COMPUTING ROBUST STYLIZED FACTS ON COMOVEMENT*

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A B S T R A C T

We propose an alternative method of obtaining stylized facts on comovement, based on the cross-correlation function of the prewhitened time series, which only depends on the purely stochastic components of the series and the cross efects between them. This approach has the property of being robust to the filtering procedure and hence to the cicle definition. The usual approach consists of obtaining the cross-correlation function of filtered variables, which reflect a mixture of both the existing cross-correlation between the variables and the autocorrelation structure of each of them. The autocorrelation structure, in turn, crucially depends on the filtering procedure. The relevance of such an approach is tested by revisiting some of the facts reported by Kydland and Prescott (1990).

JEL classification: C22; E32

Key Words: Prewhitening, Business Cycle, Stylized Facts, Comovement.

1 BACKGROUND

Since the influential work of Hodrick and Prescott (1980), many studies have addressed the statistical characterization of macroeconomic variables over the business cycle, as for example Baxter and Stockman (1989), Kydland and Prescott (1990), Baxter (1991), Backus and Kehoe (1992), or Fiorito and Kollintzas (1994). In order to perform such characterization, and given that most economic time series exhibit a nonstationary behavior, some detrending procedure or filter is needed. Choosing a particular detrending method involves a decomposition of a series into, at least, a trend and a cyclical component, by putting a different amount of weight on each business cycle frequency. Provided that such components are unobservable and do not have a precise *correct* definition, the researcher has to select a specific filter (and hence a decomposition of the series)[1] and a set of useful statistics in order to capture some elements of the phenomenon under study, depending on the aim of the analysis and the underlying economic assumptions.

As for the filtering procedure, we will focus on the Hodrick-Prescott (1980) filter (HP filter hereafter) because it has become a standard procedure in an important branch of the academic literature, as well as several economic institutions[2]. Concerning the relevant set of statistics, we are interested in the stylized facts related to the comovements of some economic variables with the Gross National Product (GNP hereafter). Such comovements are commonly measured by the cross-correlation function (CCF hereafter) between each (filtered) variable and the (filtered) GNP.

We propose and alternative method for constructing business cycle statistics on comovement by using CCF's from prewhitened (filtered) economic variables. Prewhitening is an econometric procedure that consists of filtering a variable in order to extract all the systematic autocorrelation behavior from it, so that a white noise stochastic component is obtained. The most outstanding feature of the so obtained CCF's is the fact that they do not depend on the autocorrelation which is present in the cyclical components of the series, but only on the non-systematic (unpredictable) behavior and the strictly cross effects between the variables. This way we obtain some potentially useful information that, in principle, does not depend on the definition of the cycle, as it is shown below.

2 METHODOLOGY AND RESULTS

Box et al. (1994) propose the use of prewhitening for the identification of transfer function models, given that the identification process is considerably simplified when the inputs to the system are white noise. They show that the CCF between the prewhitened input and the transformed output is directly proportional to the impulse response function. Prewhitening is also suggested by Haugh and Box (1977) for the identification of dynamic distributed lag bivariate models, and by Jenkins and Alavi (1981) for the identification of multivariate time series models. We suggest the use of prewhitening to obtain business cycle statistics concerning comovements.

To illustrate the potential effects of this procedure, consider the following experiment. First, simulate two Gaussian white noise series a_{1t}, a_{2t} of, say, T observations, with contemporaneous correlation ρ, and

[1] Canova (1998) tests the practical relevance of the filter selection by examining the cyclical properties of a set of US macroeconomic time series using a variety of detrending methods and finds that both quantitatively and qualitatively stylized facts vary widely across detrending methods.

[2] See for example, European Commission (1995) or Banca d'Italia (1999).

that are uncorrelated at any other lag. Second, generate two time series z_{it} $(i = 1, 2)$ from a_{it} and a nonstationary autoregressive data generating process of the form[3] $(1 - \phi_1 B)(1 - \phi_2 B) z_{it} = a_{it}$, where B denotes the backshift operator. Third, obtain the HP-cycle of both variables z_{it}, say C_{it}, and compute the CCF between them. Forth, prewhiten C_{it} and obtain $\hat{a}_{it} = \Pi_i(B) C_{it}$. Fifth, compute the CCF between the prewhitened series $\hat{a}_{1t}$ and $\hat{a}_{2t}$. Finally, compare the two CCF's.

Concerning the prewhitening procedure, the operators $\Pi_i(B)$ are assumed to be purely autoregressive (AR) models, and they are estimated by Ordinary Least Squares. The pure AR assumption provides a simple and useful method to obtain white noise variables for the examples presented in this paper. More sophisticated econometric models could be employed to prewhiten the series under study, depending on the available information about their generating processes[4]. The white noise null hypothesis of the obtained prewhitened series is tested by the Box-Pierce statistic $Q = n \sum_{k=1}^{10} r_k^2$, where n denotes the effective number of observations (number of observations minus the order of the autoregressive process) and r_k the k-order autocorrelation of the assumed white noise series. The autoregressive order is adjusted until the null hypothesis is not rejected at the 5 percent significance level. The results of the test are then confirmed by visual inspection of the autocorrelation function (ACF) as suggested by Box et al. (1994). In this example a third-order autoregressive model turned out to be adequate.

It should be remarked that the main difference between C_{it} and $\hat{a}_{it}$ is that the former is an autocorrelated variable aiming to measure the cyclical behavior of z_{it} whereas $\hat{a}_{it}$ tries to capture just a non-systematic component (*random shocks*) of the series, putting away all the systematic autocorrelation pattern. As a consequence, the CCF's calculated from both types of series should be quantitatively, and perhaps qualitatively, different; such differences being due to the autocorrelation (systematic) structure of the cycle[5]. Figures 1 and 2 show the results for the parameter values $\rho = 0.75$, $\phi_1 = 1$, $\phi_2 = 0$, $T = 100$, and $\lambda = 1600$, λ being the HP smoothing parameter. Approximate two-standard-deviations bands are also plotted. Note that while the series are originally constructed from a purely contemporaneous correlation pattern, the CCF between the cycles shows a strong correlation structure at both contemporaneous and lagged values. After prewhitening, we observe a merely contemporaneous correlation pattern. This result is very robust to the specific value of ρ, ϕ_1 and ϕ_2 as obtained in an exhaustive MonteCarlo experiment performed by André, Martín and Pérez (1997).

[3] For the process to be nonstationary, take $\phi_1 = 1$ or $\phi_2 = 1$.

[4] In this example, as we precisely know the generating process, the prewhitening procedure is capable of rendering a quite exact estimate of the underlying noise component of the series which is known by construction. With real data or data coming from a complex unknown generating processes, a "true" white noise component would seldom be perfectly identified. The larger the amount of information we have about the data generating process, the more accurate the estimate of the underlying stochastic component we can obtain.

[5] As proven by Bartlett (1946, 1955) the CCF of two autocorrelated stochastic processes reflects a mixture of the cross effects between both processes and the autodependence of each of them, as measured by the autocorrelation function (ACF).

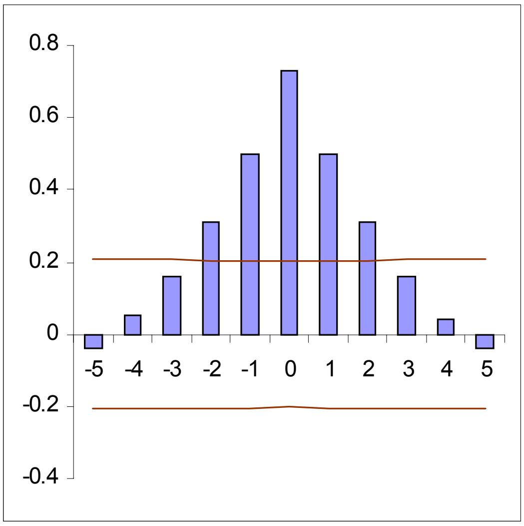

Figure 1: CCF between C_{1t} and C_{2t}

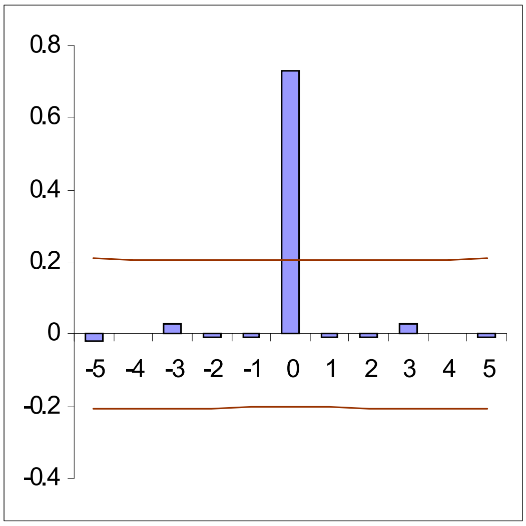

Figure 2: CCF between $\hat{a}_{1t}$ and $\hat{a}_{2t}$

Concerning the different shape of the correlograms in Figures 1 and 2, they offer different informational contents one could be interested in, depending on the aim of the study: the correlation pattern between the cycles of the variables as obtained form the HP filter (C_{1t} and C_{2t}) in Figure 1, and the correlogram between (an estimate of) the original generating noise variables ($\hat{a}_{1t}$ and $\hat{a}_{2t}$), representing the purely stochastic components, in Figure 2. An argument to be interested in the latter is the fact that it does not depend, in principle, on any specific definition of the cycle, given that all the cyclical behavior has been extracted[6].

To test the practical relevance of this point, we have replicated the results shown in the well-known article by Kydland and Prescott (1990) for the US economy without and with prewhitening. In the second case, a purely autoregressive model has been used to prewhiten, as explained in the previous example. Depending on the specific variable under analysis, the required order varies from 1 to 5. Comparing the results with and without prewhitening, we obtain two sets of results: i. on the one hand, for some variables, although the specific numerical values of the CCF's differ, the business cycle comovement results with and without prewhitening are qualitatively the same. In these cases, we can conclude that the comovement behavior of such series is mainly determined by the non-systematic component and is not qualitatively affected by the autocorrelation pattern of the cycle. ii. On the other hand, some results vary after prewhitening, meaning that the cycle (as measured by the HP-filtered series) of the series, because of its autocorrelation pattern, shows a comovement behavior qualitatively different from the one which is present in its underlying stochastic component.

For the sake of conciseness, we will only comment the results were some qualitative difference arises. Tables 1 and 2 summarize the most salient features. Table 1 shows the stylized facts reported by Kydland and Prescott (1990), while Table 2 shows the facts obtained from prewhitened series. Following Kydland and Prescott, the tables can be read as follows. Let ρ_i, $i \in \{0, \pm1, \pm2, ...\}$ denote the cross correlation

[6] Ideally, the prewhitening procedure would extract all the systematic behavior of the series, leaving just the purely ramdom component. In practice, we do not generally have enough information to perform a "perfect" prewhitening, in the sense of accurately obtaining an estimate of such a component. As a consequence, $\hat{a}_{it}$ will depend, to a certain extent, on the definition of the cycle. The more accurate is the prewhitening method, the more precise is the statement that the *white* correlogram does not depend on the definition of the cycle.

between real GNP and a given variable x_{t+i}. We say that the variable x is leading, contemporaneous or lagging the cycle of real GNP as the absolute value of ρ_i is maximum for a negative, zero, or positive i respectively. In the same fashion, according to the contemporaneous correlation of each variable with real GNP, x is said to be *procyclical* if ρ_0 is positive and close to one, *countercyclical* if ρ_0 is close to one but negative, and *uncorrelated with the cycle* if ρ_0 is close to zero.

As in Table 1, it can be observed in Table 2 that employment and hours per worker turn out to be procyclical variables after prewhitening. But note that both variables seem to lead the cycle by one and two quarters respectively, instead of lagging the cycle. One of the most remarkable facts found by Kydland and Prescott is that of the real wage being a clearly procyclical variable, as can be seen in Table 1. Once prewhitened, this variable seems to be basically uncorrelated with the business cycle, as widely held in the literature. Additionally, in Table 2, the inventory stock happens to be contemporaneous to the cycle, rather than lagging it, as in Table 1. Also in Table 2 imports seems to lag the cycle by three periods –instead of being contemporaneous in Table 1– and exports appear as contemporaneous –instead of lagging the cycle.

The second part of the title of Kydland and Prescott's article (referring to "a monetary myth") is due to the lack of evidence that the monetary base leads the cycle, as in Table 1. Once the series have been prewhitened, the monetary base seems to lead the cycle by two periods, as presented in Table 2. Kydland and Prescott report that the price level has been strongly countercyclical during the post-Korean War period. After prewhitening, it is clear from Table 2 that this evidence fades.

	Cross correlation of Real GNP with x_{t+i}										
Variable x	-5	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$+5$
Total Hours (Hous. surv.)	-.10	.05	.23	.44	.69	.86	.86	.75	.59	.38	.18
Employment	-.18	-.04	.14	.36	.61	.82	.89	.82	.67	.47	.25
Hours per worker	.08	.2	.35	.49	.66	.71	.59	.43	.29	.11	-.02
Hour Real Compensation	.30	.37	.40	.42	.40	.35	.26	.17	.05	-.08	-.20
Inventory Stock	-.37	-.33	-.23	-.05	.19	.50	.72	.83	.81	.70	.53
Exports	-.50	-.46	-.34	-.14	.11	.34	.48	.53	.53	.53	.45
Imports	.11	.18	.30	.45	.61	.71	.71	.51	.28	.03	-.19
Nominal Money Stock											
Monetary Base	-.14	.00	.12	.23	.33	.38	.37	.35	.31	.28	.26
M1	.01	.12	.22	.32	.34	.30	.21	.14	.09	.07	.07
M2	.47	.59	.66	.67	.61	.46	.25	.05	-.15	-.32	-.44
M2-M1	.53	.62	.66	.64	.56	.40	.20	-.01	-.20	-.38	-.51
Velocity											
Monetary Base	-.26	-.15	.01	.22	.41	.60	.51	.38	.22	.07	-.07
M1	-.24	-.20	-.12	-.01	.13	.31	.32	.27	.20	.10	.00
M2	-.36	-.59	-.48	-.29	-.05	.24	.33	.39	.41	.43	.42
Price Level											
Implicit GNP Deflator	-.50	-.60	-.68	-.69	-.64	-.55	-.43	-.31	-.17	-.04	.09
Consumer Price Index	-.52	-.63	-.70	-.72	-.68	-.57	-.41	-.24	-.05	.14	.29

Table 1: Comovement stylized facts reported by Kydland and Prescott (1990).

	Cross correlation of Real GNP with x_{t+i}										
Variable x	-5	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$+5$
Total Hours (Hous. surv.)	-.02	-.16	.10	.48	.23	.10	.17	-.14	-.03	.01	.19
Employment	.04	-.04	-.11	.07	.52	.28	.04	.12	-.04	-.01	.07
Hours per worker	.05	-.05	.12	.40	.19	.15	.11	-.13	.01	-.08	.05
Hour Real Compensation	.01	.06	.04	.15	.05	.10	.09	.10	-.03	.09	-.01
Inventory Stock	-.02	-.15	.04	-.11	-.30	.53	.24	.19	.08	.14	-.05
Exports	-.12	-.13	-.17	-.06	.25	.29	.05	-.08	.15	.12	.03
Imports	.14	.02	.14	.04	.05	-.02	.02	.16	.43	.11	.05
Nominal Money Stock											
Monetary Base	.02	-.07	.15	.21	-.03	.11	.03	-.01	.06	.02	.07
M1	-.09	.22	.11	.16	-.13	.09	-.07	.07	.04	-.14	.11
M2	.04	.23	.20	.09	-.12	.01	.03	-.11	-.16	-.07	-.11
M2-M1	.08	.15	.19	.02	-.10	-.04	.07	-.17	-.20	.01	-.19
Velocity											
Monetary Base	-.06	-.20	.76	-.01	-.04	.06	.10	-.09	.08	.02	-.12
M1	-.24	.58	-.02	.06	-.07	-.01	-.11	.18	-.04	.01	-.18
M2	-.13	-.20	-.26	.66	-.08	.12	.03	.13	.03	.16	.11
Price Level											
Implicit GNP Deflator	-.20	-.06	-.19	-.11	-.10	-.08	.00	-.04	.10	.04	-.04
Consumer Price Index	-.21	-.05	-.16	-.12	-.02	-.12	-.03	.01	.13	-.02	.11

Table 2: Comovement stylized facts obtained from prewhitened series

3 CONCLUDING REMARKS

In this paper we propose an alternative method to obtain stylized facts regarding the comovements among economic variables, by using the cross-correlation function between prewhitened variables. The stylized facts obtained using this procedure reflect only the non-systematic stochastic behavior of the series, and not the correlation between the cyclical components. The main advantage of the suggested approach is that it is independent, in principle, of the specific trend/cycle decomposition performed to the variables.

For a complete analysis of the stylized facts on comovement between two given economic variables, it would be worth inspecting the cross correlogram calculated using the non-prewhitened (HP-filtered) time series, and that computed using the same two time series but prewhitened. So we can obtain an insight into the extent to which the comovement results are affected by the autocorrelation pattern of the cycle.

If both cross-correlation functions were to transmit the same qualitative message, then the comovement patterns between the economic variables would not be crucially affected by the systematic autocorrelation properties of the time series, but it would rather be basically determined by the random (unpredictable) components. When both correlograms differ qualitatively we have to conclude, instead, that the stylized facts concerning comovements turn out to be crucially affected by the autocorrelation properties of the series, and consequently by the specific cycle/trend decomposition performed.

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