TIME PREFERENCE AND INDIVIDUAL HEALTH PROFILES*

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ABSTRACT

A generalization of the QALY model for general health profiles is provided in this paper. Two natural assumptions on inter-temporal preferences play a key role in arriving at our representation. The first one, *Indifference to the future after death*, is uncontestable in our framework, and the second one, *Preference Independence of the future with regard to the past,* is weaker than the usual Additive Independence or Mutual Utility Independence conditions traditionally employed. The semi-separable structure obtained for the utility function on health profiles is very similar to the discounted QALY, but unlike it, endogenous discount rates, depending on past states of health, now emerge.

Key words: Health Profiles; Time Preferences; Preference Independence.

1. Introduction

The importance of cost-utility analysis in the economic evaluation of health care has increased recently the literature on utility-based measures of health [Viscusi et al. (1991), Krupnick & Cropper (1992), Jones-Lee et al. (1995), Magat et al. (1996)]. The most popular utility-based model in health decision making is a simple additive model: the quality-adjusted life-years (QALY) model.

Strong criticisms of the foundations of the QALY model [see Loomes & McKenzie (1989), or Mehrez & Gafni (1989)] induced researchers to make some efforts to provide a basis of QALY's in utility theory. Starting with the contributions of Pliskin, Shepard & Wenstein (1980), two main branches emerge in the literature. Those papers analyzing chronic health situations [Bleichrodt (1996), Ried (1997), Miyamoto et al. (1998)], on the one hand, and those devoted to general health profiles [Bleichrodt (1996)]. They all use, however, a similar methodology: a combination of von Neumann-Morgenstern's expected utility theory (1944), and of multi-attribute evaluation theories [see Fishburn, (1974), Fishburn & Keeney, (1974), (1975), Keeney & Raiffa, (1976), Miyamoto (1983), (1988)]. Furthermore, in more recent papers [see Bleichrodt & Quiggin (1997)] the General Rank Dependent utility theory is applied to multi-attribute evaluation theories without expected utility foundations [see Miyamoto & Wakker (1996)].

Previous literature provides a solid utilitarian basis for the use of the QALY methodology in the case of chronic health situations, but the theoretical approach is not quite so satisfactory in the case of general health profiles. A particular problem arises when health profiles of a fixed (maximum) duration are considered. The use of the usual assumptions of Additive Independence or Mutual Utility Independence, lead to a different evaluation of health profiles such that, after death, the individual "enjoys" different states of health. Alternatively, health profiles have also been evaluated by using inter-temporal preferences. In particular, constant rate discounted utility models are commonly used to represent inter-temporal preferences in the evaluation of health care programs. Nevertheless, there is a general agreement on the idea that the preference structure required in order to properly do so is not the most apropriate in this context. Bleichrodt (1996), and Bleichrodt and Gafni (1996) analyze the suitability of the discounted utility model as a description of an individual inter-temporal preference for health outcomes. They conclude that the axioms underlying the individual preference structure to fit stationarity [Koopmans (1960), Koopmans et al (1964), Koopmans (1972), and Fishburn & Rubinstein (1982)] are far from being adequate in this setting. They also reject the idea that using a variable rate of discount will solve the problem, as argued by some authors [see Olsen (1993)]. Introducing variable discount rates not only fails to solve previous problems, but introduces additional problems as well, by making it possible for the individual to behave in a dynamically inconsistent way.

In this paper we analyze the problem of providing a utility-based evaluation theory for health profiles that would avoid some of the above-mentioned problems. The first step consists of choosing the space in which to work. In this respect, there are two alternatives open to us. The first one is that of considering a Cartesian space, by assuming that all profiles have identical length, and taking "death" as a feasible health state at any point in time. The second one assumes a more complicated setting, with profiles of variable duration, interpreted as "states of health before death". In this case, an additional attribute of a profile is its length, and "death" is not a feasible health state.

We follow here the first approach, which is to consider profiles of a fixed horizon, N, interpreted as the maximum life horizon for the individual. We consider two special "health states" in any given period: death, and the "perfect" health state. Furthermore, we assume that our agent has preferences on simple probability distributions (lotteries) on profiles of health, and that such preferences satisfy the usual von Neumann and Morgenstern assumptions [Assumption 1].

In our model, however, the role played by time is quite different from that of previous models. At any given point in time, we distinguish between "future" and "past". By doing so, we are able to introduce two new assumptions: indifference of the future after death [Assumption 2], and preference independence of the future with regard to the past [Assumption 3]. Assumption 2 prevents inconsistencies related to positive evaluations of health states after death. Assumption 3 requires a weak concept of independence, natural in our context. Previous assumptions are sufficient to obtain a more general result on the representation of preferences than the additive or multiplicative structures in Bleichrodt (1996). Our representation is quite similar to the discounted QALY, but, unlike it, "endogenous" discount rates, depending on past health states, more palatable than discount factors offered in previous models, now emerge.

Our approach is closely related to that presented in Keeney & Raiffa (1976, Ch.9), on the temporary evaluation of risky assets. Our assumption that the "future" is independent of the "past", fits the idea of "past" and "future" being *generalized preference independent*. Interestingly enough, the basic assumption in Ried (1997) is that of relaxing utility independence to generalized utility independence between health states and time, in the case of chronic situations. Nevertheless, in our model, we do not consider the possibility of having states of health that are worse than death.

In Section 2 we introduce the basic model and our main assumptions. In Section 3 we present the representation results. Section 4 provides possible interpretations of our model. In Section 5, we conclude with comments, remarks and open problems.

2. The Model

We face the problem of evaluating, *health profiles* from an individual point of view. Considering discrete time periods (years, months,...), and assuming that the individual faces a *maximum* number of periods, N, a health profile can be represented by a tuple $(x_1, x_2, \ldots, x_N)$, where x_t indicates the individual's health state at period t. In order to deal with health profiles of different durations, we assume that, at each period t, a possible health state is *death*, $x_t^{\mathbf{0}}$. Note that the previous notation is flexible enough to allow for both chronic and non-chronic health profiles. A health profile $(x_1, x_2, \ldots, x_N)$, such that $x_t = a \neq x_t^{\mathbf{0}}$ is constant for $1 \leq t \leq m$, and $x_{m+1} = x_{m+1}^{\mathbf{0}}$, is nothing but a chronic health profile, such that the individual lives m periods at state a, and then dies. If the states of health are different in different periods before death, we have a non-chronic health profile.

A natural way of evaluating health profiles, from an individual point of view therefore, is to consider individual inter-temporal preferences. Furthermore, and taking into account that individual's evaluation of health profiles is performed during the individual's current period of life, uncertainty appears as a natural ingredient of the problem: the agent faces uncertainty about his future health states, and about his date of death as well. We introduce uncertainty by considering lotteries on health profiles.

The most common instrument for evaluating uncertain health profiles is the *expected utility theory.* It gives us an additional payoff: it provides us with cardinal utility measures on health profiles.

2.1. Intertemporal Preferences under Uncertainty

The maximum life horizon for an individual, N, is an exogenous variable in our model. A health profile is, therefore, a vector $x = (x_1, x_2, \ldots, x_N)$, such that, for all $t \in \{1, 2, \ldots, N\}$, $x_t \in X_t$, where X_t stands for the set of possible states of health in the t-th period of life.

Let $X = \{x = (x_1, x_2, ..., x_N) : \text{for all } t \in \{1, 2, \ldots, N\},\ x_t \in X_t\}$ the set of all health profiles.

We assume that for all $t \in \{1, 2, \ldots, N\}$, there are two special states of health in X_t, namely, x_t^0, the "state of health" corresponding to death, and x_t^*, the state of "perfect" health.

Consequently, the set of health profiles is simply the Cartesian product of the sets of health states during the different periods, namely,

$$X = X_1 \times X_2 \times ..... \times X_N$$

The generic problem of an agent is to evaluate uncertain health profiles, at different moments of his life. Let $\mathcal{L}$ be the set of simple probability distributions (lotteries) on the set X. Elements in $\mathcal{L}$ are designed by $L, M, \ldots$ An element $L \in \mathcal{L}$ is a mapping $L : X \to [0, 1]$, with finite support (i.e., taking values that are different from 0 only on a finite set of health profiles), such that $\sum_X L(x) = 1$. That is, for any health profile $x \in X$, $L(x)$ can be interpreted as the probability of profile x in lottery L. A lottery L only takes a non-zero value on a finite set of health profiles, and the afore-mentioned profiles are independent.

Let us assume that the agent has preferences defined on $\mathcal{L}$, and let us denote those preferences by the binary relation $\succsim$, interpreted as a "weak preference": for any pair of lotteries $L, M \in \mathcal{L}$, $L \succsim M$ is read as "L is at least as good as M". The associated strict preference relation, $\succ$, and indifference relation, $\sim$, are defined as follows:

$$\text{For all } L, M \in \mathcal{L},\ L \sim M \Longleftrightarrow L \succsim M \text{ and } M \succsim L$$

$$\text{For all } L, M \in \mathcal{L},\ L \succ M \Longleftrightarrow L \succsim M, \text{ and not } M \succsim L$$

We now consider the following assumption on the agent's preferences:

Assumption 1: *The preference relation* $\succsim$ on $\mathcal{L}$ *satisfies the axioms of von Neumann and Morgenstern:*

(1) $\succeq$ *is a weak order, transitive and complete.*

(2) Independence: For all $L, M, H \in \mathcal{L}$, $L \succeq M \Leftrightarrow (\frac{1}{2}L + \frac{1}{2}H) \succsim (\frac{1}{2}M + \frac{1}{2}H)$.

(3) Continuity: For all $L, M, H \in \mathcal{L}$, if $L \succ M \succ H$, there exists a real number μ, $0 < \mu < 1$, such that $\mu L + (1-\mu)H \sim M$.

Assumption 1 states that (1) our agent is able to compare any pair of lotteries on health profiles, in such a way that if a lottery L is at least as good as a lottery M, and that if M is at least as good as H, then L turns out to be at least as good as H; (2) common chances are ignored, and (3) he can fill in any evaluation gaps by adjusting probabilities. In dealing with degenerated lotteries (health profiles), its significance is easier to understand: (1) says that his valuation of health profiles is consistent and complete: faced with two health profiles he is always able to compare them, and if a profile x is perceived as better than another profile y, and y as better than z, then x is better than z; (2) a profile x is considered better than y iff any lottery in which x and some z are equally likely to be better than the lottery in which y and z are equally likely, and (3) if a profile x is considered better than y, and y better than z, we can find weights μ, $(1-\mu)$ such that y is indifferent to the lottery in which x has a probability μ and z has a probability $(1-\mu)$.

Assumption 1 has a strong consequence: it implies the existence of a cardinal function $U : X \to \mathbb{R}$ such that for all $L, M \in \mathcal{L}$

$$L \succ M \Leftrightarrow \Sigma_X L(x)U(x) > \Sigma_X M(x)U(x)$$

in words, an evaluation of lotteries on health profiles can be made according to their expected utility, using utility function U. Function U can be interpreted as a utility function for $\succsim$ on X, and is unique up to affine transformations. Namely, another function $U' : X \to \mathbb{R}$ also represents $\succsim$ on X if and only if there exist real numbers, a, b, with $b > 0$, such that, for all $x \in X$, $U'(x) = a + bU(x)$.

2.2. Conditional Preferences

Consider now an individual in a particular period of his life, t. He faces the problem of evaluating his possible health profiles. It is clear that, in period t, the individual has previously enjoyed past periods, and therefore, he only faces uncertainty about his future states of health.

In order to analyze the previous situation, let us introduce some notation. For any health profile $x = (x_1, ..., x_N) \in X$, we split x into two parts, $x = (\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$, where $\overleftarrow{x}_{t-1} = (x_1, .., x_{t-1})$ stands for the subvector of past health states, and

$\overrightarrow{x}_t = (x_t, x_{t+1}, .., x_N)$ is the subvector of future health states. Let us denote $\overrightarrow{X}_t = X_t \times \cdots \times X_N$, and $\overleftarrow{X}_{t-1} = X_1 \times \cdots \times X_{t-1}$. Thus, for all $x \in X$, we have $x = (\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$, where $\overrightarrow{x}_t \in \overrightarrow{X}_t$ and $\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1}$.

At time t, the past of the agent cannot be changed, namely, $\overleftarrow{x}_{t-1}$ is fixed. His future, however, is uncertain. Consequently, the individual faces uncertain health profiles, such that they only differ in the "future" subvector, $\overrightarrow{x}_t$, or, in other words, the agent faces health profiles of the type $x = (\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) \in \{\overleftarrow{x}_{t-1}\} \times \overrightarrow{X}_t$. Once the past has been fixed at $\overleftarrow{x}_{t-1}$, the different alternatives are reduced to the space $\overrightarrow{X}_t$.

Let $\mathcal{L}_{\overrightarrow{x}_t}$ be the set of lotteries on $\overrightarrow{X}_t$. To fix a moment in the past is equivalent to consider lotteries $L \in \mathcal{L}$, such that, provided $L(y) > 0$, then, $y = (\overleftarrow{x}_{t-1}, \overrightarrow{y}_t)$, in other words, we consider lotteries that only exhibit uncertainty in the subvector of future health states. Let L_t denote the marginal probability of lottery L on $\overrightarrow{X}_t$. From previous preference relation $\succsim$ on $\mathcal{L}$, and conditioned on the past $\overleftarrow{x}_{t-1}$, we can derive a preference relation on $\mathcal{L}_{\overrightarrow{x}_t}$ that we denote $\succsim_{\overleftarrow{x}_{t-1}}$. Then, under the von Neumann Morgenstern axioms,

$$L_t \succ_{\overleftarrow{x}_{t-1}} M_t \Leftrightarrow \Sigma_{\overrightarrow{X}_t} L_t(\overrightarrow{x}_t) U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) > \Sigma_{\overrightarrow{X}_t} M_t(\overrightarrow{x}_t) U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$$

The above-mentioned relation can be interpreted in the following way: Once the past is fixed at $\overleftarrow{x}_{t-1}$, we can find a cardinal function $U_t^{\overleftarrow{x}_{t-1}} : \overrightarrow{X}_t \to \mathbb{R}$, where $U_t^{\overleftarrow{x}_{t-1}}(\overrightarrow{x}_t) = U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$, such that $\succsim_{\overleftarrow{x}_{t-1}}$is represented by the mathematical expectation of $U_t^{\overleftarrow{x}_{t-1}}$.

Consequently, $\succsim_{\overleftarrow{x}_{t-1}}$is nothing but the restriction of $\succsim$ on $\mathcal{L}_{\overrightarrow{x}_t}$, when the profile of past health states (an element of $\overleftarrow{X}_{t-1}$) is fixed at $\overleftarrow{x}_{t-1}$. This preference can be interpreted as the conditional preference on $\mathcal{L}_{\overrightarrow{x}_t}$ induced by $\succsim$ when the profile in $\overleftarrow{X}_{t-1}$ is $\overleftarrow{x}_{t-1}$.

Next, we introduce two assumptions on conditional preferences. The first one deals with health profiles that only differ from one another after death. It requires such health profiles to be indifferent between each other, since any differences in health states after death are irrelevant. Note that this property, natural in this context, does not apply in general to other types of temporary evaluation, in particular to the case of valuation of financial assets. Furthermore, it is interesting to observe that this condition is very much in the vein of the Zero-Condition in Bleichrodt et al. (1997) and Miyamoto et al. (1998). Actually, our Assumption

2 implies the Zero-Condition for chronic health profiles, and as Condition Zero itself, is a natural medical condition.

Let $\overleftarrow{X^0}_{t-1} = \{\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1} : \text{for some } k = 1, ..., t-1,\ x_k = x_k^0\}$.

Assumption 2 (*Indifference of the future after death*).- *For all* $t = 1, \ldots, N$, *for all* $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$, *and for all* $\overrightarrow{x}_t, \overrightarrow{y}_t \in \overrightarrow{X}_t$, *we have* $(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) \sim (\overleftarrow{x}_{t-1}, \overrightarrow{y}_t)$.

Assumption 2 states that, if $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$, then $\overrightarrow{x}_t \sim_{\overleftarrow{x}_{t-1}} \overrightarrow{y}_t$. Consequently, for all $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$, the conditional preference, $\succ_{\overleftarrow{x}_{t-1}}$ is empty. Namely, whenever $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$, we have that there are no health profiles $\overrightarrow{x}_t$, $\overrightarrow{y}_t$ in $\overrightarrow{X_t}$, such that $\overrightarrow{x}_t \succ_{\overleftarrow{x}_{t-1}} \overrightarrow{y}_t$. In other words, for all $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$

$$\succ_{\overleftarrow{x}_{t-1}} = \emptyset \ (\text{or } \sim_{\overleftarrow{x}_{t-1}} = \overrightarrow{X_t} \times \overrightarrow{X_t})$$

In order to justify our next assumption, let us recall that, at time t, the preferences of the agent are reduced to his future health states, conditioned by a fixed vector of past states. Taking this into account, we assume that, in period t, he will only be concerned about his potential future states of health, independently of his past. in other words, his evaluation of his future health profiles will not take particular past health states he enjoyed into account. Individuals face uncertainty about future health profiles but, for the sake of simplicity, we shall only impose the following assumption on the set of health outcomes.

Let $\overleftarrow{Y}_{t-1} = \{\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1} : \text{for all } k = 1, ..., t-1,\ x_k \neq x_k^0\ \}$.

Assumption 3 (*Preference Independence of the future with regard to the past*).- *For all* $t = 1, \ldots, N$, *for all* $\overleftarrow{x}_{t-1}, \overleftarrow{y}_{t-1} \in \overrightarrow{Y}_{t-1}$, *and for all* $\overrightarrow{x}_t, \overrightarrow{y}_t \in \overrightarrow{X}_t$, *we have*

$$(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) \succsim (\overleftarrow{x}_{t-1}, \overrightarrow{y}_t) \Leftrightarrow (\overleftarrow{y}_{t-1}, \overrightarrow{x}_t) \succsim (\overleftarrow{y}_{t-1}, \overrightarrow{y}_t)$$

Assumption 3 indicates that preferences on the future are independent of past health states, provided that in such a past, death was not present. In terms of conditional preferences it reads $\overrightarrow{x}_t \succsim_{\overleftarrow{x}_{t-1}} \overrightarrow{y}_t \Longleftrightarrow \overrightarrow{x}_t \succsim_{\overleftarrow{y}_{t-1}} \overrightarrow{y}_t$, provided $\overleftarrow{x}_{t-1}, \overleftarrow{y}_{t-1} \in \overleftarrow{Y}_{t-1}$.

Assumptions 2 and 3 are formulated on the space of health profiles, unlike Assumption 1, which was formulated on the space of lotteries on health profiles.

Consequently, Assumptions 2 and 3 are related to the concepts of *Preference (*and *Indifference) Independence* in multi-attribute theory [see Fishburn & Keeney (1974)]. Assumption 2 implies a similar property on the space of lotteries, $\mathcal{L}_{\overrightarrow{X}_t}$. This is not the case for Assumption 3, which is weaker than the corresponding property for the space of lotteries.

From Assumptions 2 and 3 we derive that $\overrightarrow{X}_t$ is *generalized preference independent from* $\overleftarrow{X}_{t-1}$. Note that for all $\overleftarrow{x''}_{t-1} \in \overleftarrow{X}_{t-1}$ such that $\succ_{\overleftarrow{x''}_{t-1}} \neq \emptyset$, and for all $\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1}$, we have $\succ_{\overleftarrow{x}_{t-1}} \in \{\succ_{\overleftarrow{x''}_{t-1}}, \emptyset\}$ [confront Ried (1997)].

It turns out that for any value of t, the set of attributes $\overrightarrow{X}_t$ and $\overleftarrow{X}_{t-1}$ are complementary. Assumptions 2 and 3 explicitly describe how general preferential independence works for the sets $\overleftarrow{X}_{t-1}$ and $\overrightarrow{X}_t$. If $\overleftarrow{x''}_{t-1} \in \overleftarrow{Y}_{t-1}$, then $\succ_{\overleftarrow{x''}_{t-1}} \neq \emptyset$. In such a case, and for any other $\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1}$, either $\succ_{\overleftarrow{x}_{t-1}} = \emptyset$ if death is present in vector $\overleftarrow{x}_{t-1}$, by Assumption 2, or , by Assumption 3, $\succ_{\overleftarrow{x}_{t-1}} = \succ_{\overleftarrow{x''}_{t-1}}$ if death is not present in vector $\overleftarrow{x}_{t-1}$. We never allow for the possibility of a reversal of conditional preferences because we do not consider any state worse than death. Compare Ried (1997), as well as Fishburn and Keeney (1974), (1975).

As was mentioned before, *Preference Independence of the future with regard to the past* is a property that is weaker than *Utility Independence of the future with regard to the past.* The reason for assuming Preference Independence instead of Utility Independence is twofold. On the one hand, they are easier to understand, from the individual point of view. On the other hand, these assumptions are sufficient for obtaining the representation results in the next section. In fact, as it is observed in next Section, this property together with Assumption 1 imply *Utility Independence of the future with regard to the past.*

3. Representation results

Under previous assumptions, we can get a particular functional form for the utility function $U : X \to \mathbb{R}$, representing $\succsim$. The following result is obtained:

Theorem 1: *Under Assumptions 1, 2 and 3, there exist functions* $U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$,

$$U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) = a_t(\overleftarrow{x}_{t-1}) + b_t(\overleftarrow{x}_{t-1})U_t(\overrightarrow{x}_t) \qquad \textit{for all } t = 1, .., N \qquad (1)$$

such that, if we fix $U_t(\overrightarrow{x}_t) = U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$

$$U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) = d_t(x_t) + c_t(x_t)U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1}) \tag{2}$$

where $b_t(\cdot) > 0$ *provided that for all* $k \in \{1, \ldots, t-1\}$, $x_k \neq x_k^0$, *and* $b_t(\cdot) = 0$ *if for some* $k \in \{1, \ldots, t-1\}$, $x_k = x_k^0$.

Proof:

Let us start by considering a particular element in $\overleftarrow{Y}_{t-1}$, $\overleftarrow{x^*}_{t-1}$, such that for all $k = 1, \ldots, t-1$, $x_k = x_k^*$, namely, $\overleftarrow{x^*}_{t-1}$ stands for a subprofile of states of health from period 1 to $t-1$, such that in all those periods, the state of health is "perfect". By Assumptions 2 and 3, $\succ_{\overleftarrow{x^*}_{t-1}} \neq \emptyset$.

Now consider a different $\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1}$. By Assumptions 2 and 3, two cases are possible:

(1) Preferences $\succ_{\overleftarrow{x}_{t-1}}$ and $\succ_{\overleftarrow{x^*}_{t-1}}$ coincide within $\overrightarrow{X}_t$. This means that utility functions $U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$ and $U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$ represent identical preferences within $\overrightarrow{X}_t$. Consequently, for all $\overrightarrow{x}_t, \overrightarrow{y}_t \in \overrightarrow{X}_t$, $U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) \geq U(\overleftarrow{x}_{t-1}, \overrightarrow{y}_t)$ if and only if $U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) \geq U(\overleftarrow{x^*}_{t-1}, \overrightarrow{y}_t)$, namely, the functions $U(\overleftarrow{x^*}_{t-1}, -)$ and $U(\overleftarrow{x}_{t-1}, -)$ represent identical preferences on $\overrightarrow{X}_t$. Due to the cardinality condition of both $U(\overleftarrow{x^*}_{t-1}, -)$ and $U(\overleftarrow{x}_{t-1}, -)$, for all $\overleftarrow{x}_{t-1} \in \overleftarrow{Y}_{t-1}$ there exist real numbers $b_t(\overleftarrow{x}_{t-1}) > 0$ and $a_t(\overleftarrow{x}_{t-1})$ such that for all $\overrightarrow{x}_t \in \overrightarrow{X}_t$

$$U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) = b_t(\overleftarrow{x}_{t-1})U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) + a_t(\overleftarrow{x}_{t-1})$$

(2) $\succ_{\overleftarrow{x}_{t-1}} = \emptyset$. By Assumptions 2 and 3, this happens whenever $\overleftarrow{x}_{t-1} \in \overleftarrow{X^0}_{t-1}$, that is, if for some $k = 1,..,t-1$, $x_k = x_k^0$. Therefore, for all $\overrightarrow{x}_t, \overrightarrow{y}_t \in \overrightarrow{X}_t$, $U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) = U(\overleftarrow{x}_{t-1}, \overrightarrow{y}_t)$. If, for this case, we define $b_t(\overleftarrow{x}_{t-1}) = 0$, $a_t(\overleftarrow{x}_{t-1}) = U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t)$, the formula:

$$U(\overleftarrow{x}_{t-1}, \overrightarrow{x}_t) = b_t(\overleftarrow{x}_{t-1})U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) + a_t(\overleftarrow{x}_{t-1})$$

holds true for all $\overleftarrow{x}_{t-1} \in \overleftarrow{X}_{t-1}$. □

Additionally, we get the following result:

Theorem 2: *Under Assumptions 1, 2 and 3, there exist: (1) a function* $U : X \to \mathbb{R}$, *such that* $U(x_1^*, \ldots, x_N^*) = 1$, $U(x_1^0, \overrightarrow{x}_2) = 0$, *and (2) single-period* *functions* $d_t : X_t \to \mathbb{R}$, $t = 1, \dots N$; $c_t : X_t \to \mathbb{R}$, $t = 1, \dots N-1$, *such that for all* $x = (x_1, \dots x_N) \in X$, $U(x) = \sum_{t=1}^{N} d_t(x_t) \left(\prod_{\tau=1}^{t-1} c_\tau(x_\tau) \right)$, *with the convention* $\prod_{\tau=1}^{0} c_\tau(x_\tau) = 1$.

Proof:

Starting with the formula obtained in Theorem 1, let us now analyze the cardinal function $U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$. Consider the profile $(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$. Profile $(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$ coincides with profile $(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$ within $\overrightarrow{X}_{t+1}$. Consequently, we can consider profiles $(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$ and $(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) = (\overleftarrow{x'}_t, \overrightarrow{x}_{t+1})$, where for all $k = 1, \dots, t-1$, $x'_k = x^*_k$, and $x'_t = x_t$.

Two cases are possible:

(1) If $x_t \neq x^0_t$, preferences $\succ_{\overleftarrow{x'}_t}$ and $\succ_{\overleftarrow{x^*}_t}$ coincide within $\overrightarrow{X}_{t+1}$. This occurs if and only if for all $\overrightarrow{x}_{t+1}, \overrightarrow{y}_{t+1} \in \overrightarrow{X}_{t+1}$, $U(\overleftarrow{x'}_t, \overrightarrow{x}_{t+1}) \geq U(\overleftarrow{x'}_t, \overrightarrow{y}_{t+1})$ if and only if $U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1}) \geq U(\overleftarrow{x^*}_t, \overrightarrow{y}_{t+1})$, that is, functions $U(\overleftarrow{x^*}_t, -)$ and $U(\overleftarrow{x'}_t, -)$ represent identical preferences on $\overrightarrow{X}_{t+1}$. Since both $U(\overleftarrow{x^*}_t, -)$ and $U(\overleftarrow{x'}_t, -)$ are cardinal utility functions, for all $x_t \neq x^0_t$, there exist real numbers $c_t(x_t) > 0$, and $d_t(x_t)$ such that for all $\overrightarrow{x}_{t+1} \in \overrightarrow{X}_{t+1}$

$$U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) = d_t(x_t) + c_t(x_t) U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$$

(2) Whenever $x_t = x^0_t$, Assumption 2 implies that $\succ_{\overleftarrow{x'}_t} = \emptyset$. In such a case, therefore, for all $\overrightarrow{x}_{t+1}, \overrightarrow{y}_{t+1} \in \overrightarrow{X}_{t+1}$, $U(\overleftarrow{x'}_t, \overrightarrow{x}_{t+1}) = U(\overleftarrow{x'}_t, \overrightarrow{y}_{t+1})$. If, in this case, we define $c_t(x_t) = 0$, $d_t(x_t) = U(\overleftarrow{x'}_t, \overrightarrow{x}_{t+1}) = U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$,

then formula

$$U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) = d_t(x_t) + c_t(x_t) U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$$

holds true for all $\overrightarrow{x}_{t+1} \in \overrightarrow{X}_{t+1}$.

Previous results imply that:

$$\begin{aligned}
&U(x_1, .., x_N) = d_1(x_1) + c_1(x_1) U(x^*_1, \overrightarrow{x}_2) = \\
&= d_1(x_1) + c_1(x_1)[d_2(x_2) + c_2(x_2) U(\overleftarrow{x^*}_2, \overrightarrow{x}_3)] = \\
&= d_1(x_1) + c_1(x_1) d_2(x_2) + c_1(x_1) c_2(x_2)[d_3(x_3) + c_3(x_3) U(\overleftarrow{x^*}_3, \overrightarrow{x_4})] = \\
&= ... = \sum_{t=1}^{N-1} d_t(x_t) \left(\prod_{\tau=1}^{t-1} c_\tau(x_\tau) \right) + \left(\prod_{\tau=1}^{N-1} c_\tau(x_\tau) \right) U(\overleftarrow{x^*}_{N-1}, x_N)
\end{aligned}$$

Finally, by calling

$$U(\overleftarrow{x^*}_{N-1}, x_N) = d_N(x_N),$$

we obtain

$$U(x_1, .., x_N) = \sum_{t=1}^{N} d_t(x_t) \left(\prod_{\tau=1}^{t-1} c_\tau(x_\tau) \right)$$

with the convention of the empty product $\prod_{\tau=1}^{0} c_\tau(x_\tau) = 1$, and where $c_\tau(x_\tau) > 0$ whenever $x_\tau \neq x_\tau^0$, and such that $c_\tau(x_\tau^0) = 0$ for all $\tau = 1, 2, \ldots, N$.

Normalizing, by setting $U(x_1^*, \ldots, x_N^*) = 1$, $U(x_1^0, -) = 0$, we have,

$$U(x_1^*, \ldots, x_N^*) = d_N(x_N^*) = 1$$

and furthermore,

$$U(x_1^0, -) = d_1(x_1^0) = 0. \ \square$$

4. Interpretation

From Theorem 2

$$U(x_1, .., x_N) = \sum_{t=1}^{N} d_t(x_t) \left(\prod_{\tau=1}^{t-1} c_\tau(x_\tau) \right) \tag{3}$$

with

$$\prod_{\tau=1}^{0} c_\tau(x_\tau) = 1$$

Consider now $h_t : X_t \to \mathbb{R}$, defined for all $x_t \in X_t$ as $h_t(x_t) = d_t(x_t) + c_t(x_t)$ if $x_t \neq x_t^*$ and $t < N$, and $h_N(x_N) = d_N(x_N)$. Note that h_t can be interpreted as a utility function on X_t. Thus, if $x_t \neq x_t^0$, we have,

$$U(\overleftarrow{x^*}_{t-1}, x_t, \overrightarrow{x^*}_{t+1}) = d_t(x_t) + c_t(x_t)$$

$$U(\overleftarrow{x^*}_{N-1}, x_N) = d_N(x_N)$$

And, if $x_t = x_t^0$, we get:

$$U(\overleftarrow{x^*}_{t-1}, x_t^0, \overrightarrow{x}_{t+1}) = d_t(x_t^0) + c_t(x_t^0) = d_t(x_t^0).$$

Note that $d_t(x_t^0) = U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x^0}_t)$, and consequently, $d_1(x_1^0) = 0$, whereas for $t \neq 1$, $d_t(x_t^0) \neq 0$.

Thus, $h_t(x_t)$ varies between $h_t(x_t^0) = d_t(x_t^0) = U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x^0}_t)$, and $h_N(x_N^*) = d_N(x_N^*) = 1$.

As a result of the foregoing, $h_t(x_t)$ can be interpreted as the utility of enjoying a state of health x_t during a period $t \leq N$, whereas during any other period, the individual is in a perfect state of health, for a duration of life of N periods. On the other hand, $h_t(x_t^0)$ represents the utility of living in a perfect state of health up to period $t-1$, and then dying during period t. Because of this interpretation it is not natural to normalize between 0 and 1 for these "utilities on X_t".

Alternative interpretations of both $d_t(x_t)$ and $c_t(x_t)$ can be obtained as follows: Taking into account that, for all $\overrightarrow{x}_{t+1} \in \overrightarrow{X}_{t+1}$

$$U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t) = d_t(x_t) + c_t(x_t) U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$$

we could interpret that, in order to compute $U(\overleftarrow{x^*}_{t-1}, \overrightarrow{x}_t)$, we apply a discount factor $c_t(x_t)$ to $U(\overleftarrow{x^*}_t, \overrightarrow{x}_{t+1})$ [recall that $0 \leq c_t(x_t) \leq 1$], and then add a "basic" value, $d_t(x_t)$. Both the discount factor, $c_t(.)$, and the basic value, $d_t(.)$, depend upon x_t. With this interpretation, it happens that the discount factor is endogenous in this model, while in previous models it was exogenous. Furthermore, the discount factor depends on past states of health.

The structure in Formula (3) is generally referred to as *semi-separable*, since, in general, single-period decisions fail to be totally separable. In (3) we obtain, as particular cases:

(1) The additive formula, if for all τ, $c_\tau(.)$ is constant

$$U(x_1, .., x_N) = \sum_{t=1}^{N} d_t(x_t) \left(\prod_{\tau=1}^{t-1} c_\tau \right)$$

(2) The multiplicative formula, if for all $t = 1, \dots, N-1$, $d_t(.) = 0$,

$$U(x_1, .., x_N) = \prod_{t=1}^{N-1} c_t(x_t) U_N(x_N)$$

(3) A special type of multilinear formula, in case $d_t(.)$ is constant for $t = 1, \dots, N-1$,

$$U(x_1, .., x_N) = \sum_{t=1}^{N} d_t \left(\prod_{\tau=1}^{t-1} c_\tau(x_\tau) \right) \text{ with } d_t = U(x^*_{t-1}, x_t) \text{ for all } x_t,$$

or if we interpret, for all $\tau = 1, \dots, N-1$, c_τ as unconditional single-period utilities, we get

$$U(x_1, .., x_N) = \sum_{t=1}^{N} d_t(x_t) \left(\prod_{\tau=1}^{t-1} U_\tau(x_\tau) \right)$$

The previous cases are the only ones in which unconditional single-period utilities exist. In any other case, formula (3) states that utilities in any period depend on the future health states, provided that in the past, perfect health states hold.

5. Final Remarks

The *semi-separable* structure for the utility function on health profiles we obtain in this paper is, in principle, more general than the additive and multiplicative structures analyzed so far [Bleichrodt (1996)]. Our structure is derived from substituting the *mutual utility independence* between states of health of different periods assumption, with two assumptions: *indifference of the future after death,* extremely natural in our context, and *preference independence of the future on the past,* which is weaker than mutual utility independence.

Our formulation avoids some theoretical inconsistencies from previous papers. The simultaneous use of our specific assumptions prevents the possibility of giving positive marginal utilities for health states after death. Furthermore, the semi-separable structure allows us to endogenously discount the future, by using discount factors depending on the immediate past just enjoyed.

The functional form of our utility function is similar to that of the *discounted* QALY, but in its derivation there are no ambiguities. Unlike the discounted

QALY, the resulting utility function has a semi-separable structure based on valuations of shorter health profiles (and not just one), weighted by the past health states instead of by valuations of separate health states in each time period. This function allows for the evaluation of a "complete" health profile, if we apply a backward-induction procedure up to the first period of life, as well as for the evaluation of "future" health profiles only, in the case of the evaluations of "gains" in health being our only interest.

We have not considered the existence of health states that are "worse than" death. There are some technical difficulties in attempting to do so, since the presence of such states implies a reduction in the overall evaluation. The difficulty arises because of the interconnection between different periods given by Formula (3). Solving this problem is left for future research.

An alternative way of formulating the problem is to consider profiles of different sizes. To a certain extent, such an approach would be closer to that in Ried (1997) for the evaluation of chronic situations. There are two types of difficulties in doing so: on the one hand, we would have to move to a different space of profiles, thus losing the comfortable "Cartesian" structure. On the other hand, we would have to consider an additional attribute, namely the life horizon. We leave this approach for future research.

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