

A discusión

A NEW ANALYSIS OF THE DETERMINANTS OF THE REAL DOLLAR-STERLING EXCHANGE RATE: 1871-1994*

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ABSTRACT

Nonlinear models of deviations from PPP have recently provided an important, theoretically well motivated, contribution to the PPP puzzle. In recent work the equilibrium level has been modeled either as constant or as time varying with very similar statistical fits and very different economic implications. The high persistence of both PPP deviations and the proxy variables for the equilibrium real rate might create a problem of spurious coefficient significance. This paper investigates the possibility of spurious regression within nonlinear models of PPP. Monte Carlo experiments show that standard critical values are not appropriate in such a context. To illustrate we consider the real Dollar-Sterling exchange rate over the period 1871-1994. Due to many exchange rate regime changes over the sample period we employ a Bootstrap methodology that preserves the original structure of the estimated residuals and obtain new critical values of the coefficient estimates. A nonlinear (ESTAR) process with a time varying equilibrium proxied by relative wealth and relative income per capita seems to parsimoniously fit the data. Our results provide further evidence for the nonlinear model with a shifting equilibrium and the implied speed of adjustment is found to be substantially faster than previously reported in the literature.

Keywords: ESTAR, Purchasing Power Parity, Bootstrapping.

JEL classification: F31, C22, C51.

1 Introduction

Recent empirical work (e.g. Obstfeld and Taylor, 1997; Michael, Nobay and Peel, 1997; Taylor, Peel, and Sarno, 2001; and Kilian and Taylor, 2003) has reported results, for a variety of data sets,² in which purchasing power parity deviations (PPP) deviations are parsimoniously modeled as univariate nonlinear time series processes.³ The Exponential Smooth Autoregressive model (ESTAR) model of Ozaki (1985) captures the adjustment mechanism implied or derived in the theoretical analyses of PPP by a number of authors (see e.g. Dumas, 1992; Sercu et al., 1995; O’Connell and Wei, 1997; O’Connell, 1998; and Berka, 2002). In these analyses the authors demonstrate how transactions costs, transport costs or the sunk costs of international arbitrage, induce nonlinear adjustment of the real exchange rate. Whilst globally mean reverting these nonlinear processes have the property of exhibiting near unit root behavior for small deviations from PPP. Essentially small deviations from PPP are left uncorrected if they are not large enough to cover the costs of international arbitrage.

In addition nonlinear impulse response functions derived from the ESTAR models show that whilst the speed of adjustment for small shocks around equilibrium will be highly persistent, larger shocks mean-revert much faster than the “glacial rates” previously reported for linear models (Rogoff, 1996). This property of the ESTAR models provides some solution to the PPP puzzle outlined in Rogoff (1996)-namely how to reconcile the vast short run volatility of real exchange rates with the glacially slow rate of 3-5 years at which shocks appear to damp out in linear models- far too long to be explained by nominal rigidities.⁴

In the literature cited above the equilibrium real exchange rate was modeled as a constant so that the nonlinear models were univariate in structure. How-

²In particular, monthly data for the interwar and post war floating periods, quarterly data for the post war floating period as well as annual data spanning two hundred years. Analysis of the temporal aggregation of an assumed ESTAR model at the highest data frequency demonstrates that the processes estimated at each level of aggregation in the above work are consistent (see Paya and Peel, 2003).

³A smooth rather than discrete adjustment mechanism is typically chosen for two reasons. First a smooth adjustment process is suggested by the theoretical analysis of Dumas (1992). Second, as postulated by Terasvirta (1994) and demonstrated theoretically by Berka (2002), in aggregate data regime changes may be smooth rather than discrete, given that heterogeneous agents do not act simultaneously, even if they make dichotomous decisions (as is assumed for the aggregate in the Threshold model).

⁴The two nonlinear models can also provide an explanation of why PPP deviations analyzed from a linear perspective often appear to be described by either a non-stationary integrated I(1) process, or alternatively, described by fractional processes (see, e.g. Diebold, Husted and Rush, 1991). Taylor, Peel and Sarno (2001), and Pippenger and Goering (1993) show that the Dickey Fuller tests have low power against data simulated from an ESTAR model. Michael, Nobay and Peel (1997), and Byers and Peel (2003) show that data that is generated from an ESTAR process can appear to exhibit the fractional property. That this would be the case was an early conjecture by Acosta and Granger (1995). Given that the ESTAR model has a theoretical rationale whilst the fractional process is a relatively nonintuitive process, the fractional property might reasonably be interpreted as a misleading linear property of PPP deviations (Granger and Terasvirta, 1999).

ever, as is well recognized, even in relatively short spans of data real effects on the equilibrium real exchange rate may be important, and therefore play a role in explaining the Rogoff puzzle, a point raised by Rogoff himself.⁵ A variety of theoretical models, such as that of Balassa (1964) and Samuelson (1964), Lucas (1982), Stein et al. (1995) and Stein (2004) imply a non-constant equilibrium in the real exchange rate. Such effects have been found to be important in panel data analysis, though a linear framework was assumed (see e.g., Canzoneri, Cumby and Diba, 1996; and Chinn and Johnston, 1996).⁶

Some attempts have been recently made to incorporate the determinants of the equilibrium real exchange rate in linear models of the real exchange rates. Edison (1987) incorporates the determinants of the monetary model of the real exchange rate in an error-correction mechanism. Lothian (1990) and Lothian and Taylor (2000) incorporate linear and nonlinear deterministic trends, as proxy variables. Engel and Kim (1999) employ data on relative per capita real incomes motivated by the models of Balassa (1964), Samuelson (1964) and Lucas (1982). Mark (2004) employs the relative net foreign asset position, measured by the cumulated real current account as a fraction of net national product, as a proxy for relative wealth. This proxy variable is motivated by the analysis of Stein et al. (1995) and Stein (2004). The results in Edison (1987) suggested that forces exist in the economy that drive the exchange rate towards the PPP equilibrium even though the exchange rate never quite returns to it. Lothian and Taylor (2000) found that adjustment speeds were much faster in the linear autoregressive model embodying time trends than in a model which excludes them. Similar results were also reported by Peel and Venetis (2003) when a nonlinear adjustment mechanism was employed.⁷

More recently Lothian and Taylor (2004) have examined the long-run behavior of the real dollar-sterling exchange rate in a nonlinear framework employing relative per capita real income as a proxy for the equilibrium rate. Their results suggest the long-run real dollar-sterling exchange rate mean reverts, in a nonlinear manner, to a changing real equilibrium rate.⁸ The economic implications of the estimates incorporating the variable equilibrium are radically different in terms of implied speeds of adjustment. However as with models incorporating proxies for movements in the equilibrium rate in a linear framework the statistical fits and other properties of the regression residuals are nearly indistinguishable from those obtained when the proxy variables are excluded.

Naturally one interpretation of these empirical results is that models that treat the equilibrium real rate as constant may lead to seriously biased infer-

⁵Rogoff (1996) suggests for instance that the sustained Post Bretton Woods war appreciation of Japan's real exchange rate against the Dollar is consistent with the Balasa-Samuelson (BS) effects- in fact he calls it the "canonical" example of BS effects.

⁶We know from the analysis of Taylor (2001) that if the true data generation process is nonlinear then the use of the linear models can severely underestimate the speed of adjustment.

⁷Paya, Venetis and Peel (2003) incorporate the relative price of nontradables to tradables in the home and foreign countries as a proxy and report results that appear to parsimoniously fit post-Bretton Woods data for the main real exchange rates.

⁸Contrasting with the analysis of Engel and Kim (1999) where deviations are non-stationary.

ences regarding the speeds of adjustment of PPP deviations to shocks.⁹ This empirical inference may well be correct and must be theoretically near a truism. However there are reasons to be sceptical about the empirical analysis. Many of the proxy variables employed to measure the equilibrium real rate are either non-stationary by construction, for instance deterministic trends, or appear non-stationary on the basis of standard unit root tests. If in fact stationary, some of the proxies are certainly very persistent. It is therefore perhaps surprising, given the dramatically different estimated speeds of adjustment, that exclusion of these variables from the ESTAR model led to apparently parsimonious regression estimates, with residual diagnostics that were not indicative of misspecification, and that inclusion of the variables barely changes the statistical fit of the regressions or the residual properties.

Our concern is that the nonlinear framework for the estimates of the relationship between PPP deviations, which are very persistent, and the proxy variables for the equilibrium real rate, which are also very persistent, may lead to spurious significance of the proxy determinants. That this is potentially the case was demonstrated by Paya and Peel (2004) for the case of deterministic trends. Bootstrapped critical values demonstrated that, in monthly data, t -values greater than three were usually needed to reject the null at the five percent level of significance.

An understanding of the PPP adjustment process is naturally important for the specification of macroeconomic models and policy advice. It consequently seems of importance to examine further the robustness of the statistical methods employed in this important applied area of research. In this paper we choose to re-examine PPP dynamics employing a long run of annual data for the Dollar-Sterling over the period 1871-1994. For this period we can employ the two well motivated proxies for the equilibrium real rate, namely, relative per capita real income as employed by Engel and Kim (1999), and Lothian and Taylor (2004) and the proxy for relative wealth used by Mark (2004).¹⁰

We model PPP deviations as a nonlinear ESTAR process incorporating the proxies for the equilibrium real rate. A number of studies have reported empirical evidence that the volatility of real exchange rates tends to vary across nominal exchange rate regimes (see e.g. Mussa, 1986; Eichengreen, 1988; Baxter and Stockman, 1989; Flood and Rose, 1993; and Frankel and Rose, 1995). The data set we employ spans several different exchange rate regimes. As a consequence we need to allow for shifts in volatility in the error term of the empirical model in both estimation and simulation. Since in this study the major concern

⁹In this regard the empirical results of Hegwood and Papell (2002) for the Gold Standard period are particularly interesting. Balassa-Samuelson effects are one of the major arguments for the numerous equilibrium mean shifts found in Hegwood and Papell (2002) for the real exchange rates in the sixteen real exchange rate series analyzed in Diebold, Husted and Rush (1991) for the period 1792-1913 under the Gold Standard. They conclude that long-run PPP (LRPPP) does not hold but instead it is quasi PPP (QPPP) theory the one supported by their analysis of the data. They also state that the slow convergence of LRPPP is due to the unaccounted mean shifts in the equilibrium rate and that a reduction of more than fifty percent is achieved in the half-lives of shocks when those shifts are included in the model.

¹⁰We thank Nelson Mark for kindly providing us with this data set.

is the magnitude and significance of the speed of adjustment and the proxies for real equilibrium effects in the mean function we do not wish to be constrained to a particular parametric specification of the variance function. Engel and Kim (1999) and Lothian and Taylor (2004) impose a parametric model of conditional heteroskedasticity.¹¹ This approach however is not problem free. The parametric form may not adequately capture the conditional heteroskedasticity in the data. This is particularly problematical, *a priori*, when there are so many changes in regime within the sample period. We also know that different parametric specifications may yield different results (see, e.g., Wolf 2000). For these reasons, a non parametric specification of conditional variance appears particularly appropriate in our context (see, e.g., Goncalves and Killian, 2003). In this setting the wild bootstrap (see, e.g., Wu, 1986; Mammen, 1993; and Davidson and Flachaire, 2001) is an appropriate method for determining critical values. We also investigate the potential for spurious correlation via a variety of bootstrap and Monte Carlo simulations.

The empirical results we report suggest that the proxy determinants for the equilibrium real rate are significant on the basis of our bootstrapped critical values. Consequently our results provide further support for the hypothesis that the Dollar-Sterling real exchange rate is a symmetric nonlinear process that reverted to a changing equilibrium real rate in this time period. We investigate the speeds of adjustment obtained from nonlinear impulse response functions in these estimated models and compare them to the estimated models that exclude equilibrium determinants. The half life of shocks implied by the nonlinear impulse response functions were found to be faster than those obtained in models that do not include the structural determinants of the real rate.

The rest of the paper is organized as follows. In section 2 we discuss the ESTAR model considered in our empirical applications and report empirical estimates of ESTAR models where the real exchange rate long run path is modelled both as a variable or a constant. This section also presents the Monte Carlo simulation exercise for the confidence interval of the statistics. Section 3 presents the results of the estimated impulse response functions for the nonlinear models. Finally, section 4 summarizes our main conclusions.

¹¹In their long span real exchange rate study-where Balasa-Samuelson effects are excluded, Lothian and Taylor (1996) allow for shifts in volatility by using Newey-West (1987) heteroskedastic-robust estimation methods. In their recent study-which incorporates proxy variables for Balasa-Samuelson effects (Lothian and Taylor, 2004) jointly estimate the parameters of the mean function and variance function. Motivated in part by the work of Reinhart and Rogoff (2002), which suggests that the actual behavior of exchange rates may not accord exactly with the officially recorded dates of exchange rate regimes, they allow for regime shifts in a flexible, data-instigated manner. The variance function is specified as a smooth transition model in the variable time. Their specification allows for N switches in variance and allows the data to determine *de facto* rather than *de jure* both the periods of shift in variance and the amount of the shift. Their analysis, and also that of Grilli and Kaminsky (1991) and Engel and Kim (1999), clearly suggests that the variance of the real exchange cannot be modelled as fixed over the sample period.

2 Nonlinear Real Exchange Rate

We assume the true data generating process for the purchasing power parity deviations (y_t) modified for equilibrium real determinants has the form of ESTAR model reported in Michael, Nobay and Peel (1997) and Kilian and Taylor (2003):

$$y_t = \alpha + \delta x_t + e^{-\gamma(y_{t-1} - \alpha - \delta x_{t-1})^2} \left[\sum_{i=1}^n \beta_i (y_{t-i} - \alpha - \delta x_{t-i}) \right] + u_t \quad (1)$$

$$y_t = s_t + p_t - p_t^*$$

where y_t is the real exchange rate, s_t is the logarithm of the spot exchange rate (the foreign price of domestic currency), p_t is the logarithm of the domestic price level and p_t^* the logarithm of the foreign price level. α , is a constant, x_t are the determinants of the equilibrium level of the real exchange rate, γ is a positive constant -the speed of adjustment, β_i are constants and u_t is a random disturbance term.

We note that if y_t and x_t are I(1) processes then estimation of (1) is effectively a cointegration regression were the nonlinear terms act, in a constrained manner, to filter serial correlation. It seems feasible that in this case, or more generally the case where y_t and x_t are very persistent stationary series, the nonlinear ESTAR form, which has the property of being able to model regions of near unit root behavior, might engender spuriously significant estimates of δ . To investigate spurious relationships within the ESTAR model, we undertake the following Monte Carlo exercise.

2.1 Spurious regression?

We simulate an ESTAR model for the real exchange rate where the variables are calibrated to match the behavior of the real rate Dollar-Sterling over the sample period with a constant equilibrium. The true DGP is given by:

$$y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-1}-a)^2} + u_t$$

where $B(L) = (1.32, -0.32)$, $\gamma = 6$, and 1, the sample size is 1,120, discarding the first 1,000, and $u_t \sim N(0, 0.058)$ based on actual estimates of the real exchange rate Dollar-Sterling detailed in the following section.

To explore the characteristics of the coefficient estimates when a persistent variable is included as the equilibrium level of the real exchange rate, we create a variable x_t that replicates the productivity differential proxies used in previous studies (see Lothian and Taylor, 2004). The following ESTAR model with variable equilibrium is estimated:

$$y_t = \tilde{\alpha} + \delta x_t + e^{-\gamma(y_{t-1} - \tilde{\alpha} - \delta x_{t-1})^2} \left(\sum_{i=1}^n \tilde{\beta}_i (y_{t-i} - \tilde{\alpha} - \delta x_{t-i}) \right) + u_t \quad (2)$$

where $x_t = \rho_0 + \rho_1 x_{t-1} + v_t$

The autoregressive coefficient ρ_1 will take different values (1, and 0.96), and the error term v_t is distributed as a normal variable with standard error 0.05.¹² We estimate equation (2) ten thousand times and compute the confidence interval of δ at five and ten percent level as shown in Table 1.¹³ The critical values are significantly higher than the standard ones. This result highlights the possibility for spurious relationships in nonlinear models if standard critical values are considered as valid.

2.2 ESTAR model for the Dollar-Sterling real exchange rate 1871-1994

Our data consist of annual observations of the real Dollar-Sterling exchange rate from 1871 until 1994.¹⁴ The real GNP and population series are taken from Maddison (1995) and used to construct the logarithm of real income per capita. To proxy wealth we use the net foreign asset position measured by the cumulated real current account as a fraction of net national product as in Mark (2004). This variable is only available from 1900. The relative real income per capita and relative wealth between UK and US form our real equilibrium proxies.

2.2.1 Unit root Testing

Augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) unit root tests as well as the Kwiatkowski et al. (KPSS) stationarity test for PPP, the productivity and wealth differentials are reported in Panel A, Table 2. We note that the null of a unit root cannot be rejected on the basis of the PP test statistics for any of the variables whilst the ADF test suggests that only wealth is non-stationary. The KPSS rejects the null of stationarity in all three cases.

We know that unit root tests have low power if the true data generating process is ESTAR (see Taylor et al, 2001), so the results for PPP deviations have to be interpreted with that caveat. However it is clear that the variables are all very persistent.

Recent research has developed new testing procedures of the null of a unit root process against an alternative of a nonlinear exponential smooth transition

¹²This is again to match empirical evidence.

¹³Please note that as ρ_1 approaches 0, the confidence interval of δ would be similar to classical inference. We do not report those results due to space considerations.

¹⁴See Lothian and Taylor (1996) for a detailed description of the real exchange rate data.

autoregressive (ESTAR) process, which is globally mean reverting.¹⁵ Kapetanios et al. (2003, KSS hereafter) derived a unit root tests of nonlinear (and asymptotically stationary) alternatives¹⁶ that has better power than the standard Dickey-Fuller test in the region of the null. They test the null hypothesis of a linear model, $H_0 : \gamma = 0$.

Under the null hypothesis, and of unit root ($\varphi_1 = 1$), using a first order Taylor approximation to the transition function around point $\gamma = 0$, they get the following auxiliary regression where lags of the dependent variable might be included in the case of error autocorrelation

$$\Delta y_t^* = \sum_{j=1}^p \Delta y_{t-j}^* + \delta y_{t-1}^{*3} + error \quad (3)$$

Testing for $\delta = 0$ against $\delta < 0$ corresponds to testing the null hypothesis (3), and the t -statistic is given by

$$t_{NL}(\hat{c}') = \frac{\hat{\delta}}{s.e(\hat{\delta})} \quad (4)$$

where $s.e(\hat{\delta})$ denotes the estimator standard error. The asymptotic distribution of (4) is not standard since under the null, the underlying process is nonstationary.¹⁷

Kiliç (2003) developed an alternative testing method to detect the presence of nonstationarity against nonlinear but globally stationary STAR process that differs from KSS in the way it deals with the nuisance parameter that occurs under the null. As the author claims, the advantage of Kiliç procedure over KSS is twofold. First, it computes the test statistic even when the threshold parameter needs to be estimated in addition to the transition parameter. Second, it claims to have higher power. Kiliç test applies to the following expression,

$$\Delta y_t^* = \phi y_{t-1}^{*3} (1 - \exp(-\gamma(z_t - c)^2)) + error \quad (5)$$

where z_t is the transition variable, in this case (Δy_{t-1}^*). He tests the null hypothesis of $H_0 : \phi = 0$ (unit root case) against $H_1 : \phi < 0$. To overcome identification problems (of γ and c) under the null hypothesis, he uses the largest possible t-value for ϕ over a space of values for γ and c , in particular ($\sup -t$).

¹⁵ Escribano and Jorda (1999) extended the familiar nonlinearity tests procedure formulated by Terasvirta (1994) and proposed a new specification strategy to choose between ESTAR and LSTAR models with higher power. We then applied the Escribano and Jorda methodology and found that we could reject the null of linearity for both the model with constant equilibrium and varying equilibrium with p-values of 0.029 and 0.007 respectively. In both cases the ESTAR specification was preferred over the LSTAR specification.

¹⁶ KSS examine the properties of their tests under three different assumptions of stochastic processes with nonzero means and/or linear deterministic trends. In the cases where y_t^* exhibits significant constant or trend, y_t^* should be viewed as the de-measured and/or de-trended variable.

¹⁷ The test statistic (4) converges weakly to a functional of Brownian motions (see KSS).

We apply both the KSS and Kilic¹⁸ tests to our data set. In both cases the null hypothesis that y_t is a random walk process could be rejected at 5% in favor of an STAR alternative.

2.2.2 ESTAR Estimation

Estimates of the PPP relationship with the equilibrium level assumed constant are reported in Table 3. This table reports the Newey-West “t-ratio” for the estimated transition parameter γ . However, empirical marginal significance levels need to be computed through simulation since under the null y_t follows a unit root and standard t-values are not valid. The last row in table 3 presents the Monte Carlo p-value of γ assuming that the true DGP process for y_t is a second-order unit root process.¹⁹ According to these values the real Dollar-Sterling exchange rate follows a ESTAR model with constant equilibrium as estimated in table 3. The diagnostic statistics also suggest no remaining structure in the residuals.

In Table 4 we report empirical results in which our two proxies for the equilibrium real exchange rate are included separately in the ESTAR model and in Table 5 when both are included. In either case, the real Dollar-Sterling rate appears to be parsimoniously modeled as a symmetric nonlinear process with a time varying equilibrium. We also note that the fits of the regressions in terms of coefficients of determination are very close to those in Table 3 raising again the issue of possible spurious relationships.

2.2.3 Wild Bootstrap and Robustness Analysis

Given the simulation results reported in section 2.1 it is important we employ appropriate standard errors to determine the significance of the equilibrium estimates of the real exchange rate. Accordingly we employ a bootstrap procedure.

Our null hypothesis is that the coefficients (δ) on the proxy variables for the equilibrium real exchange rate are zero. Accordingly we simulate an “artificial” series for y_t , denoted by \hat{y}_t^b , with the coefficients on the equilibrium determinants set equal to zero:

$$\hat{y}_i^b = \hat{\alpha} + e^{-\hat{\gamma}(y_{t-1} - \hat{\alpha})^2} \left(\sum_{k=1}^n \hat{\beta}_k (y_{t-k} - \hat{\alpha}) + u_i^b \right) \quad (6)$$

where $i = 1, \dots, 10,000$

¹⁸Kilic suggests that making the interval too wide could make the transition function to be flat for large values of γ . We have then decided to use an interval for γ according to values usually found in our simulation results for each degree of aggregation. The values of C have been selected as the corresponding to the ordered values of $|z|$ and discard 10% of the highest and smallest values.

¹⁹We first estimate y_t as a linear AR process. We used the estimated coefficients and variance to calibrate the DGP in the Monte Carlo simulations. The empirical distribution of the “t-ratio” is obtained with 10,000 replications.

The residuals u_i^b are obtained from bootstrapping, the estimated residuals obtained (\hat{u}_t) from the ESTAR models reported in those tables which include the equilibrium determinants (tables 4 and 5). However to allow for heteroskedasticity of changing form due to either changes in exchange rate regimes or particular historical periods we employ the wild bootstrap (see, e.g., Wu, 1986; Mammen, 1993; and Davidson and Flachaire, 2001, Goncalves and Killian, 2003).

Employing each time the actual residuals from regression (1) we create a new series of residuals based on these estimated residuals as

$$u_i^b = \hat{u}_t \epsilon_i$$

where ϵ_i is drawn from the two-point distribution

$$\begin{aligned} \epsilon_i &= \{-5^{0.5} - 1\}/2 \text{ with probability } p = \frac{(1 + 5^{0.5})}{2(5^{0.5})} \\ \epsilon_i &= \{5^{0.5} + 1\}/2 \text{ with probability } (1 - p) \end{aligned}$$

The ϵ_i are mutually independent drawings from a distribution independent of the original data. The distribution has the properties that $E\epsilon_i = 0$, $E(\epsilon_i^2) = 1$, and $E(\epsilon_i^3) = 1^{20}$. As a consequence any heteroskedasticity in the estimated residuals, \hat{u}_t , is preserved in the created residuals, u_i^b . We create 10,000 sets of these residuals.

As indicated by equation (6), the generated sequence of artificial PPP data has a true δ coefficient of zero. However, when we regress the artificial PPP data for a given bootstrap sample (\hat{y}_i^b) on x_i within the ESTAR model, estimated values of δ that differ from zero will result. This procedure provides an empirical distribution for δ and their associated standard errors that is based solely on resampling the residuals of the original regression. The idea in 10,000 replications is to determine the appropriate t -values and F -statistic so we do not reject the null of $\hat{\delta} = 0$. These critical values can then be employed to determine whether the estimates of $\hat{\delta}$ obtained in estimates of (1) reject the null. The last row in tables 4 and 5 report the p-values obtained through the simulation exercise for the estimated t and F values. The hypothesis that the real Dollar-Sterling rate follows an ESTAR process with time varying equilibrium proxied by productivity differential and or wealth cannot be rejected at the usual significance level.²¹

Figure 1 plots the real Dollar-Sterling rate along with the estimated constant and our estimate of the variable equilibrium (PPPEQ) reported in tables 3 and 5 respectively. This figure appears to be consistent with the stylized historical facts of the real equilibrium Dollar-Sterling real exchange rate over the sample period. An overvalued sterling at the beginning of the twenties (return to Gold Standard) and forties (Bretton Woods system) when the pound was set to the

²⁰We also employed the wildbootstrap

where $\epsilon_i = 1$ with $p=0.5$ and $\epsilon_i = -1$ with $p=0.5$. The results were not changed.

²¹The highest p-value is obtained when the proxy variable for wealth is considered as the only equilibrium level and in that case the null can be rejected at 5.9%.

Gold Standard and pre-WWII (\$4.86) parities is widely acknowledged. The US deflation of the thirties forced the real sterling rate under its equilibrium level. Undervaluation came back in the fifties when, within the Bretton Woods system, British inflation was progressively increasing with respect to US inflation. Finally the pound had to be devalued (to \$2.40). The strong dollar in the first half of the eighties and a weak dollar in the second half appears to explain the latter part of the graph.

We also computed the least squares relationships between observed PPP and our estimates of the equilibrium PPP rate. We obtained an R^2 of 37% demonstrating the quantitative importance of the variable real exchange rate equilibrium in explaining PPP movements.

3 Nonlinear Impulse Response Functions

In this section we compare the speed of mean reversion of the nonlinear model of real exchange rates with constant equilibrium and with shifting equilibrium. To calculate the half-lives of PPP deviations within the nonlinear framework we must take into account that a number of properties of the impulse response functions of linear models do not carry over to the nonlinear models.²² Koop, Pesaran and Potter (1996) introduced the Generalized Impulse Response Function (GIRF) for nonlinear models. The GIRF is defined as the average difference between two realizations of the stochastic process $\{y_{t+h}\}$ which start with identical histories up to time $t-1$ (initial conditions). The first realization is hit by a shock at time t while the other one is not:

$$GIRF_h(h, \phi, \omega_{t-1}) = E(y_{t+h}|u_t = \phi, \omega_{t-1}) - E(y_{t+h}|u_t = 0, \omega_{t-1}) \quad (7)$$

where $h = 1, 2, \dots$, denotes horizon, $u_t = \phi$ is an arbitrary shock occurring at time t and ω_{t-1} defines the history set of y_t . Given that ϕ and ω_{t-1} are single realizations of random variables, (7) is considered to be a random variable. Since analytic expressions for the conditional expectations involved in (4) are not available for $h > 1$, we used stochastic simulation (Gallant et al., 1993; and Koop et al., 1996; for a detailed description) to approximate (7). For each history, we construct 5000 replications of the sample paths $\hat{y}_0^*, \dots, \hat{y}_h^*$ based on $u_t = \delta$ and $u_t = 0$ by randomly drawn residuals as noise for $h \geq 1$. The difference of these paths is averaged across the 5000 replications and it is stored. At the end, we average across histories.²³ We set $\phi = 5\%, 20\%$, and 30% . The different values of ϕ 's would allow us to compare the persistence of very large and small shocks.

²²In particular, impulse responses produced by nonlinear models are a) history dependent, so they depend on initial conditions b) they are dependent on the size and sign of the current shock and c) they depend on the future shocks as well. That is, nonlinear impulse responses critically depend on the "past", "present" and the "future".

²³We set to $\max\{h\} = 48$.

The persistence of the shocks could be evaluated as suggested by Koop et al. (1996), using the dispersion of the distribution of (7) as horizon h increases.²⁴ However, the main issue is to compute how many periods (h) are necessary for the impulse response function to be “significantly” reduced. The complexity of the unit root exponential model produced the following peculiar behavior. In the case of nonlinear models, monotonicity need not hold.²⁵ Hence, we calculate the *λ-life* of shocks for $(1 - \lambda) = 0.25, 0.50$ and 0.80 where $1 - \lambda$ corresponds to the fraction of the initial effect u_t that has been absorbed.²⁶

For the nonlinear model with constant equilibrium the half life of the shocks is one year more than the half life of the nonlinear model with varying equilibrium. Moreover, these half life shocks estimated under the nonlinear model are significantly lower (two to three years) than the ones obtained within a linear framework by Lothian and Taylor (1996).

3.1 Misleading Speeds of Adjustment

In this section we investigate the implications for estimates of the adjustment parameter if the true data-generating process is described by equation (1) including equilibrium determinants, but the ESTAR is misspecified and estimated without them. We generated 10,000 samples simulating 1,120 observations, discarding the first 1,000, from an ESTAR model which included equilibrium determinants employing the coefficient estimates from table 5, and the bootstrapped residuals from the estimated non-linear model. We then estimated the ESTAR model with only the constant term as the equilibrium variable. Table 7 displays the mean, median and standard deviation of the estimated speed of adjustment as well as the percentage of times that the model would be considered significant. Omitting the equilibrium variables in the estimation of the nonlinear model would suggest slower speed of adjustment in the real exchange rate.

4 Conclusions

Nonlinear adjustment of PPP deviations is theoretically well motivated and recent research suggests that it provides a parsimonious empirical fit to a variety of PPP data sets. These studies show that modelling the equilibrium level of the PPP as a time-varying process rather than as a constant generates radically different estimates of the speed of adjustment of the real exchange rate to shocks.

²⁴For example, if (7) based on two initial shocks, $\delta_1 < \delta_2$, produces $\widehat{GIRF}_h(h, \delta_1, \omega_{t-1})$ more dispersed at $h = 2$ than $\widehat{GIRF}_h(h, \delta_2, \omega_{t-1})$ then smaller shocks are more persistent than larger shocks. For any given horizon, impulses based on larger shocks were less dispersed. But as horizon h increases, the dispersion of the (7) distribution is getting larger. This is due to the random walk behavior of the model for small deviations from equilibrium. Indeed, it was the case that increasing the shock magnitude reduced the dispersion of estimated impulse response functions at all horizons.

²⁵For a full discussion on different measures of half-life shocks and estimating procedures see Murray and Papell (2002) and Kilian and Zha (2002).

²⁶See Van Dijk, Franses and Boswijk (2000, p.7)

In particular adjustment speeds are much faster. From the perspective of the Rogoff real exchange rate puzzle these results are perhaps welcome. However we noted the possibly disturbing feature of the empirical results that the estimates of nonlinear models where the equilibrium is treated as a constant are near statistically indistinguishable from those where it is allowed to vary. Given that both real exchange rate deviations and the proxies employed to measure the equilibrium real exchange rate are very persistent series there appears to be the potential for a spurious regression problem. Our Monte Carlo simulations ratify that this could be the case.

Given this we analyze the important case of the real exchange rate Dollar-Sterling over the period 1871-1992. The real exchange rate is modeled as an ESTAR process in which we model the equilibrium real exchange rate as dependent upon differences in real income per capita and wealth. There were numerous changes in exchange rate regime and other historical periods that suggest the assumption of constant real exchange rate volatility is untenable over our sample period. Accordingly we construct critical values for the coefficients of the nonlinear model based on wild bootstrap simulation which accounts nonparametrically for any non-normality and heteroskedasticity in the equation residuals.

Our results suggest that the real Dollar-Sterling rate is well described by a nonlinear ESTAR process that reverts to a time-varying equilibrium level which is well proxied by either or both of relative wealth and income per capita. The real equilibrium implied by our estimates appears to be consistent with the stylized historical facts. In addition the speed of adjustment to the equilibrium level is significantly faster than the ones obtained within a constant equilibrium framework.

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Table 1. Results for artificially generated ESTAR model
and estimated with productivity differential variable

True DGP:	$y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-1}-a)^2} + u_t \quad u_t \sim N(0, 0.058)$			
	$x_t = \rho_0 + \rho_1 x_{t-1} + v_t \quad v_t \sim N(0, 0.05)$			
Estimated model:	$y_t = a + \delta x_t + [\beta_1(y_{t-1} - a - \delta x_{t-1}) + \beta_2(y_{t-2} - a - \delta x_{t-2})]e^{-\gamma(y_{t-1}-a-\delta x_{t-1})^2}$			
	Dollar-Sterling 1870-1994			
Confidence	$\gamma = 1$	$\gamma = 1$	$\gamma = 6$	$\gamma = 6$
Interval	$\rho_1 = 1$	$\rho_1 = 0.96$	$\rho_1 = 1$	$\rho_1 = 0.96$
level: 95%	(-3.00,3.00)	(-2.35,2.35)	(-2.92,2.95)	(-2.36,2.34)
level: 90%	(-2.31,2.31)	(-2.00,2.00)	(-2.32,2.36)	(-1.98,1.97)

Table 2. Unit root Tests

Panel A	ADF	PP	KPSS
y_t	-3.06*	-2.71	0.64*
x_t^i	-3.58*	-2.69	1.18**
x_t^w	-1.75	-1.30	0.36*
Panel B	KSS	Kiliç	
y_t	-3.58*	-2.79*	

An asterisk *(**) denotes significance at 5(1) percent level. x_t^i denotes income differential and x_t^w wealth differential

Table 3. Results for estimated ESTAR model

Estimated model: $y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-1}-a)^2}$					
Dollar-Sterling 1870-1992					
a	β_1	β_2	γ	s	R^2
1.55	1.36	$1 - \beta_1$	5.67	0.067	0.75
(0.02)	(0.14)		(2.12)		
			[0.045]		
Diagnosics:	$JB = 0.39$		$Q(1) = 0.25$		
$Q(4) = 0.20$	$A(1) = 0.88$		$A(4) = 0.56$		

Notes: Figures in brackets are the Newey-West standard errors. s denotes standard error of . regression $Q(1)$, $A(1)$ and JB are the p-values of the Eitrheim and Terasvirta (1996) LM test for autocorrelation in nonlinear series for l number of lags, LM test for ARCH effects up to l lags, and the normality Jarque-Bera test, respectively. Figures in square brackets represent the p-value of the γ parameter obtained through Bootstrap simulation.

Table 4. Results for estimated ESTAR model with productivity variable

Estimated model: $y_t = a + \delta_1 x_t + [\beta_1(y_{t-1} - a - \delta_1 x_{t-1}) + \beta_2(y_{t-2} - a - \delta_1 x_{t-2})]e^{-\gamma(y_{t-1} - a - \delta_1 x_{t-1})^2}$						
Dollar-Sterling 1870-1992 (income per capita)						
a	δ_1	β_1	β_2	γ	s	R^2
1.75	0.37	1.32	$1 - \beta_1$	6.20	0.066	0.76
(0.05)	(0.09)	(0.10)		(1.91)		
	[0.007]			[0.015]		
Diagnostics:		$Q(1) = 0.23$	$Q(4) = 0.35$			
$A(1) = 0.72$		$A(4) = 0.28$	$JB = 0.03$			
Dollar-Sterling 1900-1992 (wealth)						
a	δ_1	β_1	β_2	γ	s	R^2
1.55	1.52	1.34	$1 - \beta_1$	8.74	0.066	0.76
(0.02)	(0.51)	(0.11)		(2.93)		
	[0.059]			[0.026]		
Diagnostics:		$Q(1) = 0.54$	$Q(4) = 0.06$			
$A(1) = 0.32$		$A(4) = 0.86$	$JB = 0.20$			

Diagnostic statistics are the same as in previous table. Figures in square brackets represent the p-value of the δ_1, γ parameters obtained through Bootstrap and Monte Carlo simulation, respectively

Table 5. Results for estimated ESTAR model with productivity variables

Estimated model: $y_t = a + \delta_1 x_t^i + \delta_2 x_t^w + [\beta_1(y_{t-1} - a - \delta_1 x_{t-1}^i - \delta_2 x_{t-1}^w) + \beta_2(y_{t-2} - a - \delta_1 x_{t-2}^i - \delta_2 x_{t-2}^w)]e^{-\gamma(y_{t-1} - a - \delta_1 x_{t-1}^i - \delta_2 x_{t-1}^w)^2}$								
Dollar-Sterling 1870-1994 (wealth and income per capita)								
a	δ_1	δ_2	β_1	β_2	γ	F	s	R^2
1.66	1.30	0.21	1.37	$1 - \beta_1$	10.35	32.91	0.065	0.77
(0.065)	(0.55)	(0.10)	(0.12)		(3.39)	(0.000)		
					[0.023]	[0.004]		
Diagnostics:		$Q(1) = 0.75$	$Q(2) = 0.24$					
$A(1) = 0.20$		$A(4) = 0.70$	$JB = 0.17$					

The F-statistic tests the jointly significance of x_t^i and x_t^w in the equation. Figures in brackets are the corresponding p-value according to classical inference. Figures in square brackets represent the p-value of the t and F statistic obtained through Bootstrap simulation.

Table 6. Estimated half-lives shocks in years for Dollar-Pound 1900-1994

<i>Shock</i>	<i>absorbtion</i> $(1 - \lambda)$	Real exchange rate	
		Constant equilibrium	With wealth and income
		$\hat{\gamma} = 5.67$	$\hat{\gamma} = 10.35$
5%	25%	2	2
	50%	4	3
	80%	9	8
20%	25%	1	1
	50%	4	2
	80%	9	7
30%	25%	1	0
	50%	3	1
	80%	8	5

Table 7: Estimated ESTAR model without equilibrium determinants when the true ESTAR model contain a time-varying equilibrium

Simulated model	$y_t = a + \delta_1 x_t + \delta_2 z_t + [\beta_1(y_{t-1} - a - \delta_1 x_{t-1} - \delta_2 z_{t-1}) + \beta_2(y_{t-2} - a - \delta_1 x_{t-2} - \delta_2 z_{t-2})]e^{-10(y_{t-1} - a - \delta_1 x_{t-1} - \delta_2 z_{t-1})^2} + u_t^b$			
Estimated model	$y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-1} - a)^2}$			
	<i>Mean</i> $\hat{\gamma}$	<i>Median</i> $\hat{\gamma}$	<i>Std Dev</i>	<i>Significance</i>
u_t bootstrap	2.79	1.40	3.45	30.55%

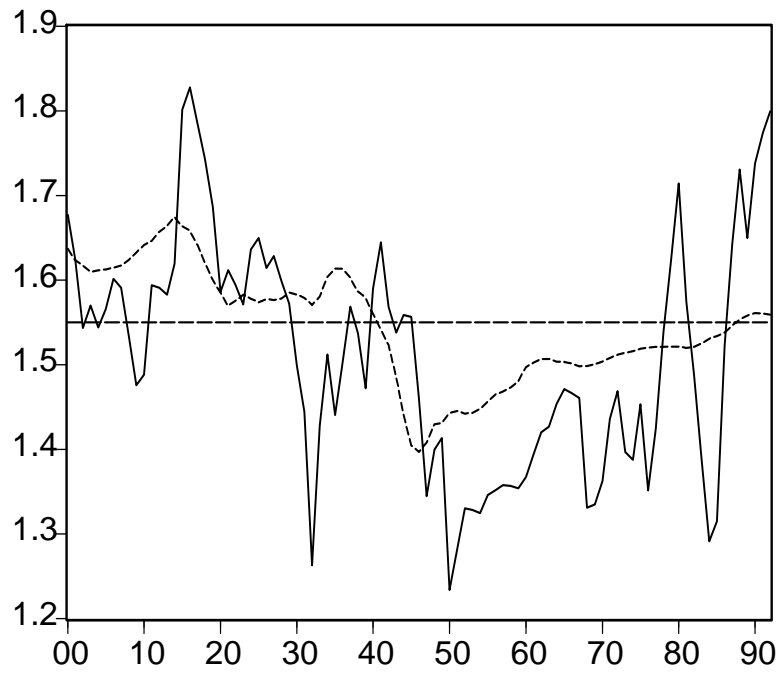


Figure 1. Solid line: Real Dollar-Sterling rate. Dashed line: Constant equilibrium. Dotted line: Variable equilibrium