) are electricity intensive industries and plants tend to locate in the vicinity of power sources, with an upper limit to growth, that depends on the limited energy resources available. Newsprint mills are part of an industry that is rather decentralised in the US, due to the localised nature of print media in component states.

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Appendix A

Table 3: *Critical values of C_4 for the no-feedback process*

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
5	86.32	89.46	96.82	99.75	99.95	**6**	77.71	81.91	91.77	98.02	99.16
7	71.10	75.56	86.74	95.28	97.40	**8**	65.73	70.26	82.04	92.12	95.05
9	61.27	65.76	77.75	88.84	92.40	**10**	57.50	61.89	73.87	85.60	89.63
11	54.25	58.53	70.37	82.48	86.85	**12**	51.43	55.58	67.21	79.51	84.14
13	48.94	52.96	64.33	76.71	81.51	**14**	46.73	50.61	61.72	74.08	79.00
15	44.74	48.51	59.33	71.61	76.61	**16**	42.95	46.60	57.14	69.29	74.33
17	41.33	44.86	55.12	67.12	72.18	**18**	39.84	43.26	53.26	65.08	70.13
19	38.48	41.80	51.53	63.17	68.20	**20**	37.23	40.45	49.93	61.37	66.37
21	36.07	39.20	48.43	59.67	64.63	**22**	34.99	38.03	47.03	58.07	62.98
23	33.99	36.95	45.72	56.56	61.42	**24**	33.05	35.93	44.49	55.13	59.93
25	32.17	34.98	43.34	53.78	58.51	**26**	31.35	34.09	42.25	52.49	57.17
27	30.58	33.25	41.22	51.27	55.89	**28**	29.85	32.45	40.24	50.11	54.66
29	29.15	31.70	39.32	49.01	53.49	**30**	28.50	30.99	38.44	47.95	52.38
31	27.88	30.31	37.61	46.95	51.31	**32**	27.29	29.67	36.82	45.99	50.29
33	26.73	29.06	36.06	45.07	49.31	**34**	26.20	28.48	35.34	44.19	48.37
35	25.69	27.92	34.65	43.35	47.46	**36**	25.20	27.39	33.99	42.54	46.60
37	24.73	26.88	33.35	41.76	45.76	**38**	24.29	26.40	32.75	41.02	44.96
39	23.86	25.93	32.16	40.30	44.19	**40**	23.45	25.48	31.61	39.61	43.44
41	23.06	25.05	31.07	38.95	42.72	**42**	22.68	24.64	30.55	38.31	42.03
43	22.31	24.24	30.05	37.69	41.36	**44**	21.96	23.85	29.57	37.09	40.71
45	21.62	23.48	29.11	36.51	40.09	**46**	21.30	23.13	28.66	35.96	39.48
47	20.98	22.78	28.23	35.42	38.90	**48**	20.68	22.45	27.81	34.90	38.33
49	20.38	22.13	27.41	34.39	37.78	**50**	20.10	21.82	27.02	33.90	37.25
51	19.82	21.51	26.64	33.43	36.73	**52**	19.56	21.22	26.27	32.97	36.23
53	19.30	20.94	25.91	32.52	35.75	**54**	19.05	20.67	25.57	32.09	35.27
55	18.80	20.40	25.23	31.67	34.81	**56**	18.57	20.14	24.91	31.26	34.37
57	18.34	19.89	24.59	30.86	33.93	**58**	18.11	19.64	24.29	30.48	33.51
59	17.90	19.41	23.99	30.10	33.10	**60**	17.69	19.18	23.70	29.73	32.70
61	17.48	18.95	23.41	29.38	32.31	**62**	17.28	18.73	23.14	29.03	31.93
63	17.09	18.52	22.87	28.69	31.56	**64**	16.90	18.31	22.61	28.36	31.19
65	16.71	18.11	22.36	28.04	30.84	**66**	16.53	17.91	22.11	27.73	30.50
67	16.36	17.72	21.87	27.42	30.16	**68**	16.18	17.53	21.63	27.12	29.83
69	16.02	17.35	21.40	26.83	29.51	**70**	15.85	17.17	21.17	26.54	29.20
71	15.69	17.00	20.95	26.27	28.89	**72**	15.54	16.83	20.74	25.99	28.60

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
73	15.38	16.66	20.53	25.73	28.30	**74**	15.24	16.50	20.32	25.47	28.02
75	15.09	16.34	20.12	25.21	27.74	**76**	14.95	16.18	19.93	24.96	27.46
77	14.81	16.03	19.73	24.72	27.20	**78**	14.67	15.88	19.54	24.48	26.93
79	14.54	15.73	19.36	24.25	26.68	**80**	14.41	15.59	19.18	24.02	26.42
81	14.28	15.45	19.00	23.79	26.18	**82**	14.15	15.31	18.83	23.58	25.94
83	14.03	15.18	18.66	23.36	25.70	**84**	13.91	15.04	18.50	23.15	25.47
85	13.79	14.92	18.33	22.94	25.24	**86**	13.67	14.79	18.17	22.74	25.01
87	13.56	14.66	18.02	22.54	24.80	**88**	13.45	14.54	17.86	22.35	24.58
89	13.34	14.42	17.71	22.15	24.37	**90**	13.23	14.30	17.56	21.97	24.16
91	13.12	14.19	17.42	21.78	23.96	**92**	13.02	14.08	17.28	21.60	23.76
93	12.92	13.96	17.14	21.42	23.56	**94**	12.82	13.85	17.00	21.25	23.37
95	12.72	13.75	16.86	21.08	23.18	**96**	12.62	13.64	16.73	20.91	22.99
97	12.53	13.54	16.60	20.74	22.81	**98**	12.44	13.44	16.47	20.58	22.63
99	12.34	13.34	16.35	20.42	22.46	**100**	12.25	13.24	16.22	20.26	22.28
101	12.16	13.14	16.10	20.11	22.11	**102**	12.08	13.04	15.98	19.95	21.94
103	11.99	12.95	15.86	19.80	21.78	**104**	11.91	12.86	15.75	19.66	21.61
105	11.82	12.77	15.63	19.51	21.45	**106**	11.74	12.68	15.52	19.37	21.30
107	11.66	12.59	15.41	19.23	21.14	**108**	11.58	12.50	15.30	19.09	20.99
109	11.50	12.42	15.19	18.95	20.84	**110**	11.43	12.33	15.09	18.82	20.69
111	11.35	12.25	14.98	18.69	20.55	**112**	11.28	12.17	14.88	18.56	20.40
113	11.20	12.09	14.78	18.43	20.26	**114**	11.13	12.01	14.68	18.30	20.12
115	11.06	11.93	14.58	18.18	19.98	**116**	10.99	11.86	14.49	18.06	19.85
117	10.92	11.78	14.39	17.94	19.72	**118**	10.85	11.71	14.30	17.82	19.58
119	10.78	11.63	14.21	17.70	19.45	**120**	10.72	11.56	14.12	17.58	19.33
121	10.65	11.49	14.03	17.47	19.20	**122**	10.59	11.42	13.94	17.36	19.08
123	10.52	11.35	13.85	17.25	18.95	**124**	10.46	11.28	13.77	17.14	18.83
125	10.40	11.21	13.68	17.03	18.71	**126**	10.34	11.15	13.60	16.92	18.60
127	10.28	11.08	13.51	16.82	18.48	**128**	10.22	11.02	13.43	16.72	18.37
129	10.16	10.95	13.35	16.61	18.25	**130**	10.10	10.89	13.27	16.51	18.14
131	10.04	10.83	13.20	16.41	18.03	**132**	9.99	10.76	13.12	16.32	17.93
133	9.93	10.70	13.04	16.22	17.82	**134**	9.88	10.64	12.97	16.12	17.71
135	9.82	10.58	12.89	16.03	17.61	**136**	9.77	10.52	12.82	15.94	17.51
137	9.71	10.47	12.75	15.84	17.40	**138**	9.66	10.41	12.68	15.75	17.30
139	9.61	10.35	12.60	15.66	17.20	**140**	9.56	10.30	12.53	15.57	17.11
141	9.51	10.24	12.47	15.49	17.01	**142**	9.46	10.19	12.40	15.40	16.91
143	9.41	10.13	12.33	15.31	16.82	**144**	9.36	10.08	12.26	15.23	16.73
145	9.31	10.03	12.20	15.15	16.63	**146**	9.26	9.98	12.13	15.06	16.54
147	9.22	9.93	12.07	14.98	16.45	**148**	9.17	9.87	12.01	14.90	16.36

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
149	9.13	9.82	11.94	14.82	16.28	**150**	9.08	9.77	11.88	14.74	16.19
151	9.03	9.73	11.82	14.67	16.10	**152**	8.99	9.68	11.76	14.59	16.02
153	8.95	9.63	11.70	14.51	15.93	**154**	8.90	9.58	11.64	14.44	15.85
155	8.86	9.53	11.58	14.36	15.77	**156**	8.82	9.49	11.52	14.29	15.69
157	8.77	9.44	11.47	14.22	15.61	**158**	8.73	9.40	11.41	14.15	15.53
159	8.69	9.35	11.35	14.07	15.45	**160**	8.65	9.31	11.30	14.00	15.37
161	8.61	9.26	11.24	13.93	15.29	**162**	8.57	9.22	11.19	13.87	15.22
163	8.53	9.18	11.14	13.80	15.14	**164**	8.49	9.13	11.08	13.73	15.07
165	8.45	9.09	11.03	13.66	14.99	**166**	8.41	9.05	10.98	13.60	14.92
167	8.38	9.01	10.93	13.53	14.85	**168**	8.34	8.97	10.88	13.47	14.78
169	8.30	8.93	10.82	13.40	14.71	**170**	8.27	8.89	10.77	13.34	14.64
171	8.23	8.85	10.73	13.28	14.57	**172**	8.19	8.81	10.68	13.22	14.50
173	8.16	8.77	10.63	13.15	14.43	**174**	8.12	8.73	10.58	13.09	14.36
175	8.09	8.69	10.53	13.03	14.30	**176**	8.05	8.66	10.49	12.97	14.23
177	8.02	8.62	10.44	12.91	14.17	**178**	7.98	8.58	10.39	12.86	14.10
179	7.95	8.55	10.35	12.80	14.04	**180**	7.92	8.51	10.30	12.74	13.97
181	7.88	8.47	10.26	12.68	13.91	**182**	7.85	8.44	10.21	12.63	13.85
183	7.82	8.40	10.17	12.57	13.79	**184**	7.79	8.37	10.13	12.52	13.73
185	7.75	8.33	10.08	12.46	13.67	**186**	7.72	8.30	10.04	12.41	13.61
187	7.69	8.26	10.00	12.36	13.55	**188**	7.66	8.23	9.96	12.30	13.49
189	7.63	8.20	9.91	12.25	13.43	**190**	7.60	8.16	9.87	12.20	13.37
191	7.57	8.13	9.83	12.15	13.32	**192**	7.54	8.10	9.79	12.09	13.26
193	7.51	8.07	9.75	12.04	13.20	**194**	7.48	8.03	9.71	11.99	13.15
195	7.45	8.00	9.67	11.94	13.09	**196**	7.42	7.97	9.63	11.89	13.04
197	7.39	7.94	9.59	11.85	12.98	**198**	7.36	7.91	9.56	11.80	12.93
199	7.34	7.88	9.52	11.75	12.88	**200**	7.31	7.85	9.48	11.70	12.83
201	7.28	7.82	9.44	11.65	12.77	**202**	7.25	7.79	9.41	11.61	12.72
203	7.23	7.76	9.37	11.56	12.67	**204**	7.20	7.73	9.33	11.51	12.62
205	7.17	7.70	9.30	11.47	12.57	**206**	7.15	7.67	9.26	11.42	12.52
207	7.12	7.64	9.23	11.38	12.47	**208**	7.09	7.62	9.19	11.33	12.42
209	7.07	7.59	9.16	11.29	12.37	**210**	7.04	7.56	9.12	11.25	12.32
211	7.02	7.53	9.09	11.20	12.28	**212**	6.99	7.50	9.05	11.16	12.23
213	6.97	7.48	9.02	11.12	12.18	**214**	6.94	7.45	8.99	11.08	12.13
215	6.92	7.42	8.95	11.03	12.09	**216**	6.89	7.40	8.92	10.99	12.04
217	6.87	7.37	8.89	10.95	12.00	**218**	6.84	7.34	8.85	10.91	11.95
219	6.82	7.32	8.82	10.87	11.91	**220**	6.80	7.29	8.79	10.83	11.86
221	6.77	7.27	8.76	10.79	11.82	**222**	6.75	7.24	8.73	10.75	11.77
223	6.73	7.22	8.70	10.71	11.73	**224**	6.70	7.19	8.67	10.67	11.69

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
225	6.68	7.17	8.63	10.63	11.64	**226**	6.66	7.14	8.60	10.59	11.60
227	6.64	7.12	8.57	10.56	11.56	**228**	6.61	7.09	8.54	10.52	11.52
229	6.59	7.07	8.51	10.48	11.48	**230**	6.57	7.05	8.49	10.44	11.44
231	6.55	7.02	8.46	10.41	11.40	**232**	6.53	7.00	8.43	10.37	11.35
233	6.50	6.98	8.40	10.33	11.31	**234**	6.48	6.95	8.37	10.30	11.27
235	6.46	6.93	8.34	10.26	11.23	**236**	6.44	6.91	8.31	10.23	11.20
237	6.42	6.88	8.28	10.19	11.16	**238**	6.40	6.86	8.26	10.16	11.12
239	6.38	6.84	8.23	10.12	11.08	**240**	6.36	6.82	8.20	10.09	11.04
241	6.34	6.80	8.17	10.05	11.00	**242**	6.32	6.77	8.15	10.02	10.97
243	6.30	6.75	8.12	9.98	10.93	**244**	6.28	6.73	8.09	9.95	10.89
245	6.26	6.71	8.07	9.92	10.85	**246**	6.24	6.69	8.04	9.88	10.82
247	6.22	6.67	8.02	9.85	10.78	**248**	6.20	6.65	7.99	9.82	10.75
249	6.18	6.63	7.96	9.79	10.71	**250**	6.16	6.60	7.94	9.75	10.68
251	6.14	6.58	7.91	9.72	10.64	**252**	6.12	6.56	7.89	9.69	10.60
253	6.11	6.54	7.86	9.66	10.57	**254**	6.09	6.52	7.84	9.63	10.54
255	6.07	6.50	7.81	9.60	10.50	**256**	6.05	6.48	7.79	9.57	10.47
257	6.03	6.46	7.77	9.54	10.43	**258**	6.02	6.44	7.74	9.51	10.40
259	6.00	6.43	7.72	9.48	10.37	**260**	5.98	6.41	7.69	9.45	10.33
261	5.96	6.39	7.67	9.42	10.30	**262**	5.94	6.37	7.65	9.39	10.27
263	5.93	6.35	7.62	9.36	10.24	**264**	5.91	6.33	7.60	9.33	10.20
265	5.89	6.31	7.58	9.30	10.17	**266**	5.88	6.29	7.55	9.27	10.14
267	5.86	6.27	7.53	9.24	10.11	**268**	5.84	6.26	7.51	9.21	10.08
269	5.82	6.24	7.49	9.18	10.05	**270**	5.81	6.22	7.46	9.16	10.02
271	5.79	6.20	7.44	9.13	9.98	**272**	5.77	6.18	7.42	9.10	9.95
273	5.76	6.17	7.40	9.07	9.92	**274**	5.74	6.15	7.38	9.05	9.89
275	5.73	6.13	7.35	9.02	9.86	**276**	5.71	6.11	7.33	8.99	9.83
277	5.69	6.10	7.31	8.97	9.80	**278**	5.68	6.08	7.29	8.94	9.77
279	5.66	6.06	7.27	8.91	9.75	**280**	5.65	6.05	7.25	8.89	9.72
281	5.63	6.03	7.23	8.86	9.69	**282**	5.62	6.01	7.21	8.83	9.66
283	5.60	6.00	7.19	8.81	9.63	**284**	5.59	5.98	7.17	8.78	9.60
285	5.57	5.96	7.15	8.76	9.57	**286**	5.56	5.95	7.13	8.73	9.55
287	5.54	5.93	7.11	8.71	9.52	**288**	5.53	5.91	7.09	8.68	9.49
289	5.51	5.90	7.07	8.66	9.46	**290**	5.50	5.88	7.05	8.63	9.44
291	5.48	5.87	7.03	8.61	9.41	**292**	5.47	5.85	7.01	8.58	9.38
293	5.45	5.83	6.99	8.56	9.36	**294**	5.44	5.82	6.97	8.53	9.33
295	5.42	5.80	6.95	8.51	9.30	**296**	5.41	5.79	6.93	8.49	9.28
297	5.40	5.77	6.91	8.46	9.25	**298**	5.38	5.76	6.89	8.44	9.22
299	5.37	5.74	6.88	8.42	9.20	**300**	5.35	5.73	6.86	8.39	9.17

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
301	5.34	5.71	6.84	8.37	9.15	**302**	5.33	5.70	6.82	8.35	9.12
303	5.31	5.68	6.80	8.32	9.10	**304**	5.30	5.67	6.78	8.30	9.07
305	5.29	5.65	6.77	8.28	9.05	**306**	5.27	5.64	6.75	8.26	9.02
307	5.26	5.63	6.73	8.23	9.00	**308**	5.25	5.61	6.71	8.21	8.97
309	5.23	5.60	6.70	8.19	8.95	**310**	5.22	5.58	6.68	8.17	8.93
311	5.21	5.57	6.66	8.15	8.90	**312**	5.19	5.55	6.64	8.13	8.88
313	5.18	5.54	6.63	8.10	8.85	**314**	5.17	5.53	6.61	8.08	8.83
315	5.15	5.51	6.59	8.06	8.81	**316**	5.14	5.50	6.58	8.04	8.78
317	5.13	5.49	6.56	8.02	8.76	**318**	5.12	5.47	6.54	8.00	8.74
319	5.10	5.46	6.53	7.98	8.71	**320**	5.09	5.44	6.51	7.96	8.69
321	5.08	5.43	6.49	7.94	8.67	**322**	5.07	5.42	6.48	7.92	8.65
323	5.05	5.40	6.46	7.89	8.62	**324**	5.04	5.39	6.44	7.87	8.60
325	5.03	5.38	6.43	7.85	8.58	**326**	5.02	5.37	6.41	7.83	8.56
327	5.01	5.35	6.40	7.81	8.54	**328**	4.99	5.34	6.38	7.79	8.51
329	4.98	5.33	6.36	7.78	8.49	**330**	4.97	5.31	6.35	7.76	8.47
331	4.96	5.30	6.33	7.74	8.45	**332**	4.95	5.29	6.32	7.72	8.43
333	4.94	5.28	6.30	7.70	8.41	**334**	4.92	5.26	6.29	7.68	8.38
335	4.91	5.25	6.27	7.66	8.36	**336**	4.90	5.24	6.26	7.64	8.34
337	4.89	5.23	6.24	7.62	8.32	**338**	4.88	5.21	6.23	7.60	8.30
339	4.87	5.20	6.21	7.58	8.28	**340**	4.86	5.19	6.20	7.57	8.26
341	4.85	5.18	6.18	7.55	8.24	**342**	4.83	5.17	6.17	7.53	8.22
343	4.82	5.15	6.15	7.51	8.20	**344**	4.81	5.14	6.14	7.49	8.18
345	4.80	5.13	6.12	7.47	8.16	**346**	4.79	5.12	6.11	7.46	8.14
347	4.78	5.11	6.10	7.44	8.12	**348**	4.77	5.10	6.08	7.42	8.10
349	4.76	5.08	6.07	7.40	8.08	**350**	4.75	5.07	6.05	7.38	8.06
351	4.74	5.06	6.04	7.37	8.04	**352**	4.73	5.05	6.02	7.35	8.02
353	4.72	5.04	6.01	7.33	8.00	**354**	4.71	5.03	6.00	7.32	7.98
355	4.70	5.02	5.98	7.30	7.97	**356**	4.69	5.01	5.97	7.28	7.95
357	4.68	4.99	5.96	7.26	7.93	**358**	4.66	4.98	5.94	7.25	7.91
359	4.65	4.97	5.93	7.23	7.89	**360**	4.64	4.96	5.92	7.21	7.87
361	4.63	4.95	5.90	7.20	7.85	**362**	4.62	4.94	5.89	7.18	7.84
363	4.61	4.93	5.88	7.16	7.82	**364**	4.60	4.92	5.86	7.15	7.80
365	4.59	4.91	5.85	7.13	7.78	**366**	4.58	4.90	5.84	7.11	7.76
367	4.58	4.89	5.82	7.10	7.75	**368**	4.57	4.88	5.81	7.08	7.73
369	4.56	4.87	5.80	7.07	7.71	**370**	4.55	4.86	5.79	7.05	7.69
371	4.54	4.84	5.77	7.03	7.68	**372**	4.53	4.83	5.76	7.02	7.66
373	4.52	4.82	5.75	7.00	7.64	**374**	4.51	4.81	5.74	6.99	7.62
375	4.50	4.80	5.72	6.97	7.61	**376**	4.49	4.79	5.71	6.96	7.59

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
377	4.48	4.78	5.70	6.94	7.57	**378**	4.47	4.77	5.69	6.93	7.56
379	4.46	4.76	5.67	6.91	7.54	**380**	4.45	4.75	5.66	6.89	7.52
381	4.44	4.74	5.65	6.88	7.51	**382**	4.43	4.73	5.64	6.86	7.49
383	4.42	4.72	5.63	6.85	7.47	**384**	4.42	4.71	5.61	6.83	7.46
385	4.41	4.70	5.60	6.82	7.44	**386**	4.40	4.69	5.59	6.81	7.42
387	4.39	4.69	5.58	6.79	7.41	**388**	4.38	4.68	5.57	6.78	7.39
389	4.37	4.67	5.55	6.76	7.37	**390**	4.36	4.66	5.54	6.75	7.36
391	4.35	4.65	5.53	6.73	7.34	**392**	4.34	4.64	5.52	6.72	7.33
393	4.34	4.63	5.51	6.70	7.31	**394**	4.33	4.62	5.50	6.69	7.30
395	4.32	4.61	5.49	6.68	7.28	**396**	4.31	4.60	5.47	6.66	7.26
397	4.30	4.59	5.46	6.65	7.25	**398**	4.29	4.58	5.45	6.63	7.23
399	4.28	4.57	5.44	6.62	7.22	**400**	4.28	4.56	5.43	6.61	7.20
401	4.27	4.55	5.42	6.59	7.19	**402**	4.26	4.55	5.41	6.58	7.17
403	4.25	4.54	5.40	6.56	7.16	**404**	4.24	4.53	5.39	6.55	7.14
405	4.23	4.52	5.37	6.54	7.13	**406**	4.23	4.51	5.36	6.52	7.11
407	4.22	4.50	5.35	6.51	7.10	**408**	4.21	4.49	5.34	6.50	7.08
409	4.20	4.48	5.33	6.48	7.07	**410**	4.19	4.48	5.32	6.47	7.05
411	4.19	4.47	5.31	6.46	7.04	**412**	4.18	4.46	5.30	6.44	7.02
413	4.17	4.45	5.29	6.43	7.01	**414**	4.16	4.44	5.28	6.42	7.00
415	4.15	4.43	5.27	6.40	6.98	**416**	4.15	4.42	5.26	6.39	6.97
417	4.14	4.42	5.25	6.38	6.95	**418**	4.13	4.41	5.24	6.37	6.94
419	4.12	4.40	5.23	6.35	6.93	**420**	4.12	4.39	5.22	6.34	6.91
421	4.11	4.38	5.21	6.33	6.90	**422**	4.10	4.37	5.20	6.31	6.88
423	4.09	4.37	5.19	6.30	6.87	**424**	4.08	4.36	5.18	6.29	6.86
425	4.08	4.35	5.17	6.28	6.84	**426**	4.07	4.34	5.16	6.27	6.83
427	4.06	4.33	5.15	6.25	6.81	**428**	4.05	4.32	5.14	6.24	6.80
429	4.05	4.32	5.13	6.23	6.79	**430**	4.04	4.31	5.12	6.22	6.77
431	4.03	4.30	5.11	6.20	6.76	**432**	4.02	4.29	5.10	6.19	6.75
433	4.02	4.28	5.09	6.18	6.73	**434**	4.01	4.28	5.08	6.17	6.72
435	4.00	4.27	5.07	6.16	6.71	**436**	4.00	4.26	5.06	6.14	6.70
437	3.99	4.25	5.05	6.13	6.68	**438**	3.98	4.25	5.04	6.12	6.67
439	3.97	4.24	5.03	6.11	6.66	**440**	3.97	4.23	5.02	6.10	6.64
441	3.96	4.22	5.01	6.09	6.63	**442**	3.95	4.22	5.00	6.07	6.62
443	3.95	4.21	4.99	6.06	6.61	**444**	3.94	4.20	4.98	6.05	6.59
445	3.93	4.19	4.98	6.04	6.58	**446**	3.92	4.18	4.97	6.03	6.57
447	3.92	4.18	4.96	6.02	6.56	**448**	3.91	4.17	4.95	6.01	6.54
449	3.90	4.16	4.94	5.99	6.53	**450**	3.90	4.16	4.93	5.98	6.52
451	3.89	4.15	4.92	5.97	6.51	**452**	3.88	4.14	4.91	5.96	6.49

n	1%	5%	50%	95%	99%	n	1%	5%	50%	95%	99%
453	3.88	4.13	4.90	5.95	6.48	**454**	3.87	4.13	4.89	5.94	6.47
455	3.86	4.12	4.89	5.93	6.46	**456**	3.86	4.11	4.88	5.92	6.45
457	3.85	4.10	4.87	5.91	6.43	**458**	3.84	4.10	4.86	5.89	6.42
459	3.84	4.09	4.85	5.88	6.41	**460**	3.83	4.08	4.84	5.87	6.40
461	3.82	4.08	4.83	5.86	6.39	**462**	3.82	4.07	4.82	5.85	6.37
463	3.81	4.06	4.82	5.84	6.36	**464**	3.80	4.05	4.81	5.83	6.35
465	3.80	4.05	4.80	5.82	6.34	**466**	3.79	4.04	4.79	5.81	6.33
467	3.78	4.03	4.78	5.80	6.32	**468**	3.78	4.03	4.77	5.79	6.30
469	3.77	4.02	4.77	5.78	6.29	**470**	3.77	4.01	4.76	5.77	6.28
471	3.76	4.01	4.75	5.76	6.27	**472**	3.75	4.00	4.74	5.75	6.26
473	3.75	3.99	4.73	5.74	6.25	**474**	3.74	3.99	4.72	5.73	6.24
475	3.73	3.98	4.72	5.72	6.23	**476**	3.73	3.97	4.71	5.71	6.21
477	3.72	3.97	4.70	5.70	6.20	**478**	3.72	3.96	4.69	5.69	6.19
479	3.71	3.95	4.68	5.68	6.18	**480**	3.70	3.95	4.68	5.67	6.17
481	3.70	3.94	4.67	5.66	6.16	**482**	3.69	3.93	4.66	5.65	6.15
483	3.69	3.93	4.65	5.64	6.14	**484**	3.68	3.92	4.64	5.63	6.13
485	3.67	3.91	4.64	5.62	6.12	**486**	3.67	3.91	4.63	5.61	6.10
487	3.66	3.90	4.62	5.60	6.09	**488**	3.66	3.89	4.61	5.59	6.08
489	3.65	3.89	4.60	5.58	6.07	**490**	3.64	3.88	4.60	5.57	6.06
491	3.64	3.88	4.59	5.56	6.05	**492**	3.63	3.87	4.58	5.55	6.04
493	3.63	3.86	4.57	5.54	6.03	**494**	3.62	3.86	4.57	5.53	6.02
495	3.61	3.85	4.56	5.52	6.01	**496**	3.61	3.84	4.55	5.51	6.00
497	3.60	3.84	4.54	5.50	5.99	**498**	3.60	3.83	4.54	5.49	5.98
499	3.59	3.83	4.53	5.48	5.97	**500**	3.59	3.82	4.52	5.47	5.96

Appendix B - Extensions of Polya's urn and the Poisson-Dirichlet distribution

Polya's urn model has been extended to accommodate entry. Consider the version analysed by Hoppe (1984, 1987). An urn initially contains one white ball of mass θ>0 and various numbers of balls of other colours (non-white) each of mass 1. At each point of discrete time a ball is drawn at random (in proportion to its mass) from the urn. If the selected ball is white it is returned with a ball of a previously unused colour (and mass 1). Otherwise the selected ball is returned with one additional ball of the same colour and mass.

The extent of entry is captured by the parameter θ. Each jth draw is taken either by a new entrant or an incumbent with probabilities $\theta/(\theta+j-1)$ and $(j-1)/(\theta+j-1)$, respectively. The probability of new entry is strictly decreasing in j, incumbents on aggregate have an increasing probability of success. In the limit (as $j\to\infty$) the probability of entry converges to zero. Asymptotically the number of firms in a market is hence a random number and its distribution depends on the parameter θ.

The most important result on generalised Polya's urn is Ewens' sampling formula (Ewens 1972). Consider an urn process described above in which n draws are made. The probability of obtaining an arbitrary partition given by $a=\{a_1,a_2,...,a_n\}$ is given by:

$$\Pr[\Pi_n = a] = \frac{n!}{[\theta]^n}\prod_{i=1}^{n}\frac{\theta^{a_i}}{i^{a_i}a_i!} \tag{3}$$

where Π_n is the sample size partition, a is the given size partition, each a_i is a non-negative integer and denotes the number of times integer i appears in the occurrences of the k different colours (i.e. the number of colours that have i balls in the urn after n draws), and $[\theta]^n = \theta(\theta+1)\ldots(\theta+n-1)$ is the ascending factorial. Note that k is an arbitrary positive number such that $0 < k \leq n$. A complete description of the size partition requires:

$$\sum_{i=1}^{n} a_i = k \qquad \text{and} \qquad \sum_{i=1}^{n} i a_i = n$$

A generalized urn model (with entry) is, however, of limited use for our purposes. First, the random number of colours in the urn has a distribution that cannot be derived explicitly. We seek to establish a benchmark for the order statistics in an urn model and hence the number of firms (colours) must be fixed (as a long-run equilibrium). Second, implicit asymptotic results can only be obtained under certain parameter restrictions. Even though Ewens (1972), Kingman (1975) and Hoppe (1987) extend the results of Athreya (1969), they do not derive the precise asymptotic distribution of the *ordered* shares of colours in Polya's urn. We have no knowledge of subsequent work that explicitly derive distributions from generalized Polya's urn processes that can readily be used in this analysis and it appears that integration and asymptotic results are impossible to obtain (Hoppe, 1987).

Appendix C - Approximating the bounds

The law of proportional effects (Gibrat, 1931) implies that firm size distribution (FSD) are lognormal. Since then, several studies have looked at patterns of growth and their implication for FSD (e.g. Ijiri and Simon, 1977; Simon and Bonnini, 1958; Mansfield, 1962). These studies generally conclude that FSDs are lognormal and stable. Analytical modelling of stochastic firm growth have predicted Pareto and Yule firm size distributions. Sutton (1998) has predicted that his model of 'equal opportunities' yields an exponential distribution of the mean concentration. We want to explore if the concentration bounds provided by Mauldon (1951) are consistent with either Pareto or exponential FSD.

We attempt to compare the results of the no-feedback process, presented in Section 2, with two well-known firm size distributions: Pareto and exponential. The fact that we are working with share distributions, and not size distributions, makes this a little complicated. Given a share distribution, it is not possible to derive its generating size distribution; however, the converse is possible and we proceed along that route. The comparisons in this chapter are based on share distributions, in particular on the Lorenz curve generated by the given share distribution. Given a cumulative density function *F* and its corresponding mean μ, the asymptotic Lorenz curve is given by:

$$L(p) = 1 - \mu^{-1} \int_0^{1-p} F^{-1}(t)dt \tag{4}$$

where *L(p)* gives the cumulative share of the largest *p* percentile of the distribution.

While in income distribution Lorenz curves map starts with the smallest percentiles, in the case of industrial concentration it is more sensible to invert the Lorenz curve in order to study the behaviour of the largest units. Furthermore, our study is focused on C_4 so we provide an easy way of translating our tables and graphs into measures of C_4. The mean and cdf of both Pareto and exponential distributions are well tabulated and it is easy to derive their respective asymptotic Lorenz curves (Table 4).

To appraise the generating size distribution of the no-feedback process we fit a special case (a market with 100 firms; *n*=100) to the Lorenz curves generated by the two common firm size distributions. We generate the empirical Lorenz curve for such an industry based on Mauldon's distribution and fit the data to the Lorenz curves of the Pareto and exponential firm size distributions given in Table 4. The parameter fitting procedure is based on non-linear least squares.

Table 4: *Distributions and asymptotic Lorenz curves*

Distribution	C.D.F.	Lorenz curve
Pareto	$F(x) = 1 - \left(\frac{a}{x}\right)^{\alpha}$ $x > a;\ \alpha > 1$	$L(p) = p^{\frac{\alpha-1}{\alpha}}$
Exponential	$F(x) = 1 - e^{-\lambda x}$ $\lambda > 0;\ x > 0$	$L(p) = p - p \ln p$
Shifted - Exponential	$F(x) = 1 - e^{-\lambda(x-a)}$ $\lambda > 0;\ x > a;\ a > 0$	$L(p) = p - (1+\lambda a)^{-1} p \ln p$

Pareto fitting

The Pareto distribution is popular in empirical studies of firm size distribution (e.g. Ijiri and Simon, 1977). The asymptotic Lorenz curve generated by a Pareto size distribution is given Table 4, where p denotes the largest p firms and α is the parameter of the distribution. We compare this curve with the Lorenz curve generated by the different significance bounds of Mauldon's distribution for an industry with 100 firms. Table 5 presents the results of the Pareto parameter fitting for the Lorenz curves generated by five concentration bounds:

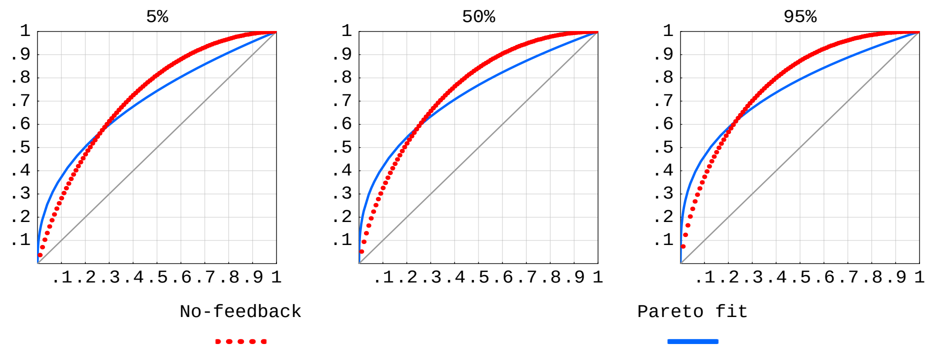
Table 5: *Pareto parameter fits for no-feedback concentration bounds*

	1%	5%	50%	95%	99%
α estimate	1.80652	1.74373	1.61061	1.49893	1.45782
R^2	.9485	.9437	.9319	.9200	.9153

Source: Authors' computation

However, even though the R^2 are high, the Pareto distribution yields significantly different Lorenz curves, shifted more towards the right end of the Lorenz curve. Figure 2 shows that the no-feedback process and the Pareto distribution generate significantly different Lorenz curves.

Figure 2: Pareto Lorenz curve fits for no-feedback concentration bounds



Exponential fitting

Sutton (1998) has shown that the expected value of the firm share distribution under "equality of opportunities" is asymptotically exponential. The median C_k in an industry of n firms is given by:

$$C_{k,median} = \frac{k}{n}\left(1 + \sum_{i=k+1}^{n} \frac{1}{i}\right)$$

which is approximated by:

$$C_{k,median} = \frac{k}{n}\left(1 - \ln(k/n)\right) = p - p\ln p \qquad (5)$$

where $p=k/n$ is the percentile of the largest firms. Equation (5) provides a formula for the share of the largest k firms in an industry of size n and also defines the Lorenz curve of the exponential distribution (Table 4).

The standard exponential distribution yields an asymptotic Lorenz curve that is independent of the distribution parameter λ (Table 4). Therefore it has limited relevance for us, since it does not allow parameter fitting with the data. However, as shown earlier, the median of the no-feedback case (as well as Sutton's lower bound) yields a distribution that is exponential. The shifted exponential, however, does allow fitting the no-feedback bounds to an asymptotic exponential Lorenz curve.

Using the formula in Table 4 we attempt to fit the parameter $\beta = \lambda\, a$ to the data. The parameter a denotes the smallest possible firm size and can be assumed without loss of generality to be 1. In practice firms must have a minimum size (in terms of sales, employment, capital, etc.) to be included in any official statistics. Firms of size zero are excluded. By using this assumption ($a=1$) the estimate for β is in fact an estimate of the parameter λ of the exponential distribution, which drives the inequality within the distribution. Notice, however, that both λ and a are positive, hence their product must be positive. Due to this, the standard exponential distribution yields the most unequal exponential distribution, i.e. the asymptotic Lorenz curve for the shifted exponential lies below the Lorenz curve of the standard exponential, regardless of the (positive) values of the parameters λ and a. The parameter estimates are given in Table 6.

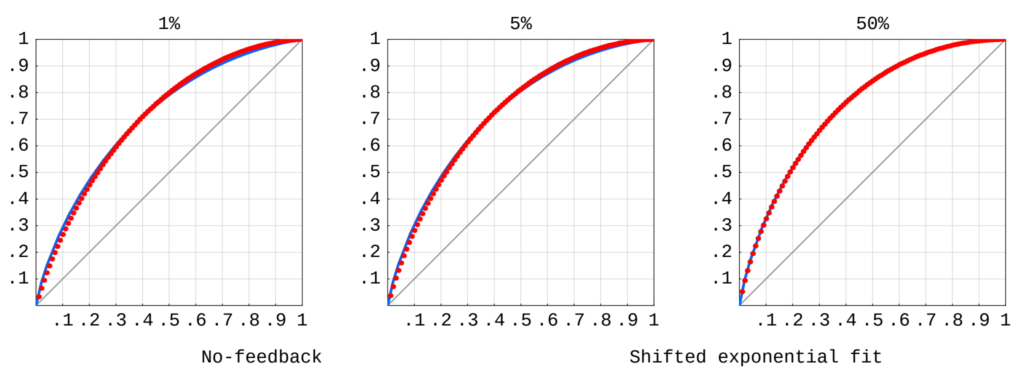
Table 6: *Exponential parameter fits for no-feedback concentration bounds*

	1%	5%	50%
λ estimate (α =1)	0.1907762	0.131953	0.00968
CI	[.1757,.2058]	[.1212,.1427]	[.0082,.0112]
R^2	.9971	.9981	.9999

Source: Authors' computation

Besides R^2, we illustrate the goodness of fit for the three different significance levels in Figure 3. We could not compute the parameter fits for the 95% and 99% bounds because the fitting yield negative values for the parameter $\beta = \lambda\, a$, cannot be assigned to any shifted exponential distribution.

Figure 3: Exponential Lorenz curve fits to the no-feedback concentration bounds



Generalized exponential concentration bounds?

Consider an extension to the standard exponential distribution, where the cumulative density function (cdf) is given by:

$$F(x) = (1 - e^{-\lambda x})^a \tag{6}$$

This distribution uses parameter α to generalize the exponential distribution. Setting α=1 we get the standard exponential. For parameter values in the [0,1] interval the cdf lies above the standard exponential, whereas parameter values in excess of unity generate cdfs below the standard exponential. We derive the Lorenz curve of this family of distribution starting from the Lorenz curve generated by the exponential distribution (eq. 2 in Table 4). Arnold et al. (1987) show that the Lorenz curve of an exponential transformation of a cdf ($F_\alpha(x)=F^\alpha(x)$) is given by:

$$L_\alpha(p) = [L(p^{\frac{1}{\alpha}})]^\alpha \tag{7}$$

where $L(u)$ is the Lorenz curve generated by $F(x)$. Hence the Lorenz curves generated by the generalized exponential distribution family described in equation 6 is given by:

$$L(p) = 1 - [(1-p)^{\frac{1}{\alpha}} + (1-(1-p)^{\frac{1}{\alpha}})\ln(1-(1-p)^{\frac{1}{\alpha}})]^\alpha \tag{8}$$

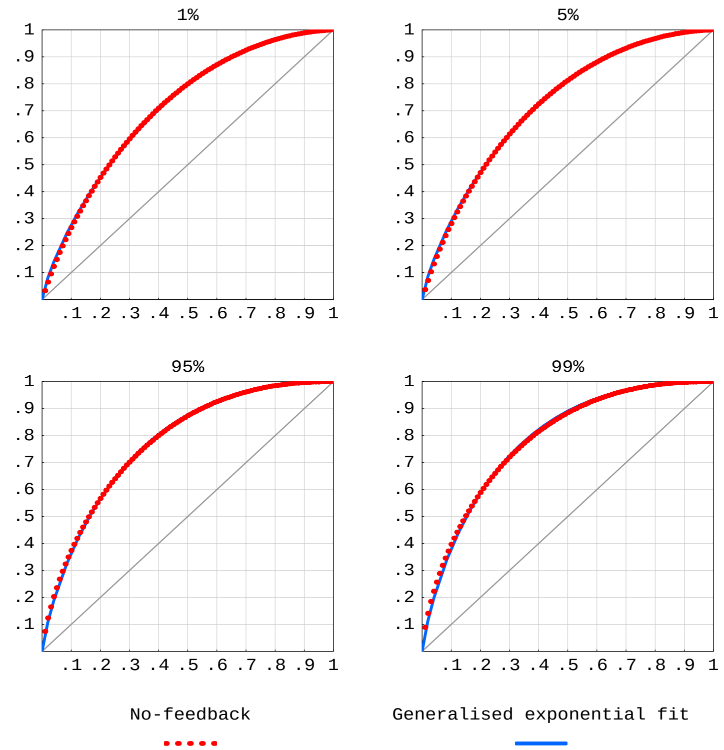
Table 7 presents the results of the non-linear least squares fitting of the empirical Lorenz curve to the asymptotic Lorenz curve described in equation (8), using parameter α as a control variable. We further document the goodness of fit by plotting the empirical Lorenz curves derived from Mauldon's distribution against the fitted Lorenz curves, i.e. Lorenz curves with corresponding fitted parameters given in Table 7. Figure 4 below displays excellent curve fit, reinforcing the high R^2 ratios observed in Table 7.

Table 7: *Generalized exponential parameter fits for no-feedback concentration bounds*

	1%	5%	95%	99%
α estimate	0.318422	0.444925	1.92542	2.61997
CI	[.308,.329]	[.433,.456]	[1.880,1.971]	[2.512,2.728]
R^2	.9995	.9996	.9995	.9985

Source: Authors' computation

Figure 4: Generalised exponential Lorenz curve fits to the
no-feedback concentration bounds



We have presented evidence that the concentration bounds generated in a no-feedback firm growth process described by Mauldon's share distributions could be approximated by an exponential distribution of firm sizes. The different levels of concentration are approximated by generalized exponential distributions with different parameter values α.

Exponential firm size distributions

What does the firm size distribution look like for the different bounds? To answer this question we use the estimated parameters in the previous section to derive the firm size distributions. Given the cumulative density function in equation (6), the probability density function for firm size distribution is given by the derivative of *F(x)* with respect to *x*:

$$f(x) = \alpha e^{-x}(1-e^{-x})^{\alpha-1} \tag{9}$$

Note that we have set λ =*1*\ without loss of generality, considering a standard exponential distribution. We use the parameter values in Table 7 to plot the density functions. For the median concentration we simply plot the asymptotic exponential distribution, i.e. $\alpha=1$ and $\lambda=1$. Figure 5 depicts the size distribution plots.

Figure 5: Firm size distributions for the no-feedback concentration bounds