

# Schumpeterian Foundations of Real Business Cycles

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## ABSTRACT

Technology shocks are at the core of real business cycle models. Although traditionally described as exogenous, technology shocks can be the result of the endogenous decisions by economic agents under uncertainty. To demonstrate it, in this paper I develop a dynamic stochastic general equilibrium model that incorporates Schumpeterian endogenous growth features that affect the convergence to the steady-state. In this model, technology advances are due to the introduction of vertical innovations by entrepreneurs who try to become monopolists in different economic sectors. Entrepreneurs' ventures are financed by banks. The model is solved and estimated by bayesian methods for the United States economy to compute the value of some of its structural parameters. Results show that for a country close to the technology frontier, the presented innovation mechanism is roughly equivalent in terms of volatilities, correlations and impulse responses to technology shocks in real business cycle models. Therefore, the behavior of the productivity can be due not only to technology considerations but also to financial and entrepreneurial reasons. (JEL C50, E27, O40)

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## 1. Introduction

Joseph A. Schumpeter’s economic thought offers a different perspective of the process of economic growth and its fluctuations from the neoclassical and keynesian traditions. In contrast to both schools, Schumpeter presented the economy as a dynamic system characterized by a Darwinian competition between firms. He emphasized the central roles of the entrepreneur as the agent of economic development and of the banker as the figure that replaces the traditional capitalist as the main provider of funds (Schumpeter, 1934). Therefore, schumpeterian theory may provide a foundation to productivity changes, denoted as “technology shocks” in real business cycles (RBC) literature. According to Schumpeter, business cycles are a consequence of the growth process, as new entrepreneurs do not appear continuously, but in groups or swarms. This initial “innovation ”shock is transmitted to the rest of the economy through a “secondary boom” that involves the banks, the producers of capital and consumption goods and finally the workers, from where it “...oozes into every economic channel...” (Schumpeter, 1934 pp. 223-226).

Schumpeter’s endogenous growth theory was introduced in the field of neoclassical economics by the initial works of Aghion and Howitt (1992) and Grossman and Helpman (1991). These models enhanced the framework à la Solow (1956) and Swan (1956) to explain long-run productivity growth as the result of successive “vertical” technology innovations that result from uncertain research activities<sup>1</sup>. However these models were mainly used to analyze the evolution of economies in the long-run and in general paid little attention to the interaction with the business cycles<sup>2</sup>.

Most real business cycles models follow the seminal work of Kydland and Prescott (1982) and are based in the neoclassical Solow model. However, several authors have intended to expand this framework to include some endogenous growth models in order to explore the links between growth and business cycles. Fatás (1998), for example, integrated a RBC model with an AK endogenous growth theory and there has been considerable work in the development of RBC models with learning by doing growth<sup>3</sup>. Nevertheless, the most important contribution so far has been Comin and Gertler (2006), who developed a Dynamic Stochastic General Equilibrium Model (DSGE) model that combined some neokeynesian features, such as monopolistic competition or wage markups, with an endogenous growth

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<sup>1</sup>Others authors have produce valuable research in this line, as Aghion and Howitt (1998, Ch. 12), Dinopoulos and Thompson (1998), Howitt (2000), and Segerstrom (1998).

<sup>2</sup>An exception is Aghion and Howitt (1998, Chapter 8).

<sup>3</sup>See, for example, Stadler (1990), and Blackburn and Pelloni (2005).

theory with R&D expanding the variety of intermediate goods à la Romer (1990).

There have also been some preliminary attempts to test whether Schumpeterian growth may constitute a plausible source of economic fluctuations, such as Phillips and Wrase (2006). In their study, the authors calibrate a simple model of vertical innovations and compare it against U.S. data. They conclude that the process alone, without other diffusion mechanisms such as labor-leisure choice or rigidities, is not appropriate to replicate the statistical properties found in the data. However, The Phillips and Wrase (2006) model does not include the presence of a financial sector. Financial frictions are an important source of business fluctuations<sup>4</sup> and recently there has been a considerable amount of research devoted to explore the links between financial frictions and economic growth, such as Banerjee and Duflo (2005) or Buera and Shin (2008). The relationship between growth and finance has also been explored for Schumpeterian endogenous growth models, such as King and Levine (1993) or Aghion, Howitt and Mayer-Foulkes (2005).

The aim of this paper is to explore whether the introduction of Schumpeterian growth provides some mechanisms that help to explain economic fluctuations. I depart from the traditional persistence explanations such as labor-leisure substitution, sticky prices or adjustment costs and present a model where the propagation mechanism is the decision of the banks to finance new enterprises that may replace the current incumbents in a creative destruction process. There is no doubt that such a simplified model cannot describe the full working of a real economy, but it may introduce a link with growth that is missing in the current generation of DSGE models, such as Christiano, Eichenbaum and Evans (2005) where the economy is exposed to “technology shocks” that are independent of the general economic performance, in contrast to what almost all the endogenous growth theory state.

The structure of the paper is as follows. In section 2 it is presented a theoretical model based in Howitt (2000) and Aghion and Howitt (1998, Chapter 12) that enhances a simple RBC model to include a financial sector and a creative destruction process. In this model, technology advances are not exogenous, but they happen due to innovations by entrepreneurs. These entrepreneurs innovate in an attempt to replace the current monopolists that populate the different economic sectors. To fund the innovation process, entrepreneurs should rely on the financial sector, whose role is to allocate resources in the most profitable way. The model exhibits a balanced growth path in its two state variables: effective capital per worker and productivity relative to the technology frontier. In section 3 this theoretical model is taken to the data. To do so, I estimate a subset of the model structural parameters by bayesian methods, as in Smets and Wouters (2003). Bayesian estimation allows to combine

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<sup>4</sup>See, for example, Bernanke, Gertler and Gilchrist (1999) or Knoop (2008).

the a priori information that could be used in calibration with a full information approach. To explore the relevance of the new proposed mechanism, the model is compared with a simple RBC one with exogenous technology shocks. Results show that the proposed model seem to replicate the behaviour of a RBC with technology shocks, thus providing some microfoundations for these technology shocks. In section 4 I summarize the main findings of the paper and conclude by proposing future lines of research.

## 2. The Model Economy

This section introduces a general equilibrium model based on Schumpeter (1934). The model is annual as opposed to quarterly as its focus is in fluctuations over a longer horizon than is typically studied in business cycle research, in line with Comin and Gertler (2006). As commented above, the model abstracts from a number of complication factors that otherwise might be useful for understanding quarterly dynamics, such as money or nominal rigidities, and it assumes them as embedded into a set of stochastic shocks.

The model presented here is inspired by the endogenous growth models with capital accumulation of Howitt (2000) and Aghion and Howitt (1998, Chapter 12). The economy is populated by final goods producers who use intermediate goods along with labor as inputs in a perfectly competitive framework. Each intermediate good is produced by an incumbent monopolist using capital. Intermediate sectors differ in their different technologies. Monopolists rent the capital from households and earn a flow of profits by selling their products. Innovations are targeted at specific intermediate products and allow successful entrepreneurs to replace incumbent monopolists. When a successful innovation arrives, a new technologically enhanced version of the intermediate good is produced, and this new technology may be applied via a spill-over effect by potential entrepreneurs in other sectors to improved their own products. As a consequence, there is a process of “creative destruction” that fuels economic growth. The financial sector allocates resources to entrepreneurs in different sectors. It is assumed to be risk neutral profit-maximizing, and it is the only source of resources for entrepreneurs. If entrepreneurs fail to achieve an innovation, all the funds are lost. If they succeed the financial sector receives the flow of profits accrued by the intermediate firms. Finally, households are conventional risk-averse consumption-maximizers who rent capital to intermediate firms, received wages from final goods firms and dividends from the financial sector.

I first describe the final and the intermediate goods firms and characterize the innovation process. I next introduce the financial sector and households. Finally I describe the complete equilibrium and its steady-state.

## 2.1. Final Good Firms

In the model, a country economy produces a final good under perfect competition by using labor and a continuum of intermediate products, according to the production function

$$Y_t = \int_0^{J_t} A_t(j) \varphi_t^\alpha(j) (L_t/J_t)^{1-\alpha} dj, \quad (2-1)$$

where  $J_t$  measures the number of different intermediate products produced and used in the country,  $\varphi_t(j)$  is the flow output of intermediate product  $j \in [0, J_t]$ , and  $A_t(j)$  is a productivity parameter attached to the latest version of intermediate product  $j$ . The number of intermediate products is assumed to be proportional to the labor-force size, so the number of workers per product  $N \equiv L_t/J_t$  is constant. The form of the production function (2-1) ensures that the growth in product variety does not affect aggregate productivity model. This and the fact that the number of intermediate products grows linearly with the market size guarantees that the model does not exhibit the sort of scale effects criticized in Jones (1995). In this model, neither a bigger population by itself nor its rate of growth will raise the incentive to innovate by raising the size of the market captured by the entrepreneur, because each innovation is restricted to a unique intermediate good, and the number of consumers per product does not increase with the size of the population.

Solving the profit-maximization problem for the final-good firms the price of intermediate goods results in

$$p_t(j) = \alpha A_t(j) (\varphi_t(j)/N)^{\alpha-1}, \quad (2-2)$$

and the wages per labor-unit

$$w_t = (1 - \alpha) y_t, \quad (2-3)$$

where  $y_t \equiv \frac{Y_t}{L_t}$  is the output per labor unit. Equation (2-3) indicates that the income share devoted to workers is the constant  $(1 - \alpha)$ , the same as in the Solow model.

## 2.2. Intermediate Good Firms

Final output can be used interchangeably as a consumption or capital good, or as an input to innovation. Each intermediate product is produced by an incumbent monopolist using capital, according to the production function:

$$\varphi_t(j) = (u_t K_t(j)) / A_t(j), \quad (2-4)$$

where  $K_t(j)$  is the input of capital in sector  $j$  and  $u_t$  is its utilization rate. Division by  $A_t(j)$  indicates that successive vintages of the intermediate product are produced by increasingly

capital-intensive techniques. The incumbent monopolist of each sector operates with a price schedule given by (2-2) and a cost function equal to  $(r_t + \delta)K_t(j)$ , where  $r_t$  is the rate of interest.

**Proposition 1** *The intermediate good firms of all the sectors in the economy produce the same amount of intermediate good  $\varphi_t = \varphi_t(j)$ .*

PROOF. As both marginal cost  $A_t(j)(r_t + \delta)\varphi_t(j)/u_t$  and marginal revenues  $A_t(j)\alpha N^{\alpha-1}\varphi_t(j)^\alpha$  are proportional to  $A_t(j)$  and this is the only difference between sectors, they all choose to supply  $\varphi_t = \varphi_t(j) = u_t \hat{k}_t N$ ,  $\forall j$ , where  $\hat{k}_t$  is the capital stock per effective labor unit  $\hat{k}_t \equiv K_t/(A_t L_t)^5$  and  $A_t$  is the average productivity across all sectors  $A_t \equiv \frac{1}{J_t} \int_0^{J_t} A_t(j) dj$  Q.E.D.

The output per labor-unit results in

$$y_t = \int_0^{J_t} A_t(j) \varphi_t^\alpha L_t^{-\alpha} J_t^{\alpha-1} dj = A_t \left( u_t \hat{k}_t \right)^\alpha. \quad (2-5)$$

Instead of providing explicit microfoundations for the capital utilization, the model is simplified by assuming  $u_t$  to follow an autoregressive stochastic process

$$\log u_{t+1} \equiv (1 - \rho_u) \log u + \rho_u \log u_t + \sigma_u \varepsilon_{t+1}^u, \quad |\rho_u| < 1, \quad u > 0, \quad \sigma_u > 0, \quad (2-6)$$

where  $\varepsilon_t^u$  is a normally distributed i.i.d. process with mean zero and variance unity.  $u_t$  tries to account for shocks that affect the degree of capital utilization in an economy, some of them may be consequence of monetary or fiscal policies and others due to supply-side disruptions, such as a rise on commodity prices, for example.

Profit maximization implies that the equilibrium interest rates are:

$$r_t = \alpha^2 u_t^\alpha \hat{k}_t^{\alpha-1} - \delta, \quad (2-7)$$

and the flow of profits that each incumbent earns is:

$$\pi_t(j) = A_t(j) \alpha (1 - \alpha) \left( u_t \hat{k}_t \right)^\alpha N = A_t(j) u_t^\alpha N \bar{\pi}_t(\hat{k}_t), \quad (2-8)$$

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<sup>5</sup>Throughout the paper, the accent  $\hat{\cdot}$  above a variable denotes that it has been normalized by  $(A_t L_t)$ :  $\hat{x}_t \equiv X_t/(A_t L_t)$ .

where  $\bar{\pi}_t(\hat{k}_t) \equiv \alpha(1 - \alpha)\hat{k}_t^\alpha$ . The total profits per labor unit are:

$$\pi_t = \frac{1}{L_t} \int_0^{J_t} \pi_t(i) di = A_t u_t^\alpha \bar{\pi}_t(\hat{k}_t) = \alpha(1 - \alpha) y_t, \quad (2-9)$$

so income is distributed according to  $(1 - \alpha)$  per cent as wages,  $\alpha(1 - \alpha)$  per cent as monopolist profits and  $\alpha^2$  per cent as returns to capital.

### 2.3. Entrepreneurs and Banks

Innovations result from entrepreneurship that uses technological knowledge. Each period there is one entrepreneur per sector devoting resources to innovation. Each innovation creates an improved version of the existing product by raising  $A_t(j)$  to the technology frontier. Once an innovation happens, it allows the entrepreneur to replace the incumbent monopolist until the next innovation in that sector arrives. At any date there is a “leading-edge technology”

$$A_t^{\max} \equiv \max\{A_t(j) | j \in [0, J_t]\}. \quad (2-10)$$

This technology frontier just represents the most advanced technology across all the sectors. The probability of occurrence of a successful innovation in a sector  $j$  during a time period is:

$$P(\text{1 innovation at time } t + 1 \text{ in sector } j) = n_t(j). \quad (2-11)$$

This is the discrete-time version of a Poisson arrival rate of innovations, under the assumption that the probability of two successful innovations in a time period is negligible. The variable  $n_t(j)$  is a function of the quantity of final output devoted to entrepreneurship  $Q_t(j)$ :

$$n_t(j) = \sqrt{\frac{2Q_t(j)}{\bar{\lambda}_t A_t^{\max}}}, \quad n_t(j) \geq 0. \quad (2-12)$$

Equation (2-12) displays decreasing returns to scale in innovation<sup>6</sup>. The parameters  $\bar{\lambda}_t$  accounts for the productivity of resources devoted to innovation. The amount of resources is adjusted by the technology frontier variable  $A_t^{\max}$  to represent the increasing complexity of progress: as technology advances, the resource cost of further advances increases proportionally. If an innovation arrives at time  $t$  the technology level  $A_t(j)$  of this sector “jumps” to the technology frontier in  $t - 1$ :  $A_{t-1}^{\max}$ .

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<sup>6</sup>Several studies have found decreasing returns in R&D expenditure, such as Kortum (1993).

**Proposition 2** *There exists a symmetric equilibrium solution where the probability of an innovation at time  $t + 1$  is the same in all the sectors:  $n_t(j) = n_t$ .*

PROOF. Let define  $v_t(A_t, n_t(j))$  as the value of being the incumbent monopolist in sector  $j$  at time  $t$  given  $A_t$  and  $n_t(j)$ . Therefore, the value of becoming an incumbent at time  $t$  by making an innovation  $v_t(A_{t-1}^{\max}, n_t(j))$  can be expressed recursively as:

$$v_t(A_{t-1}^{\max}, n_t(j)) = A_{t-1}^{\max} u_t^\alpha N \bar{\pi}_t(\hat{k}_t) + (1 - n_t(j)) \frac{E_t [v_{t+1}(A_{t-1}^{\max}, n_{t+1}(j))]}{(1 + r_t)}. \quad (2-13)$$

That is, the value of an innovation  $v_t(A_{t-1}^{\max}, n_t(j))$  at time  $t$  that raises productivity to the technology frontier  $A_{t-1}^{\max}$  is equal to the flow of profits obtained this period plus the discounted expected value of  $v_{t+1}(A_{t-1}^{\max}, n_{t+1}(j))$ , which is the value of retaining the monopolist position in  $t + 1$ . The cost associated follows (2-12). From the point of view of entrepreneurs, all the sectors are identical in costs and benefits, with the only difference being  $n_t(j)$ . Therefore there should be a symmetric equilibrium solution  $n_t(j) = n_t = \sqrt{\frac{2Q_t}{\lambda A_t^{\max}}}$ ,  $\forall i$ , Q.E.D.

The average productivity across all sectors  $A_t$  is the average of the sectors that experience an innovation and of the sectors that do not:

$$A_{t+1} = \frac{1}{J_t} \int_0^{J_t} [n_t(j) A_t^{\max} + (1 - n_t(j)) A_t(j)] dj = n_t (A_t^{\max} - A_t) + A_t. \quad (2-14)$$

Growth in the technology frontier  $A_t^{\max}$  occurs as a result of the knowledge spillovers produced by innovations, in the same line of Aghion and Howitt (1998). At any moment of time, potential entrepreneurs may access to the technology frontier as it is publicly available, but no costless. The growth of the technology frontier is assumed to exogenously grow at a rate  $g_t = g$ . It tries to reflect the small impact of an individual country on the world technology frontier, whose evolution should depend on the aggregate rate of innovations all around the world, as commented in Howitt (2000).

Following the idea of Schumpeter’s “swarms of entrepreneurs”, I assume  $\bar{\lambda}_t$  to follow a stochastic process:

$$\log \bar{\lambda}_{t+1} \equiv (1 - \rho_\lambda) \log \lambda + \rho_\lambda \log \bar{\lambda}_t + \sigma_\lambda \varepsilon_{t+1}^\lambda, \quad |\rho_\lambda| < 1, \lambda > 0, \sigma_\lambda > 0, \quad (2-15)$$

where  $\varepsilon_t^\lambda$ , the innovation shock, is a normally distributed i.i.d. process with mean zero and variance unity. Therefore, its evolution should reflect structural changes in the economy, such as the arrival of general purpose technologies, or an improvement in the efficiency of the financial sector.

According to Schumpeter the banker (i.e. the financial system) is the centerpiece in the capitalist system, assuming the role of the capitalist for the classics. The role of the



banks is to allocate resources to entrepreneurs in different sectors by assuming the risk. As Schumpeter (1934, pp. 137) clearly puts it:

“The entrepreneur is never the risk bearer.[...] The one who gives credit comes to grief if the undertaking fails. For although any property possessed by the entrepreneur may be liable, yet such possession of wealth is not essential, even though advantageous. But even if the entrepreneur finances himself out of former profits, or if he contributes the means of production belonging to his “static” business, the risk falls on him as capitalist or as possessor of goods, not as entrepreneur. Risk-taking is in no case an element of the entrepreneurial function.”

Each period, the bank decides which amount of resources  $Q_t J_t$  it should invest in new entrepreneurial projects in each sector. If at time  $t$  an innovation successfully arrives in sector  $j$ , the financial sector becomes the final owner of the new incumbent firm and obtains all its profits  $\pi_t(j)$ . If no innovation arrives, the financial sector loses all the resources invested when failed entrepreneurs go to bankruptcy.

The bank chooses  $Q_t$  to maximize the discounted flow of dividends  $D_t \equiv (\pi_t L_t - Q_t J_t)$ :

$$E_t \sum_{i=0}^{\infty} \left( \prod_{l=t+1}^{t+i} (1 + r_l)^{-1} \right) D_t, \quad (2-16)$$

subject to (2-8), (2-12) and (2-14). Banks are therefore assumed to be risk-neutral maximizers of the expected value of the profits they obtain from monopolists minus the cost of new entrepreneurial projects. Their discount rate is the real interest rate in the economy, as this is the opportunity cost of resources, which otherwise could be invested in capital.

It is usually more convenient to work with a set of stationary variables. Thus let  $a_t(j) \equiv A_t(j)/A_t^{\max}$  be a country’s average productivity relative to the leading edge (the “distance to the frontier”), rescale  $\lambda_t = \frac{\bar{\lambda}_t}{N}$  so that  $\hat{q}_t \equiv (Q_t J_t) / (A_t L_t) = \frac{1}{2a_t} \lambda_t n_t^2$  is the effective amount of resources devoted to innovation per labor unit and let  $G_t^L \equiv L_{t+1}/L_t$  be the growth of the labor force.

The first order condition associated with the bank’s choice of  $Q_{t+i}$  is:

$$n_t = E_t \left[ \frac{\alpha (1 - \alpha) (1 + g) \left( u_{t+1} \hat{k}_{t+1} \right)^\alpha G_t^L (1 - a_t)}{(1 + r_{t+1}) \lambda_t} \right]. \quad (2-17)$$

This equation describes the dynamic trade-offs that face the banker. It will increase its investment in innovation if the expected profits  $E_t \left[ \alpha (1 - \alpha) (1 + g) \left( u_{t+1} \hat{k}_{t+1} \right)^\alpha G_t^L \right]$  grow,

which can be due to higher capital utilization, a higher amount of capital in the economy or a faster growth of the labor force. Investment will decrease it if the discount rate  $(1 + r_{t+1})$  is expected to rise or if the economy gets closer to the technology frontier  $(1 - a_t)$  and therefore the benefits by technology adoption (“leapfrogging”) are reduced. The shocks  $\lambda_t$  to the innovation production function also affect  $n_t$  as they alter the probability of success by a given amount of invested resources  $\hat{q}_t$ . As it will be shown later, these shocks play an equivalent role to technology shocks un real business cycle models.

## 2.4. Households

The formulation of the household sector is standard. Households maximize their utility, which derived from consumption per adult. Let  $C_t$  be total consumption in the economy, and  $c_t = C_t/L_t$  consumption per labor unit, then households maximize the present discounted utility as given by the following expression:

$$E_t \sum_{i=0}^{\infty} \beta^i \frac{(c_{t+i})^{1-\vartheta}}{1-\vartheta}, \quad (2-18)$$

with  $0 < \beta < 1$ , subject to the budget constraint:

$$C_t = w_t L_t + D_t + (1 + r_t) K_t - K_{t+1} + \xi_t, \quad (2-19)$$

where  $\xi_t$  is an exogenous shock that guarantees that the budget identity is always satisfied. From an accounting perspective, it represents the current account balance, although our model does not consider an open economy. To simplify the structure, I assume  $\hat{\xi}_t = \frac{\xi_t}{(A_t L_t)}$  to follow an independent autoregressive process:

$$\log(\hat{\xi}_{t+1}) = (1 - \rho^\xi) \log(\xi) + \rho^\xi \log(\hat{\xi}_t) + \sigma_\xi \varepsilon_{t+1}^\xi, \quad |\rho^\xi| < 1, \quad \sigma_\xi > 0, \quad (2-20)$$

where  $\varepsilon_t^\xi$  is a normally distributed i.i.d. process with mean zero and variance unity.

The Euler equation for households is:

$$1 = \beta E_t \left[ \frac{\hat{c}_t^\vartheta (1 + r_{t+1}) a_t^\vartheta}{\hat{c}_{t+1}^\vartheta G_t^L a_{t+1}^\vartheta (1 + g)^\vartheta} \right], \quad (2-21)$$

where  $\hat{c}_t \equiv \frac{C_t}{(A_t L_t)}$  is the effective consumption per labor unit.

As the model does not include fertility choices<sup>7</sup> or labor-leisure decisions, the labor-force just represents the demographic evolution of the population and it is assumed to follow

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<sup>7</sup>Such as the ones presented in Barro and Sala-i-Martin (2004, Chapter 9).

autoregressive processes

$$\log(G_{t+1}^L) = (1 - \rho^L) \log(G^L) + \rho^L \log(G_t^L) + \sigma_L \varepsilon_{t+1}^L, \quad |\rho^L| < 1, \quad \sigma_L > 0 \quad (2-22)$$

where  $\varepsilon_t^L$  is a normally distributed i.i.d. processed with mean zero and variance unity.

## 2.5. Market Equilibrium and Balanced Growth

The final goods market is in equilibrium if production equals demand by households for consumption and capital accumulation. The capital rental market is in equilibrium when the demand for capital by the intermediate good producers equals the supply by the households. The labor market is in equilibrium if firms' demand for labor equals labor supply by households (which is inelastic). The innovation market is in equilibrium if the entrepreneurs' demand for funds equals the supply by the bank.

**Proposition 3** *The presented model exhibits a balanced growth path where output, consumption, innovation investment and capital per labor unit grow at the constant rate  $g$ .*

PROOF. The model economy can be characterized by two state variables:  $\hat{k}_t$ , and  $a_t$  that display constant steady-state values, as in Howitt (2000). In the case of  $\hat{k}_t$ , its evolution is given by:

$$\hat{k}_{t+1} = \frac{a_t \left( u_t^\alpha \hat{k}_t^\alpha + (1 - \delta) \hat{k}_t - \hat{c}_t - \hat{q}_t - \hat{\xi}_t \right)}{a_{t+1} (1 + g) G_t^L}, \quad (2-23)$$

so its steady state value  $\hat{k} = \left( \frac{\alpha^2 u^\alpha}{r + \delta} \right)^{1/(1-\alpha)}$  where  $r = \frac{G^L(1+g)^\theta}{\beta} - 1$  from (2-7) and (2-21).

In the case of  $a_t$ , its evolution is given by:

$$a_{t+1} = \frac{n_t (1 - a_t) + a_t}{1 + g}, \quad (2-24)$$

so its steady state value  $a = \frac{n}{n+g}$ , where  $n = \frac{g}{2} \left( -1 + \sqrt{1 + 4 \frac{\alpha(1-\alpha)(u\hat{k})^\alpha G^L}{g(1+r)\lambda}} \right)$ , from (2-14) and (2-17).

The existence of these steady states implies that  $K_t$ ,  $A_t$ , and  $L_t$  grow at the steady state at the rate  $(1 + g) G^L$ ,  $(1 + g)$  and  $G^L$  respectively. Therefore  $Y_t$ ,  $C_t$ , and  $Q_t I_t$  grow at a rate of  $(1 + g) G^L$  when the system is in its steady state. Q.E.D.

### 3. Numerical Results

#### 3.1. Observed Series

To take the model to the data, I take yearly data for the United States economy in the period 1960 to 2005<sup>8</sup>. Output  $Y_t$  is proxied by the gross domestic product (GDP) at constant prices. Total consumption  $C_t$  is the sum of private and government consumption. Total investment  $I_t^T$  includes government and private gross fixed capital formation and changes in stocks. The difference between output and consumption plus investment equals the difference between exports and imports  $Y_t - (C_t + I_t^T) = \xi_t$ . The labor force  $L_t$  is proxied by the population aged 15-64. The reason to take the population and not the labor force provided by the National Accounts is because the population 15-64 (the population who can work) is approximately exogenous to economic conditions whereas the official labor force (the population who can and want to work) fluctuates with economic output. It can be correctly argue that the population also fluctuates due to the immigration flows, but as a simplifying assumption I neglect this issue.

Data is not detrended, neither by a Hodrik-Prescott filter nor by any other mechanism. Instead, rates of growth are computed to avoid working with non stationary series. Observed series are  $dy_t$ ,  $dc_t$ ,  $dL_t$  and  $di_t^T$ .  $dy_t \equiv \log(y_t/y_{t-1})$  with  $y_t \equiv \frac{Y_t}{L_t}$  is the growth in output per labor unit,  $dc_t \equiv \log(c_t/c_{t-1})$  with  $c_t \equiv \frac{C_t}{L_t}$  is the growth of consumption per labor unit,  $di_t^T \equiv \log(i_t^T/i_{t-1}^T)$  with  $i_t^T \equiv \frac{I_t^T}{L_t}$  is the growth of total investment per labor unit and  $dL_t \equiv \log(L_t/L_{t-1})$  is the growth of the labor force.

Which is the relationship between real variables and the simulated ones? In the case of output  $dy_t = \log(y_t/y_{t-1}) = \log(\frac{\hat{y}_t a_t}{\hat{y}_{t-1} a_t}) + \log(1 + g)$  so:

$$dy_t = \log(1 + g) + (\tilde{a}_t - \tilde{a}_{t-1}) + \alpha(\tilde{y}_t - \tilde{y}_{t-1}), \quad (3-1)$$

and labor force :

$$dL_t = \log(G^H) + \tilde{G}_{t-1}^L. \quad (3-2)$$

The variables denoted as  $\tilde{x}_t$  represents log-linearized deviations from the steady state  $\tilde{x}_t = \log(x_t/x)$ , where  $x$  is the value of  $x_t$  in the nonstochastic steady state. The description of the log-linearized variables is presented in Appendix A.

The main question here is to define what is  $Q_t J_t$ , the amount of resources devoted to innovation in terms of observed economic series. On one hand, it does not accumulate in

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<sup>8</sup>Economic data comes from the OECD National Accounts. Population comes from the World Development Indicators.

any form of capital, which would indicate that it is better described as a consumption item. On the other hand, its goal is to increase the productivity of new capital vintages, so it might be regarded as investment. The standard definition of investment is the increment in installed capital  $K_{t+1} - K_t = I_t - C(K_t)$ , where  $C(K_t) = \delta K_t$ , and the output per capita equals  $A_t \left(\frac{K_t}{A_t L_t}\right)^\alpha$  with  $A_t$  exogenous. It would mean that agents invest in new capital, whose productivity grows exogenously. An alternative explanation would be to assume that the investment in new, more advanced, capital has a higher cost, so  $C(K_t) = \delta K_t + Q_t J_t$  but now the average productivity level grows with the improvement in the capital technology level  $A_t = A_t(Q_t J_t)$ . Following this second interpretation, in line with the idea of endogenous technology change, I decide to assume that total investment is composed by “traditional” capital formation plus investment in innovation  $I_t^T = K_{t+1} - (1 - \delta)K_t + Q_t J_t$ .

Another alternative would be to consider  $Q_t J_t$  as R&D expenditure. However, R&D expenditure is not the same as innovation expenditure. As an example, let’s assume that an old building is restored and becomes a luxury hotel. The accounting record would register an increase in the capital of the hotel (new beds, swimming pool, televisions and so on) and zero R&D expenditure. However, if the business is profitable, the productivity of the capital would increase above the depreciation expenses and interest rate payments. Part of this increment would come from the increment in the capital stock (more beds), but another part would come from productivity gains due to a more successful business model for this capital. The key is that the model concept of “technology” does not necessarily implies real technology (computers, machines and so on) but what Comin and Mulani (2007) denoted as “disembodied innovations” such as managerial and organizational techniques, personnel, accounting and work practices, and financial innovations. A new cafe in the neighborhood that displaces the current incumbent via a better quality of its imported world-class coffee has improved the productivity of its products without any official R&D. All successful R&D produces innovations, whereas not all innovations come from R&D expenditure.

Therefore, observed consumption results in  $dc_t = \log(c_t/c_{t-1}) = \log\left(\frac{\tilde{c}_t a_t}{\tilde{c}_{t-1} a_t}\right) + \log(1 + g)$  so:

$$dc_t = \log(1 + g) + (\tilde{c}_t - \tilde{c}_{t-1}), \quad (3-3)$$

and investment  $di_t^T = \log(i_t^T/i_{t-1}^T) = \log((\hat{i}_t + \hat{q}_t)/(\hat{i}_{t-1} + \hat{q}_{t-1}))$ . The value of the capital formation  $\hat{i}_t$  is

$$\hat{i}_t = \frac{K_{t+1} - (1 - \delta) K_t}{L_t A_t} = \frac{G_t^L (1 + g) a_{t+1}}{a_t} \hat{k}_{t+1} - (1 - \delta) \hat{k}_t, \quad (3-4)$$

and the observed investment

$$di_t = \log(1 + g) + \frac{\hat{q}}{\hat{q} + \hat{i}} (\tilde{q}_t - \tilde{q}_{t-1}) + \frac{\hat{i}}{\hat{q} + \hat{i}} (\tilde{i}_t - \tilde{i}_{t-1}), \quad (3-5)$$

where  $\hat{q} = \frac{\lambda n^2}{2a}$  and  $\hat{i} = [(1 + g) G^L - (1 - \delta)] \hat{k}$ .

### 3.2. Estimation

The model is estimated following a bayesian approach similar to the one presented in Smets and Wouters (2003). In addition to all the reasons exposed there, such as the possibility of integrating a priori information that may come from micro and macroeconomic studies or the stability of the optimization algorithm in short samples, it should be added that due to the novel structure of the model, it is almost impossible to find in the literature values for many of the parameters, making calibration impossible.

Estimation involves the following steps. In the first step, the linear rational expectations model in Appendix A is solved for the reduced form state equation in its predetermined variables. In the second step, the model is written in its state space form. This involves augmenting the state equation in the predetermined variables with the observation equations that links the predetermined state variables to observable variables. In the present model, no measurement error is considered, so this step employs the equations (3-1), (3-2), (3-3) and 3-5). The third step consists of using the Kalman filter to form the likelihood function. I have firstly estimated the parameters by maximum likelihood to have a guess about plausible initial values for all the parameters that lack a priori information about. In the final step, a priori information is introduced and the parameters are estimated by bayesian methods, such as Monte-Carlo Markov-Chain <sup>9</sup>.

As in Smets and Wouters (2003), a number of parameters were kept fixed from the start of the exercise. Most of these parameters can be directly related to the steady-state values of the state variables and could therefore be estimated from the means of the observable variables. The discount factor,  $\beta$ , is calibrated to be 0.99, a typical value in the literature. The values of  $g$ , the long-term growth rate of output, and  $G^L$ , the long-term growth rate of the labor force, are set to 0.0191 and 1.0135 respectively to guarantee that the simulated means are the same as the observed ones. The coefficient  $\alpha$  is set to 0.35, which roughly implies a steady-state share of labor income in total output of 65 per cent, as it is observed in the period. Finally, the quotient  $\xi/\hat{y}$  is set to -0.016, as the average current account deficit has been 1.6 percent.

The first three columns of Table 1 give an overview of the assumptions regarding the prior distribution of the other 11 estimated parameters. All the variances of the shocks are

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<sup>9</sup>The code is based on the Dynare software.

assumed to be distributed as an inverted Gamma distribution. This distribution guarantees a positive variance with a rather large domain. The precise mean for the prior distribution was based on previous estimation outcomes by maximum likelihood and trials with a very weak prior. The same distribution is assumed for the risk aversion coefficient and the capital depreciation. The precise means and standard errors are also taken from previous estimation, except the depreciation mean that is taken as 10 per cent as it is the typical value in the literature. The distribution of the autoregressive parameters in the shocks is assumed to follow a beta distribution. The beta distribution covers the range between 0 and 1. The standard errors were set so that the domain covers a reasonable range of parameter values, also based on previous estimations. The steady state probability of a successful innovation  $n$  is also assumed to follow a beta distribution due to its range between 0 and 1. Its mean value is taken 3.6 per cent, as in Howitt (2000), from Caballero and Jaffe (1993).

TABLE 1. PARAMETER ESTIMATES

	Prior Distribution			Estimated Posterior Distribution		
	Type	Mean	Std. Error	Mean	5%	95%
$n$	Beta	0.036	0.033	0.017	0.000	0.036
$\vartheta$	Inv. Gamma	2.500	0.500	2.479	1.857	3.153
$\delta$	Inv. Gamma	0.100	2.000	0.115	0.073	0.156
$\rho_\lambda$	Beta	0.900	0.050	0.946	0.905	0.983
$\rho_u$	Beta	0.800	0.050	0.783	0.717	0.842
$\rho_\xi$	Beta	0.900	0.050	0.949	0.915	0.986
$\rho_L$	Beta	0.900	0.050	0.928	0.878	0.969
$\sigma_\lambda$	Inv. Gamma	0.200	5.000	0.247	0.159	0.325
$\sigma_u$	Inv. Gamma	0.050	5.000	0.053	0.043	0.061
$\sigma_\xi$	Inv. Gamma	0.200	5.000	0.289	0.238	0.331
$\sigma_L$	Inv. Gamma	0.001	0.001	0.001	0.001	0.001

In addition to the prior distribution, Table 1 reports the results regarding the parameter estimates. Results report the mean and the 5% and 95% values. Most parameters are estimated to be significantly different from zero. This is true for the standard errors of all the shocks, with the exception of the demographic shock, which does not seem to play much of a role. This will also be clear in the forecast error variance decomposition discussed next. The shocks are estimated to have autoregressive parameters that lie between 0.78 (for the capital utilization shock) and 0.95 for the current account shock. A high level of persistence in the current account and demographic shocks is natural as they reproduce exogenous time series. In the case of the shock to the efficiency in the innovation spending  $\lambda_t$ , a high level of persistence may proxy for non-stationary structural change. The values for the depreciation and risk-aversion coefficients, 0.11 and 2.48 respectively, are in line with the

literature. Finally, the estimation of the steady-state rate of creative destruction  $n$  is 0.017, less than the value estimated in microeconomic studies such as Caballero and Jaffe (1993). This value suggests that the steady-state value of  $a$  is 0.47 so the average productivity of the economy would be below a half of the technology frontier.

### 3.3. What is Behind Technology Shocks?

Following the seminal work by Kydland and Prescott (1982), technology shocks are considered as one important source of business cycles. Technology shocks are stochastic processes that affect the productivity of a country. As a consequence, productivity evolves according to a stochastic trend independently of the rest of economic variables. In contrast, in the model presented in this paper, productivity is an endogenous variable that reflects the decision by entrepreneurs and agents, so its stochastic behavior not only should reflect the impact of shocks to the efficiency in the innovation spending  $\lambda_t$  but it is also related to other economic variables, such as the distance to the technology frontier or the level of profits.

In this section it is shown how for a country close to the technology frontier, as the United States is assumed to be, the endogenous stochastic growth process seems to reproduce the behavior of exogenous technology shocks. It has interesting implications, as it suggests that issues that cannot be classified as “technological”, such as any shock to the financial system that affects the capacity of financial intermediaries to channel funds to entrepreneurs, may have a severe impact on productivity that would be reflected as a technology shock.

To analyze if this effect is significant, I compare the Schumpeterian model with the equivalent RBC with technology shocks and analyze the differences. The RBC follows a similar structure of the schumpeterian model, including the shocks  $\tilde{u}_t$ ,  $G_t^H$  and  $\tilde{\xi}_t$ . Investment now equals capital accumulation and there are no dividends in the budget constraint. In contrast to the endogenous growth model, it neglects the role of entrepreneurs and banks. Therefore, the increase in productivity is given by an equation of the form  $\log A_{t+1} = \log A_t + \log Z_{t+1}$  with

$$\log Z_{t+1} = (1 - \rho_A) \log(1 + g) + \rho_A \log(Z_t) + \sigma_A \varepsilon_{t+1}^A, \quad |\rho^A| < 1, \quad \sigma_A > 0, \quad (3-6)$$

where  $\varepsilon_t^A$ , the technology shock, is a normally distributed i.i.d. process with mean zero and variance unity. In sum it is a traditional stochastic Ramsey model with 4 exogenous shocks.

To compare it, I assume that both models have the same values for  $\vartheta$ ,  $\delta$  and the autoregressive coefficients  $\rho_u$ ,  $\rho_\xi$  and  $\rho_L$ , which are equal to the mean values of the posterior distribution estimated above. The rest of parameters are estimated in the same way as it was done for the schumpeterian model. Results are presented in Table 2.



TABLE 2. PARAMETER ESTIMATES RBC MODEL

	Prior Distribution			Estimated Posterior Distribution		
	Type	Mean	Std. Error	Mean	5%	95%
$\rho_A$	Beta	0.900	0.050	0.925	0.876	0.971
$\sigma_A$	Inv. Gamma	0.005	5.000	0.003	0.003	0.004
$\sigma_u$	Inv. Gamma	0.050	5.000	0.052	0.044	0.061
$\sigma_\xi$	Inv. Gamma	0.200	5.000	0.237	0.237	0.334
$\sigma_L$	Inv. Gamma	0.001	0.001	0.001	0.001	0.001

To analyze the weight of the different shocks in the variance of the observed time series I conduct a variance decomposition by running 1,000 simulations of the state-space solution of the two models with the coefficients taken as the mean of the respective posterior distributions. Table 3 reports the result for the Schumpeterian model and its reduced RBC model. The main source of variance is the capital utilization shock, which tries to capture issues such as rigidities in prices and wages as those presented in Christiano, Eichenbaum and Evans (2005). Secondly comes the innovation/technology shock. The other two shocks (external balance and demographic) are marginal in the term of their contributions to the total variance. This result confirms the work of Ireland (2004) where technology shocks are second to sources such as a cost-push shocks to justify most of the observed variance in output.

TABLE 3. VARIANCE DECOMPOSITION

	Schumpeterian Model				RBC Model			
	$\lambda$	$u$	$\xi$	$L$	$A$	$u$	$\xi$	$L$
$dy_t$	13.24	86.63	0.02	0.11	15.47	84.40	0.03	0.09
$dc_t$	21.57	69.77	8.46	0.20	25.27	66.38	8.20	0.15
$di_t$	21.35	78.48	0.17	0.01	20.26	79.46	0.28	0.00
$dL_t$	0.00	0.00	0.00	100.00	0.00	0.00	0.00	100.00

To analyze the differences between the schumpeterian/endogenous productivity model and the RBC/exogenous productivity one I decide to follow a calibration approach, where I simulate the estimated models and compare some of their moments, such as standard deviations, correlations coefficients and impulse response functions. These exercises are meant simply as a first pass at exploring which is the relationship between the mechanism I emphasize and the traditional technology shocks: they are not formal statistical tests. The idea is to test if there is any major difference between both models. Table 4 reports the standard deviations and correlations. Both model perform similar in all the consider moments, and their differences with the data are higher than between them. Their main limitation is their low one-lag and their high second-lag autocorrelations compared to the data. From a calibration perspective, both models are almost identical so the introduction

of endogenous productivity does not seem to bring any change in term of moments.

TABLE 4. CALIBRATION RESULTS

	Data	Schumpeter	RBC
Std. Deviations			
$dy_t$	0.0185	0.0197	0.0198
$dc_t$	0.0146	0.0158	0.0159
$di_t$	0.0707	0.0734	0.0757
$dL_t$	0.0034	0.0028	0.0027
Correlations			
$(dy_t, dc_t)$	0.8823	0.9248	0.9252
$(dy_t, di_t)$	0.8661	0.8720	0.8692
$(dy_t, dL_t)$	-0.1835	-0.0230	-0.0190
$(di_t, dc_t)$	0.5954	0.6996	0.6977
$(dy_t, dy_{t-1})$	0.3143	0.1342	0.1585
$(dy_t, dy_{t-2})$	-0.0642	0.1224	0.1449

Figure 1, 2 and 3 display the impulse response of  $dy_t$ ,  $dc_t$ ,  $di_t$  and  $r_t$  to capital utilization shock  $u_t$ , current account  $\xi_t$  and labor force  $L_t$  shocks for both models. Numerical results do not find any significant difference, neither in term of impulse responses nor in simulated variances and correlations, between a traditional exogenous technology shock and a endogenous productivity model with an innovation shock. A first conclusion is that the current model provides richer macro foundations to technology shocks, indicating why these kind of hocks do not necessarily come from “technology” in a narrow sense, but from any impediment in the financing process of entrepreneurs and the working of the innovation market, such as barriers to entry, bureaucracy costs, underdeveloped financial systems or lack of human capital. The financial issues may be specially interesting. Bernanke and Parkinson (1991), for example, conclude that the evolution in the Solow’s residual in the Great Depression cannot be easily explained assuming that it only reflects technology progress. Under the present model, it could indicate a fall in capital utilization and a shock to the innovation process, for example due to the wave of bankruptcies that temporarily stopped the efficient working of the bank as provider of funds for entrepreneurship, highlighted in the present model.

#### 4. Conclusions

Technology development and adoption is the result of economic processes central to the correct working of market economies. Entrepreneurs and bankers, typically absent in most theories of growth and fluctuations, play a major role in this process, as confirmed by several

Fig. 1.— IMPULSE RESPONSES TO A CAPITAL UTILIZATION SHOCK (solid Schumpeterian model, stars RBC).

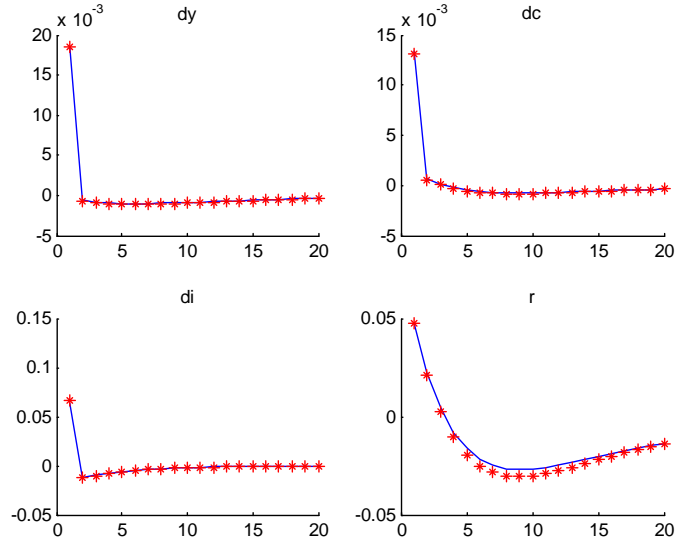


Fig. 2.— IMPULSE RESPONSES TO A CURRENT ACCOUNT SHOCK (solid Schumpeterian model, stars RBC).

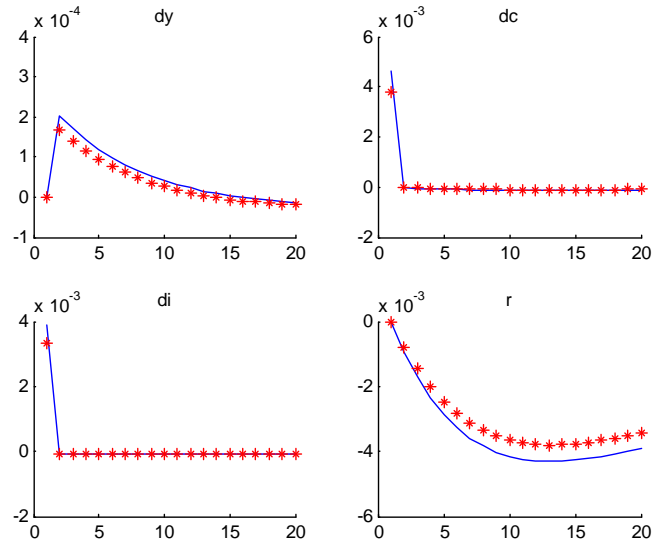
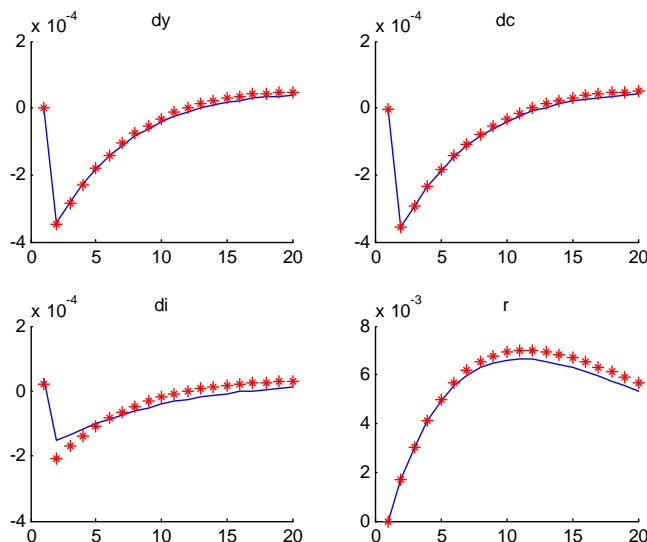


Fig. 3.— IMPULSE RESPONSES TO A LABOR FORCE SHOCK (solid Schumpeterian model, stars RBC).



empirical studies<sup>10</sup>. Ignoring these facts can only yield to the development of models that describe economies unrelated with the reality. This paper has tried to advance a small step in the right direction, but much is left to do.

The paper continues the line of Blackburn and Pelloni (2005) and Comin and Gertler (2006) by integrating endogenous growth (in this case due to vertical innovations) into a simple stochastic general equilibrium model. The main originality of the model is that it underlines the importance, not only of entrepreneurs, but also of bankers in the process of economic growth. Although the model considers a constant steady-state growth of the technology frontier, it can be extended to a context where the technology frontier evolves dynamically as a consequence of spill-over effects as in Aghion and Howitt (1998). By estimating the model, I follow the research agenda of Klenow and Rodriguez-Clare (1997) and Wacziarg (2002), who have suggested that growth econometrics should evolve from linear regressions to DSGE models that allow the researcher to take their theories to the data.

The main empirical result of the paper is to show how technology shocks can be the result of shocks to the innovation production function of entrepreneurs which may be produced by reasons different to what is typically defined as “technology”, such as the efficiency

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<sup>10</sup>For example, Aghion, Fally, and Scarpetta (2007).

of the financial system in redirecting resources to entrepreneurship. The paper shows how the endogenous model can replicate the behavior of a RBC model for the case of the United States, a country assumed to be close to the technology frontier. Results should be extended to other countries with a lower technology level before making a general statement. Notwithstanding, these conclusions seem to indicate that part of the observed evolution in the productivity of a country may be due to financial considerations.

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### A. Appendix: Log-linear Approximation of the Model

To approximate the solution to the model, I first log-linearize the Euler equations and various model identities about the steady state of the model. The 12 variables in the system are contained in a vector  $z_t$ :

$$z_t = \left[ \tilde{r}_t, \tilde{a}_t, \tilde{n}_t, \tilde{q}_t, \tilde{y}_t, \tilde{u}_t, \tilde{c}_t, \tilde{k}_t, \tilde{u}_t, \tilde{\lambda}_t, \tilde{\xi}_t, \tilde{G}_t^L \right]'$$

Recall,  $\tilde{x}_t = \log(x_t/x)$ , where  $x$  is the value of  $x_t$  in nonstochastic steady state.

I solve for the dynamics of these variables using 12 Euler and other equations. The first one derives from (2-7):

$$\tilde{r}_t = -\frac{(1-\alpha)(r+\delta)}{r}\tilde{k}_t + \frac{\alpha(r+\delta)}{r}\tilde{u}_t, \quad (\text{A1})$$

so the interest rates decrease with the effective capital per labor unit and increase with the utilization rate.

The second one is the linearized version of the increase in productivity equation (2-24):

$$\tilde{a}_{t+1} = \frac{(1-n)}{1+g}\tilde{a}_t + \frac{g}{1+g}\tilde{n}_t. \quad (\text{A2})$$

The third equation is results from the Euler equation from the bank (2-17) by taking into account that  $\tilde{k}_{t+1}$  is known at time  $t$  and that  $E_t[\tilde{u}_{t+1}] = E_t[\rho_u\tilde{u}_t + \sigma_u\tilde{c}_{t+1}^u] = \rho_u\tilde{u}_t$ :

$$\tilde{n}_t = \alpha\rho_u\tilde{u}_t + \alpha\tilde{k}_{t+1} - \frac{a}{1-a}\tilde{a}_t - \frac{r}{1+r}E_t[\tilde{r}_{t+1}] + \tilde{G}_t^L - \tilde{\lambda}_t, \quad (\text{A3})$$

and the fourth equation combines (A3) with a linearized version of (2-12) and considering that  $\hat{q}_t = \frac{Q_t I_t}{A_t L_t}$  is the effective resources devoted to entrepreneurship per labor unit:

$$\tilde{q}_t = -\tilde{a}_t + 2\tilde{n}_t + \tilde{\lambda}_t. \quad (\text{A4})$$

The effective output per labor unit is  $\hat{y}_t \equiv \frac{Y_t}{A_t L_t}$  and therefore the fifth equation results in

$$\tilde{y}_t = \alpha\tilde{k}_t + \alpha\tilde{u}_t. \quad (\text{A5})$$

Defining capital formation  $I_t$  as  $I_t = K_{t+1} - (1-\delta)K_t$  and  $\hat{i}_t \equiv \frac{I_t}{A_t L_t}$  as in (3-4) then the budget constraint (2-19) results in:

$$\tilde{i}_t = \frac{\hat{y}}{\hat{i}}\tilde{y}_t - \frac{\hat{c}}{\hat{i}}\tilde{c}_t - \frac{\hat{q}}{\hat{i}}\tilde{q}_t - \frac{\hat{\xi}}{\hat{i}}\tilde{\xi}_t, \quad (\text{A6})$$



where  $y = (u\hat{k})^\alpha$ , and  $\hat{c} = \hat{y} - \hat{i} - \hat{q} - \xi$ .

The seventh equation describes the dynamics of consumption and it is derived from the Euler equation (2-21):

$$\tilde{c}_t = \tilde{a}_{t+1} - \tilde{a}_t + E_t [\tilde{c}_{t+1}] - \frac{r}{\vartheta(1+r)} E_t [\tilde{r}_{t+1}] + \frac{1}{\vartheta} \tilde{G}_t^L. \quad (\text{A7})$$

The eight one is the capital equation, it is obtained by taking a linear approximation of (3-4):

$$\hat{k}_{t+1} = -\tilde{a}_{t+1} + \tilde{a}_t - \tilde{G}_t^L + \frac{\hat{i}}{\hat{k}(1+g)G^L} \tilde{t}_t + \frac{(1-\delta)}{(1+g)G^{\tilde{N}}} \tilde{k}_t. \quad (\text{A8})$$

The following four equations are the stochastic specification of  $\tilde{u}_t$ ,  $\tilde{\lambda}_t$ ,  $G_t^H$  and  $\tilde{\xi}_t$  and follow the structure  $\tilde{x}_{t+1} = \rho_x \tilde{x}_t + \sigma_x \varepsilon_{t+1}^x$ , derived from equations (2-6), (2-15), (2-22) and (2-20).

The vector of structural parameters of the model is  $\theta \in \mathbb{R}^{16}$ :

$$\theta = [\alpha, \beta, \delta, g, \lambda, \xi, u, G^L, \rho_\lambda, \rho_\xi, \rho_u, \rho_L, \sigma_\lambda, \sigma_\xi, \sigma_u, \sigma_L]'. \quad (\text{A9})$$