DUOPOLY PRICE COMMUNICATION*

Dolores Alepuz, José J. Sempere-Monerris and Amparo Urbano**

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Correspondence to:
Amparo Urbano. Universitat de València. Facultad de Ciencias Económicas.
Dpto. de Análisis Económico. Edificio Departamental Oriental.
Campus dels Tarongers, s/n.
46071 VALENCIA-SPAIN
E-mail: Amparo.Urbano@uv.es

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** D. Alepuz, J.J. Sempere, A. Urbano: University of Valencia.

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ABSTRACT

We investigate the role of price communication in imperfect information environments by setting up a dynamic differentiated duopoly where actions are not observable and where firms decide, before pricing, whether to communicate their choices to the rivals.When firms play simultaneously in the pricing stages, communication across them is a dominant strategy allowing firms to coordinate prices, thus reducing competition. However, when communication takes place within pricing stages, this meaning that firms are given the opportunity to choose roles, the above firms' coordination in prices is mitigated. This is because of the existence of a "second mover advantage" effect. Communication by the leader acts as a pre-commitment device to a "price umbrella" that the follower will undercut. As a result, we end up with a more competitive situation although price levels will not go down to those without communication.

Key words: Commitment; Price Communication; "Second Mover Advantage".

1 INTRODUCTION

Output or price communication has often been seen as a device to promote collusion among firms. Maybe this could be the case in a world of certainty where the typical models of collusion are formulated. However, when considerations of imperfect information are introduced, this link is not so clear. On the one hand, in models of non-cooperative collusion under imperfect sales information, price communication could be used as a monitoring device to detect deviations.[1] On the other, in models of information manipulation - "signal-jamming" -,[2] communication may lead firms to coordinate on the non-manipulative outcome. However, in both classes of models, such communication need not satisfy firms' incentive compatibility constraints and it may be just cheap talk.[3]

It is our purpose to investigate the problem of price communication in simple models of information manipulation. We set up a dynamic differentiated duopoly where actions are not observable and where firms decide before pricing whether to communicate their choices to the rivals. Communication may serve as a coordination mechanism. By giving firms the opportunity to make their prices observable to rivals, they can avoid incurring in signal-jamming -indeed, the most competitive outcome. Furthermore, price communication by both firms (referred to as *full communication*) or just by one of them (*asymmetric communication* pattern) will arise depending on the simultaneity or sequentiality of pricing decisions in the first pricing stage,

[1]See, for instance, Green and Porter (1984) and Porter (1983), where firms can observe only their own production level and a common (stochastic) market price. There, an unexpected low price may signal either deviations from collusive output levels or a downward demand shock.

[2]The term signal-jamming is first used in Fudenberg and Tirole (1986a, b). Though the use of the term seems to vary (see Tirole, 1988, for a survey), we are adopting fairly standard terminology in using signaling to refer to cases in which an agent controls an *observed* action upon which other agents condition beliefs, and using signal-jamming to refer to cases in which an agent can alter an unobserved action in order to manipulate the nontrivial *distribution* of some variable observed by a rival. Mirman, Samuelson and Urbano (1993b), study the case of pure signal-jamming, when there is no possibility of experimentation. See also Mirman and Urbano (1992).

[3]Note that both kind of models are concerned with imperfect (but complete) information, and with public signal. For models with private signal see Lehrer (1992) and Compte (1997), among others.

respectively.[4]

Our model is related to but different from the literature on information sharing. These models typically feature a multi-stage game in which first the oligopolistic firms commit to share or not to share information about an unknown demand or cost parameter and then pricing or output decisions take place. The question is whether information sharing can arise as a subgame perfect equilibrium of such a non-cooperative game.[5] However, these models are static and they also disregard the exchange of information on decision variables such as output or price levels.

In most practical cases, information exchange is on past prices or production and this requires to abandon the static analytical framework and to consider at least two pricing periods. Furthermore, and contrary to the single period information sharing models, equilibrium prices can be unobservable by rivals, particularly in uncertain environments. In such an eventuality, price observations from the first period are then signals about future market conditions. There appear incentives for a firm to manipulate the beliefs of its rivals, i.e. signal-jamming. There is however a cost attached to the activity of information manipulation. In particular, present expected profits must be traded for future expected profits. Therefore, profits under signal-jamming are lower than those when equilibrium price levels are announced by firms. This unfavourable situation could be bypassed if firms made their prices observable to rivals.

The above reasoning motivates the study of the role of communication as a coordination device. As long as firms may set higher prices and achieve higher

[4]The exchange of information on prices between competitors is considered by the European Commission as a proof of collusion, as illustrated by the Woodpulp (1985) and the ICI-Solvay (1990) cases. The seasonal pattern of pricing by an important number of producers from North America and Scandinavia suggested a case of price coordination in the wood pulp industry. In the second case, we have the two largest producers of soda-ash, an alkaline chemical commodity, with plants in continental Europe (Solvay) and in the UK (ICI). Neither ICI nor Solvay invaded each other's market even when UK prices rose by 15-20% above those of continental Europe and Solvay did not enter the higher price market. The Container Corporation of America case (late 60s) is an example of price information exchange in the US.

[5]The results vary depending on the nature of competition (prices or quantities) and the type of uncertainty (common or idiosincratic shocks to demand or to costs). See e.g. Novshek and Sonnenschein (1982), Clarke (1983), Gal-Or (1985b, 1986), Sakai (1985), and Shapiro (1986).

profits by exchanging information, price communication becomes more valuable to firms. We propose a three stage non-cooperative game, which consists of two pricing stages preceded by an initial one in which the duopolists decide on whether to communicate their stage one equilibrium prices. To identify to which extent communication may be valuable to both firms, we model firms' choices under two possible scenarios: the first pricing stage consist of either one or two potential pricing periods. In the latter, we assume that firms can set prices in either period one or two but not in both, a premise in line with the endogenous timing literature.[6] At the beginning of stage one, if any of the duopolists chose to communicate, then it must specify both an action and the date of this action (when both firms communicate, this is similar to the extended game with action commitment described by Hamilton and Slutsky, 1990). Finally, the second pricing stage where firms play Bertrand provides the incentives for information manipulation.

We observe that if communication takes place across pricing stages the equilibrium in dominant strategies of the full game entails communication by both firms. However, when it takes place within pricing stages and hence, it affects the current price choices of the rivals, a richer behavior may arise. In particular, when firms are given the opportunity to choose roles, the above firms' coordination in prices is mitigated, since now the firm which plays late gets a "second mover advantage". Thus, communication acts as a pre-commitment device to a "price umbrella" lower than the price when both communicate. The firm which communicates, -the leader- will set up a price and the follower will undercut it. Yet, price levels will never go down to those without communication.[7]

Additionally, our analysis contributes to the literature on information transmission in two respects: first, firms exchange information about market variables and second, the equilibria of initially symmetric firms may entail asymmetric communication patterns. In this context, though, information

[6] Contributors to this literature are Gal-Or, 1985a; Dowrick, 1986; Saloner, 1987; Hamilton and Slutsky, 1990; Mailath, 1993; among others.

[7] By contrast, in a homogeneous substitute products duopoly in which firms compete in quantities, incentives to communicate equilibrium outputs involve a "first mover advantage" effect. Besides, for the case where roles are not an endogenous choice, non-communication is a dominant strategy to both firms. See our companion paper Alepuz et al. (1997).

sharing becomes a coordination device rather than a purely informative one.[8] Our model can also be seen as an extension of the literature on the strategic choice of timing. Firstly, uncertainty in demand parameters and unobservable prices are introduced, and secondly, pricing takes place in two stages what provides an incentive for information manipulation. Finally, by allowing firms whether to make their first stage equilibrium prices observable, we endogenize the possibility of signal-jamming.

Next Section describes the model. Section three is devoted to the pricing subgames, whereas the equilibria for the full game are presented in Section four. Section five concludes.

2 DESCRIPTION OF THE GAME

Consider a symmetric duopoly model. The firms denoted by 1 and 2 produce heterogeneous products with constant marginal costs which without loss of generality are assumed to be zero. Symmetric market demands are given by,[9]

$$Q_1 = \tilde{\theta} - bp_1 + cp_2 + \tilde{\varepsilon} \quad (1)$$

$$Q_2 = \tilde{\theta} - bp_2 + cp_1 + \tilde{\varepsilon} \quad (2)$$

where Q_i are sales in market i and p_i denotes firm $i's$ price, $i = 1,\ 2$, with $b > c > 0$. The random variables are $\tilde{\theta}$, the fixed intercept parameter and $\tilde{\varepsilon}$, a time dependent shock[10] on total demand that captures all the

[8] Notice that a different equilibrium concept should be used in order for information sharing to be truly informative. However, alternative equilibrium concepts such as subjective equilibrium or self-confirming equilibrium do not apply to our model since here equilibrium prices are not observable (see Fudenberg and Levine, 1993, and Kalai and Lehrer, 1993).

[9] Notice that we could have assumed general symmetric market demands:

$$Q_1 = \gamma_1(\theta, p_1, p_2) + \tilde{\varepsilon}_1$$
$$Q_2 = \gamma_2(\theta, p_1, p_2) + \tilde{\varepsilon}_2$$

with $\gamma_i(\theta, p_i, p_j)$ a decreasing (continuous) function of p_i and an increasing function of p_j. The linearity has been assumed to obtain close-form solutions that allow us comparison of profits under different scenarios. Also, we abstract from active learning phenomena such as experimentation by just assuming the intercept unkown. See Mirman, Samuelson and Urbano (1993a) and Alepuz and Urbano (1994).

[10] For the sake of the exposition we suppress the time subindex in $\tilde{\varepsilon}$.

random influences that affect both firms in an equal way.[11] It is assumed that each of these two random variables has full support on $\mathbb{R}$ and that they are independently and normally distributed. In particular, $\tilde{\theta} \sim N(m,\ h)$ and $\tilde{\varepsilon} \sim N(0,\ \tau)$ where $h = \frac{1}{\sigma_\theta^2}$ is $\tilde{\theta}'s$ precision and $\tau = \frac{1}{\sigma_\varepsilon^2}$ is $\tilde{\varepsilon}'s$ precision. These distributions are known by the firms. However, firms are unable to observe their rivals actions (even equilibrium prices) unless the competitors have decided in advance to communicate them.

Assume that both firms face a three stage game. In the initial stage (stage zero) each firm decides whether to communicate its stage one equilibrium price choice to its rival. The decision "to communicate" (C) or "not to communicate" (NC) prices is made simultaneously by both firms and before market competition takes place (see Figure 1). Notice that this is a model of imperfect but complete information. This means that there is no private information and hence firms do not have any reason to lie about their equilibrium outcomes.

Once the communication decision has been made, firms must decide about pricing. The pricing decisions take place in the last two stages of the game (stages one and two). In stage two, firms choose prices, p_{12}, p_{22}, simultaneously and independently (where the first subscript denotes the firm and the second one the stage). This stage has the purpose of providing the dynamic market setting of the duopoly. In particular, it gives an incentive for the manipulation of the information (signal-jamming).

In stage one we consider two scenarios. The first one comprises two potential pricing periods, where firms must decide when to fix prices and the price levels they will set. Firms can take pricing decisions in either period 1 or 2, but not in both. The decision "to wait" or "to set a price" is made simultaneously by both firms at the beginning of period 1. If a firm chooses "to set a price", it is committed to that particular price immediately, while choosing "to wait" means delaying the pricing decision until period 2. Notice that if a firm chose C, in the initial stage, and it chooses "to set a price" in period 1, its price is observable by any firm which has chosen "to wait" (see

[11] If we had assumed instead a pair $(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2)$ of uncorrelated random shocks on demands, with some joint distribution function, then the market signal would have been a vector of sales, which in turn would have led to more cumbersome computations, yet without adding any further insight. See Alepuz and Urbano (1993), for a general model with product differentiation.

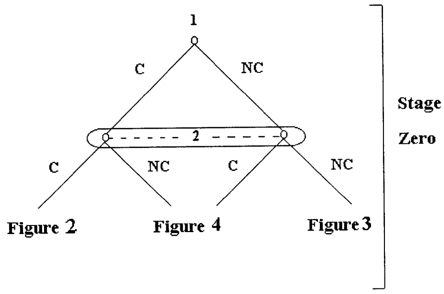

Figure 1:

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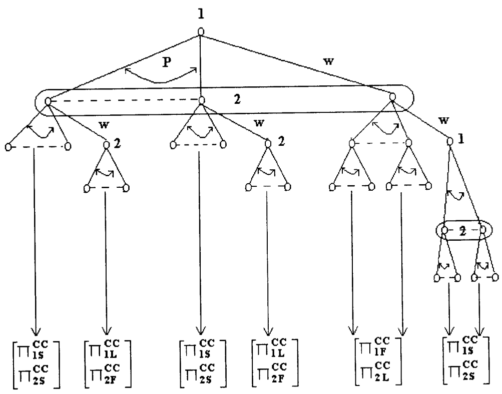

Figure 2: (C,C) history.

Figure 2:

Figure 2).[12] However, this price would not be observable if a firm chose NC. In the second scenario, stage one is a simultaneous Bertrand pricing game. We will refer to scenario one as the *full game with sequential moves,* while scenario two will be called the *full game with simultaneous moves.*

Prices in this stage are denoted by p_{11}, p_{21}, and sales Q_{11} and Q_{21} are determined at the end of period 2, according to $Q_{11} = \tilde{\theta} - bp_{11} + cp_{21} + \tilde{\varepsilon}$, and $Q_{21} = \tilde{\theta} - bp_{21} + cp_{11} + \tilde{\varepsilon}$, and they are announced to the firms.

This is a dynamic game of imperfect information and the appropriate equilibrium concept is the subgame perfect equilibrium. However, we will refine this concept by imposing iterated deletion of dominated strategies whenever possible. We are particularly interested in the subgame perfect equilibria that may arise in the initial stage of the game, where firms decide whether to communicate. The equilibrium is calculated in the standard

[12]For the case where both firms chose to communicate in the initial stage, the description of stage one would correspond to an extended game of action commitment as introduced by Hamilton and Slutsky (1990).

backward way.

3 ANALYSIS OF THE PRICING SUBGAMES

We begin by solving first for the last stage of the game. Since this is a standard signal-jamming simultaneous move game, most of its resolution is relegated to the Appendix. Then, we will deal with the analysis of the stage one where we isolate the equilibrium timing/pricing patterns for all the continuation subgames under the above mentioned scenarios.

3.1 Stage two equilibrium

In the last stage, after observing the stage one sales, Q_{11} and Q_{21}, both firms choose prices simultaneously and independently. Moreover, after the stage one play, an information set for firm $i = 1,\ 2$ is characterized by either $I_i^{NC} = \{Q_{11},\ Q_{21},\ p_{i1}\}$ or $I_i^{C} = \{Q_{11},\ Q_{21},\ p_{i1},\ p_{j1}\}$ $i \neq j$, depending on whether firm j has decided in the initial stage to communicate its stage one equilibrium price. Firms choose equilibrium prices $p_{12}^e,\ p_{22}^e$ on the basis of this information such that

$$p_{12}^e = \arg\max_{p_{12}} E\left[Q_{12}(p_{12}, p_{22}^e \mid I_1)\right] p_{12} \tag{3}$$

$$p_{22}^e = \arg\max_{p_{22}} E\left[Q_{22}(p_{12}^e, p_{22} \mid I_2)\right] p_{22} \tag{4}$$

where $I_i = \{I_i^{NC},\ I_i^{C}\}$, $i = 1,\ 2$ and $Q_{12}(\cdot)$ and $Q_{22}(\cdot)$ are the stage two market demands.

To solve this problem, we find out first what is $E[Q_{i2}(p_{i2}, p_{j2}^e \mid I_i)]$, i.e. $E[\tilde{\theta} - bp_{i2} +\ cp_{j2}^e + \tilde{\varepsilon} \mid I_i]$, which depends on I_i. Since $E[\tilde{\theta} - bp_{i2} +\ cp_{j2}^e + \tilde{\varepsilon} \mid I_i] = E[\tilde{\theta} \mid I_i] - bp_{i2} +\ cp_{j2}^e$, it is sufficient to find $E[\tilde{\theta} \mid I_i]$.

We now proceed to take a candidate for a Nash equilibrium for the stage one and show which conditions it has to satisfy.

Consider the stage one pair of equilibrium prices, $p_{11}^e,\ p_{21}^e$.[13] In equilibrium, firm 1 and firm 2 price p_{11}^e, and p_{21}^e, respectively, and use their

[13]These prices may be the Bertrand or Stackelberg equilibrium ones. However, this is not relevant now.

observation of stage one sales, along with the fact that equilibrium prices are p_{11}^e and p_{21}^e, to solve (using Bayes' rule) for stage two posterior beliefs. These may be common or not across firms and this fact is common knowledge. In the event that both firms have decided to communicate their stage one equilibrium prices, the posterior beliefs would be common, that is $E\left[\tilde{\theta} \mid I_i\right] = E\left[\tilde{\theta} \mid I_j\right] = \hat{\theta}$. We obtain the standard Bertrand prices, $p_{12}^e = p_{22}^e = p(\hat{\theta}) = \frac{\hat{\theta}}{2b-c}$ and the expected value of stage two profits, $V_i(\hat{\theta}) = V_j(\hat{\theta}) = V(\hat{\theta}) = \frac{b\hat{\theta}^2}{(2b-c)^2}$.

Alternatively if firm i does not communicate its stage one equilibrium price to firm j, then firm j cannot observe it. In equilibrium firm $j's$ information set is characterized by $I_j^{NC} = \{Q_{11},\ Q_{21},\ p_{j1}^e\}$ and on the basis of this information it chooses p_{j2} so as to maximize the stage two expected profits, so $p_{j2} = p_{j2}(Q_{11},\ Q_{21},\ p_{j1}^e)$. If firm i deviates from p_{i1}^e to p_{i1} the same possible information sets are obtained and firm j is neither going to change $p_{j2}(Q_{11},\ Q_{21},\ p_{j1}^e)$ nor its beliefs, since its strategy only depends on $(Q_{11},\ Q_{21},\ p_{j1}^e)$ and not on the actions of firm i. The next lemma summarizes the above discussion. Calculations are in the Appendix.

Lemma 1 *Under non-common posterior beliefs, the expected value of stage-two profits to firm i is given by,*

$$V_i(\hat{\theta}_i, \hat{\theta}^e) = \frac{\left(\hat{\theta}_i(2b-c) + c\hat{\theta}^e\right)^2}{4b(2b-c)^2} \tag{5}$$

that depends both on $\hat{\theta}_i$ and $\hat{\theta}^e$, which are firm $i's$ own and the equilibrium posterior beliefs, respectively.

3.2 Stage one equilibria: simultaneous vs. sequential play.

As discussed above, the stage one considers two scenarios: a) firms may choose their roles, b) firms play simultaneously.

3.2.1 Endogenous selection of roles

Under this scenario the stage one pricing subgame consists of two potential pricing periods, denoted periods 1 and 2, with the proviso that a firm can set a price in either period, but not in both. Thus, the decisions for each firm consist of whether "to set a price" in period 1, or to wait; if a firm chose to communicate in the initial stage, and it chooses "to set a price" in period 1 its price is observed by any firm which has chosen "to wait".

There are four possible histories associated with the stage one pricing subgame. The notation is as follows: $(C,\ C)$ denotes the history in which both firms decided to communicate; similarly $(NC,\ NC)$ denotes the history in which neither firm decided to communicate; finally, $(C,\ NC)$ and $(NC,\ C)$ denote histories in which one of the firms communicated its stage one equilibrium price whereas the rival did not. We shall analyze each of these in turn, specifying how equilibrium play unfolds for each possible history.

In the $(C,\ C)$ history both firms decided to communicate their equilibrium prices and hence, these are common knowledge; while in the $(NC,\ NC)$ history both firms will try to manipulate the rival's market information through the price choice, i.e. both firms will practice signal-jamming. The existence of two pricing periods implies that there may be three price equilibria in pure strategies, however only two of them are in undominated pure strategies, the two which involve sequential play (Hamilton and Slutsky, 1990).

We require that firms' strategies[14] form a subgame perfect equilibrium in undominated strategies. This means that the continuation game considered following any out-of-equilibrium action that occurs during period 1 will be solved assuming sequential rationality. Thus, players who can act in period 2 are assumed to respond optimally.

In stage one, the posterior beliefs $\hat{\theta}_i,\ \hat{\theta}^e,\ i=1,\ 2$ are random variables when firms choose their prices. Given the above analysis of stage two, one can write firm $i's$, two stage (expected) profits as a function of its stage one price and the equilibrium prices. Denote first the stage one expected profits for firm i by,

$$\Pi_{i1}(p_{11}, p_{21}) = E\left[\tilde{\theta} - bp_{11} + cp_{21}) + \tilde{\varepsilon}\right] p_{i1}, \quad i = 1,\ 2 \tag{6}$$

[14]Let P_i (resp. P_j) denote the action space for firm i (resp. firm j), and let w be the action of waiting until the second period to choose an action from P_i (resp. P_j). In our extended game the set of strategies for, say firm i, is $S_i = \{p_{i1}, w \times \zeta_i(p_{j1})\}$ where $p_{i1} \in P_i$ and ζ_i is the set of functions that maps $P_j \cup w$ into P_i.

and let[15]

$$\Pi_i(p_{11},p_{21}) = \Pi_{i1}(p_{11},p_{21}) + E\left[V_i(\hat{\theta}_i,\hat{\theta}^e)\right] \quad i=1,\ 2 \tag{7}$$

be the two stage expected profits for firm i, where by (A4) and (A6) in the Appendix, $\hat{\theta}_i = \hat{\theta}_i(\bar{Q}_1,\bar{p}_1)$, $\hat{\theta}^e = \hat{\theta}^e(\bar{Q}_1,\bar{p}_1^e)$, and thus[16],

$$E\left[V_i(\hat{\theta}_i,\hat{\theta}^e)\right] = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}_i(\bar{Q}_1,\bar{p}_1),\hat{\theta}^e(\bar{Q}_1,\bar{p}_1^e)\right) f\left(\bar{Q}_1+(b-c)\bar{p}_1\right) d\bar{Q}_1 \tag{8}$$

where f is the density function of the random variable $\bar{Q}_1+(b-c)\bar{p}_1$. Since $\bar{Q}_1+(b-c)\bar{p}_1 = \tilde{\theta}+\tilde{\varepsilon}$, then, for a given value p_{j1}^e and for each p_{i1},

$$\bar{Q}_1+(b-c)\bar{p}_1 \sim N\left(m, \frac{h\tau}{h+\tau}\right)$$

Let $\Gamma_i(p_{i1},\bar{p}_1^e) = E\left[V_i(\hat{\theta}_i,\hat{\theta}^e)\right]$, $i=1,\ 2,\ i\neq j$. The function Γ_i describes firm $i's$ stage two expected profits as a function of stage two beliefs and hence of stage one equilibrium prices and firm $i's$ stage one choice, p_{i1}.

Let p_{i1}^S, p_{i1}^L and p_{i1}^F denote firm $i's$ stage one prices when it plays Bertrand, Stackelberg leader and Stackelberg follower, respectively.

Also let $\Pi_{iS}^{C,NC}$, $\Pi_{iL}^{C,NC}$, $\Pi_{iF}^{C,NC}$, denote firm $i's$ sum of the two pricing stage profits, when firm i communicates and the rival does not, and firm i is a Bertrand player, a Stackelberg leader and a follower, respectively. The same obvious notation applies for the remaining possible combination of profits.

We consider first the situation where both firms are willing to communicate their stage one equilibrium prices to each other, i.e. $(C,\ C)$. Before showing and characterizing the equilibria for this history, we prove first that their existence is trivially guaranteed.

Since firms communicate stage one equilibrium prices, stage two posterior beliefs are common and common knowledge, say $\hat{\theta}^e = \hat{\theta}^e(\bar{Q}_1,\bar{p}_1^e)$, and

[15] Without loss of generality, we abstract from discounting.

[16] $\bar{Q}_1 = \frac{Q_{11}+Q_{21}}{2}$, is the average of sales in stage one, $\bar{p}_1 = \frac{p_{i1}+p_{j1}^e}{2}$ and $\bar{p}_1^e = \frac{p_{i1}^e+p_{j1}^e}{2}$ are stage one price averages as defined in the Appendix.

the associated value function is given by $V_1(\hat{\theta}^e) = V_2(\hat{\theta}^e) = V(\hat{\theta}^e)$, with $E[V(\hat{\theta}^e)] = \Gamma(\bar{p}_1^e)$.

Starting with the simultaneous move problem, firm $i's$ stage one problem is to solve:

$$\max_{p_{i1}} \Pi_{i1}(p_{i1}, p_{j1}^e) + \Gamma(\bar{p}_1^e) \tag{9}$$

The objective function in (9) is continuous and the maximization can be taken to be in the compact set $[0,\ \bar{p}_{i1}]$ for some $\bar{p}_{i1}$, since by (8), $\Gamma(\bar{p}_1^e)$ is bounded above by the monopoly profits and $\lim_{p_{i1}\to\infty} \Pi_i = -\infty$. Hence a solution must exist. Firm $i's$ reaction function is denoted $p_{i1}(p_{j1}^e)$. An equilibrium is a pair (p_{11}^e, p_{21}^e) with the property that

$$p_{11}^e = p_{11}(p_{21}^e) \tag{10}$$

$$p_{21}^e = p_{21}(p_{11}^e) \tag{11}$$

More specifically,

Lemma 2 *An equilibrium pair (p_{11}^e, p_{21}^e) for a simultaneous move must satisfy,*

$$m - 2bp_{11}^e + cp_{21}^e + \frac{d\Gamma(\bar{p}_1^e)}{dp_{11}} = 0 \tag{12}$$

$$m - 2bp_{21}^e + cp_{11}^e + \frac{d\Gamma(\bar{p}_1^e)}{dp_{21}} = 0 \tag{13}$$

Furthermore, the function $\Gamma(\bar{p}_1^e)$ is constant in $\bar{p}_1^e$. Hence (12) and (13) yield functions $p_{11}(p_{21}^e)$ and $p_{21}(p_{11}^e)$ which are linear and thus there exists at most one equilibrium and by symmetry $p_{11}^e = p_{21}^e$. Also, (12) and (13) are sufficient conditions for such an equilibrium.

Proof. See the Appendix.

A similar argument applies to the sequential move equilibria. In this situation, the firms' problem amounts to solving a static Stackelberg duopoly and the existence and uniqueness of the equilibrium is guaranteed for any of the two possible configurations.

The next proposition shows the subgame equilibria for the (C, C) history and characterizes them.

Proposition 1 *For the case (C, C), the following two timing/price patterns are the unique subgame perfect equilibria in undominated pure strategies.*

(a) *Firm i sets* $p_{i1}^L = \frac{m(2b+c)}{2(2b^2-c^2)}$ *in the first period and firm j sets* $p_{j1}^F = \frac{m}{2b}(1 + \frac{c(2b+c)}{2(2b^2-c^2)})$ *in the second period, yielding, respectively, two-stage profits to firm i (the "leader") and two stage profits to firm j (the "follower"),*

$$\Pi_{iL}^{C,C} = \frac{m^2(2b+c)^2}{8b(2b^2-c^2)} + \frac{b}{(2b-c)^2}\left(m^2 + \frac{\tau}{h(h+\tau)}\right)$$
$$\Pi_{jF}^{C,C} = \frac{m^2((2b+c)^2-2c^2)}{16b(2b^2-c^2)^2} + \frac{b}{(2b-c)^2}\left(m^2 + \frac{\tau}{h(h+\tau)}\right)$$

(b) *Firm j sets* p_{j1}^L *in the first period and firm i set* p_{i1}^F *in the second period, yielding two stage profits to firm j (the "leader") of* $\Pi_{jL}^{C,C}$ *and two-stage profits to firm i (the "follower") of* $\Pi_{iF}^{C,C}$.

Proof. First, consider (p_{11}^S, p_{21}^S), the simultaneous play equilibrium. Since p_{21}^S is a best reply to p_{11}^S, neither waiting nor any other choice in the first period can raise 2's profits, and similarly for firm 1. Thus, this is an equilibrium.

Second, consider the situation when firm 1 plays p_{11}^L in the first period and 2 waits and then plays its follower strategy p_{21}^F. When 2 waits, 1 knows that it is a leader with an optimal strategy of p_{11}^L. To p_{11}^L, p_{21}^F is the best reply, which can be played at the first turn or after observation. However, if it were played at the first turn, p_{11}^L would not be the best reply to it. Similarly, 2 playing p_{21}^L and 1 waiting and then playing its following strategy p_{11}^F is an equilibrium.

Third, no other pair can be a pure strategy equilibrium. Assume $(p_{11}, p_{21}) \neq (p_{11}^S, p_{21}^S)$ is an equilibrium. Then, p_{21} must be a best reply to p_{11} and p_{11} must be a best reply to p_{21}. This contradicts (p_{11}^S, p_{21}^S) being the unique simultaneous move equilibrium. If 1 waits, the only possible equilibrium action is 2 playing p_{21}^L, and similarly if 2 waits. Thus, these are the only equilibria in pure strategies.

The above set of pure strategy equilibria can be refined by sequentially deleting dominated strategies. Recall that our basic duopoly game has a unique equilibrium (simultaneous), in the interior of the action space, (p_{11}^S, p_{21}^S).

If firm 2 plays p_{21}^S at the first turn or waits to play at the second turn, firm 1 is indifferent between playing p_{11}^S at the first or second turn. For all other actions of firm 2, 1 does better by waiting and observing 2's play in order to play according to its reaction function at the second turn. Thus, p_{11}^S is dominated by waiting. If 2 waits and plays p_{21}^F, then 1 does strictly better by playing p_{11}^L rather than any other action. Thus, the Stackelberg equilibria with sequential play do not use dominated strategies.

In the Appendix it is shown that the equilibria are as displayed in the proposition. Note that all the equilibria are the unique "myopic" short run best responses, since under mutual price communication there is no room for manipulation of information, i.e. signal jamming. ■

Next, consider the situation when firms are not willing to communicate their stage one equilibrium prices. In this case firms have an incentive to manipulate their rival's posterior beliefs through adjustments of price levels away from the single-period optima, i.e. by signal-jamming. Proposition 2 will show that the unique subgame perfect equilibrium for the (NC, NC) case is the simultaneous move. We prove first that the existence of this equilibrium with signal-jamming is also guaranteed.

Notice that since firms do not communicate stage one equilibrium prices, firm $i's$ second stage value function is given by (5), i.e. $V_i(\hat{\theta}_i, \hat{\theta}^e)$, where $\hat{\theta}_i$ and $\hat{\theta}^e$ are defined in (A4) and (A6), respectively, in the Appendix.

Under simultaneous move, firm $i's$ stage one problem is:

$$\max_{p_{i1}} \Pi_{i1}(p_{i1}, p_{j1}^e) + \Gamma_i(p_{i1}, p_{i1}^e, p_{j1}^e) \tag{14}$$

where $\Gamma_i(p_{i1}, p_{i1}^e, p_{j1}^e)$ is given by (8).

By a similar reasoning to the simultaneous move in the (C, C) case, a solution to (14) must exist, which we shall denote by $p_{i1}(p_{11}^e, p_{21}^e)$.[17]

An equilibrium is a pair (p_{11}^e, p_{21}^e) with the property that, $p_{11}^e = p_{11}(p_{11}^e, p_{21}^e)$ and $p_{21}^e = p_{21}(p_{11}^e, p_{21}^e)$. More specifically,

[17]Notice that this is not now a conventional duopoly problem, as we cannot write reaction functions as simply $p_{11}(p_{21}^e)$ and $p_{21}(p_{11}^e)$. We can write them as either $p_{11}(p_{11}^e, p_{21}^e) : \mathbb{R}^2 \to \mathbb{R}$ and $p_{21}(p_{11}^e, p_{21}^e) : \mathbb{R}^2 \to \mathbb{R}$; or $p_{11}^e(p_{21}^e) : \mathbb{R} \to \mathbb{R}$ and $p_{21}^e(p_{11}^e) : \mathbb{R} \to \mathbb{R}$, where the latter incorporates incentive constraints.

Lemma 3 *An equilibrium pair* (p^e_{11}, p^e_{21}) *for a simultaneous move must satisfy,*

$$m - 2bp^e_{11} + cp^e_{21} + \frac{d\Gamma(p^e_{11}, p^e_{11}, p^e_{21})}{dp_{11}} = 0 \tag{15}$$

$$m - 2bp^e_{21} + cp^e_{11} + \frac{d\Gamma(p^e_{21}, p^e_{11}, p^e_{21})}{dp_{21}} = 0 \tag{16}$$

Furthermore, the terms $\frac{d\Gamma(p^e_{11}, p^e_{11}, p^e_{21})}{dp_{11}}$ *and* $\frac{d\Gamma(p^e_{21}, p^e_{11}, p^e_{21})}{dp_{21}}$ *are constant in* p^e_{11} *and* p^e_{21}.[18] *Hence (15) and (16) yield functions* $p^e_{11}(p^e_{21})$ *and* $p^e_{21}(p^e_{11})$ *which are linear, and thus there exists at most one equilibrium and by symmetry* $p^e_{11} = p^e_{21}$. *Also, (15) and (16) are sufficient conditions for such an equilibrium.*

Proof. See the Appendix.

It is important to notice that stage one equilibrium prices are lower than the ones obtained without signal-jamming. This is so because firms by decreasing prices, make output increase. As a consequence, $\hat{\theta}^e$ is also increased implying that stage two firm's expected profits $(V_i(\hat{\theta}_i, \hat{\theta}^e))$ raises.

Two aspects of this analysis are interesting. The first is that after the signal-jamming considerations have been incorporated into the problem, the reaction functions are linear. This is a result of the fact that the demand curves are specified so as to make experimentation valueless. The second is that the existence issues are concerned not with duopoly considerations but with the solution to the single-firm problem. If the second-order conditions for the single-firm optimization problem hold (as in our case), then a duopoly equilibrium exists.

The next proposition shows the subgame equilibria when neither firm wants to communicate its stage one equilibrium price.

Proposition 2 *For the case (NC, NC); the following outcome is the unique subgame perfect equilibria in pure strategies, and hence it is also the unique subgame perfect equilibrium in undominated pure strategies:*

[18] We will abuse terminology somewhat, though without causing any confusion, by using the phrase "$\frac{d\Gamma(p^e_{i1}, p^e_{i1}, p^e_{j1})}{dp_{i1}}$ is constant in p^e_{i1}" to mean that: if p^e_{i1} changes and the value of p_{i1} at which this derivative is evaluated also changes so as to preserve equality with p^e_{i1}, then the derivative does not change.

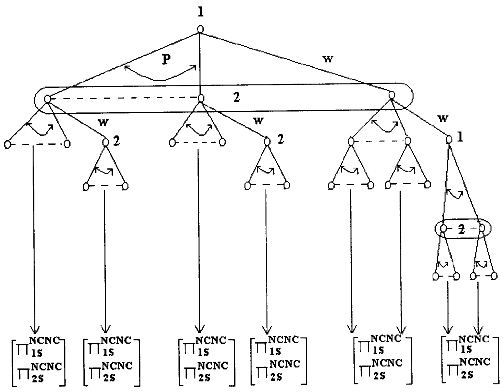

Figure 3: (NC, NC) history.

Figure 3:

Both firms play their simultaneous Bertrand prices,

$$p_{i1}^S = p_{j1}^S = \frac{m}{(2b-c)}\left(1 - \frac{\tau c(b-c)}{2(2b-c)^2(h+\tau)}\right)$$

with two stage profits:

$$\Pi_{iS}^{NC,NC} = \Pi_{jS}^{NC,NC} = \tfrac{b}{(2b-c)^2}\left(2m^2 + \tfrac{\tau}{h(h+\tau)}\right) - \tfrac{\tau c(b-c)m^2}{2(2b-c)^4(h+\tau)}\left(c + \tfrac{\tau c(b-c)^2}{2(2b-c)^4}\right)$$

Proof. Firms do not communicate, and hence regardless of pricing or waiting, actions are not observed. Hence, simultaneous play results for any possible timing configuration (see Figure 3). ■

Since the asymmetric histories have a parallel analysis, we will only address the $(C,\ NC)$ case in detail.

Proposition 3 *For the case $(C,\ NC)$, the following timing/price pattern is the unique subgame perfect equilibria in undominated pure strategies. Firm*

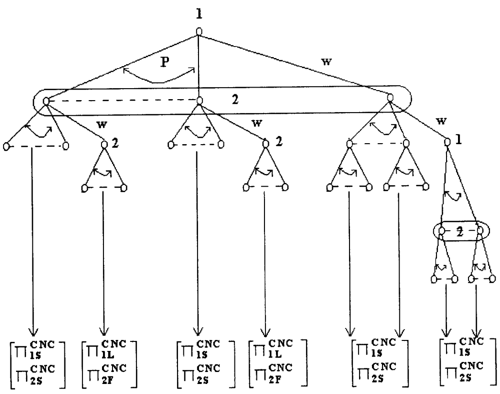

Figure 4: (C, NC) history.

Figure 4:

1 prices, $p_{11}^{L} = \frac{m(2b+c)}{2(2b^2-c^2)}\left(1 - \frac{\tau c^2(b-c)}{2(h+\tau)(2b^2-c^2)(2b+c)}\right)$ *in the first period and firm 2 prices* $p_{21}^{F} = \frac{m}{2b}[1 + \frac{c(2b+c)}{2(2b^2-c^2)} - \frac{\tau c(b-c)}{2(h+\tau)(2b-c)^2}\left(1 + \frac{c^2}{2(2b^2-c^2)}\right)]$ *in the second period, yielding two stage profits to the leader and the follower of*

$$\Pi_{1L}^{C,NC} = \frac{b}{(2b-c)^2}\left(m^2 + \frac{\tau}{h(h+\tau)}\right) + \frac{bm^2}{2(2b^2-c^2)}\left(\frac{2b+c}{2b} - \frac{\tau c^2(b-c)}{4b(h+\tau)(2b-c)^2}\right)^2$$

$$\Pi_{2F}^{C,NC} = \frac{b}{(2b-c)^2}\left(m^2 + \frac{\tau}{h(h+\tau)}\right) + \frac{m^2}{4b}\left(\frac{(2b+c)^2-2c^2}{2(2b^2-c^2)} - \frac{\tau c(b-c)(2b+c)}{4(h+\tau)(2b-c)(2b^2-c^2)}\right)^2$$

Proof. It goes along the lines of the proof of proposition 1, just noticing that the firm which does not communicate, firm 2, either plays Bertrand or waits and observes firm 1's action in the first period. Also, notice that firm 2 will always practice signal jamming in either of the two equilibria (see Figure 4). The sequential equilibrium is the only one in undominated pure strategies. ■

3.2.2 Firms play simultaneously

Now, firms choose prices simultaneously and independently in stage one of the game. The next proposition gathers the subgame equilibrium outcomes under each possible communication pattern.

Proposition 4 *The following outcomes are the unique subgame perfect equilibria in pure strategies:*
i) For the (C,C) history, the subgame perfect equilibrium Bertrand prices and two-stage profits are:

$$p_{i1}^S = p_{j1}^S = \frac{m}{2b-c},$$
$$\Pi_{iS}^{C,C} = \Pi_{jS}^{C,C} = \frac{b}{(2b-c)^2}\left(2m^2 + \frac{\tau}{h(h+\tau)}\right)$$

ii) The subgame perfect equilibrium prices for the (C,NC) history are:

$$p_{11}^S = \frac{m}{2b-c}\left(1 - \frac{\tau c^2(b-c)}{2(h+\tau)(2b-c)^2(2b+c)}\right)$$
$$p_{21}^S = \frac{m}{2b-c}\left(1 - \frac{\tau bc(b-c)}{2(h+\tau)(2b-c)^2(2b+c)}\right)$$

yielding two stage profits of:

$$\Pi_{1S}^{C,NC} = \frac{b}{(2b-c)^2}\left[2m^2 + \frac{\tau}{h(h+\tau)}\right] -$$
$$\frac{\tau bc^2(b-c)m^2}{(2b-c)^4(2b+c)(h+\tau)}\left[1 - \frac{\tau c^2(b-c)}{4(h+\tau)(2b-c)^2(2b+c)}\right]$$
$$\Pi_{2S}^{C,NC} = \frac{b}{(2b-c)^2}\left[2m^2 + \frac{\tau}{h(h+\tau)}\right] -$$
$$\frac{\tau c^2(b-c)m^2}{2(2b-c)^2(2b+c)(h+\tau)}\left[c + \frac{\tau b(b-c)(2b^2-c^2)}{(h+\tau)(2b-c)^2(2b+c)}\right]$$

iii) The subgame perfect equilibrium prices and two stage profits for the (NC,NC) history are as displayed in proposition 2.

		firm 2	
		C	NC
firm 1	C	$\Pi_{1S}^{C,C}$ $\Pi_{2S}^{C,C}$	$\Pi_{1S}^{C,NC} = \Pi_{1S}^{NC,NC} + t$ $\Pi_{2S}^{C,NC} = \Pi_{2S}^{C,C} - k$
	NC	$\Pi_{1S}^{NC,C} = \Pi_{1S}^{C,C} - k$ $\Pi_{2S}^{NC,C} = \Pi_{2S}^{NC,NC} + t$	$\Pi_{1S}^{NC,NC}$ $\Pi_{2S}^{NC,NC}$

Table 1: The simultaneous game. The equilibrium is (C, C) in dominant strategies, where $k = \frac{\tau c^2(b-c)m^2}{2(2b-c)^2(2b+c)(h+\tau)}\left[c + \frac{\tau b(b-c)(2b^2-c^2)}{(h+\tau)(2b-c)^2(2b+c)}\right]$, and $t = \frac{\tau c(b-c)m^2}{(2b-c)^4(h+\tau)}\left[\frac{c^2}{2(2b+c)} + \frac{\tau c(b-c)}{4(2b-c)^2}\left(\frac{b-c}{(2b-c)^2} + \frac{bc^2}{(h+\tau)(2b+c)}\right)\right]$, with $k > 0$, $t > 0$.

4 EQUILIBRIA FOR THE FULL GAMES

We now proceed to analyze the stage zero decision, which amounts to choosing an communication structure. Recall that in this stage firms choose whether to communicate their price levels and since this is a model of imperfect but complete information, firms do not have any incentive to lie about their equilibrium outcomes.

4.1 The simultaneous play scenario

In order to highlight the role of price communication, let us consider first the simultaneous pricing subgame in stage one. The communication game is as displayed in table 1. For the random variables precisions different from zero and infinity the two stage profits are ranked as follows, $\Pi_{1S}^{C,C} > \Pi_{1S}^{C,NC} > \Pi_{1S}^{NC,C} > \Pi_{1S}^{NC,NC}$ and $\Pi_{2S}^{C,C} > \Pi_{2S}^{NC,C} > \Pi_{2S}^{C,NC} > \Pi_{2S}^{NC,NC}$.

Note that if $\tau = 0$, then signal-jamming does not take place: i.e. if the variance of the demand noise is infinite there is no way to manipulate each other firms' beliefs about $\tilde{\theta}$. This is so because by Bayesian updating (see (A4) and (A6)), posterior beliefs equal prior beliefs regardless of the price choice. Also, when $h = 0$ (with $\tau > 0$) posterior beliefs are equal to their corresponding market signals, so that there is still room for information manipulation. However, when the variance of the unknown demand parameter is infinite, the expected second stage profits, $\Gamma(q_{11}^e, q_{21}^e)$, goes to infinity (see (A18)).

In words, under the simultaneous play scenario the profits under full communication are higher than those under non-communication. Moreover, under the simultaneous play scenario, it is a dominant strategy to communicate, and hence the unique equilibrium would be (C, C). This is due to the fact that under Bertrand in all the continuation games, non-communication entails a more aggressive behavior, i.e. lower prices. Thus, the coordination in (C, C) means a commitment to less competition in the market place, that is, higher prices, and hence higher profits.[19] Although the joint maximization most collusive outcome is not achieved, assuming simultaneity throughout the two pricing stages allows firms the best coordination ever possible. From a policy viewpoint, it is worth noting that such full communication pattern has been obtained from a non-cooperative game. Though an interesting finding in itself, it does not seem to correspond very well with reality. As we noted in the introduction, real world examples suggest the sequential exchange of price information and that firms' behavior might respond to leader-follower patterns. This point can be incorporated into the analysis by unfolding stage one in two potential pricing periods. Then, two questions emerge: if firms are given the opportunity to choose roles, will full communication be the equilibrium of the game? in case not, will price competition be tougher or weaker? The next subsection is precisely devoted to answer these questions.

4.2 The sequential play scenario

Given the undominated pure strategy subgames equilibria in the (C, C), (C, NC), (NC, C), (NC, NC) subgames, there are two possible continuation payoffs in the overall game.[20]

[19] The nature of the products strategic complementarity is important for this result, i.e. best responses sloping upwards. With strategic substitutes, even though the (C, C) outcome yields higher profits, the firms would have NC as a dominant strategy and they would be unable to coordinate in the (C, C) cell.

[20] van Damme (1983) demonstrates for two-player non-zero sum matrix games that trembling hand perfect equilibria use only undominated strategies. If players might make errors in playing their strategies, opponents consider this in choosing their own strategies. Here, we consider the stronger property that strategies remain undominated after iterated dele-

We find that the only equilibrium for each of the two games entails price communication by only one firm, the leader. In fact, communication is a dominant strategy for this firm. Besides, the firm which does not communicate obtains a "second mover advantage": it will become the follower and set lower prices than the leader.[21]

The game under each of the two possible continuation payoffs is displayed and analyzed below in tables 2 and 3. Each variation is indexed by the conjectures about which of the two equilibria of the (C, C) subgame is played, using the notation $\{1(C),\ 2(C)\}$, where the entry is L or F as appropriate. Thus, $G(\{L, F\})$ where $1(C) = L,\ 2(C) = F$, is the game wherein if the subgame (C, C) is reached, then the common conjecture is that firm 1 will set its leader price in the first period and firm 2 will set its follower price in the second period. Since the other three subgames only have one equilibrium, these conjectures need not be specified.

The following Proposition (derived by examining tables 2 and 3) summarizes the central result of the paper. It states the conditions under which an asymmetric communication patterns is the unique outcome of the full game.

Proposition 5 *Provided that* $\tau \neq 0$ *and* $h \neq 0$ *and both different from infinity, the following communication outcomes are equilibria under the indicated conditions:*
(NC, C) *is a Nash equilibrium when in the* (C, C) *subgame, firm 1 is the leader and 2 is the follower.*
(C, NC) *is a Nash equilibrium when in the* (C, C) *subgame, firm 2 is the leader and 1 is the follower.*

In $G1$, C is a dominant strategy for firm 2, since by communicating it may get to be either a follower (if firm one also chooses C) or a price leader. Since firm 1 chooses its most preferred role, being the follower even without communication, the (NC, C) outcome results. The opposite incentives work out in $G2$ and consequently, C is a dominant strategy for firm 1 and the

tion of dominated strategies.

[21]Recall that in a symmetric Bertrand market with differentiated products, the follower has the advantage of being able to undercut the leader's price and thus does better than the leader. The leader in turn must do as well in the sequential game as in the simultaneous play game, since as a leader, he could play his simultaneous play equilibrium strategy. Hence a follower in the sequential game does better than a simultaneous player.

		firm 2: C	firm 2: NC
firm 1	C	$\Pi_{1L}^{C,C}$ $\Pi_{2F}^{C,C}$	$\Pi_{1L}^{C,NC} = \Pi_{1S}^{NC,NC} + l$ $\Pi_{2F}^{C,NC} = \Pi_{2F}^{C,C} - p$
	NC	$\Pi_{1F}^{NC,C} = \Pi_{1L}^{C,C} + q$ $\Pi_{2L}^{NC,C} = \Pi_{2S}^{NC,NC} + l$	$\Pi_{1S}^{NC,NC}$ $\Pi_{2S}^{NC,NC}$

Table 2: G1= G({L,F},{L,F},{F,L}). There is a NE, (NC, C). Where $l = \frac{bm^2}{2(2b^2-c^2)}\left(\frac{2b+c}{2b} - \frac{\tau c^2(b-c)}{4b(h+\tau)(2b-c)^2}\right)^2 + \frac{\tau c(b-c)m^2}{2(2b-c)^4(h+\tau)}\left(c + \frac{\tau c(b-c)^2}{2(2b-c)^4}\right) - \frac{m^2 b}{(2b-c)^2}$ $q = \frac{m^2}{4b}\left(\frac{(2b+c)^2-2c^2}{2(2b^2-c^2)} - \frac{\tau c(b-c)(2b+c)}{4(h+\tau)(2b-c)(2b^2-c^2)}\right)^2 - \frac{m^2(2b+c)^2}{8b(2b^2-c^2)}$, and $p = \frac{m^2((2b+c)^2-2c^2)}{16b(2b^2-c^2)^2} - \frac{m^2}{4b}\left(\frac{(2b+c)^2-2c^2}{2(2b^2-c^2)} - \frac{\tau c(b-c)(2b+c)}{4(h+\tau)(2b-c)(2b^2-c^2)}\right)^2$, $l > 0, q > 0$ and $p > 0$.

(C, NC) takes place. Notice that communication acts as a pre-commitment device to a "price umbrella" lower than the price with full communication. The firm which communicates will set up a price and the follower will undercut it. Observe that even though the (C, C) outcome in $G1$ and $G2$ would be Pareto optimal, given firms' symmetry, they are unable to coordinate on it. The possibility given to the firms to choose their role makes the market more competitive since there is one firm that has an incentive to undercut the price set by leader and to become a follower.

However compared with Bertrand in all the continuation games, NC entails a more aggressive behavior, i.e. lower prices. Since when the choice of roles is not permitted, there is no possibility that by choosing NC a firm becomes a follower (or a leader), it is a dominant strategy for either firm to choose C, and hence they coordinate in the (C, C) outcome. This coordination means a commitment to less competition in the market place, i.e. higher prices and hence higher profits.

In our extended game, leadership means committing to a particular price whether the rival attempts to lead or to follow. Thus, we deal with an "extended game with action commitment", where the properties of the basic duopoly game are irrelevant as long as the simultaneous and sequential move

		firm 2	
		C	NC
firm 1	C	$\Pi_{1F}^{C,C}$ $\Pi_{2L}^{C,C}$	$\Pi_{1L}^{C,NC} = \Pi_{1S}^{NC,NC} + l$ $\Pi_{2F}^{C,NC} = \Pi_{2L}^{C,C} + q$
	NC	$\Pi_{1F}^{NC,C} = \Pi_{1F}^{C,C} - p$ $\Pi_{2L}^{NC,C} = \Pi_{2S}^{NC,NC} + l$	$\Pi_{1S}^{NC,NC}$ $\Pi_{2S}^{NC,NC}$

Table 3: G2=G({F,L}, {L,F},{F,L}). There is a NE, (C, NC).

equilibria all differ. And, regardless of Pareto dominance or slopes of reaction functions, both sequential games are outcomes in undominated strategies of the extended game. This is in clear contrast with extended games, where firms announce at which time they will choose an action and are committed to this choice, but they need not specify the action they will take if they choose in the first period. In these "games with observable delay", the nature of the basic duopoly game is important for the results. The Pareto dominance of the simultaneous move equilibrium is an important consideration and this in turn depends on the slopes of reaction functions.[22]

4.3 Robustness

Although we have analyzed the undiscounted case, i.e. $\delta = 1$, for δ the discount factor, our results remain valid for any $0 < \delta \leq 1$. This is in clear contrast with discounted dynamic models of collusion where δ has to be big enough to support collusive behavior. Note that when $\delta = 0$ our model collapses into a model of perfect information since now the stage two of the

[22]More recently, Amir and Grilo (1994) have shown that for the Cournot duopoly, log-concavity of the (inverse) demand function alone leads to simultaneous play as the endogenous timing, regardless of the cost functions. On the other hand if the demand function is log-convex and production is costless, the endogenous timing leads to sequential play with both leader-follower configurations. The third possible outcome is sequential play with a specific leader-follower assignment. This outcome prevails when firm 1 (says) has constant marginal cost c_1, firm 2 has no costs, the demand function is log-convex while this demand minus c_1 is log-concave. In this case, firm 1 emerges as the endogenous leader.

game does not matter and therefore no equilibrium (in stage one) entails manipulation of information by any firm. Therefore, there is no need to coordinate by communicating.

Our results are robust to mean-preserving contractions of the density functions around the means of θ and ε. In fact, they are robust to more general distribution functions as long as the existence[23] of a stage one signal-jamming equilibrium is guaranteed. In our case, existence and uniqueness are driven by the fact that the distribution function of the variables are normally distributed.[24]

Unlike collusive models, where symmetric optimal non-cooperative solutions emerge from symmetric duopoly structures, our model predicts asymmetric outcomes from communication.

The gain from non-communication (second mover advantage) of say firm 1 when the rival does (see $G1$) is $\Pi_{1F}^{NC,C} - \Pi_{1L}^{C,C} = q$, where q is a positive function of the precisions of the random variables, h and τ, provided that $\tau \neq 0$ and $h \neq 0$ and both different from infinity. In particular, q is increasing in h and decreasing in τ. The reason is that the price decrease of a follower who does not communicate when the rival does with respect to a leader communicating duopolist is an increasing function of the precision of θ and a decreasing function of the precision of ε. Thus, the best situation for a follower is to face a very precise θ, because it can better adjust its price (the price of the follower increases with h), and a noisy demand (a sufficiently small τ), because it undercuts less the leader's price (the price of the follower is a decreasing function of τ). In the same way, the communicating leader prefers a precise θ and a noisy demand, because the price umbrella is higher and the follower will undercut him less.

[23] As noted above and shown in Mirman, Samuelson and Urbano (1993b), under general distribution functions the equilibrium existence is guaranteed as long as each firm's problem has a solution, and this will be the case whenever the different possible values of the unknown intercept are not too far away from each other.

[24] In Porter's (1983) dynamic collusive model, the distribution function of the demand shock has to be convex, in order to guarantee the concavity of the value function (given the symmetric linear structure of inverse demand and costs), and hence uniqueness of the equilibrium output in cooperative periods.

5 CONCLUSIONS

We have analyzed the subgame perfect equilibria of a three stage game where firms have to decide whether to communicate their stage one equilibrium prices. The equilibrium outcome is that just one firm communicates. Communication enables pre-commitment to a price umbrella by a leader which a follower will undercut. This firm will choose not to communicate since in this way it gets a second mover advantage.

In contrast to the outcome of the simultaneous game, the possibility of choosing roles mitigates price coordination making the market more competitive. However, there is not a clear conclusion as to which communication scheme attains the social welfare optimum. Nevertheless, several considerations can be made. First of all, consumer surplus is increased when firms practise signal-jamming regardless of the simultaneity or sequentiality of stage one moves. Secondly, when both firms communicate, it can be shown that a sequential timing is always welfare improving compared with the simultaneous one.

Considering the case where firms choose prices simultaneously, it is not possible to find an unambiguous ranking. However, there is a sufficient condition for the case where both firms communicate to be the socially preferred situation. It entails the products not being too differentiated, (precisely, $b > c > \frac{3b}{4}$). Finally, a similar condition is found when firms choose prices sequentially, communication by both firms is socially preferred to communication by only one firm if $b > c > 0.784b$.

It is generally said that Cournot is less competitive than Bertrand. We have not compared both outcomes in our dynamic model of communication. Yet, it is worth emphasizing what the role of communication is when firms may choose roles in stage one of the game. In the case of output communication, a firms wants to become a leader and this can be achieved due to the pre-commitment effect of communication within production stages. With price communication though, leader-follower patterns also arise but the reason behind such a behavior lies in trying to avoid tougher competition intensity. With prices as strategic variables, there exists a second mover advantage effect. But if both firms decide to wait and not to communicate prices, then they will end up practising signal-jamming. Hence, one firm has the incentive to communicate and become a leader while the rival prefers not to communicate and become a follower. In either case, both firms are better off relative to the signal-jamming equilibrium. From the private point of view of firms, the final situation is halfway between the most competitive one and the most coordinated one.

A APPENDIX

A.1 Proof of Lemma 1.

In order to ensure that p_{11}^e and p_{21}^e are an equilibrium for the second stage of the game, firm i, $i=1,\ 2$, must be able to calculate the profits from setting a price $p_{i1} \neq p_{i1}^e$ (and find this profit level inferior), assuming the firm $j's$ stage one equilibrium price is p_{j1}^e and assuming, since firm j cannot observe $i's$ price, that firm j believes firm i to have set p_{i1}^e in the first stage.

Given this information, firm i uses Bayes' rule to form the posterior beliefs about $\tilde{\theta}$, i.e. $E\left[\tilde{\theta} \mid Q_{11},\ Q_{21},\ p_{i1},\ p_{j1}^e\right]$. Namely, by (1), firm i observes, after the stage one, the market signals

$$\begin{aligned} Q_{11} + bp_{i1} - cp_{j1}^e &= \tilde{\theta} + \tilde{\varepsilon} \\ Q_{21} + bp_{j1}^e - cp_{i1} &= \tilde{\theta} + \tilde{\varepsilon} \end{aligned} \tag{A1}$$

or what is the same

$$\frac{Q_{11} + Q_{21} + (b-c)(p_{i1} + p_{j1}^e)}{2} = \tilde{\theta} + \tilde{\varepsilon} \tag{A2}$$

Let $\bar{Q}_1 = \frac{Q_{11}+Q_{21}}{2}$, be the average of sales in the stage one, and let $\bar{p}_1 = \frac{p_{i1}+p_{j1}^e}{2}$, be the stage one average of prices, then firm i observes

$$\bar{Q}_1 + (b-c)\bar{p}_1 = \tilde{\theta} + \tilde{\varepsilon}$$

Therefore, for each value of the parameter θ,

$$\bar{Q}_1 + (b-c)\bar{p}_1 \sim N(\theta, \tau) \tag{A3}$$

The value of θ is unknown to firm i, but it a priory believes that $\tilde{\theta} \sim N(m, h)$. Then, after observing the sales average $\bar{Q}_1$ and knowing p_{i1}, and that firm j prices p^e_{j1}, its new beliefs about $\tilde{\theta}$ are given by (see Degroot, 1970, page 167) the normal distribution, $N\left(\hat{\theta}_i, \hat{h}_i\right)$, where for $i = 1,\ 2,\ i \neq j$:[25]

$$\hat{\theta}_i = \hat{\theta}_i\left(\bar{Q}_1, \bar{p}_1\right) = \frac{mh + \tau\left(\bar{Q}_1 + (b-c)\bar{p}_1\right)}{h+\tau} \tag{A4}$$

$$\hat{h}_i = h + \tau \tag{A5}$$

In order to choose a stage two price, firm i, must also compute $j's$ posterior beliefs about $\tilde{\theta}$. In particular, given that firm j observes Q_{11} and Q_{12}, knows p^e_{j1}, and believes that firm i prices p^e_{i1}, by (A1)-(A5), its posterior beliefs about $\tilde{\theta}$ are given by the normal distribution $N\left(\hat{\theta}^e, \hat{h}^e\right)$, where,

$$\hat{\theta}^e = \hat{\theta}\left(\bar{Q}_1, \bar{p}^e_1\right) = \frac{mh + \tau\left(\bar{Q}_1 + (b-c)\bar{p}^e_1\right)}{h+\tau} \tag{A6}$$

$$\hat{h}^e = h + \tau \tag{A7}$$

and $\bar{p}^e_1 = \frac{p^e_{i1}+p^e_{j1}}{2}$.

If p_{i1} and p^e_{i1} differ, so that firm i deviated from the equilibrium in the stage one, then $\hat{\theta}_i \neq \hat{\theta}^e$ so that firm $i's$ posterior beliefs do not match firm $j's$ beliefs about $\tilde{\theta}$.

Given these posterior beliefs, firm i first calculates firm $j's$ stage two price, that is,

$$\hat{p}_{j2} = \frac{\hat{\theta}^e}{2b-c} \tag{A8}$$

Firm $i's$ choice is found by maximizing the stage two profits, given that firm i has calculated that firm j prices $\hat{p}_{j2}$ and given that firm i expects $\tilde{\theta}$ to be $\hat{\theta}_i$.[26] Hence firm i maximizes:

$$E\left[Q_{12}(p_{i2}, \hat{p}_{j2}) \mid \bar{Q}_1,\ p_{i1},\ p^e_{j1}\right] p_{i2} = \left(\hat{\theta}_i - bp_{i2} + c\hat{p}_{j2}\right) p_{i2} \tag{A9}$$

[25] Note that given our normality assumption, the signal and the updated beliefs, $\hat{\theta}_i$, may take negative values. Firms are constrained to choose positive prices. For convenience, this is ignored and given the firms' strategies that are derived, we can get negative prices and outputs for certain conditions of the signal and $\hat{\theta}_i$. However, the probability of such an event can be made arbitrarily small by appropriately choosing the variances of the model.

[26] Notice that there is some asymmetry in the way that we treat firms i and j. This arises from the fact that we are solving firm $i's$ optimization problem and must consider the case in which firm i has deviated from $\hat{p}_{i1}$ to p_{i1}, in the stage one; while firm i presumes that firm j produced $\hat{p}_{j1}$ in the stage one.

Firm i differentiates (A9) with respect to p_{i2}, yielding,

$$\hat{p}_{i2} = \frac{\hat{\theta}_i + c\hat{p}_{j2}}{2b} \tag{A10}$$

and inserting (A8) in (A10)

$$\hat{p}_{i2} = \frac{\hat{\theta}_i(2b-c) + c\hat{\theta}^e}{2b(2b-c)} \tag{A11}$$

which is firm $i's$ stage two price. Notice that (A11) coincides with (A8) when posteriors are common.

A.2 Equilibria Characterization

A.2.1 Characterization of equilibria for the (C,C) History

Proof of Lemma 2:

Conditions (12) and (13), follow from (10) and (11) and the first order conditions for (9) (along with an analogous first order condition for firm j). To see that $\Gamma(\bar{p}_1^e)$ is constant in $\bar{p}_1^e$, notice that,

$$\Gamma(\bar{p}_1^e) = E\left[V(\hat{\theta}^e)\right] = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}^e(\bar{Q}_1, \bar{p}_1^e)\right) f\left(\bar{Q}_1 + (b-c)\bar{p}_1^e\right) d\bar{Q}_1 \tag{A12}$$

we can write (A12) as:[27]

$$\Gamma(\bar{p}_1^e) = E\left[V(\hat{\theta}^e)\right] = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}^e(\theta+\varepsilon)\right) f\left(\theta+\varepsilon\right) d(\theta+\varepsilon) \tag{A13}$$

where $\hat{\theta}^e(\theta+\varepsilon)$ is the value of $\hat{\theta}^e$ that will appear if the random variable ε is drawn and the true state is θ. Thus, $\hat{\theta}^e(\theta+\varepsilon)$ is obtained from (A6) by substituting $\bar{Q}_1 + (b-c)\bar{p}_1^e$. Upon performing this substitution, we find that,

$$\hat{\theta}^e(\theta+\varepsilon) = \frac{mh + \tau(\theta+\varepsilon)}{h+\tau} \tag{A14}$$

that does not depend on $\bar{p}_1^e$. This result is driven by two facts. First, our setting is one of parallel demand curves and indicates that variations in firm $i's$

[27]This result does not depend on the specific distribution $f(\cdot)$.

stage one price cannot affect firm $i's$ posterior beliefs.[28] This ensures that any adjustment of price levels away from the single-period optima arise out of the strategic interaction between firms. Second, when firms announce their stage one equilibrium prices, there is no possibility to manipulate rival's posterior beliefs. Hence, the firms' stage one problem reduces to solving a single-period Bertrand duopoly, and given the demand structure, reaction curves are linear and they intersect just once. Also notice that second conditions are trivially satisfied, (see that (12) and (13) are sufficient for existence). ■

The computation of the equilibria displayed in Proposition 1 follows:

a) Simultaneous move (Bertrand)

By lemma 2 in the text, an equilibrium (p_{11}^e, p_{21}^e) must satisfy ,

$$m - 2bp_{11}^e + cp_{21}^e + \frac{d\Gamma(p_{11}^e, p_{21}^e)}{dp_{11}} = 0 \quad \text{(A15)}$$

$$m - 2bp_{21}^e + cp_{11}^e + \frac{d\Gamma(p_{11}^e, p_{21}^e)}{dp_{21}} = 0 \quad \text{(A16)}$$

where

$$\Gamma(p_{11}^e, p_{21}^e) = E\left[V(\hat{\theta}^e)\right] = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}^e(\bar{Q}_1, \bar{p}_1^e)\right) f\left(\bar{Q}_1 + (b-c)\bar{p}_1^e\right) d\bar{Q}_1 \quad \text{(A17)}$$

where $\bar{Q}_1 = \frac{Q_{11}+Q_{21}}{2}$, $\bar{p}_1^e = \frac{p_{11}^e + p_{21}^e}{2}$.

We show first

Lemma 4 (A1)

A1

[28] In other words, there is no active learning or experimentation in this model. If it were, p_{i1}^e would appear explicitly in both (A13) and (A14) (see Mirman, Samuelson and Urbano, 1993a). We could have considered such a model, by alternatively assuming the slope demand parameter unknown. However, this just would complicate both the computations and comparison among equilibria without adding more insight to the problem at hand. For a model with both signal-jamming and experimentation, see Urbano (1993).

Proof. Since $V_i\left(\hat{\theta}^e(\bar{Q}_1,\bar{p}_1^e)\right)=\frac{b(\hat{\theta}^e)^2}{(2b-c)^2}$, (A17) is

$$\Gamma(p_{11}^e,p_{21}^e)=\frac{b}{(2b-c)^2}\int_{-\infty}^{+\infty}(\hat{\theta}^e)^2 f\left(\bar{Q}_1+(b-c)\,\bar{p}_1^e\right)d\bar{Q}_1 \tag{A19}$$

and by the definition of $\hat{\theta}^e$(see A6), (A16) is

$$\begin{aligned}\Gamma(p_{11}^e,p_{21}^e) &= \frac{b}{(2b-c)^2(h+\tau)^2}\int_{-\infty}^{+\infty} m^2h^2+\tau^2\left[(\bar{Q}_1+(b-c)\,\bar{p}_1^e\right]^2 \\ &\quad +2mh\tau(\bar{Q}_1+(b-c)\,\bar{p}_1^e)f\left(\bar{Q}_1+(b-c)\,\bar{p}_1^e\right)d\bar{Q}_1\end{aligned} \tag{A20}$$

Using the fact that $E(x^2)=var(x)+(E(x))^2$, for x a random variable, (A20) is

$$\Gamma(p_{11}^e,p_{21}^e)=\frac{b}{(2b-c)^2(h+\tau)^2}\left[m^2h^2+\tau^2\left[\frac{h+\tau}{h\tau}+m^2\right]+2m^2h\tau\right] \tag{A21}$$

that rearranging yields (A18). ■

Hence $\frac{d\Gamma}{dp_{i1}}=0,\ i=1,2$ and inserting this result in (A15) and (A16) and solving simultaneously, gives the equilibrium outputs: $p_{11}^e=p_{21}^e=p_1^e=\frac{m}{2b-c}$, with first stage profits $\Pi_{i1}=\Pi_{j1}=\frac{bm^2}{(2b-c)^2}$, and total profits for the two-stages:$\Pi_i^{C,C}(p_{11}^s,p_{21}^s)=\Pi_{i1}+\Gamma(p_{11}^s,p_{21}^s)=\frac{b}{(2b-c)^2}[2m^2+\frac{\tau}{h(h+\tau)}]$.

b) Sequential play

As above, by lemma A1, the stage one prices that maximize the sum of the two stage profits are given by the single-period Stackelberg prices for our problem: $p_{i1}^L=\frac{m(2b+c)}{2(2b^2-c^2)}$ and $p_{j1}^F=\frac{m}{2b}\left[1+\frac{c(2b+c)}{2(2b^2-c^2)}\right]$, for $i=1,2$ and $i\neq j$ (note that $p_{i1}^L>p_{j1}^F>p_1^S$ under (C,C)); with stage one profits: $\Pi_{i1}^L=\frac{m^2(2b+c)^2}{8b(2b^2-c^2)}$, $\Pi_{j1}^F=\frac{m^2}{4b}\left[1+\frac{c(2b+c)^2}{2(2b^2-c^2)}\right]^2$; and with the sum of the two stages profits equal to: $\Pi_{iL}^{C,C}=\frac{m^2(2b+c)^2}{8b(2b^2-c^2)}+\frac{b}{(2b-c)^2}\left[m^2+\frac{\tau}{h(h+\tau)}\right]$, $\Pi_{jF}^{C,C}=\frac{m^2((2b+c)^2-2c^2)}{16b(2b^2-c^2)^2}+\frac{b}{(2b-c)^2}\left[m^2+\frac{\tau}{h(h+\tau)}\right]$.

A.2.2 Characterization of the Bertrand equilibrium for the (NC,NC) History

Proof of Lemma 3:

Conditions (15) and (16) are the first order conditions for (14) (along with an analogous first order condition for firm j). To see that $\frac{d\Gamma(p_{i1}^e, p_{i1}^e, p_{j1}^e)}{dp_{i1}}$ is constant in p_{i1}^e and p_{j1}^e, consider (8). We can write (8) as:

$$\Gamma_i(p_{i1}, p_{i1}^e, p_{j1}^e) = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}_i(\theta, \varepsilon), \hat{\theta}^e(\theta, \varepsilon, (p_{i1}^e - p_{i1})\right) f(\theta + \varepsilon)\, d(\theta + \varepsilon) \tag{A22}$$

where $\hat{\theta}_i(\theta, \varepsilon)$, and $\hat{\theta}^e(\theta, \varepsilon, (p_{i1}^e - p_{i1}))$ [29] are the value of $\hat{\theta}_i$ and $\hat{\theta}^e$ that will appear if the random variable ε is drawn and the true state is θ. Thus, $\hat{\theta}_i(\theta, \varepsilon)$ and $\hat{\theta}^e(\theta, \varepsilon, (p_{i1}^e - p_{i1}))$ are obtained from (A4) and (A6) respectively, by substituting $\bar{Q}_1 + (b - c)(\bar{p}_1)$. Upon performing this substitution, we find that,

$$\begin{aligned} \hat{\theta}_i(\theta, \varepsilon) &= \frac{mh + \tau(\theta + \varepsilon)}{h + \tau} \\ \hat{\theta}^e(\theta, \varepsilon, (p_{i1}^e - p_{i1})) &= \frac{mh + \tau(\theta + \varepsilon + (b - c)(p_{i1}^e - p_{i1})}{h + \tau} \end{aligned}$$

Notice that $\hat{\theta}_i$ does not depend on p_{i1}, p_{i1}^e or p_{j1}^e and that $\hat{\theta}^e$ does not depend on p_{j1}^e and depends only on the difference $p_{i1}^e - p_{i1}$. The derivative $\frac{d\Gamma(p_{i1}^e, p_{i1}^e, p_{j1}^e)}{dp_{i1}}$ then similarly depends only on $p_{i1}^e - p_{i1}$. Since this derivative appears in (15) and (16), with $p_{i1}^e - p_{i1}$ fixed at zero, (15) and (16) are linear ensuring that they have at most one intersection and hence at most one equilibrium exists. Symmetry ensures that the final terms of the left hand side of (15) and (16) evaluated at $p_{11}^e - p_{11} = p_{21}^e - p_{21} = 0$ are equal. Any solution of (15) and (16), given symmetry, must then satisfy $p_{11}^e = p_{21}^e$.

However, intersection of the best response functions supplies only a necessary condition for an equilibrium. Sufficiency requires second-order conditions to be satisfied. More precisely:

$$-2b + \frac{d^2\Gamma(p_{i1}, p_{i1}^e, p_{j1}^e)}{dp_{i1}^2} < 0 \tag{A23}$$

[29] This result does not depend on the specific Bayesian updating rule.

for all $p_{i1}, p_{i1}^e, p_{j1}^e$, with a similar condition for firm j.

It is easy to check that in our model (A23) is reduced to: $-2b+\frac{\tau^2 c^2 (b-c)^2}{8b(2b-c)^2(h+\tau)^2} < 0$, that is always satisfied. Hence (15) and (16) are not only necessary but sufficient conditions for the existence of the equilibrium. ■

By lemma 3 in the text, an equilibrium (p_{11}^e, p_{21}^e) must satisfy ,

$$m - 2bp_{11}^e + cp_{21}^e + \frac{d\Gamma_1(p_{11}, p_{11}^e, p_{21}^e)}{dp_{11}} = 0 \quad \text{(A24)}$$

$$m - 2bp_{21}^e + cp_{11}^e + \frac{d\Gamma_2(p_{21}, p_{11}^e, p_{21}^e)}{dp_{21}} = 0 \quad \text{(A25)}$$

where

$$\Gamma_i(p_{i1}, p_{i1}^e, p_{j1}^e) = \int_{-\infty}^{+\infty} V_i\left(\hat{\theta}_i(\bar{Q}_1, \bar{p}_1), \hat{\theta}^e(\bar{Q}_1, \bar{p}_1^e)\right) f\left(\bar{Q}_1 + (b-c)\bar{p}_1\right) d\bar{Q}_1 \quad \text{(A26)}$$

with $\bar{p}_1 = \frac{p_{i1}+p_{j1}^e}{2}$, and

$$V_i\left(\hat{\theta}_i, \hat{\theta}^e\right) = \frac{(\hat{\theta}_i(2b-c) + c\hat{\theta}^e)^2}{4b(2b-c)^2} \quad \text{(A27)}$$

We show first,

Lemma 5 (A2)

A2

Proof. By (A22) and the definitions of $\hat{\theta}_i$ and $\hat{\theta}^e$(see (A4) and (A6)), (A21) is

$$\Gamma_i(p_{i1}, p_{11}^e, p_{21}^e) = \frac{1}{4b(2b-c)^2(h+\tau)^2} \int_{-\infty}^{+\infty} [2b(mh + \tau(\bar{Q}_1 + (b-c)\bar{p}_1) \quad \text{(A29)}$$
$$-c(b-c)\tau(\bar{p}_1 - \bar{p}_1^e)]^2 f\left(\bar{Q}_1 + (b-c)\bar{p}_1\right) d\bar{Q}_1$$

Tedious computations using the fact that $E(x^2) = var(x) + (E(x))^2$, for x a random variable, and rearranging gives (A28). ■

Using (A28) to compute the final terms of the left-hand-side of (A24) and (A25), and evaluating these derivatives at the equilibrium, i.e. when $(p_{i1}^e - p_{i1}) = 0$, give the following reaction functions:

$$p_{11}^e = \frac{m}{2b} + \frac{cp_{21}^e}{2b} - \frac{\tau mc(b-c)}{4b(2b-c)^2(h+\tau)} \quad \text{(A30)}$$

$$p_{21}^e = \frac{m}{2b} + \frac{cp_{11}^e}{2b} - \frac{\tau mc(b-c)}{4b(2b-c)^2(h+\tau)} \quad \text{(A31)}$$

that solving simultaneously yield equilibrium prices,

$$p_{11}^e = p_{21}^e = p_{11}^s = p_{21}^s = \frac{m}{2b-c}[1 - \frac{\tau c(b-c)}{2(2b-c)^2(h+\tau)}] \quad \text{(A32)}$$

with stage one profits $\Pi_{i1} = \Pi_{j1} = \frac{m^2}{(2b-c)^2}[b - \frac{\tau c^2(b-c)}{2(2b-c)^2(h+\tau)} - \frac{\tau^2 c^2(b-c)^3}{4(2b-c)^6(h+\tau)}]$, and total profits for the two-stages: $\Pi_{iS}^{NC,NC} = \Pi_{jS}^{NC,NC} = \frac{b}{(2b-c)^2}[2m^2 + \frac{\tau}{h(h+\tau)}] - \frac{\tau c(b-c)m^2}{2(2b-c)^4(h+\tau)}[c + \frac{\tau c(b-c)^2}{2(2b-c)^4}] < \Pi_{iS}^{C,C} = \Pi_{jS}^{C,C}$.

Note that equilibrium prices are lower than under Bertrand for the (C, C) history, meaning that signal-jamming considerations force the firms to price less than the single-period Bertrand equilibrium. Consequently firms' profits are lower.

A.2.3 Characterization of equilibria for the (C,NC) History

a) Simultaneous move

Notice that in this case firm 1 communicates its stage one equilibrium price to firm 2, while firm 2 does not do it. Hence an equilibrium here is a pair (p_{11}^e, p_{21}^e) that

$$p_{11}^e \in \arg\max_{p_{11}} \Pi_{11}(p_{11}^e, p_{21}) + \Gamma_1(p_{11}, p_{21}^e) \quad \text{(A33)}$$

$$p_{21}^e \in \arg\max_{p_{21}} \Pi_{21}(p_{11}^e, p_{21}) + \Gamma_2(p_{21}, p_{11}^e, p_{21}^e) \quad \text{(A34)}$$

where

$$\Gamma_1(p_{11}^e, p_{21}^e) = \int_{-\infty}^{+\infty} V_1(\hat{\theta}^e(\bar{Q}_1, \bar{p}_1^e)) f(\bar{Q}_1 + (b-c)\bar{p}_1^e) d\bar{Q}_1 \quad \text{(A35)}$$
$$= \frac{b}{(2b-c)^2}\left[m^2 + \frac{\tau}{h(h+\tau)}\right]$$

by lemma A1. And by lemma A2,

$$
\begin{aligned}
\Gamma_2(p_{21}, p_{11}^e, p_{21}^e) &= \int_{-\infty}^{+\infty} V_2(\hat{\theta}_2(\bar{Q}_1, \bar{p}_1), \hat{\theta}^e(\bar{Q}_1, \bar{p}_1^e)) f(\bar{Q}_1 + (b-c)\bar{p}_1) d\bar{Q}_1 \\
&= \frac{b}{(2b-c)^2}[m^2 + \frac{\tau}{h(h+\tau)}] \qquad \text{(A36)} \\
&\quad + \frac{\tau c(b-c)(p_{21} - p_{21}^e)}{8b(2b-c)^2(h+\tau)}[\frac{\tau c(b-c)(p_{21} - p_{21}^e)}{2(h+\tau)} - 4bm]
\end{aligned}
$$

The first order conditions for (A33) and (A34) are:

$$
\begin{aligned}
m - 2bp_{11}^e + cp_{21}^e + \frac{d\Gamma_1}{dp_{11}} &= 0 \qquad \text{(A37)} \\
m - 2bp_{21}^e + cp_{11}^e + \frac{d\Gamma_2}{dp_{21}} &= 0 \qquad \text{(A38)}
\end{aligned}
$$

Using (A35) and (A36) to compute (A37) and (A38), yield

$$
\begin{aligned}
p_{11}^e &= \frac{m + cp_{21}^e}{2b} \qquad \text{(A39)} \\
p_{21}^e &= \frac{m + cp_{21}^e}{2b} - \frac{\tau c(b-c)m}{4b(2b-c)^2(h+\tau)} \qquad \text{(A40)}
\end{aligned}
$$

and solving simultaneously, gives the equilibrium prices: $p_{11}^S = p_{11}^e = \frac{m}{2b-c}[1 - \frac{\tau c^2(b-c)}{2(2b-c)^2(2b+c)(h+\tau)}]$ and $p_{21}^S = p_{21}^e = \frac{m}{2b-c}[1 - \frac{\tau cb(b-c)}{(2b-c)^2(2b+c)(h+\tau)}]$ with $p_{11}^S > p_{21}^S$. Note that $p_i^{NC,NC} < p_i^{NC,C} < p_i^{C,NC} < p_i^{C,C}$.

Notice that p_{21}^S in (C, NC) is higher than p_{21}^S in (NC, NC) (see A32), and that firm 1 prices less than under Bertrand in the (C, C) case.

Profits for the stage one are $\Pi_{11} = \frac{bm^2}{(2b-c)^2}[1 - \frac{\tau c^2(b-c)}{2(2b-c)^2(2b+c)(h+\tau)}]^2$ and $\Pi_{21} = \frac{m^2}{(2b-c)^2}[b - \frac{\tau c^2(b-c)}{2(2b-c)^2(2b+c)(h+\tau)}[c + \frac{\tau b(b-c)(2b^2-c^2)}{(2b-c)^2(2b+c)(h+\tau)}]]$, and total profits for the two-stages:$\Pi_{1S}^{C,NC} = \frac{b}{(2b-c)^2}[2m^2 + \frac{\tau}{h(h+\tau)}] - \frac{\tau bc^2(b-c)m^2}{2(2b-c)^4(2b+c)(h+\tau)}[1 - \frac{\tau c^2(b-c)}{4(2b-c)^2(2b+c)(h+\tau)}] < \Pi_{1S}^{C,C}$ and $\Pi_{1S}^{C,NC} > \Pi_{1S}^{NC,NC}$, $\Pi_{2S}^{C,NC} = \frac{b}{(2b-c)^2}[2m^2 + \frac{\tau}{h(h+\tau)}] - \frac{\tau c^2(b-c)m^2}{2(2b-c)^2(2b+c)(h+\tau)}[c + \frac{\tau b(b-c)(2b^2-c^2)}{(2b-c)^2(2b+c)(h+\tau)}] < \Pi_{2S}^{C,C}$.

b) Sequential play

The follower, firm 2, calculates its reaction function, that by (A40) above is

$$p_{21}^e = \frac{m + cp_{21}^e}{2b} - \frac{\tau c(b-c)m}{4b(2b-c)^2(h+\tau)}$$

Firm 1, the leader, introduces the above reaction function in its profits function and chooses a p_{11} such that it maximizes

$$\Pi_{11}(p_{11}, p_{21}^e(p_{11}^e)) + \Gamma_1(p_{11}^e, p_{21}^e(p_{11}^e))$$

where by lemma A1,

$$\Gamma_1(p_{11}^e, p_{21}^e(p_{11}^e)) = \frac{b}{(2b-c)^2}\left[m^2 + \frac{\tau}{h(h+\tau)}\right]$$

Hence, firm 1 chooses the single period Stackelberg leader price but with p_{21} given above. This yields, $p_{11}^L = \frac{m(2b+c)}{2(2b^2-c^2)}[1 - \frac{\tau c^2(b-c)}{2(2b^2-c^2)(2b+c)(h+\tau)}]$ and $p_{21}^F = \frac{m}{2b}[1 + \frac{c(2b+c)}{2(2b^2-c^2)} - \frac{\tau c(b-c)}{2(2b-c)^2(h+\tau)}[1 + \frac{c^2}{2(2b^2-c^2)}]]$, Note that p_{11}^L in (C, NC) is smaller than p_{11}^S in (C, C). Stage one profits are: $\Pi_{11}^L = \frac{m^2 b}{2(2b^2-c^2)}[\frac{2b+c}{2b} - \frac{\tau c^2(b-c)}{4b(2b-c)^2(h+\tau)}]^2$, $\Pi_{21}^F = \frac{m^2}{4b}[1 + \frac{cb}{(2b^2-c^2)}[\frac{2b+c}{2b} - \frac{\tau c^2(b-c)}{4b(2b-c)^2(h+\tau)}] - \frac{\tau c(b-c)}{2(2b-c)^2(h+\tau)}]^2$; and with the sum of the two stages profits equal to: $\Pi_{1L}^{C,NC} = \frac{b}{(2b-c)^2}[m^2 + \frac{\tau}{h(h+\tau)}] + \frac{m^2 b}{2(2b^2-c^2)}[\frac{2b+c}{2b} - \frac{\tau c^2(b-c)}{4b(2b-c)^2(h+\tau)}]^2$, $\Pi_{2F}^{C,C} = \frac{b}{(2b-c)^2}[m^2 + \frac{\tau}{h(h+\tau)}] + \frac{m^2}{4b}[\frac{(2b+c)^2-2c^2}{2(2b^2-c^2)} - \frac{\tau c(b-c)(2b+c)}{4(2b-c)(2b^2-c^2)(h+\tau)}]^2$.

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