PHYSICAL AND HUMAN CAPITAL INVESTMENT: RELATIVE SUBSTITUTES IN THE ENDOGENOUS GROWTH PROCESS*

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A B S T R A C T

This paper aims at studying the interaction between growth of real output and human capital accumulation when education requires investment of physical resources. To this end we investigate the aggregate implications of individual specific uncertainty about returns to investment in education in the absence of insurance markets. We do so in a general equilibrium OLG model in which physical resources must be devoted to education in order to accumulate human capital. We find that uncertainty with incomplete financial markets may strongly affect individual behavior but not the aggregate of the economy: different degrees of uncertainty will induce different intensities of human to physical capital but will not have a significant impact on the long run growth rate of the economy. This framework allows us to conclude that investing less in education in relative terms does not necessarily lead to less growth: the accumulation of physical and human capital display some degree of substitutability as an engine for long run growth.

Keywords: Overlapping generations, Investment in education, Uninsured shocks, Human capital, Sustained growth.

JEL\ classification numbers: E13, O41, I29.

1 Introduction

The objective of this paper is to better understand the interaction between growth of real output and human capital accumulation when education requires investment of physical resources. To this end we explore the role of uncertainty in individual education decisions, and ultimately in long run growth patterns. The departure point will be a model economy in which agents invest real resources in education, thus leaving the door open to the possibility that output growth causes the accumulation of human capital by creating resources afterwards allocated to education. In this framework we will examine whether different degrees of uncertainty may generate differences in investment in education and in capital intensities while leaving the output growth rate unchanged.

Ever since the seminal contributions of Schultz (1960, 1961) and Becker (1962), investment and accumulation of human capital has been widely regarded as a way to increase individual labor productivity and therefore labor earnings, on one hand, and as an engine for sustained aggregate growth on the other hand. If individual investment in education is positively affected by the environment (the current stock of human capital), the individual act of investment becomes an accumulation process at the aggregate. In the mainstream view, education increases labor productivity which in turn causes the economy to grow. A recent U.S. Congress report reflects the view that "formal education is an important determinant of individual earnings as well as economic growth." Institutions like the World Bank or the United Nations stress the importance of investment in education for growth and foster policies directed towards the increase of formal education.

There is, however, evidence that the causal relationship between education, earnings, and growth is far from clear and that it may be more subtle. At the individual level there are serious qualifications as for the role of formal education in productivity. Recent contributions regard education as a consequence rather than a cause: education would result from characteristics already determined in earlier stages of the individual's life (Stokey (1998) or Heckman (2000)). At the aggregate level, we observe sharp differences both in schooling and in expenses in education between developed and underdeveloped countries. U.N. estimates for school life expectancy in the nineties are roughly 10 years for developing countries and 16 years for industrialized countries; expenses in education as a percentage of GDP were around 3.9% for developing countries and around 5.4% for industrialized countries. Nevertheless, recent research suggests that there is no absolute poverty trap: the poorest countries in 1960-1985 have been growing on average at similar rates as the richest countries (Maddison (1995) and Parente and Prescott (1995)). Further, there is some theoretical controversy as for the causal relationship between education and growth. Even if education is accepted as one of the determinants of the steady growth of labor productivity, a growing economy generates resources that can be allocated ex post to education (Bénabou (1996)).

In this paper we focus on the aggregate implications of education for growth and viceversa. The evidence outlined above suggests both (a) that there are ways to accumulate human capital other than formal training and (b) that time and resources devoted to education relative to GDP cannot fully account for long run growth in actual economies as suggested by schooling theories. The first observation motivates the search for mechanisms of human capital accumulation complementary to that of formal education. For instance, Lucas (1993) stressed the potential role of learning-by-doing in development processes. The second observation motivates a closer look at the relationship between real (resources) investment in education and growth. In particular, to the possibility that growing economies may generate the real resources necessary to invest in education and to accumulate human capital which in turn will cause the economy to grow.

The real nature of investment in human capital has already been emphasized in early contributions to this literature. Schultz (1960) and Uzawa (1963) have a broad notion of investment in human capital of which education is only one aspect: they include health care and any other real expenditure affecting the quality of labor.[2] From the individual point of view education is also regarded as a long run project characterized by uncertainty. Different innate abilities to

[2]Schultz (1961) traces the idea back to John Stuart Mill. This view is shared by modern approaches to the question of education at the individual level. Stokey (1998) emphasizes that expenditures in school fees are just a part of the parent's investment in their children.

take advantage of formation, life length, the impact of family environment, or simply unpredictable events, more likely to occur in such a long period, are some of the identified sources of uncertainty affecting educational choices (see Schultz (1961) and Becker (1962) or more recently Kodde (1986)). Many authors have analyzed the impact of uncertainty on individual decisions of education: Levhari and Weiss (1974) carried out the first formal analysis and were followed by many others like Williams (1978, 1979) or more recently Snow and Warren (1990). Under various financial frameworks, they basically confirm the intuition that uncertainty negatively affects the level of investment in education. These are, however, partial equilibrium analyses.

This paper is intended as a first step in the development of a general equilibrium model in which similar rates of growth are compatible with different fractions of output devoted to education. Following Michel (1993) we analyze an OLG model in which agents have to invest resources in their education to produce human capital. The accumulation of human capital is the source of sustained growth. Furthermore, individuals make their choice under uncertainty about the returns to education and with no insurance (an idiosyncratic shock affects the individual production of human capital). We will discuss that different degrees of uncertainty can generate differences in expenses in education (as a fraction of output) while leaving the growth rate relatively unchanged: The negative impact of uncertainty on education at the individual level is not translated to the aggregate because the accumulation of physical and human capital act as relative substitutes as an engine for growth.

2 Individual investment in human capital

The economy is represented by an overlapping generations model, a stochastic version of that described in Michel (1993). The demand-side of the economy is represented by a sequence of generations, each composed of a large number of ex ante identical agents. The supply side of the economy is represented by a single competitive firm endowed with a technology of constants returns to scale in physical and human capital.

2.1 Households and human capital

Agents live for three periods. Consider the typical agent born in period $t-1$. In the first period she decides educational investment $E_t \geq 0$. To finance her education she borrows in the financial market: she issues the single asset of the economy (there is no insurance). The resulting level of individual human capital will be conditional on a productivity shock $z_t \sim U[a,b]$ with $a > 0$, and on the aggregate level of human capital H_{t-1}, and it is given by

$$h_t(z_t) = z_t H_{t-1}^{1-\beta} E_t^{\beta} \tag{1}$$

where $\beta \in (0,1)$. Returns to scale are assumed to be constant to ensure feasibility of sustained growth.[3] All agents are identical ex ante and their productivity shock is independent from the others.[4] Then E_t is the same for all agents of generation $t-1$. Further, there is a large number of agents with mass normalized to one. Hence, ex ante expectations are ex post averages and aggregates. For example, aggregate human capital is $H_t = \mu H_{t-1}^{1-\beta} E_t^{\beta}$ where μ is the expectation of the shock.

Hereafter we will often work with variables per unit of stock of human capital writing $e_t = E_t / H_{t-1}$. Preferences over consumption will be homogeneous so that this transformation applies to both consumptions as described below.

There is no aggregate uncertainty and prices are perfectly forecasted by the

[3]This function meets the condition in Levhari and Weiss (1977, page 956) for uncertainty to have a negative effect on education. The assumption is quite appealing: it would be difficult to justify a positive effect of z_t on returns and a negative effect on marginal returns because it would suggest that higher skills (human capital) would be associated with lower marginal ability to increase skills (further accumulation of human capital through schooling).

[4]As discussed in the introduction, this individual specific ability to accumulate human capital can be interpreted to be partially innate or can be assumed to represent external factors: Schultz (1961) cites access to health services, Card and Krueger (1992) and Kodde (1986) point out the quality of schooling while Altonji and Dunn (1996) stress the family background (see again Stokey (1998) for a related discussion). These interpretations seem to fit well the assumption of independence.

agent. The real interest rate $r_t, r_{t+1} > 0$ and the real wage per efficiency unit $w_t > 0$ are taken as given by the agent. Leisure is not valued: the agent supplies inelastically her human capital stock. Dividing by H_{t-1}, net income next period (labor earnings minus debt) will be $m_t(z_t) = z_t w_t e_t^{\beta} - (1 + r_t)e_t$. The agent chooses consumption in the second and third periods of her life $c_t(z_t), d_{t+1}(z_t) \geq 0$ as well as savings $s_t(z_t)$ contingent to the realization of z_t so as to verify the set of contingent budget constraints

$$c_t(z_t) + s_t(z_t) \leq m_t(z_t) \tag{2}$$

$$d_{t+1}(z_t) \leq (1 + r_{t+1})s_t(z_t). \tag{3}$$

These variables are expressed in terms of per unit of human capital. Benefits will be zero so that equilibrium prices of shares will be zero as well. Since there is no insurance (and bankruptcy is not allowed) the non negativity constraints on consumption and the budget constraints impose $m_t(z_t) \geq 0$ for all z_t. This is equivalent to impose $m_t(a) \geq 0$ or $e_t \leq \bar{e}_t$ where $\bar{e}_t$ is defined implicitly as $a w_t \bar{e}_t^{\beta} - (1 + r_t)\bar{e}_t = 0$. That is, affordable choices of educational investment should not induce negative net income in any state of nature.

We shall refer to this constraint $e_t \leq \bar{e}_t$ as the limit to borrowing: any higher borrowing would lead the agent to bankruptcy in the worst states of nature. This limit to borrowing stems from the no bankruptcy assumption underlying any general equilibrium model together with the particular market structure considered.[5]

2.2 The saving rule and indirect utility

The agent has preferences defined over contingent plans of consumption $c_t(z_t)$ and $d_{t+1}(z_t)$ for all $z_t \in [a, b]$ represented by the expected utility function

$$U(c_t, d_{t+1}) = \int_a^b \frac{[c_t(z_t)^{\varepsilon} d_{t+1}(z_t)^{1-\varepsilon}]^{1-\theta} - 1}{1-\theta} dz_t$$

[5]The limit to borrowing is a consequence of the absence of insurance (incomplete markets) and not exogenously imposed: it should not be mistaken with an (exogenously imposed) borrowing constraint (generally considered a capital markets' imperfection).

where $\varepsilon \in (0,1)$ and $\theta \geq 0$. The density of z_t is a positive constant and therefore ignored. When $\theta = 1$ the integrand is interpreted to be the logarithm (in the sense that U converges pointwise to the integral of the logarithm $\log(c_t(z_t)^{\varepsilon} d_{t+1}(z_t)^{1-\varepsilon})$). We choose these preferences because the saving rate is independent of the relative degree of risk aversion θ.[6]

Since consumption choices across states of nature are separable we can solve the problem backwards. Fixed some z_t and e_t, the agent maximizes $c_t(z_t)^{\varepsilon} d_{t+1}(z_t)^{1-\varepsilon}$ subject to (2) and (3). As soon as $m_t(z_t) > 0$ the solution is interior. Further, the budget constraints must be binding at the optimum as the objective function is increasing in both arguments. Hence, the optimal saving rule is

$$s_t(z_t) = (1-\varepsilon)m_t(z_t), \tag{4}$$

a linear function of income because preferences are homothetic. This is also the optimal rule when $m_t(z_t) = 0$ because in that case $s_t(z_t) = 0$. Since the objective function is strictly quasiconcave and the budget set convex, expression (4) describes the unique solution to the problem given net income.

Introducing this optimal rule in the budget constraints and these in the objective function yields $\chi m_t(z_t)$ for all $m_t(z_t) \geq 0$ where $\chi > 0$ is a constant from the individual point of view. Use the definition of $m_t(z_t)$ in terms of e_t and introduce this expression in the utility function above to obtain

$$V(e_t) = \int_a^b \frac{(z_t w_t e_t^{\beta} - (1+r_t)e_t)^{1-\theta} - 1}{1-\theta}\, dz_t,$$

the indirect utility function.[7] The function V is twice continuously differentiable, strictly concave, and $V'(0) = \infty$. Moreover, it is continuous and differentiable as a function of parameters a, b, and θ. The proof of these statements is standard and therefore omitted.

[6] If the degree of risk aversion were related to the elasticity of intertemporal substitution, it would be difficult to disentangle the effect of risk aversion from that of the savings rate in the aggregate.

[7] We have omitted the term $\chi^{1-\theta}$, a positive constant: add and substract $\chi^{1-\theta}(1-\theta)^{-1}$ and operate to conclude that this is the relevant indirect utility function in the sense that it represents the same preferences.

2.3 Optimal choices of education

Avoid the trivial case of no uncertainty assuming hereafter that $a < b$ so that the standard deviation of the shock is positive: $\sigma > 0$. Unless otherwise stated we shall also assume that $\theta > 0$, that is, we concentrate on the case of risk aversion. The optimal rule of investment in education has constant elasticity with respect to the ratio of prices:

Proposition 1 *For all parameters specifications and prices $w_t/(1+r_t) > 0$ there is a unique optimal choice $e_t^* > 0$. Optimal education is given by the rule*

$$e^* = \varphi\left(\frac{w_t}{1+r_t}\right) = \left(Q\frac{w_t}{1+r_t}\right)^{\frac{1}{1-\beta}} \tag{5}$$

where $Q = a$ when the limit to borrowing is binding and $Q = M > 0$ when the solution is interior; M will then be a function of parameters a, b, β, α, and θ.

Proof: Since V is continuous and $[0, \bar{e}_t]$ compact, an optimal solution e_t^* exists. Uniqueness follows from strict concavity of V. This solution cannot be zero because $V'(0) = \infty$. When the solution is interior the first order condition is

$$\int_a^b (z_t w_t e_t^{*\beta} - (1+r_t)e^*)^{-\theta}(z_t w_t \beta e_t^{*\beta-1} - (1+r_t))\, dz_t = 0. \tag{6}$$

Since $r_t > -1$ and $e_t^* > 0$ this condition can be rewritten as

$$\int_a^b \left(z_t\frac{w_t}{1+r_t}e_t^{*\beta-1} - 1\right)^{-\theta}\left(z_t\frac{w_t}{1+r_t}\beta e_t^{*\beta-1} - 1\right) dz_t = 0. \tag{7}$$

Implicitly $e_t^* = \varphi(w_t/(1+r_t))$. Since the left hand side of the equation is a continuously differentiable function of the ratio of prices and of education, the implicit function theorem ensures that

$$\int_a^b (-\theta)\left(z_t\frac{w_t}{1+r_t}e_t^{*\beta-1} - 1\right)^{-\theta-1}$$
$$\times\, e^{*\beta-1}\left(z_t + z_t\frac{w_t}{1+r_t}(\beta-1)\frac{1}{e_t^*}\varphi'\left(\frac{w_t}{1+r_t}\right)\right)\left(z_t\frac{w_t}{1+r_t}\beta e_t^{*\beta-1} - 1\right) dz_t$$
$$+\int_a^b \left(z_t\frac{w_t}{1+r_t}e_t^{*\beta-1} - 1\right)^{-\theta}\beta e_t^{*\beta-1}\left(z_t + z_t\frac{w_t}{1+r_t}(\beta-1)\frac{1}{e_t^*}\varphi'\left(\frac{w_t}{1+r_t}\right)\right) dz_t = 0.$$

Rearranging terms and simplifying it turns out that φ' can be expressed as

$$\varphi'\left(\frac{w_t}{1+r_t}\right) = \frac{e_t^*}{\frac{w_t}{1+r_t}}\frac{1}{1-\beta}.$$

But this implies that the optimal rule of educational investment is of the form

$$\varphi\left(\frac{w_t}{1+r_t}\right) = \left(M\frac{w_t}{1+r_t}\right)^{\frac{1}{1-\beta}}$$

where $M > 0$ is some constant. Direct substitution of this expression in (7) yields

$$\int_a^b \left(zM^{-1}-1\right)^{-\theta}\left(zM^{-1}\beta-1\right)\,dz = 0. \tag{8}$$

This equation has a unique solution because optimal education is unique and the rule φ is strictly increasing in M. The solution depends on the values of a, b, β, and θ, and indirectly of μ and σ through a and b.

The optimal rule is completed setting, when the limit to borrowing is binding,

$$\varphi\left(\frac{w_t}{1+r_t}\right) = \left(a\frac{w_t}{1+r_t}\right)^{\frac{1}{1-\beta}}. \tag{9}$$

That is, the limit to borrowing $\bar{e}_t$ expressed in terms of its value as a function of w_t, r_t, β, and a. ■

An immediate consequence of the proposition above is that $M < a$ when the solution is interior.

The effect of prices on the choice of education is obvious. Increasing the wage (or decreasing the interest rate) uniformly increases net returns to education over all states of nature inducing higher levels of education. More interesting is the effect of risk aversion and the incomplete structure of financial markets: Risk aversion reduces (interior choices of) investment in education as it will make the agent care relatively more about the bad states of nature.

Proposition 2 *Interior optimal choices of investment in education are strictly decreasing in risk aversion as represented by θ. Border solutions are insensitive to infinitesimal changes of θ.*

Proof: When $V'(\bar{e}_t) > 0$ the limit to borrowing is binding. Since V' is a continuous function of θ, infinitesimal changes in this parameter will not affect this inequality while the limit to borrowing itself does not depend on θ.

Now suppose that M is the solution to (8) for some $\theta \geq 0$ and let $\theta' > \theta$. Let $\tilde{z}$ be the only value of the shock for which $(\tilde{z}M^{-1} - 1)^{-\theta}(\tilde{z}\beta M^{-1} - 1) = 0$. Then (8) can be written as

$$\int_{\tilde{z}}^{b} (\tilde{z}M^{-1} - 1)^{-\theta}(\tilde{z}\beta M^{-1} - 1)\, dz = -\int_{a}^{\tilde{z}} (\tilde{z}M^{-1} - 1)^{-\theta}(\tilde{z}\beta M^{-1} - 1)\, dz.$$

Moreover, since $\theta' - \theta > 0$ we have

$$0 < (z'M^{-1} - 1)^{-(\theta'-\theta)} < (z''M^{-1} - 1)^{-(\theta'-\theta)}$$

for all $z' > z''$ and in particular for all $z' \in (\tilde{z}, b]$ and $z'' \in [a, \tilde{z})$. Since

$$(zM^{-1} - 1)^{-\theta}(z\beta M^{-1} - 1)(zM^{-1} - 1)^{-(\theta'-\theta)} = (zM^{-1} - 1)^{-\theta'}(z\beta M^{-1} - 1)$$

for all z it must be the case that

$$\int_{\tilde{z}}^{b} (zM^{-1} - 1)^{-\theta'}(z\beta M^{-1} - 1)\, dz < -\int_{a}^{\tilde{z}} (zM^{-1} - 1)^{-\theta'}(z\beta M^{-1} - 1)\, dz.$$

The overall effect is negative because multiplying the integrand by $(zM^{-1} - 1)^{-(\theta'-\theta)}$ amounts to assign a bigger weight to the negative part of the integral relative to the positive part. This inequality can be written as

$$\int_{a}^{b} (zM^{-1} - 1)^{-\theta'}(z\beta M^{-1} - 1)\, dz < 0$$

implying that M is not optimal for θ'. Since V' is strictly decreasing in e_t it must be the case that the left hand side of this expression is strictly decreasing in M. Hence, the new optimal M' must be such that $M' < M$. ■

Some notation: for any $z \in [a, b]$ we will write $e_{z,t}$ to denote the value of investment in education that verifies $zw_t\beta e_{z,t}^{\beta-1} - (1 + r_t) = 0$.

Lemma 1 *For all parameters specifications it is true that* $e_{a,t} < e_t^* \leq \min\{\bar{e}_t, e_{\mu,t}\}$. *In other words, it is true that* $\beta a < M \leq \min\{a, \beta\mu\}$.

Proof: Suppose that $e_{a,t} > e_t^*$, then $z_t w_t \beta e_t^{*\beta-1} - (1+r_t) \geq a w_t \beta e_t^{*\beta-1} - (1+r_t) > 0$ for all z_t. An infinitesimal increase of e_t^* would induce a marginal increase of net income in all states of nature thus increasing expected indirect utility and therefore contradicting that e_t^* is optimal.

That $e_t^* \leq \bar{e}_t$ follows from the limit to borrowing. Finally, suppose that $e_{\mu,t} < \bar{e}_t$, then follow the proof of proposition 2 for $\theta = 0$ to show that $e_t^* < e_{\mu,t}$. ■

Observe that the proof of proposition 2 could have been written in terms of the first order condition and e_t^* with any other distribution function. First, the constant elasticity optimal rule of proposition 1 holds for any distribution F with support infimum $a > 0$; second, in the proof of proposition 2, whenever $(zM^{-1} - 1)^{-\theta}$ appears, it can be exchanged by marginal utility in state z and $(z\beta M^{-1} - 1)$ by marginal income in expression (6). In short, the proofs above can be reproduced without the uniform distribution assumption.

The uniform distribution was assumed for simplicity in the next proof and in the interpretation of the numerical simulations: the effect of σ is readily interpreted when the shock is uniformly distributed because other moments of the distribution are very simple.

Proposition 3 *Optimal investment in education, binding or not, is strictly decreasing in σ (the standard deviation of the shock z_t), μ constant.*

Proof: When the solution is $e_t^* = \bar{e}_t$, decreasing a directly decreases the optimal choice as it is clear from expression (9). When the solution is interior, with a uniform distribution (8) can be written in terms of σ as

$$\int_{\mu-\sigma\sqrt{3}}^{\mu+\sigma\sqrt{3}} (zM^{-1} - 1)^{-\theta}(z\beta M^{-1} - 1)\, dz = 0.$$

We need the left hand side of this expression to be decreasing in σ for optimal M to be decreasing in σ because the integral is decreasing in M (see the end of the previous proof). Applying Leibniz rule it is clear that the integral is decreasing in σ if and only if

$$-(aM^{-1} - 1)^{-\theta}(a\beta M^{-1} - 1) > (bM^{-1} - 1)^{-\theta}(b\beta M^{-1} - 1).$$

The rest of the proof is devoted to show that this inequality holds.

Let M solve (8) for some given value $\sigma > 0$ and let n be the (only) linear function of z that verifies

$$n(a) = (aM^{-1} - 1)^{-\theta}(a\beta M^{-1} - 1) \text{ and } n(b) = (bM^{-1} - 1)^{-\theta}(b\beta M^{-1} - 1).$$

where $n(a) < 0$ and $n(b) > 0$ because by lemma 1 interior solutions verify $\beta a < M < \beta\mu < \beta b$. Suppose that $n(a) + n(b) > 0$ contradicting the inequality above. Let $\tilde{z}$ be the critical value for which $\tilde{z}\beta M^{-1} - 1 = 0$ and note that $M < \beta\mu$ so that $\tilde{z} < \mu$. On one hand, we have

$$\begin{aligned}\int_a^b n(z)\,dz = \frac{1}{2}(n(b)(b-\tilde{z}) + n(a)(\tilde{z}-a)) &> \frac{1}{2}(n(b)(b-\mu) + n(a)(\mu - a)) \\ &= \frac{b-a}{4}(n(b) + n(a)) > 0.\end{aligned}$$

On the other hand, since the integrand is strictly concave or decreasing it must be the case that

$$n(z) < (zM^{-1} - 1)^{-\theta}(z\beta M^{-1} - 1)$$

for all $z \in (a, b)$. We conclude that

$$0 < \int_a^b n(z)\,dz < \int_a^b (zM^{-1} - 1)^{-\theta}(z\beta M^{-1} - 1)\,dz$$

thus contradicting that M is a solution to (8). ■

Observe that (in view of the last inequality in the proof) the negative effect of σ becomes stronger as the concavity of the integral, as measured by θ, increases. In other words, a higher degree of risk aversion will worsen the effect of σ on the optimal choice of investment in education. Figure 1 plots the typical simulation of the individual decision for an array of values of θ and σ.[8] We set $\mu = 8$ and

[8] The routines used to simulate individual and aggregate behavior in this economy are based in a simple bisection procedure to find a solution to (8). The code is written in Matlab and is available from the authors upon request.

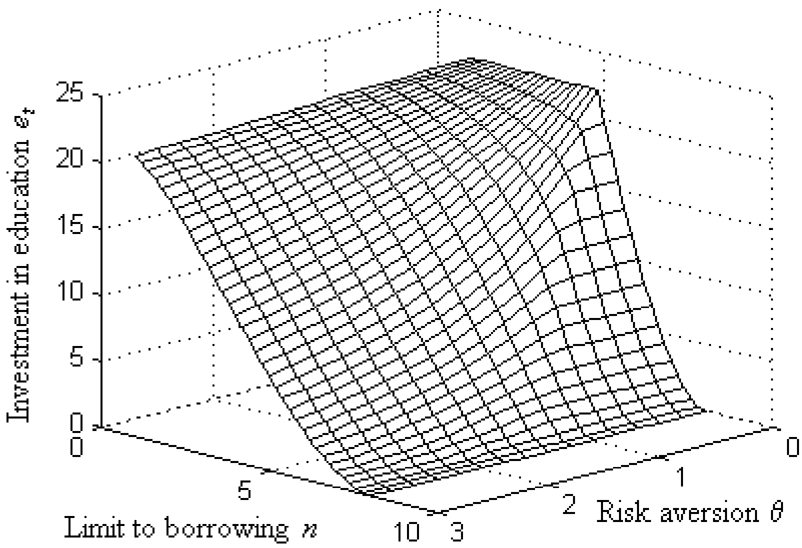

Figure 1: The effect of risk aversion and the limit to borrowing

let $a = \mu - n$ and $b = \mu + n$. In the third axis we plot different values n from zero to 8 so that $n = 8$ represents $a = 0$. The degree of risk aversion ranges from risk neutral $\theta = 0$ to $\theta = 3$. The strong effect of the lack of insurance markets at the individual level is reflected in this figure: When the agent is risk neutral she chooses e_μ up to the point where a is so low that she has to choose the corner solution $\bar{e}$. As the agent becomes risk averse, the effect of lowering a will operate sooner: her risk aversion makes her care very soon about the worst states of nature. She therefore lowers e^* in an effort to increase income in those bad states of nature. In a world with insurance, although one would expect education to decrease with σ, the effect would be less obvious and not as dramatic when a is driven to zero.[9]

[9]It may be worth stressing that whether the limit to borrowing is binding or not is irrelevant. In the plot it is obvious where the constraint is binding: for low values of θ and low values of a (high values of n). It can proven that for $\theta \geq 1$ the constraint is never binding. What matters, however, is that the agent is led to choose zero investment when $a = 0$ because of the contraction of the choice set.

In the absence of insurance, uncertainty strongly discourages investment in education. As we will see in the next section, however, at the aggregate this negative effect is attenuated and may even be reversed. As a consequence the long run growth rate of the economy will be rather insensitive to the degree of uncertainty.

3 The equilibrium of the economy

In this section we introduce the representative firm and define a competitive equilibrium for this economy. We will prove existence of a unique equilibrium and long run convergence of transformed stationary variables to a unique steady state.

3.1 The firm

The supply side of the economy is represented by a standard single competitive firm producing output in t from output in $t-1$ (physical capital) and effective labor in t (human capital). The firm is endowed with technology represented by a Cobb-Douglas production function with share of physical capital $\alpha \in (0,1)$, scale factor $A > 0$, and full depreciation for simplicity.

The firm borrows its stock of capital $K_t \geq 0$ in the credit market in period t and returns $1 + r_t$ the next period. Since agents do not care about leisure, they inelastically supply their stock of human capital so that in equilibrium H_t is also the effective labor hired by the firm. Define $k_t = K_t/H_t$, then the firm's first order conditions can be written as $w_t = A(1-\alpha)k_t^{\alpha}$ and $1 + r_t = A\alpha k_t^{\alpha-1}$.

3.2 Competitive equilibrium

The credit market clearance requires $K_{t+1} + E_{t+1} = S_t$ where $S_t/H_t = (1 - \varepsilon)(w_t - (1/\mu)(1+r_t)e_t^{1-\beta})$ is obtained integrating the optimal saving rule (4) over z_t (that is, over individuals) and dividing it by μe_t^{β} making use of the fact that $H_{t/}H_{t-1} = \mu e_t^{\beta}$. Then, in terms of variables per unit of human capital the credit market clears when

$$(k_{t+1}\mu e_{t+1}^{\beta}+e_{t+1})=(1-\varepsilon)\left(w_t-\frac{1}{\mu}(1+r_t)e_t^{1-\beta}\right). \tag{10}$$

This equation, the agent's optimal rule φ and the firm's first order conditions, which incorporate the labor market clearance condition, describe competitive equilibria for this economy.

Definition 1 *Given an initial stock of physical capital $k_0>0$, a competitive equilibrium (for the stationary variables) is an allocation $(k_{t+1},e_t)_{t=0}^{\infty}$ and a sequence of prices $(w_t,r_t)_{t=0}^{\infty}$ such that $e_t=\varphi(w_t/(1+r_t))$, $w_t=A(1-\alpha)k_t^{\alpha}$, $1+r_t=A\alpha k_t^{\alpha-1}$, and such that (10) holds for all $t\geq 0$.*

Consumption and savings can be recovered from the optimal saving rule (4) and the agent's budget constraints (2) and (3). Substituting prices for their values as given by the firm's first order conditions yields an equilibrium system of two equations

$$e_t = \left(Q\frac{1-\alpha}{\alpha}k_t\right)^{\frac{1}{1-\beta}} \tag{11}$$

$$k_{t+1}\mu e_{t+1}^{\beta}+e_{t+1} = (1-\varepsilon)A\left[(1-\alpha)k_t^{\alpha}-\frac{\alpha}{\mu}k_t^{\alpha-1}e_t^{1-\beta}\right] \tag{12}$$

where $Q=a$ or M when the solution induced by k_t is corner or interior respectively. Equilibrium exists, is unique, and converges to a steady state $(\tilde{k},\tilde{e})$.

Proposition 4 *For all $k_0>0$ there is a unique competitive equilibrium. Competitive equilibrium allocations converge monotonically to a single interior steady state $(\tilde{k},\tilde{e})$ independently of initial conditions.*

Proof: Let $k_t>0$ be any stock of capital. At interior solutions $Q=M$ while M exists and is unique by proposition 1. At corner solutions $Q=a$. In both cases a unique e_t of equilibrium is determined by equation (11). Direct substitution of (11) into (12) yields the aggregate equilibrium transition

$$k_{t+1}=\left[\frac{(1-\varepsilon)(1-\alpha)A\left(1-\frac{Q}{\mu}\right)}{\mu\left(Q\frac{1-\alpha}{\alpha}\right)^{\frac{\beta}{1-\beta}}+\left(Q\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\beta}}}\right]^{1-\beta}k_t^{\alpha(1-\beta)}. \tag{13}$$

Suppose that $a < \mu$. Then, if the solution is interior $Q = M < \mu$ because $M < \beta\mu$ by lemma 1, otherwise $Q = a < \mu$. Alternatively suppose that $a = \mu$, then the solution is interior and $Q = M = \beta\mu < \mu$. In both cases $Q < \mu$ so that (13) is strictly positive and well defined. In short, k_t uniquely determines k_{t+1} which in turn, again through equation (11), uniquely determines e_{t+1}: a unique competitive equilibrium exists.

Write $k_{t+1} = \eta(k_t)$ and observe that η is a differentiable, strictly concave, strictly increasing function of k_t with $\eta'(0) = \infty$ and $\eta'(\infty) = 0$. Hence, there is a unique steady state value for capital

$$\tilde{k} = \left[\frac{(1-\varepsilon)(1-\alpha)A\left(1-\frac{Q}{\mu}\right)}{\mu\left(Q\frac{1-\alpha}{\alpha}\right)^{\frac{\beta}{1-\beta}} + \left(Q\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\beta}}} \right]^{\frac{1-\beta}{1-\alpha(1-\beta)}} \tag{14}$$

and $k_t \to \tilde{k}$ monotonically. The steady state value of education $\tilde{e}$ is simply given by equation (11) evaluated at $\tilde{k}$. ■

An alternative approach to the question of existence of a competitive equilibrium consists of having a closer look at the left hand side of equation (12). Given any equilibrium allocation (k_t, e_t), equation (12) implicitly describes the combinations of education and capital $e_{t+1} = \psi(k_{t+1}, k_t, e_t)$ for the next period that clear the credit market. For any allocation (k_{t+1}, e_{t+1}) to be an equilibrium for the next period it must be true that $e_{t+1} = \psi(k_{t+1}, k_t, e_t)$ and that $e_{t+1} = \varphi(k_{t+1})$ (the abuse of notation is justified because $w_{t+1}/(1 + r_{t+1})$ is a function of k_{t+1}). Since φ is exponentially increasing in k and zero at $k = 0$, it would suffice that ψ is positive at zero and decreasing to prove that there is a unique equilibrium. If $k_{t+1}\mu e_{t+1}^{\beta} + e_{t+1}$ is to remain equal to a constant, when $k_{t+1} = 0$ it must be the case that $e_{t+1} > 0$ while as $k_{t+1} \to \infty$ education should converge to zero. Hence, an equilibrium exists. It is unique because $k_{t+1}\mu e_{t+1}^{\beta} + e_{t+1}$ is increasing in education and capital so that $\psi_1 < 0$. With this reasoning in mind the proof of the following result is straightforward:

Proposition 5 *Fixed an equilibrium (k_t, e_t), the equilibrium allocation for capital k_{t+1} (resp. education e_{t+1}) next period is strictly increasing (resp. decreasing) with θ when e_{t+1} is interior, and strictly increasing (resp. decreasing) with σ.*

Proof: Note that ψ is independent of θ and σ while φ is strictly decreasing with θ when the solution is interior (proposition 2) and strictly decreasing with σ (proposition 3). An increase in σ, and/or in θ when interior, would cause φ to shift downwards uniformly: the new equilibrium allocation would result from a shift to the right along the graph of ψ, a decreasing function. ■

Fixed any equilibrium allocation (k_t, e_t), an increase of σ in period t will induce individuals next period to invest less in education e_{t+1}, and more in physical capital k_{t+1}. The increase in k_{t+1} may stimulate future investment in education e_{t+2}. Indeed, at the aggregate the increased ratio of physical to human capital will result in higher savings. From equation (10) we can write

$$\frac{S_{t+1}}{H_{t+1}} = (1-\varepsilon)(1-\alpha)k_{t+1}^{\alpha}\left(1-\frac{Q}{\mu}\right).$$

An higher σ increases k_{t+1} and decreases Q. As a consequence, aggregate savings per unit of human capital rise and so do the available resources to be allocated to k_{t+2} and e_{t+2}. This is the resources effect. The subsequent increase in k_{t+2} will have in turn consequences as for the individual behavior: a higher wage w_{t+2} and lower interest rate r_{t+2} make investment in human capital e_{t+2} relatively more attractive. This is the price effect.

In short, less investment in education today means relatively more physical resources available for the future, and thus potentially allows more future investment in education. As we shall see, this mechanism underlies the ambiguous effect of σ on the long run growth rate.

3.3 The steady state growth rate

In models of schooling increasing physical capital for tomorrow increases the wage and therefore the incentive to devote time to schooling today. Here the price effect is analogous: it increases the wage and reduces the interest rate and therefore stimulates investment in education. However, there is an additional effect: the higher ratio of physical to human capital increases savings, and thus increases the amount of resources available for education. In contrast with the (fixed) time endowment model, in our economy resources can be created (and subsequently allocated to education) in an unbounded manner, leaving room for a sustained resources effect. In the long run this mechanism will cause the aggregrate impact of uncertainty to be ambiguous despite its strong negative effect at the individual level.

Along a balanced growth path $\tilde{k} = K_t/H_t$ will be describing whether an economy is more or less physical capital intensive (relative to human capital) while $\tilde{e}$ will be a proxy for the long run growth rate because all aggregate variables will be growing at the same rate as the stock of human capital $H_{t+1}/H_t = \mu\tilde{e}^{\beta}$.[10] Higher degrees of uncertainty always increase $\tilde{k}$, and therefore they are associated with more physical capital intensive balanced paths.

Proposition 6 *For all parameters specifications, $d\tilde{k}/d\sigma > 0$. That is, economies with high σ are more physical capital intensive than economies with low σ.*

Proof: Direct differentiation of (14) with respect to Q yields

$$\frac{d\tilde{k}}{dQ}\frac{Q}{\tilde{k}} = -\frac{1-\beta}{1-\alpha(1-\beta)}\frac{\left(1+\frac{1-\alpha}{\alpha}\frac{Q}{\mu}\right)+\left(1-\frac{Q}{\mu}\right)\left(\frac{\mu}{Q}\frac{\beta}{1-\beta}+\frac{1}{1-\beta}\frac{1-\alpha}{\alpha}\right)}{\left(1-\frac{Q}{\mu}\right)\left(\mu+\frac{1-\alpha}{\alpha}\right)} < 0. \quad (15)$$

To see why the negative sign recall the proof of proposition 4 where it is shown that $Q < \mu$ so that $(1-(Q/\mu)) > 0$. Finally observe that $\tilde{k}$ only depends on σ indirectly through Q. Then

$$\frac{d\tilde{k}}{d\sigma} = \frac{d\tilde{k}}{dQ}\frac{dQ}{d\sigma} > 0$$

because $dQ/d\sigma < 0$: when $Q = a$ trivially, and when $Q = M$ by proposition 3. ■

In short, an increase of σ, by decreasing Q, will increase $\tilde{k}$. This is further also reflected in that the long run share of physical investment in savings

$$\tilde{\rho}_k = \frac{K}{S} = \frac{\alpha\mu}{\alpha\mu + Q(1-\alpha)},$$

[10] Observe that unless μ and A are high enough there is no guarantee that $\mu\tilde{e}^{\beta} > 1$. For the growth rate to be positive the (expected) scale of the human capital production function and the scale of the physical good production function should be high enough to generate the resources necessary to sustain growth.

is increasing with σ (because $dQ/d\sigma < 0$) combined with the fact that aggregate savings S/H also increases with σ. Hence, the reasoning of the previous subsection holds in the long run: economies characterized by a higher σ are more physical capital intensive and have more resources to invest.

Contrary to the determinate effect of σ on $\tilde{k}$, its effect on $\tilde{e}$, and therefore on the growth rate, is ambiguous. Observe that the increase of $\tilde{\rho}_k$ implies a decrease of $\tilde{\rho}_e$, the share of education in savings. Nevertheless, aggregate savings S/H are increasing with σ. These two opposite effects explain why the effect of σ on $\tilde{e}$ remains ambiguous. More formally, differentiating (11) with respect to σ yields

$$\frac{d\tilde{e}}{d\sigma}\frac{\sigma}{\tilde{e}} = \frac{1}{1-\beta}\left(\frac{dQ}{d\sigma}\frac{\sigma}{Q} + \frac{d\tilde{k}}{d\sigma}\frac{\sigma}{\tilde{k}}\right). \tag{16}$$

This expression renders explicit the two mechanisms operating in the impact of σ on $\tilde{e}$. The first term inside the brackets is negative as $dQ/d\sigma < 0$, while the second is positive because $d\tilde{k}/d\sigma > 0$ by the proposition above. The first term summarizes the effect of the absence of insurance at the individual level while the second captures the price and resources effects. For reasonable parameters' specifications the steady state growth rate appears to be relatively insensitive with respect to small changes in σ. Indeed, using (15) and after some cumbersome (and therefore omitted) algebra it can be shown that either of the two effects can be dominating. For example, in the case where

$$\frac{d\tilde{k}}{d\sigma}\frac{\sigma}{\tilde{k}} > -\frac{dQ}{d\sigma}\frac{\sigma}{Q},$$

the price and resources effect dominate the no insurance effect, and therefore increase $\tilde{e}$ despite the higher degree of uncertainty. The verification of this inequality implies that $\tilde{\rho}_k$ is growing with σ to a smaller extent than aggregate savings. That is, both available resources and investment in physical capital (both per unit of human capital) increase with σ, but the former more than the latter. When the no insurance effect dominates the price and resource effects, the reverse reasoning applies.

Figure 2 plots the typical numerical simulation for the long run growth rate for an array of values for θ and σ (we considered the same parameters' values as in

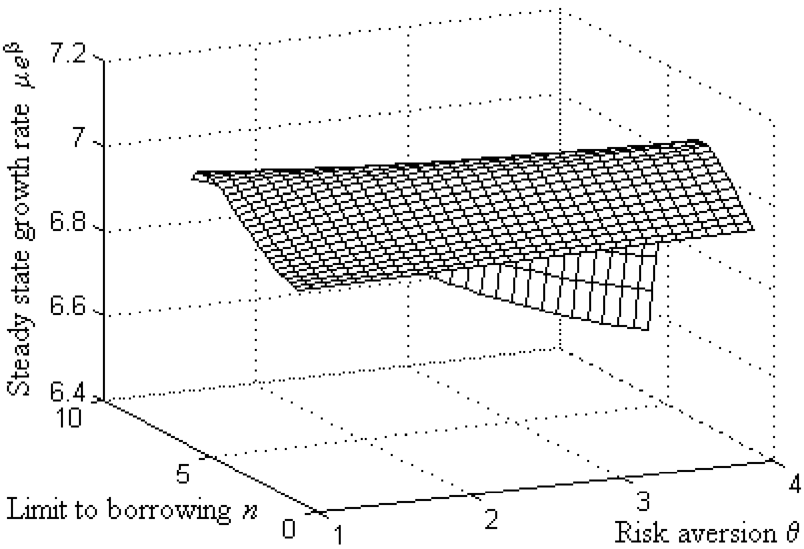

Figure 2: The impact of no insurance in the long run

section 2). The ambiguous impact of σ contrasts with the strong negative impact we observed at the individual level. Of course, as a tends to zero the negative impact will finally dominate as agents are forced by their budget constraints to invest zero in education.

3.4 Education expenditures as a percentage of GDP: a suitable indicator for economic performance?

Investment in education is, not surprisingly, an important indicator when evaluating the economic and social performance of a country. From the economic point of view, education enlarges the stock of human capital widely regarded as an engine of endogenous growth. From the social point of view education is a way of emancipating people; and more specifically, when publicly provided, education generally allows to reduce inequalities. The relevant figures on education in the data often refer to investment in education as a percentage of GDP. In this subsection we will have a look at this indicator in the light of our model. More particularly, we will examine the relation between the proportion of available resources devoted to education E/Y, and the long run growth rate.

In period t the proportion of national product Y_t devoted to individual investment in education E_{t+1} is given by $E_{t+1}/Y_t = e_{t+1}/(Ak_t^\alpha)$, or in the long run by $E/Y = e/(Ak^\alpha)$. The effect of an increase in the degree of uncertainty σ on this indicator can be written as

$$\frac{dE/Y}{d\sigma}\frac{\sigma}{E/Y} = \frac{de}{d\sigma}\frac{\sigma}{e} - \alpha\frac{dk}{d\sigma}\frac{\sigma}{k}$$

As observed before, for reasonable parameters the long run growth rate, determined by e, is relatively insensitive to σ; hence $(de/e)/(d\sigma/\sigma) \simeq 0$. In that case an increase in σ has a negative effect on E/Y given the fact that $dk/d\sigma > 0$. Stated differently, similar growth rates are compatible with different proportions of national product invested in education. This leads us to the following lemma:

Lemma 2 *Economies that invest a smaller proportion of their GDP in education do not necessarily display lower growth rates.*

Indeed, when this smaller proportion is accompanied by a higher physical capital intensity k, the growth rate does not need to be negatively affected: human and physical capital are relative substitutes as an engine for growth, making the link between educational investment and growth less straightforward.

According to our model the proportion of GDP devoted to education is thus less suitable an indicator for economic performance as far as economic growth is concerned. Yet this does not in any way temper the rationale for providing public education. As mentioned before, education does not only serve economic goals; it can also constitute a powerful tool to fight social exclusion and to smooth inequality.

4 Conclusions

In this paper we have analyzed a general equilibrium model of investment in education when the returns to this investment are uncertain and agents cannot insure themselves. Confirming previous partial equilibrium analyses, at the individual level the impact of uncertainty is negative and very strong. Nevertheless, at the aggregate other mechanisms operate compensating the initial individual incentive to reduce investment in education when uncertainty is introduced. The model economy describes a world in which different degrees of uncertainty can yield different capital intensities but similar long run growth rates.

It is commonly accepted that the accumulation of human capital is at least one of the characteristics that allow modern market economies to grow sustainably. In this paper it is shown that the mechanisms by which human capital induces growth may be subtle. When education is modeled as an investment in terms of physical resources, rather than schooling, education causes growth but growth also causes education: making resources available that will eventually be allocated to education in the future. In short, the accumulation of physical and human capital display some degree of substitutability as an engine for long run growth: two economies identical except for the variance of the productivity shock may grow at the same rate along two different paths: one will be more physical capital intensive than the other, and will invest a smaller proportion of its GDP in education. In terms of economic policy, these results suggest that policies of public education should be conceived as a mean to smooth inequality, rather than to foster growth.

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