## **VETO IN FIXED AGENDA SOCIAL CHOICE CORRESPONDENCES\***

Mª Carmen Sánchez and José E. Peris\*\*

WP-AD 95-08

<sup>\*</sup> We would like to thank B. Subiza and J.V. Llinares for their helpful suggestions and comments. Any remaining errors are our exclusive responsability. This author acknowledges financial support from the Spanish DGICYT, under project PB92-0342.

<sup>\*\*</sup> M.C. Sánchez and J. Peris: University of Alicante.

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.

Primera Edición Marzo 1995.

ISBN: 84-482-0903-6

Depósito Legal: V-1202-1995

Impreso por Copisteria Sanchis, S.L., Quart, 121-bajo, 46008-Valencia.

Printed in Spain.

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ABSTRACT

In this paper we analyze the relationship between acyclic social

decision functions and fixed agenda social choice correspondences which

some rationality conditions (such verify as Pareto, independence,

monotonicity or neutrality). This enables us to translate known results of

existence of individuals with veto from the social decision functions

context into the fixed agenda framework, such as those of Blau and

Deb (1977), Blair and Pollak (1982),...

Keywords: Veto; Fixed Agenda SSC; Acyclic SDF

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#### 0. INTRODUCTION

One of the aims of social choice theory is to analyze collective choices within a feasible set of alternatives; that is, to decide which are the "best alternatives" for society from individual preferences. In order to do this, we need to specify which subsets of the universal set of alternatives are potential feasible sets. Sometimes it is assumed that there exists a social preference relation whose maximization defines the choice set. Therefore, the choice rule operates on different subsets of the universal set of alternatives. In these cases it is usually assumed that the family of feasible sets consists of all nonempty finite subsets of the universal set (we will refer to it as intra agenda framework). However, at other times it is assumed that individuals have a unique subset of feasible alternatives known in advance (given by the particular restrictions of the problem); so collective choice is analyzed in the context of fixed agenda, that is, when the set of alternatives presented for choice is fixed.

Denicolò (1993) analyzes the relationship between fixed agenda social choice correspondences and social decision functions in the particular cases whereby the social preference relation is considered as a preorder or a quasiorder. Concretely, he translates Arrow's Impossibility Theorem and Gibbard's oligarchy results into a fixed agenda framework. There are other important results which could also be translated into the context of fixed agenda, such as those of Blau and Deb (1977), Blair and Pollak (1982), Kelsey (1985),... among others. However, all of these results are stated in terms of acyclic social decision functions, that is, when the social preference relation is considered to be acyclic.

As Denicolò mentions in his work (1993), "...some further weakening of the weak Independence condition would correspond to Independence of Irrelevant Alternatives plus acyclicity. If this conjecture were correct, then it should be possible to prove fixed agenda counterparts of the results for SDFs obtained by, among others, Blau and Deb and by Blair and Pollak". Thus, the aim of this paper is to state equivalence results between acyclic social decision functions and fixed agenda social choice correspondences in order to translate most of these results.

The paper is organized as follows: Firstly, basic definitions, properties and notation which are used throughout the work are presented. In Section 2 suitable properties of independence, neutrality and monotonicity for social choice correspondences are introduced. In Section 3 equivalence results between social decision functions and social choice correspondences verifying some of these conditions are stated and finally, Section 4 is devoted to translating well known results which prove the existence of veto, from the context of social decision functions to the context of fixed agenda social choice correspondences.

#### 1. PRELIMINARIES

Let X be a finite set of alternatives such that |X|>2 and  $N = \{1,2,...,n\}$  the finite set of individuals. Let W(X) be the family of weak orderings on X, and A(X) the family of acyclic binary relations on X. Given a weak order R, the strict preference P, and indifference I, are defined in

the usual way:  $xP_iy \Leftrightarrow xR_iy$  and  $no[yR_ix]$ ;  $xI_iy \Leftrightarrow xR_iy$  and  $yR_ix$ . A profile will be any n-tuple of weak orderings,  $(R_1,R_2,...,R_n) \in W^n(X)$ .

Formally a social choice correspondence (SCC) is a functional relationship that selects a nonempty subset of alternatives for each and every profile of individual preferences, C:  $W^n(X) \longrightarrow X$ .

On the other hand, a social decision function (SDF) is a functional relationship that associates an acyclic social preference relation to each and every profile of individual preferences,  $F \colon \operatorname{W}^n(X) \longrightarrow \operatorname{A}(X)$ .

In order to simplify the notation, henceforth we will refer to the social preference as R,  $F(R_1,R_2,...,R_n)=R$ , and P and P and P and P and P and P are associated social strict preference and social indifference relations, respectively.

It is clear that a SDF always defines a SCC in a natural way: by maximizing the social binary relation provided by it. Since X is finite and the social preference relation is acyclic, the set of maximal elements is always nonempty; therefore it is always well-defined. However a SCC does not always define an acyclic binary relation. In the intra agenda framework, we could define the base relation by stating that an alternative x is preferred or indifferent to another y if and only if x belongs to the choice set when the set of alternatives presented for choice is  $\{x,y\}$ . Under some conditions, this is an acyclic binary relation. But in the fixed agenda context it is not possible to do the same, because the set of alternatives presented for choice is always the whole set X. So, we need to use "artificial" profiles in order to obtain an acyclic preference relation (in

general a non trivial one) from a fixed agenda SCC. Moreover, by making use of additional properties which usually appear in the literature (Pareto, monotonicity,...), the relationship between the existence of a fixed agenda SCC and the existence of an SDF defined from it which verifies these properties will be stated. First of all the weak Pareto principle and weak Pareto optimality are formally defined as follows:

(P1). A SDF satisfies the weak Pareto principle if for all  $x,y \in X$ 

$$x \stackrel{P}{=} y \quad \forall i \in \mathbb{N} \quad \text{implies} \quad x \stackrel{P}{=} y$$

(P2). A SCC satisfies weak Pareto optimality if for all  $x,y \in X$ 

$$x P_i y \quad \forall i \in \mathbb{N} \quad \text{implies} \quad y \notin C(R_1, R_2, ..., R_n)$$

In order to present some additional definitions, we introduce the following notation:

- a) Given a profile  $(R_1, R_2, ..., R_n)$  and a subset  $S \subseteq X$  we will denote by  $(R_1, R_2, ..., R_n) : S \text{ the restriction of } (R_1, R_2, ..., R_n) \text{ to } S$
- b) The relation  $\boldsymbol{R}_{i}^{\text{S}}$  is defined from  $\boldsymbol{R}_{i}$  as follows:

if 
$$x \in S$$
 and  $a \notin S$  then  $x P_i^S$  a

if  $x,y \in S$  then  $x R_i^S$   $y \iff x R_i$   $y$ 

if  $x,y \notin S$  then  $x I_i^S$   $y$ 

By making use of this notation, we present the notion of *veto* for a fixed agenda social choice correspondence.

#### Definition 1.1.

An individual  $i \in N$  is said to be a veto for a SDF if for every  $x,y \in X$ 

# Definition 1.2. (1)

An individual  $i \in N$  is said to be a veto for a SCC if for every  $x,y \in X$ ,

$$x P_i y$$
 implies  $C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) \neq \{y\}$ 

## 2. INDEPENDENT, NEUTRAL AND MONOTONE SCC.

Most of the results which are going to be translated require independence, neutrality or monotonicity properties, so we devote this section to introducing these notions.

On the one hand, and in the context of SDF, the independence notion which is used to obtain impossibility results or the existence of vetoes, is the well known axiom of "independence of irrelevant alternatives" (AIIA), which can be stated as follows:

(A1). AIIA: If  $F:W^n(X) \longrightarrow A(X)$  is a SDF,  $x,y \in X$  and  $(R_1,R_2,...,R_n)$ ,  $(\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  are profiles such that:

<sup>&</sup>lt;sup>1</sup> The notion of veto introduced here is different from that introduced by Denicolò (1993). In general, our definition is weaker, but in the context of Denicolò's work both definitions coincide.

$$(R_1, R_2, ..., R_n): (x, y) = (\bar{R}_1, \bar{R}_2, ..., \bar{R}_n): (x, y)$$

then

$$x R y \Leftrightarrow x \overline{R} y$$

On the other hand, in the context of fixed agenda SCC, different notions of independence can be used. Denicolò (1993) presents two such notions which allow him to translate Arrow's and Gibbard's results. In particular, Denicolò proves that there exists a quasitransitive social decision function (respectively a social welfare function) which satisfies independence of irrelevant alternatives and weak Pareto principle if and only if there exists a social choice correspondence which verifies quasi-independence (2) (respectively independence) and weak Pareto optimality.

In particular, quasi-independence states that if  $C:W^n(X) \longrightarrow X$  is a SCC,  $x,y \in X$ ,  $(R_1,R_2,...,R_n), (\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  are profiles such that  $C(R_1,R_2,...,R_n) = \{x\}$  and  $(R_1,...,R_n):\{x,y\} = (\bar{R}_1,\bar{R}_2,...,\bar{R}_n):\{x,y\},$  then  $y \notin C(\bar{R}_1,\bar{R}_2,...,\bar{R}_n)$ . The next example shows that the SCC defined in a natural way from a SDF which satisfies (P1) and (A1), does not necessarily verify this condition. Therefore we will need to introduce a new notion of independence in order to obtain the corresponding equivalence result in the case of social decision functions.

## Example 2.1.

Let us consider  $X = \{x,y,z\}$ ,  $N = \{1,2\}$  and  $F:W^n(X) \longrightarrow A(X)$  a SDF defined as follows:

<sup>&</sup>lt;sup>2</sup> Denicolò calls this property *weak-independence*. Since we weaken it, we have called it *quasi-independence* since it is used to characterize quasitransitive SDF.

$$y P a \iff [y P_i a \text{ for at least one individual}] \quad \forall a \in X-\{y\}$$
 
$$a P b \iff a P_i b \quad \forall i \in N \qquad \forall a \in X-\{y\}, \ \forall b \in X, \ a \neq b$$

It is easy to prove that this SDF verifies (P1) and (A1). However, if we define the associated SCC by maximizing this SDF, it does not verify quasi-independence. To show this, consider the following profiles:

R 1	R <sub>2</sub>
z	х
x	у
у	z

$\overline{R}_{1}$	$\bar{R}_2$
Z	х
х	z
у	у

given by strict preference relations (that is:  $zP_1x$ ,  $xP_1y$ ,... and so on).

In this case it is observed that:

$$C(R_1, R_2) = \{x\}, (R_1, R_2) : \{x, z\} = (\overline{R}_1, \overline{R}_2) : \{x, z\}$$
 but  $C(\overline{R}_1, \overline{R}_2) = \{x, z\}$ 

The new notion of independence which yields to the corresponding equivalence result for SDF is as follows:

(A2). Weak Independence: If 
$$C:W^n(X) \longrightarrow X$$
 is a SCC,  $x,y \in X$ ,  $(R_1,R_2,...,R_n)$ ,  $(\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  are profiles such that:

$$C(R_1, R_2, ..., R_n) = \{x\} \text{ and } (R_1, ..., R_n) : \{a, y\} = (\bar{R}_1, \bar{R}_2, ..., \bar{R}_n) : \{a, y\}$$
 
$$\forall a \in A_y \cup \{x\} \text{ where } A_y = \{a \in X \mid y \notin C(R_1^{(a, y)}, ..., R_n^{(a, y)})\},$$

then

$$y \notin C(\bar{R}_1, \bar{R}_2, ..., \bar{R}_n)$$

That is, if given a profile of individual preferences, x is the only choice, y is another alternative and we consider another profile which coincides with the first one not only in  $\{x,y\}$ , but also in the position of y with respect to other alternatives ("better than y"), then y is not chosen in the new profile either.

The neutrality and monotonicity conditions which will be used in the context of social decision functions are those used by Blair and Pollak (1982). Now we introduce the translation of these properties to the fixed agenda context. In order to define the neutrality condition we will use the following notation: for every binary relation R and every permutation  $\sigma$  of X, a binary relation  $\sigma(R)$  is defined as follows:

$$x \sigma(R) y \iff \sigma^{-1}(x) R \sigma^{-1}(y)$$

Moreover, if F is a SDF and R =  $F(R_1, R_2, ..., R_n)$ , we will denote

$$R_{\sigma} = F(\sigma(R_1), \sigma(R_2), ..., \sigma(R_n))$$

(N1). (Blair and Pollak, 1982): A SDF  $F:W^n(X) \longrightarrow A(X)$  is neutral if for every  $(R_1,R_2,...,R_n) \in W^n(X)$  and every permutation  $\sigma$  of X it is verified that  $R_{\sigma} = \sigma(R)$ .

That is, a permutation of the names of alternatives in every individual preference originates the same permutation in the social preference

relation. So a symmetric treatment of alternatives is required. The idea of neutrality for SCC is exactly the same.

(N2). A SCC  $C:W^n(X) \longrightarrow X$  is neutral if for every permutation  $\sigma$  of X and every  $(R_1,R_2,...,R_n) \in W^n(X)$  it is verified that

$$C\left[\sigma(R_{1}), \sigma(R_{2}), ..., \sigma(R_{n})\right] = \sigma\left[C(R_{1}, R_{2}, ..., R_{n})\right]$$

Finally, monotonicity conditions are introduced as follows:

(M1). (Blair and Pollak, 1982): A SDF  $F:W^n(X) \longrightarrow A(X)$  is monotonic if  $\forall x,y \in X, (R_1,R_2,...,R_n), (\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  if

$$\left.\begin{array}{ccccc} x & I_{i} & y & \Rightarrow & x & \bar{R}_{i} & y \\ x & P_{i} & y & \Rightarrow & x & \bar{P}_{i} & y \end{array}\right\}$$

then

x P y implies x 
$$\bar{P}$$
 y

This condition (which some authors call *positive responsiveness*) requires that if an alternative x is socially preferred to another y and the position of x is improved with respect to individual preferences, then it has to be preferred to y in the new social preference.

$$(M2). \ \ \, \text{A SCC } \ \, \text{C:W}^n(X) \longrightarrow X \ \, \text{is monotonic} \ \, \text{if} \ \, \forall \ \, x,y \in X, \ \, (R_1,R_2,...,R_n),$$
 
$$(\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X) \ \, \text{such that} \ \, \text{C}(R_1,R_2,...,R_n) = \{x\}$$
 
$$\text{a } I_i \ \, y \Rightarrow \text{a } \bar{R}_i \ \, y$$
 
$$\text{a } P_i \ \, y \Rightarrow \text{a } \bar{P}_i \ \, y$$
 
$$\text{implies } \ \, y \not\in C(\bar{R}_1,\bar{R}_2,...,\bar{R}_n)$$
 
$$\forall \ \, \text{a} \in A_y \cup \{x\}$$

where  $A_{y}$  is defined as in axiom (A2).

In words, if given a profile of individual preferences in which x is the only choice, we consider another alternative y and another profile such that its position gets worse with respect to some alternatives and does not change with respect to others, then y is not chosen in the new profile either.

# 3. THE RELATIONSHIP BETWEEN ACYCLIC SDF AND FIXED AGENDA SCC.

Firstly we prove an equivalence result between the existence of fixed agenda social choice correspondences and the existence of social decision functions which verify independence and Pareto conditions.

#### Theorem 3.1.

- a. Every SDF which verifies (A1) and (P1) defines a SCC that satisfies (A2) and (P2).
- b. Conversely, every SCC which verifies (A2) and (P2) defines a SDF that satisfies (A1) and (P1).

#### Proof.

a. Let  $F:W^n(X) \longrightarrow A(X)$  be a SDF which satisfies (P1) and (A1). We define the SCC by maximizing the social preference relation associated to each profile of individual preferences, and we prove that this correspondence verifies (A2) and (P2). Let us define  $C:W^n(X) \longrightarrow X$  by

$$C(R_1,...,R_n) = \{a \in X \mid a R y \forall y \in X\}$$

Since X is finite and R acyclic, it is always well defined,  $(C(R_1,\ldots,R_n) \neq \emptyset \qquad \forall (R_1,\ldots,R_n) \in W^n(X)). \text{ It only remains to prove that it satisfies (P2) and (A2). Weak Pareto optimality is obvious by definition of the SCC: if <math>x \ P_i \ y \ \forall i \in N, \ by \ (P1) \ x \ P \ y \ and \ therefore \ y \not\in C(R_1,\ldots,R_n).$ 

To prove (A2), let us consider  $x,y \in X$  and  $(R_1,R_2,...,R_n)$ ,  $(\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  such that:  $C(R_1,R_2,...,R_n) = \{x\}$  and  $(R_1,...,R_n):\{a,y\} = (\bar{R}_1,\bar{R}_2,...,\bar{R}_n):\{a,y\},$  for every  $a \in A_y \cup \{x\}.$  Since  $y \notin C(R_1,...,R_n)$ , there exists  $z \in X$  such that z P y. If we show that  $z \in A_y$ , then as

$$(R_1,...,R_n):\{z,y\} = (\bar{R}_1,\bar{R}_2,...,\bar{R}_n):\{z,y\},$$

by applying (A1) it is obtained that  $z \bar{P} y$ , which in turn implies that  $y \notin C(\bar{R}_1, \bar{R}_2, ..., \bar{R}_n)$ . But as z P y, (A1) implies that  $z P^* y$ , where

$$R^* = F(R_1^{(z,y)}, R_2^{(z,y)}, \dots, R_n^{(z,y)})$$

and then  $y \notin C(R_1^{(z,y)}, R_2^{(z,y)}, \dots, R_n^{(z,y)})$ . So  $z \in A_y$ .

b. Conversely, if  $C:W^n(X) \longrightarrow X$  is a SCC which verifies (A2) and (P2), we can define a SDF  $F:W^n(X) \longrightarrow A(X)$  by

$$x \ R \ y \iff x \in C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)})$$

It is a complete binary relation since by (P2)

$$C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) \subset \{x,y\}$$

and by definition  $C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)}) \neq \emptyset$ . In order to prove that this relation is acyclic, let us consider  $x_1, x_2, \dots, x_p \in X$  such that  $x_1 P x_2, x_2 P x_3, \dots, x_{p-1} P x_p$ . Then

$$x_{k+1} \notin C(R_1^{(x_k, x_{k+1})}, R_2^{(x_k, x_{k+1})}, \dots, R_n^{(x_k, x_{k+1})}) = x_k$$

for all k = 1, 2, ..., p-1, and

$$(R_1^{(x_k, x_{k+1})}, \dots, R_n^{(x_k, x_{k+1})}) : (x_k, x_{k+1}) = (R_1^S, R_2^S, \dots, R_n^S) : (x_k, x_{k+1})$$

where  $S = \{x_1, ..., x_n\}.$ 

By the way in which  $(R_1^{(x_k, x_{k+1})}, R_2^{(x_k, x_{k+1})}, \dots, R_n^{(x_k, x_{k+1})})$  is defined,  $A_{x_{k+1}} = \{x_k\}$ ; therefore we can apply (A2) and we obtain that  $x_{k+1} \notin C(R_1^S, R_2^S, \dots, R_n^S)$  for all  $k = 1, 2, \dots, p-1$ . However, by applying (P2) we know that  $C(R_1^S, R_2^S, \dots, R_n^S) \subset S$ , so by (P2)  $C(R_1^S, R_2^S, \dots, R_n^S) = x_1$ . Thus, if we assume that  $x_p P x_1$ , then  $C(R_1^{(x_1, x_p)}, R_2^{(x_1, x_p)}, \dots, R_n^{(x_1, x_p)}) = x_p$  and by (A2) we would obtain that  $x_1 \notin C(R_1^S, R_2^S, \dots, R_n^S)$  which is a contradiction. Then R is an acyclic relation.

To prove that it verifies (P1), consider  $x,y\in X$  and  $(R_1,...,R_n)\in W^n(X)$  such that  $x P_i y \quad \forall i\in N;$  by considering  $(R_1^{(x,y)},R_2^{(x,y)},...,R_n^{(x,y)})$  we have that  $x P_i^{(x,y)} y \quad \forall i\in N,$  and by (P2) we obtain that

$$y \notin C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}),$$

so  $C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)}) = \{x\}$  and therefore  $x \ P \ y$ .

In order to show that (A1) is verified, let us consider  $x,y \in X$  and  $(R_1,R_2,...,R_n),\; (\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X) \text{ such that}$ 

$$(R_1, R_2, ..., R_n): \{x, y\} = (\bar{R}_1, \bar{R}_2, ..., \bar{R}_n): \{x, y\},$$

hence  $C(R_1^{(\mathbf{x},\mathbf{y})},R_2^{(\mathbf{x},\mathbf{y})},\ldots,R_n^{(\mathbf{x},\mathbf{y})}) = C(\bar{R}_1^{(\mathbf{x},\mathbf{y})},\bar{R}_2^{(\mathbf{x},\mathbf{y})},\ldots,\bar{R}_n^{(\mathbf{x},\mathbf{y})}).$  Thus, by the way we have defined R, x R y  $\iff$  x  $\bar{R}$  y.

In the following theorem we prove that monotonicity and neutrality conditions can be also transferred from one context to another.

#### Theorem 3.2.

- 1. Every SDF which verifies (A1), (P1) and (N1) defines a SCC that satisfies (A2), (P2) and (N2). Conversely every SCC which verifies (A2), (P2) and (N2) defines a SDF that satisfies (A1), (P1) and (N1).
- 2. Every SDF which verifies (A1), (P1) and (M1) defines a SCC that satisfies (A2), (P2) and (M2). Conversely every SCC which verifies (A2), (P2) and (M2) defines a SDF that satisfies (A1), (P1) and (M1).

#### Proof.

1. Let us consider a SDF which satisfies (A1), (P1) and (N1). From Theorem 3.1. we can define a SCC which verifies (A2) and (P2). So, we only need to show that it also verifies (N2). Consider  $(R_1, R_2, ..., R_n) \in W^n(X)$  and  $\sigma$  a permutation of X. If  $a \in C(R_1, R_2, ..., R_n)$ , by definition of C,  $a R z \forall z \in X$  and if we take  $x = \sigma(a)$  by applying (N1) we obtain that

$$x R_{\sigma} z \iff x \sigma(R) z \iff \sigma^{-1}(x) R \sigma^{-1}(z) \iff a R \sigma^{-1}(z) \quad \forall z \in X$$

so it is clear that

$$\begin{split} \mathbf{x} \; \in \; \mathbf{C} \bigg[ (\sigma(\mathbf{R}_1), \sigma(\mathbf{R}_2), \dots, \sigma(\mathbf{R}_n) \bigg] \; &\Longleftrightarrow \; \sigma^{-1}(\mathbf{x}) \; = \; \mathbf{a} \; \in \; \mathbf{C}(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n) \; \Longleftrightarrow \\ \\ & \iff \; \mathbf{x} \; \in \; \sigma \bigg[ \mathbf{C}(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n) \bigg] \end{split}$$

Conversely, if we have a SCC which verifies (A2), (P2) and (N2), by applying Theorem 3.1. we can define a SDF which verifies (A1) and (P1). In order to prove that it also verifies (N1), consider  $(R_1, R_2, ..., R_n) \in W^n(X)$ .

Since  $x R y \Leftrightarrow x \in C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)})$ , by applying (N2) we know that

$$\begin{split} &C(\sigma(R_1)^{\{x,y\}},\sigma(R_2)^{\{x,y\}},...,\sigma(R_n)^{\{x,y\}}) = \\ &= \sigma\bigg[C(R_1^{\{\sigma^{-1}(x),\sigma^{-1}(y)\}},R_2^{\{\sigma^{-1}(x),\sigma^{-1}(y)\}},...,R_n^{\{\sigma^{-1}(x),\sigma^{-1}(y)\}})\bigg], \end{split}$$

hence,

$$\begin{array}{lll} & x \ R_{\sigma} \ y & \Longleftrightarrow \ x \in C(\sigma(R_{1})^{(x,y)}, \sigma(R_{2})^{(x,y)}, ..., \sigma(R_{n})^{(x,y)}) & \Longleftrightarrow \\ \\ & \Leftrightarrow \ x \in \sigma \bigg[ C(R_{1}^{(\sigma^{-1}(x), \sigma^{-1}(y))}, R_{2}^{(\sigma^{-1}(x), \sigma^{-1}(y))}, ..., R_{n}^{(\sigma^{-1}(x), \sigma^{-1}(y))}) \bigg] & \Leftrightarrow \\ \\ & \Leftrightarrow \ \sigma^{-1}(x) \in C(R_{1}^{(\sigma^{-1}(x), \sigma^{-1}(y))}, R_{2}^{(\sigma^{-1}(x), \sigma^{-1}(y))}, ..., R_{n}^{(\sigma^{-1}(x), \sigma^{-1}(y))}) & \Leftrightarrow \\ \\ & \Leftrightarrow \ \sigma^{-1}(x) \ R \ \sigma^{-1}(y). \end{array}$$

2. Let us consider now a SDF which satisfies (A1), (P1) and (M1). By applying Theorem 3.1. we can define a SCC which verifies (A2) and (P2). So, we only need to show that it also verifies (M2). In order to do this, we take  $x,y \in X$  and  $(R_1,R_2,...,R_n)$ ,  $(\bar{R}_1,\bar{R}_2,...,\bar{R}_n) \in W^n(X)$  such that

$$C(R_1, R_2, ..., R_n) = \{x\}$$

and for every  $a \in A_y \cup \{x\}$ ,

$$\text{a } I_{i} \text{ } y \text{ } \Rightarrow \text{a } \bar{R}_{i} \text{ } y \quad \text{ and } \quad \text{a } P_{i} \text{ } y \text{ } \Rightarrow \text{a } \bar{P}_{i} \text{ } y$$

Since  $y \notin C(R_1, R_2, ..., R_n)$  and C has been defined by maximizing the SDF, there exists  $w \in X$  such that w P y. By considering now  $(R_1^{(w,y)}, R_2^{(w,y)}, ..., R_n^{(w,y)})$  and  $(R_1, R_2, ..., R_n)$  and by denoting  $F(R_1^{(w,y)}, R_2^{(w,y)}, ..., R_n^{(w,y)}) = R^*$ , if we apply (A1) it is obtained that  $w P^* y$ . Therefore

$$y \notin C(R_1^{(w,y)}, R_2^{(w,y)}, ..., R_n^{(w,y)}) = w,$$

which implies that  $w \in A_y$ ; by applying (M1) we have  $w \bar{P} y$ , which in turn implies  $y \notin C(\bar{R}_1, \bar{R}_2, ..., \bar{R}_n)$ .

Conversely, if we assume a SCC which satisfies (A2), (P2) and (M2), by Theorem 3.1. we know that there exists a SDF which verifies (A1) and (P1). Now we will show that it also verifies (M1). Consider  $x,y\in X$ , and  $(R_1,R_2,...,R_n)$ ,  $(\bar{R}_1,\bar{R}_2,...,\bar{R}_n)\in W^n(X)$  such that

$$x I_i y \Rightarrow x \bar{R}_i y; x P_i y \Rightarrow x \bar{P}_i y$$
 and  $x P y.$ 

Since x P y we know that  $C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)}) = \{x\}$ ; moreover, if we consider  $(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)})$  and  $(\bar{R}_1^{(x,y)}, \bar{R}_2^{(x,y)}, \dots, \bar{R}_n^{(x,y)})$ , since individual preferences between x and y are the same in

$$(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)})$$
 and  $(\bar{R}_1^{(x,y)}, \bar{R}_2^{(x,y)}, ..., \bar{R}_n^{(x,y)})$ 

than in  $(R_1, R_2, ..., R_n)$  and  $(\bar{R}_1, \bar{R}_2, ..., \bar{R}_n)$  respectively,

$$x I_{i}^{(x,y)} y \Rightarrow x \bar{R}_{i}^{(x,y)} y; x P_{i}^{(x,y)} y \Rightarrow x \bar{P}_{i}^{(x,y)} y$$

Moreover, if we consider any other alternative  $a \in X$ ,  $a \neq x,y$  we know that

$$y P_i^{(x,y)} a, y \bar{P}_i^{(x,y)} a \forall i \in N,$$

therefore, we can apply (M2) and obtain that

$$y \notin C(\bar{R}_{1}^{(x,y)}, \bar{R}_{2}^{(x,y)}, \dots, \bar{R}_{n}^{(x,y)}),$$

which implies by (P2) that  $C(\bar{R}_1^{(x,y)}, \bar{R}_2^{(x,y)}, \dots, \bar{R}_n^{(x,y)}) = x$ , that is  $x \bar{P} y$ .

In the next result, the relationship between the axioms of independence, neutrality and monotonicity for SCCs and for SDFs when Pareto properties are not assumed is proved.

#### Theorem 3.3.

a. Every SDF which verifies (A1), (M1) and (N1) defines a SCC that satisfies (A2), (M2) and (N2).

b. Conversely, every SCC which verifies (A2), (M2) and (N2) defines a SDF that satisfies (A1), (M1) and (N1).

## Proof.

a. As in Theorem 3.1 we define the SCC by maximizing the social preference relation associated to each profile and we prove that it verifies (A2). Moreover it is not difficult to prove, with a similar argument to that used in Theorem 3.2, that this SCC also satisfies (M2) and (N2).

b. Given a SCC which satisfies (A2), (M2) and (N2). We define a SDF  $F:W^n(X) \longrightarrow A(X) \text{ as follows:}$ 

$$x P y \iff C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) = \{x\}$$

Note that this definition is different from the one given in the previous Theorems. When the SCC satisfies (P2), as is the case of the former results, both definitions coincide. Now we complete the social preferences as usual: x R y if not [y P x]. We are going to prove that this SDF is acyclic and verifies (A1), (M1) and (N1).

To prove the acyclicity of this SDF let  $x_1, x_2, ..., x_p \in X$  such that  $x_1 P x_2, x_2 P x_3, ..., x_{p-1} P x_p$ . Then, by the way in which P has been defined,

$$C(R_1^{(x_{k+1})}, R_2^{(x_{k+1})}, R_2^{(x_{k+1})}, \dots, R_n^{(x_{k+1})}) = \{x_k\} \quad \forall k = 1, 2, \dots, p-1\}$$

By applying (M2) to profiles  $(R_1^{(x_k,x_{k+1})}, R_2^{(x_k,x_{k+1})}, \dots, R_n^{(x_k,x_{k+1})})$  and  $(R_1^S, R_2^S, \dots, R_n^S)$ , where  $S = \{x_1, x_2, \dots, x_p\}$ , we obtain that

$$x_{k+1} \notin C(R_1^S, R_2^S, ..., R_n^S) \quad \forall k = 1, 2, ..., p-1.$$

If we suppose that  $C(R_1^{(x_1,x_p)},R_2^{(x_1,x_p)},...,R_n^{(x_1,x_p)})=\{x_p\}$ , we have that  $x_1 \notin C(R_1^S,R_2^S,...,R_n^S)$ , therefore  $C(R_1^S,R_2^S,...,R_n^S) \in X$ -S. But if we consider an alternative  $z \in X$ -S and the profiles  $(R_1^{(x_1,x_p)},R_2^{(x_1,x_p)},...,R_n^{(x_1,x_p)})$  and  $(R_1^S,R_2^S,...,R_n^S)$ , by applying (M2) we will obtain that  $z \notin C(R_1^S,R_2^S,...,R_n^S)$ , which would imply that  $C(R_1^S,R_2^S,...,R_n^S)=\emptyset$ , a contradiction. Therefore we can conclude that

$$C(R_1^{(x_1,x_p)}, R_2^{(x_1,x_p)}, ..., R_n^{(x_1,x_p)}) \neq (x_p)$$

and so  $x_1 R x_p$ . Thus the relation is acyclic.

Now, and by following a similar argument to the one used in Theorems 3.1 and 3.2, it is not difficult to prove that this SDF verifies (A1), (M1) and (N1).

## 4. VETO EXISTENCE RESULTS IN FIXED AGENDA SCC.

Finally, and by making use of the results from the previous section, we prove the existence of *veto* for fixed agenda SCC. The first result we

present is the counterpart of the following Blau and Deb's result (1977). First we give the definition of a veto hierarchy.

#### Definition 4.1.

A partition  $V_1, V_2, ..., V_t$  of the set of individuals N is said to be a veto hierarchy if, disregarding order, it is satisfied that:

- 1. each member of  $V_1$  is a veto
- 2. each member of  $V_2$  is a veto when all in  $V_1$  are indifferent
- 3. each member of  $V_3$  is a veto when all in  $V_1$   $\cup$   $V_2$  are indifferent; etc.

## Theorem 4.1. (Hierarchy Theorem, Blau and Deb [1977])

If F is a SDF such that it verifies (A1), (M1) and (N1) and  $|X| \ge n$ , then there is a veto hierarchy.

The equivalent result in the context of fixed agenda SCC is as follows:

#### Theorem 4.2.

If C is a SCC such that it verifies (A2), (M2) and (N2) and  $|X| \ge n$ , then there is a veto hierarchy.

#### Proof.

By applying Theorems 3.3 and 4.1 we obtain the existence of a hierarchy of veto for the SDF defined by the SCC. We need to show that it is also a hierarchy of veto for the SCC. If we proved that an individual who is a veto for the SDF is also a veto for the SCC, we would obtain the result. But this is obvious since if individual i belongs to  $V_1$  and therefore has a veto for the SDF, then whenever  $x P_i$  y it is verified that x R y and by definition of R it implies that  $C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) \neq \{y\}$ . By

reasoning in the same way for  $V_2$ ,  $V_3$ ,... we would obtain the existence of the hierarchy of veto for the SCC.

Before translating the existence of veto for SDF obtained by Blair and Pollak (1982) to the context of fixed agenda SCC, we present the following proposition which states the relationship between the notion of *veto* in SDF

and SCC which both verify independence and weak Pareto conditions.

#### Proposition 4.1.

a. Let F be a SDF verifying (A1) and (P1) such that individual i is a veto, then individual i is also a veto in the SCC defined by F.

b. Conversely, if C is a SCC verifying (A2) and (P2) such that individual i is a veto, then individual i is also a veto in the SDF defined by C.

#### Proof.

a. Let F be a SDF which has a veto and verifies (A1) and (P1). We define a SCC from it (as in Theorem 3.1) by maximizing the social preference relation on X. Let individual i be veto for the SDF and assume that  $x P_i$  y, so x R y. If  $C(R_1^{(x,y)},R_2^{(x,y)},...,R_n^{(x,y)})=\{y\}$ , since  $x \notin C(R_1^{(x,y)},R_2^{(x,y)},...,R_n^{(x,y)})$  there exists an alternative  $z \in X$  such that  $z P^* x$ , where  $R^* = F(R_1^{(x,y)},...,R_n^{(x,y)})$ , but since by applying (P1) we know that  $x P^* t V \in X-\{x,y\}$ , then the only possibility is that  $y P^* x$ , which implies by (A1) that y P x, a contradiction.

b. Let C be a SCC which verifies (A2) and (P2) such that individual i is a veto. We define the SDF as in Theorem 3.1:

$$x R y \iff x \in C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)})$$

It is obvious that i is veto for this SDF, since if x P, y, then

$$C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) \neq \{y\},$$

but by applying (P2),  $C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)}) \subseteq \{x,y\}$ , therefore

$$x \in C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)})$$

which implies that x R y.

Now we present the counterpart to the following Blair and Pollak's result (1982).

## Theorem 4.3. [Blair and Pollak, 1982]

If  $|X| = \alpha > n$ , under every SDF which satisfies (A1), (P1) and (N1), there exists a veto.

## Theorem 4.4.

If  $|X| = \alpha > n$ , under every SCC which satisfies (A2), (P2) and (N2), there exists a veto.

## Proof.

Let  $C: W^n(X) \longrightarrow X$  be a SCC which satisfies (A2), (P2) and (N2). By applying Theorem 3.2. we know that there exists a SDF  $F: W^n(X) \longrightarrow A(X)$  which verifies (A1), (P1) and (N1) defined as follows:  $x \ R \ y \iff x \in C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)})$ . By applying Theorem 4.3 it is

obtained that there exists a veto for it. Hence if individual i has a veto, whenever  $x \ P_i$  y it is verified that  $x \ R$  y, but by definition of R it implies that  $x \in C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)})$ . Therefore we can conclude that  $C(R_1^{(x,y)}, R_2^{(x,y)}, \dots, R_n^{(x,y)}) \neq \{y\}$ .

Finally, we translate some results of Blair and Pollak (1982) in which the existence of an individual who is veto over a subset of X, but not for every alternative in X, is stated. Thus, in this case, there does not exist a symmetric treatment of alternatives.

## Theorem 4.5. [Blair and Pollak, 1982]

If  $|X| = \alpha > n$  and  $\alpha \ge 4$ , then for every SDF which satisfies (A1) and (P1) there exists at least one individual who is veto over at least  $(\alpha-n+1)(\alpha-1)$  pairs of alternatives.

## Theorem 4.6. [Blair and Pollak, 1982]

If  $|X| = \alpha \ge 4n$ , then for every SDF which satisfies (A1), (P1) and (M1) there exists at least one individual who is veto over at least  $[\alpha-4(n-1)](\alpha-1)$  pairs of alternatives.

In order to translate these results to fixed agenda social choice correspondences, we need to define the notion of veto over a subset of alternatives<sup>(3)</sup> for a fixed agenda SCC. However, it is easily done by

<sup>&</sup>lt;sup>3</sup> In this case some authors have called it *individual semi-decisive* over that subset of alternatives.

restricting the definition of veto only to the subset of alternatives which are going to be vetoed, as follows:

#### Definition 4.1.

Let C be a fixed agenda social choice correspondence, individual i is veto or semi-decisive over (x,y) if and only if  $x P_i$  y implies that

$$C(R_1^{(x,y)}, R_2^{(x,y)}, ..., R_n^{(x,y)}) \neq \{y\}.$$

It is important to note that, especially in this case, (when the individual does not have veto power over all of the alternatives) the definition of veto used by Denicolò<sup>(4)</sup> has no sense as the following example shows.

## Example 4.1.

Let us consider the same SDF as the one we used in Example 2.1. but with the set of alternatives given by  $X = \{x,y,w,z\}$  (since we are going to apply Theorem 4.5.). It is easy to prove that it is a SDF which verifies (A1), (P1) and (M1) (Blair and Pollak, 1982). Therefore we can apply Theorem 4.5. and obtain the existence of an individual who is veto over at least 9 pairs of alternatives (in fact, both individuals are veto over that number of pairs). However we are going to show that, in general, the said individual is not a veto (in the sense of Denicolò) over these pairs of alternatives in the associated fixed agenda SCC. Consider the SCC  $C:W^n(X) \longrightarrow X$  given by maximizing  $F(R_1,R_2)$  and the following profile:

The notion of veto used by Denicolò (1993) is as follows: an individual i is a veto for the SCC if  $x > P_i y$  implies  $C(R_1, R_2, ..., R_n) \neq \{y\}$ 

$$R_1$$
:  $x P_1 y P_1 w P_1 z$   
 $R_2$ :  $w P_2 x P_2 y P_2 z$ 

The social (acyclic) preference relation is given by:

and then  $C(R_1,R_2)=\{x\}$ . Note that, by the way in which the SDF has been defined, individual 2 is a veto for the SDF over the pair (w,x); however he is not a veto in the sense of Denicolò for the SCC, since  $w P_2 x$  but  $C(R_1,R_2)=\{x\}$ .

However if we consider the notion of veto we have defined, then  $C(R_1^{(x,w)},R_2^{(x,w)},...,R_n^{(x,w)}) = \{w,x\} \neq \{x\} \text{ and in this case individual 2 is a veto over the pair } (w,x) \text{ for the SCC.}$ 

Now a similar equivalence result to the one presented in Proposition 4.1 can be stated in terms of this notion of veto over pairs of alternatives. The proof is analogous to that of Proposition 4.1, so it is omitted here.

#### Proposition 4.2.

a. Let F be a SDF verifying (A1) and (P1) such that individual i is veto for x against y, then individual i is also a veto over (x,y) in the SCC defined by F.

b. Conversely, if C is a SCC verifying (A2) and (P2) such that individual i is veto over (x,y), then individual i is also a veto over (x,y) in the SDF defined by C.

The following two results are the straightforward translation of Theorems 4.5 and 4.6. respectively. Since the proof is done by reasoning in the same way as the previous results, it is omitted.

#### Theorem 4.7.

If  $|X| = \alpha > n$  and  $\alpha \ge 4$ , then for every SCC which satisfies (A2) and (P2) there exists at least one individual who is veto over at least  $(\alpha-n+1)(\alpha-1)$  pairs of alternatives.

#### Theorem 4.8.

If  $|X| = \alpha \ge 4n$ , then for every SCC which satisfies (A2), (P2) and (M2), there exists an individual who is veto over at least  $[\alpha-4(n-1)](\alpha-1)$  pairs of alternatives.

## 5. FINAL COMMENTS

In this paper we have introduced a weak notion of independence for a social choice correspondence which allows us to translate most results of existence of veto for acyclic social decision functions to the context of fixed agenda social choice correspondences. To do this, we have also introduced a notion of veto for SCC which turns out to be equivalent to the usual notion of veto in SDF in the context of acyclic social preferences.

Most of the results which ensure the existence of veto for social decision functions need to assume that there are more alternatives than

individuals. However there are other results in which it is assumed that the number of individuals is greater than the number of alternatives and which prove the existence of coalitions which have veto power. In particular we have to mention an extension of Blau and Deb's results and Blair and Pollak's results obtained by Kelsey (1985). On the one hand Kelsey proves that if |X| > t, where  $G_1, G_2, ..., G_t$  is a partition of N of disjointed groups and we have a SDF which satisfies (A1), (N1) and (M1), then there exists b such that  $G_b$  has a veto. On the other hand he proves that, under the same conditions but by requiring the SDF to satisfy (A1) and (P1), there exists b such that  $G_b$  is semi-decisive over at least  $\left(|X|-t-1\right)\left(|X|-1\right)$  pairs of alternatives. Both results could be translated to the context of fixed agenda social choice correspondences by defining the notion of group veto in this context: ASN has a veto if for every x,yeX whenever  $xP_1y$   $\forall i \in A$  implies that  $C(R_1^{(x,y)},...,R_n^{(x,y)}) \neq \{y\}$ . Thus the results we would obtain are as follows:

"Let  $C: W^n(X) \longrightarrow X$  a SCC such that  $G_1, G_2, ..., G_t$  is a partition of N into t disjointed subgroups such that  $|X| \ge t$  and C satisfies (A1), (N1) and (M1), then there exists b such that  $G_b$  has a veto"

"Let  $C:W^n(X) \longrightarrow X$  a SCC such that  $G_1, G_2, \dots, G_t$  is a partition of N into t nonempty disjoint subgroups such that  $|X| \ge t$  and C satisfies (A1) and (P1), then there exists b such that  $G_b$  is semidecisive over at least  $\Big(|X|-t-1\Big)\Big(|X|-1\Big)$  pairs of alternatives"

Apart from this we have to note that Mas-Colell and Sonnenschein (1971) have a result which proves the existence of an individual with veto for SDFs

which verifies (A1), (P1) and a very strong monotonicity condition (they call it *positive responsiveness*) which has been criticized by many authors, and a result of group vetos by weakening this condition. They could also be translated by defining the counterpart to these assumptions in the fixed agenda context.

In any case, it is important to note that, although the set of alternatives is restricted to be always the whole space (fixed agenda), the results of existence of veto are exactly the same in this case as in the case of considering that the social choice correspondence operates on many different subsets of the universal set of alternatives.

To sum up all of the results which have been obtained we present the following diagram:

# SUMMARY OF AXIOMS AND EQUIVALENCE RESULTS

AXIOMS	ACYCLIC SDF	FIXED AGENDA SCC
Pareto	(P1)	(P2)
I nde pendence	(A1)	Independence [Denicolò, 1993]
		Quasi-Independence [Denicolò, 1993]
		(A2) [Weak-Independence]
Monotonicity	(M1)	(M2)
Neutrality	(N1)	(N2)

EQUIVALENCE RESULTS	
ACYCLIC SDF+(P1)+(A1) $\equiv$ SCC+(P2)+(A2)	[Theorem 3.1]
ACYCLIC SDF+(P1)+(A1)+(M1) $\equiv$ SCC+(P2)+(A2)+(M2)	[Theorem 3.2]
ACYCLIC SDF+(P1)+(A1)+(N1) = SCC+(P2)+(A2)+(N2)	[Theorem 3.2]
ACYCLIC SDF+(A1)+(M1)+(N1) $\equiv$ SCC+(M2)+(A2)+(N2)	[Theorem 3.3]
$Q-SDF+(P1)+(A1) \equiv SCC+(P2)+(Quasi-Independence)$	[Denicolò, 1993]
$SWF+(P1)+(A1) \equiv SCC+(P2)+(Independence)$	[Denicolò, 1993]

A. D	[D 1 1) 1000]
Arrow-Dictatorial	[Denicolò,1993]
Gibbard-Oligarchy	[Denicolò,1993]
Blau and Deb-Veto Hierarchy	[Theorem 4.2]
Blair and Pollak (1)-Global Vetoer	[Theorem 4.4]
Blair and Pollak (2)-Veto	[Theorem 4.7]
Blair and Pollak (3)-Veto	[Theorem 4.8]
Kelsey-Group Veto	[Final comments]

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