

# A Rational Expectations Model for Simulation and Policy Evaluation of the Spanish Economy\*

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## Abstract

This paper describes a Rational Expectations Model of the Spanish economy, REMS, which is in the tradition of small open economy dynamic general equilibrium models, with a strongly micro-founded system of equations. The model is built on standard elements, but incorporates some distinctive features to provide an accurate description of the Spanish economy. We contribute to the existing models of the Spanish economy by adding search and matching rigidities to a small open economy framework. Our model also incorporates habits in consumption and rule-of-thumb households. As Spain is a member of EMU, we model the interaction between a small open economy and monetary policy in a monetary union. The model is primarily constructed to serve as a simulation tool at the Spanish Ministry of Economic Affairs and Finance. As such, it provides a great deal of information regarding the transmission of policy shocks to economic outcomes. The paper describes the structure of the model in detail, as well as the estimation and calibration technique and some examples of simulations.

*Keywords:* general equilibrium, rigidities, policy simulations.

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## 1. Introduction

REMS is a small open economy dynamic general equilibrium (DGE) model of the Spanish economy. It builds upon the existing literature on macroeconomic models<sup>1</sup>. The model is primarily intended to serve as a simulation tool at the Spanish Ministry of Economic Affairs and Finance, with a focus on the economic impact of alternative policy measures over the medium term. A comprehensive analysis of the transmission channels linking policy options with economic outcomes should be regarded as one of the model's most valuable assets.

Regarding country coverage, modelling the Spanish case as a small open economy appears to be a fair compromise between realism and tractability. Presumably, the small open economy paradigm means that a number of foreign variables are given from the perspective of the country in question. It also implies that the magnitude of spillover effects is assumed to be of second-order importance.

As already mentioned, REMS is a DGE model. As such, REMS departs from MOISEES<sup>2</sup> -the preceding simulation tool available at the Spanish Ministry of Economic Affairs and Finance-, in a number of modelling routes. Unlike MOISEES, REMS is endowed with microfoundations insofar as all behavioral equations can be traced to dynamic optimization problems faced by representative households and firms. As agents are assumed to be rational, all relevant economic decisions explicitly incorporate a forward-looking behaviour, i.e., the dynamic responses are governed by theoretical considerations and any ad-hoc dynamics have been avoided. Whereas MOISEES adopted the Keynesian tradition of econometric models, strongly stressing the demand side of the economy and modelling economic agents' behaviour in backward-looking fashion, REMS is in the vein of new neoclassical-Keynesian synthesis models, with the optimizing behavior of households and firms being deeply rooted in the Rational Expectations hypothesis. The supply side of the economy is modelled through a neoclassical production function, so that, in the long run, the model chiefly behaves in accordance with the neoclassical growth model. However, it is assumed that the final goods sector and the labour market do not work in a competitive fashion. As a result, the levels of employment and economic activity in the long run are lower than in a competitive framework.

In the short run, the model incorporates standard elements from the Keynesian tra-

<sup>1</sup> Most central banks and other international institutions have designed DGE models. These include the SIGMA model for the US (Erceg, Gurrieri and Gust, 2006), the BEQM for the UK (Harrison *et al.*, 2005), the TOTEM for Canada (Murchison, Rennison and Zhu, 2004), AINO for Finland (Kilponen, Ripatti and Vilmunen, 2004), or the models by Smets and Wouters (2003) for EMU, Lindé, Nessén and Söderström (2004) for Sweden or Cadiou *et al.* (2001) for 14 OECD countries. There are two complementary models to REMS for the Spanish economy: BEMOD (Andrés, Burriel and Estrada, 2006) and MEDEA (Burriel, Fernández-Villaverde and Rubio, 2007).

<sup>2</sup> See Molinas *et al.* (1990)

dition and introduces some distinctive features to provide an accurate description of the Spanish economy. Modelling efforts to develop REMS have been accompanied by similar research activities in the field. Notable examples in the related literature are represented by BEMOD (Andrés, Burriel and Estrada, 2006) and MEDEA (Burriel, Fernández-Villaverde and Rubio, 2007). None of these models are capable of adequately capturing all the relevant information but rather complement each other. The greatest value added of REMS is the specification of the labour market block, achieved by adding search and matching rigidities to a small open economy framework. This is done to account for equilibrium unemployment. Also, because the Spanish economy is highly open, current account dynamics is needed to provide a realistic picture. The model also allows for habits in consumption and rule-of-thumb households. Price rigidities, on the other hand, apply to intermediate good producers so that a Phillips curve is derived under the well-known Calvo assumption.

The model is parametrized using Spanish data. A database (REMSDB<sup>3</sup>) has been specifically elaborated to fit the model's estimation and calibration requirements as well as to generate a baseline scenario for REMS.

The paper is organized as follows. Section 2 provides a detailed description of the theoretical model. Section 3 deals with the calibration strategy. Section 4 discusses the transmission channels at work following standard simulation experiments. The last section presents the main conclusions.

## 2. Theoretical framework

We model a decentralized, small open economy where households, firms, policymakers and the external sector actively interact each period by trading one final good, two financial assets and three production factors. In order to produce gross output, firms employ physical capital (public and private), labour and an intermediate input (energy). While private physical capital and energy are exchanged in a context of perfect competition, the labour market is not Walrasian. Households possess the available production factors. They also own all the firms operating in the economy. In such a scenario, each representative household rents physical capital and labour services out to firms, for which they are paid income in the form of interests and wages. New jobs are created after investing in searching activities. The fact that exchange in the labour market is resource and time-consuming generates a monopoly rent associated with each job match. It is assumed that the worker and the firm bargain over these monopoly rents in a Nash fashion.

Each period the government faces a budget constraint where expenditure items are

<sup>3</sup> See Bosca *et al.*, 2007, for further details.

financed by means of public debt and various distortionary taxes. Intertemporal sustainability of fiscal balance is ensured by a conventional policy reaction function, whereby a lump-sum tax transfer responds to the deviation of the debt to GDP ratio from its long-term target level.

Monetary policy is geared by the European Central Bank (ECB) by means of interest rates movements, which target EMU inflation in which the weight of the Spanish economy is approximately 10 per cent.

Each household is made of working-age agents who may be active or inactive. In turn, active workers participating in the labour market may either be employed or unemployed. If unemployed, agents are actively searching for a job. Firms' investment in vacant posts is endogenously determined and so are job inflows. Finally, job destruction is taken as exogenous.

## 2.1 Consumption behavior

Following Galí et al. (2007), liquidity-constrained consumers are incorporated into the standard Keynesian model. This extension is consistent with the large body of empirical work that finds substantial deviations of consumption behaviour from the permanent-income hypothesis. There are, hence, two types of representative households. One representative household, of size  $N_t^o$ , enjoys unlimited access to capital markets, so its members substitute consumption intertemporally in response to changes in interest rates. We will refer to these households as "Ricardian or optimizing consumers". Another representative household, of size  $N_t^r$ , does not have access to capital markets, so its members can only consume out of current labour income. We will refer to these liquidity-constrained consumers as "rule-of-thumb (*RoT*) consumers". The size of the working-age population is given by  $N_t = N_t^o + N_t^r$ . Let  $1 - \lambda^r$  and  $\lambda^r$  denote the fractions in the working-age population of Ricardian and *RoT* consumers, which, for simplicity, are assumed to be constant over time. With the working-age population growing at the exogenous (gross) rate of  $\gamma_N = N_t/N_{t-1}$ , a similar growth rate can be predicated for each representative household.

Let  $\bar{A}_t$  represent the trend component of total factor productivity at time  $t$ , which will be assumed to grow at the exogenous rate of  $\gamma_A = \bar{A}_t/\bar{A}_{t-1}$ . Balanced growth in the model can be ensured by transforming variables in a convenient way. More specifically, any flow variable  $X_t$  is made stationary through  $x_t \equiv X_t/\bar{A}_{t-1}N_{t-1}$ . Similarly, any stock variable  $X_{t-1}$  is made stationary through  $x_{t-1} \equiv X_{t-1}/\bar{A}_{t-1}N_{t-1}$ .

Both types of households maximize intertemporal utility by selecting streams of consumption and leisure. Household members may be either employed or unemployed, but are able to fully insure each other against fluctuations in employment, as in Andolfatto (1996) or Merz (1995). In this regard, our specification for *RoT* consumers largely differs from Galí et al. (2007). For the sake of simplicity, we assume that only optimizing con-

sumers hold money balances, as well as foreign and domestic bonds. However, taxes are levied on both Ricardian and the liquidity-constrained consumers.

### Optimizing households

Ricardian households face the following maximization programme:

$$\max_{\substack{c_t, n_t, j_t, k_t, \\ b_t^o, b_t^{o,emu}, m_t^o}} E_t \sum_{t=0}^{\infty} \beta^t \left[ \ln (c_t^o - h c_{t-1}^o) + n_{t-1}^o \phi_1 \frac{(T - l_{1t})^{1-\eta}}{1-\eta} + (1 - n_{t-1}^o) \phi_2 \frac{(T - l_{2t})^{1-\eta}}{1-\eta} + \chi_m \ln (m_t^o) \right] \quad (1)$$

subject to

$$\begin{aligned} & \left( r_t (1 - \tau_t^k) + \tau_t^k \delta \right) k_{t-1}^o + w_t \left( 1 - \tau_t^l \right) \left( n_{t-1}^o l_{1t} + r r_t s (1 - n_{t-1}^o) l_{2t} \right) + \left( \left( 1 - \tau_t^l \right) g_{st} - tr h_t \right) + \\ & \frac{m_{t-1}^o}{1 + \pi_t^c} + (1 + r_{t-1}^n) \frac{b_{t-1}^o}{1 + \pi_t^c} + (1 + r_{t-1}^{emu}) \frac{b_{t-1}^{o,emu}}{1 + \pi_t^c} \\ & - (1 + \tau_t^c) c_t^o \frac{P_t^c}{P_t} - \frac{P_t^j}{P_t} j_t^o \left( 1 + \frac{\phi}{2} \left( \frac{j_t^o}{k_{t-1}^o} \right) \right) - \gamma_A \gamma_N \left( m_t^o + b_t^o + \frac{b_t^{o,emu}}{\phi_{bt}} \right) = 0 \end{aligned} \quad (2)$$

$$\gamma_A \gamma_N k_t^o = j_t^o + (1 - \delta) k_{t-1}^o \quad (3)$$

$$\gamma_N n_t^o = (1 - \sigma) n_{t-1}^o + \rho_t^w s (1 - n_{t-1}^o) \quad (4)$$

$$k_0^o, n_0^o, b_0^o, b_0^{ow}, m_0^o \quad (5)$$

All variables in the maximisation problem above are stationary. In our notation, variables indexed by  $r$  and  $o$  respectively denote  $RoT$  and optimizing households. Non-indexed variables apply indistinctly to both types of households. Thus,  $c_t^o, n_{t-1}^o$  and  $s(1 - n_{t-1}^o)$  represent, consumption, the employment rate and the unemployment rate of Ricardian households.  $s$  is the share of the non-employed searching for a job, which is assumed to be exogenous.<sup>4</sup>  $T, l_{1t}$  and  $l_{2t}$  are the time endowment, hours worked per employee, and hours devoted to job search by the unemployed. Note that, whereas the household decides over

<sup>4</sup> For simplicity, we assume that the leisure utility of the unemployed searching for a job is the same as for the non-active:

$$s(1 - n_{t-1}^o) \phi_2 \frac{(T - l_{2t})^{1-\eta}}{1-\eta} = (1 - s)(1 - n_{t-1}^o) \phi_3 \frac{(T - l_{3t})^{1-\eta}}{1-\eta} \quad (6)$$

$l_{1t}$ , the same cannot be said of  $l_{2t}$ : time dedicated to job search is assumed to be a function of the overall economic activity, so that individual households take it as given<sup>5</sup>.

Several parameters are present in the utility function of Ricardian households. Future utility is discounted at a rate of  $\beta \in (0, 1)$ . The parameter  $-\frac{1}{\eta}$  measures the negative of the Frisch elasticity of labour supply. As consumption is subject to habits, the parameter  $h$  takes a positive value. In general  $\phi_1 \neq \phi_2$ , i.e., the subjective value of leisure imputed by workers may vary across employment statuses.

For simplicity, we adopt the money-in-the-utility function approach to incorporate money into the model. The timing implicit in this specification assumes that this variable is the household's real money holdings at the end of the period ( $m_t^o = \frac{M_t^o}{P_t A_t N_t^o}$  where  $P_t$  is the aggregate price level), thus after having purchased consumption goods, that yields utility<sup>6</sup>.

Maximization of (1) is constrained as follows. First, the budget constraint (2) describes the various sources and uses of income. The term  $w_t (1 - \tau^l) n_{t-1}^o l_{1t}$  captures net labour income earned by the fraction of employed workers, where  $w_t$  stands for hourly real wages. The product  $rr_t w_t (1 - \tau^l) s (1 - n_{t-1}^o) l_{2t}$  measures unemployment benefits accruing to the unemployed, where  $rr_t$  denotes the (exogenous) replacement rate of the unemployment subsidy to the market wage. There are four assets in the economy, namely private physical capital ( $k_t^o$ ), domestic and Euro-zone bonds ( $b_t^o$  and  $b_t^{ow}$ ) and money balances ( $M_t^o$ ). All assets are owned by Ricardian households. Barring money, the remaining assets yield some remuneration. Net return on capital is captured by  $r_t k_{t-1}^o (1 - \tau^k) + \tau^k \delta k_{t-1}^o$ , where  $r_t$  represents the gross return on physical capital. Note that depreciation is tax-deductible as reflected in  $\tau^k \delta k_{t-1}^o$ . Interest payments on domestic and foreign debt are respectively captured by  $r_{t-1}^n \frac{b_{t-1}^o}{1 + \pi_t^o}$ , and  $r_{t-1}^{emu} \frac{b_{t-1}^{ow}}{1 + \pi_t^o}$ , where  $r^n$  and  $r^{emu}$  represent the nominal interest rates on domestic and EMU bonds, which may differ because of a risk premium. The other two expenditure categories are lump-sum transfers,  $trh_t$ , and other government transfers,  $g_{st}$ .

<sup>5</sup> More specifically, we assume that the search effort undertaken by unemployed workers increases during expansions, depending positively on the GDP growth rate:

$$l_{2t} = \left( \bar{l}_2 \left( \frac{gdp_t}{gdp_{t-1}} \right)^{\phi_e} \right)^{(1-\rho_e)} l_{2t-1}^{\rho_e}$$

where  $\phi_e$  is the elasticity of search effort with respect to the rate of growth of GDP and  $\rho_e$  captures the strength of inertia in the search effort. The reason for endogenizing search effort in this way is an empirical one, making possible to obtain a reasonable volatility of vacancies.

<sup>6</sup> Carlstrom and Fuerst (2001) have criticized this timing assumption on the grounds that the appropriate way to model the utility from money is to assume that money balances available *before* going to purchase goods yield utility. However, we follow the standard approach in the literature whereby the end-of-period money holdings yield utility.

Total revenues can either be invested in private capital or spent on consumption. The household's consumption and investment are respectively given by  $(1 + \tau^c) \frac{P_t^c}{P_t} c_t^o$  and  $\frac{P_t^i}{P_t} j_t^o \left(1 + \frac{\phi}{2} \left(\frac{j_t}{k_{t-1}}\right)\right)$ , where  $\tau^c$  is the consumption income tax. Note that total investment outlays are affected by the increasing marginal costs of installation  $j_t^o \left(1 + \frac{\phi}{2} \left(\frac{j_t}{k_{t-1}}\right)\right)$ . Also, the presence in the model of the relative prices  $P_t^c/P_t$  and  $P_t^i/P_t$  implies that a distinction is made between the three deflators of consumption, investment and aggregate output.

The remaining constraints faced by Ricardian households concern the laws of motion for capital and employment. Each period the private capital stock  $k_t^o$  depreciates at the exogenous rate  $\delta$  and is accumulated through investment,  $j_t^o$ . Thus, it evolves according to (3). Employment obeys the law of motion (4), where  $n_{t-1}^o$  and  $s(1 - n_{t-1}^o)$  respectively denote the fraction of employed and unemployed optimizing workers in the economy at the end of period  $t - 1$ . Each period employment is destroyed at the exogenous rate  $\sigma$ . Likewise new employment opportunities come at the rate  $\rho_t^w$ , which represents the probability that one unemployed worker will find a job. Although the job-finding rate  $\rho_t^w$  is taken as exogenous by individual workers, at the aggregate level it is endogenously determined according to the following Cobb-Douglas matching function<sup>7</sup>:

$$\rho_t^w (1 - n_{t-1}) = \vartheta_t (v_t, n_{t-1}) = \chi_1 v_t^{\chi_2} [s(1 - n_{t-1}) l_{2t}]^{1-\chi_2}. \quad (7)$$

Finally,  $k_0^o, n_0^o, b_0^o, b_0^{o,emu}, m_0^o$  in (4) represent the initial conditions for the corresponding stock variables.

The solution to the optimization programme above generates the following first order conditions for consumption, employment, investment, capital stock, government debt, foreign debt and money holdings:

$$\lambda_{1t}^o = \frac{1}{(P_t^c/P_t)(1 + \tau_t^c)} \left( \frac{1}{c_t^o - hc_{t-1}^o} - \beta \frac{h}{c_{t+1}^o - hc_t^o} \right) \quad (8)$$

$$\gamma_N \lambda_{3t}^o = \beta E_t \left\{ \begin{aligned} & \phi_1 \frac{(T-l_{1t+1})^{1-\eta}}{1-\eta} - \phi_2 \frac{(T-l_{2t})^{1-\eta}}{1-\eta} + \lambda_{1t+1}^o w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ & + \lambda_{3t+1}^o [(1 - \sigma) - \chi_1 \rho_{t+1}^w] \end{aligned} \right\} \quad (9)$$

$$\gamma_A \gamma_N \frac{\lambda_{2t}^o}{\lambda_{1t}^o} = \beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left\{ \left[ r_{t+1} (1 - \tau_{t+1}^k) + \tau_{t+1}^k \delta \right] + \frac{\phi}{2} \frac{P_{t+1}^i}{P_{t+1}} \frac{j_{t+1}^{o2}}{k_t^{o2}} + \frac{\lambda_{2t+1}^o}{\lambda_{1t+1}^o} (1 - \delta) \right\} \quad (10)$$

<sup>7</sup> Note that this specification presumes that all workers are identical to the firm.

$$\lambda_{2t}^o = \lambda_{1t}^o \frac{P_t^i}{P_t} \left[ 1 + \phi \left( \frac{j_t^o}{k_{t-1}^o} \right) \right] \quad (11)$$

$$\gamma_A \gamma_N E_t \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} = \beta E_t \frac{1 + r_t^n}{1 + \pi_{t+1}^c} \quad (12)$$

$$\gamma_A \gamma_N \lambda_{1t}^o \frac{1}{\phi_{bt}} = \beta E_t \frac{\lambda_{1t+1}^o (1 + r_t^{emu})}{1 + \pi_{t+1}^c} \quad (13)$$

$$\frac{\chi_m}{m_t^o} = \gamma_A \gamma_N \lambda_{1t}^o \frac{r_t^n}{1 + r_t^n} \quad (14)$$

as well as the three households' restrictions (2), (3) and (4).

According to equation (8) the current-value shadow price of income is equal to the difference between the marginal utility of consumption in two consecutive periods  $t$  and  $t + 1$ .

Equation (9) ensures that the intertemporal reallocation of labour supply cannot improve the life-cycle household's utility. This optimizing rule distinguishes search models from the competitive framework, as it substitutes for the conventional labour supply. It tells us that as search is a costly process there is a premium on being employed,  $\lambda_{3t}^o$ , which measures the marginal contribution of a newly created job to the household's utility.  $\lambda_{3t}^o$  is equal to the sum of its return from labour net of current and expected disutility arising from work provided it is not destroyed in the meantime. Therefore  $\lambda_{3t}^o$  includes three terms. The first term on the right hand side of (9) represents the net disutility arising from the newly created job. The second term captures the present discounted value of the cash-flow generated by the new job in  $t + 1$ , defined as the after-tax labour income minus the foregone unemployment benefits. The current-value shadow price of income,  $\lambda_{1t+1}^o$ , evaluates this cash-flow according to its purchasing power in terms of consumption. The third term represents the capital value in  $t + 1$  of an additional employed worker corrected for the probability that the new job will be destroyed between  $t$  and  $t + 1$ .

Expression (10) ensures that the intertemporal reallocation of capital cannot improve the household's utility.  $\frac{\lambda_{2t}^o}{\lambda_{1t}^o}$  denotes the current-value shadow price of capital. This arbitrage condition includes two terms. The first term represents the present discounted value of its cash-flow in  $t + 1$ , defined as the sum of the after-tax rental cost net of the tax-deductible depreciation investment and total adjustment costs evaluated in terms of consumption. The second term represents the present value in  $t + 1$  of an additional unit



of productive capital corrected for the depreciation rate.

Equation (11) states that investment is undertaken to the extent that the opportunity cost of a marginal increase in investment in terms of consumption is equal to its marginal expected contribution to the household's utility.

The marginal utility of consumption evolves according to expression (12), which is obtained by deriving the Lagrangian with respect to domestic government bonds  $b_t^o$ . (12) and (8) jointly yield the Euler condition for consumption.

Expression (13) results from deriving the Lagrangian with respect to foreign debt  $b_t^{o,emu}$ . Note that the specification above assumes that Ricardian households incur in a risk premium when buying foreign bonds ( $\phi_{bt}$ ). This line is taken in Turnovsky (1985), Benigno (2001), Schmitt-Grohe and Uribe (2003) or Erceg et al. (2005), as a means to ensure that net foreign assets are stationary. Specifically, the risk premium is made a function of net foreign assets holdings in the following way

$$\ln \phi_{bt} = -\phi_b (\exp (b_t^{o,emu}) - 1) \quad (15)$$

The rest of the section simply rearranges first order conditions to facilitate the economic interpretation of Ricardian households' behaviour. As physical capital, domestic and foreign bonds perfectly substitute one another, there must be two arbitrage relations. In order to obtain the arbitrage relation between domestic and foreign bonds we proceed to combine (13) with (12). This algebra yields

$$1 + r_t^n = \phi_{bt} (1 + r_t^{nw}) \quad (16)$$

implying that the interest parity condition holds between domestic and EMU bonds to the extent that they are perfect substitutes. Note that (16) slightly differs from the standard uncovered interest parity condition in that there is no risk associated with exchange rate movements, as both domestic and foreign bonds are expressed in the same currency.<sup>8</sup>

To obtain the arbitrage relation between physical capital and government bonds. it

<sup>8</sup> For simplicity, it is assumed that foreign bonds are expressed in euros. We could assume instead that some bonds could be from the rest of the world, expressed in a foreign currency. In this case, the UIP for the euro area ensures that

$$1 + r_t^{emu} = E_t \frac{er_{t+1}}{er_t} (1 + r_t^{rw}) \quad (17)$$

where  $er$  is the nominal exchange rate. Given the relative small size of Spain in EMU, we assume that the euro exchange rate with the rest of the world is unaffected by Spanish variables, even though Spanish inflation has a small influence on ECB interest rates. This assumption is additionally supported by the empirical evidence since, as documented by many authors (see, for example, Adolfson et al, 2007, and the references there in), the uncovered interest parity condition cannot account for the forward premium puzzle shown by the data. For these reasons, all foreign prices, including those of foreign bonds, are taken to be exogenous and are expressed in euros.

is convenient to define  $q_t \equiv \lambda_{2t}^o / \lambda_{1t}^o$ , which allows us to rewrite equation (10) as

$$q_t = \frac{1 + \pi_{t+1}^c}{1 + r_t^n} \left[ r_{t+1}(1 - \tau_{t+1}^k) + \tau_{t+1}^k \delta + \frac{\phi}{2} \frac{j_{t+1}^2}{k_t^2} + q_{t+1}(1 - \delta) \right] \quad (18)$$

This is a Fisher-type condition in a context characterised by the adjustment costs of installation weighing on physical capital. In order to see this more clearly let  $\phi = 0$ , implying that the investment process is not subject to any installation costs. In this case, (11) boils down to  $q_t = q_{t+1} = 1$  and expression (18) becomes

$$E_t \left[ 1 + (r_{t+1} - \delta) (1 - \tau^k) \right] = E_t \left( \frac{1 + r_{t+1}^n}{1 + \pi_{t+1}^c} \right) \quad (19)$$

which is the conventional Fisher parity condition.

Finally, expression (14) can be easily rewritten as a money demand function by using the current-value shadow price of income (8)

$$m_t^o = \frac{1}{\gamma_A \gamma_N} \chi_m (1 + \tau_t^c) \frac{P_t^c}{P_t} \frac{1 + r_t^n}{r_t^n} \frac{1}{\left( \frac{1}{c_t^o - h c_{t-1}^o} - \beta \frac{h}{c_{t+1}^o - h c_t^o} \right)} \quad (20)$$

#### *Rule-of-thumb households*

RoT households do not benefit from access to capital markets, so that they face the following maximization programme:

$$\max_{c_t^r, n_t^r} E_t \sum_{t=0}^{\infty} \beta^t \left[ \ln (c_t^r - h c_{t-1}^r) + n_{t-1}^r \phi_1 \frac{(T - l_{1t})^{1-\eta}}{1 - \eta} + (1 - n_{t-1}^r) \phi_2 \frac{(T - l_{2t})^{1-\eta}}{1 - \eta} \right]$$

subject to the law of motion of employment (4) and the specific liquidity constraint whereby each period's consumption expenditure must be equal to current labour income and government transfers, as reflected in:

$$w_t (1 - \tau_t^l) (n_{t-1}^r l_{1t} + r r_t^s (1 - n_{t-1}^r) l_{2t}) + g_{st} (1 - \tau_t^l) - tr h_t - (1 + \tau_t^c) c_t^r \frac{P_t^c}{P_t} = 0 \quad (21)$$

$$\gamma_N n_t^r = (1 - \sigma) n_{t-1}^r + \rho_t^w s (1 - n_{t-1}^r) \quad (22)$$

$$n_0^r \quad (23)$$

where  $n_0^r$  represents the initial aggregate unemployment rate, which is the sole stock variable in the above programme. Note that *RoT* consumers do not save and, as a result, they do not hold any assets. This feature of *RoT* consumers considerably simplifies the solution to the optimization programme, which is characterized by the following equations concerning optimal consumption,  $c_t^r$ , and optimal employment,  $n_t^r$ :

$$\lambda_{1t}^r = \frac{1}{(P_t^c/P_t)c_t^r(1+\tau_t^c)} \left( \frac{1}{c_t^r - hc_{t-1}^r} - \beta \frac{h}{c_{t+1}^r - hc_t^r} \right) \quad (24)$$

$$\gamma_N \lambda_{3t}^r = \beta E_t \left\{ \begin{aligned} & \phi_1 \frac{(T-l_{t+1})^{1-\eta}}{1-\eta} - \phi_2 \frac{(T-l_{2t})^{1-\eta}}{1-\eta} + \lambda_{1t+1}^r w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ & + \lambda_{3t+1}^r [(1-\sigma) - \chi_1 \rho_{t+1}^w] \end{aligned} \right\} \quad (25)$$

It is worth mentioning that the optimizing behaviour of *RoT* households preserves the dynamic nature of the model, due to consumption habits and the dynamic nature of the employment decision.

#### Aggregation

Aggregate consumption and employment can be defined as a weighted average of the corresponding variables for each household type:

$$c_t = (1 - \lambda^r) c_t^o + \lambda^r c_t^r \quad (26)$$

$$n_t = (1 - \lambda^r) n_t^o + \lambda^r n_t^r \quad (27)$$

For the variables that exclusively concern Ricardian households, aggregation is merely performed as:

$$k_t = (1 - \lambda^r) k_t^o \quad (28)$$

$$j_t = (1 - \lambda^r) j_t^o \quad (29)$$

$$b_t = (1 - \lambda^r) b_t^o \quad (30)$$

$$b_t^{emu} = (1 - \lambda^r) b_t^{oemu} \quad (31)$$

$$m_t = (1 - \lambda^r) m_t^o \quad (32)$$

## 2.2 Factor demands

Production in the economy takes place at two different levels. At the lower level, an infinite number of monopolistically competing firms produce differentiated intermediate goods ( $y_i$ ), which imperfectly substitute each other in the production of the final good. These differentiated goods are then aggregated by competitive retailers into a final domestic good ( $y$ ) using a CES aggregator.

Intermediate producers solve a two-stage problem. In the first stage, each firm faces a cost minimization problem which results in optimal demands for production factors. When choosing optimal streams of capital, energy, employment and vacancies intermediate producers set prices by varying the mark-up according to demand conditions. Variety producer  $i \in (0, 1)$  uses three inputs: a composite input of private capital and energy, labour and public capital, so that technological possibilities are given by:

$$y_{it} = z_{it} \left\{ \left[ a k_{it-1}^{-\rho} + (1-a) e_{it}^{-\rho} \right]^{-\frac{1}{\rho}} \right\}^{1-\alpha} (n_{it-1} l_{it})^\alpha (k_{it-1}^p)^\zeta \quad (33)$$

where all variables are scaled by the trend component of total factor productivity and  $z_t$  represents a transitory technology shock. Each variety producer rents physical capital,  $k_{t-1}$ , and labour services,  $n_{t-1} l_{1t}$ , from households, and uses public capital services,  $k_{t-1}^p$ , provided by the government. Intermediate energy inputs  $e_t$  can be either imported from abroad or produced at home. The technical elasticity of substitution between private capital and energy is given by  $\frac{1}{1+\rho}$ .  $\alpha \in (0, 1)$  is a distribution parameter: it determines relative factor shares in the steady state. Furthermore, it is convenient to denote capital services by  $k_{iet}$  as:

$$k_{iet} = \left[ a k_{it-1}^{-\rho} + (1-a) e_{it}^{-\rho} \right]^{-\frac{1}{\rho}} \quad (34)$$

This specification is quite general in that private capital and energy can be seen as either complements or substitutes, depending on the value of  $\rho$ . Our calibration strategy will nevertheless pin down the value of  $\rho$  so as to ensure that the elasticity of substitution between private capital and energy is smaller than the elasticity of substitution between the capital-energy composite and labour.

Factor demands are obtained by solving the cost minimization problem faced by each variety producer (we drop the industry index  $i$  when no confusion arises)

$$\min_{k_t, n_t, v_t, e_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left( r_t k_{t-1} + w_t (1 + \tau^{sc}) n_{t-1} l_{1t} + \kappa_v v_t + \frac{P_t^e}{P_t} e_t (1 + \tau^e) \right) \quad (35)$$

subject to

$$y_t = z_{it} \left( \left[ a k_{t-1}^{-\rho} + (1-a) e_t^{-\rho} \right]^{-\frac{1}{\rho}} \right)^{1-\alpha} (n_{t-1} l_{1t})^\alpha (k_{t-1}^p)^\zeta - \kappa_f \quad (36)$$

$$\gamma_N n_t = (1 - \sigma) n_{t-1} + \rho_t^f v_t \quad (37)$$

$$n_0 \quad (38)$$

where, in accordance with the ownership structure of the economy, future profits are discounted at the household relevant rate  $\beta$ .  $\kappa_v$  captures recruiting costs per vacancy,  $\kappa_f$  is an entry cost which ensures that extraordinary profits vanish in imperfectly-competitive equilibrium,  $\tau^{sc}$  is the social security tax rate levied on gross wages<sup>9</sup>, and  $\rho_t^f$  is the probability that a vacancy will be filled in any given period  $t$ . It is worth noting that the probability of filling a vacant post  $\rho_t^f$  is exogenous from the firm's perspective. However, from the perspective of the overall economy, this probability is endogenously determined according to the following Cobb-Douglas matching function:

$$\rho_t^w (1 - n_{t-1}) = \rho_t^f v_t = \chi_1 v_t^{\chi_2} [s (1 - n_{t-1}) l_{2t}]^{1-\chi_2} \quad (39)$$

Under the assumption of symmetry, the solution to the optimization programme above generates the following first order conditions for private capital, employment, energy and the number of vacancies

$$r_{t+1} = (1 - \alpha) m c_{t+1} \frac{y_{t+1}}{k_{et+1}} a \left( \frac{k_{et+1}}{k_t} \right)^{1+\rho} \quad (40)$$

$$\gamma_N \lambda_t^{nd} = \beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left( \alpha m c_{t+1} \frac{y_{t+1}}{n_t} - w_{t+1} (1 + \tau_{t+1}^{sc}) l_{1t+1} + \lambda_{t+1}^{nd} (1 - \sigma) \right) \quad (41)$$

<sup>9</sup> Note that, in our specification, firms bear the statutory incidence of social security contributions.

$$(1 - \alpha)(1 - a)mc_t \frac{y_t}{ke_{t-1}} \left( \frac{ke_t}{e_t} \right)^{1+\rho} = \frac{P_t^e}{P_t} (1 + \tau_t^e) \quad (42)$$

$$\kappa_v v_t = \lambda_t^{nd} \chi_1 \chi_2 v_t^{\chi_2} (s(1 - n_{t-1})l_{2t})^{1-\chi_2} \quad (43)$$

where the real marginal cost ( $mc_t$ ) corresponds to the Lagrange multiplier associated with the first restriction (36), whereas  $\lambda_t^{nd}$  denotes the Lagrange multiplier associated with the second restriction (37).

The demand for private capital is determined by (40). It is positively related to the marginal productivity of capital  $(1 - \alpha) \frac{y_{t+1}}{k_{t+1}} a \left( \frac{k_{t+1}}{k_t} \right)^{1+\rho}$  times the firm's marginal cost  $mc_{t+1}$ <sup>10</sup> which, in equilibrium, must equate the gross return on physical capital.

The intertemporal demand for labour (41) requires the marginal contribution of a new job to profits to be equal to the marginal product net of the wage rate plus the capital value of the new job in  $t + 1$ , corrected for the job destruction rate between  $t$  and  $t + 1$ .

Energy demand is defined by (42). It is positively related to the marginal productivity of energy  $(1 - \alpha)(1 - a) \frac{y_t}{ke_{t-1}} \left( \frac{ke_t}{e_t} \right)^{1+\rho}$  times the marginal cost  $mc_t$  which, in equilibrium, must equate the real price of energy including energy taxes.

Expression (43) reflects that firms choose the number of vacancies in such a way that the marginal recruiting cost per vacancy,  $\kappa_v$ , is equal to the expected present value of holding it,  $\lambda_t^{nd} \frac{\chi_1 \chi_2 v_t^{\chi_2} (s(1 - n_{t-1})l_{2t})^{1-\chi_2}}{v_t}$ , where  $\lambda_t^{nd}$  denotes the shadow price of an additional worker, and  $\frac{\chi_1 \chi_2 v_t^{\chi_2} (s(1 - n_{t-1})l_{2t})^{1-\chi_2}}{v_t}$  is the transition probability from an unfilled to a filled vacancy.

### 2.3 Pricing behavior of intermediate firms: the New Phillips curve

Intermediate firms enjoy market power and are, therefore, price setters. Each intermediate firm produces a variety  $y_i$  and faces a downward-sloping demand curve of the form:

$$y_{it} = y_t \left( \frac{P_{it}}{P_t} \right)^{-\varepsilon} \quad (44)$$

where  $\left( \frac{P_{it}}{P_t} \right)$  is the relative price of variety  $y_i$ ,  $\varepsilon$  can be expressed in terms of the elasticity of substitution between intermediate goods,  $\zeta \geq 0$ , as  $\varepsilon = (1 + \zeta) / \zeta$ , and  $y_t$  represents the

<sup>10</sup> Under imperfect competition conditions, cost minimization implies that production factors are remunerated by the marginal revenue times their marginal productivity. In our specification for factor demands, the marginal revenue has been replaced by the corresponding marginal costs. We are legitimated to proceed in this manner because in equilibrium these two marginal concepts are made equal by the imperfectly-competitive firm.

production of the final product as defined by

$$y_t = \left( \int_0^1 y_{it}^{1/1+\zeta} di \right)^{1+\zeta} \quad \text{and} \quad P_t = \left( \int_0^1 P_{it}^{-\frac{1}{\zeta}} di \right)^{-\zeta} \quad (45)$$

Variety producers act as monopolists and choose prices when allowed. We use the well-known Calvo hypothesis (Calvo, 1983), thereby assuming some overlapping adjustment in prices. Those firms that do not reset their prices optimally at a given date adjust them according to a simple indexation rule to catch up with lagged inflation. Thus, each period a proportion  $\theta$  of firms simply set  $P_{it} = (1 + \pi_{t-1})^\varkappa P_{it-1}$  (with  $\varkappa$  representing the degree of indexation), while only a measure  $1 - \theta$  of firms set their prices,  $\tilde{P}_{it}$ , to maximize the present value of expected profits. Consequently,  $1 - \theta$  represents the probability of adjusting prices each period, whereas  $\theta$  can be interpreted as a measure of price rigidity. Thus, the maximization problem of the representative variety producer can be written as:

$$\max_{\tilde{P}_{it}} E_t \sum_{j=0}^{\infty} \rho_{it,t+j} (\beta\theta)^j \left[ \tilde{P}_{it} \bar{\pi}_{t+j} y_{it+j} - P_{t+j} mc_{it,t+j} (y_{it+j} + \kappa_f) \right] \quad (46)$$

subject to

$$y_{it+j} = \left( \tilde{P}_{it} \bar{\pi}_{t+j} \right)^{-\varepsilon} P_{t+j}^\varepsilon y_{t+j} \quad (47)$$

where  $\tilde{P}_{it}$  is the price set by the optimizing firm at time  $t$ ,  $\beta$  is the discount factor,  $mc_{t,t+j}$  represents the marginal cost borne at  $t+j$  by the firm that last set its price in period  $t$ ,  $\bar{\pi}_{t+j} = \prod_{h=1}^j (1 + \pi_{t+h-1})^\varkappa$ , and  $\rho_{t,t+j}$  is a price kernel which captures the marginal utility of an additional unit of profits accruing to Ricardian households at  $t+j$ , i.e.,

$$\frac{E_t \rho_{t,t+j}}{E_t \rho_{t,t+j-1}} = \frac{E_t (\lambda_{1t+j}^0 / P_{t+j})}{E_t (\lambda_{1t+j-1}^0 / P_{t+j-1})} \quad (48)$$

The first order condition of the optimization problem above is

$$\tilde{P}_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{\infty} (\beta\theta)^j E_t \left[ \rho_{it,t+j} P_{t+j}^{\varepsilon+1} mc_{it,t+j} y_{t+j} \bar{\pi}_{t+j}^{-\varepsilon} \right]}{\sum_{j=0}^{\infty} (\beta\theta)^j E_t \left[ \rho_{it,t+j} P_{t+j}^\varepsilon y_{t+j} \bar{\pi}_{t+j}^{(1-\varepsilon)} \right]} \quad (49)$$

and the corresponding aggregate price index is equal to

$$P_t = \left[ \theta (\pi_{t-1}^\varkappa P_{t-1})^{1-\varepsilon} + (1 - \theta) \tilde{P}_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (50)$$

As is standard in the literature<sup>11</sup>, equation (50) can be used to obtain an expression for aggregate inflation of the form:

$$\pi_t = \frac{\beta}{1 + \varkappa\beta} E_t \pi_{t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta(1 + \varkappa\beta)} \widehat{mc}_t + \frac{\varkappa}{1 + \varkappa\beta} \pi_{t-1} \quad (51)$$

where  $\widehat{mc}_t$  measures the deviation of the firm's marginal cost from the steady state, i.e.,  $mc_t = \frac{\varepsilon-1}{\varepsilon}(1 + \widehat{mc}_t)$ . Equation (51) is known in the literature as the New Phillips curve. It participates of the conventional Phillips-curve philosophy that inflation is influenced by activity in the short run. Unlike the conventional specification, the New Phillips Curve emphasizes real marginal costs as the relevant variable to the inflation process, which is seen as a forward-looking phenomenon: when opportunities to adjust prices arrive infrequently, a firm will be concerned with future inflation. A second departure of the New Phillips curve from the traditional one is that it is derived from the optimizing behavior of firms. Thus, it is possible to define the marginal cost elasticity of inflation,  $\lambda$ , as a function of the structural parameters in the model,  $\beta$  and  $\theta$ :

$$\lambda = \frac{(1 - \beta\theta)(1 - \theta)}{\theta(1 + \varkappa\beta)} \quad (52)$$

Equation (52) shows that an increase in the average time between price changes,  $\theta$ , makes current inflation less responsive to  $\widehat{mc}_t$ . Output movements will therefore have a smaller impact on current inflation, holding expected future inflation constant. The reduced form of the New Phillips curve can be written as:

$$\pi_t = \beta^f E_t \pi_{t+1} + \lambda \widehat{mc}_t + \beta^b \pi_{t-1} \quad (53)$$

Notice that our model, as is standard in the literature, does not take into account how the equilibrium is affected by the distribution of prices determined by the Calvo hypothesis. Burriel, Fernández-Villaverde and Rubio (2007) show that the effects of this simplification are almost negligible, both upon the dynamics and also the steady state of a DGE model with several nominal rigidities.

#### 2.4 Trade in the labour market: the labour contract

The key departure of search models from the competitive paradigm is that trading in the labour market is subject to transaction costs. Each period, the unemployed engage in search activities in order to find vacant posts spread over the economy. Costly search in the labour market implies that there are simultaneous inflows into and outflows out of the

<sup>11</sup> See, for example, Galí, Gertler and López-Salido (2001).



state of employment, so that an increase (reduction) in the stock of unemployment results from the predominance of job destruction (creation) over job creation (destruction). Stable unemployment occurs whenever inflows and outflows cancel out one another, i.e.,

$$\rho_t^f v_t = \rho_t^w (1 - n_{t-1}) = \chi_1 v_t^{\chi_2} [(1 - n_{t-1}) l_{2t}]^{1-\chi_2} = (1 - \sigma) n_{t-1} \quad (54)$$

Because it takes time (for households) and real resources (for firms) to make profitable contacts, some pure economic rent emerges with each new job, which is equal to the sum of the expected transaction (search) costs the firm and the worker will further incur if they refuse to match. The emergence of such rent gives rise to a bilateral monopoly framework. On the one hand, both sides have incentives to cooperate in the formation of a job match, as there are monopoly rents associated with it; on the other hand, they compete for the appropriation of such rents.

Because all jobs are equally productive and all workers have the same reservation wage a new job is created whenever a job contact occurs. Once a job-seeking worker and vacancy-offering firm match they negotiate a labour contract in hours and wages, so that we stick to the efficient-bargaining hypothesis instead of the right-to-manage hypothesis (see Trigari, 2006, for further details about the implications of these two different hypotheses). Note that, because homogeneity holds across all job-worker pairs in the economy, the outcome of this negotiation will be the same everywhere. However an individual firm and worker are too small to influence the market. As a result when they meet they negotiate the terms of the contract by taking as given the behaviour in the rest of the market.

Several wage and hours determination schemes can be applied to a bilateral monopoly framework. In particular, we will assume that firms and workers negotiate by means of a Nash bargain, so the outcome of the bargaining process maximizes the product

$$\max_{w_{t+1}, l_{t+1}} \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w} \left( \lambda_t^{nd} \right)^{(1-\lambda^w)} \quad (55)$$

where  $\lambda^w \in (0, 1)$  reflects the worker's bargaining power. The first term in brackets represents the worker surplus while and the second is the firm surplus. More specifically,  $\lambda_{3t}^o / \lambda_{1t}^o$  and  $\lambda_{3t}^r / \lambda_{1t}^r$  respectively denote the earning premium (in terms of consumption) of employment over unemployment for a Ricardian and a *RoT* worker. Similarly,  $\lambda_t^{nd}$  represents the profit premium of a filled over an unfilled vacancy for the representative firm. Note that this bargaining scheme features the same wage for all workers, irrespective of whether they are Ricardian or *RoT*.

Optimal real wage and hours worked (55) satisfy the following conditions (see Ap-

pendix 1 for further details):

$$(1 + \tau_t^{sc})w_t l_{1t} = \frac{\lambda^w}{\left[1 - (1 - \lambda^w) rr_t s_{1t}^{l_{2t}}\right]} \left[ \alpha m c_t \frac{y_t}{n_{t-1}} + (1 - \sigma) \frac{\kappa_v}{\rho_t^f} \right] \quad (56)$$

$$+ \frac{(1 - \lambda^w)}{\left[1 - (1 - \lambda^w) rr_t s_{1t}^{l_{2t}}\right]} \frac{(1 + \tau_t^{sc})}{(1 - \tau_t^l)} \left[ \frac{\left(\frac{\lambda_{3t}}{\lambda_{1t-1}}\right)}{\left(\frac{\lambda_{1t}}{\lambda_{1t-1}}\right)} [\rho_t^w - (1 - \sigma)] - \frac{\left(\frac{1}{\lambda_{1t-1}}\right)}{\left(\frac{\lambda_{1t}}{\lambda_{1t-1}}\right)} u_t \right]$$

$$\left(\frac{\lambda_{1t}}{\lambda_{1t-1}}\right) \alpha m c_t \frac{y_t}{n_{t-1} l_{1,t}} = \left(\frac{1}{\lambda_{1t-1}}\right) \frac{(1 + \tau_t^{sc})}{(1 - \tau_t^l)} \phi_1 [1 - l_{1t}]^{-\eta} \quad (57)$$

where:

$$u_t = \frac{P_t^c}{P_t} c_t (1 + \tau_t^c) \left[ \phi_1 \frac{(T - l_{1t+1})^{1-\eta}}{1 - \eta} - \phi_2 \frac{(T - l_{2t})^{1-\eta}}{1 - \eta} \right] \quad (58)$$

$$\left(\frac{\lambda_{1t}}{\lambda_{1t-1}}\right) = \left[ \lambda^r \frac{\lambda_{1t}^r}{\lambda_{1t-1}^r} + (1 - \lambda^r) \frac{\lambda_{1t}^o}{\lambda_{1t-1}^o} \right]$$

$$\left(\frac{1}{\lambda_{1t-1}}\right) = \left[ \lambda^r \frac{1}{\lambda_{1t-1}^r} + (1 - \lambda^r) \frac{1}{\lambda_{1t-1}^o} \right]$$

$$\left(\frac{\lambda_{3t}}{\lambda_{1t-1}}\right) = \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t-1}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t-1}^o} \right]$$

where we see that, unlike the Walrasian outcome, the wage prevailing in the search equilibrium is at some point between the marginal rate of substitution of consumption for leisure and the marginal productivity of labour, depending on the worker bargaining power  $\lambda^w$ . Put differently, the wage is a weighted average between the highest feasible wage (i.e., the marginal productivity of labour plus the cost of posting a vacancy corrected by the probability that the vacancy will be filled) and the lowest acceptable wage (i.e., the reservation wage as given by the disutility from work corrected by the probability of finding a job). Consequently, the equilibrium wage depends on a number of policy parameters and institutional variables describing labour market performance. Notice that when  $\lambda^r = 0$ , all consumers are Ricardian, and, therefore, the solutions for the wage rate and hours simplify to the standard ones.

The first bracket in expression (56) includes a number of technological and institutional factors that have an influence on the bargained wage. Holding all other things constant, an increase in the marginal productivity of labour results in a higher bargained wage. The bargained wage also increases following a rise in the cost of posted vacancies, which makes the match more profitable, thereby reducing the firms bargaining power. The opposite occurs when the probability of filling a vacancy increases. Last, but not least, wage dynamics are influenced by the indicator of labour market tightness, with a lower  $\frac{(1-\sigma)}{\rho_t^f}$  having an adverse impact on the negotiated wages insofar as more congestion depresses the probability that a worker may find a job.<sup>12</sup>

The second bracket in expression (56) includes additional factors related to households' preferences and institutions having an impact on the bargained wage. The reservation wage depends positively on the value imputed for leisure by an unemployed worker, as well as the probability of finding a job. Both elements increase, *ceteris paribus*, the relative bargaining power of workers. Conversely, the value imputed for leisure by an employed worker and the survival rate  $(1 - \sigma)$  both reduce the bargained wage. Finally, there are a number of fiscal variables that influence the division of the surplus arising from a new job. This is the case of the replacement rate, which increases the bargained wage because it raises income from unemployment thereby improving the worker's threat point in the bargain process<sup>13</sup>. Both consumption and labour marginal taxes influence equilibrium wages because the imputed value of leisure is not taxed. An increase in either  $\tau_t^c$  or  $\tau^l$  make leisure more attractive in relation to work and, by doing so, increase wages in equilibrium. By contrast, an increase in social security contributions reduces wages by making recruiting an additional worker more expensive.

## 2.5 Government

Each period the government decides the size and composition of public expenditure and the mix of taxes and new debt holdings required to finance total outlays. It is assumed that government purchases of goods and services ( $g_t^c$ ) and public investment ( $g_t^i$ ) follow an exogenously given pattern. Conversely, interest payments on government bonds ( $1 +$

<sup>12</sup> Note that equilibrium in the labour market implies that

$$\rho_t^w (1 - n_{t-1}) = \rho_t^f v_t = \chi_1 v_t^{\chi_2} [(1 - n_{t-1}) l_2]^{1-\chi_2} = (1 - \sigma) n_{t-1}$$

so  $\frac{v_t}{n_{t-1}} = \frac{\rho_t^w}{\rho_t^f}$ , where the latter ratio is a measure of congestion in the labour market.

<sup>13</sup> Note that the replacement rate influences the division of the surplus arising from a new job, not the definition of the worker's or the firm's threat points in the bargain as captured by the two brackets in equation (56). This is because the replacement ratio is a marginal element in unemployment compensation, that is, the benefit paid depends on the prevailing wage in the economy.

$r_t)b_{t-1}$ , unemployment benefits  $g_{ut}(1 - n_{t-1})$ , and government social transfers  $g_{st}$  are assumed to be endogenous. The two latter expenditure categories are given by

$$g_{ut} = rr_t w_t \quad (59)$$

$$g_{st} = tr_t gdp_t \quad (60)$$

whereby  $g_{ut}$  and  $g_{st}$  are made proportional to the level of real wages,  $w_t$ , and activity,  $gdp_t$ , through  $rr_t$  and  $tr_t$ .

Government expenditure is financed by direct taxation, levied on either labour income (personal labour income tax,  $\tau_t^l$ , and social security contributions,  $\tau_t^{sc}$ ) or capital income ( $\tau_t^k$ ), as well as indirect taxation, represented by consumption ( $\tau_t^c$ ) and energy taxes ( $\tau_t^e$ ). Government revenues are therefore given by

$$\begin{aligned} t_t = & (\tau_t^l + \tau_t^{sc})w_t(n_{t-1}l_{1t}) + \tau_t^k(r_t - \delta)k_{t-1} \\ & + \tau_t^c \frac{P_t^c}{P_t} c_t + \tau_t^e \frac{P_t^e}{P_t} e_t + trh_t + \tau_t^l \bar{r} w_t (1 - n_{t-1})l_{2t} + \tau_t^l g_{st} \end{aligned} \quad (61)$$

where  $trh_t$  stands for lump-sum transfers as defined below.

Each period total receipts and outlays are made consistent by means of the government's budget constraint

$$\gamma_A \gamma_N b_t = g_t^c + g_t^i + g_{ut}(1 - n_{t-1}) + g_{st} - t_t + \frac{(1 + r_t^m)}{1 + \pi_t} b_{t-1} \quad (62)$$

Equation (62) reflects that the gap between total receipts and outlays is financed by variations in lump-sum transfers to households,  $trh_t$  (which enter the fiscal budget rule through the term  $t_t$ ), and/or the issue of domestic bonds ( $b_t - b_{t-1}$ ). As it stands, equation (62) has an intertemporal dynamic nature. Note that government income from seniorage is nil.

Dynamic sustainability of public debt requires the introduction of a debt rule that makes one or several fiscal categories an instrument for debt stabilization. In order to enforce the government's intertemporal budget constraint, the following fiscal policy reaction function is imposed

$$trh_t = trh_{t-1} + \psi_1 \left[ \frac{b_t}{gdp_t} - \overline{\left( \frac{b}{gdp} \right)} \right] + \psi_2 \left[ \frac{b_t}{gdp_t} - \frac{b_{t-1}}{gdp_{t-1}} \right] \quad (63)$$

where  $\overline{\left( \frac{b}{gdp} \right)}$  is the long-run target for the debt-to-GDP ratio and  $\psi_1 > 0$  captures the speed of adjustment from the current ratio towards the desired target. The value of  $\psi_2 >$

0 is chosen to ensure a smooth adjustment of actual debt towards its steady-state level. Note that while in the baseline specification debt stabilization is accomplished through variations in lump-sum transfers, nothing precludes other receipt or spending categories from playing this role.

Government investment augments public capital, which in turn depreciates at the rate  $\delta^p$  and thus follows the law of motion:

$$\gamma_A \gamma_N k_t^p = g_t^i + (1 - \delta^p) k_{t-1}^p \quad (64)$$

## 2.6 Monetary policy

Monetary policy is geared by the European Central Bank (ECB), which targets EMU inflation by means of movements in interest rates. More specifically, short-term interest rates are governed by the following reaction function

$$\ln \frac{1 + r_t^{emu}}{1 + r^{emu}} = \rho^r \ln \frac{1 + r_{t-1}^{emu}}{1 + r^{emu}} + \rho^\pi (1 - \rho^r) \ln(\pi_t^{emu} - \overline{\pi^{emu}}) + \rho^y (1 - \rho^r) \ln \Delta \ln y_t^{emu} \quad (65)$$

where all the variables indexed by *emu* refer to EMU aggregates. Thus,  $r_t^{emu}$  and  $\pi_t^{emu}$  are the Euro-zone nominal short-term interest rate and consumption price deflator (to which the Spanish economy contributes according to its relative size), and  $\Delta \ln y_t^{emu}$  measures the deviation of GDP growth from its trend. As explained in Woodford (2003), (65) is the optimal outcome of a rational central bank facing and objective function under general equilibrium conditions.

Finally, the disappearance of national currencies since the inception of the monetary union means that the intra-euro-area real exchange rate is simply given by the ratio of relative prices between the domestic economy and the remaining EMU members, so real appreciation/depreciation developments are driven by the inflation differential vis-à-vis the euro area:

$$\frac{rer_{t+1}}{rer_t} = \frac{1 + \pi_{t+1}^{emu}}{1 + \pi_t^{emu}} \quad (66)$$

## 2.7 The External Sector

The small open economy hypothesis adopted in REMS implies that world prices and world demand are taken as given. It also means that feedback linkages between the domestic economy, EMU and the rest of the world are ignored. Another simplifying assumption concerns the nature of final and intermediate goods produced at home, which are all considered to be tradable.

*The allocation of consumption and investment between domestic and foreign produced goods*

Let us think of aggregate consumption (investment) as a composite basket of home and foreign produced goods. There is a representative consumption (investment) distributor whose role is to determine the share of aggregate consumption (investment) to be satisfied with home produced goods  $c_h$  ( $i_h$ ) and foreign imported goods  $c_f$  ( $i_f$ ). This is done so on the basis of a CES technology:

$$c_t = \left( (1 - \omega_{ct})^{\frac{1}{\sigma_c}} c_{ht}^{\frac{\sigma_c-1}{\sigma_c}} + \omega_{ct}^{\frac{1}{\sigma_c}} (c_{ft})^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c-1}} \quad (67)$$

$$i_t = \left( (1 - \omega_{it})^{\frac{1}{\sigma_i}} i_{ht}^{\frac{\sigma_i-1}{\sigma_i}} + \omega_{it}^{\frac{1}{\sigma_i}} (i_{ft})^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i-1}} \quad (68)$$

where  $\sigma_c$  ( $\sigma_i$ ) is the consumption (investment) elasticity of substitution between domestic and foreign goods.

Each period, the representative consumption distributor chooses  $c_{ht}$  and  $c_{ft}$  so as to minimize production costs subject to the technological constraint given by (67). The Lagrangian of this problem can be written as:

$$\begin{aligned} \min_{c_{ht}, c_{ft}} \left\{ \left( P_t c_{ht} + P_t^m c_{ft} \right) \right. \\ \left. + P_t^c \left[ c_t - \left( (1 - \omega_c)^{\frac{1}{\sigma_c}} c_{ht}^{\frac{\sigma_c-1}{\sigma_c}} + \omega_c^{\frac{1}{\sigma_c}} (c_{ft})^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c-1}} \right] \right\} \quad (69) \end{aligned}$$

where  $P_t$  and  $P_t^m$  are respectively the prices of home and foreign produced goods. Note that  $P_t^c$  represents both the price of the consumption good borne by households and the shadow cost of production borne by the aggregator.

The optimal allocation of aggregate consumption between domestic and foreign goods,  $c_{ht}$  and  $c_{ft}$ , satisfies the following conditions:

$$c_{ht} = (1 - \omega_c) \left( \frac{P_t}{P_t^c} \right)^{-\sigma_c} c_t \quad (70)$$

$$c_{ft} = \omega_c \left( \frac{P_t^m}{P_t^c} \right)^{-\sigma_c} c_t \quad (71)$$

Proceeding in the same manner as with the investment distributor problem, similar expressions can be obtained regarding the optimal allocation of aggregate investment

between domestic and foreign goods,  $i_{ht}$  and  $i_{ft}$

$$i_{ht} = (1 - \omega_i) \left( \frac{P_t}{P_t^i} \right)^{-\sigma_i} i_t \quad (72)$$

$$i_{ft} = \omega_i \left( \frac{P_t^m}{P_t^i} \right)^{-\sigma_i} i_t \quad (73)$$

### Price formation

In the preceding analysis, the price of domestically produced consumption and investment goods is equal to the GDP deflator,  $P_t$ . In order to obtain the consumption price deflator, one needs to further incorporate the demand schedules provided by (70) and (71) for home and foreign consumption goods into the cost of producing one unit of aggregate consumption goods ( $P_t c_{ht} + P_t^m c_{ft}$ ). Bearing in mind that the production cost per unit equates to the price of production, it is straightforward to express the consumption (investment) price deflator as a function of the GDP and import deflators

$$P_t^c = \left( (1 - \omega_{ct}) P_t^{1-\sigma_c} + \omega_{ct} P_t^{m1-\sigma_c} \right)^{\frac{1}{1-\sigma_c}} \quad (74)$$

$$P_t^i = \left( (1 - \omega_{it}) P_t^{1-\sigma_i} + \omega_{it} P_t^{m1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}} \quad (75)$$

The exogenous world price is a weighted average calculated on the basis of final good and intermediate good prices,  $\overline{PFM}$  and  $\overline{P}^e$ , both expressed in terms of the domestic currency. Given the small open economy assumption, the relevant foreign price is defined as:

$$P_t^m = (\tilde{\alpha}_e P_t^e + (1 - \tilde{\alpha}_e) \overline{PFM}_t) \quad (76)$$

where  $\tilde{\alpha}_e$  stands for the ratio of energy imports to overall imports.

Let us consider that export prices charged by Spanish firms deviate from prices charged by competitors in foreign markets, at least temporarily. This well-known pricing-to-market hypothesis is consistent with a model of monopolistic competition among firms where each firm regards its influence on other firms as negligible. It can be assumed that, for fraction of  $(1 - ptm)$  goods, firms set different prices in home and foreign markets. Prices for these goods are set in the local currency. The remaining  $ptm$  goods can be freely traded by consumers, i.e., the law of one price holds, so firms must set a unified price

across countries. Their pricing behavior would be consistent with high competition between domestic and foreign firms in markets for very homogenous products. Prices for these goods are set in the currency of the seller. In light of the arguments above, we may define the Spanish export price deflator as

$$P_t^x = P_t^{(1-ptm)} (\overline{PFM}_t)^{ptm} \quad (77)$$

where  $P_t^x$  is the export price deflator,  $\overline{PFM}_t$  is a competitors price index expressed in euros and the parameter  $ptm$  determines the extent to which there is pricing-to-market.

#### *Exports and Imports*

Aggregate imports include two final goods, foreign consumption and investment, and one intermediate commodity, energy:

$$im_t = c_{ft} + i_{ft} + \alpha_e e_t \quad (78)$$

where  $\alpha_e$  represents the ratio of energy imports over total energy consumption.

Exports demand can be defined in terms of aggregate consumption and investment from abroad,  $\bar{y}_t^w$ , and the ratio of the export price deflator to the competitors price index expressed in euros,  $P_t^x / \overline{PFM}_t$ :

$$ex_t = s_t^x \left( \frac{P_t^x}{\overline{PFM}_t} \right)^{-\sigma_x} \bar{y}_t^w \quad (79)$$

Plugging (77) into (79) yields the exports demand under the small open economy assumption and the pricing-to-market hypothesis:

$$ex_t = s_t^x \left( \frac{\overline{PFM}_t}{P_t} \right)^{(1-ptm)\sigma_x} \bar{y}_t^w \quad (80)$$

Note that if the law of one price is completely absent, then  $ptm = 0$ ,  $P_t^x = P_t$  as dictated by (77) and expression (79) boils down to

$$ex_t = s_t^x \left( \frac{P_t}{P_t^m} \right)^{-\sigma_x} \bar{y}_t^w = s_t^x \left( \frac{P_t}{\overline{PFM}_t} \right)^{-\sigma_x} \bar{y}_t^w \quad (81)$$

Conversely, if the law of one price holds for all consumption and investment goods, then  $ptm = 1$ ,  $P_t^x = P_t^m = \overline{PFM}_t$ , as dictated by (77), and expression (79) boils down to

$$ex_t = s_t^x \bar{y}_t^w \quad (82)$$



A comparison of expressions (81) and (82) reveals that, if the law of one price holds, exports demand is simply a fraction of total aggregate consumption and investment from abroad. But if the law of one price is not satisfied ( $ptm = 0$ ), relative prices also affect the exports demand. Relative prices also play a role, although relatively less important, under the assumption of partial pricing-to-market, ( $0 < ptm < 1$ ).

*Stock-flow interaction between the current account balance and the accumulation of foreign assets*

In the model, the current account balance is defined as the trade balance plus interest rate receipts/payments from net foreign assets:

$$ca_t = \frac{P_t^x}{P_t} ex_t - \frac{P_t^m}{P_t} im_t + (r_t^{emu} - \pi_t) b_{t-1}^{oemu} \quad (83)$$

Following standard practice in the literature (see, for example, Obstfeld and Rogoff, 1995, 1996), net foreign assets are regarded as a stock variable resulting from the accumulation of current account flows. This is illustrated by the following dynamic equation:

$$\frac{\gamma_A \gamma_N b_t^{oemu}}{\phi_{bt}} = \frac{(1 + r_t^{emu})}{1 + \pi_t^c} b_{t-1}^{oemu} + \frac{P_t^x}{P_t} ex_t - \frac{P_t^m}{P_t} im_t \quad (84)$$

(84) is obtained by combining the Ricardian households' budget constraint (assuming a zero net supply for domestic bonds and money), the government's budget constraint and the economy's aggregate resource constraint (see Appendix 2 for details).

## 2.8 Accounting identities in the economy

Gross output can be defined as the sum of (final) demand components and the (intermediate) consumption of energy:

$$y_t = c_{ht} + i_{ht} + g_t + \frac{P_t^x}{P_t} ex_t + \kappa_v v_t + \frac{P_t^e}{P_t} (1 - \alpha_e) e_t + \kappa_f \quad (85)$$

whereas value added generated in the economy is given by:

$$gdp_t = y_t - \frac{P_t^e}{P_t} e_t - \kappa_f - \kappa_v v_t \quad (86)$$

Note that, in accordance with previous definitions,  $c_{ht}$  and  $i_{ht}$  are equal to overall domestic consumption and investment minus consumption and investment goods imported from abroad. Thus,  $c_{ht}$  and  $i_{ht}$  are consistent with the definitions above for gross output and value added.

### 3. Model solution method and parameterization

#### 3.1 Model solution method

The number of equations involved in the model and the presence of nonlinearities make it impossible to derive a closed-form solution for the dynamic stable path. In order to provide a solution for dynamic systems of this nature it has become a common procedure in the literature to use a numerical method. There are several ways of solving forward-looking models with rational expectations. Most of them rely on algorithms that are applied to the linearized version of the system around the steady state. This approach, which is very popular in the literature, was first introduced by Blanchard and Kahn (1980).

In order to solve the model we follow the method developed by Laffarque (1990), Boucekine (1995) and Juillard (1996). As various endogenous variables in the model have leads, representing expectations of these variables in future time periods, an assumption has to be made regarding the formation of expectations. In REMS expectations are rational and, therefore, model-consistent. This means that each period's future expectations coincide with the model's solution for the future. In simulations this implies that the leads in the model equations are equal to the solution values for future periods. This rational expectations solution is implemented by applying a stacked-time algorithm that solves for multiple time periods simultaneously, i.e., it stacks the time periods into one large system of equations and solves them simultaneously using Newton-Raphson iterations. The method is robust because the number of iterations is scarcely affected by convergence criteria, the number of time periods or the size of the shock. The model is simulated using the Dynare software system.

REMS obeys the necessary local condition for the uniqueness of a stable equilibrium in the neighborhood of the steady state, i.e., there are as many eigenvalues larger than one in modulus as there are forward-looking variables in the system.

#### 3.2 The database REMSDB

The database REMSDB includes the main Spanish economic aggregates (see Boscá et al., 2007, for further details). The complete set of series covers the period 1980 to 2010, which in turn can be divided into two sub-periods depending on the nature of the data. The first one ranges from 1980 to the last available data released by the various statistical sources, i.e., 2006 in the current version of the database. The second one, which spans over a five-year period, relies on the official forecasts based upon the Stability and Growth Programme (SGP). In the existing version of REMSDB the last available year of the forecast period corresponds to 2010. Finally, in order to generate a baseline scenario for the REMS model, the whole set of variables are prolonged further towards a 2050 horizon. Despite not being part of REMSDB, this forward extrapolation obviously builds on the database and further

complies with the balanced-growth hypothesis of the model.

All series are quarterly data, which is a very convenient property for simulation purposes. When quarterly data were not readily available from existing statistical sources, high-frequency series were obtained by applying the Kalman filter and smoother to an appropriate state-space model in which observations correspond to low-frequency data. The frequency of monthly series has also been accommodated and converted into quarterly data with techniques that are specific to each series. Needless to say, all series take the form of seasonally-adjusted data. Whenever the series provided by official statistical sources were not so, they were seasonally-adjusted through TRAMO-SEATS procedures.

The dataset has not been subject to any transformation other than the extraction of the seasonal component or the mere application of linking-back techniques. This is not the case of the variables utilized to construct a baseline scenario for REMS model, most of which have been expressed in efficiency units for them to show a stationary pattern. That is to say that every series included in the baseline exhibits a number of statistical properties that comply with the balanced-growth hypothesis.

The database considers five types of variables. While each of these groups is somewhat stylized, they gather a set of variables of a different nature. The taxonomy is as follows. The first category includes various production and demand aggregates along with their corresponding deflators. A second group brings together population and labour-market series. The third block is made up of monetary and financial variables, whereas the fourth one includes relevant government aggregates. A final set gathers a number of heterogeneous variables that play a role in the baseline scenario and for which no direct statistical counterpart is available from official sources.

### 3.3 Model parameterization

Model parameters have been fixed using a hybrid approach of calibration and estimation. Some parameter values are taken from QUEST II and other related DGE models. Several other parameters are calculated from the sample averages counterpart of long-run conditions. The remaining parameters have been estimated on the basis of selected model's equations. Altogether, these parameters produce a baseline solution that accurately resembles the behaviour of the Spanish economy over the last two decades.

The data used in the calibration come from the REMSDB database. All series cover the period 1985:3 2006:4. Beginning the sample the third quarter of 1985 displays adequate cyclical properties for most of the endogenous variables (see Puch and Licandro, 1997, and Boscá et al, 2007). Several variables included in the model have no direct statistical counterpart from official sources. Such variables include consumption and employment of *RoT* and optimizing consumers, Lagrange multipliers, the Tobin's *q*, the composite capital stock, marginal cost and total factor productivity. In order to sidestep the lack of data

Table 1 – Parameter Values

|                  |        |                  |        |                              |        |                    |        |
|------------------|--------|------------------|--------|------------------------------|--------|--------------------|--------|
| $\bar{g}$        | 0.0905 | $\bar{l}_2$      | 0.2183 | $\beta^b$                    | 0.5035 | $\bar{\pi}^{emu}$  | 0.0000 |
| $\tau^c$         | 0.1069 | $\phi_1$         | 2.9483 | $\left(\frac{b}{gdp}\right)$ | 2.4000 | $T$                | 1.3690 |
| $\tau^l$         | 0.1107 | $\eta$           | 2.0000 | $\bar{r}\bar{r}$             | 0.3832 | $\lambda^w$        | 0.4274 |
| $\tau^k$         | 0.2080 | $\sigma_c$       | 1.2063 | $\tau_t^e$                   | 0.2000 | $\omega_i$         | 0.1381 |
| $\bar{t}\bar{r}$ | 0.1381 | $\phi_b$         | 0.0060 | $\psi_1$                     | 0.0100 | $\sigma_i$         | 0.9305 |
| $\tau^{sc}$      | 0.2215 | $\phi_2$         | 2.2389 | $\psi_2$                     | 0.2000 | $\lambda^r$        | 0.5000 |
| $P^e$            | 0.8620 | $\rho$           | 0.9571 | $\sigma_x$                   | 1.3000 | $h$                | 0.6000 |
| $\beta$          | 0.9908 | $a$              | 0.9985 | $\bar{r}^{emu}$              | 0.0158 | $\alpha_e$         | 0.4980 |
| $\alpha$         | 0.5938 | $\chi_m$         | 0.1987 | $\zeta$                      | 0.0600 | $\tilde{\alpha}_e$ | 0.1049 |
| $\delta$         | 0.0141 | $\bar{m}\bar{c}$ | 0.7961 | $\gamma_A$                   | 1.0018 | $\omega_c$         | 0.0930 |
| $\phi$           | 5.5000 | $\gamma$         | 1.7500 | $\gamma_N$                   | 1.0047 | $\kappa_f$         | 0.2730 |
| $\kappa_v$       | 1.4966 | $s$              | 0.2600 | $\delta^p$                   | 0.0105 | $s^x$              | 0.0142 |
| $\sigma$         | 0.0180 | $\rho^r$         | 0.7500 | $ptm$                        | 0.5767 | $\rho^w$           | 0.7600 |
| $\chi_1$         | 0.6115 | $\lambda$        | 0.2006 | $\omega_c$                   | 0.0930 | $\phi_e$           | 25.000 |
| $\chi_2$         | 0.5725 | $\beta^f$        | 0.4965 | $\bar{y}^w$                  | 7.7855 | $\rho_e$           | 0.8500 |

availability affecting these variables, we use the model's related behavioural equations to compute them (see Appendix 3 for further details).

Table 1 lists the values of parameters and exogenous variables. The implied steady state values of the endogenous variables are given in Table 2<sup>14</sup>. Roughly speaking, the calibration strategy follows a sequence by which one starts by setting the value of a number of parameters which are subsequently used to obtain a measure of the level of total factor productivity. This makes it possible to express all variables in the model in terms of efficiency units. The remaining parameters are then fixed on the basis of the model's equations with variables measured in efficiency units.

The Cobb-Douglas parameter  $\alpha$  matches the labour share, as measured by the ratio of compensation of employees to GDP. The public capital elasticity of output,  $\zeta$ , has been set at 0.06, within the range of the estimated values obtained by Gramlich (1994). With  $\alpha$  and  $\zeta$  it is straightforward to obtain the level of technological efficiency,  $A_t$ , by means of expression (33). HP-filtered total factor productivity is then used to obtain variables in efficiency units. Technical progress,  $\gamma_A$ , is given by the growth rate of trend productivity.

The values of a number of parameters come from QUEST II. This is the case of the subjective discount rate,  $\beta$ , the adjustment costs parameter in the investment function,  $\phi$ , the long-run price elasticity of exports,  $\sigma_x$ , and fiscal rule parameters,  $\psi_1$  and  $\psi_2$ .

<sup>14</sup> The model has been programmed in relative prices. This means that all prices are relative to the deflator price index,  $P_t$ , and the real exchange rate is defined as  $rer_t = \frac{PFM_t}{P_t}$ .

Table 2 – Steady State

|            |        |                                       |        |  |        |             |        |
|------------|--------|---------------------------------------|--------|--|--------|-------------|--------|
| $b_t$      | 1.3951 | $i_{ht}$                              | 0.0987 | $\left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)$ | 1.0000 | $r_t$       | 0.0335 |
| $b_t^o$    | 1.3951 | $im_t$                                | 0.1565 | $\left(\frac{\lambda_{3t+1}}{\lambda_{1t}}\right)$ | 0.2539 | $r_t^m$     | 0.0142 |
| $c_t$      | 0.3192 | $j_t$                                 | 0.1582 | $\lambda_{3t}^o$                                   | 0.1997 | $g_{st}$    | 0.0803 |
| $c_{ft}$   | 0.0542 | $j_t^o$                               | 0.3163 | $\lambda_{3t}^r$                                   | 1.4439 | $trh_t$     | 0.0067 |
| $c_{ht}$   | 0.2659 | $k_t$                                 | 7.5943 | $\lambda_t^{nd}$                                   | 0.4648 | $t_t$       | 0.2024 |
| $c_t^o$    | 0.3570 | $ke_t$                                | 6.3208 | $\lambda_{1t}^o$                                   | 2.6355 | $U_t^n$     | -1.258 |
| $c_t^r$    | 0.2814 | $k_t^o$                               | 15.189 | $\lambda_{1t}^r$                                   | 3.3428 | $g_{ut}$    | 0.0600 |
| $e_t$      | 0.0477 | $l_{1t}$                              | 0.4487 | $\frac{p_t^c}{p_t}$                                | 0.9734 | $v_t$       | 0.0595 |
| $rer_t$    | 0.8390 | $m_t$                                 | 2.6116 | $\pi_t$  | 0.0000 | $w_t$       | 1.2830 |
| $ex_t$     | 0.1431 | $\widetilde{m}c_t$                    | 0.0000 | $gdp_t$  | 0.5813 | $y_t$       | 0.9861 |
| $b_t^w$    | 0.0000 | $m_t^o$                               | 5.2232 | $\pi_t^c$  | 0.0000 | $\phi_{bt}$ | 1.0000 |
| $b_t^{ow}$ | 0.0000 | $n_t$                                 | 0.6102 | $\frac{p_t^l}{p_t}$                                | 0.9228 | $k_t^p$     | 1.1131 |
| $g_t^c$    | 0.0905 | $n_t^o$                               | 0.6102 | $\frac{p_t^m}{p_t}$                                | 0.8267 | $l_{2t}$    | 0.2183 |
| $i_t$      | 0.1672 | $n_t^r$                               | 0.6102 | $\frac{p_t^i}{p_t}$                                | 0.9037 | $g_t^i$     | 0.0173 |
| $i_{ft}$   | 0.0785 | $\left(\frac{1}{\lambda_{1t}}\right)$ | 0.3393 | $q_t$  | 1.0285 |             |        |

Following Andolfatto (1996), we choose a value of 2 for the intertemporal labour substitution,  $\eta$ , whereas the amount of time devoted to looking for a job,  $\bar{l}_2$ , is estimated as half as much the quarterly average of working time.  $\kappa_v$  is calibrated to match an overall cost of vacant posts equal to 0.5 percentage points of GDP. As in Burnside, Eichenbaum, and Rebelo (1993) we fix total productive time endowment  $T$  at 1369 hours a quarter. The exogenous job destruction rate,  $\sigma$ , is calibrated from the law of motion of the employment (4). The share of non-employed workers actively searching for a job,  $s$ , is obtained from the ratio between the unemployment rate and  $(1 - n)$ .

The value of the parameters that enter the monetary policy reaction function  $\gamma_{EMU}$  and  $\rho_{EMU}^r$  are standard (see, for instance, Doménech, Ledo and Taguas, 2002). The weight of the Spanish economy in the Euro-zone inflation,  $\omega_{sp}$ , is set at 10 per cent. The public debt to GDP target ratio has been set at 2.4, which corresponds to a 60 per cent value on an annual basis, highly coherent with the upper bound established in the Stability and Growth Pact. Tax rates on labour and capital income and consumption expenditure ( $\tau^l$ ,  $\tau^k$ ,  $\tau^{sc}$ ,  $\tau^c$ ) have been constructed following the methodology developed in Boscá, García and Taguas (2005). Tax rate on energy,  $\tau_t^e$ , has been set at 0.20. The value of 0.6 for  $h$ , the parameter that captures habits in consumption, has been taken from Smets and Wouters (2003). The risk premium parameter,  $\phi_b$ , is fixed at 0.006. This is somewhat higher than

the value in Erceg, Guerrieri and Gust (2005), so as to ensure a faster convergence of the foreign asset position to the steady state. The fraction of *RoT* consumers in the Spanish economy,  $\lambda^r$ , is assumed to be 0.5. The scale parameter of the matching function,  $\chi_1$ , and the elasticity of matchings to vacant posts,  $\chi_2$ , have been estimated at 0.61 and 0.57, respectively. Following Hosios (1990), the workers' bargaining power is set at  $1 - \chi_2$ .

The entry cost,  $\kappa_f$ , is fixed at 0.35. The distributional parameter in the energy-capital composite,  $a$ , has been set to 0.9985, whereas the estimated value of  $\rho$  is 0.96, which corresponds to an elasticity of substitution between energy and private capital of 0.51, implying that these two inputs are complements in production. Value added is then computed using the accounting identity (86). The growth rate of population,  $\gamma_N$ , has been computed by quarterly averaging the rate of growth of the working-age population. We use the ratio of unemployment benefits (corrected by unemployment) to labour compensation (corrected by working hours) to calibrate the replacement rate,  $\bar{r}$ . The ratio of energy imports to total energy consumption,  $\alpha_e$ , is taken from the Input-Output Tables (year 2000). Private and public capital depreciation rates,  $\delta$  and  $\delta^p$ , are those implicit in the capital series in the BDREMS. These are, roughly, 6 and 4 percent per year, respectively.

As regards preference parameters in the households utility function,  $\phi_1$  and  $\phi_2$ , the former is estimated from the hours schedule equation (57), whereas the latter has been computed as a weighted average (with weights  $\lambda^r$  and  $(1 - \lambda^r)$ ) of the estimates arising from the two labour supply conditions, (9) and (25). Overall, we obtain values for  $\phi_1$  and  $\phi_2$  similar to those calibrated by Andolfatto (1996) and other related research in the literature, implying that the imputed value for leisure by an employed worker is situated well above the imputed value for leisure by an unemployed worker.

We use the employment demand (41) to obtain a series for the firm's marginal cost,  $mc_t$ . The steady state value,  $\bar{mc}$ , is set at the sample mean. We fix the degree of price indexation  $\varkappa$  at 1 (we can not reject this value when  $\varkappa$  is estimated), and use the expression (51) to obtain a GMM estimation of the fraction of firms that do not optimally change prices,  $\theta$ . The estimated value is equal to 0.54. Jointly with the subjective discount rate, these two parameters imply a value of 0.20 for the marginal cost elasticity of inflation,  $\lambda^{15}$ , 0.49 for the parameter on the forward component of inflation,  $\beta^f$ , and 0.50 for the parameter on the backward component of inflation,  $\beta^b$ . Values of these parameters about one half are a key feature of the hybrid Phillips curve with full indexation, and closer to some recent empirical evidence.<sup>16</sup>

<sup>15</sup> This value is significantly higher than that obtained by Galí and López-Salido (2001).

<sup>16</sup> Bils and Klenow (2004) employ data used by the US Bureau of Labor Statistics to obtain the consumer price index and find that the average duration between price adjustments is over six months. The evidence on price setting in the euro area at the individual level by Álvarez et al (2006) shows that prices in the euro area are stickier than in the US.

With respect to the external sector, the pricing-to-market parameter,  $ptm$ , and the ratio of energy imports to overall imports,  $\tilde{\alpha}_e$ , have been estimated from the exports and imports price equations (77) and (76). Our estimate for  $\tilde{\alpha}_e$  suggests that energy represents around 10 percent of total imports. The estimated value of  $ptm$  parameter (0.57) is slightly lower than in QUEST II (0.61). The weight of domestic consumption goods in the CES aggregator,  $\omega_c$ , and the consumption elasticity of substitution between domestic and foreign goods,  $\sigma_c$ , are estimated simultaneously using conditions (70) and (71). Similarly, for the case of investment goods,  $\omega_i$  and  $\sigma_i$  are estimated simultaneously using equations (72) and (73). The estimated elasticities suggest a slightly higher elasticity of substitution between domestic and foreign goods for consumption goods as compared with investment goods. We use the export equation (79) to calibrate the scale parameter  $s^x$ . The energy price index,  $P^e$ , and the world price index,  $\overline{PFM}$ , match sample averages calculated over the calibration period.

The value of two parameters has been accommodated to ensure the desirable long-run properties of the model. Firstly, the world interest rate,  $\overline{r^{emu}}$ , is set to satisfy the static version of the Euler equation given by (12), so that the current account is balanced in the steady state. Second, foreign output,  $\bar{y}^w$ , is adjusted to obtain a steady state value of (close to) one for the real exchange rate.

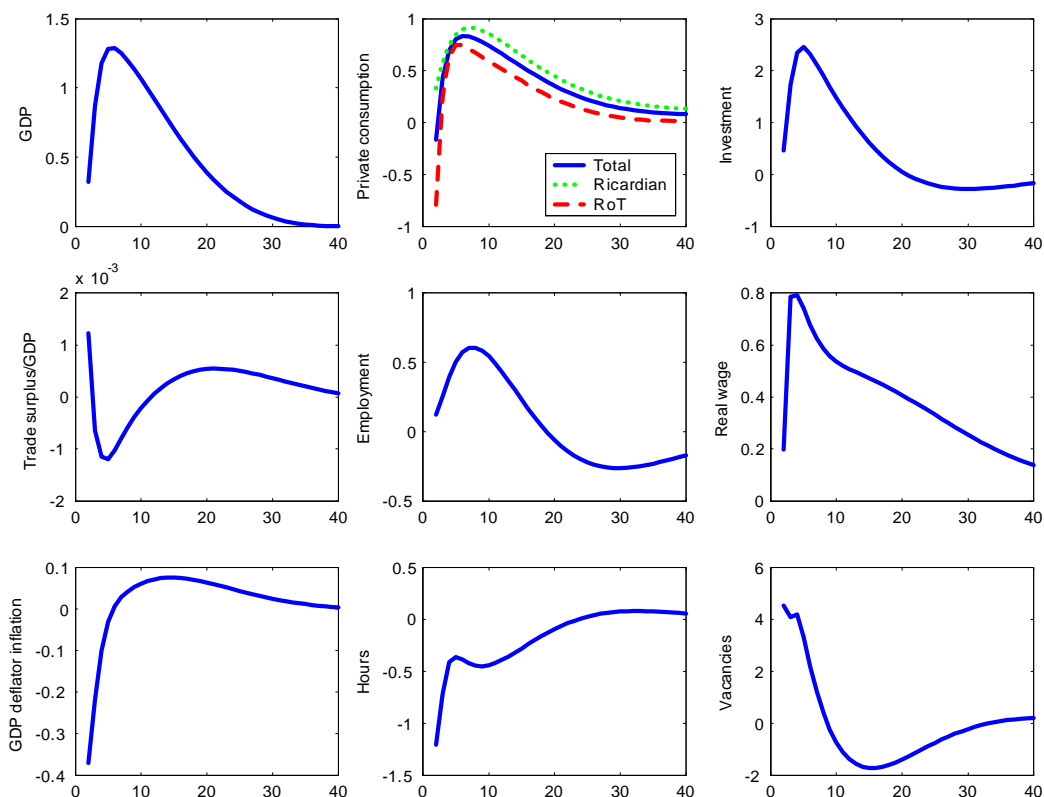
Finally, the value of some additional parameters has been chosen for labour market variables to display a plausible dynamics. Namely, we assume partial inertia in the search effort,  $\rho_e$ . As in Blanchard and Galí (2006), we also allow for slow adjustment in wages according to the expression:  $w_t = w_{t-1}^{\rho^w} \tilde{w}_t^{(1-\rho^w)}$  where  $\tilde{w}_t$  stands for the Nash-bargained wage. Note that, although somewhat ad-hoc, this hypothesis enjoys empirical support as suggested by our estimated value of  $\rho^w$ , which is equal to 0.76.

#### 4. Some standard simulations

The calibration procedure described in the previous section enables us to conduct simulation experiments with our artificial economy. This section presents two standard simulations. The primary goal of these exercises is to examine the transmission channels at work in the model. Shocks considered under this section are assumed to be of a temporary nature and fully anticipated by economic agents.

##### 4.1 Transitory technology shock

The first exercise considers an exogenous transitory technology shock,  $z_i$ , such that the level of total factor productivity increases by 1 per cent on impact. Given that the shock is assumed to be highly persistent (a first-order autoregressive process with a parameter of 0.9), the level of total factor productivity five years later is as high as 0.2 percentage points



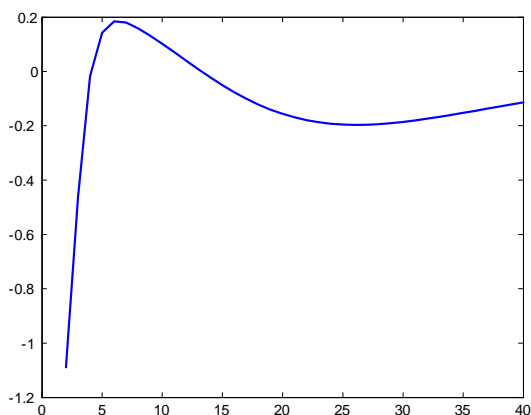
**Figure 1:** Effects of a transitory technological shock

above the steady state level.

Figure 1 displays the (quarterly) dynamic responses of some relevant macroeconomic variables in the model. Simulation results are presented in percentage deviations from baseline values, except for the trade surplus and the GDP deflator inflation, for which we report absolute deviations from steady-state levels.

Figure 1 shows that the impact multiplier on GDP is close to one third and very persistent, reaching a peak after four quarters. Much in the same manner, there are positive and long-lasting effects on consumption, investment and wages. Slow adjustment in Nash-bargained wages is reflected in a positive albeit muted effect on wages, which generates marked responses in employment and consumption. Consumption dynamics are hump-shaped, so that this variable displays steady increases before reaching a peak five quarters from the shock. Consumption dynamics can be rationalised in terms of the mix of





**Figure 2:** *Effects of a technology shock on full-time equivalent employment*

*RoT* and Ricardian households: as compared with Ricardian households, *RoT* households display relatively more volatile and less persistent consumption behaviour, closely related to the behaviour of wages. It is worth mentioning that our impulse-response functions closely resemble those presented in Andrés, Doménech and Fatás (2008) who, extending the research of Andrés and Domenech (2006), also find small effects in the very short run following a technology shock in the context of a model with price rigidities and *RoT* consumers.

The technology shock also has a sizable effect on marginal costs and the Tobin's  $q$ : marginal costs fall on impact whereas the increase in Tobin's  $q$  encourages capital accumulation through investment. The fall in marginal costs tends to reduce inflation, thereby improving competitiveness and export demand. Imports also increase due to the rise in private consumption and investment, which partially offsets real exchange rate depreciation.

The effect on vacancies is quite pronounced. In order to illustrate this, note that the response of vacancies on impact is 6.5 times larger than the effect on GDP. This result is explained by the positive impact that enhanced GDP growth has on search effort. However, our model cannot accommodate the evidence documented by Fujita (2004) and Ravn and Simonelli (2007) whereby vacancies display a hump-shaped pattern following a technology shock, with a peak around 3 quarters from the shock. By contrast, our model features a rapid decline in vacancies as the impact on search effort vanishes and labour market tightness increases. Although short-lived, the marked increase in vacancies encourages job creation, thereby fostering employment, whose effects are more persistent.

As expected, Figure 2 reveals that the behaviour of full-time-equivalent employ-

ment implied by our model is in sharp contrast with the implications of a standard RBC model with flexible prices. In the presence of price rigidities, the path displayed by employment in our model reflects responses in both the intensive and extensive margins of labour which differ in direction and size: while the employment rate rises slightly over time, hours per worker sketches a stark fall on impact. Overall, full-time equivalent employment falls over the first four quarters, and only begins to recover one year after the shock. These predictions match the empirical findings of a growing literature beginning with Galí (1999) (see, for instance, section 2 in Galí and Rabanal, 2004, and Andrés, Doménech and Fatás, 2008, for an overview of some relevant research that reaches similar conclusions).

#### 4.2 A transitory public consumption shock

So as to illustrate further dynamic mechanisms at work in REMS, let us discuss an exogenous transitory shock affecting the steady-state level of public consumption. The shock amounts to 1 percent of initial output (or 6 percent of  $g^c$ ) and, as in the case of the technology shock, it is assumed to follow a highly persistent autorregressive process, with an autocorrelation coefficient equal to 0.9. Figure 3 displays the quarterly dynamic responses of the group of macroeconomic variables shown in the preceding simulation.

The government expenditure impact multiplier on GDP ( $\Delta GDP / \Delta g^c$ ) is approximately 1.03. As for final demand components, consumption increases by 0.41 percentage points on impact, while private investment registers an increase immediately after the shock but falls 0.70 percent after four quarters. The trade balance worsens on impact by 0.13 percent of GDP.

Looking in more detail at the dynamics of private consumption, we find that a positive transitory public consumption shock leads to a significant increase in consumption, that lasts for two quarters. This evidence is consistent with related models in the literature where a fraction of consumers are liquidity-constrained (see Blanchard and Galí (2006) and Galí, López-Salido and Vallés (2007)). Indeed, the second panel in the Figure reveals that the dynamics of overall consumption is driven by the behaviour of RoT households, whose consumption increases on impact by more than one percentage point. In contrast, Ricardian households, who choose the stream of consumption optimally on the basis of their intertemporal budget constraints, revise current consumption downwards in response to the rise in government purchases of goods and services.

The government spending shock is shown to have a sizable crowding-out effect on investment. As reflected in Figure 3, a trough in investment occurs three quarters after the shock. It is accompanied by slightly higher Euro-zone nominal interest rates (insofar higher inflation in the Spanish economy affects Euro-zone inflation in proportion to its relative economic size), a higher the risk premium (which leads to lower accumulation of

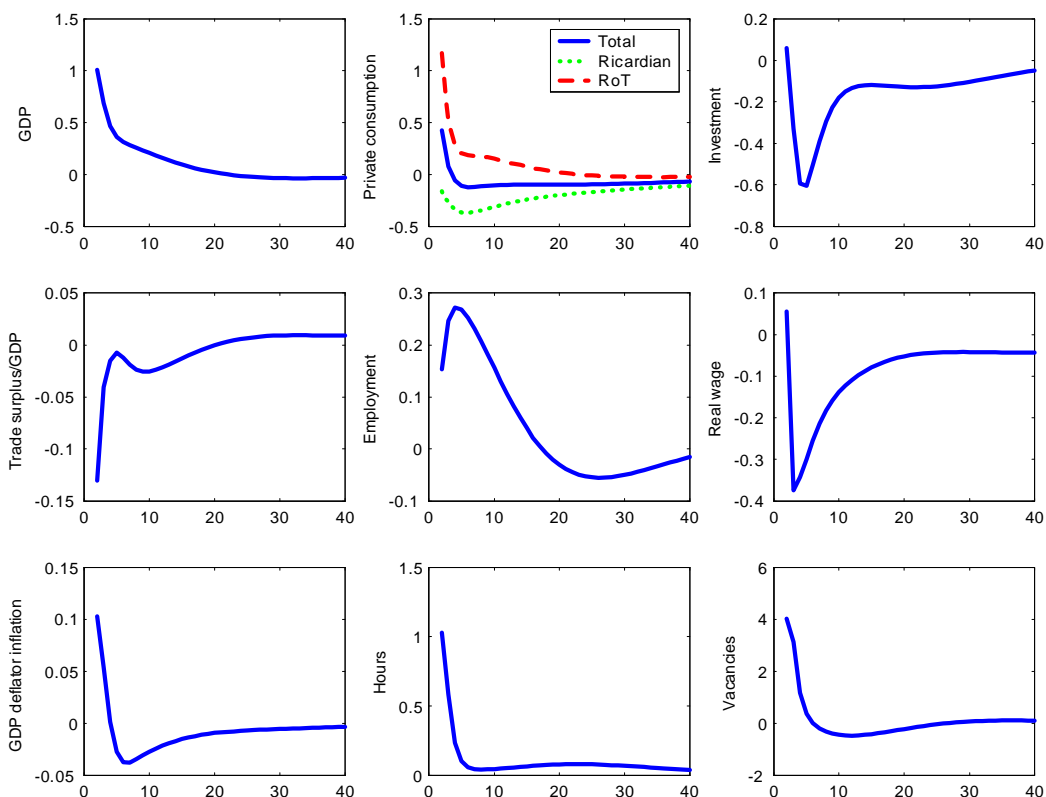


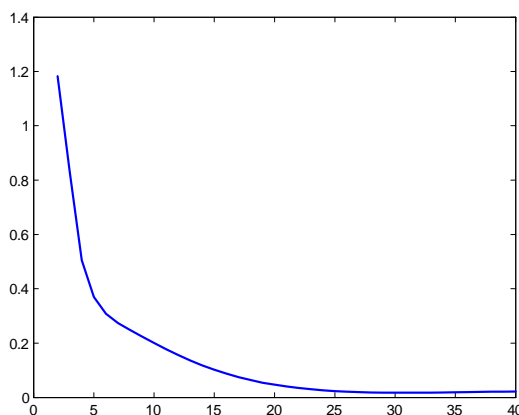
Figure 3: Effects of a transitory public consumption shock

external debt) and a decline in Tobin's  $q$ .

As regards labour market variables, Figure 3 shows that employment, hours, real wages and vacancies increase on impact, then gradually return to normal. A jump in vacant posts is brought about by an increase in search effort and the shadow price of employment,  $\lambda^{nd}$ . Full-time equivalent employment (see Figure 4) is enhanced by the positive response of both employment in head counts and hours worked.

## 5. Conclusions

This paper presents a Rational Expectations Model for Simulation and Policy Evaluation of the Spanish Economy (REMS). REMS is a dynamic general equilibrium model for a small open economy. It is primarily constructed to serve as a tool for simulation and policy evaluation of alternative scenarios. This means that REMS is not used for forecasting, but



**Figure 4:** *Effects of a public consumption shock on full-time equivalent employment*

rather to analyse how the effects of policy shocks are transmitted over the medium term. It also means that the emphasis of the model is on the transmission channels through which policy action affects the domestic economy.

As far as economic theory is concerned, REMS can be characterized as a New Keynesian model with the optimizing behavior of households and firms being deeply rooted in the Rational Expectations hypothesis. The supply side of the economy is modelled through a neoclassical production function, implying that the long-term behavior of the model closely reproduces the Solow growth model, that is, the economy reaches a steady state path with a growth rate determined by the rate of exogenous technical progress plus the growth rate of the population. However, some prominent features differentiate this model from the neoclassical paradigm in the long term. First, trading both in the goods and the labour market is not achieved under Walrasian conditions. Firms in goods markets operate under monopolistic competition, setting prices in a sluggish manner. In the labour market, firms and workers negotiate how to distribute the rents generated in the matching process. Consequently, in our model equilibrium unemployment will persist in the long term. Second, a share of households behaves as myopic consumers that do not optimize intertemporally. Additionally, consumers take into account past consumption (habits) in their decisions. As a result, the behaviour of aggregate consumption after a fiscal shock departs considerably from what could be expected in a neoclassical setting. In the short term REMS is also influenced by the New Keynesian literature allowing for a stabilizing role of demand-side policies, as shown by the simulations conducted in Section 4.

In order to actually parameterize the model, we use Spanish quarterly data cover-

ing the period 1985 to 2006. The parameters of the model have been calibrated and/or estimated in order to reproduce the behavior of the Spanish economy, implying that the steady state solution of the model reflects the main features of the data. With our artificial economy we conduct simulations of technological and fiscal shocks. The dynamic effects shown by the model are not very different to those obtained with other related models for the Spanish economy, although there are some important qualitative and quantitative differences that can be attributed to some of the mechanisms present in our model.

### Appendix 1: Nash bargaining with RoT consumers

Wage setting: The first order condition for the hourly wage is given by:

$$\begin{aligned} \frac{\partial}{\partial w_{t+1}} = & \lambda^w \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w - 1} \left( \lambda_t^{nd} \right)^{(1 - \lambda^w)} \left[ \frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dw_{t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dw_{t+1}} \right] \\ & + (1 - \lambda^w) \left( \lambda_t^{nd} \right)^{-\lambda^w} \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w} \frac{d\lambda_t^{nd}}{dw_{t+1}} = 0 \end{aligned}$$

Thus, rearranging terms,

$$\left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w - 1} \left( \lambda_t^{nd} \right)^{-\lambda^w} \left[ \begin{aligned} & \lambda^w \lambda_t^{nd} \left[ \frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dw_{t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dw_{t+1}} \right] + \\ & (1 - \lambda^w) \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w} \frac{d\lambda_t^{nd}}{dw_{t+1}} \end{aligned} \right] = 0$$

Simplifying further,

$$\lambda^w \lambda_t^{nd} \left[ \frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dw_{t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dw_{t+1}} \right] + (1 - \lambda^w) \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w} \frac{d\lambda_t^{nd}}{dw_{t+1}} = 0$$

Taking into account that,

$$\frac{d\lambda_{3t}^o}{dw_{t+1}} = \frac{\beta}{\gamma_N} E_t \lambda_{1t+1}^o (1 - \tau_{t+1}^l) l_{1t+1}$$

$$\frac{d\lambda_{3t}^r}{dw_{t+1}} = \frac{\beta}{\gamma_N} E_t \lambda_{1t+1}^r (1 - \tau_{t+1}^l) l_{1t+1}$$

$$\frac{d\lambda_t^{nd}}{dw_{t+1}} = -\gamma_A \frac{\pi_{t+1}}{1 + r_t^m} E_t (1 + \tau_{t+1}^{sc}) l_{1t+1} = -\frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} E_t (1 + \tau_{t+1}^{sc}) l_{1t+1}$$

We obtain

$$\begin{aligned} & \lambda^w \lambda_t^{nd} \left[ \frac{\lambda^r}{\lambda_{1t}^r} \left( \frac{\beta}{\gamma_N} E_t \lambda_{1t+1}^r (1 - \tau_{t+1}^l) l_{1t+1} \right) + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \left( \frac{\beta}{\gamma_N} E_t \lambda_{1t+1}^o (1 - \tau_{t+1}^l) l_{1t+1} \right) \right] \\ & - (1 - \lambda^w) \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]^{\lambda^w} \left( \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} E_t (1 + \tau_{t+1}^{sc}) l_{1t+1} \right) = 0 \end{aligned}$$

Simplifying,

$$\begin{aligned} & \lambda^w \lambda_t^{nd} (1 - \tau_{t+1}^l) E_t \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \right] \\ & - (1 - \lambda^w) E_t \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right] \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} (1 + \tau_{t+1}^{sc}) = 0 \end{aligned}$$

which can be written in a more stylised way as,

$$\lambda^w \lambda_t^{nd} E_t \left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) (1 - \tau_{t+1}^l) - (1 - \lambda^w) \left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} (1 + \tau_{t+1}^{sc}) = 0$$

where:

$$\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) = \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \right]$$

$$\left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) = \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]$$

Now, let us divide the expression by  $\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right)$  to obtain

$$\lambda^w \lambda_t^{nd} E_t (1 - \tau_{t+1}^l) - (1 - \lambda^w) \left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \frac{1}{\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right)} (1 + \tau_{t+1}^{sc}) = 0$$

Clearing for  $\left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right)$  one obtains,

$$\left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) = \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} E_t \left( \frac{\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right)}{\frac{\lambda_{1t+1}^o}{\lambda_{1t}^o}} \right) \lambda_t^{nd}$$

Taking into account the definition of  $\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right)$ , the previous expression can be written as,

$$\left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) = \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} E_t \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} + (1 - \lambda^r) \right] \lambda_t^{nd} \quad (1.1)$$

Let us elaborate further on the left hand side of (1.1). Recall that,

$$\left( \frac{\widetilde{\lambda}_{3t}}{\lambda_{1t}} \right) = \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right]$$

Thus, bearing in mind that,

$$\frac{\lambda_{3t}^r}{\lambda_{1t}^r} = \frac{\beta}{\gamma_N} \frac{1}{\lambda_{1t}^r} E_t \left[ \begin{array}{l} u_t + \lambda_{1t+1}^r w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ + \lambda_{3t+1}^r [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right]$$

$$\frac{\lambda_{3t}^o}{\lambda_{1t}^o} = \frac{\beta}{\gamma_N} \frac{1}{\lambda_{1t}^o} E_t \left[ \begin{array}{l} u_t + \lambda_{1t+1}^o w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ + \lambda_{3t+1}^o [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right]$$

where  $\rho_{t+1}^w = \frac{\chi_1 v_{t+1}^{\chi_2} (s(1-n_t)l_{2t})^{1-\chi_2}}{1-n_t}$  is the probability of an unemployed worker finding a new job.

It follows that  $\left( \frac{\widetilde{\lambda}_{3t}}{\lambda_{1t}} \right)$  equals,

$$\left( \frac{\widetilde{\lambda}_{3t}}{\lambda_{1t}} \right) = \left[ \begin{array}{l} \lambda^r \frac{\beta}{\gamma_N} \frac{1}{\lambda_{1t}^r} E_t \left[ \begin{array}{l} u_t + \lambda_{1t+1}^r w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ + \lambda_{3t+1}^r [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right] + \\ (1 - \lambda^r) \frac{\beta}{\gamma_N} \frac{1}{\lambda_{1t}^o} E_t \left[ \begin{array}{l} u_t + \lambda_{1t+1}^o w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) \\ + \lambda_{3t+1}^o [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right] \end{array} \right]$$

This expression can be rewritten as,

$$\left( \frac{\widetilde{\lambda}_{3t}}{\lambda_{1t}} \right) = \left[ \begin{array}{l} \frac{\beta}{\gamma_N} E_t \left[ \lambda^r \frac{1}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{1}{\lambda_{1t}^o} \right] u_t + \\ \frac{\beta}{\gamma_N} E_t \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \right] w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) + \\ \frac{\beta}{\gamma_N} E_t \left[ \lambda^r \frac{\lambda_{3t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t+1}^o}{\lambda_{1t}^o} \right] [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right]$$

On the right hand side of (1.1) the term  $\lambda_t^{nd}$  is,

$$\lambda_t^{nd} = \frac{\beta}{\gamma_N} E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left( \alpha m c_{t+1} \frac{y_{t+1}}{n_t} - w_{t+1} (1 + \tau_{t+1}^{sc}) l_{1t+1} + \lambda_{t+1}^{nd} (1 - \sigma) \right)$$

Plugging now the expressions for  $\left( \frac{\widetilde{\lambda}_{3t}}{\lambda_{1t}} \right)$  and  $\lambda_t^{nd}$  into (1.1) and simplifying further it fol-



lows that,

$$\left[ \begin{array}{c} E_t \left[ \lambda^r \frac{1}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{1}{\lambda_{1t}^o} \right] u_t + \\ E_t \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \right] w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} s l_{2t}) + \\ E_t \left[ \lambda^r \frac{\lambda_{3t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t+1}^o}{\lambda_{1t}^o} \right] [(1 - \sigma) - \rho_{t+1}^w] \end{array} \right] = \\ \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} E_t \left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) \left( \alpha mc_{t+1} \frac{y_{t+1}}{n_t} - w_{t+1} (1 + \tau_{t+1}^{sc}) l_{1t+1} + \lambda_{t+1}^{nd} (1 - \sigma) \right)$$

Gathering terms on wages,

$$\left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) w_{t+1} (1 - \tau_{t+1}^l) (l_{1t+1} - rr_{t+1} l_{2t}) + \frac{\lambda^w}{(1 - \lambda^w)} (1 - \tau_{t+1}^l) E_t \left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) w_{t+1} s l_{1t+1} = \\ \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} E_t \left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) \left( \alpha mc_{t+1} \frac{y_{t+1}}{n_t} + \lambda_{t+1}^{nd} (1 - \sigma) \right) - \\ E_t \left[ \lambda^r \frac{1}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{1}{\lambda_{1t}^o} \right] u_t - E_t \left[ \lambda^r \frac{\lambda_{3t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t+1}^o}{\lambda_{1t}^o} \right] [(1 - \sigma) - \rho_{t+1}^w] \quad (2)$$

Taking into account that the first order condition for vacancies arising from the firm's optimisation problem is

$$\lambda_t^{nd} = \kappa_v \frac{v_t}{\chi_1 v_t^{\chi_2} ((1 - n_{t-1}) s l_{2t})^{1 - \chi_2}}$$

and defining the probability that a firm may fill a vacancy as  $\rho_t^f = \frac{\chi_1 v_t^{\chi_2} ((1 - n_{t-1}) s l_{2t})^{1 - \chi_2}}{v_t}$ , then  $\lambda_{t+1}^{nd} = \frac{\kappa_v}{\rho_{t+1}^f}$ . Using this result to feed (1.2) and simplifying the expression further one is left with,

$$\left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) w_{t+1} (1 - \tau_{t+1}^l) l_{1t+1} \left[ \frac{1}{1 - \lambda^w} - rr_{t+1} s \frac{l_{2t}}{l_{1t+1}} \right] = \\ \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} \left( \widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}} \right) \left( \alpha mc_{t+1} \frac{y_{t+1}}{n_t} + (1 - \sigma) \frac{\kappa_v}{\rho_{t+1}^f} \right) - \\ \left( \widetilde{\frac{1}{\lambda_{1t}}} \right) u_t - \left( \widetilde{\frac{\lambda_{3t+1}}{\lambda_{1t}}} \right) [(1 - \sigma) - \rho_{t+1}^w]$$

where:

$$\left(\widetilde{\frac{1}{\lambda_{1t}}}\right) = \left[\lambda^r \frac{1}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{1}{\lambda_{1t}^o}\right]$$

$$\left(\widetilde{\frac{\lambda_{3t+1}}{\lambda_{1t}}}\right) = \left[\lambda^r \frac{\lambda_{3t+1}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t+1}^o}{\lambda_{1t}^o}\right]$$

Finally, rearranging terms we obtain the following solution for real wages:

$$(1 + \tau_t^{sc})w_{t+1}l_{1t+1} = \frac{\lambda^w}{\left[1 - (1 - \lambda^w)rr_{t+1}s_{t+1}^l\right]} \left[\alpha mc_{t+1} \frac{y_{t+1}}{n_t} + (1 - \sigma) \frac{\kappa_v}{\rho_{t+1}^f}\right]$$

$$+ \frac{(1 - \lambda^w)}{\left[1 - (1 - \lambda^w)rr_t^l\right]} \frac{(1 + \tau_{t+1}^{sc})}{(1 - \tau_{t+1}^l)} \left[ \frac{\left(\widetilde{\frac{\lambda_{3t+1}}{\lambda_{1t}}}\right)}{\left(\widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}}\right)} [\rho_{t+1}^w - (1 - \sigma)] - \frac{\left(\widetilde{\frac{1}{\lambda_{1t}}}\right)}{\left(\widetilde{\frac{\lambda_{1t+1}}{\lambda_{1t}}}\right)} u_t \right]$$

Hours setting: Thus, the first order condition for hours is

$$\frac{\partial}{\partial l_{1,t+1}} = \lambda^w \left[\lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right]^{\lambda^w - 1} \left(\lambda_t^{nd}\right)^{(1 - \lambda^w)} \left[\frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dl_{1,t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dl_{1,t+1}}\right]$$

$$+ (1 - \lambda^w) \left(\lambda_t^{nd}\right)^{-\lambda^w} \left[\lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right]^{\lambda^w} \frac{d\lambda_t^{nd}}{dl_{1,t+1}} = 0$$

Thus, rearranging terms

$$\left[\lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right]^{\lambda^w - 1} \left(\lambda_t^{nd}\right)^{-\lambda^w} \left[ \frac{\lambda^w \lambda_t^{nd}}{(1 - \lambda^w)} \left[\frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dl_{1,t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dl_{1,t+1}}\right] + \frac{d\lambda_t^{nd}}{dl_{1,t+1}} \right] = 0$$

Simplifying further

$$\lambda^w \lambda_t^{nd} \left[\frac{\lambda^r}{\lambda_{1t}^r} \frac{d\lambda_{3t}^r}{dl_{1,t+1}} + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \frac{d\lambda_{3t}^o}{dl_{1,t+1}}\right] + (1 - \lambda^w) \left[\lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right] \frac{d\lambda_t^{nd}}{dl_{1,t+1}} = 0$$

Taking into account that

$$\frac{d\lambda_{3t}^o}{dl_{1,t+1}} = \frac{\beta}{\gamma_N} \left[-\phi_1 [1 - l_{1t+1}]^{-\eta} + \lambda_{1t+1}^o w_{t+1} (1 - \tau_{t+1}^l)\right]$$

$$\frac{d\lambda_{3t}^r}{dl_{1,t+1}} = \frac{\beta}{\gamma_N} \left[-\phi_1 [1 - l_{1t+1}]^{-\eta} + \lambda_{1t+1}^r w_{t+1} (1 - \tau_{t+1}^l)\right]$$

$$\frac{d\lambda_t^{nd}}{dl_{1,t+1}} = \frac{\beta}{\gamma_N} \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left[ \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} - w_{t+1} (1 + \tau_{t+1}^{sc}) \right]$$

we obtain

$$\lambda^w \lambda_t^{nd} \left[ \frac{\lambda^r}{\lambda_{1t}^r} \left( \frac{\beta}{\gamma_N} \left[ -\phi_1 [1 - l_{1t+1}]^{-\eta} + \lambda_{1t+1}^r w_{t+1} (1 - \tau_{t+1}^l) \right] \right) \right. \\ \left. + \frac{(1 - \lambda^r)}{\lambda_{1t}^o} \left( \frac{\beta}{\gamma_N} \left[ -\phi_1 [1 - l_{1t+1}]^{-\eta} + \lambda_{1t+1}^o w_{t+1} (1 - \tau_{t+1}^l) \right] \right) \right] \\ - (1 - \lambda^w) \left[ \lambda^r \frac{\lambda_{3t}^r}{\lambda_{1t}^r} + (1 - \lambda^r) \frac{\lambda_{3t}^o}{\lambda_{1t}^o} \right] \left( \frac{\beta}{\gamma_N} \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left[ \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} - w_{t+1} (1 + \tau_{t+1}^{sc}) \right] \right) = 0$$

Notice that, as in Pissarides (2000) and Andolfatto (1996) we assume that the marginal product of labour is taken as given by the agents during the bargaining process.

Simplifying

$$\lambda^w \lambda_t^{nd} \left\{ - \left( \frac{1}{\lambda_{1t}} \right) \phi_1 [1 - l_{1t+1}]^{-\eta} + \left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) w_{t+1} (1 - \tau_{t+1}^l) \right\} \\ + (1 - \lambda^w) \left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left[ \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} - w_{t+1} (1 + \tau_{t+1}^{sc}) \right] = 0$$

If we now make use of the expression for  $\left( \frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}} \right)$  in (1.1), we can rewrite previous equation as,

$$\lambda^w \lambda_t^{nd} \left\{ - \left( \frac{1}{\lambda_{1t}} \right) \phi_1 [1 - l_{1t+1}]^{-\eta} + \left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) w_{t+1} (1 - \tau_{t+1}^l) \right\} \\ + (1 - \lambda^w) \left( \frac{\lambda^w}{(1 - \lambda^w)} \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} + (1 - \lambda^r) \right] \lambda_t^{nd} \right) \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \\ \left[ \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} - w_{t+1} (1 + \tau_{t+1}^{sc}) \right] = 0$$

Simplifying further, the expression takes the form,

$$\left\{ - \left( \frac{1}{\lambda_{1t}} \right) \phi_1 [1 - l_{1t+1}]^{-\eta} \right\} + \frac{(1 - \tau_{t+1}^l)}{(1 + \tau_{t+1}^{sc})} \left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} = 0$$

Thus,

$$\left( \frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}} \right) \alpha mc_{t+1} \frac{y_{t+1}}{n_t l_{1,t+1}} = \left( \frac{1}{\lambda_{1t}} \right) \frac{(1 + \tau_{t+1}^{sc})}{(1 - \tau_{t+1}^l)} \phi_1 [1 - l_{1t+1}]^{-\eta}$$

## Appendix 2: Net foreign assets accumulation

From the government budget constraint:

$$\gamma_A \gamma_N b_t = g_t^c + g_t^i + g_{ut}(1 - n_{t-1}) + g_{st} - t_t + \frac{(1 + r_t^n)}{1 + \pi_t} b_{t-1}$$

and assuming  $\gamma_A \gamma_N b_t - \frac{(1 + r_t^n)}{1 + \pi_t} b_{t-1} = 0$  we obtain

$$t_t = g_t^c + g_t^i + g_{ut}(1 - n_{t-1}) + g_{st} \quad (2.1)$$

Recall the aggregate resource constraint:

$$c_{ht} + i_{ht} + g_t^c + g_t^i + \frac{P_t^x}{P_t} ex_t = y_t - \frac{P_t^c}{P_t} (1 - \alpha_e) e_t - \kappa_v v_t - \kappa_f \quad (2.2)$$

Multiplying the optimizing household budget constraint by  $(1 - \lambda^r)$  and the *RoT* budget constraint by  $\lambda^r$ , we obtain after aggregation

$$\begin{aligned} & (r_t(1 - \tau_t^k) + \tau_t^k \delta) k_{t-1} + w_t (1 - \tau_t^l) (n_{t-1} l_{1t} + \bar{r} \bar{r} (1 - n_{t-1}) l_{2t}) + (1 - \tau_t^l) g_{st} - trh_t + \\ & \frac{m_{t-1}}{1 + \pi_t^c} + (1 + r_{t-1}^n) \frac{b_{t-1}}{1 + \pi_t^c} + er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^w}{1 + \pi_t^c} \\ & - (1 + \tau_t^c) c_t \frac{P_t^c}{P_t} - \frac{P_t^i}{P_t} j_t \left( 1 + \frac{\phi}{2} \left( \frac{j_t}{k_{t-1}} \right) \right) - \gamma_A \gamma_N \left( m_t + b_t + \frac{er_t^n b_t^w}{\phi_{bt}} \right) = 0 \end{aligned}$$

and rearrange terms to leave taxes on the left hand side

$$\begin{aligned} & r_t \tau_t^k k_{t-1} - \tau_t^k \delta k_{t-1} + w_t \tau_t^l n_{t-1} l_{1t} + \bar{r} \bar{r} w_t \tau_t^l (1 - n_{t-1}) l_{2t} - (1 - \tau_t^l) g_{st} + trh_t + \\ & \tau_t^c c_t \frac{P_t^c}{P_t} = r_t k_{t-1} + w_t n_{t-1} l_{1t} + \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} - \frac{P_t^c}{P_t} c_t \\ & - \frac{P_t^i}{P_t} j_t \left( 1 + \frac{\phi}{2} \left( \frac{j_t}{k_{t-1}} \right) \right) + er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^w}{1 + \pi_t^c} - \gamma_A \gamma_N \frac{er_t^n b_t^w}{\phi_{bt}} \end{aligned} \quad (2.3)$$

where we have assumed that  $\gamma_A \gamma_N b_t - \frac{(1 + r_t^n)}{1 + \pi_t} b_{t-1} = 0$  and  $\frac{m_{t-1}}{1 + \pi_t^c} - \gamma_A \gamma_N m_t = 0$ . From the definition of government tax revenues,

$$\begin{aligned} t_t &= (\tau_t^l + \tau_t^{sc}) w_t (n_{t-1} l_{1t}) + \tau_t^k (r_t - \delta) k_{t-1} \\ &+ \tau_t^c \frac{P_t^c}{P_t} c_t + \tau_t^e \frac{P_t^e}{P_t} e_t + trh_t + \tau_t^l \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} + \tau_t^l g_{st} \end{aligned}$$

and using (2.1) it follows that

$$\begin{aligned} g_t^c + g_t^i + \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} + g_{st} &= (\tau_t^l + \tau_t^{sc}) w_t (n_{t-1} l_{1t}) + \tau_t^k (r_t - \delta) k_{t-1} \\ &+ \tau_t^c \frac{P_t^c}{P_t} c_t + \tau_t^e \frac{P_t^e}{P_t} e_t + trh_t + \tau_t^l \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} + \tau_t^l g_{st} \end{aligned}$$

Rearranging terms, the left hand side of (2.3) turns out to be:

$$r_t \tau_t^k k_{t-1} - \tau_t^k \delta k_{t-1} + w_t \tau_t^l n_{t-1} l_{1t} + \bar{r} \bar{r} w_t \tau_t^l (1 - n_{t-1}) l_{2t} - \\ (1 - \tau_t^l) g_{st} + tr h_t + \tau_t^c c_t \frac{P_t^c}{P_t} = g_t^c + g_t^i + \bar{r} \bar{r} w_t (1 - n_{t-1}) - \tau_t^e \frac{P_t^e}{P_t} e_t - \tau_t^{sc} w_t n_{t-1} l_{1t}$$

Introducing this result in (2.3) we obtain

$$g_t^c + g_t^i + \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} - \tau_t^e \frac{P_t^e}{P_t} e_t - \tau_t^{sc} w_t n_{t-1} l_{1t} = \\ = r_t k_{t-1} + w_t n_{t-1} l_{1t} + \bar{r} \bar{r} w_t (1 - n_{t-1}) l_{2t} - \frac{P_t^c}{P_t} c_t \\ - \frac{P_t^i}{P_t} i_t + er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^{ow}}{1 + \pi_t^c} - \gamma_A \gamma_N \frac{er_t^n b_t^{ow}}{\phi_{bt}}$$

Rearranging terms:

$$\frac{P_t^c}{P_t} c_t + \frac{P_t^i}{P_t} i_t + g_t^c + g_t^i - \tau_t^e \frac{P_t^e}{P_t} e_t - \tau_t^{sc} w_t n_{t-1} l_{1t} \\ - r_t k_{t-1} - w_t n_{t-1} l_{1t} = \\ er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^{ow}}{1 + \pi_t^c} - \gamma_A \gamma_N \frac{er_t^n b_t^{ow}}{\phi_{bt}}$$

Now, using (2.2) and taking into account that  $P_t^c c_t = P_t c_{ht} + P_t^m c_{ft}$ ,  $P_t^i i_t = P_t i_{ht} + P_t^m i_{ft}$  and  $P_t^m im_t = P_t^m c_{ft} + P_t^m i_{ft} + P_t^e \alpha_e e_t$ :

$$y_t - (1 + \tau_t^e) \frac{P_t^e}{P_t} e_t - \kappa_v v_t - \kappa_f - \tau_t^{sc} w_t n_{t-1} l_{1t} \\ - r_t k_{t-1} - w_t n_{t-1} l_{1t} + \frac{P_t^m}{P_t} im_t - \frac{P_t^x}{P_t} ex_t = \\ er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^{ow}}{1 + \pi_t^c} - \gamma_A \gamma_N \frac{er_t^n b_t^{ow}}{\phi_{bt}}$$

Notice that GDP measured at factor costs is given by:

$$gdp_t = y_t - (1 + \tau_t^e) \frac{P_t^e}{P_t} e_t - \kappa_v v_t - \kappa_f - \tau_t^{sc} w_t n_{t-1} l_{1t}$$

or, alternatively:

$$gdp_t = r_t k_{t-1} + w_t n_{t-1} l_{1t}$$

Therefore:

$$\gamma_A \gamma_N \frac{er_t^n b_t^{ow}}{\phi_{bt}} - er_t^n (1 + r_{t-1}^{nw}) \frac{b_{t-1}^{ow}}{1 + \pi_t^c} = \frac{P_t^x}{P_t} ex_t - \frac{P_t^m}{P_t} im_t$$

### Appendix 3: Using behavioural equations to produce non-observable data

Several variables included in the model have no direct statistical counterparts from official sources. Such variables include consumption and employment of both *RoT* and optimizing consumers, Lagrange multipliers, Tobin's  $q$ , the composite capital stock, the marginal cost and a measure of total factor productivity. In order to sidestep the lack of data availability affecting these variables, one may use the model's related behavioural equations. For instance, consumption of *RoT* consumers is obtained from the first order condition represented by the budget restriction:

$$c_t^r = \frac{1}{(1 + \tau_t^c)} \frac{P_t}{P_t^c} \left[ w_t (1 - \tau_t^l) (n_{t-1}^r l_{1t} + r r_t (1 - n_{t-1}^r) l_{2t}) + g_{st} \right]$$

while consumption of optimizing households is obtained from consumption of *RoT* consumers and aggregate consumption according to:

$$c_t^o = \frac{c_t - \lambda^r c_t^r}{(1 - \lambda^r)}$$

As regards the employment rates of the different household types, under the assumption of identical  $\rho_t^w$  and  $\sigma$ , it follows that  $n_t = n_t^o = n_t^r$ . To see this more clearly, one may write the law of motion of employment in terms of the number of employees  $E_t^o$  and  $E_t^r$ :

$$E_1^o = (1 - \sigma) E_0^o + \rho_t^w (N_0^o - E_0^o)$$

$$E_1^r = (1 - \sigma) E_0^r + \rho_t^w (N_0^r - E_0^o)$$

Let the initial condition satisfy  $\frac{N_0^o}{N_0} = 1 - \lambda^r$  and  $\frac{E_0^o}{E_0} = 1 - \lambda^r$ . Then it is convenient to write  $E_1^o$  in terms of  $E_1^r$  as:

$$E_1^o = \frac{1 - \lambda^r}{\lambda^r} [(1 - \sigma) E_0^r + \rho_t^w (N_0^r - E_0^o)] = \frac{1 - \lambda^r}{\lambda^r} E_1^r$$

that in per capita terms turns out to be:

$$\frac{N_0^o}{N_0} \frac{E_1^o}{N_0^o} = \frac{1 - \lambda^r}{\lambda^r} \frac{E_1^r}{N_0^r} \frac{N_0^r}{N_0}$$

or

$$(1 - \lambda^r) n_1^o = \frac{1 - \lambda^r}{\lambda^r} n_1^r \lambda^r \rightarrow n_t^o = n_t^r = n_t$$

The marginal utility of consumption for both Ricardian and *RoT* consumers is obtained from the following conditions,

$$\lambda_{1t}^o = \frac{1}{(P_t^c/P_t)c_t^o(1+\tau_t^c)}$$

$$\lambda_{1t}^r = \frac{1}{(P_t^c/P_t)c_t^r(1+\tau_t^c)}$$

that in turn allow us to compute both  $\left(\frac{\widetilde{\lambda_{1t+1}}}{\lambda_{1t}}\right)$  and  $\left(\frac{1}{\lambda_{1t}}\right)$ .

The shadow price of an additional worker for the firm,  $\lambda_t^{nd}$ , can be computed using the free-entry condition affecting the creation of vacancies:

$$\lambda_t^{nd} = \frac{\kappa_v v_t}{mat_t} \quad (3.1)$$

being  $\kappa_v$  the flow cost of an open vacancy,  $v$  the number of vacancies and  $mat$  the number of new matches. The flow cost of an open vacancy has been estimated assuming that, on average, the overall cost of posted vacancies in the economy,  $\kappa_v v$ , represents 0.5 percent of GDP (see Andolfatto, 1996).

In order to obtain the shadow price of a new job match one may use the expression:

$$\left(\frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}}\right) = \frac{\lambda^w}{(1-\lambda^w)} \frac{(1-\tau_{t+1}^l)}{(1+\tau_{t+1}^{sc})} E_t \left[ \lambda^r \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} + (1-\lambda^r) \right] \lambda_t^{nd} \quad (3.2)$$

This expression, which concerns the derivation of the equilibrium wage, equates the marginal utility of a new match for firms and workers and allows us to identify the weighted average  $\left(\frac{\widetilde{\lambda_{3t}}}{\lambda_{1t}}\right)$ . In order to pin down  $\lambda_{3t}^o$  and  $\lambda_{3t}^r$ , we impose the restriction that (3.2) has to be satisfied. Thus, we use the following estimators:

$$\left(\frac{\lambda_{3t}^r}{\lambda_{1t}^r}\right) = \frac{\lambda^w}{(1-\lambda^w)} \frac{(1-\tau_{t+1}^l)}{(1+\tau_{t+1}^{sc})} E_t \left[ \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} \right] \lambda_t^{nd} \quad (3.3)$$

$$\left(\frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right) = \frac{\lambda^w}{(1-\lambda^w)} \frac{(1-\tau_{t+1}^l)}{(1+\tau_{t+1}^{sc})} \lambda_t^{nd} \quad (3.4)$$

Notice that the following condition can be derived from the above two expressions:

$$\left(\frac{\lambda_{3t}^r}{\lambda_{1t}^r}\right) = E_t \left[ \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \frac{\lambda_{1t}^o}{\lambda_{1t+1}^o} \right] \left(\frac{\lambda_{3t}^o}{\lambda_{1t}^o}\right)$$

The intuition behind this is straightforward. When the ratio of marginal utilities of consumption is the same for both types of consumers,  $\frac{\lambda_{1t}^r}{\lambda_{1t}^o} = \frac{\lambda_{1t+1}^r}{\lambda_{1t+1}^o}$ , the value of a job match is also equal. However, the more restricted the *RoT* consumer is in terms of consumption smoothing, the larger the difference is between  $\frac{\lambda_{1t+1}^r}{\lambda_{1t}^r}$  and  $\frac{\lambda_{1t+1}^o}{\lambda_{1t}^o}$ , and the more valuable finding a job is for the *RoT* consumer. From expressions (3.3) and (3.4) we obtain  $\lambda_{3t}^r$  and  $\lambda_{3t}^o$  and hence  $\left(\frac{\lambda_{3t+1}^o}{\lambda_{1t}^o}\right)$ .

Time series for the Tobin's  $q$  have been constructed from the following expression:

$$q_t = \left(1 + \phi \left(\frac{j_t}{k_{t-1}}\right)\right) \quad (3.5)$$

This behavioural equation, which is derived from the household maximization problem, states that the amount of investment net of adjustment costs,  $j_t$ , is undertaken to the extent that the opportunity cost of a marginal increase in investment in terms of consumption,  $1 + \phi \left(\frac{j_t}{k_{t-1}}\right)$ , is equal to its marginal expected contribution to household utility. The adjustment costs parameter,  $\phi$ , has been set at 5.5, which is the value estimated for Spain within the QUEST II model.

Capital stock is modelled as a composite of physical capital and energy, according to the following CES technology:

$$ke_t = \left[ak_{t-1}^{-\rho} + (1-a)e_t^{-\rho}\right]^{-\frac{1}{\rho}} \quad (3.6)$$

As is standard in the empirical literature, we rely on a Cobb-Douglas production function to residually compute an indicator for total factor productivity,  $A_t$ . The production function in levels is given by:

$$Y_{it} = \left\{ \left[ a\tilde{k}_{it-1}^{-\rho} + (1-a)En_{it}^{-\rho} \right]^{-\frac{1}{\rho}} \right\}^{1-\alpha} (A_t E_{it-1} l_{it})^\alpha \left( k_{it-1}^p \right)^\zeta \quad (3.7)$$

where  $E$  represents the employed population and  $k_{it-1}^p$  appears in efficiency units. The same expression in per capita terms can be written as:

$$\tilde{y}_{it} = \left\{ \left[ a\tilde{k}_{it-1}^{-\rho} + (1-a)\tilde{e}_{it}^{-\rho} \right]^{-\frac{1}{\rho}} \right\}^{1-\alpha} (A_t n_{it-1} l_{it})^\alpha \left( \frac{K_{it-1}^p}{A_t N_t} \right)^\zeta \quad (3.8)$$

and clearing for  $A_t$  :

$$A_t = \left( \frac{\tilde{y}_{it}}{\tilde{k}_{it-1}^{1-\alpha} (n_{it-1} l_{it})^\alpha \tilde{k}_{it-1}^{p\zeta}} \right)^{\frac{1}{\alpha-\zeta}}$$



where  $n_t l_t$  represents overall hours worked and  $y_t$  reflects (per capita) gross output, which includes gross value added, imported energy and the (time-varying) fixed costs weighing on non-competitive firms. Furthermore, in the calibration exercise we decompose  $A_t$  into a permanent and a transitory component ( $z$ ):

$$\ln A_t = \overline{\ln A_t} + z_t \quad (3.9)$$

where the permanent component is used to obtain variables in efficiency units.

Finally, we ascertain the value of the marginal cost series by using the inter-temporal demand for labour condition, i.e.

$$\gamma_N \lambda_t^{nd} = \frac{1 + \pi_{t+1}}{1 + r_{t+1}^n} \left[ \alpha mc_{t+1} \frac{y_{t+1}}{n_t} - w_{t+1} (1 + \tau_{t+1}^{sc}) l_{1t+1} + \lambda_{t+1}^{nd} (1 - \sigma) \right] \quad (3.10)$$

Whereby optimality requires the marginal contribution of a newly created job to the firm's profit being equal to the marginal product of labour net of the wage rate plus the capital value of the new job in  $t + 1$ , corrected by the job destruction rate between  $t$  and  $t + 1$ . Leaving the marginal cost series aside, any variable in the expression above is taken to the data in our calibration procedure, meaning that the former can be expressed as a function of the latter. More precisely,  $\gamma_N$ ,  $\pi$ ,  $r^n$ ,  $y$ ,  $w$ ,  $\tau^{sc}$ ,  $l$  and  $\sigma$  respectively match the actual values of the labour force (gross) growth rate, inflation rate, nominal interest rate and gross output as defined above, the hourly real wage, the payroll tax rate, the number of working hours per employee and the exogenous rate of job destruction.

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