

# Stochastic labour market shocks, labour market programmes, and human capital formation: a theoretical and empirical analysis

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## Abstract

This paper develops a life-cycle model of labour supply that captures endogenous human capital formation allowing for individual's heterogeneous responses to stochastic labour market shocks. The shocks determines conditions in the labour market and sort individuals into three labour market regimes; employment, unemployment with and unemployment without participation in labour market programmes. The structural model entails time independent stochastic shocks that have transitory effects on monetary returns while the effect on human capital formation may be permanent. The permanent effect may justify the existence of active labour market programmes if these programmes imply non-depreciating human capital and human capital depreciation is detected for the non-participant unemployed. Using several years of the Swiss Labour Force Survey (SAKE, 1991 – 2004) the empirical section compares the dynamic formation of human capital between labour market regimes. The results are consistent with the assumptions of the structural model and suggest human capital depreciation for unemployment without programme participation. They further show that labour programmes may act as a buffer to reduce human capital loss while unemployed.

**Keywords:** Human capital formation, life-cycle labour supply models, active labour market policies, search activities, productivity shocks, unemployment.

**JEL – Classification:** D31, D91, J24, J68

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# 1 Introduction<sup>1</sup>

The aim of most Active Labour Market Programmes (ALMP) is to increase and improve the employment chances of programme participants by increasing (or avoiding a reduction of) their productive capacity. Microeconomic evaluation studies usually focus on the participant's success in improving his labour market status with respect to the programme's targeted labour market outcome (e.g., Lechner and Gerfin, 2002). But this classic partial equilibrium approach may undervalue the long-term intrinsic benefits of ALMP participation in terms of human capital (productive capacity). This paper aims at modelling and estimating human capital gains in the presence of ALMP using the Swiss labour market for the empirical illustration.

We develop a structural life-cycle model of labour supply with endogenous human capital formation. Drawing from Magnac and Robin (1991, 1996) we define a behavioural model where optimizing individuals chose among mutually exclusive labour market regimes that determine their participation decisions in the labour market. Heterogeneous individuals (with respect to skill formation) make labour market and consumption decisions accounting for idiosyncratic stochastic shocks that inform them about the contemporaneous state of nature in the labour market. Although the shocks are purely transitory in nature, they are modelled to have a permanent effect on the stock of human capital. Assuming a learning-by-doing environment, the contemporaneous accumulation of human capital increases the productive capacity of individuals within their skill class thus reducing the risk from labour market participation in the future. The model implies that the opposite may be true if human capital depreciates as a result of un-

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employment spells where lost skill specific knowledge may increase the risk associated with the choice of working in future periods. ALMP can be thought as reducing the risk of human capital depreciation. The structural model in this paper allows for three labour market regimes (employment, unemployment with ALMP, and unemployment without ALMP). It determines identifying conditions for the estimation of the parameters associated with human capital growth and depreciation.

Using the 14 waves (1991 to 2004) from the Swiss Labour Force Survey (SAKE) – a rotating panel – we estimate human capital appreciation and depreciation for Switzerland. For example., we show that having experienced unemployment for up to 1.5 years leads to a loss in productivity equal to about 10% for the lowest skill class (if they have previous labour market experience lasting for at most 2 years). However, for an otherwise identical individual with higher skills the loss in productive capacity increases to more than 40%. Although our estimates are not always precise enough, they are consistent with the underlying assumptions in our model.

The paper is organized as follows: Section 2 presents an overview of modelling ALMP and relates that to the situation in Switzerland. Section 3 introduces a dynamic labour supply model with endogenous human capital formation. Section 4 derives the econometric identification conditions and describes the estimation methods. Section 5 presents the data. Section 6 shows the results and Section 7 concludes. Further technical material is relegated to the appendices that can be downloaded from the internet at [www.siaaw.unisg.ch/lechner/lifecycle](http://www.siaaw.unisg.ch/lechner/lifecycle).

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the need to contrast our work with that of endogenous human capital in the macroeconomic literature. Financial support from the Swiss National Funds under project No. 1214-066928 is gratefully acknowledged.

## **2 ALMP, stochastic shocks and human capital formation**

During the 1990s many continental European countries introduced wide-ranging active labour market policies (ALMP) to combat the then rising levels of unemployment, e.g., the 1997 expansion of active labour market policies in Switzerland. Following on the footsteps of program evaluation in North America (see for example Ashenfelter and Card, 1985, Angrist and Krueger, 1999, or the survey by Heckman, LaLonde and Smith, 1999) alongside the widespread introduction of ALMP in western economies, research aimed at evaluating ALMP effectiveness has surged on both sides of the Atlantic. Specifically in Switzerland, studies by Gerfin and Lechner (2000, 2002) or Gerfin, Lechner and Steiger (2001) have focused on evaluating the direct effect on employment of specific policies using as key identification strategy the assumption that labour market outcomes and the selection process into the programme are independent events conditional on observed heterogeneity, while the outcome of interest is the direct effect on the participation's labour status at some point in the future. In such fairly nonparametric econometric studies, the effects of the programme on the accumulation of human capital have to remain implicit, as human capital is a theoretical concept that cannot be directly measured in the data. Yet, it is the stock of human capital that matters when determining individual's chances of employment.

At a micro-economic level, studying human capital accumulation requires life-cycle models that capture the interaction between individual's preferences and the dynamics in the labour market (e.g., Bell, Blundell and Van Reenen, 1990, or Browning, Hansen and Heckman, 1999). Our structural model aims at modelling such dynamics drawing from the framework in Magnac and Robin (1991, 1996) to define a behavioural problem where optimizing individuals chose among mutually exclusive labour market regimes and participation decisions. In doing so, we extend the structural model in Costa-Dias (2002) by introducing human capital depreciation

in the event of unemployment. Conditional on skill class the model by Costa-Dias (2002) assumes that new arrivals to the unemployment pool (with or without ALMP) are identical to the long term unemployed regarding productive capacity (human capital). However, in European economies unemployment duration is a key determinant of future employment chances and unemployment outflow rates. In these economies, long-term unemployment has detrimental effects on individual's productive capacity as well as on the physical and psychological wellbeing (Machin and Manning, 1998). Thus, our structural model introduces ALMP as a policy measure designed to combat these 'side-effects' of unemployment.

A key issue in this paper is individual heterogeneity, a concept that is undermined in life-cycle models with macroeconomic foundations even if these later aim at capturing the dynamics of labour market choices allowing for endogenous human capital formation. Models that focus on macroeconomic fluctuations often estimate endogenous human capital formation conditional on changes defined by key macroeconomic variables. For example, Perroni (1995) models human capital formation endogenously conditional on individual's aggregate investment choices which may differ in the presence of different income or consumption taxes; Nunziata (2003) considers the effect of macroeconomic shocks in a dynamic model of unemployment determination; Albrecht (2002) builds a theoretical model that determines skill specific human capital formation conditional on key macroeconomic parameters while simulating human capital formation conditional on the economy's demand for skills. In contrast to our assumptions, Perroni's (1995) set-up does not assume stochastic shocks whereas common to our assumptions both Nunziata (2003) and Albrecht (2002) allow for labour market shocks. However, whereas Nunziata (2003) and Albrecht (2002) assume common shocks to all individuals over time, we assume labour market shocks to differ among individuals with such heterogeneity playing a

central role in the choice of labour market regimes, a choice that eventually determines endogenous human capital formation.<sup>2</sup>

The key dynamics in our model imply that individuals enter the market with an exogenously given skill class (i.e., start-up education) and their human capital (i.e., productive capacity) may increase within skill class as result of working in a learning-by-doing environment. Different investment decisions result from individual's idiosyncratic labour market histories combined with their evaluation of stochastic changes in the state of the labour market resulting from time independent stochastic labour market shocks. Thus, individuals with similar skills and labour market experience may differ in preferences and human capital investment choices. This reasoning may explain the empirically observed difference in returns between individuals with similar skills (see Heckman, Lochner and Taber, 1998).

Clearly, unemployment can lead to a loss of human capital. Being unemployed may imply losing touch with the up-to-date skill specific knowledge demanded by constantly incoming new vacancies. Thus, c.p. new arrivals to the pool of unemployment are closer to fulfilling the skill-specific knowledge required by new incoming vacancies. This contrasts with participating in ALMP that may lead to maintaining (or increase) pre-unemployment productivity levels.

A consequence of allowing for stochastic labour market shocks in our model is that choices of labour market regimes are not-deterministic following closely the key modelling assumptions in Huggett (1997) and Huggett and Ventura (1999) where idiosyncratic shocks determine individual's state of nature at each point in time. The shocks affect individual's

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<sup>2</sup> Other models (e.g., Bontemps, Robin and van den Berg, 1999) allow for individual's heterogeneity within their dynamic framework but the aim in these papers focus on search theory and not on individual specific human

contemporaneous opportunity cost of participating (or not) in paid labour market activities. As in Huggett (1997) it is assumed that prices (e.g. wages, interest rates, etc) are deterministic with price fluctuations resulting from the shocks that directly determine individual's capital holding over time. We differ from Huggett (1997) in that we allow for alternative labour market regimes when setting up the individual's decision problem thus following closely to Magnac and Rubin (1991, 1996) where the representative agent can choose between alternative working modes (in their case, between wage-work and self-employment). Our model is such that individuals hold 'latent' or hidden valuations on each possible labour market regime. These valuations reflect for example the individual's perceived cost of participation in ALMP that could change if the state of the world changes as result of idiosyncratic stochastic labour market shocks. Before the shock is realized the agent is uncertain about his or her labour endowments and total asset holdings. Once the shock is realized the state of the world (i.e., the individual's idiosyncratic value of each alternative regime) is known and the agent chooses the optimal labour market regime according to the new state. The arguments are similar to those in Kihlstrom and Laffont (1979) and Magnac and Robin (1991) where it is also assumed that individual's uncertainty on future labour market returns can be explained by attitudes towards risk, where 'risk' is thought as 'opportunity costs' among alternatives choices. In our model, we consider the 'risk' of different alternatives as the differences in opportunity cost between them, where such cost may depend on personal characteristics and the idiosyncratic view of the state of nature in the labour market (see Cossa, Heckman and Lochner, 2003). The shocks are not observed by the econometrician. Instead, individuals' choices with regards to labour market regimes reveal the magnitude of the shocks relative to individuals' reservation entry values into each of the different labour market regimes.

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capital formation. See Regerson, Shimer and Wright (2005) for a recent survey on dynamic labour market literature.

A positive and sufficiently large shock implies a working decision. If the shock is not sufficiently large to imply a working regime the individual will choose unemployment. In the event that the shock is 'sufficiently bad' it will place the individual in a regime of no work and no ALMP participation that may include the option to search in the open market, or not to search and leave the labour market. Our model is such that whereas the permanent component of the transitory shock may differ by labour market regime, in all three regimes the effects of the shock are permanently reflected by their effects on human capital. This argument implies that ALMP can be viewed not just as a set of instruments to make the unemployed more marketable but also as a mean to help them keep their human capital stock relative to their most recent employment spell. Taking all the above arguments into account, we think of accumulated human capital as providing an insurance against risk (i.e., it lowers the relative opportunity cost of employment by increasing its returns) while each individual's taste for risk depends on individual characteristics and past labour market history.

The above arguments imply that evaluating the impact of ALMP requires us to examine the dynamics of the labour market in the economy where both 'earnings' and 'benefits' are thought as best signals in terms of disentangling the behaviour of the active population with regards to labour market choices. Our proposed structural life-cycle model mimics the arguments just mentioned with regards to the dynamics of labour-supply behaviour, of human capital formation, earnings and unemployment insurance. Ultimately, the structural model provides the benchmark to determine the necessary conditions to identify and estimate key human capital parameters (growth and depreciation) conditional on skill class.



### 3 A model of labour supply with stochastic shocks

#### 3.1 Notation and model outline

The fundamental problem for the representative individual  $i$  is to maximise utility ( $u$ ) coming from consumption ( $c$ ) subject to the endogenous evolution of assets ( $a$ ) and human capital ( $h$ ) and subject to exogenous idiosyncratic labour market shocks ( $\pi$ ):

$$\max_{\{c, I\}_{t=0}^T} E_t \sum_{t=0}^T \left[ \beta^t u_t(c_{it}) \mid (\tilde{X}_{it}, \psi_t)_{t=0}^T \right];$$

where,

$$\tilde{X}_{it} = (s_t, a_{it}, h_{it}, \pi_{it});$$

$$\psi_t := (r_t, W_{1t}, \dots, W_{st}, \dots, W_{st}, B_{1t}, \dots, B_{st}, \dots, B_{st}, P_{1t}, \dots, P_{st}, \dots, P_{st}, \tau_t);$$

$$a_{i,t+1} = (1 + r_t) a_{i,t} + \sum_{s=1}^S \mathbf{1}(s = s_i) \left\{ I_{it}^w W_{st} (1 - \tau_t) h_{it} \pi_{it} + I_{it}^q (B_{st} - P_{st}) h_{it} + I_{it}^n B_{st} h_{it} \right\} - c_{it};$$

$$h_{i,t+1} = h_{it} \cdot \exp\{v(s_i, h_{it}) \pi_{it}\}, \quad \text{if } I_{it}^w = 1; \quad (1)$$

$$h_{i,t+1} = h_{it}, \quad \text{if } I_{it}^q = 1;$$

$$h_{i,t+1} = h_{it} \cdot \exp\{-\sigma(s_i, h_{it}) \pi_{it}\}, \quad \text{if } I_{it}^n = 1.$$

According to (1), individual  $i$  enters the labour market at time  $t = 0$  and retires at  $T$ . Optimal allocation of lifetime resources implies maximizing expected discounted utility over the entire working horizon (i.e. from  $t = 0$  to  $t = T$ ) where  $\beta$  is the discount factor. The individual's objective is summarized with a separable time-variant utility function  $u_t = u_t(c_{it})$  where consumption  $c_{it}$  at  $t$  is the only argument. Beyond contemporaneous  $t$  future realisations of the shocks  $\pi_{it}$  are unknown (only the distribution is assumed known). Thus, at each point in time the individual evaluates each labour market choice  $I_{it}$  and takes action accounting for the remaining lifetime subject to the contemporaneous state of nature that contains information from all the stochastic realisations in the past. At the point of labour market entry individual's skill class  $s$  is fully determined by their start-up education, thus  $s$  enters exogenously and remains unchanged

throughout life. Thus, labour market history will explain ‘skill-specific knowledge’ within the skill class. Together with the price vector  $\psi_t$ , the vector  $\tilde{X}_{it}$  defines *the state of nature* at  $t$  for the individual  $i$ , where  $\tilde{X}_{it}$  indicates that the state of the world is a function of his skill class  $s$ , accumulated returns at  $t$   $(a_{it}, h_{it})$  and the time-independent idiosyncratic labour market shock received at  $t$ ,  $\pi_{it}$ . We also define  $X_{it} = (a_{it}, h_{it})$  as the vector containing the endogenous state variables for an individual  $i$  with skill class  $s$ . The vector of prices  $\psi_t$  describes a deterministic sequence of wages and benefits for any  $s$  skill class at time  $t$ ,  $W_{st}$  and  $B_{st}$ , earnings tax at  $t$ ,  $\tau_t$ , and the rate of return from asset investments at  $t$ ,  $r_t$ . Furthermore, we assume that participants in active labour market programmes face a cost  $P_{st}$  (non-pecuniary or otherwise) of programme participation that is also assumed to vary over time and by skill-class. The indicator  $I_{it}^j = 1$ ,  $j = w, n, q$  explains the individual’s labour market choice at  $t$ ; at any given  $t$ ,  $I_{it}^w = 1$  implies working whereas either  $I_{it}^q = 1$  (unemployed with ALMP) or  $I_{it}^n = 1$  (unemployed without ALMP) indicate non-working. The three choices are mutually exclusive and add-up to one for each individual at any  $t \in [0, T]$ . The model in (1) shows that consumption and labour market participation are the only choice variables. Unlike consumption, labour market choices are modelled discretely so that the solution to the problem in (1) solves for three mutually exclusive optimal consumption paths, one for each of the three labour choices in the current period. The indicator function  $\underline{1}(s = s_i)$  clarifies that earnings are skill specific. They vary over time for each individual within skill class. For example, the term  $W_{st}$  is a vector of prices of dimension equal to the number of skills in the population; allowing for  $\underline{1}(s = s_i)$  implies that  $W_{st}$  becomes  $W_{s(i)t}$ ,  $B_{st}$  becomes  $B_{s(i)t}$  and  $P_{st}$  becomes  $P_{s(i)t}$ .

The structural set-up in (1) is a dynamic problem where stochastic shocks determine labour market choices according to the combined effect that such shocks have on physical and human capital. In the absence of shocks the model is a deterministic partial equilibrium model where agents would have remained from  $t = 0$  to  $T$  in one of the three labour market regimes defined according to initial stocks of assets and skill specific choices at the point of entering the labour market. Stochastic shocks and individual's evaluation of these stochastic changes lead to the potential to shift between labour market regimes. In allowing for stochastic labour market shocks the framework follows that of Kihlstrom and Laffont (1979) and Magnac and Robin (1991, 1996). As in their case, the dynamics in (1) are such that at the beginning of each discrete period the shocks are revealed and the individual chooses an optimal labour market regime. Earning's uncertainty (i.e., price uncertainty) plays no role in evaluating present and future values of the available choice set. Instead, observing stochastic shocks reveals information on the contemporaneous state of nature thus helping the individual to exercise a labour market choice relative to the individual's risk attitude towards each of the labour market alternatives. The risk attitude itself is the result of individual's own valuation of personal characteristics and past labour market history. For example, at time  $t$  a low skill individual has a valuation for each of the three labour market regimes that may lead to a choice of ALMP participation as opposed to working. The choice would signal that conditional on the individual's contemporaneous productivity level, working is a riskier choice when comparing the returns from work (i.e., combined assets and human capital) to those received when participating in ALMP. We think of labour choices as 'risky' choices because contemporaneous decisions imply permanent investment effects in terms of human capital stocks. For example, assume that at time  $t + 1$  the same individual receives a positive shock  $\pi_{it+1}$  perceived as relatively high (e.g., a wage subsidy that pays above the individual's productive capacity). The structural model in (1) implies that the

choice to work (i.e.,  $I_{it+1}^w = 1$ ) could become more attractive relative to receiving either  $\{I_{it+1}^q (B_{st+1} - P_{st+1}); h_{it+1}\}$  or  $\{I_{it+1}^n (B_{st+1}); h_{it+1} \cdot \exp\{-\sigma(s_i, h_{it+1})\pi_{it+1}\}\}$ , thus, the perceived ‘high’ (idiosyncratic) shock implies that the combined returns from assets and human capital from a working choice,  $\{I_{it}^w W_{st} (1 - \tau_t) h_{it} \pi_{it}; h_{it} \cdot \exp\{\nu(s_i, h_{it})\pi_{it}\}\}$ , makes the latter the optimal ‘investment’ decision. The result is that physical assets ( $a_{it+1}$ ) and human capital ( $h_{it+1}$ ) accumulate where the latter does so at the rate of  $\nu(s, h_t)$  with obvious permanent positive effects in terms of increased future productivity capacity. The parameter  $\nu(s_i, h_{it})$  depends on  $h_{it}$  because ‘learning-by-doing’ depends on the individual’s contemporaneous stock of knowledge (i.e., stock of human capital  $h_{it}$ ). But if today’s human capital increases, tomorrow’s opportunity cost of not working increases relative to the gains implied by either of the two unemployment regimes and, therefore, working becomes more likely in periods ahead. The example illustrates the permanent effect of a transitory positive shock. A contrasting example is that where the shock  $\pi_{it}$  may be perceived as ‘low’ so that  $\{I_{it}^w W_{st} (1 - \tau_t) h_{it} \pi_{it}; h_{it} \cdot \exp\{\nu(s_i, h_{it})\pi_{it}\}\} < \{B_{st} |_{I(i,t,q)=0}, h_{it} |_{I(i,t,q)=0}\}$ , that is, benefits resulting from any of the two unemployment alternatives may imply a lower risk (opportunity cost) than working for wages. Choosing unemployment may lead to spells where productive capacity either remains constant or depreciates at a rate of  $\sigma(s_i, h_{it})$ , thus making working in the future a riskier option, i.e., unemployment could lower productive capacity  $h_{it}$  and with this reduce the total net gains (i.e.,  $I_{it+1}^w W_{st+1} (1 - \tau_{t+1}) h_{it+1} \pi_{it+1}$ ) from working in periods ahead. The structural model in (1) implies that for unemployment alternatives the effect of the shocks have only a direct effect on human capital. In sum, the shocks determine the state of nature at which point individuals evaluate and chose a labour market regime. The choice involves an attitude towards risk that involves comparing individual’s preference and characteristics. For individual

that take unemployment as the optimal choice, the shock determines the benefit of seeing  $h_{it}$  decrease relative to that associated with the cost implied by ALMP, that is,  $P_{st}$ .

### 3.2 The Bellman representation

The dynamic problem described in (1) is formulated in terms of multiplicative stochastic shocks  $\pi_{it}$  and the two endogenous state variables  $(a_{it}, h_{it})$ . The solution to the problem is a sequence of consumption choices  $\{c_{it}\}_{t=0}^T$  among all admissible sequences for each of the discrete labour market regimes, conditional on initial and final conditions (i.e.,  $a_{t=0} = 0, h_{t=0}^s > 0$  and  $a_T \geq 0, h_T > 0$ , respectively); these conditions pin down sets of admissible consumption paths. We choose to characterise the problem with recursive methods in terms of a value function. The model in (1) allows for time independent shocks with the permanent effects of these picked up by physical and human capital, the only two components that carry information from today to the future. Thus, as function of these two variables the model in (1) provides the classic set up for the Bellman representation that relates current value functions  $V_t(a, h, \pi)$  – i.e., the value of the maximised problem conditional on all possible paths at  $t$  – to expectations of future value functions  $V_{t+1}(a, h, \pi)$  assuming knowledge of the shocks up to  $t$  and discounted back to contemporaneous values:

$$V_{it}^s(a_{it}, h_{it}, \pi_{it}; \psi_t) = \max_{\{c_{it}, J_{it}\}_{t=0}^T} \left\{ u(c_{it}) + \beta \cdot E_{\pi} \left[ V_{it+1}^s(a_{it+1}, h_{it+1}, \pi_{it+1}; \psi_{t+1}) \right] \right\}. \quad (2)$$

The value function in (2) summarizes the skill-specific individual's problem representing current and future values of the optimal consumption path that changes as the state variables change over the planning horizon. However, a unique solution characterizing the individual's optimal choice is only possible if the value function in (2) is well behaved, that is, if expression

(2) complies with a set of regularity conditions that imply a unique solution for the individual's optimal consumption path for each of the discrete labour market choices.

### 3.3 Assumptions

We now turn to list a set of necessary assumptions that provide the necessary conditions to derive a set of premises to proof that the problem in (2) is well behaved.

**Assumption 1** (shocks): Stochastic labour market shocks  $\pi$  are assumed to be *iid* independent across time and individuals with known and continuously (at least once) differentiable distribution function on a bounded non-negative support  $[\underline{\pi}, \bar{\pi}]$ .

**Assumption 2** (utility function):  $u_t = u_t(c_{it})$  depends on consumption only and is strictly increasing, twice differentiable, and concave in its argument.

**Assumption 3** (state space): Both state space vectors spanned by the state variables  $\tilde{X}_{it} = (a_{it}, h_{it}, \pi_{it})$  or  $X_{it} = (a_{it}, h_{it})$  are assumed to be continuous, bounded and convex. Skill type ( $s_i$ ) is also part of the individual's state space but we assume it to be exogenous and constant throughout the planning horizon.

**Assumption 4** (initial and final conditions): Initially,  $a_{i0} = 0$  and  $h_{i0} = \underline{h}^{s(i)} > 0$ . Terminal conditions are assumed to be such that  $a_{iT} \geq 0$  and  $h_{iT} > 0$ .

**Assumption 5** (non-crossing): The value function is assumed to have a derivative in the neighbourhood of zero that tends towards  $-\infty$  from the right.

**Assumption 6** (absolute risk aversion): Individuals display decreasing absolute risk aversion, with risk attitudes towards labour market choices that change in the opposite direction of assets, but with changes that are never far from zero in magnitude. Technically, this implies degrees of risk aversion such that  $(\partial \pi_a^R / \partial a) \leq 0$  and  $(\partial \pi_b^R / \partial a) \leq 0$ , where  $\pi_a^R, \pi_b^R$  stand for the reservation levels that determine choices between different regimes in the labour market and 'a' stands for 'capital assets'.

**Assumption 7** (human capital growth and depreciation):  $\nu(\cdot) \geq 0$  and  $\sigma(\cdot) > 0$ , where the parameter  $\nu(\cdot)$  stands for the human capital growth rate and  $\sigma(\cdot)$  stands for human capital depreciation rate.

**Assumption 8** (prices):  $B_{st} > 0, W_{st} > 0, P_{st} > 0$  at any point in time.

**Assumption 9** (uniqueness): Consumption and savings are normal goods.

Assumptions 1 to 9 are necessary requirements to determine the uniqueness of the solution to the problem in (2). At any time  $t$ , the only source of uncertainty is next period stochastic shocks (Assumption 1). Nature draws at each  $t \in [0, T]$  to reveal the state of the world and individuals compare alternative choices at the new state conditional on own preference and characteristics to value the opportunity cost among alternative regimes (Assumption 6). A choice of labour market and consumption are made by rational agents that maximize a well defined objective conditional on a monotonically changing dynamic state space (Assumptions 2 and 3). Excluding leisure from the objective function precludes wealth effects (i.e., backward bending labour supply functions), a requirement for a sub-population that may be subject to ALMP. Lifetime constraints in assets (Assumption 4) pin down a feasible set of consumption paths from which to choose the optimal one. Individuals are allowed to borrow (no liquidity constraints) although they are bounded to retire without debt ( $a_{iT} \geq 0$ ). At entry point ( $t = 0$ ) human capital is at its lowest (thus the lower bar in  $\underline{h}^{s(i)}$ ), positive and identical among individuals with equal skills, while human capital stocks can never be negative, i.e., individuals will always hold some skill specific knowledge. Finally, a concave function that goes through the origin allows for monotonic changes to the unique solution if exogenous parameters shift the function in particular directions (Assumption 5), whereas positive prices and positive human capital parameters (Assumptions 7 and 8) also define monotonic conditions for the dynamics in (2).

### 3.4 Some properties of the optimisation problem

Based on the previous formulation, the following intermediate results on model properties are obtained (all proofs are relegated to the Appendix):

**Lemma 1** (choice of labour market states): Assumptions 1-8 hold. Given  $\pi$ , an optimal choice of labour market regime is characterized by a monotonic labour market reservation policy that is determined conditional on each individual's characteristics at any  $t$  such that,

- (a) An agent prefers  $I_{it}^w = 1$  to either  $I_{it}^q = 1$  or  $I_{it}^n = 1$  at  $t$  if
- $$\pi_{it} > \pi_{it[b]}^R(X_{it} | (\psi_t)_t^T),$$
- (b) An agent prefers  $I_{it}^q = 1$  to either  $I_{it}^w = 1$  or  $I_{it}^n = 1$  at  $t$  if
- $$\pi_{it[a]}^R(X_{it} | (\psi_t)_t^T) < \pi_{it} \leq \pi_{it[b]}^R(X_{it} | (\psi_t)_t^T) \quad (3)$$
- (c) An agent prefers  $I_{it}^n = 1$  to either  $I_{it}^q = 1$  or  $I_{it}^w = 1$  at  $t$  if
- $$\pi_{it} \leq \pi_{it[a]}^R(X_{it} | (\psi_t)_t^T)$$

where  $\pi_{it[a]}^R < \pi_{it[b]}^R$ .

**Lemma 2** (properties of the value function): Allow for Assumptions 1-8. Then, the expected value function  $E_\pi V^s(a, h, \pi)$  is strictly increasing, twice differentiable and a concave function of  $a$  (assets).

The Bellman representation in (2) shows that a realization  $\pi_{it}$  is the only variable that implies changes in the state of the world, thus, it is the fundamental determinant of a regime choice. But utility comes only from  $c_{it}$  with labour market choices acting as a 'conditional' determinant of the optimal consumption path: to obtain this latter is the unique objective of the individual, i.e., the solution to the problem in (2) is the optimal consumption path for a given regime choice as characterized by an Euler Equation that explains the intertemporal consumption rule. The Euler Equation, however, is only a valid characterization if it fulfils the set of regularity



conditions as determined in Lemmas 1 and 2. Lemma 1 characterizes the working decision and interprets the value function as the sum of mutually exclusive value functions, each associated with a unique labour market regime over the support of the labour market shocks. Lemma 2 establishes the continuity, differentiability and concavity of the value function *in assets* alone. Thus, the regularity conditions in Lemma 2 are sufficient and necessary to define optimal consumption decisions for fixed labour market conditions as determined by the following individual specific Euler Equation:

$$\frac{\partial u_t(c_{it})}{\partial c_{it}} \Big|_{I_{it}} = E \left[ \beta(1+r) \frac{\partial u_t(c_{it})}{\partial c_{it+1}} \Big|_{I_{it}} \right]. \quad (4)$$

However, the problem (2) implies a more complex set of dynamics than just assets. The Euler Equation in (4) conditional on fixed labour market regimes is not sufficient to ‘characterize’ the consumption decision rule because for a given history of shocks and for a given skill class, assets move along with human capital. The combination of the Euler Equation in (4) together with Lemma 3 and its proof in Appendix 1 provide an interpretation of the necessary and sufficient conditions for the Euler Equation in (4) to represent the optimal consumption path conditional on given levels of human capital:

**Lemma 3** (Identification of the optimal consumption path). Allow for Assumptions 1-9. Then, the Euler Equation in (4) is not sufficient to identify an optimal consumption path, since identification further requires that both consumption and savings are normal goods for fixed labour market decisions.

## 4 Identification and estimation

Our aim is to use the structural set-up in (1) to provide identifying conditions for the parameters underlying human capital formation, namely  $\nu(\cdot)$  and  $\sigma(\cdot)$ . Once these conditions are

well specified estimation relies on the use of informative data at the individual level concerning labour market choices together with socio-economic background information that provide insights into individuals preferences and labour market histories.

#### 4.1 Specifications by labour market regime

The structural model in (1) implies three labour market regimes and therefore three sets of behavioural conditions leading to the same number of reduced form specifications each of which explains the gains (total receipts) from respective labour market choices. We start by studying the conditions implied by observing an individual as working.<sup>3</sup>

##### Employment

From (1), at any time  $t$  a working individual ( $I_t^w = 1$ ) receives skill specific total assets  $E_t$  equal to  $\tilde{W}_t h_t \pi_t$  where  $\tilde{W}_t = W_t(1 - \tau_t)$  is the average net return from working during  $t$  and  $h_t$  is the contribution to total assets from human capital stocks for a given productivity shock  $\pi_t$ .<sup>4</sup> At levels the components in  $E_t$  do not provide direct information on  $\nu(\cdot)$ . However, the model in (1) determines earning's growth partly as the consequence of human capital growth – i.e.,  $\Delta \ln h_{t,t+1} = \nu(s, h_t) \pi_t$  – resulting from working activities over a working spell of at least two consecutive periods, that is:

$$\begin{aligned}
 E_t &= \tilde{W}_t h_t \pi_t \quad \text{where} \quad \Delta \ln E_{t,t+1} = \ln E_{t+1} - \ln E_t; \\
 \Rightarrow \quad \Delta \ln E_{t,t+1} &= \Delta \ln(\tilde{W}_{t,t+1}) + [\ln h_{t+1}(\pi(t)) - \ln h_t(\pi(t-1))] + \Delta \ln \pi_{t,t+1}; \\
 \Rightarrow \quad \Delta \ln E_{t,t+1} &= \Delta \ln(\tilde{W}_{t,t+1}) + \nu(s, h_t) \pi_t + \Delta \ln \pi_{t,t+1}.
 \end{aligned} \tag{5}$$

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<sup>3</sup> All that follows from this point onwards makes constant reference to the individual  $i$  with skill level  $s_i$ , therefore we suppress the suffixes  $(i, s)$  to simplify the notation.

<sup>4</sup> Following Huggett (1997) the expression allows for human capital to enter multiplicatively towards total receipts from labour market activities and it is seen as the efficiency unit's productive capacity.

Expression (5) shows earning's growth between periods ( $\Delta \ln E_{t,t+1}$ ) as the sum of three components; skill specific wage growth,  $\Delta \ln(\tilde{W}_{t,t+1})$ , growth due to idiosyncratic changes in human capital,  $[\ln h_{t+1}(\pi(t)) - \ln h_t(\pi(t-1))]$ , and growth due to between periods changes on stochastic shocks,  $[\ln \pi_{t+1} - \ln \pi_t]$  where the latter represents stochastic changes in productivity gains as direct result of a working spell. Notice that (5) characterizes  $h_{t+j}$  as determined by  $\pi(t-1+j)$ ,  $j=0,1$  which does not stand for the stochastic shock  $\pi_{t-1+j}$  but instead characterises the labour market history up to the period  $(t-1+j)$ ,  $j=0,1$ : recall that the stochastic shock is what finally determines the state of nature and therefore the choice of regime. If so, the sequence of shocks up to  $t$  have a decisive effect on  $h$  at  $t+1$  as emphasised by the notation  $h_{t+j}(\pi(t-1+j))$ ,  $j=0,1$ , with this latter as the only component in (5) to relate directly to the characteristics of the individual, that is, Lemma 1 states that a particular choice depends on the individual's perception of the shock at  $t$  *relative to some individual specific reservation level set*  $[\pi_{t,a}^R, \pi_{t,b}^R]$ , where the latter summarize the individual's labour market preferences. These preferences summarize individual's characteristics (e.g., household and living conditions, health status, age, gender, etc.) as well as labour market history. Thus, characterizing  $h_{t+j}$  with  $\pi(t-1+j)$ ,  $j=0,1$ , links observed individual specific information in the data to time varying unobserved human capital stocks  $h_{t+j}(\pi(t-1+j))$ ,  $j=0,1$ , a link that becomes crucial when identifying the two human capital parameters in later sections. The last expression in (5) makes use of the dynamics in (1) to explicitly interpret human capital growth at the rate  $\nu(\cdot)$  that varies over time as consequence of its dependence on contemporaneous human capital stock;  $\nu(\cdot)$  is a key parameter that we aim to estimate for each of the skill classes  $s$ . We interpret  $\nu(h_t)$  as the

‘skill specific’ ability to learn since in our model ‘learning’ is the only reason for human capital to grow between periods.

### Unemployment without ALMP

Let  $\Gamma_t$  define any gains at  $t$  for the unemployed without ALMP participation (i.e.,  $I_t^n = 1$  leads to  $\Gamma_t$  at  $t$ ). The model in (1) implies that  $\Gamma_t = B_t h_t$  where  $B_t$  stands for the average receipts (e.g., social benefits, social assistance, etc.) for skill class  $s$  at time  $t$  and  $h_t$  are human capital stocks at  $t$ . Comparing outcomes between  $I_t^w = 1$  and  $I_t^n = 1$  (i.e., comparing  $\tilde{W}_t h_t \pi_t$  and  $B_t h_t$ ) shows a difference regarding the direct effect of  $\pi_t$ , i.e., unemployed individuals do not experience gains in productive capacity implied by the stochastic changes that affect those in employment. Instead, the unemployed without ALMP become subject to human capital depreciation, thus, if  $I_t^n = 1$  the effect of the shock is picked up in full by the human capital stock,  $h_t$ . Following an equal interpretation as for those in employment, the following expressions define the change in gains when  $I_t^n = I_{t+1}^n = 1$  is the result of labour market choices over two consecutive periods ( $t, t+1$ ):

$$\begin{aligned}
 \Gamma_t &= B_t h_t \quad \text{where} \quad \Delta \ln \Gamma_{t,t+1} = \ln \Gamma_{t+1} - \ln \Gamma_t; \\
 &\Rightarrow \Delta \ln \Gamma_{t,t+1} = \Delta \ln(B_{t,t+1}) + [\ln h_{t+1}(\pi(t)) - \ln h_t(\pi(t-1))]; \\
 &\Rightarrow \Delta \ln \Gamma_{t,t+1} = \Delta \ln(B_{t,t+1}) - \sigma(s, h_t) \pi_t.
 \end{aligned} \tag{6}$$

Expression (6) explicitly shows the human capital depreciation parameter,  $\sigma(h_t)$ , to be interpreted as the skill specific loss in learning, since it is this loss (given low or no-contact with

working environments) that may explain human capital depreciation over time. Of course, the negative sign for that rate is a modelling assumption that needs to be empirically verified.<sup>5</sup>

### Unemployment with ALMP

Expressions (5) and (6) relate observed outcomes to the key parameters of interest,  $v(h_t)$  and  $\sigma(h_t)$ . However, the structural model defines an alternative unemployment regime for participants in ALMP. Define  $\Omega_t = (B_t - P_t)h_t$ , where  $(B_t - P_t)$  describes unemployment insurance ( $B_t$ ) net from the individual specific cost of participating in ALMP ( $P_t$ )<sup>6</sup> for individuals with labour market choices such that  $I_t^q = I_{t+1}^q = 1$ . From the dynamics of assets in (1) the following net gains result:

$$\begin{aligned} \Omega_t &= (B_t - P_t)h_t \quad \text{where} \quad \Delta \ln \Omega_{t,t+1} = \ln \Omega_{t+1} - \ln \Omega_t; \\ \Rightarrow \Delta \ln \Omega_{t,t+1} &= [\ln(B_{t+1} - P_{t+1}) - \ln(B_t - P_t)] + [\ln h_{t+1}(\pi(t)) - \ln h_t(\pi(t-1))]; \\ \Rightarrow \Delta \ln \Omega_{t,t+1} &= \Delta \ln(B_{t,t+1} - P_{t,t+1}). \end{aligned} \quad (7)$$

Clearly, expression (7) brings no information regarding changes in human capital. However, relative to the changes implied in (6), the identity in (7) should provide us with an empirical test for the null of no changes in human capital as result ALMP participation. We notice that the potentially non-pecuniary value  $P_t$  differs for all in population; empirically one can only observe the identity  $\Delta(\ln \Omega_{t,t+1}) \equiv \Delta(\text{change in remuneration over time} \mid I_t^q = I_{t+1}^q = 1)$ , but

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<sup>5</sup> Theoretically, the structural model is only consistent with the underlying assumptions if both  $\sigma \geq 0$  and  $\underline{\pi} \geq 0$  hold. It is the only this combination that implies potential changes between labour market regimes while leaving the monotonic condition in Lemma 1 intact.

<sup>6</sup> For example,  $P_t$  may be the individual specific monetary cost of attending the course – as is the case of transport cost – but can also be thought as the individual's perceived cost of signalling to the market in the even of taking up course within the ALMP range (that is, stigmatization, low ability, etc.)

it is the assumption that  $P > 0$  which leads to two distinct choice of unemployment in the labour market (see the proof to Lemma 1 in Appendix 1).

## 4.2 Identification

From the structural model in (1) and the two informative conditions (5) and (6), we define the following population moment conditions (for any given skill class  $s$ , omitted):

*From (5), Employment spells:*

$$\begin{aligned} E\left[\Delta(\ln E_{t,t+1}) \middle| I_t^w = I_{t+1}^w = 1\right] = \\ E\left[\Delta(\ln(\tilde{W}_{t,t+1})) \middle| I_t^w = I_{t+1}^w = 1\right] + E\left[\nu(h_t)\pi_t \middle| I_t^w = I_{t+1}^w = 1\right] + E\left[\Delta(\ln(\pi_{t,t+1})) \middle| I_t^w = I_{t+1}^w = 1\right]. \end{aligned}$$

*From (6), Unemployment spells & no ALMP:*

$$\begin{aligned} E\left[\Delta(\ln \Gamma_{t,t+1}) \middle| I_t^n = I_{t+1}^n = 1\right] = \\ E\left[\Delta(\ln(B_{t,t+1})) \middle| I_t^n = I_{t+1}^n = 1\right] - E\left[\sigma(h_t)\pi_t \middle| I_t^n = I_{t+1}^n = 1\right]. \end{aligned} \tag{8}$$

Our final aim is to obtain skill specific estimates for  $\nu(h_t)$  and  $\sigma(h_t)$  using the sample analogues for each of the population conditions in (8) having shown that these are informative with regards to human capital formation. The vectors  $(\nu(h_t), \pi_t)$  and  $(\sigma(h_t), \pi_t)$  imply sets of random variables dependent on unobserved  $h_t$ . Unless we can disentangle the effects between  $\nu(h_t)$  and  $\pi_t$ , and likewise between  $\sigma(h_t)$  and  $\pi_t$ , we cannot identify the parameter set  $(\nu(h_t), \sigma(h_t))$ . Thus, we need to expand our set of assumptions to elicit the effect of  $\nu(h_t)$  and  $\sigma(h_t)$  away from the idiosyncratic effects that the shock  $\pi_t$  may have on  $\Delta(\ln E_{t,t+1})$  and  $\Delta(\ln \Gamma_{t,t+1})$ , respectively. Let  $Z_t$  be a set of variables that determines  $h_t$ . For example,  $Z_t$  may include age and labour market experience (within specific skill class  $s$ ). Assume that for each value of  $Z_t$  individual's human capital is sufficiently homogenous so that

$E[\nu(h_t)\pi_t | Z_t, I_t^w = I_{t+1}^w = 1] = \nu_t E[\pi_t | Z_t, I_t^w = I_{t+1}^w = 1]$ , where  $\nu_t = \nu_t(s)$  represents the skill specific ability to learn. Notice that for an appropriate choice of  $Z_t$  similar arguments would apply to those in unemployment spells so that  $E[\sigma(h_t)\pi_t | Z_t, I_t^n = I_{t+1}^n = 1] = \sigma_t E[\pi_t | Z_t, I_t^n = I_{t+1}^n = 1]$  applies. With this, define  $\Xi_{E,t} := (Z_{E,t}, I_t^w = I_{t+1}^w = 1)$  and  $\Xi_{\Gamma,t} := (Z_{\Gamma,t}, I_t^n = I_{t+1}^n = 1)$ : within each subgroup  $\Xi_{E,t}$  (or  $\Xi_{\Gamma,t}$ ) the rate of change of  $h_t$  (and  $h_t$  itself) is assumed to be identical even if we cannot directly observe such stocks  $h_t$ . These arguments lead to the following assumption:

**Assumption 10** (homogeneity in human capital): Within each skill class  $s$ ,  $\nu(h_t)$  and  $\sigma(h_t)$  determine human capital changes. Let  $Z_{E,t}$  and  $Z_{\Gamma,t}$  be observed information defining subgroups in the population with homogeneous stocks of human capital  $h_t$ . Conditional on membership in these groups, individual's human capital stocks are identical between periods, so that both  $E[\nu(h_t) | Z_t, I_{t+1}^w = I_t^w = 1] = \nu_t$  and  $E[\sigma(h_t) | Z_t, I_{t+1}^n = I_t^n = 1] = \sigma_t$ <sup>7</sup> apply.

Assumption 10 implies that  $\nu$  and  $\sigma$  are ‘sub-group constant’ conditional on the information set  $Z$  at  $t$  that may be potentially different for different regime choices (i.e., difference in information between  $Z_{E,t}$  and  $Z_{\Gamma,t}$ ). Thus, taking expectations over sub-populations defined by corresponding vectors of  $Z_t$  solves the problem of unobserved human capital  $h_t$  and Assumption 10 becomes the key identifying assumption disentangling the effects of  $h_t$  and  $\pi_t$ . With this, the following applies to substitute the previous moment conditions:

$$\begin{aligned} E[\Delta(\ln E_{t,t+1}) | \Xi_{E,t}] &= E[\Delta(\ln(\tilde{W}_{t,t+1})) | \Xi_{E,t}] + \nu_t \cdot E[\pi_t | \Xi_{E,t}] + E[\Delta(\ln(\pi_{t,t+1})) | \Xi_{E,t}]; \\ E[\Delta(\ln \Gamma_{t,t+1}) | \Xi_{\Gamma,t}] &= E[\Delta(\ln(B_{t,t+1})) | \Xi_{\Gamma,t}] - \sigma_t \cdot E[\pi_t | \Xi_{\Gamma,t}]. \end{aligned} \quad (9)$$

<sup>7</sup> It is not necessary to condition on  $Z_{t+1}$ , because  $Z_t$  explains human capital stocks at  $t$  for individuals whose choice is identical at  $t$  and  $t+1$ . Therefore, by definition, they experience identical changes in human capital.

The moment conditions in (9) cannot directly be used to identify the unknown parameters, because we do not observe the shocks that determine the expectations  $E[\pi_t | \Xi_{E,t}]$ ,  $E[\Delta(\ln(\pi_{s,t,t+1})) | \Xi_{E,t}]$  and  $E[\pi_t | \Xi_{\Gamma,t}]$ . However, we do observe individual's labour market choices that result directly from individual's comparison of the state of the world (i.e., the transitory shock  $\pi_t$ )<sup>8</sup> with their labour reservation policies  $\pi_{t[a]}^R$  and  $\pi_{t[b]}^R$ . In Section 3 it has been shown that these policies are functions of both labour market histories and individual's specific characteristics that determine the individual's labour market choices. Thus, observing  $I_t^w = 1$  at  $t$  implies  $\pi_t \geq \pi_{t[b]}^R$ , whereas observing  $I_t^n = 1$  at  $t$  implies  $\pi_{t[a]}^R \geq \pi_t$ ; for all individuals  $\pi_{t[a]}^R < \pi_{t[b]}^R$  (see Lemma 1). If we are able to retrieve the reservation values  $\pi_{t[a]}^R$  and  $\pi_{t[b]}^R$  from the data, they would provide one-sided bounding information on the magnitude of the unobserved shocks  $\pi_t$ . Furthermore, Assumption 1 interprets the shocks as stochastic draws from some known distribution with positive bounded support  $[\underline{\pi}, \bar{\pi}]$ , bounds that are otherwise required to establish the unique solution to the problem in expression (2) (see Lemma 2). Taking these conditions together implies the following bounds:

$$\begin{aligned}
\text{Employment :} & & I_t^w = 1 & \Rightarrow & \pi_{t[b]}^R \leq \pi_t \leq \bar{\pi} \\
\text{Unemployment, no ALMP:} & & I_t^n = 1 & \Rightarrow & \underline{\pi} \leq \pi_t \leq \pi_{t[a]}^R.
\end{aligned} \tag{10}$$

The conditions in (10) define bounds for the unknown stochastic shocks conditional on a given labour market regime in period  $t$ , bounds that are consistent with the structural model in

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<sup>8</sup> The structural model in (1) determines that the only stochastic component in the state space is the contemporaneous and transitory labour market shock. At the beginning of the period the shock is revealed and individuals evaluate the state of the world to make an optimal labour market (and consumption) decision. Thus, saying that individuals evaluate the state of the world against the reservation policies ( $\pi_{t[a]}^R$ ) and ( $\pi_{t[b]}^R$ ) is the same as suggesting that individuals evaluate the labour market shock against such reservation parameters.



(1). These conditions imply that we can identify  $E[\pi_t | \Xi_{E,t}]$ ,  $E[\Delta(\ln(\pi_{s,t,t+1})) | \Xi_{E,t}]$  and  $E[\pi_t | \Xi_{\Gamma,t}]$  up to an interval if we can use the observed behaviour in the population to identify the reservation policies ( $\pi_{t[a]}^R$  and  $\pi_{t[b]}^R$ ) and the two limiting values ( $\underline{\pi}$  and  $\bar{\pi}$ ). Consider only regimes associated with changes in the stock of human capital and define the following conditions:

$$\begin{aligned}
& \text{For any individual,} & \pi_{t[a]}^R < \pi_{t[b]}^R \\
& \text{and for all in the population,} & \pi_t \in [\underline{\pi}, \bar{\pi}] \\
& \text{Then, we define the following unconditional population moment conditions:} & 
\end{aligned} \tag{11}$$

$$\begin{aligned}
& \text{Employment:} & P(I_t^w = 1) = P(\pi_{t[b]}^R \leq \pi_t \leq \bar{\pi}) \\
& \text{Unemployment, no ALMP:} & P(I_t^n = 1) = P(\pi_{t[a]}^R \geq \pi_t \geq \underline{\pi}).
\end{aligned}$$

Expression (11) needs further structure to become operational. We have a clear difference between the sets  $[\underline{\pi}, \bar{\pi}]$  and  $[\pi_{t[a]}^R, \pi_{t[b]}^R]$ ; whereas the limits  $[\underline{\pi}, \bar{\pi}]$  are independent from individual's characteristics (thus identical to all), the reservation values  $\pi_{t[a]}^R$  and  $\pi_{t[b]}^R$  are heterogeneous and depend on observables representing individual's preference in the population. Let these preferences and characteristics be explained by a set of variables  $K_t$  such that  $\pi_{t[a]}^R = \pi_{t[a]}^R(K_{\Gamma,t})$  and  $\pi_{t[b]}^R = \pi_{t[b]}^R(K_{E,t})$  applies.<sup>9</sup> To formalize these relations we specify a function that relates the reservation values to the observed characteristics. The function is such that for a given  $K_t$  set the reservation values are always projected on the positive line, a

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<sup>9</sup> The set  $K_{\Gamma}$  may differ from  $K_E$ . These information sets would include all those variables that we may think determines the fixed cost for entry into a given labour market regime (e.g., individuals household characteristics, motivation, the value of time, etc.) as well as personal characteristics that may explain individual's preference when choosing to participate in the labour market (e.g., assets, debts, family composition, gender, etc).

requirement because the structural model in (1) establishes  $0 < \underline{\pi} < \pi_{t,[a]}^R \leq \pi_{t,[b]}^R < \bar{\pi} < \infty$ . Thus, we chose an exponential relation between the pair  $(\pi_{t,[a]}^R, \pi_{t,[b]}^R)$  and the information set  $K_t$ :

**Assumption 11:** The following relations hold for every individual in the population:

$$\pi_{t,[a]}^R = \exp(K_t \gamma_a) \quad \text{and} \quad \pi_{t,[b]}^R = \exp(K_t \gamma_b) \quad \Rightarrow \quad \ln \pi_{t,[a]}^R = K_t \gamma_a \quad \text{and} \quad \ln \pi_{t,[b]}^R = K_t \gamma_b.$$

The vectors of parameters  $\gamma_a$  and  $\gamma_b$  in Assumption 11 are weights that determine the importance of each of the variables in  $K_t$  with regards to the reservation values (i.e., relative labour market choices). The logarithmic transformation is monotonic thus preserving the relation between the values  $(\underline{\pi}, \bar{\pi}, \pi_{t,[a]}^R, \pi_{t,[b]}^R)$ , i.e.,  $0 < \ln \underline{\pi} < \ln \pi_{t,[a]}^R \leq \ln \pi_{t,[b]}^R < \ln \bar{\pi} < \infty$  preserves  $0 < \underline{\pi} < \pi_{t,[a]}^R \leq \pi_{t,[b]}^R < \bar{\pi} < \infty$ . At this point we can modify the condition in (11). Observing that for any individual we have that  $\pi_{t,[a]}^R < \pi_{t,[b]}^R$ , we obtain the following representation for all shocks in the support  $\pi_t \in [\underline{\pi}, \bar{\pi}]$ :

$$\begin{aligned} \text{Employment:} & \quad P(I_t^w = 1) = P(\ln \pi_{t,[b]}^R \leq \ln \pi_t \leq \ln \bar{\pi}) \\ \text{Unemployment, no ALMP:} & \quad P(I_t^n = 1) = P(\ln \underline{\pi} \leq \ln \pi_t \leq \ln \pi_{t,[a]}^R). \end{aligned} \tag{12}$$

The structural model in (1) assumes individuals make choices to maximise utility. Thus, we assume that labour market choices are governed by an underlying unobserved latent process that describes the utility associated with each potential labour market choice conditional on the individual's observed characteristics. What we observe (the actual labour market choice) is the outcome of such utility valuation in the form of a realized labour market regime. The following formalizes this argument:

$$\begin{aligned} \text{Employment:} & \quad I_t^w = 1 \Rightarrow I_t^{w*} \geq 0 \quad \text{where} \quad I_t^{w*} = -\ln \pi_{t,[b]}^R + \ln \pi_t; \\ \text{Unemployment, no ALMP:} & \quad I_t^n = 1 \Rightarrow I_t^{n*} \leq 0 \quad \text{where} \quad I_t^{n*} = -\ln \pi_{t,[a]}^R + \ln \pi_t. \end{aligned} \tag{13}$$

Applying Assumption 11 to the two second conditions in (13) leads to the following probabilities:

$$\begin{aligned}
\text{Employment :} & & P(I_t^w = 1 | K_t, \gamma_b) &= P(I_t^{w*} \geq 0 | K_t, \gamma_b) = P(\ln \pi_t \geq K_t \gamma_b) \\
\text{Unemployment, no ALMP:} & & P(I_t^n = 1 | K_t, \gamma_a) &= P(I_t^{n*} \leq 0 | K_t, \gamma_a) = P(\ln \pi_t \geq K_t \gamma_a).
\end{aligned} \tag{14}$$

Assumption 11 implies that having estimated the vectors  $\gamma_a$  and  $\gamma_b$ , we can predict the reservation values, i.e.,  $\hat{\pi}_{t,a}^R = \exp(K_t \hat{\gamma}_a)$  and  $\hat{\pi}_{t,b}^R = \exp(K_t \hat{\gamma}_b)$ . The latent processes in (13) treat the stochastic shocks ( $\ln \pi_t$ ) as independent error terms summarizing all the stochastic variability that adjusts labour market choices to the deterministic preferences ( $\ln \pi_{t,l}^R, l = a, b$ ). Moreover, the specifications in (14) – given (13) – justify the use of an index model to estimate the weighting vectors ( $\gamma_a$  and  $\gamma_b$ ) as long as we make some distributional assumptions on the shock  $\ln \pi_t$ :

**Assumption 12** (characterization of labour market shocks): The productivity shock  $\pi$  follows a truncated lognormal distribution  $LnN(\mu_\pi, sig_\pi)$  in the support  $[\underline{\pi}, \bar{\pi}]$ , where  $\underline{\pi} < \bar{\pi}$ ,  $\underline{\pi} > 0$ . This implies that  $\ln \pi$  follows a truncated normal distribution  $N(\mu_{\ln \pi}, sig_{\ln \pi})$  in the support  $[\ln \underline{\pi}, \ln \bar{\pi}]$ , where we assume  $\mu_{\ln \pi} = 0$ .

Together with (13) and (14), Assumptions 11 and 12 identify the reservation values  $(\pi_{t,[a]}^R, \pi_{t,[b]}^R)$  for each individual in the population. The unknown bounds  $(\underline{\pi}, \bar{\pi})$  establish the truncation of the distribution in Assumption 11 and are estimated jointly with  $\pi_{t,[a]}^R$  and  $\pi_{t,[b]}^R$ . According to (10), knowledge of  $(\underline{\pi}, \bar{\pi}, \pi_{t,[a]}^R, \pi_{t,[b]}^R)$  together with the log-normality assumption leads to identification of  $E[\pi_t | \Xi_{E,t}]$ ,  $E[\Delta(\ln(\pi_{s,t,t+1})) | \Xi_{E,t}]$  and  $E[\pi_t | \Xi_{\Gamma,t}]$  so that the parameters  $\nu_t$  and  $\sigma_t$  are also identified from (9). Point identification of these parameters is possible if we

assume mean values of the intervals in (10) to represent individual's unobserved stochastic shocks.

### 4.3 Estimation

Assumption 12 imposes that  $\ln \pi_t \sim N(0, \sigma_{\ln \pi})$ , thus we estimate the vectors  $\gamma_a$  and  $\gamma_b$  (up to scale) at distinct times  $t+j, j=0,1$  using probit models with dependent variable  $I_{t+j}^w = 1, j=0,1$ , (against  $I_{t+j}^w \neq 1, j=0,1$ , respectively) conditional on  $K_{t+j}^w, j=0,1$ , and for the outcome  $I_{t+j}^n = 1, j=0,1$  (against  $I_{t+j}^n = 0, j=0,1$ , respectively) conditional on  $K_{t+j}^n, j=0,1$ , with obvious suffix for the information sets.<sup>10</sup> In estimating the probit models we allow for the distribution function of the error term to be truncated from both the left and right hand side. Thus, for every time period  $t$  and employment state, the likelihood function conditional on employment state is defined as follows:

$$\begin{aligned}
& \text{Employment: } I_t^w = 1 \quad \Rightarrow \quad I_t^{w*} \geq 0 \quad P(I_t^w = 1 | K_t; \gamma_b) \\
& \Rightarrow P(I_t^w = 1 | K_t; \gamma_b) = \left( \frac{\Phi(\ln \bar{\pi}) - \Phi(K_t \gamma_b)}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right) \quad \& \quad P(I_t^w \neq 1 | K_t; \gamma_b) = \left( \frac{\Phi(K_t \gamma_b) - \Phi(\ln \underline{\pi})}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right) \\
& \Rightarrow L(\gamma_b | \underline{\pi}, \bar{\pi}, K_t) = \prod_N \left( \frac{\Phi(\ln \bar{\pi}) - \Phi(K_t \gamma_b)}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right)^{(I_t^w=1)} \left( \frac{\Phi(K_t \gamma_b) - \Phi(\ln \underline{\pi})}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right)^{(I_t^w \neq 1)} . \\
\\
& \text{Unemployment, no ALMP: } I_t^n = 1 \quad \Rightarrow \quad I_t^{n*} \leq 0 \quad P(I_t^n = 1 | K_t; \gamma_a) \\
& \Rightarrow P(I_t^n = 1 | K_t; \gamma_a) = \left( \frac{\Phi(K_t \gamma_a) - \Phi(\ln \underline{\pi})}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right) \quad \& \quad P(I_t^n \neq 1 | K_t; \gamma_a) = \left( \frac{\Phi(\ln \bar{\pi}) - \Phi(K_t \gamma_a)}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right) \quad (15) \\
& \Rightarrow L(\gamma_a | \underline{\pi}, \bar{\pi}, K_t) = \prod_N \left( \frac{\Phi(K_t \gamma_a) - \Phi(\ln \underline{\pi})}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right)^{(I_t^n=1)} \left( \frac{\Phi(\ln \bar{\pi}) - \Phi(K_t \gamma_a)}{\Phi(\ln \bar{\pi}) - \Phi(\ln \underline{\pi})} \right)^{(I_t^n \neq 1)} .
\end{aligned}$$

In expression (15) the index  $I_t^j \neq 1$  stands for the alternative labour market regime to choice  $j$ ,  $\Phi$  is the cumulative density function from the standard normal distribution and  $N$  defines a random sample representative of the underlying population. The bounds  $(\underline{\pi}, \bar{\pi})$  are not included in the parameter space so that the likelihood function is given as  $L(\gamma, | \underline{\pi}, \bar{\pi}, K_t)$  instead of  $L(\gamma, \underline{\pi}, \bar{\pi} | K_t)$ . Allowing for the set  $(\underline{\pi}, \bar{\pi})$  to enter the parameter space turned out to be numerically unstable: the information matrix would depend on the cross derivatives  $(\partial/\partial \gamma \partial \underline{\pi})$ ,  $(\partial/\partial \gamma \partial \bar{\pi})$  and  $(\partial/\partial \bar{\pi} \partial \underline{\pi})$  entering as off diagonals. It is now easy to show that  $E(\partial/\partial \gamma \partial \underline{\pi}) = E(\partial/\partial \gamma \partial \bar{\pi}) = 0$ . At the same time, the second derivatives  $(\partial^2/\partial \bar{\pi} \partial \underline{\pi})$  and  $(\partial/\partial \underline{\pi} \partial \underline{\pi})$  as well as  $(\partial/\partial \bar{\pi} \partial \underline{\pi})$  rely multiplicatively on the values of the probability density functions  $\phi(\ln \underline{\pi})$  and  $\phi(\ln \bar{\pi})$  that can be arbitrarily close to zero for small changes in  $(\underline{\pi}, \bar{\pi})$  as the process iterates towards the maximum. Thus, the information matrix becomes singular very fast for small changes in  $(\underline{\pi}, \bar{\pi})$ . As an alternative estimation procedure, we fix the values  $(\underline{\pi}, \bar{\pi})$  since the likelihood function is well specified for given values  $(\underline{\pi}, \bar{\pi})$ , i.e., the bounding intervals systematically shift the likelihood function but leave the properties of the maximum likelihood estimates of  $\gamma_a$  and  $\gamma_b$  intact. We choose the set  $(\underline{\pi}^*, \bar{\pi}^*)$  and the resulting  $\gamma_a^{mle}$  (or  $\gamma_b^{mle}$ ) such that the pair  $(\underline{\pi}^*, \bar{\pi}^*)$  implies the highest estimated value of the likelihood function among a finite number of  $(\underline{\pi}, \bar{\pi})$  pairs (see Section 6 for further details).

Once the values  $(\underline{\pi}^*, \bar{\pi}^*)$  and the pair  $\gamma_a^{mle}$  and  $\gamma_b^{mle}$  are estimated from the data reservation values  $\hat{\pi}_{t,a}^R = \exp(K_t^n \gamma_a)$  and  $\hat{\pi}_{t,b}^R = \exp(K_t^n \gamma_b)$  are also identified and, therefore, so is the

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<sup>10</sup> It would be misleading to index the reservation values  $\pi_{t,n}^R$  and  $\pi_{t,w}^R$  as opposed to the established indexes  $\pi_{t,a}^R$  and  $\pi_{t,b}^R$ : the suffixes  $[a, b]$  clearly indicate that irrespective of market choice  $(w, n)$ , all individuals at each point in time hold a pair of reservation values to identify their three distinct labour market valuations,  $(w, n, q)$ .

distribution of the unobserved shocks  $\pi_t$ . With these, the conditional expectations  $E[\pi_t | \Xi_{E,t}]$ ,  $E[\Delta(\ln(\pi_{s,t,t+1})) | \Xi_{E,t}]$  and  $E[\pi_t | \Xi_{\Gamma,t}]$  are approximated using their sample analogues such that  $(\frac{1}{N})\sum_{i=1}^N(\tilde{\pi}_{it} | \Xi_{E,t})$ ,  $(\frac{1}{N})\sum_{i=1}^N((\ln \tilde{\pi}_{it+1} - \ln \tilde{\pi}_{it}) | \Xi_{E,t})$  and,  $(\frac{1}{N})\sum_{i=1}^N(\tilde{\pi}_{it} | \Xi_{\Gamma,t})$ , where  $\tilde{\pi}_{it}$  is the midpoint of the interval in (10) for the individual  $i$ .<sup>11</sup> With this all the elements in expression (9) are identified and sample analogues to the population conditions in (9) can be used to estimate the key human capital parameters  $\nu_t$  and  $\sigma_t$ . Inference from the sample to the population is possible using a bootstrap procedure that re-samples randomly from the data with replacement (i.e., a naïve bootstrap) to obtain intervals as given in (10) for each bootstrap sample (see Section 6 for more details on the full bootstrap estimation process).

## 5 Data

We use 13 waves from the Swiss Labour Force Survey (SAKE, 1991 – 2004) to empirically quantify human capital growth and depreciation rates. The Swiss Labour Force Survey is a 5 year rotating panel that collects information from a representative sample of working age individuals (ages 16 and above) living in Switzerland. Questions in the panel refer to labour market outcomes, extensive cover of labour market histories and key socio-economic indicators. Between the start of the panel in 1991 and 2004, 152,010 distinct individuals have participated.

In practice, estimating (9) requires only two consecutive waves. Using all 13 waves increases the sample size within each of the three sub-samples (i.e., in spells of employed, unemployment with and unemployment without ALMP) while allowing to control for time dependent macro-economic changes. The 13 waves determine at most 12 sets of consecutive years

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<sup>11</sup> Note that although the midpoint of the interval is an inconsistent estimator if the interval is not symmetric around the truncated mean, it provides a computationally convenient and fast approximation.

$t, t+1$ . Thus, individuals may appear as repeated units in our data set.<sup>12</sup> Such repeated observations from an individual at different intervals  $(t, t+1)$  count as distinct units due to changing labour market histories that contribute differently over time towards distinct human capital sub-cells.

Assumption 10 requires selecting units of working age (16 to 65). To attain a homogeneous sample regarding the implication of active and passive labour market policies, our sample criteria consists of selecting non-disabled males, full time workers if employed or, in the case of the unemployed, they claimed to have worked full time in their last employment. All selected individuals declare to have finished their formal education and are Swiss or foreigners with have a permanent working permit. Furthermore, individuals with a working status at  $t$  ( $t+1$ ) are selected only if they also claim a working status at  $t-1$  ( $t$ ) to further ensure Assumption 10, i.e., individual with similar human capital stocks must have similar labour market histories. Conditioning on employment at  $t-1$  ( $t$ ) for those working at  $t$  ( $t-1$ ) guarantees that the human capital stock of those in current employment has not been affected by unemployment histories in the most recent past.<sup>13</sup> To estimate the parameters in (9) we need to consider only the two main sub-samples, i.e., consecutively employed and consecutively unemployed without ALMP. However, estimating index models as in (14) requires counterfactual outcomes defined at  $t$  independently of the outcome one period ahead. We take this into account when selecting the sample (see Table 1).

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<sup>12</sup> Because of the 5-year rotating nature of the panel, an individual can appear (at most) 5 consecutive times, while leaving the panel at any time before years 5 implies not being surveyed any more in the future. By definition an individual that completes the 5 years may appear at most 4 times as a unit observed at  $t$  and at  $t+1$ .

<sup>13</sup> We apply this criteria only to individuals at least 19 years old at  $t$  ( $t+1$ ), because individuals who are 16 to 18 at time  $t$  may still complete their start up education at time  $t-1$ .

Our analysis considers three distinct skill groups defined by pre-labour market education. The lowest skill class ( $s = 1$ ) has not finished compulsory education with a degree (either primary or secondary). The next lowest skill class ( $s = 2$ ) is composed of individuals who have finished up to secondary education and may have completed some complementary vocational education (e.g., apprenticeship or low 1<sup>st</sup> tier vocational college). They have not completed high school (Matura). The third and final skill class ( $s = 3$ ) are medium-skill individuals with completed high school or equivalent and include those with 2<sup>nd</sup> tier vocational college. We exclude individuals with university or higher technical college (either case, completed or otherwise) because they are unlikely to be part of the ALMP system and thus we won't observe any reasonable numbers in the respective cells. Skill class is determined at  $t$ .

The sub-group  $I_t^w = I_{t+1}^w = 1$  in full-time work can easily be identified in the data. The data also distinguishes the 'unemployed' (registered at official unemployment offices) from the 'not employed' (those claiming a non-working status while being of working age). The unemployed enter the sub-group  $I_t^q = I_{t+1}^q = 1$  if observed consecutively over any two periods. The not-employed make up the pool from which to draw our unemployed *without* ALMP, i.e., those in  $I_t^n = I_{t+1}^n = 1$ . However, instead of selecting everyone in this pool, we select units to guarantee that members in  $I_t^n = I_{t+1}^n = 1$  have sufficiently low latent reservation wages so that working in the future is highly likely. This is done by including units who are not registered as unemployed,<sup>14</sup> but search or/and claim willingness to work immediately if offered work in short notice. Furthermore, since the subgroups help to measure human capital depreciation we require from all individuals to have some labour market history. In all, these restrictions in the pool of 'not-

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<sup>14</sup> The SAKE includes information on the elapse of the benefit period. We use this variable when available. More regular information involves asking the unemployed about their status as registered or not. If not registered, they should declare intensity of job search and willingness to work if offered work immediately.



employed' implies a substantial cut in the size of the sample but guarantees the non-selection of true outsiders to the labour market. It is important to note that in this set-up we treat the regular services by the case workers as ALMP as well (contrary to some conventional definitions).

*Table 1: Distribution of consecutively observed units between  $t, t+1$  among alternative labour market regimes*

State in period $t$	State in period $t+1$	Observations
Working ( $I_t^w = 1$ )  45,826	Working ( $I_{t+1}^w = 1$ )	42,438
	Registered unemployed ( $I_{t+1}^q = 1$ )	545
	Unemployed without ALMP ( $I_{t+1}^n = 1$ )	607
	Not classified at $t+1$	2,236
	Total	45,826
Registered Unemployed ( $I_t^q = 1$ )  980	Working ( $I_{t+1}^w = 1$ )	322
	Registered unemployed ( $I_{t+1}^q = 1$ )	236
	Unemployed without ALMP ( $I_{t+1}^n = 1$ )	128
	Not classified at $t+1$	294
	Total:	980
Unemployed without ALMP ( $I_t^n = 1$ )  1,847	Working ( $I_{t+1}^w = 1$ )	264
	Registered Unemployed ( $I_{t+1}^q = 1$ )	97
	Unemployed without ALMP ( $I_{t+1}^n = 1$ )	585
	Not classified at $t+1$	901
	Total	1,857

Note: Full time workers ages 19 or more at  $t$  must declared to have worked for pay during all last calendar year (i.e., during the last 12 months). The same selection condition applies to full time workers at  $t+1$ . Registered unemployed claim to be officially registered as unemployed. Unemployed without ALMP are not officially registered but indicate willingness to take up work immediately if offered the right vacancy.

There are 56,390 males observed at least once during the period 1991-2004. They are all candidates at  $t$  to become units observed between  $t$  and  $t+1$ . Applying our selection criteria reduces the number of (unique) individuals to 24,041. These individuals make 48,653 distinct contributions observed consecutively between  $t$  and  $t+1$ . Each of the 24,041 can appear at most

4 times.<sup>15</sup> Table 1 shows the distribution of these units by regime and the changing distribution of these among regimes one period ahead.

Estimates based on (9) require two mutually exclusive groups: those classified as working ( $I_t^w = I_{t+1}^w = 1$ ) (42,438) and those classified as non-working-non-ALMP ( $I_t^n = I_{t+1}^n = 1$ ) (585). Table 2 shows the distribution of these two groups and the group ( $I_t^q = I_{t+1}^q = 1$ ) by skill class, because skill, together with the idiosyncratic labour market shock, is the key variable that defines heterogeneity of human capital formation conditional on human capital stocks.

*Table 2: Distribution of skill class by ( $I_t^w = I_{t+1}^w = 1$ ), ( $I_t^n = I_{t+1}^n = 1$ ) and ( $I_t^q = I_{t+1}^q = 1$ )*

	Skill class 1 (lowest skill) ( $s = 1$ )	Skill class 2 (second lowest skill) ( $s = 2$ )	Skill class 3 (medium skill) ( $s = 3$ )
Working Spell ( $I_t^w = I_{t+1}^w = 1$ )	4,974	24,781	12,683
Registered unemployed spell ( $I_t^q = I_{t+1}^q = 1$ )	66	117	53
Unemployed without ALMP spell ( $I_t^n = I_{t+1}^n = 1$ )	228	190	167
Mixed spells and/or outsiders at $t + 1$	1,174	2,795	1,445
Total	6,445	27,883	14,335
Skill class % based on <i>unique individuals</i>			
( $I_t^w = I_{t+1}^w = 1$ )	13.8%	56.0%	30.2%
( $I_t^q = I_{t+1}^q = 1$ )	36.9%	39.8%	23.3%
( $I_t^n = I_{t+1}^n = 1$ )	45.0%	24.1%	31.0%
Percentage of total	16.3%	54.1%	29.6%

Note: Skill class is identical over the two periods. The last 4 rows show the distribution by sub-groups for each skill class with each of the rows adding up to 100%.

Section 4 defines two sets of observables;  $Z_t$  with variables that help to proxy for human capital stocks at  $t$ , and  $K_t$ , the set of variables that determines individual's preferences (utility) regarding a labour market choice. The SAKE provides information to construct the two sets. With obvious notation, we define  $Z_t^E, Z_t^\Gamma$  and  $K_t^w, K_t^n$  to clarify difference in sets according to the

<sup>15</sup> 11,352 distinct individuals appear only once between  $t$  and  $t + 1$ , and leave the panel after two years; 5,181 participate over three consecutive years and contribute as two distinct units between different  $t$  and  $t + 1$ ; 3,093

two main sub-groups included in (9).<sup>16</sup> Variables that appear in  $K_t^w$  and  $K_t^n$  are age (and age square), cantonal dummies (Germanic, French or Italian cantons), household size, marital status, household ownership, most recent industrial sector (primary, secondary or tertiary), skill class (skill class 1, or 2 or 3), the natural log of net real household income, dummy variables to indicate the length of the most recent labour market experience (up to 6 months, between 6 months and 1 year, 1 to 1.5 years, 1.5 to 2 years, 2 to 4 years, 4 to 6 years and more than six years), and time dummies for each year between 1991 and 2004. Beside these,  $K_t^n$  includes information on unemployment duration (i.e., currently unemployed for at most 6 months, unemployed 6 to 12 months, 12 to 18 months, 18 to 24 months and unemployed for 24 months or more). The variables in  $Z_t^E$  and  $Z_t^\Gamma$  allow to divide any sub-group according to homogeneity in unobserved human capital stocks  $h_t$ . The set  $Z_t^E$  contains age and the same dummies explaining labour market experience as defined in  $K_t^w$ : further we recall that all 42,438 units of workers have worked continuously for one year before being observed at  $t$ . Conditional on skill class, dividing the sub-group of workers among cells with a similar age and similar experience increases within cell homogeneity of human capital, i.e., within cell by skill class, the rate of change in human capital (i.e.,  $\nu$ ) is similar (Assumption 10). The set  $Z_t^\Gamma$  does not contain age: instead, it allows for labour market experience but most importantly it contains duration of current unemployment spells. It is assumed that individuals with similar experience (before

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determine three distinct units between  $t$  and  $t + 1$ , and 4,415 determine four distinct units between  $t$  and  $t + 1$  having participated in the panel for the full 5 years term.

<sup>16</sup> The difference in subscript (i.e.,  $(E, \Gamma)$  for sets  $Z$ , and  $(w, n)$  for sets  $K$ ) indicate that these sets are used for populations defined according regime choices over different lengths. An employed (unemployed) individual over consecutive periods provides information for the set  $Z_t^E$  ( $Z_t^\Gamma$ ), whereas choosing a regime at  $t$  or  $t + 1$  is based on preferences  $K_{t+j}^{choice}$ ,  $j = 0, 1$ ;  $choice = n, w$  irrespective of next periods spell.

becoming unemployed) in the labour market have similar rates of human capital depreciation if they have experienced similar lengths of unemployment.

Table 3 shows sample statistics for the variables in  $K_t^w$  and  $K_t^n$  according to labour market regimes. Table 4 does the same for  $Z_t^E$  and  $Z_t^I$ . Both tables include information for the subgroup  $I_t^q = 1$  or  $I_{t+1}^q = 1$  as well.

Table 3:  $K_t^w$  and  $K_t^n$  by labour market regime choice

VARIABLES	Period $t$			Period $t+1$		
	$I_t^w = 1$ (45,826)	$I_t^q = 1$ (980)	$I_t^n = 1$ (1,847)	$I_{t+1}^w = 1$ (43,024)	$I_{t+1}^q = 1$ (878)	$I_{t+1}^n = 1$ (1,320)
Age	.41 (.11)	.38 (.13)	.37 (.17)	.42 (.11)	.39 (.13)	.43 (.17)
German Canton	.70 (.002)	.59 (.016)	.59 (.011)	.69 (.002)	.60 (.015)	.61 (.013)
French Canton	.24 (.002)	.34 (.015)	.31 (.011)	.24 (.002)	.32 (.015)	.30 (.012)
Household size	2.8 (1.4)	2.6 (1.4)	2.8 (1.4)	2.8 (1.4)	2.5 (1.4)	2.6 (1.3)
Partner present at home	.64 (.002)	.42 (.016)	.33 (.011)	.65 (.002)	.46 (.015)	.46 (.013)
Household owner	.37 (.002)	.15 (.011)	.33 (.011)	.39 (.002)	.18 (.012)	.35 (.012)
Primary industry	.05 (.001)	.03 (.006)	.02 (.003)	.05 (.001)	.04 (.006)	.03 (.005)
Secondary industry	.37 (.002)	.69 (.015)	.68 (.011)	.37 (.002)	.67 (.015)	.66 (.012)
Labour market experience						
< 6 months	.01 (.001)	.14 (.011)	.54 (.011)	.006 (.001)	.14 (.011)	.50 (.013)
6 - 12 months	.01 (.0001)	.02 (.005)	.03 (.004)	.007 (.000)	.016 (.004)	.018 (.004)
1 - 2 years	.02 (.001)	.03 (.006)	.02 (.003)	.011 (.001)	.018 (.005)	.026 (.005)
2 - 4 years	.03 (.001)	.09 (.009)	.04 (.005)	.03 (.001)	.066 (.008)	.040 (.005)
4 - 6 years	.04 (.001)	.09 (.009)	.04 (.004)	.038 (.001)	.058 (.007)	.030 (.005)
> 6 years	.89 (.001)	.63 (.015)	.32 (.011)	.901 (.001)	.70 (.014)	.39 (.013)
Unemployment duration						
< 6 months	--	.58 (.016)	.47 (.011)	--	.55 (.015)	.39 (.013)
6 - 12 months	--	.24 (.013)	.18 (.009)	--	.22 (.013)	.21 (.011)
12 - 18 months	--	.18 (.010)	.08 (.006)	--	.14 (.011)	.098 (.008)
18 - 24 months	--	.08 (.008)	.07 (.006)	--	.08 (.008)	.071 (.007)
> 24 months	--	.00 (.00)	.20 (.009)	--	.00 (.00)	.21 (.011)

Note: Most of the variables are part of the probit models (see Appendix). Monetary quantities are in year 2000 CHF. All variables relating to spells are binary. Variables in  $K_t^w$  omit unemployment duration whereas variables in  $K_t^n$  include them. Bracketed numbers are standard deviations.

Table 4:  $Z_t^E$  and  $Z_t^U$  by sub-groups with identical labour market regimes

VARIABLES	Employment spells	Unemployment spells with ALMP	Unemployment. Spells, no ALMP
	$(I_t^w = I_{t+1}^w = 1)$ (42,438)	$(I_t^u = I_{t+1}^u = 1)$ (236)	$(I_t^n = I_{t+1}^n = 1)$ (585)
Age	41.37 (10.74)	40.5 (13.02)	37.10 (16.77)
Labour market experience below 6 months	.007 (.0004)	.097 (.019)	.62 (.02)
6 to 12 months	.007 (.0004)	.021 (.009)	.014 (.005)
12 to 24 months	.013 (.0006)	.034 (.012)	.026 (.007)
more than 2 years	.97 (.0008)	.85 (.020)	.34 (.020)
Unemployment duration below 6 months	--	.60 (.032)	.43 (.020)
6 to 12 months	--	.29 (.03)	.17 (.015)
12 to 18 months	--	.064 (.016)	.051 (.009)
18 to 24 months	--	.047 (.014)	.038 (.008)
more than 24 months	--	.00 (.00)	.32 (.019)

Note: The statistics relate to period  $t$ . Since all individuals described in this table chose identical regimes between periods, changes from the information set  $Z_t$  to  $Z_{t+1}$  are identical for all individuals. See also note below Table 3.

Table 5: Means and variances for net annual earnings and net annual receipts

VARIABLES	Employment spells ( $I_t^w = I_{t+1}^w = 1$ ) (42,438)		UE spells without ALMP ( $I_t^n = I_{t+1}^n = 1$ ) (585)	
	$t$	$t+1$	$t$	$t+1$
Net annual labour earnings				
Group Average	66,575 (33,698)	74,180 (45,463)	N/A	N/A
Skill Class 1	51,106 (25,103)	56,856 (32,839)		
Skill Class 2	64,109 (30,684)	71,540 (44,077)		
Skill Class 3	77,460 (38,583)	86,132 (49,225)		
Net annual per capita household income				
Group Average	65,802 (98,594)	67,651 (98,594)	30,022 (20,573)	24,225 (20,573)
Skill Class 1	50,937 (92,129)	54,174 (99,521)	26,853 (19,628)	22,801 (18,956)
Skill Class 2	64,654 (97,324)	67,026 (101,790)	30,491 (22,613)	25,146 (21,973)
Skill Class 3	73,874 (93,655)	74,158 (91,017)	33,813 (25,072)	25,123 (21,058)

Note: Information on labour earnings is only available for individuals classified as working. About 10% of the sample is affected by non-response for earnings and income variables. Net annual per capita household income equals net household income divided by the square root of household size. See also comments in Table 3.

Other observed variables required by our analysis are net yearly earnings for working observations and net receipts for the unemployed observations. Working individuals are asked about labour earnings while all individuals should declare total net household's income. The latter is an important variable because the SAKE is such that for the non-working net household income is the information to approximate net income receipts. Thus,  $B_{s,t}$  is approximated by estimating per capita net real household income. In principle, this could include incomes that are

not associated with benefit receipts (e.g., capital income), but we are dealing with a population with relatively low skills who claim a strong willingness to work while being unemployed. Thus, it is more likely that the net per capita income of these individuals reflects benefits receipts (e.g., family allowance, social assistance, etc.) that capture the individual's ability to survive economically while in unemployment. As it is common in survey data, all monetary quantities in the SAKE data are subject to response problems: whereas non-response is a larger problem for the sub-group of unemployed (approximately 20% of these do not declare household incomes) non-response for the working sub-group is less of a problem (4.6% do not declare labour income). We use the classic Mahalanobis imputation procedure (see Rubin, 1987) to impute missing money values using the respondents as support information for the non-respondents. Table 5 shows that net receipts increases on average by 6% in real terms. The increase, however, differs by skills with earnings of skill class  $s = 1$  experiencing a real increase of 4% while skill class  $s = 3$  experience a real earnings increase of 7%.<sup>17</sup>

## 6 Results

This section presents estimates for the parameters  $\nu_i$  and  $\sigma_i$  according to cells defined by  $(Z_i^E, Z_i^\Gamma)$ , as explained in Section 5. Before hand, we need to follow a sequence of intermediate steps to obtain estimates for  $\underline{\pi}, \bar{\pi}, \pi_{i,[a]}^R$  and  $\pi_{i,[b]}^R$ .

As a first step we use Assumption 11 and expression (15) to elicit estimates of  $\pi_{i,[b]}^R, \pi_{i,[a]}^R$  for a given choice of  $\underline{\pi}$  and  $\bar{\pi}$  (see Section 4.3). Allowing for a combination  $[\underline{\pi}, \bar{\pi}]$  in (15),

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<sup>17</sup> We notice that those who work in Skill classes 1 and 2 receive per capita net income below their net labour earnings. This outcome may be the result of normalizing net household income by the square root of household size: lower skills households may include lower numbers of individual contributing with earnings towards total net household income.

where  $\underline{\pi} > 0$ ,  $\underline{\pi} < \bar{\pi}$ ,  $\bar{\pi} < \infty$  leads to the maximum likelihood estimates  $\hat{\gamma}_b$  and  $\hat{\gamma}_a$ .<sup>18</sup> The vectors  $\hat{\gamma}_b$  and  $\hat{\gamma}_a$  (either at  $t$  or  $t+1$ ) are associated with respective values of the likelihood function. Allowing for all possible combinations of  $[\underline{\pi}, \bar{\pi}]$  results in two vectors of estimated values for the likelihood functions, one for each of the conditional outcomes  $P(I_t^w = 1 | \cdot)$  and  $P(I_t^n = 1 | \cdot)$ ; each vector has a dimension equal to the number of possible  $[\underline{\pi}, \bar{\pi}]$  combinations. For each of the outcomes  $P(I_t^w = 1 | \cdot)$  and  $P(I_t^n = 1 | \cdot)$  we chose the combined pair  $[\underline{\pi}^*, \bar{\pi}^*]$  implying the highest value of the likelihood function: but the true and unknown  $[\underline{\pi}, \bar{\pi}]$  values are the same for all in the population (see Assumption 1, Lemma 1 and Assumption 12). Thus, we choose the truncation points such that  $[\underline{\pi}^{**}, \bar{\pi}^{**}] = \left\{ \min(\underline{\pi}_n^*, \underline{\pi}_w^*), \max(\bar{\pi}_n^*, \bar{\pi}_w^*) \right\}$ . This procedure results in values  $\underline{\pi}^{**} = 0.000001$  and  $\bar{\pi}^{**} = 23.8 (0.15)$ , where the bracketed number is a standard error.<sup>19</sup>

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<sup>18</sup> With two time periods, the full set of results implies four vectors, namely,  $(\hat{\gamma}_{b,t}, \hat{\gamma}_{b,t+1})$  and  $(\hat{\gamma}_{a,t}, \hat{\gamma}_{a,t+1})$ . The reason why these are not the same for distinct time periods is because of potential regime changes of the population between periods  $t$  and  $t+1$ , a change that is reflected in the sample distribution shown in Table 1.

<sup>19</sup> We grid-search in a selected region starting with combinations in the two dimension region defined such that  $\underline{\pi} \in (0.00001, 30]$  and  $\bar{\pi} \in (0.001, 30]$  if the condition  $\underline{\pi} < \bar{\pi}$  applies, i.e., we take combinations in the lower triangular part of the two dimensional grid. Assuming initial equidistant increments of 0.5 our search starts with 1,740 combinations. We choose the value 30 because  $\Phi(\ln 30)$  is sufficiently close to one. Using the two distinct time periods as defined in Table 1, our initial search determines that  $[\underline{\pi}^{**}, \bar{\pi}^{**}]$  can be approximated with the values  $[0.000001, 24.0]$ , since  $[\underline{\pi}^{**}, \bar{\pi}^{**}]_{t \text{ or } t+1}^w = [0.000001; 3.0]$  and  $[\underline{\pi}^{**}, \bar{\pi}^{**}]_{t \text{ or } t+1}^n = [0.03; 23.0]$ . From this initial estimate we fix the lower limit to be 0.000001 (thus there is no standard error associated with this value) and let the value for  $\bar{\pi}$  vary from 1 to 25, this time in steps of 0.2 to end up with a choice for 23.8. This last grid-search procedure is performed using only the individuals in the sub-groups  $I_t^n = 1$  and  $I_{t+1}^n = 1$ , since these are the sub-groups providing the highest value of the upper bound in the  $[\underline{\pi}^{**}, \bar{\pi}^{**}] = \left\{ \min(\underline{\pi}_n^*, \underline{\pi}_w^*), \max(\bar{\pi}_n^*, \bar{\pi}_w^*) \right\}$  step. The standard error for the upper bounds is based on a naïve bootstrap procedure with 500 replications that re-samples with replacement from the original data. This bootstrap procedure is part of the overall bootstrap procedure that aims at eliciting the final parameters.

The upper and lower limits  $[\underline{\pi}^{**}, \bar{\pi}^{**}]$  enter expression (15) as limiting values in the truncated likelihood functions leading to estimates of the vectors  $\hat{\gamma}_{b,t}, \hat{\gamma}_{b,t+1}$  and  $\hat{\gamma}_{a,t}, \hat{\gamma}_{a,t+1}$ , where the  $K_t^w, K_{t+1}^w$  and  $K_t^n, K_{t+1}^n$  are the covariates underlining the respective probit specifications (see Table 3). The complete results of these probit can be found in the appendix. Table 6 shows means and variance for the resulting projections of the time dependent reservation values.

*Table 6: Summary statistics for the projected reservation values*

	WORKING		UNEMPLOYED	
	Reservation value for employed at time $t$ : $\hat{\pi}_{t,b}^R$	Reservation value for employed at time $t+1$ : $\hat{\pi}_{t+1,b}^R$	Reservation value for unemployed at time $t$ : $\hat{\pi}_{t,a}^R$	Reservation value for unemployed at time $t+1$ : $\hat{\pi}_{t+1,a}^R$
Full Population at $t$ or at $t+1$	mean / std = .19 / .24 min / max = .02 / 8.6	mean / std = .17 / .16 min / max = .022 / 5.5	mean / std = .15 / .41 min / max = .010 / 17	mean / std = .13 / .28 min / max = .013 / 6.6
$I_t^w = 1, I_{t+1}^w = 1$ or $I_t^n = 1, I_{t+1}^n = 1$	mean / std = .16 / .12 min / max = .02 / 2.4	mean / std = .16 / .11 min / max = .02 / 2.7	mean / std = 1.0 / 1.3 min / max = .02 / 7.6	mean / std = .89 / 1.0 min / max = .043 / 6.7

Note: See note on previous tables.

The reservation values  $\pi_{t,a}^R, \pi_{t,b}^R, \pi_{t+1,a}^R, \pi_{t+1,b}^R$  indicate individual's preferences with regards to alternative labour market regimes. But ' $\pi_{t,a}^R, \pi_{t,b}^R, \pi_{t+1,a}^R, \pi_{t+1,b}^R$ ' are only meaningful in relative terms. For example, we can compare the relative value of the option 'work' for the subgroup  $I_t^w = 1, I_{t+1}^w = 1$  between periods  $t$  and  $t+1$ : The second row in Table 6 shows that relative to the first period, on average, the mean reservation value that describes the sub-groups reservation threshold has decreased from 0.163 to 0.158 thus, working at  $t$  increases both assets and human capital and working tomorrow becomes less risky, thus the reduced value in the reservation policy. These estimates are consistent with Assumption 6 and the conditions for Lemma 2.

Individual's reservation values (i.e.,  $\hat{\pi}_{it,b}^R, \hat{\pi}_{it+1,b}^R$  and  $\hat{\pi}_{it,a}^R, \hat{\pi}_{it+1,a}^R$ ) joint with  $\underline{\pi}^{**} = 0.000001$  and  $\bar{\pi}^{**} = 23.8$  identify the intervals in (10) for each unit in Table 1. Thus, applying Assumptions 11 and 12 joint with expression (15) have elicited the necessary information to determine



intervals  $\hat{\pi}_{it,b}^R \leq \pi_{it} \leq \bar{\pi}^{**}$  and  $\hat{\pi}_{it+1,b}^R \leq \pi_{it} \leq \bar{\pi}^{**}$  for the employed, and intervals  $\underline{\pi}^{**} \leq \pi_{it} \leq \hat{\pi}_{it,a}^R$  and  $\underline{\pi}^{**} \leq \pi_{it} \leq \hat{\pi}_{it+1,a}^R$  for the unemployed without ALMP. We approximate each individual's idiosyncratic shock  $\pi_{it}$  by  $\tilde{\pi}_{it}$ , the midpoint value for each of the constructed intervals. Table 7 shows the resulting means and variances.

Now we estimate the parameters  $\nu_t$  and  $\sigma_t$  from expression (9) using within skill class sub-cells determined by the information in Table 4 (i.e., variables in  $Z$ -vectors described in Section 5). Although defining cells is subjective, our definitions group individuals with similar potential for human capital changes (growth and depreciation). Table 8 shows the estimates of human capital growth  $\nu_t$  based on sub-cells by skills in the sub-group  $I_t^w = 1, I_{t+1}^w = 1$ . Table 9 shows estimates of human capital depreciation  $\sigma_t$  based on sub-cells defined by the sub-sample  $I_t^n = 1, I_{t+1}^n = 1$ . Each cell estimate is based on the sample analogue for respective expressions in (9), where  $\hat{\nu}(s | \Xi_{Et}) = (1/N) \left[ \sum_{i=1}^{N \in w} \Delta \ln E_{it,t+1} - \sum_{i=1}^{N \in w} \Delta \ln \tilde{W}_{s,t,t+1} - \sum_{i=1}^{N \in w} \Delta \ln \tilde{\pi}_{it,t+1} \right] / (1/N) \sum_{i=1}^{N \in w} \tilde{\pi}_{it}$ , while  $\hat{\sigma}(s | \Xi_{\Gamma}, t) = -(1/N) \left[ \sum_{i=1}^{N \in n} \Delta \ln \Gamma_{it,t+1} - \sum_{i=1}^{N \in n} \Delta \ln B_{s,t,t+1} \right] / (1/N) \sum_{i=1}^{N \in n} \tilde{\pi}_{it}$  estimate human capital growth and depreciation, respectively. In both cases  $\Xi_{type}$  explains cell-division by  $Z_{type}$  for given time period  $t$  and skill class  $s$ .

Table 7: Means and variances for the projected reservation shocks ( $\pi_t, \pi_{t+1}$ )

	Mean value of the approximation for the stochastic shock at time $t$			Mean value of the approximation for the stochastic shock at time $t + 1$		
	$I_t^w = 1$	$I_t^q = 1$	$I_t^n = 1$	$I_{t+1}^w = 1$	$I_{t+1}^q = 1$	$I_{t+1}^n = 1$
Full sample	12.00 (.066)	.500 (.76)	.83 (.96)	11.98 (.058)	.49 (.60)	.80 (.70)
Skill Class 1	12.03 (.099)	.821 (.90)	.97 (.90)	12.02 (.087)	.55 (.62)	.73 (.60)
Skill Class 2	11.97 (.061)	.510 (.76)	.76 (.89)	11.97 (.052)	.45 (.70)	.60 (.65)
Skill Class 3	11.98 (.060)	.550 (.85)	1.10 (.95)	11.98 (.052)	.45 (.60)	.73 (.54)
$I_t^w = 1, I_{t+1}^w = 1$		12.02 (.059)			00.00 (.000)	
$I_t^n = 1, I_{t+1}^n = 1$		1.21 (.91)			.00 (.00)	

Note: Individual specific stochastic shocks for the employed and the unemployed (without ALMP) are approximated by the midpoints of the intervals. Approximating to the shock for the sub-sample  $I_t^q = 1$  is based on the midpoint of the interval  $\hat{\pi}_{t,a}^R < \pi_t < \hat{\pi}_{t,b}^R$ . Standard errors are reported in brackets.

Table 8 presents estimates based on the number of units as given in the first row for each of the cells. They help to understand how human capital grows. The values within each cell are cell-wise estimates for  $\nu$ , based on the gradient in the relation  $\ln(h_{it+1}/h_{it}) = \nu\pi_{it}$ , that is,  $\ln(h_{t+1}/h_t) = \nu\tilde{\pi}_{t,\Xi_E}$  or  $(h_{t+1}/h_t) = \exp(\nu\tilde{\pi}_{t,\Xi_E})$ . According to the structural model  $\nu$  should be positive. Table 8 shows although some negative values occur they are extremely small and not statistically significant.<sup>20</sup> Indeed, Table 8 shows that despite the positive sign for most cells, only few estimates are significantly different than zero. In terms of magnitudes, the values suggest that while working, growth rates are larger during earlier stages in the lifecycle.

<sup>20</sup> Significance is based on the already mentioned bootstrap procedure. The actual estimates for  $\nu$  are based on applying expressions (9) and having fixed the lower limit to 0.000001 and the upper limit to 23.8. For sampling variance we still fix the lower interval to 0.000001 and allow for the upper limit to be determined freely based on the sample selected with replacement (always using an interval close to 23.8 and in steps of 0.2 as previously described). The bootstrap estimate for the upper limit is used to estimate the probit outcomes as defined in (15) where these are also estimated allowing for the bootstrap sample to obtain the reservation values, and with this the intervals in (10). The bootstrap sample for the parameters  $\nu$  (and likewise  $\sigma$ ) follow straightforwardly from (9). At each step intermediate values from bootstraps re-samples are kept and these are later used to estimate standard errors given in Tables 6 to 9 (some cells are too small in size to be used for inference).

Table 8: Estimates of  $v$  by cells with common human capital stocks and skill class

	Experience in the labour market	Age below 21	Age between 21 -- 25	Age between 26 -- 30	Age between 31 – 40	Age between 41 – 50	Age above 50
Lowest Skill Class	6 months or less	N/A	N/A	N/A	N/A	23 -0.003 (.001)	24 .0011 (.0009)
						12.03	12.07
	7 – 12 months	N/A	N/A	N/A	12 .0002 (.0002)	13 .0016 (.0012)	6 -0.001 (.0005)
					12.00	12.00	12.06
$s = 1$	13 – 24 month	19 .042 (.036)	11 .0051 (.0042)	11 .0015 (.0016)	22 .0003 (.0003)	16 .0006 (.0004)	14 .0173 (.0134)
		12.3	12.1	12.02	12.00	12.00	12.01
	greater than 24 months	36 .004 (.001)	157 .0001 (.001)	342 -0.001 (.001)	1,399 .0020 (.001)	1,313 .001 (.001)	1,521 .001 (.001)
		12.30	12.20	12.1	12.00	12.0	12.04
Next lowest skill class	6 months or less	N/A	15 .001 (.0006)	28 .0032 (.0030)	39 .0010 (.0011)	41 .0007 (.0010)	33 .0004 (.0008)
			12.20	12.10	12.00	12.00	12.10
	between 7 – 12 months	N/A	36 .0015 (.0005)	31 .0053 (.0037)	34 .0058 (.0030)	29 .0014 (.0014)	17 -0.004 (.0011)
			12.10	12.00	12.00	12.00	12.00
$s = 2$	between 13 – 24 month	N/A	79 .0000 (.0002)	50 .0000 (.0001)	78 .0030 (.0020)	35 .0013 (.0010)	36 .0028 (.0027)
			12.03	12.00	12.00	12.00	12.00
	greater than 24 months	174 .0021 (.0006)	1,548 -0.009 (.0008)	3,014 -0.008 (.0005)	7,714 .00011 (.00037)	6,208 -0.0001 (.0004)	5,493 .0011 (.0007)
		12.00	12.05	12.00	12.00	12.00	12.00

Table 8 to be continued

Table 8 continued...

	Experience in the labour market	Age below 21	Age between 21 -- 25	Age between 26 -- 30	Age between 31 – 40	Age between 41 – 50	Age above 50
Medium (semi-skill) class	6 months or less	N/A	N/A	17 <b>.0020</b> (.0011)	28 .0300 (.0266)	20 .0007 (.0020)	15 .0004 (.0004)
$s = 3$	Between 7 – 12 months	N/A	19 .0022 (.0019)	23 -.0264 (.0200)	25 <b>.0013</b> (.0007)	15 <b>.0034</b> (.0001)	N/A
	between 13 – 24 month	N/A	50 .0010 (.0005)	61 .0002 (.0004)	44 -.0100 (.0079)	24 .0001 (.0009)	13 .00013 (.0009)
	greater than 24 months	18 .00000 (.0003)	330 .0063 (.0044)	1,387 .0009 (.0007)	4,349 .0006 (.0004)	3,461 .0003 (.0005)	2,763 .0004 (.0005)
		12.00	12.10	12.01	12.00	12.00	12.00

Note: Estimates in classes with less than 10 observations are not shown. The first row in each cell shows the units in the cell. The second row in any cell shows the estimate of  $\nu$  for that cell with bootstrap standard errors in brackets. The last row within each cell is the mean value of the shock per cell (i.e., the mean  $\bar{\pi}_t$  within cell). Values significant at the 5% level are **bold**.

For example, in skill class 3 human capital grows at 0.2% for low experience (less than 6 months) and age bracket 26-30. Moving to more than 6 months of experience and older workers (31-40) human capital grows at a rate 0.13%. Once we move to older ages and more experience (i.e., one to two years or more than two years, and ages 41 to 50 or more than 50) the magnitudes of human capital growth (0.01% and 0.04%) suggest that individuals are already at the flat section of their concave human capital curve.

Table 9 shows the estimates for the human capital depreciation rates. We display these values such that a positive value implies a negative estimate. The partition within cells is based on experience and unemployment duration at the present spell of unemployment. The partitions displayed in Table 9 are the finest partitions possible; allowing more sub-cells would lower the size of the units within each cell considerably while leaving other cells empty. As with rates of growth, we interpret these parameters as the gradient in a function explaining the human capital

change (depreciation rates) between consecutive periods such that  $(h_{t+1}/h_t) = \exp(-\sigma\tilde{\tau}_{t,\Xi})$ . Most of the cells provide estimates consistent with our model, while estimates with negative signs (there are only 3 such values) are not statistically different from zero.

Table 9: Estimates of  $\sigma$  by cells with common human capital stocks and skill class

	Unemployment duration in months	Experience in the labour market below or equal to 2 years	Experience in the labour market greater than 2 years	
Lowest Skill Class	below 6 months	98	13	
		<b>.168</b> (.073)	.094 (.052)	
	$s = 1$	between 7 – 18 months	.721	.912
			28	30
			<b>.092</b> (.047)	.048 (.041)
between 19 – 24 months	2.11	1.64		
greater than 2 years	N/A	N/A		
		41	28	
		.42 (.065)	.38 (.206)	
		.158	.129	
Next lowest skill class	below 6 months	19	34	
		-0.046 (.313)	.128 (.206)	
	$s = 2$	between 7 – 18 months	.260	.359
			11	46
			<b>.464</b> (.218)	.184 (.128)
between 19 – 24 months	.800	.620		
greater than 2 years	N/A	N/A		
		52	24	
		<b>1.59</b> (.875)	.605 (.91)	
		.074	.058	
Medium (semi-skill) class	below 6 months	68	26	
		<b>.53</b> (.264)	.19 (.22)	
	$s = 3$	between 7 – 18 months	.34	.57
			18	24
			-0.047 (.041)	-0.043 (.070)
between 19 – 24 months	.89	.641		
greater than 2 years	N/A	N/A		
		28	11	
		<b>2.66</b> (1.2)	1.1 (2.03)	
		.080	.084	

Note: Estimates in classes with less than 10 observations are not shown. The first row in each cell is the number of units in the cell; the second row shows the estimated human capital parameter  $\sigma$  and its standard error in brackets. The final row in each of the cells is the average of the projected stochastic shock in each cell. All estimates are based on the sample observed consecutively unemployed without ALMP over time. Figures in bold show estimates of  $\sigma$  that are significantly different from zero at least at the 10% level.

Table 9 shows that ‘more experience’ decreases the rate at which human capital depreciates for similar unemployment durations. For example, skill class 2 individuals who have been unemployed for a period lasting between 7 and 18 months with labour market experience for at

most 2 years experience depreciating human capital at the rate of 46.4% between two consecutive years, while similar individuals with a longer labour market experience (more than 2 years) experience a human capital depreciation at the rate of 18.4% (between 2 consecutive years). Table 8 shows that comparing less to more experience for similar unemployment spells implies the same monotonic condition as described in our example, thus suggesting that entering unemployment with more experience reduces the rate of human capital depreciation relative to low labour market experience. However, looking at individuals with similar experience in the labour market but increasing unemployment spells (between rows within column and skill class) does not suggest a monotonic gradient for relative human capital changes over consecutive periods.

Overall, Table 9 provides weak evidence for our predictions of human capital depreciation in the event of unemployment without ALMP. The signs are correct as required (at least not significantly different than zero if otherwise), but the estimated magnitudes are only weakly significant. At best our estimates suggest that in the Swiss economy the data cannot reject the potential of human capital depreciation in the event of unemployment without ALMP.

## **7 Conclusions**

The paper develops a structural framework to theoretically and empirically analyse endogenous human capital formation in the presence of three distinct labour market regimes: employment, unemployment sheltered by passive and active labour market policies, and unemployment without sheltering, i.e., limited or no access to labour market policies like counselling or training programmes. The three regimes characterize the actual dynamics in the labour markets of Western Economies in a stylised way, while the structural model reflects behavioural dynamics with respect to the evolution of assets and human capital formation. A

crucial assumption in the model is that stochastically changing labour market conditions are accounted for by individuals before these make a labour decision. The presence of ALMP can act as a buffer against bad stochastic shocks for those whose preferences, characteristics and labour market histories imply a choice of participation in ALMP. The theoretical model suggest that such buffer may translate into non-depreciating human capital in the presence of ALMP while not participating in ALMP may lead to a period of human capital depreciation.

The implications from the theoretical model can be directly tested. We do this by applying a labour market panel data representative of the Swiss population (the SAKE or Swiss Labour Force Survey, 1991-2004). Our data is ideal for that purpose because it allow to clearly distinguishing between individuals according to their labour market regime while providing sufficient information on individual's labour market history as well as a battery of key socio-economic indicators. Our estimates suggest that human capital depreciates while unemployed without ALMP. No such losses can be detected for unemployed receiving the services of the unemployment insurance system. On the other hand, the econometric estimates suggest a clear effect of working on learning capacity. In particular, younger cohorts learn faster than older cohorts. Overall, our estimates suggest that working and likewise participating in ALMP while unemployed does not reduce potential gains from future engagements in the labour market and may in fact increment the gains from future active participation.

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## Appendix 1

### A1.1 Proof of Lemma 1

Suppose that for a given compact space  $X_t$  for some agent  $i$  (this index will be suppressed in this section) at time  $t$  employment is the preferred labour market regime for some value  $\pi = \pi'$ .<sup>21</sup> This particular choice of the agent implies the following:

$$V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w \neq 1) \quad (\text{L.1})$$

Since a larger value of the shock strictly increases future human capital while working (something that does not happen in the other states) and in turn this (strictly) increases future earnings and thus future consumption possibilities, and because the period's returns from wages increase as well, for any larger value of the shock ( $\pi'' \geq \pi'$ ), the person works as well:

$$\begin{aligned} & V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w \neq 1) \\ \Rightarrow & \\ & V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w \neq 1) \end{aligned} \quad (\text{L.2})$$

This establishes that there is a value of  $\pi$ , say  $\pi'$ , beyond which the agent will always choose employment ( $w$ ) among all other labour market options. But then there is a range of values in the distribution of  $\pi$  below which contemporaneous and future earnings from employment are so low that the agent's optimal choice would be non-employment. Say this happens at  $\pi = \pi^*$ . Then for any lower value ( $\pi^{**}, \pi^{**} < \pi^*$ ), the individual won't work either, because when the value of the shock declines employment becomes less attractive compared to the non-employment options. Thus, a threshold  $\pi_b^R$  defined in terms of  $X_{vt}$  exists that completely

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<sup>21</sup> The first part of this proof is similar to Costa-Dias (2002), but allows for a third labour market regime. The second part of the proof refers to the third regime explicitly.

characterizes the decision between choosing employment or not. The threshold  $\pi_b^R$  depends on assets and human capital accumulated so far as well as on state of nature (i.e. the realisations of the shock), and determines the circumstances upon which the agent is willing to work.

For the case  $\pi \leq \pi_b^R$ , it remains to analyse the choice between the two non-employment alternatives. From the financial capital accumulation equation we see that the shock does not influence current period physical returns for the non-employment states. If there would be no effect of the shock on human capital accumulation, then individuals would all choose state  $I_{it}^n = 1$ . However, the larger shock, the less attractive alternative ‘ $n$ ’ becomes in terms of human capital, because the depreciation is increasing in the shock. Suppose there is a value  $\pi_a^R$  ( $\pi_a^R \leq \pi_b^R$ ) such that individuals are just indifferent between  $q$  and  $n$ . Because of Assumption 8 (i.e., positive prices including  $P_{st} > 0$ ), if  $\pi$  decreases below  $\pi_a^R$  alternative  $n$  become more valuable, i.e., *further loss of human capital declines* such that below  $\pi_a^R$ :  $h_{it} | I_{it}^n \rightarrow h_{it} | I_{it}^q$  and  $P_{st} | I_{it}^n = 0$ . If the shock increases above  $\pi_a^R$ , the alternative ‘ $q$ ’ gains in value. Thus the monotone reservation policy is proved.

### **Proof of Lemma 2**

This proof extends that in Lemma 2 Costa-Dias (2002) to cover a third labour market regime. In both cases the proof uses backward induction starting with the valued function at age  $T$  and showing similar properties for ages 0 to  $T - 1$  (the index  $i$  is suppressed for simplicity, so that for any  $i$ ,  $W_{s(i)t} = W_{st}$ , etc.)

**At age  $T$**  the agent maximizes the contemporaneous utility only as function of consumption such that  $c_T^* = (1 + r_T)a_T + I_T^w \pi_T h_T W_{sT} + I_T^q (B_{sT} - P_{sT}) + I_T^n B_{sT}$ ; the agent decides to work or not according to the realization of  $\pi_T$  conditional on past labour market history and

characteristics. Whatever labour market regime the agent decides to select,  $E_\pi V_{T+1}^s(\cdot) = 0$  and each of the (partitioned) value functions are characterized by the utility of final time period resources:

$$\begin{aligned}
V_T^s(a_T, h_T, \pi_T) &= u\left((1+r_t)a_T + \pi_T h_T W_{sT}(1-\tau_T)\right) & \text{if } I_T^w = 1; \\
V_T^s(a_T, h_T, \pi_T) &= u\left((1+r_t)a_T + (B_{sT} - P_{sT})h_T\right) & \text{if } I_T^q = 1; \\
V_T^s(a_T, h_T, \pi_T) &= u\left((1+r_t)a_T + B_{sT}h_T\right) & \text{if } I_T^n = 1.
\end{aligned} \tag{L.4}$$

Allow for Assumption 2 at age  $T$ : the same properties for the utility function carry through for the value function for all the three labour market regimes. Allow for Assumptions in 3.3 and use the conditions in Lemma 1. Let  $V_T^s(\cdot | I_T^j = 1)$  be the short hand notation of the conditional (on  $j = w, n, q$ ) value function:

$$\begin{aligned}
E_\pi V_T^s(a_T, h_T) &= V_T^s(\cdot | I_T^w = 1)P(I_T^w = 1) + V_T^s(\cdot | I_T^q = 1)P(I_T^q = 1) + V_T^s(\cdot | I_T^n = 1)P(I_T^n = 1) = \\
&= \int_{\underline{\pi}}^{\pi_a^R} V_T^s(a_T, h_T | I_T^w = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\pi_a^R} V_T^s(a_T, h_T | I_T^q = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\underline{\pi}} V_T^s(a_T, h_T | I_T^n = 1) f(\pi) d\pi
\end{aligned} \tag{L.5}$$

But (L.4) implies that  $V_T^s(\cdot | I_T^j = 1)$  is strictly increasing, twice differentiable and concave in assets for any of the  $j \in \{w, q, n\}$  labour market alternative, therefore, so is the expectation  $E_\pi V_T^s(a_T, h_T)$ ; notice that this is also taking into account that at any point in the lifetime of individuals, including at  $T$ , the reservation thresholds depend on past information and not in the present levels of assets (as determined in Lemma 1).

**At ages 0 to  $T-1$ :** The proof has four steps (following Costa-Dias (2002) and adapting Stokey and Lucas (1989) to be applicable to any number of labour market regimes)

Let  $E_\pi V_{i+j}^s(a, h) = E_\pi V_T^s(a, h)$ : The previous step shows that given Lemma 2, the RHS is strictly increasing, twice differentiable and a concave function in assets  $(a_t)$ .

Step 1: We show that the conditional value functions  $V_t^s(\cdot | I_t^j = 1)$  are increasing, twice differentiable and concave in (physical) assets. Given that  $u(c_{t+j})$  is concave (Assumption 2) and  $E_\pi V_{i+j}^s(\cdot | \cdot)$  are strictly increasing, concave and twice differentiable in  $c_{t+j}$  and  $a_{t+j}$ , standard recursive methods show that for bounded objective functions,  $V_{t+j-1}^s(\cdot | \cdot)$  has identical properties that  $E_\pi V_{i+j}^s(\cdot | \cdot)$ .

The proof can be found in Stokey and Lucas (1989), Chapter 9, page 261. Furthermore, take expectations on  $V_{t+j-1}^s(\cdot | \cdot)$  over the support so that we define  $E_\pi V_{t+j-1}^s(\cdot | \cdot)$ . The latter could be represented as  $E_\pi V_{t+1}^s(\cdot | \cdot)$  for any  $t$  in the working life of an individual. Then, the same standard recursive methods in Stokey and Lucas (1989) imply that with  $u(c_{t+1})$  and  $E_\pi V_{t+1}^s(\cdot | \cdot)$  strictly increasing, twice differentiable and concave in  $c_{t+1}$  and  $a_{t+1}$ , respectively, the value function  $V_t^s(k, h, \pi | \cdot)$  is strictly increasing, twice differentiable and a concave function in assets  $(a_t)$ .

Step 2: We show that the reservation value  $\pi_b^R$  for the labour market shock  $\pi_t$  is continuous in assets  $(a_t)$ . The monotonic relation between  $\pi_a^R$  and  $\pi_b^R$  implies that both reservation values are continuous and differentiable (at least once) in assets  $(a_t)$ .

The reservation values  $\pi_a^R$  and  $\pi_b^R$  both solve the equalities between the three value-functions determined by the three labour market choices. Furthermore, Step 1 implies the continuous differentiability (with respect to assets) of the value functions for any given labour market regime. Since assets are an increasing, continuous and differentiable function of human capital  $h_{vt}$ , the value functions are also strictly increasing, twice differentiable, concave functions

with respect to human capital. Take, for example, the threshold  $\pi_b^R$ . We know from Lemma 1 that this threshold solves the equality given by  $V_t^s(a, h, \pi_b^R | I_t^w = 1) = V_t^s(a, h, \pi_b^R | I_t^q = 1)$ , where the latter is a function of the same arguments in the neighbourhood of  $\pi_b^R$ . All the above implies the following:

- (a) The partial derivatives  $V_h(|I)$ ,  $V_a(|I)$ , and  $V_\pi(|I)$  exist. That is, Assumption 1 and Step 1 guarantee the existence of these partial derivatives for any labour market option (notice that for  $V_h(|I) = V_a \cdot (\partial a / \partial h)$  so that the existence of the partial derivative with respect to human capital is also guaranteed.)
- (b) Suppose we can define a point  $(a^R, h^R, \pi_b^R)$ . From Lemma 1 we know that  $\pi_b^R$  solves the equality  $V_t^s(a, h, \pi_b^R | I_t^w = 1) = V_t^s(a, h, \pi_b^R | I_t^q = 1)$ , therefore, this must also happen so that  $V_t^s(a^R, h^R, \pi_b^R | I_t^w = 1) = V_t^s(a^R, h^R, \pi_b^R | I_t^q = 1)$ . That is, at this point the equality is also true. Since the value function is continuous and differentiable over the support of  $\pi$ , and  $\pi_b^R$  is in the support  $[\underline{\pi}, \bar{\pi}]$ , then the derivative  $\frac{\partial V(a^R, h^R, \pi_b^R | I)}{\partial \pi} \neq 0$  in the neighbourhood of that point.

The Implicit Function Theorem says that if a function  $V(a, h, \pi): D^n \rightarrow \mathbb{R}^m, m < n$ , complies with conditions (a) and (b), then, there exists a function  $g(h, a)$  such that  $V_t^s(a^R, h^R, g(a, h) | I_t^w = 1) = V_t^s(a^R, h^R, g(a, h) | I_t^q = 1)$  in the neighbourhood of  $(a^R, h^R, \pi_b^R)$ . This function has an implicit representation, say  $\pi^R = g(a, h)$ , satisfies  $\pi_b^R = g(a^R, h^R)$ , and is continuous and at least once differentiable in its arguments. Notice also that in our model  $a = a(h)$ , and not the other way around. Assume both  $(a, h)$  follow monotonically the same direction as is the case for fixed labour market regimes. Stokey and Lucas (1989, page 290) show that the model can be reformulated in terms of only one endogenous variable with the recursive

solution applying identically to the reformulated problem. Thus, we can let  $\pi_b^R = \pi_b^R(a)$ . The one-to-one mapping is guaranteed.

The same argument can be applied to the reservation value  $\pi_a^R$  that solves for the equality between the value functions  $V_t^s(a, h, \pi_a^R | I_t^q = 1) = V_t^s(a, h, \pi_a^R | I_t^n = 1)$ . In both cases we have shown that Assumptions 1 and Step 1 allow for the application of the Implicit Function Theorem, and this ensures that both reservation policies are continuous differentiable functions (at least once) of assets ( $a_t$ ). This is to be used in further steps.

*Step 3:* Allowing for Assumption 1 and the interpretation of the reservation policies in Lemma 1, the expected value function at time  $t$  can be written as follows:

$$E_\pi V_t^s(a_t, h_t) = \int_{\underline{\pi}}^{\pi_a^R} V_t^s(a_t, h_t | I_t^w = 1) f(\pi) d\pi + \int_{\pi_a^R}^{\pi_b^R} V_t^s(a_t, h_t | I_t^q = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\bar{\pi}} V_t^s(a_t, h_t | I_t^n = 1) f(\pi) d\pi \quad (\text{L.6})$$

Step 1 determines that  $V_t^s(\cdot | I_t^j = 1)$  is strictly increasing, twice differentiable and concave in physical assets for all three labour market regimes. Step 2 determines that the reservation policies are continuous differential functions of assets, and the differentiability of the joint density function of the productivity shocks is also guaranteed in Assumption 1. Therefore,  $E_\pi V_t^s(a_t, h_t)$  is also twice differentiable with respect to assets  $a_t$ . This is a necessary condition for Step 4 below.

*Step 4:* We show that the value function  $E_\pi V_t^s(a_t, h_t)$  is an increasing and concave function of assets  $a_t$ .

Step 3 allows for the following representation for the first derivative of  $E_\pi V_t^s(a_t, h_t)$ :

$$\begin{aligned}
\frac{\partial EV_t^s(a, h)}{\partial a_t} &= \int_{\underline{\pi}}^{\pi_a^R} \frac{\partial(V_t^s(\cdot | I^n = 1))}{\partial a_t} dF(\pi) + \int_{\pi_a^R}^{\pi_b^R} \frac{\partial(V_t^s(\cdot | I^q = 1))}{\partial a_t} dF(\pi) + \int_{\pi_b^R}^{\bar{\pi}} \frac{\partial(V_t^s(\cdot | I^w = 1))}{\partial a_t} dF(\pi) \\
&+ \frac{\partial \pi_b^R}{\partial a} \left\{ (V_t^s(\cdot | I^q = 1) - V_t^s(\cdot | I^w = 1)) \right\} dF(\pi_{b,t}^R) + \\
&+ \frac{\partial \pi_a^R}{\partial a} \left\{ (V_t^s(\cdot | I^n = 1) - V_t^s(\cdot | I^q = 1)) \right\} dF(\pi_{a,t}^R). \tag{L.7}
\end{aligned}$$

The last two terms in the RHS vanish at the reservation value in the density function of  $\pi$  (the value functions are identical), while the first derivatives with respect to assets are all positive since Step 1 ensures that the conditional value function is strictly increasing. Therefore it follows that  $(\partial EV_t^s(a, h)/\partial a_t) > 0$ . All what is needed for concavity is to show that  $(\partial^2 EV_t^s(a, h)/\partial^2 a_t) < 0$ . From (L.7), the second order derivative is given by:

$$\begin{aligned}
\frac{\partial^2 EV_t^s(a, h)}{\partial^2 a_t} &= \int_{\underline{\pi}}^{\pi_a^R} \frac{\partial^2(V_t^s(\cdot | I^n = 1))}{\partial^2 a_t} dF(\pi) + \int_{\pi_a^R}^{\pi_b^R} \frac{\partial^2(V_t^s(\cdot | I^q = 1))}{\partial^2 a_t} dF(\pi) + \int_{\pi_b^R}^{\bar{\pi}} \frac{\partial^2(V_t^s(\cdot | I^w = 1))}{\partial^2 a_t} dF(\pi) \\
&+ \frac{\partial \pi_b^R}{\partial a} \left\{ \left( \frac{\partial(V_t^s(\cdot | I^q = 1))}{\partial a_t} - \frac{\partial(V_t^s(\cdot | I^w = 1))}{\partial a_t} \right) \right\} dF(\pi_{b,t}^R) + \\
&+ \frac{\partial \pi_a^R}{\partial a} \left\{ \left( \frac{\partial(V_t^s(\cdot | I^n = 1))}{\partial a_t} - \frac{\partial(V_t^s(\cdot | I^q = 1))}{\partial a_t} \right) \right\} dF(\pi_{a,t}^R). \tag{L.8}
\end{aligned}$$

The first three terms in the RHS of (L.8) are negative because of the concavity of the conditional value functions. But the value of the last two terms in (L.8) depend on the relative degree of concavity between paired labour market regimes (i.e. between  $I_t^q$  and  $I_t^w$ , and between  $I_t^n$  and  $I_t^q$ ), and the degree of absolute risk aversion (given by the derivatives  $\left( \frac{\partial \pi_b^R}{\partial a} \right)$  and



$\left(\frac{\partial \pi_a^R}{\partial a}\right)$ .<sup>22</sup> Assumption 4 states that individuals are risk averse in the sense that an increase in assets reduces the reservation policy (subjective valuation of labour market choice) thus making employment more likely than non-employment in the future for any random shock. Likewise, an increase in assets as result of non-decreased in human capital (rather than depreciation) implies that program participation becomes more likely than ‘unemployment without program participation’, also for any given random productivity shock. Therefore,  $\left(\frac{\partial \pi_b^R}{\partial a}\right) < 0$  and  $\left(\frac{\partial \pi_a^R}{\partial a}\right) < 0$  are implied by Assumption 4 as well as being consistent with our model (see introductory notes). But, if an increase in physical assets implies reducing the respective reservation policies through an increase in the willingness to take risk the implication is that for any given assets level,  $a_t$ , comparing the value functions between labour market regimes implies that  $\left(\frac{\partial V(\cdot | I_t^w = 1)}{\partial a}\right) > \left(\frac{\partial V(\cdot | I_t^q = 1)}{\partial a}\right) > \left(\frac{\partial V(\cdot | I_t^n = 1)}{\partial a}\right)$ .<sup>23</sup> Decreasing absolute risk aversion

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<sup>22</sup> That is, as stated in the introduction, individual’s hold latent valuation on each of the labour market regimes that we define as ‘reservation valuation policy set’. These sets depend on individual’s taste for risk possible determined by individual’s history, characteristics, etc: Lemma 1 embodies this idea. Each time the agent has to evaluate the labour market conditions as the shock is realized, they compare the realized shock  $\pi_t$  to own reservation policy that explains individual’s taste for risk ( $\pi_t^R$ ), and make a labour market choice. Since the risk attitude is given by the set of reservation policies ( $\pi_t^R$ ), risk aversion is measured by the change on this with respect to assets, where assets includes human capital as part of the individuals wealth. This justifies that the derivatives  $\left(\frac{\partial \pi_t^R}{\partial a}\right)$  explain the concept of risk aversion (coefficient of risk aversion).

<sup>23</sup> That is, expected value of a choice is the weighted sum of the three possible choices so that expectations of the value function is  $EV = V(| I = w)P(w) + V(| I = q)P(q) + V(| I = n)P(n)$ , and the choice among the three alternative depends on the realization of the shock that will determine the weight (probability). But independently, each of the value functions is an increasing, twice differentiable and concave function of assets, while the value of the value function for the working choice has to be steeper than for the non-employment alternatives and in turn. At this point is when we need to apply Assumption 5 (no crossing of the value functions).

and derivatives of value functions that are increasing as taste for risk increases implies that the second and third terms in the RHS of (L.8) can be positive and overtake the negative value of the first three terms. Then, concavity of the valued function can only be guaranteed if we assume ‘constant absolute risk aversion’ in which case  $\left(\frac{\partial \pi_b^R}{\partial a}\right) = \left(\frac{\partial \pi_a^R}{\partial a}\right) = 0$ . This would imply that the reservation policies are not responsive to changing wealth that is neither a realistic assumption, nor is it completely consistent with our structural model. Thus, Assumption 4 is required so that ‘decreasing absolute risk aversion’, i.e.,  $\left(\frac{\partial \pi_b^R}{\partial a}\right) < 0$  and  $\left(\frac{\partial \pi_a^R}{\partial a}\right) < 0$ , but by a magnitude that is ‘not too large’ (both values are assumed to be bounded from below in the neighbourhood of zero) guarantees that the positive terms in the last two parts of the RHS in (L.8) never overtake the negative values of the set of second derivatives. This is the only way to guarantee concavity.

### A1.3 Proof of Lemma 3

Given Lemma 2 (i.e., having established the conditions for a well behaved value function), the Euler Equation is the necessary and sufficient condition for the optimal consumption decision ‘for fixed labour market regimes’ (since it is within labour market regimes that the value function is continuous, twice differentiable and concave function of assets). Recall the Euler Equation:

$$\frac{\partial u}{\partial c_t} \Big|_{I(j,t)} = E \left[ \beta(1+r) \frac{\partial u}{\partial c_{t+1}} \Big|_{I(j,t)} \right] \quad \forall j \quad (\text{L.9})$$

But (L.9) gives the optimal intertemporal relation for the choice variable assuming concavity of the value function only with respect to assets, when in reality the problem in (2) implies a more complex set of dynamics in the state space. Then, there must be as many optimal

consumptions paths that are consistent with (L.9) as possible values of  $h_{t+1}$  that are consistent with the assets path ( $a_t$ ) that underlines (L.9). Then identification/characterization of the optimal consumption path is only possible if we find an expression analogous to (L.9) such that the new expression implies restrictions for human capital. Recall Step 1 in the proof of Lemma2. This step states that under the regularity assumptions for  $u(c)$  and  $E_\pi V_{t+1}^s(\cdot)$  in  $c_t$  and  $a_{t+1}$ , standard recursive methods show that  $V_t^s(\cdot | I_t^j = 1)$  has identical properties than  $E_\pi V_{t+1}^s(\cdot)$ . First we apply the envelope theorem to  $V_t^s(\cdot | I_t^j = 1)$  so that at the optimal consumption choice and for fixed labour regime, a change in assets implies zero additions from future changes in the value function:

$$\begin{aligned} \left( \frac{\partial V_t^s(\cdot | I^j)}{\partial a} \right) &= \frac{\partial u}{\partial a} \Big|_{c^*(t), I(j,t)} + \beta E_t \frac{\partial V_{t+1}^s(\cdot | I^j)}{\partial a_{t+1}} \Big|_{c^*(t), I(j,t)} \\ &= \frac{\partial u}{\partial c} \cdot \frac{\partial c}{\partial a} \Big|_{c^*(t), I(j,t)} \quad \text{since } E_t \frac{\partial V_{t+1}^s(\cdot | I^j)}{\partial a_{t+1}} \Big|_{c^*(t), I(j,t)} = 0 \quad (\text{L.10}) \\ &= u'(c_t)(1 + r_t) \end{aligned}$$

Since  $\left( \frac{\partial V_t^s(\cdot | I^j)}{\partial a_t} \right) = (1 + r_t)u'(c_t)$  and  $V_t^s(\cdot | I_t^j = 1)$  has identical properties than  $E_\pi V_{t+1}^s(\cdot)$ , we take expectations so that  $E_t \left( \frac{\partial V_{t+1}^s(\cdot | I^j)}{\partial a_{t+1}} \right) = (1 + r_{t+1})E_t u'(c_{t+1})$ ; the result is labour market regime and skill specific. The result is then applied to the Euler Equation in (L.9):

$$\frac{\partial u(c_t)}{\partial c_t} \Big|_{I(j)} = \beta \cdot E_t \left[ \frac{\partial V_{t+1}^s(a_{t+1}, h_{t+1})}{\partial a_{t+1}} \Big|_{I(j)} \right] \quad (\text{L.11})$$

Expression (L.11) maintains the same properties as the Euler condition in (L.9) but we have now established a relation between current consumption and the other dynamic variable in the system, human capital. We are now closer to identifying the optimal condition for consumption (optimal consumption path) taking into account the full dynamic system. Notice

from the dynamics in (1) that the two endogenous state variables always follow the same direction, while the value function is concave in assets. This means that the derivative in the RHS of (L.11) is positive for any value of  $h_{t+1}$ , with this latter variables also increasing as  $a_{t+1}$  increases. At the same time (L.11) explains that any marginal change in utility today has to be matched by an equal but weighted expected marginal change in tomorrow's utility establishing a precise relation between the concavity of  $u(\cdot)$  and  $EV(\cdot)$  with respect to the variables  $c_t$  and  $a_t$ . From the dynamics in (1) we see that this must imply that we are pinning down the optimal human capital path. That is  $a_{t+1} = (1+r)a_t + INC(h, W, \pi)|_{I(j)} - c_t$ . Then, for fixed working conditions, any increase in assets has to be met by an increase in consumption so that (L.11) is satisfied, and this leaves no room for  $h_{t+1}$  to move other than whatever value satisfies (L.11). In other words, (L.11) can be re-written as:

$$\left. \frac{\partial u(c_t)}{\partial c_t} \right|_{I(j)} = \beta \cdot E_t \left[ \left. \frac{\partial V_{t+1}^s(a_{t+1}, h_{t+1} | h_{t+1})}{\partial a_{t+1}} \right|_{I(j)} \right] \quad (L.12)$$

Then, given the properties of the value function, the values of the state variables and for fixed skills and working decisions, the optimal condition for consumption is given by (L.12). With this (allowing for all regularity conditions and assumptions above), the problem in (2) has a unique solution 'for fixed labour market regimes' and for given skill type. In the development of (L.12) we have seen that agents are restricted to be risk averse. Expression (L.12) places further restriction in the variables that determine the behaviour of individuals: consumption ( $c_t$ ) and savings ( $a_{t+1}$ ) must both be *normal goods* in the sense that an increase in net income must be followed by an increase in both consumption and assets for fixed labour market regimes. The reason is the following: suppose 'total net income' increases (for example as result of an increase in human capital, but also as result of any other change in the state space). From the law of

motion in assets (see (1)), the implication is that either  $(a_{t+1})$  or  $(c_t)$  increase. But both  $u$  and  $EV$  are concave functions, therefore, both must increase to keep the equality in (L.12) satisfied. Another way to interpret this is as follows: allowing for  $EV(a, h)$  in L.11 does not pin down a specific optimal path among all possible optimal paths given all admissible  $h$  paths, so L.11 is necessary but not sufficient. Conditioning on  $h$  implies that the Euler is now based on  $EV(a, h | h)$  thus restricting the relation between assets and consumption so that the marginal intertemporal gains are now fixed for given labour market conditions. This latter is what allows to identify the optimal path but at the expense of further restrictions on the type of consumption and savings that individuals are allowed to consume and hold.

Lemmas 1, 2 and 3 complete the set of regularity conditions that allow for expression (2) to represent the individual's problem, for the problem to be well defined and for this to have a unique solution (identification of an optimal consumption path). At the same time, expression (2) is based on (1) and we have shown that the structural model as specified in (1) is well behaved. This is what allows us to use the characterization of the endogenous variables to specify the reduced form specification, and with this to estimate the parameters. In reality, what is crucial is to make sure that for fixed labour market regimes the dynamic endogenously changing variables change all monotonically in the same direction. Our specification is correct because the newly introduced labour market regime still maintains such monotonic relation. Assuming a well behaved bounded functions in a bounded support (for anyone of the three labour market regimes), the problem boils down to 'maximising a concave function' subject to a set of constrain that 'do not jump in different directions in some unspecified form': this is also guaranteed. Because in our case these constrains also behave monotonically, the problem can be placed in the shape of a value function with behaviour that is driven by the dynamics in the model, thus the value function is also well behaved. The regularity conditions for the value

function implies three constraints (risk aversion, consumption is normal and savings is also a normal good). This completes the theoretical part (the structural model and its conditions).

## Appendix 2

Probit estimates for the conditional probabilities  $P(I_t^w = 1 | K_t^w; \gamma_{t,b})$ ,  $P(I_{t+1}^w = 1 | K_{t+1}^w; \gamma_{t,b})$  and  $P(I_t^n = 1 | K_t^n; \gamma_{ta})$ ,  $P(I_{t+1}^n = 1 | K_{t+1}^n; \gamma_{t+1a})$ . The samples used in each of the four cases are based on the distribution from Table 1 so that estimates for period  $t$  are based on 48,653 units, and estimates for period  $t+1$  are based on 45,222. The 3,431 drop in sample between periods results from those who move to be non-classified in one of the labour market regimes after period  $t$ .

Table A2.1: Probit Estimates for the outcomes working (versus not working) and unemployment without ALMP (versus working and unemployed with ALMP)

Dependent Variable	Working				Unemployed without ALMP			
	<i>t</i>		<i>t</i> + 1		<i>t</i>		<i>t</i> + 1	
Time Period	Coefficient	s.e	Coefficient	s.e	Coefficient	s.e	Coefficient	s.e
Constant	5.373	0.184	<b>4.828</b>	0.205	4.104	0.261	3.714	0.32
Age	-0.218	0.006	-0.195	0.007	-0.231	0.009	-0.191	0.011
Age square	0.003	0	0.002	0	0.003	0	0.002	0
Lives in German Canton	-0.232	0.023	-0.185	0.024	-0.19	0.032	-0.191	0.065
Household size	0.028	0.01	0.01	0.011	0.048	0.015	-0.053	0.07
Permanent Partner present	-0.463	0.029	-0.368	0.031	-0.436	0.041	0.04	0.016
Household ownership	-0.195	0.026	-0.164	0.027	0.019	0.033	-0.334	0.045
Primary Industry (Agro, fishery, mine)	0.118	0.068	0.193	0.068	-0.173	0.083	0.067	0.037
Secondary Industry (Manufacturing)	<b>0.649</b>	0.021	<b>0.625</b>	0.023	<b>0.454</b>	0.026	-0.089	0.087
Skill Class 1	-0.008	0.03	0.012	0.033	0.064	0.039	<b>0.421</b>	0.031
Skill Class 2	-0.325	0.025	-0.263	0.026	-0.404	0.032	0.018	0.047
Natural Logs, Net household income	-0.241	0.012	-0.247	0.013	-0.157	0.016	-0.3	0.036
Labour market experience <= 6 months	0.797	0.117	<b>1.06</b>	0.151	0.162	0.195	-0.223	0.018
Labour market experience <= 12 months	0.286	0.14	-0.331	0.17	0.089	0.198	-0.285	0.255
Labour market experience <= 18 months	0.056	0.146	<b>0.645</b>	0.176	0.281	0.258	0.225	0.245
Labour market experience <= 2 years	-0.01	0.109	-0.27	0.136	-0.038	0.216	0.148	0.186
Labour market experience <= 4 years	-0.14	0.051	0.03	0.058	-0.793	0.101	-0.372	0.113
Labour market experience <= 2 years	-0.485	0.059	-0.12	0.053	-0.408	0.072	-0.138	0.071
Labour market experience <= 4 years	-0.218	0.05	0.045	0.048	-0.338	0.063	-0.108	0.072
Time dummy, 1991	-0.137	0.048	-0.016	0.05	-0.295	0.065	-0.116	0.071
Time dummy, 1992	-0.005	0.045	-0.04	0.049	-0.261	0.064	-0.092	0.074
Time dummy, 1993	-0.152	0.052	-0.331	0.059	-0.171	0.069	-0.313	0.105
Time dummy, 1994	-0.261	0.05	-0.216	0.053	-0.281	0.077	-0.347	0.098
Time dummy, 1995	-0.281	0.049	-0.308	0.057	-0.308	0.076	-0.265	0.098
Time dummy, 1996	-0.244	0.049	-0.392	0.059	-0.137	0.071	-0.135	0.082
Time dummy, 1997	-0.36	0.05	-0.418	0.059	-0.195	0.072	-0.076	0.075
Time dummy, 1998	-0.484	0.057	-0.453	0.064	-0.17	0.073	-0.08	0.072
Time dummy, 1999	-0.32	0.05	-0.274	0.053	-0.07	0.061	-0.031	0.077
Time dummy, 2000	-0.32	0.05	-0.274	0.053	-0.07	0.061	-0.031	0.077
Time dummy, 2001	-0.259	0.036	-0.129	0.039	0.031	0.049	-0.03	0.058
Time dummy, 2002	-0.259	0.036	-0.129	0.039	-0.29	0.086	-0.317	0.085
Unemployed for 6 or less moths					-0.095	0.126	0.175	0.115
Unemployed for 12 or less months					0.012	0.154	-0.133	0.143
Unemployed for 18 or less months					2.133	0.114	2.168	0.112
Unemployed for 24 or less months								

Note: All estimates are based on truncated probits (expressed, 15, Section 4) with points  $[\underline{\pi}, \bar{\pi}] = [0.000001, 23.8]$  as the points used for truncating the likelihood function. Table 1 explains the sample sizes used for estimating each of the specifications in the table. For example, estimates for the working outcome at period  $t$ , conditionally compares 45,826 working males to (980+1,847) non-working males to estimate the coefficients in column 2, whereas for column 3 (at  $t + 1$ ) the comparison is between 42,438+322+264 working males against (545 + 607 + 236 + 128 + 97 + 585) non-working individuals who are still active labour market participants. Similarly, Table 1 shows the sizes involved in estimating the coefficients in columns 4 and 5. The omitted variables are 'lives in a non-German speaking canton', tertiary (service) sector, skill class 3 (the highest skill considered), 'has working experience greater than 48 months' and specifically for columns 4 and 5 'has been unemployed for more than 2 years'. Furthermore we omit the time dummies for 2002 and 2003. We consider significance at 5% level or below with bold coefficient suggesting such level of significance. All p-values for the diagnostics suggest rejecting overall heteroscedasticity and acceptance of the specification by means of the likelihood ratio.