THE ROLE OF UNIONS IN HIRING PROCEDURES FOR JOB MARKETS*

Jose Alcalde and Pablo Revilla**

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Correspondence to:
José Alcalde. University of Alicante. Facultad de Ciencias Económicas.
Dpto. Fundamentos del Análisis Económico. Ctra. San Vicente del Raspeig, s/n.
03071 ALICANTE

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** University of Alicante.

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ABSTRACT

This paper studies simple hiring procedures for job markets. We show that when agents act strategically only individual rational outcomes should be expected. Moreover, if agents can form unions, thereby gaining the possibilitly to commit on the decision to be chosen, only stable allocations are implemented.

Key words: Job Matching Markets, Mechanism Design.

1. INTRODUCTION

This paper studies simple hiring procedures for job markets from a game-theoretical point of view. In several OECD countries, contractual processes are centralized. There is an employment office receiving demands and supplies for job positions. This office decides how to match workers and firms depending on the information it receives from the agents. Each worker usually reports the characteristics of the job she is looking for. These characteristics can be divided into two groups. The first one is the salary she is asking for, whereas the second one is a physical description of the job. Concerning the firms, their report includes characteristics of the workers to be hired and the salary to be paid. Assume that firms and workers are heterogeneous enough. In such a case, we can identify each firm's report with the set of workers it is looking for and a salary to be paid to each one. Similarly, we can think of workers' reports as a salary to be received and the firm for which she wants to work.

In this framework each agent's action is taken without knowing the others' decisions. Thus we can think of a game where players are firms and workers, and their actions are taken simultaneously. The outcome function is decided by the employment office. The aim of this paper is to analyze the agents' behavior in this game. More than that, we are interested in knowing about the properties of the outcomes that we can expect when agents act strategically.

Once the employment office has the information given by agents, it has to match workers and firms and to decide about the salaries to be paid. A reasonable way to proceed is the following. A worker has to be contracted by a firm if the two agree. A firm-worker agreement holds only if that worker wants to be hired by this firm, the firm likes to hire this worker, and salaries reported by them are compatible. Compatibility of salaries is stated in such a way that the firm declares to be able to pay at least the salary that the worker is asking for. Concerning the salary to be paid, the employment office has several possibilities to proceed. There is a requirement which should be satisfied by the employment office's recommendation. When the salaries that worker and firm reported coincide, this has to be the salary paid by the firm to the worker. In any other case, the (positive) difference between the salary required by the worker and that offered by the firm can be shared among them and the employment office according to some "surplus sharing" rule established by the office. Nevertheless, as we will see, no difference between these salaries will hold in equilibrium.

Once the surplus sharing rule is established, we can think that workers and firms are faced with a specific job matching mechanism. We will analyze two types of agents' behavior. The first one is related to an individualistic decision-making procedure. In such a case, agents do not commit on the action to be taken, and the outcomes that we will expect, when agents are faced with this mechanism, are those that are supported by Nash equilibria. The second one is a collective decision-making procedure. Think of partial agreements involving any set of workers (a workers union) and a firm or any set of workers and firms (workers unions and firms unions). The set of expected outcomes will coincide with the set of rematching-proof equilibrium outcomes (if only workers unions are allowed) or strong Nash equilibrium outcomes (when the two classes of unions are allowed).

The answer to the generic question we want to discuss is given by employing the approach of implementation theory. In Maskin's [7] words, "the theory of implementation concerns the problem of designing game forms (sometimes called "mechanisms" or "outcome functions") the equilibria of which have properties that are desirable according to a specified criterion of social welfare called a social choice rule." Thus, we look for mechanisms whose equilibrium outcomes satisfy some properties. More than that, we will analyze a contractual procedure and characterize the set of equilibrium outcomes.

Following this approach in frameworks that are close to our model we find some results. For instance, Kara and Sönmez [4] analyze the problem of implementation in many-to-one matching markets. They show that the set of stable allocations is implementable in Nash equilibrium, while no particular subset of the core is Nash implementable. Alcalde and Romero-Medina [2] present two simple sequential mechanisms implementing in subgame perfect Nash equilibrium the core correspondence in the college admission problem. That is, their analysis concerns many-to-one matching models where monetary transfers are not allowed.

In the same framework considered in this paper, Alcalde, Pérez-Castrillo, and Romero-Medina [1] present a sequential mechanism implementing the core of job matching markets in subgame perfect Nash equilibrium. Nevertheless, nothing is said about mechanism design where agents' decisions have to be taken simultaneously.

The paper is organized as follows. Section 2 introduces some definitions and the model. Section 3 presents the mechanism to be analyzed. Section 4 studies our mechanism.

2. THE MODEL

Consider a job market with n workers and ℓ firms. Let $W = \{w_1, \ldots, w_n\}$ and $F = \left\{f^1, \ldots, f^\ell\right\}$ be the sets of workers and firms, respectively. Each worker's preferences depend on two variables. The first one is the firm that she is working for, whereas the second relevant aspect is the wage that this firm pays to her. Let us assume that worker w_i's preferences are representable by the utility function $U_i\left(f^j, s_i\right)$, non-decreasing and continuous in $s_i \in \mathbb{R}$, where s_i is the salary that worker w_i receives, and f^j is the firm in which she is working. A worker who is not engaged by any firm reaches a utility level $U_i(\emptyset, 0)$. That is, we represent by $f = \emptyset$ the situation in which the worker is not hired by any firm. We also assume that for each firm f^j there is a reservation salary r_i^j such that $U_i\left(f^j, r_i^j\right) = U_i(\emptyset, 0)$ and that $U_i\left(f^j, s_i\right)$ has the same limit as s_i tends to $+\infty$ independently of the identity of the firm f^j. Each firm's profit depends on the set of workers it contracts, say $W^j \subseteq W$, and the salaries it pays to them, say $S^j = (s_i^j)_{w_i \in W^j}$. For notational convenience, we will sometimes treat S^j as a vector in $\mathbb{R}^n$. Let $\pi^j : 2^W \times \mathbb{R}^n \longrightarrow \mathbb{R}$ be the profit function of firm f^j. $\pi^j(W^j, S^j)$ is assumed to be decreasing in s_i^j whenever $w_i \in W^j$ and independent of s_i^j if $w_i \notin W^j$. A firm which does not hire any worker obtains $\pi^j(\emptyset, 0)$.

We can describe an allocation by means of two variables. The first one is a vector $S \in \mathbb{R}^n$ representing the wage that each worker gets, where $s_i = 0$ if w_i is not engaged by any firm. The second one is a correspondence, to be called *matching*, that states which firm (if any) hires each worker and vice-versa. More precisely, a matching μ is a correspondence that maps $W \cup F$ into itself such that (i) for each $w_i \in W$, if $\mu_i = \mu(w_i)$ does not belong to F, then $\mu_i = \emptyset$; (ii) for each f^j in F, $\mu^j = \mu(f^j)$ is contained in W, and (iii) for each pair $(w_i, f^j) \in W \times F$, $\mu_i = f^j$ if, and only if, $w_i \in \mu^j$.

We are interested in the job market allocations that are stable. Stability of an allocation depends on the possibilities that agents have to improve their utility level (for workers) or their profits (for firms). Since job matching markets can be viewed as a particular class of cooperative games, the stability concept that we are going to consider is the core.

Stability of an allocation can be easily checked in the case of job markets. An allocation is stable if there exist no subset of workers and a firm that can improve their own utility (respectively profit, for firms) by themselves. That is, (S, μ) is stable if, and only if, $\nexists(\widehat{W}, f^j) \in 2^W \times (F \cup \emptyset)$, and $\widehat{S} \in \mathbb{R}^n$ such that

(i) $U_i\left(f^j, \widehat{s}_i\right) > U_i\left(\mu_i, s_i\right)$ for all $w_i \in \widehat{W}$, with $\widehat{s}_i = 0$ if $f^j = \emptyset$, and

(ii) $\pi^j\left(\widehat{W}, \widehat{S}\right) > \pi^j\left(\mu^j, S\right)$, with $\widehat{S} = 0$ if $\widehat{W} = \emptyset$.

Notice that our definition of stability includes two main features. The first one is that of individual rationality. We say that an allocation is individually rational if (i) each worker weakly prefers the payoff that she gets in this allocation to being unmatched, i.e. $U_i(\emptyset, 0) \leq U_i(\mu_i, s_i)$ and (ii) no firm has an interest to dismiss any worker it has assigned, i.e. $\pi^j\left(\mu^j, S\right) = \max_{W^j \subseteq \mu^j} \pi^j\left(W^j, S\right)$. The second one is that of collective rationality in the following sense. There is no possibility for a firm and a group of workers to form a *new* matching, in such a way that both the firm and the new workers it hires find the new situation profitable.

3. THE EMPLOYMENT OFFICE'S MECHANISM

This section presents a family of mechanisms doubly implementing the stable correspondence in rematching-proof and strong Nash equilibria. In fact, when the analysis is restricted to mechanisms belonging to this family, the behavior that agents present in all these mechanism will not depend on the specific mechanism. We will denote by M^{sm} the set of such mechanisms.

Let us consider the following mechanism. The set of agents coincides with $W \cup F$, all the agents play simultaneously. Each worker's message (and strategy) space coincides with $[F \cup \{\emptyset\}] \times \mathbb{R}$. A message for worker w_i, namely $m_i = (m_{i1}, m_{i2})$, specifies the firm for which she is ready to be hired, m_{i1}, and the salary she requests, m_{i2}. Firms' message space is $2^W \times \mathbb{R}^n$. A message for firm f^j, namely $m^j = (m^{j1}, m^{j2})$, specifies the set of workers it wants to hire, m^{j1}, and the salary it is ready to pay to each worker[1], m^{j2}. In order to introduce the outcome function, let us fix non-negative real numbers α_i^j, β_i^j, $i = 1, \ldots, n$; $j = 1, \ldots, l$, such that, for all i, j, $\alpha_i^j + \beta_i^j \leq 1$. Given these numbers, the outcome function will select, for each strategy profile, say $\tilde{m}$, a matching $\mu(\tilde{m})$ and salaries to be payed by each firm, s^j , and earned by workers, s_i. The matching $\mu(\tilde{m})$ is defined in such a way that, for any worker, $\mu_i(\tilde{m}) = \emptyset$ if $m_{i1} = \emptyset$; if $m_{i1} = f^j \in F$, then

[1]In our description of a firm's strategy we require the salaries to belong to $\mathbb{R}^n$ for notational simplicity. In fact, a firm only needs to state a salary for each worker it wants to hire.

$$\mu_i(\tilde{m}) = \begin{cases} f^j & \text{if } w_i \in m^{j1} \text{ and } m_{i2} \leq m_i^{j2} \\ \emptyset & \text{otherwise} \end{cases}$$

Salaries are defined as follows. Worker w_i will earn 0 if $\mu_i(\tilde{m}) = \emptyset$ and $m_{i2} + \alpha_i^j \left(m_i^{j2} - m_{i2}\right)$ if $\mu_i(\tilde{m}) = f^j$. Firm f^j will pay 0 to a worker which is not matched to it and $m_i^{j2} - \beta_i^j \left(m_i^{j2} - m_{i2}\right)$ to any worker w_i matched to that firm.

A simple interpretation can be given concerning the working of this mechanism. Each worker has to decide on the firm for which she wants to work and some minimal contractual conditions (in terms of salary). Once the firm is selected by the worker, the employment office studies if the two agents could agree on some contractual conditions. Whenever such a contract exists, the worker is hired by the firm. The firm pays to the worker a salary which is decided by the employment office, and some fee to that office. That fee, if any, will depend on the difference between worker's and firm's expected salary. Following this interpretation, let us suppose that worker w_i and firm f^j are matched. In such a case, the per unit fee that the employment office gets is $1 - \alpha_i^j - \beta_i^j$.

The result we can expect from agents' interaction depends on the proportional fee. Notice that it is the employment office who chooses the level of all these variables by selecting the particular mechanism to be employed. Nevertheless, in equilibrium, whenever a worker is hired by a firm, the salary that the worker demands coincides with that offered by the firm.

Since we are interested in studying the set of allocations that might be expected when such mechanisms are employed, we need to be more precise on the solution concepts we will analyze.

Concerning to the situation where only individualistic behavior is assumed, the natural equilibrium concept is the Nash equilibrium. On the other hand, when agents can agree on the strategies to be played, one can study the next two equilibrium concepts, namely strong Nash and rematching-proof equilibrium.

Aumann [3] introduces the strong Nash equilibrium as a Nash equilibrium in which no coalition formed by any possible group of players (workers and firms) can jointly change their reported preferences (keeping other agents' reports fixed) so as to make them better off. In other words, in a strong Nash equilibrium there is no way to improve the welfare of any agent by changing her behavior even if she does it coordinated with other agents that at least will be not worse off. The rematching-proof equilibrium was introduced by Ma [6] for the marriage market. Here we adapt that concept to the many-to-one matching market in the following way. A rematching-proof equilibrium is a Nash equilibrium in which no coalition formed by one firm and a group of workers can jointly change their reported preferences (keeping other agents' reports fixed) so as to make all of them better off. If a Nash equilibrium is not rematching-proof, then there exists at least a coalition like we have defined above that can make all of its members better off by coordinating their behavior. Note that, in our framework, the only difference presented by the two equilibrium concepts comes from allowing each firm to commit with any other firm on the strategy to be played or not.

4. THE RESULTS

The first result we will present in this section tells us that agents want to participate in a hiring procedure described by our mechanism. This is because only individually rational allocations should be expected when employing this mechanism.

Theorem 4.1. *Any mechanism in M^{sm} implements in Nash equilibrium the individually rational correspondence.*

Proof. Consider a job matching market and a mechanism belonging to M^{sm}. First, we will show that any Nash equilibrium leads to an individually rational allocation.

Let $\tilde{m}$ be a Nash equilibrium. Suppose that the allocation $[\mu(\tilde{m}), s(\tilde{m})]$ is not individually rational. Then, there should be a worker w_i which prefers not being hired by the firm which she is assigned to, or a firm preferring to hire less workers than it does. We will see that none of both possibilities holds at equilibrium. First, suppose that $U_i(\mu_i(\tilde{m}), s_i(\tilde{m})) < U_i(\emptyset, 0)$. Then, worker w_i can get a higher utility level by playing $m_i^{'} = (\emptyset, 0)$. Note that when playing strategy $m_i^{'}$, that worker guarantees herself to get her reservation utility level. Second, suppose that the individual rationality condition is not satisfied for firm f^j. Thus, there is a set of workers contained in $\mu^j(\tilde{m})$, say W^j, such that $\pi^j(W^j, s(\tilde{m})) > \pi^j(\mu^j(\tilde{m}), s(\tilde{m}))$. Since such a firm can hire only workers in W^j, and pay salaries $s(\tilde{m})$, by playing $m^{j\prime} = (W^j, s(\tilde{m}))$, this fact contradicts that $\tilde{m}$ was a Nash equilibrium. Thus, every Nash equilibrium has to lead to an individually rational allocation.

On the other hand, let (μ, s) be an individually rational allocation. We are going to build a Nash equilibrium supporting such an allocation. Consider the following strategies for the agents. Worker w_i's strategy is $m_i = (\mu_i, s_i)$, and firm f^j will play strategy $m^j = (\mu^j, \hat{s}^j)$, with $\hat{s}_i^j = s_i$ if $w_i \in \mu^j$ and $\hat{s}_j^j = r_i^j$ otherwise. It is easy to check that these strategies constitute a Nash equilibrium, whose outcome is (μ, s). Notice that, given agents' strategies, any unilateral deviation will provide a new outcome $(\overline{\mu}, \overline{s})$ such that: (i) for all $f^j \in F$, $\overline{\mu} \subseteq \mu$ and (ii) for each $w_i \in W$, $\overline{s} = s_i$ if $\overline{\mu} = \mu$ or $\overline{s} = 0$ otherwise. Since (μ, s) is individually rational, no agent will benefit from unilateral deviations. ∎

Previous theorem states that, when agents behavior is individualistic, any individually rational allocation can be expected from the agents' interaction. Even if such an allocation does not satisfy Pareto efficiency. A possible way to avoid this problem is by leading the agents to commit to the strategy to be employed. This job can be done by allowing the agents to form unions. In such a case, the role of these unions is to negotiate the strategy to be played by each agent in order to reach the highest utility level (profit for the firm) they can. When only workers' unions are allowed, the equilibrium concept to be analyzed is rematching-proof; in addition, if firms' unions are also formed, we only can expect allocations that are supported by strong Nash equilibrium. The next result will show that efficiency can be reached by this collusive behavior. Moreover, such an efficiency is obtained whenever workers' unions are formed, no matter if firms' unions are formed or not.

Theorem 4.2. *Any mechanism in M^{sm} doubly implements in rematching-proof and strong Nash equilibrium the stable correspondence.*

Proof. To prove that these mechanisms implement the stable set in rematching-proof equilibrium in the first place, let us consider a job matching market and a mechanism belonging to M^{sm}. First, we are going to show that every rematching-proof equilibrium yields a stable allocation. And second, for any stable allocation, we will build a rematching-proof equilibrium supporting it.

Let $\tilde{m}$ be a rematching-proof equilibrium for such a mechanism. Suppose that allocation $[\mu(\tilde{m}), s(\tilde{m})]$ is unstable. Since every rematching-proof equilibrium is a Nash equilibrium, we know from Theorem 4.1 that such an allocation has to be individually rational. Thus, there should be a firm, say f^j, a set of workers, say

W^j, and a salary vector S such that

(i) $U_i(f^j, s_i) > U_i(\mu_i(\tilde{m}), s(\tilde{m}))$ for all $w_i \in W^j$, and

(ii) $\pi^j(W^j, S) > \pi^j(\mu^j(\tilde{m}), s(\tilde{m}))$.

This is contradictory with the fact that $\tilde{m}$ was a rematching-proof equilibrium, because workers in W^j can increase their utility level by committing with firm f^j in order to be hired by that firm at salaries in S. Note that such a firm will also increase its profit level.

On the other hand, let $[\mu, s]$ be a stable allocation. We are going to build a rematching-proof equilibrium supporting such an allocation. Consider the following strategies of the agents. Worker w_i's strategy is $m_i = (\mu_i, s_i)$, whereas firm f^j plays strategy $m^j = (\mu^j, s)$. It is easy to check that these strategies constitute a rematching-proof equilibrium whose outcome is (μ, s).

As the reader can see, the proof for implementation in strong Nash equilibrium is similar to the above proof for the rematching-proof equilibrium case. In fact, if we change rematching-proof equilibrium for strong Nash equilibrium in the previous paragraphs, the proof is done. ∎

The previous result pointed out that when agents are allowed to commit in the strategy to be employed stability of the markets, and hence Pareto efficient allocations, can be assured in job markets. In order to obtain stability it is essential that firms might commit to some workers' union. The existence of the workers' union guarantees the necessary coordination among workers who will be employed by the firm. Thus, workers' unions have an interesting social role of establishing coordination among agents to get stability and efficiency.

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