DUOPOLY EXPERIMENTATION:
COURNOT AND BERTRAND COMPETITION[*]

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ABSTRACT

This paper analyzes how learning behaviour can modify the outcome of competition in an industry facing demand uncertainty. We consider a duopoly game where firms have imperfect information about market demand and learn through observing prices (Cournot competition) or sales (Bertrand).

The main body of the paper consists in showing how duopoly experimentation is affected by the type of market competition. We find that, if the goods are substitutes, firms will experiment more under Bertrand than under Cournot. If the goods are complements, the result is reversed. Furthermore, there is less experimentation under Cournot with product substitution.

1.- INTRODUCTION.

It is well known that in an uncertain context firms may devote much effort to the acquisition of information. This improvement in the information may have a favourable impact on a firm's profitability and competitive advantage in the marketplace. In this paper we study an aspect of the acquisition of information and its effect on firm behaviour known as experimentation, that is, changes in present actions induced by a desire to vary the amount of information available in the future. We deal with a multifirm setting where a strategic dimension is introduced: by varing its present actions a firm not only affects the amount of information available to it, but also affects the inferences drawn by rival firms. This paper focuses on the complications introduced by these strategic considerations. In particular, we are interested in analyzing how the incentives to experiment depend on the type of competition prevailing in the market, i.e. quantity (Cournot) and price (Bertrand) competition.

We propose to examine these questions in the context of a two-stage game. Since we want to compare firms' experimental behaviour in both quantity and price competition we set up two models. To analyze Cournot competition we consider a very simple nondifferentiated duopoly. This simple model highlights the intuition behind the incentives to experiment. Also, it allows us to extend the results to the cases when the firm's prior information is not common, which help us to understand strategic experimentation better. Thus, our Cournot model deals with two firms that face a lineal demand function

whose slope parameter is both a random variable as well as random noise. The random slope has an unknown mean, and the noise term masks the true value of the mean of this parameter. Each firm is assumed to have prior beliefs about this mean that are common and common knowledge.

In period one, each firm chooses a level of production. A market price is then realized. We assume that both firms observe this price as well as industry output. This information is used to update prior beliefs, yielding posterior beliefs that will be used in the second period. Since both firms use the same signal - namely the price realization - to revise beliefs, any attempt to vary the informative nature of this signal must take into account the effects of such action on the beliefs of rivals (since such beliefs affect future expected profits).

To better analyze Bertrand competition we study a symmetric differentiated duopoly where firms set up prices, and where stochastic linear demands are as above, i.e. the slopes are random variables, but we keep the spirit as close as possible to the Cournot model. In particular, we assume that the firms observe the same signal, average industry sales, as well as average prices. We deal mainly with product substitution, but we extend the analysis to cover complements. Due to the symmetry and linearity of the demand structure, Bertrand competition with substitute (complement) products is the perfect dual of Cournot competition with complements (substitutes), (see Vives, 1984). We thus compare Bertrand and Cournot under product differentiation, and relate Cournot experimentation with the degree of product substitution.

A hypothesis we want to explore is whether better informed firms are nicer or more unpleasant competitors. We would also like to know, if an improvement of the information is always best for the firms, i.e., if the value of information is positive to the firms, and under which type of competition (Bertrand and Cournot) it is higher.

Experimentation has been thoroughly analyzed for single agent decision problems. Mirman, Samuelson and Urbano (1989 a, henceforth MSU), for example, recently investigated experimentation in a monopoly model. They study a model with an uncertain demand function containing a parameter (with two possible values) which is unknown to the monopolist. In this case all observations lead to partial learning about the true value of the unknown parameter. The monopoly makes output decisions in the first period of a two period horizon model. Each output decision implies two possible price distributions. Observations of the price leads to Bayesian updating about the unknown parameter.

By experimenting a firm increases the informative content of its observations of, e.g. price. However, in order to do that, it must give up present period profits. In our duopoly setting firms affect the informative nature of the commonly observed signal (where "more informative" and "less informative" are defined in the sense of Blackwell's (1951) comparison of experiments), thus they may affect the degree to which a rival is likely to update[1]. Since the strategic informative choice that firms face is how

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[1] The models probably closest in spirit to one of duopoly experimentation are the "signal-jamming" models of Riordan (1985), Fudenberg and Tirole (1986),

informative to make the publicly observed signal, an important determinant of firm behaviour is whether future expected profits increase with an increase of information. It is well known that, in contrast to a single agent problem, an increase in information in a game may make some players be worse off. We demostrate that this cannot happen in models like ours, where the demand structure is linear and a priori information is common.

The value of information in oligopoly games has been the subject of intensive research. These studies typically assume either that the firms transmit information by means of "certifiable announcements" or that the signals that yield information to the firms are generated exogenously. Our model differs from these in that the amount of information generated is determined endogenously by the choice of actions in the first period. The model closest to ours is the duopoly experimentation model of Aghion, Espinosa, and Jullien (1988). However, their main concern is to explain the existence of price dispersion in an oligopolistic industry. To get this result they have to assume that the degree of product substitution is unknown by the duopolists.

Our first concern in this paper is how experimentation affects the first period decisions of the firms in a subgame perfect equilibrium of a two period

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and MSU (1989,b). In these models, like ours, no firm is perfectly informed about the state of nature and each firm may have an incentive to manipulate the inferences drawn by rival firms. The principal difference between these signal-jamming models and ours lies in the assumption made about the observability of actions: in signal-jamming models, firms do not know - even ex-post - the action chosen by rivals. Therefore, firms attempt to influence the direction in which a rival updates its beliefs.

game. We contrast the behaviour of the duopolists under Cournot and under Bertrand with the experimenting monopolist in MSU (1989, a).

We find that the incentives to experiment in a duopoly depend on the informative properties of market competition. On one hand we have the incentives to revise a firm's own beliefs. This is similar to the monopoly's incentives. But we also have the incentive to influence revisions of the rival firms' beliefs. This, in turn, depends on both: how information and competition are related in the market, i.e., if a better informed firm is a "nicer" or a "more unpleasant" competitor, and how this relationship affects a firm's own revision, i.e., if firms are informationally substitutes or complements. Given our lineal demand setting the first incentive is the same under Cournot as under Bertrand. However, the strategic incentives depend on the type of competition (quantity or price) and on the degree of product substitution.

Vives (1988) has shown that in an uncertain context, where firms try to gain a competitive advantage via strategic investments, an improvement in the information precision of the firm has always had a favourable impact on its competitive position. The rivals are hurt, if the market is characterized by quantity competition, but favoured, if the price competition prevails.

Our model differs from Vives' model in that our signals are endogenous and that the firms' investment in information, i.e. strategic experimentation, also takes endogenously into account the relationship and spillover effects between information and competition. However, our findings can be taken as

the experimenting behaviour counterparts of its strategic investments.

Thus, our basic results are that under Bertrand competition (with product substitutes) a better informed firm is a "prefered" competitor. Moreover, firms are informationally complements so that the experimentation that they undertake is higher than under monopoly. This is translated to best responses that represent higher prices than the "myopic" or short run reaction functions. However, under Cournot competition (with product substitutes) a better informed firm is disliked, and are also informationally substitutes so that firms experiment less than under monopoly (although they always do it). Therefore, we find that firms' experimenting behaviour is more aggressive under Bertrand than under Cournot. Moreover the experimentation under Cournot is less intense the higher the degree of product substitution. Hence, Cournot competition with homogeneous product gives us the smaller level of experimentation in our duopoly setting.

The plan of the paper is as follows. The Cournot model is considered in Section 2 where the basic demand and information structure assumptions are first laid down. Analysis of the informative characteristic and the experimenting behaviour of firms follows. Extensions to the different a priori case are discussed. Section 3 deals with the Bertrand model. It analyzes the value of information for this model. Finally, the main results, i.e. comparison of experimentation in both Cournot and Bertrand models, are considered, as well as their relation with the degree of product substitution. Concluding remarks follow.

2..- THE COURNOT MODEL.

We consider a two period duopoly model. The firms produce a nondifferentiated product over the two periods. The inverse stochastic demand function is given by $P_t = a - \theta Q_t + \varepsilon_t$, $t = 1,2$, where P_t is the market price in the period t, Q_t is the industry output, θ is the slope parameter that is unknown by the firms and ε_t is the realization in period t of a random variable $\tilde{\varepsilon}$. These error terms are i.i.d. and independent of the parameter θ.

Firms assume that θ and ε are normally distributed. Since it is more convenient for our purposes to represent distributions by their means and their precisions, let $\varepsilon \sim N(0,\tau)$; where $E(\varepsilon) = 0$ and $\tau = \dfrac{1}{\sigma_\varepsilon^2}$ is the precision of ε and σ_ε^2 is its variance. In like fashion, let the agents initial beliefs about θ be described by the normal distribution $\theta \sim N(\theta,h)$, where $E(\theta) = m_i > 0$, and $h = \dfrac{1}{\sigma_\theta^2}$ is its precision and σ_θ^2 is its variance. These prior beliefs are revised as firms gather more information. Each $m_i \in [\underline{m}, \overline{m}] \subset \mathbb{R}_+$ with $\overline{m} < 2\underline{m}$ in order to ensure interior solutions. Although we will assume that prior beliefs are common, we will keep them notationally different for the most part of the analysis, in order to highlight the intuition of our results. Moreover this will allow us to consider $m_1 \neq m_2$ in the extensions.

Firms choose outputs to maximize the sum of the two period expected profits. For simplicity, we assume that production cost is zero (or that P is the net price received after substracting a constant marginal cost that is the

same for both firms), so that firm i's current expected profits in period one are:

$$\pi_i(Q_{1,}Q_2;m_i) = E\left[a - \theta(Q_1+Q_2) + \varepsilon\right]Q_i = \left(a -m_i(Q_1+Q_2)\right)Q_i \qquad (1)$$

The flow of information and the updating procedure.

After the first period quantities are chosen, values of both $\tilde{\varepsilon}$ and θ, and therefore a value of P, are realized. We assume that firms observe first period price and aggregate output, but not the realization of both $\tilde{\varepsilon}$ and θ. Consequently firms may not be able to determine the value of the mean of θ after the first period. However, the observation of P, together with knowledge of the aggregate output, leads each firm to revise its beliefs regarding the mean of the random variable θ. We assume that such revision proceeds according to the Bayesian procedure for the conjugate family of distributions of the signals that are observed.

In particular, after the first period firms observe the signal:

$$\frac{a - P}{Q_1+Q_2} = \theta - \frac{\varepsilon}{Q_1+Q_2} \qquad (2)$$

That for each value of θ is distributed as

$$\frac{a - P}{Q_1+Q_2} \sim N(\theta,\tau(Q_1+Q_2)^2) \qquad (3)$$

where θ is the mean, and $\tau(Q_1+Q_2)^2$ is the precision.

The value of θ is unknown by the firms, but they have a prior distribution on it, that is a normal distribution with mean $m_i > 0$, and precision $h > 0$, i.e.

$$\theta \sim N(m_i,h)$$

Then, the posterior distribution of θ (see DeGroot, 1970, page 167), after seeing the signal $\left(\dfrac{a - P}{Q_1 + Q_2}\right)$ is a normal distribution with mean $\hat{m}_i$ and precision $\hat{h}$, where[2]

$$\hat{m}_i = \frac{hm_i + \tau(Q_1+Q_2)^2\left[\dfrac{a - P}{Q_1 + Q_2}\right]}{h + \tau(Q_1+Q_2)^2} = \frac{hm_i + \tau(Q_1+Q_2)(a - P)}{h + \tau(Q_1+Q_2)^2} \quad (4)$$

and

$$\hat{h} = h + \tau(Q_1+Q_2) \quad (5)$$

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[2] Note that given our normality assumptions, the signal and the updated beliefs, $\hat{m}_i$, may take negative values. Firms are constrained to choose positive prices and quantities. For convenience, we ignore this and, given the firm's strategies that we derive, we can get negative prices and outputs for certain combinations of the signal and $\hat{m}_i$. The probability of such an event can be made arbitrarily small by appropriately choosing the variances of the model.

Now, in period two, each firm again chooses a level of production Q_i, and as in (1), current expected profits are $\pi_i(Q_1,Q_2;\hat{m}_i)$ (where Q_i here denotes second period output for firm i and $\hat{m}_i$ is calculated from (4)).

We are particularly interested in the subgame perfect equilibrium of this two period game. As usual, we analyze subgame perfect equilibria by transforming the two-period game into a one-period game by specifying a value function for each firm, giving period two expected profits as a function of posterior beliefs. Therefore we deal first with the second stage equilibria, Bayesian-Nash equilibria in our case.

Suppose that the posterior $\hat{m}_1$ and $\hat{m}_2$ are given. (We consider $\hat{m}_1$ and $\hat{m}_2$ distinct to make the analysis more transparent. Under the common prior assumption $\hat{m}_1 = \hat{m}_2$ which will be used when necessary). The second period problem for firm 1 is:

$$\underset{Q_1}{Max}\left[\pi(Q_1,Q_2;\hat{m}_1)\right] = E\left[(a - \theta(Q_1+Q_2) + \varepsilon)Q_1\right] = \left(a - \hat{m}_1(Q_1+Q_2)Q_1\right) \qquad (6)$$

and for firm 2:

$$\underset{Q_2}{Max}\left[\pi(Q_1,Q_2;\hat{m}_2)\right] = E\left[(a - \theta(Q_1+Q_2) + \varepsilon)Q_2\right] = \left(a - \hat{m}_2(Q_1+Q_2)Q_2\right) \qquad (7)$$

giving the system:

$$a - 2\hat{m}_1Q_1 - \hat{m}_1Q_2 = 0$$
$$a - 2\hat{m}_2Q_2 - \hat{m}_2Q_1 = 0$$

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and hence the equilibrium outputs are unique and equal to:

$$Q_1^* = \frac{a(2\hat{m}_2 - \hat{m}_1)}{3 \ \hat{m}_1\hat{m}_2} \tag{8}$$

$$Q_2^* = \frac{a(2\hat{m}_1 - \hat{m}_2)}{3 \ \hat{m}_1\hat{m}_2} \tag{9}$$

With $Q_1 > 0$ and $Q_2 > 0$, since $\hat{m}_1 = \hat{m}_2$ (In general note the $\overline{m} < 2\underline{m}$ so that $Q_1 > 0$, $Q_2 > 0$).

Inserting (8) and (9) in (6) and (7) we get the maximum second period expected profits as a function of posterior beliefs. Let as call $V_i(\hat{m}_1,\hat{m}_2)$ to this maximum, hence:

$$V_i(\hat{m}_1,\hat{m}_2) = \left[a - \hat{m}_i Q_1^*(\hat{m}_1,\hat{m}_2) - \hat{m}_i Q_2^*(\hat{m}_1,\hat{m}_2) \right] Q_i^*(\hat{m}_1,\hat{m}_2) \tag{10}$$

is the value function for firm i, that expresses the maximum second period profits as a function of posterior beliefs.

Lemma 1: $V_i(\hat{m}_1,\hat{m}_2)$ is a decreasing function of $\hat{m}_i$, and an increasing function of $\hat{m}_j$.

Proof: It is clear from (10). For example take $V_1(\hat{m}_1,\hat{m}_2)$. From (10)

$$\frac{\delta V_1}{\delta \hat{m}_1} = \frac{-a^2}{9\hat{m}_1^2 \hat{m}_2^2}\left[4\hat{m}_2^2 - \hat{m}_1^2\right] < 0$$

$$\frac{\delta V_2}{\delta \hat{m}_2} = \frac{2a^2}{9\hat{m}_2^3}\left[2\hat{m}_2 - \hat{m}_1\right] > 0$$

The intuition is that a higher value of $\hat{m}_i$, means a "bad" mean of the stochastic demand, therefore firm i will be better off the smaller the value of $\hat{m}_i$ is. In like fashion, firm i will be better off if firm j thinks that the mean of the stochastic demand is high (high $\hat{m}_j$), since in this case firm j will produce less, allowing firm i to produce more.

Now, in period one, the posterior beliefs $(\hat{m}_1, \hat{m}_2)$ are not known, but rather are random variables whose distribution depends upon first period industry output (as well as the distribution of P induced by ε and θ). We may therefore write each firm's two period expected profit as a function of first period output:

$$\Pi_i(Q_1, Q_2; \hat{m}_1, \hat{m}_2) = E\left[(a - \theta(Q_1 + Q_2) + \varepsilon)Q_i\right] + E\left[V_i(\hat{m}_1(P,Q), \hat{m}_2(P,Q))\right] =$$

$$\Pi_i(Q_1, Q_2, m_i) + E\left[V_i(\hat{m}_1(P,Q), \hat{m}_2(P,Q))\right] \tag{11}$$

where $Q = Q + Q$ is aggregated output, and where:

$$E\left[V_i(\hat{m}_1(P,Q), \hat{m}_2(P,Q))\right] = \int_{-\infty}^{+\infty} V_i(\hat{m}_1(P,Q), \hat{m}_2(P,Q)) f(a-P) \, dP \tag{12}$$

where f is the density of (a - P). Note that since:

$$a - P = \theta(Q_1 + Q_2) - \varepsilon$$

Then, for each $Q = Q_1 + Q_2$, firm i sees (a - P) distributed:

$$(a-P) \sim N\left(m_i(Q_1+Q_2), \frac{h\tau}{h + \tau(Q_1+Q_2)^2}\right)$$

where $E(a-P) = m_i(Q_1+Q_2)$, and $\dfrac{h\tau}{h + \tau(Q_1+Q_2)^2}$ is the precision.

Therefore

$$f(a-P) = \frac{\left(\dfrac{h\tau}{h + \tau(Q_1+Q_2)^2}\right)^{1/2}}{\sqrt{2\pi}} e^{-1/2\left(\dfrac{h\tau}{h + \tau(Q_1+Q_2)^2}\right)\left(a-P-m_i(Q_1+Q_2)\right)^2} \quad (13)$$

Then, the problem that firm i has to solve in period one is to choose a $Q_i \in \underset{Q_i}{\text{Argmax}} \; \pi_i(Q_1,Q_2; m_1,m_2)$. Let $\varphi_i(Q_j) = \{Q_i \in \underset{Q_i}{\text{Argmax}} \; \pi_i \; (Q_1,Q_2; m_1,m_2)\}$ denote firm i's best reply correspondence. A Bayesian-Nash equilibrium for the first stage of the game is a pair $(Q_1^e, Q_2^e) \in \varphi_1(Q_2^e) \times \varphi_2(Q_1^e)$ and posteriors $\hat{m}_1$, $\hat{m}_2$ (with $\hat{m}_1 = \hat{m}_2$), that are consistent, i.e. that come from Bayesian updating. Any such equilibrium will be in the equilibrium path of a subgame equilibrium of the two-period game.

Let $Q_i^m \in \text{Argmax} \; \pi_i(Q_1,Q_2,m_i)$, denote firm i's "myopic" or short run choice, and $Q_i^m(Q_j)$ be the "myopic" reaction function.

First Period Best Reply Mappings.

To analyze the behaviour of firms when they take into account the effect of first period choices on the flow of information for period two, we compare the best reply mapping $\varphi_i(Q_j)$ with its "myopic" reaction function $Q_i^m(Q_j)$. This "myopic" reaction function ignores such effects, therefore a comparison of the two best reply mappings will determine whether or not the presence of such information flows lead a firm to increase or decrease first period output (for a given output level of the other firm).

First Order Conditions:

Note that the first order condition for the "myopic" best reply sets up current marginal profits equal to zero, i.e

$$\frac{\delta \pi_i(Q_1, Q_2, m_i)}{\delta Q_1} = 0,$$

meanwhile the first order condition for the dynamic best reply is by (11):

$$\frac{\delta \pi_i(Q_1, Q_2; m_i)}{\delta Q_i} + \frac{\delta}{\delta Q_i} E\left[V_i(\hat{m}_1(P,Q), \hat{m}_2(P,Q))\right] = 0 \qquad (14)$$

so that :

$$\frac{\delta \pi_i(Q_1, Q_2; m_i)}{\delta Q_i} \overset{>}{_<} 0$$

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depending on

$$\frac{\delta}{\delta Q_i} E\left[V_i(\hat{m}_1,\hat{m}_2)\right] \underset{<}{^>} 0 \tag{15}$$

Moreover, since $\pi_i(Q_1,Q_2,m_i)$ is strictly concave on Q_i, (15) means that,

$$\varphi_i(Q_j) \underset{<}{^>} Q_i^m(Q_j)$$

depending on

$$\frac{\delta}{\delta Q_i} E\left[V_i(\hat{m}_1,\hat{m}_2)\right] \underset{<}{^>} 0 \tag{16}$$

for any element of $\varphi_i(Q_j)$.

We now proceed to derive the sign of $\dfrac{\delta}{\delta Q_i} E\left[V_i(\hat{m}_1,\hat{m}_2)\right]$

Consider this problem for firm 1, and let:

$$\Gamma_1(Q_1) = E\left[V_1(\hat{m}_1,\hat{m}_2)\right] = \int_{-\infty}^{+\infty} V(\hat{m}_1(P,Q), \hat{m}_2(P,Q)) \, f(a-P) \, dP \tag{17}$$

Value Function.

We are interested in the sign of $\dfrac{\delta\Gamma_1}{\delta Q_1}$ that will tell us whether the presence of information flows lead firm 1 to increase, decrease or leave

unaltered its period one best response respect to its "myopic" counterpart.

To facilitate the interpretation of the results, let us keep on defining $\hat{m}_1$ and $\hat{m}_2$ distinct. By (4) we have that:

$$\hat{m}_1 = \frac{m_1 h + \tau(Q_1 + Q_2)(a-P)}{h + \tau(Q_1 + Q_2)^2} \tag{18}$$

$$\hat{m}_2 = \frac{m_2 h + \tau(Q_1 + Q_2)(a-P)}{h + \tau(Q_1 + Q_2)^2} \tag{19}$$

Next, to simplify our calculations let us express $\hat{m}_2$ as a function of $\hat{m}_1$. By (18) and (19)

$$\hat{m}_2 = \hat{m}_1 + \frac{h(m_2 - m_1)}{h + \tau(Q_1 + Q_2)^2} \tag{20}$$

hence $\hat{m}_2 = \gamma(\hat{m}_1)$, with $\gamma'(\hat{m}_1) = 1$, i.e. $\hat{m}_2$ and $\hat{m}_1$ have a perfectly positive correlation. Therefore we can express the value function $V_1(\hat{m}_1, \hat{m}_2)$ - that depends on $\hat{m}_1$ and $\hat{m}_2$ - as a new value function that only depends on $\hat{m}_1$. Define this new value function $W_1 : [\underline{m}, \overline{m}] \to \mathbb{R}_+$ by

$$W_1(\hat{m}_1) = V_1(\hat{m}_1, \gamma(\hat{m}_1)) \tag{21}$$

so that

$$\Gamma_1(Q_1) = \int_{-\infty}^{+\infty} W_1(\hat{m}_1(P,Q)) \, f(a-P) \, dP \tag{22}$$

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where $Q = Q_1 + Q_2$.

Next, differentiating (22) with respect to Q_1 yields

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int \left[W_1'(\hat{m}_1) \frac{\delta \hat{m}_1}{\delta Q_1} \ f(a\text{-}P) + W_1(\hat{m}_1) \frac{\delta}{\delta Q_1} \ f(a\text{-}P) \right] dP \qquad (23)$$

The manipulation of (23) is quite long and tedious, so that it is relegated to the Appendix. There, we show that (23) can be expressed in the following convenient form :

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int \frac{1}{\hat{h}} \ W_1''(\hat{m}_1) \left(-\frac{\delta \hat{m}_1}{\delta P} \right) \ f(a\text{-}P) \ dP \qquad (24)$$

At first glance, (24) seems the random slope counterpart of the experimenting monopoly of MSU (1989,a). But this is not the case, since the function $W_1(\hat{m}_1)$ is not just a single decisor function. In fact:

$$W_1(\hat{m}_1) = V_1(m_1, \gamma(\hat{m}_1)),$$

then

$$W_1''(\hat{m}_1) = \frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1^{\ 2}} + 2 \frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1 \delta \hat{m}_2} \gamma'(m_1) + \frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_2^{\ 2}} (\gamma'(m_1))^2$$

and since $\gamma'(\hat{m}_1)=1$, (24) is

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int \frac{1}{\hat{h}} \left[\frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1^{\ 2}} + 2 \frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1 \delta \hat{m}_2} + \frac{\delta^2 V_1(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_2^{\ 2}} \right]$$

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$$\left(\frac{\delta \hat{m}_1}{\delta P}\right) f(a - P) \, dP \qquad (25)$$

Where $\hat{h} = h + \tau(Q_1+Q_2)^2$, the updated precision of the random variable θ. Moreover, our lineal stochastic demand setting allows us to calculate these second derivates.

$$\frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_1^2} = \frac{8 a^2}{9\hat{m}_1^3} > 0 \qquad (26)$$

$$\frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_1 \delta \hat{m}_2} = \frac{-2 a^2}{9\hat{m}_2^3} < 0 \qquad (27)$$

and

$$\frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_2^2} = \frac{2 a^2}{9\hat{m}_2^4} (3\hat{m}_1 - 4\hat{m}_2) \qquad (28)$$

With these expressions we have the following Proposition.

Proposition 1: With common priors, firms in a Cournot duopoly with linear stochastic demand and random slope specification, will always choose higher quantities for experimental purposes than the "myopic" choice (for a given output of the rival), i.e.for any $Q_i^b(Q_j) \in \varphi_i(Q_j)$

$$Q_i^b(Q_j) > Q_i^m(Q_j)$$

Proof: when $m_1 = m_2$, then $\hat{m}_1 = \hat{m}_2 = \hat{m}$ and then by (26), (27) and (28) we have that :

$$W''(\hat{m}) = \frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta\hat{m}_1^{\,2}} + 2\,\frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta\hat{m}_1\,\delta\hat{m}_2} + \frac{\delta^2 V_1(\hat{m}_1,\hat{m}_2)}{\delta\hat{m}_2^{\,2}} = \frac{2a^2}{9\hat{m}_3} > 0$$

$$(29)$$

Moreover by (4) (or by (18) or (19))

$$\frac{\delta\hat{m}}{\delta P} = \frac{-\tau(Q_1+Q_2)}{h + \tau(Q_1+Q_2)^2} < 0$$

Hence (25) is

$$\frac{\delta\Gamma_1(Q_1)}{\delta Q_1} = \int \frac{1}{\hat{h}^2}\,\frac{2a^2\tau(Q_1+Q_2)}{9\hat{m}^3}\,f(a\text{-}P)dP =$$

$$= \frac{2a^2}{9}\int \frac{\tau(Q_1+Q_2)}{\hat{h}^2\,\hat{m}^3}\,f(a\text{-}P)dP > 0 \qquad\qquad (30)$$

where $\hat{h} = h + \tau(Q_1+Q_2)^2$.

Now by the first order condition and the strict concavity of $\pi_i(Q_1,Q_2;m)$ (see (14),(15) and (16)) the result follows. ∎

The result of proposition 1 tells us that under the common prior assumption, information is always valuable for firm i, (for a given output of firm j) and hence it increases quantity respect to the myopic choice for

experimental purposes, i.e. in order to gather more information. This result matches its monopoly counterpart (see MSU (1989,a)). However in our duopoly setting there is a link between information and competitive advantage.

Consider a monopoly in our demand specification. The monopoly second period value function, i.e. $V(\hat{m})$ is always strictly convex in $\hat{m}$, so that $\Gamma(Q)$ is always increasing in Q. In our duopoly setting this convexity condition would translate in the convexity of $V_i(\hat{m}_1,\hat{m}_2)$ in $(\hat{m}_1,\hat{m}_2)$. We have found that if priors are common $\Gamma_1(Q_1)$ is increasing in Q_1, i.e. $W_1(Q_1)$ is convex in Q_1. However, this does not mean that $V_1(\hat{m}_1,\hat{m}_2)$ is convex in both $(\hat{m}_1,\hat{m}_2)$ for every $(\hat{m}_1,\hat{m}_2)$ in $]\underline{m},\overline{m}[\ \times \ ]\underline{m},\overline{m}[$. In fact (25) shows that $W_1'(Q_1)$ consists of three terms.

The first of them, $\dfrac{\delta^2 V_1}{\delta \hat{m}_1^2}$, captures the incentive for firm 1 to produce information to revise its own beliefs. In our linear setting, convexity of V_1 in $\hat{m}_1$ is always assured and it implies that firm 1 prefers more to less information holding $\hat{m}_2$ constant.

Now, the second and third terms represent the strategic incentives for experimentation. Firms try to gain a competitive advantage via improving their information. However since $\hat{m}_1$ and $\hat{m}_2$ are perfectly positively correlated, firms have to take into account the spillovers of their experimental behaviour. Thus, the second term in (25), $\dfrac{\delta^2 V_1}{\delta \hat{m}_1 \delta \hat{m}_2}$, captures the interaction between the two firm's information. Thus $\dfrac{\delta^2 V_1}{\delta \hat{m}_1 \delta \hat{m}_2} < 0$ would imply that downward

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(upward) revisions in $\hat{m}_1$ are less valuable the lower (higher) $\hat{m}_2$ is, i.e. a negative sign would discourage the production of information and a positive sign would encourage it. Therefore we would say that firms are informationally substitutes, if this sign is negative, and informationally complements on the contrary.

Finally the third term in (25), represents whether or not firm 1 prefers firm 2 to be better informed, holding $\hat{m}_1$ constant, i.e. it captures the incentive for firm 1 to increase or decrease information to influence the revision of firm 2's beliefs. By (26), (27) and (28), since $\hat{m}_1 = \hat{m}_2$ we have the following result.

Proposition 2: Under Cournot competition, duopoly firms under the random slope linear demand specification have the following informational characteristics:

(i) The value of information is always positive for the game, i.e. they both always experiment to produce more information.

(ii) $V_i(\hat{m}_i, \hat{m}_j)$ is strictly convex in $\hat{m}_i$.

(iii) Firms are informationally substitutes, i.e. $\delta^2 V_i / \delta \hat{m}_i \delta \hat{m}_j < 0$

(iv) Firm i is better off the less informed firm j is, i.e., $V_i(\hat{m}_i, \hat{m}_j)$ is strictly concave in $\hat{m}_j$, so that a less informed rival is a better competitor.

By (iii) and (iv) we can deduce that the acquisition of information is smaller under Cournot competition that under monopoly, i.e. the experimental behaviour of firms is less intense under Cournot due to the spillover effects of the acquisition of information and that they are informationally substitutes.

Remark

A natural extension of the Cournot case would be to analyze firms' experimental behavior when we relax the common prior assumption.

In general it is difficult to characterize how experimentation by firm 1 changes as the distance $\hat{m}_1 - \hat{m}_2 \neq 0$ does it. However, it can be said that departing from $\hat{m}_1 = \hat{m}_2$, it is easy to check that the experimentation that firm 1 undertakes, decreases as $\hat{m}_2$ increases, i.e. as $\hat{m}_2 - \hat{m}_1 > 0$ increases. This is so, because firm 1 is better off the bigger $\hat{m}_2$ is and, moreover, this firm prefers firm 2 to be less informed. However, if $\hat{m}_1$ is very small compared to $\hat{m}_2$, i.e. for $\hat{m}_1 < 3/4 \, \hat{m}_2$, then firm 1 will increase its experimentation as the distance $\hat{m}_2 - \hat{m}_1 > 0$ increase, i.e. as $\hat{m}_2$ increases. Now, the reason is that since $\hat{m}_2$ is so big, firm 1 does not worry too much about firm 2's beliefs and since information is always valuable it experiments to gather it.

The other extreme is obtained when $\hat{m}_1 > 4/3 \, \hat{m}_2$. In this case $\dfrac{\delta^2 V}{\delta \hat{m}_2^{\,2}} > 0$, i.e. now firm 1 is better off the more informed firm 2 is (V_1 is strictly

convex in $(\hat{m}_1, \hat{m}_2)$). In this case firm 1 will experiment more the bigger the distance $\hat{m}_1 - \hat{m}_2 > 0$ is. The intuition is clear since now $\hat{m}_2$ is very low and given that $\dfrac{\delta V_1}{\delta \hat{m}_2} > 0$, firm 1 will gather information in order to release it and make firm 2 better informed. Note that in this case $\hat{m}_2 < 3/4\hat{m}_1$ so that by above (by symmetry) firm 2 also experiments as $\hat{m}_1 - \hat{m}_2$ increases. Therefore, when $\hat{m}_i > 4/3\hat{m}_j$ we have the maximum experimentation level undertaken by the duopoly. In general, note that since $\hat{m}_i < 2m_j$, the value of information is always positive for our duopoly model, i.e. firms will always experiment. But, firm i will experiment more the smaller the value of m_j is.

Suppose now that equilibria exist for our dynamic Cournot duopoly. We can characterize any of these equilibria in the following way.

Proposition 3: (Characterization of the equilibrium). If our dynamic problem admits Bayesian Nash solutions for the first period, any equilibrium is symmetric and has the property that

$$Q_i^e = Q^{em} + \frac{2a^2}{27m_i} \int A > Q_i^{em} \tag{31}$$

where:

$$\int A = \int \frac{\tau(Q_1^e + Q_2^e)}{\hat{h}^2 \hat{m}^3} f(a-P)\, dP \tag{32}$$

and Q_i^{em} is the myopic equilibrium output. Moreover:

$$Q^e = Q_1^e + Q_2^e = Q^{em} + \frac{4a^2}{27m} \Bigg\rfloor \quad A > Q^{em} \tag{33}$$

where $Q^{em} = Q_1^{em} + Q_2^{em}$, is the myopic industry supply.

Proof: By computation using (8) and (9), for the myopic game with $\hat{m} = m$, we have that $Q_1^{em} = Q_2^{em} = \frac{a}{3m}$, so that $Q^{em} = \frac{2a}{3m}$ and now by (14), and (30) and solving simultaneously, the result follows.   ∎

Therefore in a symmetric equilibrium, the presence of intertemporal information flows will push industry output in whichever direction generates more information.

Existence of the Equilibrium.

We now investigate the conditions under which an equilibrium exists for the first period problem of our duopoly game. Existence in pure strategies for general duopoly games of our type may be problematic without restrictions on the demand functions and the distributions of the random variables. The source of this difficulty is that without these restrictions the second period value functions V_i may not be concave in the firm's output (even though the first period payoff function is concave), which, in turn, may preclude existence.

Recall that each firm's first period problem is (by (11) and (17)):

$$\underset{Q_i}{\text{Max}} \quad \left[\pi_i(Q_1, Q_2) + \Gamma_i(Q_1, Q_2) \right] \qquad (34)$$

Given our lineal demand specification and our normality assumptions the objective function in (34) is continous and the maximization can be taken to be on the compact set $[0, \overline{Q}_i]$ for some $\overline{Q}_i$, since Γ_i is bounded above by the monopolist profits for the most favourable demand curve and $\underset{Q_1 \to \infty}{lim} \pi_1(Q_1, Q_2)$

$= -\infty$.

Hence a solution must exist $Q_1(Q_2)$ and $Q_2(Q_1)$ that is symmetric. Now note that the second order condition for our rolled back game of the original two period game is:

$$- 2m_i + \frac{d^2\Gamma_i(Q_1, Q_2)}{dQ_i^2} < 0 \qquad (35)$$

If (35) holds for all Q_1^e, Q_2^e then let

$$\beta_1(Q_1^e + Q_2^e) = \frac{\delta\Gamma}{\delta Q} (Q_1^e + Q_2^e)$$

evaluated at $Q_1 = Q_1^e$. Note that by (25) and the definition of $\hat{m}_1$, $\dfrac{\delta\Gamma}{\delta Q_1}$ depends only on the sum of $Q_1^e + Q_2^e$ and not on Q_1^e and Q_2^e separately. Define $\beta_2(Q_1^e + Q_2^e)$ analogously. Now let $g(Q_1^e + Q_2^e) \in \mathbb{R}^2$, be defined as follows. Fix a value of $Q_1^e + Q_2^e$ and hence a value of $\beta_1(Q_1^e + Q_2^e)$ and $\beta_2(Q_1^e + Q_2^e)$. Then let $g(Q_1^e + Q_2^e) = (g_1(Q_1^e + Q_2^e), g_2(Q_1^e + Q_2^e))$ be that value of (g_1, g_2) that solves:

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$$a - m(2g_1 + g_2) + \beta_1(Q_1^e + Q_2^e) = 0$$

$$a - m(2g_2 + g_1) + \beta_2(Q_1^e + Q_2^e) = 0 \tag{36}$$

if the solution falls in the first quadrant. If not, (g_1, g_2) is the appropriate boundary solution. Since $\beta_i(Q_1^e + Q_2^e)$ is constant, once Q_1^e and Q_2^e are fixed, a unique solution $(g_1(Q_1^e + Q_2^e), g_2(Q_1^e + Q_2^e))$ to (36) exits for any value of $Q_1^e + Q_2^e$. This follows because (36) is a standard linear-demand duopoly problem with reaction functions shifted by $\beta_i(Q_1^e + Q_2^e)$.

Now construct a map G: $\mathbb{R} \to \mathbb{R}$ by $G(Q) = g_1(Q) + g_2(Q)$ or $G(Q_1^e + Q_2^e) = g_1(Q_1^e + Q_2^e) + g_2(Q_1^e + Q_2^e)$. Then G is continuous (because $g_i(Q_1^e + Q_2^e)$ is) and for sufficiently large Q, $G(Q) < Q$ (because $\beta_i(Q_1^e + Q_2^e)$ does not grow without bound as Q_1^e and Q_2^e do). Thus G has a fixed point, say $\hat{Q}$. Given $\hat{Q}$, then $(\hat{Q}_1, \hat{Q}_2) \equiv (g_1(\hat{Q}), g_2(\hat{Q}))$ solves

$$a - m(2\hat{Q}_1 + \hat{Q}_2) + \frac{\delta\Gamma_1}{\delta Q_1} = 0$$

and

$$a - m(\hat{Q}_1 + 2\hat{Q}_2) + \frac{\delta\Gamma_2}{\delta Q_2} = 0$$

then $\hat{Q} = Q^e = Q_1^e + Q_2^e$.

We thus have a sufficient condition for the existence of an equilibrium, which is that (35) holds for all Q_1^e and Q_2^e. The sufficient condition is not necessary. In particular (35) is not constant in $Q_1^e + Q_2^e$. (35) can then fail for some values of $Q_1^e + Q_2^e$ without precluding existence, as long as (35) holds

at the equilibrium values of Q_1^e and Q_2^e and the firms' objective function is sufficiently well behaved as to have only one local maximum.

We can next offer sufficient conditions for existence that rely on the use of normal distributions. Since our random variables are normal we have some specification of the problem for which the equilibrium exists. In particular, since $\dfrac{\delta\Gamma}{\delta Q}$ depends on $\dfrac{\delta\hat{m}}{\delta P}$ and it in turn depends inversely on $\hat{h}$ $= h + \tau(Q_1 + Q_2)^2$, i.e. the updated precision of the random variable θ, and where h is its a priori precision and τ the random noise's precision, then $\dfrac{\delta^2\Gamma}{\delta Q_r^2}$ (that is a complicated expression) also depends inversely on $\hat{h}$. Let this precision become higher (or let the updated variance of θ, σ_θ^2 become smaller). As this happens $\dfrac{\delta^2\Gamma_i}{\delta Q_i^2}$ approaches zero. Because $\dfrac{\delta^2\Gamma_i}{\delta Q_i^2}$ is continous and converges pointwise to a continous limit (zero) on a compact set of values of Q_1, Q_2, the convergence can be taken to be uniform. A value of $\hat{h}$ can be then chosen (i.e. choosing h or τ) so that (35) is satisfied. Hence an equilibrium exists.

Our next step is to extend the above analysis to a duopoly model where firms choose prices as strategic variables (Bertrand model). This will permit us to compare the strategic experimentation and the informative characteristic of the two types of competition (Cournot and Bertrand).

3.- THE BERTRAND MODEL.

Now, we analyze a two period duopoly model where firms choose prices as strategic variables. To simplify the analysis consider a symmetric version of a duopoly model proposed by Dixit (we allow for product differentiation), i.e. the demand structure in each period is given by:

$$Q_1 = a - \theta P_1 + cP_2 + \varepsilon_1 \tag{37}$$

$$Q_2 = a - \theta P_2 + cP_1 + \varepsilon_2 \tag{38}$$

in the region of price space where quantities are positive. Firms have constant and equal marginal cost. From now on suppose prices are net of marginal cost. With $c > 0$, we have Bertrand competition with substitute products. θ is the slope parameter that is randomly distributed. To keep things simple and to conduct the analysis as close as possible to our Cournot model, we suppose that $\varepsilon_1 = \varepsilon_2 = \varepsilon$ [3] is the realization of a random variable. These error terms are i.i.d. and independents with the parameter θ. We suppose the same informative assumptions as in the Cournot model, i.e. firms assume

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[3] Note that we could also have assumed that $\varepsilon_1 = -\varepsilon_2$. But in this case adding (37) and (38) in the text, will give us a signal that depends on price dispersion. But, since the degree of product substitution is known, i.e. c is a known parameter, we do not think that this specification is the best for our purpose. Moreover, under Cournot competition with perfect substitution, both firms will follow the same policy so that no quantity dispersion is ever observed.

that the random variables θ and ε are normally distributed, $\varepsilon \sim N(0,\tau)$, $\theta \sim N(\theta,h)$ and that the a priori beliefs of the firms about $E(\theta) = m_i$, $i = 1,2$. As above, we will consider common priors, but we will keep them notionally different in most of the analysis to better explain our findings.

Since the analysis replicates the one we have done before under Cournot competition, we just supply the details that are necessary to make the informative comparisons between the two types of competitions.

The flow of information and the updating procedure:

Since we assume that firms observe both first period quantities and prices, and that $\varepsilon_1 = \varepsilon_2 = \varepsilon$, then both firms see the same signal. In particular, after the first period, by adding (37) and (38), firms observe the signal:

$$\left(\theta - \frac{\varepsilon}{P} \right) = \frac{a}{P} + c - \frac{Q}{P} \tag{39}$$

where $P = \dfrac{P_1 + P_2}{2}$ is average prices and $Q = \dfrac{Q_1 + Q_2}{2}$ is average quantities. The above signal is distributed, for each value of θ as, $\left(\theta - \frac{\varepsilon}{P} \right) \sim N(\theta, \tau P^2)$ and since for each firm $\theta \sim N(m_i, h)$, hence

$$\hat{m}_i = \frac{m_i h + \tau P(a + cP - Q)}{h + \tau P^2} \tag{40}$$

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and

$$\hat{h} = h + \tau P^2 \qquad (41)$$

As before, we solve for the second period first. Let $\hat{m}_1$ and $\hat{m}_2$ (that are equal) be second period updated beliefs for firms 1 and 2 respectively. Firms second period equilibrium prices are (in the Appendix we supply the details):

$$P_1^* = \frac{a(2\hat{m}_2 + c)}{4\hat{m}_1\hat{m}_2 - c^2} \qquad (42)$$

$$P_2^* = \frac{a(2\hat{m}_1 + c)}{4\hat{m}_1\hat{m}_2 - c^2} \qquad (43)$$

Note that since $\hat{m}_1 = \hat{m}_2 = \hat{m}$, then $P_1^* = P_2^* = \dfrac{a}{2\hat{m}-c}$. However we do not yet use this equality because it is more convenient for our analysis. Define the second period value function for firm i as:

$$V_i^{\beta}(\hat{m}_1,\hat{m}_2) = aP_i^* - \hat{m}_i P_i^{*2}(\hat{m}_1,\hat{m}_2) + cP_j^*(\hat{m}_1,\hat{m}_2) P_i^*(\hat{m}_1,\hat{m}_2) \qquad (44)$$

(where the β reads for Bertrand) with the following properties:

<u>Lemma 2</u>: $V_i^{\beta}(\hat{m}_1,\hat{m}_2)$ is a decreasing function of both $\hat{m}_1$ and $\hat{m}_2$. Moreover $V_i^{\beta}(\hat{m}_1,\hat{m}_2)$ is stricty convex in $[\hat{m}_1,\hat{m}_2] \in \,]\underline{m},\bar{m}[\,\times\,]\underline{m},\bar{m}[$, i.e.:

$$\frac{\delta^2 V_i^{\beta}}{\delta \hat{m}_1^2} > 0 \qquad \frac{\delta^2 V_i^{\beta}}{\delta \hat{m}_2^2} > 0 \qquad \frac{\delta^2 V_i^{\beta}}{\delta\hat{m}_1 \,\delta\hat{m}_2} > 0$$

Proof: See Appendix.

Next, we analyze the first period best reply mappings and compare them to the myopic best responses. In order to do that we just restate the Cournot first period problem for our Bertrand model.

Let $\Pi_i(P_1,P_2,\hat{m}_1,\hat{m}_2)$ be firm i's second period expected profits as a function of first period price, i.e.

$$\Pi_i(P_1,P_2,\hat{m}_1,\hat{m}_2) = E\left[(a - \theta P_1 + cP_2 + \varepsilon_i)P_i\right] + E\left[V_i^{\beta}\left(\hat{m}_1(P,Q),\hat{m}_2(P,Q)\right)\right] \qquad (45)$$

with $P = \dfrac{P_1 + P_2}{2}$, and $Q = \dfrac{Q_1 + Q_2}{2}$, and where:

$$E\left[V_i^{\beta}\left(\hat{m}_1(P,Q),\hat{m}_2(P,Q)\right)\right] = \int_{-\infty}^{+\infty} V_i^{\beta}\left(\hat{m}_1(P,Q),\hat{m}_2(P,Q)\right) f(a-Q+cP)\, dQ \qquad (46)$$

where f is the density of (a-Q+cP). Note that since

$$a - Q + cP = \theta P - \varepsilon = \theta \left(\frac{P_1 + P_2}{2}\right) - \varepsilon$$

Then for each P, firm i sees the random variable (a-Q+cP) distributed as:

$$(a-Q+cP) \sim N\left(m_i P, \frac{h\tau}{\tau P^2 + h}\right)$$

Where $\dfrac{h\tau}{\tau P^2 + h}$ is the precision of the signal and $m_i P$ is the mean.

Therefore:

$$f(a-Q+cP) = \frac{\left(\dfrac{h\tau}{\tau P^2 + h}\right)^{1/2}}{\sqrt{2\Pi}} e^{-1/2} \left(\frac{h\tau}{\tau P^2 + h}\right)(a-Q+cP-m_iP)^2$$

Define firm i best reply mapping as a $P_i(P_j)$ such that

$$P_i(P_j) \in \underset{P_i}{\text{Argmax}} \; E\left[(a - \theta P_i + cP_j + \varepsilon_i)P_i\right] + E\left[V_i^\beta(\hat{m}_1,\hat{m}_2)\right] \qquad (47)$$

and firm i, myopic reaction function as

$$P_i^m(P_j) \in \underset{P_i}{\text{Argmax}} \; E\left[(a - \theta P_i + cP_j + \varepsilon_i)P_i\right] \qquad (48)$$

Then $P_i(P_j) \overset{>}{<} P_i^m(P_j)$ depending $\dfrac{\delta}{\delta P_i} E\left[V_i^\beta(\hat{m}_1,\hat{m}_2)\right] \overset{>}{<} 0.$

In words, each firm will choose a higher price than the "myopic" one, for experimental purposes, for a given price of the other firm, depending on whether the firm's second period value function is increasing in the firm's price. Therefore we investigate this sign next. To this end let us express firm 2's posterior $\hat{m}_2$ as (see (40))

$$\hat{m}_2 = \hat{m}_1 + \frac{h(m_2 - m_1)}{h + \tau P^2} \qquad (49)$$

hence:

$$\hat{m}_2 = \gamma^\beta(\hat{m}_1), \text{ with } \gamma^{\beta'}(\hat{m}_1) = 1$$

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Let us consider firm 1's second period value function. By (49) it can be expressed as

$$V_1^\beta(\hat{m}_1, \hat{m}_2) = V_1^\beta(\hat{m}_1, \gamma^\beta(\hat{m}_1))$$

Let

$$W_1(\hat{m}_1) = V_1^\beta(\hat{m}_1, \gamma^\beta(\hat{m}_1)) \tag{50}$$

and let

$$\Gamma_1(P_1) = \int W_1(\hat{m}_1(P,Q)) \, f(a-Q+cP) \, dQ \tag{51}$$

Next, differentiating (51) with respect to P_1, yields:

$$\frac{\delta\Gamma_1(P_1)}{\delta P_1} = \int \left[W_1'(\hat{m}_1) \frac{\delta\hat{m}_1}{\delta P_1} f(a-Q+cP) + W(\hat{m}_1) \frac{\delta}{\delta P_1} f(a-Q+cP) \right] dQ \tag{52}$$

In the Appendix it is shown that (52) is

$$\frac{\delta\Gamma_1(P)}{\delta P_1} = \int \frac{1}{\hat{h}} W_1''(\hat{m}_1) \frac{1}{2} \left(- \frac{\delta\hat{m}_1}{\delta Q} \right) f(a-Q+cP) \, dQ =$$

$$\int \frac{1}{\hat{h}} W_1''(\hat{m}_1) \left(- \frac{\delta\hat{m}_1}{\delta Q_1} \right) f(a-Q+cP) \, dQ \tag{53}$$

Next, since by (40), $\dfrac{\delta\hat{m}_1}{\delta Q} < 0$, the sign of (53) depend on the sign of $W''(\hat{m}_1)$. If it is strictly convex, (53) is positive. By (50) $W(\hat{m}_1) = V_1^\beta(\hat{m}_1, \gamma^\beta(\hat{m}_1))$ so that (53) can be expressed as:

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$$\frac{\delta \Gamma_1(P)}{\delta P_1} = \int \frac{1}{\hat{h}} \left[\frac{\delta^2 V_1^\beta (\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_1^2} + 2 \frac{\delta^2 V_1^\beta (\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_1 \delta \hat{m}_2} + \frac{\delta^2 V_1^\beta(\hat{m}_1,\hat{m}_2)}{\delta \hat{m}_2^2} \right]$$

$$\left(- \frac{\delta \hat{m}_1}{\delta Q} \right) f(a-Q+cP) \, dQ > 0 \tag{54}$$

by the convexity of V_1^β in $(\hat{m}_1,\hat{m}_2)$ (see lemma 2). Therefore, we have the following proposition that is the Bertrand product subtitutes counterpart of proposition 1.

Proposition 4: Under Bertrand competition with product substitution a duopoly facing an stochastic demand with a random slope specification will always choose a higher price than the myopic one, for any given price level of the rival.

Moreover since $V_i^\beta(\hat{m}_i,\hat{m}_j)$ is strictly convex in $(\hat{m}_i,\hat{m}_j) \in \,]\underline{m},\overline{m}[x[\underline{m},\overline{m}[$, we have that (see lemma 2) under Bertrand competition with product substitution,

(i) Firms are informationally complements, i.e. $\dfrac{\delta^2 V_i^\beta}{\delta \hat{m}_i \delta \hat{m}_j} > 0$, so that downward (upward) revision of firm i's beliefs are more profitable the lower (the higher) the rival's beliefs.

(ii) Firm i is better off the more informed j is, i.e. $\dfrac{\delta^2 V_i^\beta}{\delta \hat{m}_j^2} > 0$, so that a better informed rival is a "nicer" competitor.

Therefore the value of information is always positive for the duopoly game, and by expression (54) firms under Bertrand will experiment more than the monopoly counterpart.

Comparing expressions (54) and (25) we can conclude that firms will experiment more under Bertrand thand under Cournot, if products are substitutes. However, our linear demand setting allows us to extend the results to complement products and to relate the level of experimentation under Cournot with the degree of product's substitution:

Thus, note that under Bertrand competition if products are complements (i.e., $c < 0$), $V_i^\beta(\hat{m}_i, \hat{m}_j)$ is convex in $\hat{m}_i$, but concave in $\hat{m}_j$, with $\dfrac{\delta^2 V_i}{\delta \hat{m}_i \, \delta \hat{m}_j} < 0$ (see Appendix), so that firms are informationally substitutes. Moreover, as the absolute value of c increases, the more concave $V_i^\beta(\hat{m}_i, \hat{m}_j)$ in $\hat{m}_j$ is, and the more negative $\dfrac{\delta^2 V_i}{\delta \hat{m}_i \, \delta \hat{m}_j}$ is.

Given that our demand structure is linear, Bertrand competition with complement (substitute) products is the perfect dual of Cournot with substitutes (complements)(See Vives, 1984). Hence, by the above results, firms under Cournot with substitutes are more informationally substitutes and informed rivals are more "unpleasant" the higher the degree of product substitution. Therefore, we have the following propositions between Bertrand and Cournot competition and homogeneous and differentiated product.

Proposition 5: The level of experimentation under Cournot competition with substitutes decreases as the degree of product substitution increases.

Proposition 6: Under Cournot (Bertrand) competition, if products are substitutes, then firms are informationally substitutes (complements) and each firm will be better off the less (the more) informed the rival is, so that firms will experiment less than (more than) the monopoly counterpart. Hence the value of information is higher under Bertrand than under Cournot and consequently a Cournot competitor will experiment less than a Bertrand competitor. The smallest level of experimentation will correspond to Cournot with homogeneous products.

If products are complements, the results are reversed, so that a Cournot duopolist will set up a higher level of experimentation than a Bertrand duopolist.

4.- CONCLUSIONS.

In this paper we have analyzed how learning behaviour can modify the outcome of competition in a duopoly industry facing demand uncertainty. We have considered two duopoly games where firms have imperfect information about market demand - in particular, the demand slope is random - and learn through observing prices (in the Cournot model) or industry sales (under Bertrand competition).

The central body of the paper has consisted in showing two main results. The first is concerned with the value of information in duopoly games and its relationship with the monopoly counterpart of MSU (1989,a). We have found that with linear demand structure, just one demand parameter unknown (i.e. the random slope) and common priors, the value of information is always positive for our duopoly models, i.e. firms will always experiment.

The second result comes from relating duopoly experimentation to market competition. We have showed that, if the goods are substitutes, firms will experiment more under Bertrand competition (with Bertrand experimentation being greater than monopoly experimentation) than under Cournot competition (with Cournot experimentation smaller than that of monopoly). If the goods are complements, the results are reversed. Furthermore, Cournot experimentation is decreasing in the degree of product substitution, so that the lower level of duopoly experimentation corresponds to the Cournot homogeneous product (perfect substitution) model.

APPENDIX.-

Derivation of equation 24:

From (23)

$$\frac{\delta \ \Gamma_1(Q_1)}{\delta Q_1} = \int \left[W'_1(\hat{m}_1) \ \frac{\delta \hat{m}_1}{\delta Q_1} \ f(a\text{-}P) + W_1(\hat{m}_1) \frac{\delta}{\delta Q_1} \ f(a\text{-}P) \right] dP \qquad (1)$$

From (13), and letting $Q = Q_1 + Q_2$ we have that

$$f'(a\text{-}P) = \frac{\delta}{\delta P} \ f(a\text{-}P) = \frac{h\tau}{h + \tau Q^2} \ (a - P - m_1 Q) \ f(a\text{-}P), \qquad (2)$$

and then

$$\frac{\delta}{\delta Q_1} \ f(a\text{-}P) = - \frac{\tau Q}{h + \tau Q^2} \ f(a\text{-}P) + \frac{h\tau^2 Q}{(h + \tau Q^2)^2}(a\text{-}P\text{-}m_1 Q)^2 f(a\text{-}P) + m_1 f'(a\text{-}P)$$

hence (1) is

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int W'_1(\hat{m}_1) \frac{\delta \hat{m}_1}{\delta Q_1} \ f(a\text{-}P)dP + \int W_1(\hat{m}_1) \left[- \frac{Q\tau}{h + \tau Q^2} + \frac{h\tau^2 Q}{(h + \tau Q^2)^2} \right.$$

$$\left. \Big(a\text{-}P + m_1 Q \Big)^2 \right] f(a\text{-}P)dP + \int W_1(\hat{m}_1)m_1 f'(a\text{-}P)dP \qquad (3)$$

Integrating the last term of the above expression by parts: i.e.

$$m_1 \int W_1(\hat{m}_1)f'(a\text{-}P)dP = -m_1 \int W'_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P} \ f(a\text{-}P)dP$$

and inserting this result in (3) and rearranging yields:

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int W'_1(\hat{m}_1)\left[\frac{\delta \hat{m}_1}{\delta Q_1} - m_1\frac{\delta \hat{m}_1}{\delta P}\right] f(a-P)dP +$$

$$\int W_1(\hat{m}_1)\left[-\frac{\tau Q}{h+\tau Q^2} + \frac{h\tau^2 Q}{(h+\tau Q^2)^2}(a-P+m_1 Q)^2\right] f(a-P)dP \qquad (4)$$

Next, by the definition of $\hat{m}_1$ (see(18))

$$\frac{\delta \hat{m}_1}{\delta Q_1} = \frac{\tau(a-P)}{h+\tau Q^2} - \frac{2\tau Q\hat{m}_1}{h+\tau Q^2} \quad \text{and} \quad \frac{\delta \hat{m}_1}{\delta P} = -\frac{\tau Q}{h+\tau Q^2} \qquad (5)$$

and we can express the first term in brackets of the RHS of (4) as

$$\frac{\delta \hat{m}_1}{\delta Q_1} - m_1\frac{\delta \hat{m}_1}{\delta P} = \frac{\tau(a-P-m_1 Q)}{h+\tau Q^2} - \frac{2\tau Q(\hat{m}_1-m_1)}{h+\tau Q^2} \qquad (6)$$

Inserting (6) in (4) and by (5):

$$\frac{\delta \Gamma_1(Q_1)}{\delta Q_1} = \int W'_1(\hat{m}_1)\frac{\tau(a-P-m_1 Q)}{h+\tau Q^2} f(a-P)\,dP +$$

$$2\int W'_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P}(\hat{m}_1-m_1)f(a-P)dP + \int W_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P} f(a-P)\,dP. +$$

$$\int W_1(\hat{m}_1)\frac{h\tau^2 Q}{(h+\tau Q^2)^2}(a-P-m_1 Q)^2 f(a-P)\,dP \qquad (7)$$

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Next, note the following facts:

By (2),the first term of (7) is

$$\frac{1}{h} \int W'(\hat{m}_1) \; f'(a-P)dP$$

and after integration by parts it yields,

$$-\frac{1}{h} \int W''(\hat{m}_1) \; \frac{\delta \hat{m}_1}{\delta P} f(a-P)dP \tag{8}$$

moreover

$$\hat{m}_1 - m_1 = \frac{\tau Q(a-P-m_1 Q)}{h + \tau Q^2} \tag{9}$$

and by (2) the last term of (7) is

$$\int W'_1(\hat{m}_1)(\hat{m}_1 - m_1) \; f'(a-P)dP$$

and after integration by parts is

$$= - \int W'_1(\hat{m}_1)(\hat{m}_1 - m_1)\frac{\delta \hat{m}_1}{\delta P} f(a-P) \; dP - \int W(\hat{m}_1) \; \frac{\delta \hat{m}_1}{dP} f(a-P) \; dP \tag{10}$$

Substitution of (8) and (10) in (7) and cancellation of terms yields

$$\frac{\delta \Gamma_1}{\delta Q_1} = - \frac{1}{h} \int W''_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P} f(a-P)dP + \int W'_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P}(\hat{m}_1 - m_1)f(a-P)dP \tag{11}$$

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Next by (9) and by (2), the last term of (11) can be written as

$$\int W_1' \frac{\delta \hat{m}_1}{\delta P} \frac{Q}{h} f'(a-P)dP,$$ that, again, by integration by parts yields

$$-\int W_1''(\hat{m}_1) \frac{Q}{h} \left(\frac{\delta \hat{m}_1}{\delta P} \right)^2 f(a-P) \, dP \qquad (12)$$

Hence by (12), (11) is

$$\frac{\delta \Gamma_1}{\delta Q_1} = -\frac{1}{h} \int W_1''(\hat{m}_1) \frac{\delta \hat{m}_1}{\delta P} f(a-P) \, dP - \int W_1''(\hat{m}_1) \frac{Q}{h} \left(\frac{\delta \hat{m}_1}{\delta P} \right)^2 f(a-P)dP$$

$$= -\frac{1}{h} \int W_1''(\hat{m}_1) \left(\frac{\delta \hat{m}_1}{\delta P} \right) \left[1 + Q \frac{\delta \hat{m}_1}{\delta P} \right] f(a-P) \, dP$$

That by (5) yields:

$$\frac{\delta \Gamma_1}{\delta Q_1} = \int W_1''(\hat{m}_1) \frac{1}{\hat{h}} \left(-\frac{\delta \hat{m}_1}{\delta P} \right) f(a-P) \, dP$$

where $\hat{h} = h + \tau Q^2$ ■

Derivation of Bertrand second period equilibrium:

Let $\hat{m}_1$ and $\hat{m}_2$ be the posterior beliefs of firm 1 and firm 2 respectively. Firm i's expected profits for period two are:

$$\pi_i = E\ [(a-\theta P_i + c P_j + \varepsilon_i)P_i] = aP_i - \hat{m}_i P_i^2 + c P_i P_j \qquad (1)$$

Hence firm i will choose a P_i that maximizes (1), i.e.

$$P_i \in \underset{P_i}{\text{Argmax}} \ (aP_i - \hat{m}_i P_i^2 + cP_i P_j) \tag{2}$$

and firm j will choose a P_j such that

$$P_j \in \underset{P_j}{\text{Argmax}} \ (aP_j - \hat{m}_j P_j^2 + cP_j P_i) \tag{3}$$

giving the reaction functions:

$$P_1(P_2) = \frac{a}{2\hat{m}_1} + \frac{c}{2\hat{m}_1} P_2 \tag{4}$$

and

$$P_2(P_1) = \frac{a}{2\hat{m}_2} + \frac{c}{2\hat{m}_2} P_1 \tag{5}$$

Both reaction functions are continuous and upward slopping (since products are substitutes) and since profits functions are quadratic in prices, then the equilibrium exist and it is unique. Solving (4) and (5) simultaneously yield the values:

$$P_1^* = \frac{a\,(\,2\hat{m}_2 + c\,)}{4\hat{m}_1 \hat{m}_2 - c^2}$$

$$P_2^* = \frac{a\,(\,2\hat{m}_1 + c\,)}{4\hat{m}_1 \hat{m}_2 - c^2}$$

Proof of lemma 2: By (44) firm 1 second period value function is:

$$V_1^\beta(\hat{m}_1, \hat{m}_2) = aP_1^* - \hat{m}_1 P_1^{*2}(\hat{m}_1, \hat{m}_2) + cP_1^*(\hat{m}_1, \hat{m}_2)P_2^*(\hat{m}_1, \hat{m}_2) \tag{1}$$

with c>0 , (The products are substitute), and since

$$P_1^* = \frac{a(2\hat{m}_2 + c)}{4\hat{m}_1\hat{m}_2 - c^2} \tag{2}$$

$$P_2^* = \frac{a(2\hat{m}_1 + c)}{4\hat{m}_1\hat{m}_2 - c^2} \tag{3}$$

then

$$\frac{\delta P_1^*}{\delta \hat{m}_1} = \frac{-4\hat{m}_2 a(2\hat{m}_2 + c)}{[4\hat{m}_1\hat{m}_2 - c^2]^2} < 0 \tag{4}$$

$$\frac{\delta P_2^*}{\delta \hat{m}_1} = \frac{-2ca(2\hat{m}_2 + c)}{[4\hat{m}_1\hat{m}_2 - c^2]^2} < 0 \tag{5}$$

$$\frac{\delta^2 P_2^*}{\delta \hat{m}_1^2} > 0 \tag{6}$$

$$\frac{\delta^2 P_2^*}{\delta \hat{m}_1 \delta \hat{m}_2} = \frac{16ca\hat{m}_1\hat{m}_2 + 16\hat{m}_1 c^2 a + 4c^3 a}{[4\hat{m}_1\hat{m}_2 - c^2]^3} > 0 \tag{7}$$

$$\frac{\delta P_1^*}{\delta \hat{m}_2} = \frac{-2ac[c + 2\hat{m}_1]}{[4\hat{m}_1\hat{m}_2 - c^2]^2} < 0 \tag{8}$$

$$\frac{\delta P_2^*}{\delta \hat{m}_2} = \frac{- 4\hat{m}_1 \, a \, [2\hat{m}_1 + c]}{[4\hat{m}_1 \hat{m}_2 - c^2]^2} < 0 \qquad (9)$$

and

$$\frac{\delta^2 P_2}{\delta \hat{m}_2^2} = \frac{32\hat{m}_1^2 a \, [2\hat{m}_1 + c]}{[4\hat{m}_1 \hat{m}_2 - c^2]^3} < 0 \qquad (10)$$

Then, by (1) and using the Envelope Theorem when applicable,

$$\frac{\delta V_1^\beta(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1} = - P_1^* \, [P_1^* - c \, \frac{\delta P_2^*}{\delta \hat{m}_1}] = - \frac{P_1^* [P_1^* + 2c^2 a \, (c + 2\hat{m}_2)]}{[4\hat{m}_1 \hat{m}_2 - c^2]^2} < 0 \qquad (11)$$

by (5).

Moreover,

$$\frac{\delta^2 V_1^\beta(\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1^2} = - 2P_1^* \frac{\delta P_1^*}{\delta \hat{m}_1} + c \, \frac{\delta P_1^*}{\delta \hat{m}_1} \frac{\delta P_2^*}{\delta \hat{m}_1} + c \, \frac{\delta^2 P_2^*}{\delta \hat{m}_1^2} P_1^* > 0 \qquad (12)$$

by (4),(5) and (6).

Next,

$$\frac{\delta^2 V \, (\hat{m}_1, \hat{m}_2)}{\delta \hat{m}_1 \delta \hat{m}_2} = - 2P_1^* \frac{\delta P_1^*}{\delta \hat{m}_2} + c \, \frac{\delta P_1^*}{\delta \hat{m}_2} \frac{\delta P_2^*}{\delta \hat{m}_1} + cP_1^* \frac{\delta^2 P_2^*}{\delta \hat{m}_1 \delta \hat{m}_2} > 0 \qquad (13)$$

by (8), (5), and (7).

Finally, by (1) and using again the Evelope Theorem when applicable

$$\frac{\delta V_1^\beta}{\delta \hat{m}_2} = cP_1^* \frac{\delta P_2^*}{\delta \hat{m}_2} < 0 \qquad (14)$$

by (9), and

$$\frac{\delta^2 V_1^\beta}{\delta \hat{m}_2^2} = c \frac{\delta P_1^*}{\delta \hat{m}_2} \frac{\delta P_2^*}{\delta \hat{m}_2} + c P_1^* \frac{\delta^2 P_2^*}{\delta \hat{m}_2^2} > 0 \qquad (15)$$

by (8), (9) and (10)

Hence by (11) and (14): $\dfrac{\delta V_i^\beta (\hat{m}_1, \hat{m}_2)}{\delta m_j} < 0$ for j=1,2 and by (12), (13) and

(15), $V_1^\beta(\hat{m}_1, \hat{m}_2)$ is strictly convex in $[\hat{m}_1, \hat{m}_2]$.

Note that if the products where complements, i.e. c<0, then we will here that:

$$\frac{\delta P_i^*}{\delta \hat{m}_i} < 0, \text{ but } \frac{\delta P_i^*}{\delta \hat{m}_j} > 0, \text{ i=1,2, j=1,2, and}$$

$$\frac{\delta^2 P_i^*}{\delta \hat{m}_i^2} > 0, \frac{\delta^2 P_i^*}{\delta \hat{m}_j^2} < 0, \frac{\delta^2 P_i^*}{\delta \hat{m}_i \delta \hat{m}_j} < 0 \text{ so that we have now by (11)-(15):}$$

$$\frac{\delta V_i^\beta}{\delta \hat{m}_i} < 0, \text{ but } \frac{\delta V_i^\beta}{\delta \hat{m}_j} > 0$$

and

$$\frac{\delta^2 V_i^\beta}{\delta \hat{m}_i^{\,2}} > 0, \text{ i.e. } V_i^\beta(\hat{m}_1, \hat{m}_2) \text{ is strictly convex in } \hat{m}_i, \text{ but}$$

$$\frac{\delta^2 V_i^\beta}{\delta \hat{m}_i \, \delta \hat{m}_j} < 0, \text{ and } \frac{\delta^2 V_i^\beta}{\delta \hat{m}_j^{\,2}} < 0, \text{ i.e. } V_i^\beta(\hat{m}_1, \hat{m}_2) \text{ is strictly concave in } \hat{m}_j.$$

∎

Derivation of expression (53)

This derivation is very similar to that of expression (24) so that we only give same details. We have that

$$\frac{\delta \Gamma_1(P)}{\delta P_1} = \int [W_1'(\hat{m}_1) \frac{\delta \hat{m}_1}{\delta P_1} f(a-Q+cP) + W_1(\hat{m}_1)\frac{\delta}{\delta P_1} f(a-Q+cP)] \, dQ \quad (1)$$

where $P = \dfrac{P_1 + P_2}{2}$, $Q = \dfrac{Q_1 + Q_2}{2}$ are average prices and quantities respectively.

Let

$$f(Q) = f(a-Q+cP) = \frac{\left(\dfrac{h\tau}{\tau P^2 + h}\right)^{1/2}}{\sqrt{2\pi}} \, e^{-1/2\left(\frac{h\tau}{\tau P^2 + h}\right)(a-Q+cP-m_1 P)^2} \quad (2)$$

then:

$$f'(Q) = \frac{h\tau}{\tau P^2 + h}(a-Q+cP-m_1 P)f(Q) \quad (3)$$

and

$$\frac{\delta}{\delta P_1} f(Q) = \frac{-\tau P}{2(\tau P^2 + h)} f(Q) + \frac{Ph\tau^2}{2(\tau P^2 + h)^2}(a-Q+cP-m_1 P)^2 f(Q) + \frac{1}{2}(m_1-c)f'(Q)$$

(4)

Inserting this result in (1),

$$\frac{\delta \Gamma_1(P)}{\delta P_1} = \int W'_1(\hat{m}_1)\frac{\delta \hat{m}_1}{\delta P_1} f(Q) \, dQ + \int W_1(\hat{m}_1)(\frac{-\tau P}{2(\tau P^2 + h)^2})f(Q) \, dQ +$$

$$\int W_1(\hat{m}_1)(\frac{Ph\tau^2}{2(\tau P^2 + h)})(a-Q+cP-m_1 P)^2 f(Q)dQ + \int W_1(\hat{m}_1)\frac{1}{2}(m_1-c)f'(Q)dQ$$

(5)

Now integration of the last term of the RHS by parts, and rearranging terms in (5), taking into account that by the definition of $\hat{m}_1$,

$$\frac{\delta \hat{m}_1}{\delta Q} = \frac{-\tau P}{h + \tau P^2} \; ; \; \frac{\delta \hat{m}_1}{\delta P_1} = \frac{\tau(a-Q+2cP)}{2[h + \tau P^2]} - \frac{\tau P \hat{m}_1}{[h + \tau P^2]}$$

and hence:

$$\frac{\delta \hat{m}_1}{\delta P_1} - (m_1 - c)\frac{1}{2}\frac{\delta \hat{m}_1}{\delta Q} = \frac{\tau(a-Q+cP-m_1 P)}{2[h + \tau P^2]} - \frac{\tau P(\hat{m}_1 - m_1)}{h + \tau P^2}$$

and where

$$\hat{m}_1 - m_1 = \frac{\tau P(a-Q+cP-m_1 P)}{h + \tau P^2}$$

(6)

then (5) is,

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$$\frac{\delta\Gamma_1(P)}{\delta P_1} = \int W_1'(m_1) \; \frac{\tau(a-Q+cP-m_1 P)}{2[h + \tau P^2]} \; f(Q) \; dQ \; +$$

$$\int W_1'(\hat{m}_1) \; \frac{\delta\hat{m}_1}{\delta Q} \; (\hat{m}_1 - m_1)f(Q)dQ \; + \int W_1(\hat{m}_1) \; \frac{1}{2} \; \frac{\delta\hat{m}_1}{\delta Q} \; f(Q)dQ \; +$$

$$\int W_1(\hat{m}_1) \frac{1}{2} \; \frac{Ph\tau^2}{(\tau P^2 + h)} \; (a-Q+cP-m_1 P)^2 \; f(Q)dQ$$

<div align="right">(7)</div>

Now, by (3), we can integrate the first term of the RHS of (7) by parts, and again by (3) and by (6) a new integratiation of the last term of RHS of (7) by parts, yields after rearranging and cancellation of terms

$$\frac{\delta\Gamma_1(P)}{\delta P_1} = - \; \frac{1}{h} \int W_1''(\hat{m}_1) \frac{1}{2} \; \frac{\delta\hat{m}_1}{\delta Q} \; f(Q)dQ \; +$$

$$\int W_1'(\hat{m}_1) \frac{1}{2} \; \frac{\delta\hat{m}_1}{\delta Q}(\hat{m}_1 - m_1)f(Q)dQ$$

<div align="right">(8)</div>

That again by (6) and after a new integration by parts is,

$$\frac{\delta\Gamma_1(P)}{\delta P_1} = \int W_1''(\hat{m}_1)\frac{1}{2}\left(- \; \frac{\delta\hat{m}_1}{\delta Q_1}\right) \frac{1}{\hat{h}} \; f(Q)dQ =$$

$$= \int \frac{1}{\hat{h}} \; W_1''(\hat{m}_1)\left(- \; \frac{\delta\hat{m}_1}{\delta Q}\right) \; f(Q) \; dQ. \qquad \blacksquare$$

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