A discusión

THE INSIDER'S CURSE*

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ABSTRACT

This paper studies an auction model in which one of the bidders, the *insider,* has better information about a common component of the value of the good for sale, than the other bidders, the *outsiders*. Our main result shows that the insider may have incentives to disclose her private information if she faces sufficiently strong competition from the outsiders. We also show that the insider can protect the value of her private information by hiding her presence in the auction to the outsiders. Finally, we analyze the implications of information revelation on the efficiency of the auction and on the auctioneer's expected revenue.

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1 Introduction

This paper provides an auction model that shows that strong competition, in the sense of either a large number of competitors or competitors with ex ante[1] greater willingness to pay, may create bidders' incentives to disclose their private information. This result provides new insights into a long pending question whose origins may be traced back to Hayek (1945): does competition in a market aggregate information privately held by the agents? Strategic models of price formation, for instance Milgrom (1979, 1981), show that there may be situations in which agents' incentives to manipulate the market can preclude the price to fully aggregate agents' private information.[2] Our results show that competition may give rise to a direct path to information aggregation alternative to the price.

The existence of incentives to disclose private information is not novel. In fact, our result builds on a well-known result in strategic games: to hold private information may be bad for an agent if known by the other agents. This is, for instance, the root of the adverse selection problem, a central concept in information economics. Similar results have also been obtained in other market settings with different intuitions. For instance, Milgrom and Weber (1982a) and Ottaviani and Prat (2001) show that full information disclosure may be beneficial to a monopolist.[3]

However, these models differ from ours in one important aspect. They assume that an agent's private information refers to the preferences of the agents on the other side of the market.[4] We, on the contrary, assume that it refers to the preferences of agents on the same side of the market. More precisely, we assume that some bidders compete

[1]By ex ante we mean taking expectations with respect to the bidders' private information.

[2]Although there are other papers that show that the price can fully aggregate agents' private information in some limit cases, for instance Pesendorfer and Swinkels (1997) and Pesendorfer and Swinkels (2000).

[3]The incentives of monopolistic auctioneers to reveal information that affects bidders' values has also been studied by Bergemann and Pesendorfer (2004), Ganuza (2004), Ganuza and Penalva-Zuasti (2004), and Hagedorn (2004).

[4]This means that if the agent is a buyer, she has private information about the sellers' preferences, and that if she is a seller, she has private information about the buyers' preferences.

for an indivisible good, and one of them, the *insider*, privately knows a common component of the bidders' values, whereas the others, the *outsiders*, only have noisy estimates. We show that the insider may get less expected utility in this situation than when the common value is common knowledge if bidders' values also have private components. It is in this sense that we say that the insider has incentives to disclose her private information about the common component.

There are many real-life auctions that are captured reasonably well with our set-up. An example is the auction of a license to operate a service, say mobile telephones, rubbish collection, or highways. The common value component may come from the demand of the service whereas the private value may be due to differences in the bidders' cost structure. The insider may be, for instance, a bidder who has been operating the same or a related service for some time before. Another example is the auction of art and antique objects. The common value may be associated with the quality of the object and the private component with the taste of the bidder. In this case, we may identify insiders with expert dealers.

Our results may also be applied to other less formalized selling processes resembling an auction. An example is a takeover battle and, as a particular case, a leveraged buy-out. The common value of the firm may be the value of the company's assets and the private value can be due to portfolio decisions, synergies or buyers' differences in managerial skills or aims. Insiders may be the management team in the case of leveraged buy-outs, a white knight[5] in the case of a hostile take-over, or simply buyers that have special links with the firm for sale.[6] Another example is the job market in which firms compete for a worker. The common component may refer to the ability of the worker, and the private component to the suitability of the matching between the firm and the worker. In this case, the obvious insider is the firm that currently hires the worker.

[5]A white knight is a bidder that has been persuaded by the management team of another company to rescue the company from a hostile takeover.

[6]Some other authors, for instance Bulow, Huang, and Klemperer (1999), have considered that insiders may have a private value advantage. We consider the complementary case that insiders' advantage may be in terms of better information.

An important aspect of our analysis is that we model the market as an open ascending auction.[7] In this mechanism, the price increases continuously starting from a sufficiently low value. Bidders may exit the auction at any time, and the auction finishes when no more than one bidder remains in the auction. Quite crucially, bidders observe at all times the identity of the bidders that still remain in the auction. Note that this is not only a standard auction mechanism, but also a reasonable model of a real-life bargaining situation in which one single seller receives public offers by several buyers.

In this auction format, the fact that the insider remains in the auction may increase the outsider's incentives to do so. For instance, if an outsider knows to have a greater private value than the insider, she may be willing to outbid any insider's bid, even bids above her expected value conditional on her private information. The reason is that such an outsider finds it profitable to win at any price at which the insider does so. Note, for instance, the following quote from Cassady (1967):

> A collector of antiques, if reasonably sure that an opponent is a dealer who must allow for a retail markup in his bidding, may consider himself safe in raising the latter's bid[...]

Of course, in a world in which bidders have uncertainty about the other bidders' private values, the above argument does not work. Nevertheless, we shall show that outsiders with large private value tend to gamble that they have a higher private value than the insider and, as a consequence, tend to bid higher whenever the insider does so. We shall explain this phenomenon as a consequence of the interplay between the winner's and the loser's curse.[8]

[7]Note, however, that our analysis would also apply to the second price sealed bid auction when there are two bidders. The reason is that in this case, the open auction and the second price sealed bid auction are strategically equivalent.

[8]The study of the incentives to bid in terms of the winner's and the loser's curse was introduced in multi-unit auctions by Pesendorfer and Swinkels (1997) in symmetric models, and by Hernando-Veciana (2004b) in asymmetric models. They also provide a formal definition of the winner's curse and the loser's curse in terms of statistical events. In the current paper, we show that these effects may also be useful to understand the incentives to bid in single-unit auctions.

Clearly, the above effect would not exist, if the outsiders knew the common value. This explains why the insider may prefer to fully reveal the common value when the insider expects to compete with outsiders with large private values. This is what happens in the two cases we study: when there is a sufficient number of outsiders, and when outsiders have sufficiently larger private values than the insider ex ante. We shall refer to the case in which the insider is better off revealing the common value as the *insider's curse*, as in some sense, the insider regrets being an insider.

It is remarkable that the insider's curse depends on the fact that the outsiders are aware of the presence of the insider and that they know her identity. This suggests that the insider may protect herself from the insider's curse if she can hide her presence to the outsiders. Quoting Cassady (1967):[9]

> Another element of the bidding strategy is to decide whether one should execute bids himself or employ an agent to do so. A buyer who is known as an outstanding figure in an important field frequently chooses the latter approach, for his own bidding, if detected, suggests that the item is highly desirable and therefore arouses strong competition[...]

Our results show that whenever there is an insider's curse, the insider is better off when the outsiders have uncertainty about her presence, than when the outsiders know regardless of whether the insider reveals the common value. Nevertheless, it is important to note that there may be many real-life cases in which it may be infeasible for the insider to hide. Often, it may be under the control of the auctioneer whether the insider can hide, for instance by allowing anonymous bids.

The game with a hidden insider is also interesting from a technical point of view. It is the first model, to the best of our knowledge, that studies a non-trivial auction game in which bidders have uncertainty not only about bidders' preferences, but also about the precision of the information that other bidders have. The technical challenge of this model is that, in equilibrium, bidders' beliefs must take into account the different

[9]Quite interestingly, Cassady (1967) also argues that buyers with high private interest in the good may profit by revealing their identity, as predicted by our model.

bid behavior of bidders with differences in the precision of their information. In our model, this problem is even richer due to the dynamic structure of our auction game.

The plan of the paper is as follows. In the next section, we review the related literature. Section 3 presents a simple model that illustrates the main insights of our results. Section 4 and Section 5 constitute the core of our theoretical analysis. Section 4 covers the study of the insider's curse and Section 5 the insider's incentives to hide. Section 6 concludes. We also include an Appendix with the most technical proofs.

2 Related Literature

In a closely related paper, Milgrom and Weber (1982b) study a model that, despite the similarity to ours, leads to opposite conclusions. In their set-up, an insider never wants to reveal her private information, and she always prefers to signal her presence to the other bidders. We think that there are two reasons for these discrepancies.

The first one is that they consider a different auction set-up, a sealed bid auction. In this format, there is less information revelation along the game and, as a consequence, a stronger winner's curse that makes outsiders less willing to outbid the insider.[10] In fact, we show that, in our model, the same results as in Milgrom and Weber (1982b) arise when the winner's curse is sufficiently strong. This happens in our model when either there are few outsiders or they have a low private value ex ante.

The other reason is that in their model there is no private value component. Clearly, in this case, once the insider reveals the common value, she has no private information and, hence, does not get positive expected utility. In fact, Larson (2004) shows that in a two bidder-second price sealed bid auction, open[11] information acquisition about the common value[12] has positive returns. Notice that our results are consistent with

[10]This idea is explored by Hernando-Veciana (2004a) to show that additional private information about a common value component is more valuable in a sealed bid second price auction than in an open ascending auction.

[11]We mean by open information acquisition that the decision of a bidder to acquire information is observed by the other bidders.

[12]Note that in our paper we do not address the issue of information acquisition but rather of information disclosure. However, both issues are closely related. In fact, if all signals are independent

these findings, since in our model, the insider's curse vanishes as the private value component becomes sufficiently small.

Technically, Larson's analysis is similar to ours since he also considers the effect of private value components. He also arrives at the conclusion that more private information about a common value component by a bidder may increase the dispersion of the other bidders' bids. However, the coincidences between our paper and his ends here as Larson's focus is on the problem of equilibrium selection in a pure common value auction. In this sense, Larson studies how the introduction of small private value perturbations allow us to select different equilibriums.

Hagedorn (2004) provides results in the same direction as Milgrom and Weber (1982b) and Larson (2004). He shows that marginal open information acquisition has positive value in an efficient auction when the number of bidders is sufficiently large. However, the key assumption that leads to opposite results seems to be that in Hagedorn's model, bidder's private information is a one dimensional signal that is informative of both the private and the common value. Hence, to acquire a more informative signal does not only mean to get more information about the common value, as in our model, but also about the private value.

Campbell and Levin (2000), and Benoit and Dubra (2003) have also shown that additional private information can decrease a bidder's expected utility. The intuition of their results is, however, quite different. In the first paper, a bidder acquires information about the other bidder's private information, and not about the common value of the good, as in our paper. In the second paper, more private information decreases the bidder's expected utility not in the auction game, but in an information revelation stage played before the auction. Benoit and Dubra (2003) also provide an example of an auction game in which a bidder is ex ante better off when she fully discloses her private information. Nevertheless, their intuition is completely different as they assume for their example a pure private value setting.

(as is the case in our Section 3, or in Larson's paper), it may be shown that a version of the revenue equivalence theorem implies that open information acquisition of an additional signal informative of the common value has negative returns if and only if there is an insider's curse with respect to the same additional signal.

3 A Simple Model

We start our analysis with a simple example. We assume that there is a pool of bidders $\mathcal{I} \equiv \{0, 1, ..., n\}$ who participate in the sale of an indivisible unit of a good through an open ascending auction that has been called the *Japanese auction.* According to Milgrom and Weber (1982a):

> [In the Japanese auction], the price is raised continuously, and a bidder who wishes to be active at the current price depresses a button. When he releases the button, he has withdrawn from the auction.

More precisely, we assume that at all times there are two types of bidders: *active* bidders and *inactive* bidders. Bidders are active until they manifest that they want to become inactive. Once a bidder has decided to become inactive her decision is irreversible. The identity of the active bidders is publicly observable along the auction. During the auction the price is publicly observable and increases continuously from zero. At any time bidders can decide to become inactive. The price stops increasing whenever there is no more than one bidder active. In this case, if a bidder remains active, she wins the auction. If no bidder remains active, the good is randomly allocated (with equal probability) among the bidders that quit at the last price. The price paid by the winner is the last price at which the bidders quit. We shall assume that there is neither an entry fee nor a reserve price.

The fact that we do not allow bidders to reenter the auction may may seem unrealistic for some relevant real-life examples. However, this assumption is not crucial for our analysis, but rather simplifies it. It allows us to solve the game by iterated elimination of weakly dominated strategies. Our main result, the existence of an insider's curse, is robust to this assumption. In fact, there are perfect Bayesian equilibria of the auction game with reentry in which bidders leave the auction according to the strategies proposed below and do not use reentry along the equilibrium path.

We assume that bidders maximize expected payoffs and that the monetary value of the object for a given bidder i is equal to $\frac{Q+T_i}{2}$. The component Q (for quality) is common to all bidders, and the component T_i (for taste) is idiosyncratic. Using the terminology of auction theory, we refer to Q as the common value and T_i as the private value. We also assume that Q and the T_i's are independent random variables with uniform distribution and support $[0, 1]$. We assume that each bidder has private information about her own private value T_i. Moreover, bidder $I \in \mathcal{I}$, whom we shall call the *insider*, also knows Q.

We shall distinguish three cases according to the information that the other bidders, which we call the *outsiders*, may have. In the first case, that we call the *symmetric information structure* (SIS), Q is common knowledge. In the second case, Q is private information of the insider, the outsiders only know its distribution; moreover, we assume that the identity of I is common knowledge. We call this model the *asymmetric information structure* (AIS). Finally, in the third case, we shall assume that the outsiders bid as if there were no insider, this is, they bid as in a model in which no bidder has any private information about Q. We refer to this last case as the *hidden insider structure* (HIS). The careful reader will realize that the last model is not logically consistent. We analyze it for its simplicity. We shall show in Section 5 that the insights we learn from this model also hold true in a logically consistent model.

The comparison between the insider's expected utility in the asymmetric information structure and the symmetric information structure tells us whether the insider has incentives to reveal his private information about the common value. As we anticipated in the Introduction, we shall say that there is an *insider's curse* if there are such incentives. Moreover, the comparison of the insider's expected utility in the asymmetric information structure and in the hidden insider structure tells us whether the insider wants the outsiders to know that she is an insider. If it is not the case, we shall say that the *insider has incentives to hide.*

To analyze the three structures, note first that in our open ascending auction, it is weakly dominant for the insider to stay in the auction until the price reaches her value. The reason is the same as in a private value auction. The insider has no uncertainty about her value, and hence the above strategy assures the bidder to win whenever the final price in the auction is below her value and that she loses otherwise.

In the symmetric information structure, outsiders also know the common value. As a consequence, and for the same reasons as above, it is weakly dominant for them to remain in the auction until the price reaches their value. The hidden symmetric structure is somewhat similar. Although the outsiders do not know the common value component, they behave as if no bidder has any private information about the common value. Under these beliefs, the outsiders find it optimal to remain in the auction until the price reaches their expected value computed by taking averages with respect to the common component. This means that an outsider with type t_i bids $\frac{t_i+E[Q]}{2}$.

The asymmetric information structure is more complicated. In its analysis we shall fix the insider's strategy to be her unique weakly dominant strategy described above. Thus, we are effectively doing one step of elimination of weakly dominated strategies.

First, we study information sets in which the insider is no longer active in the auction because she quit, say at a price p. An outsider learns from this information that the insider's value is equal to p. The expected value of an outsider with type t_i conditional on this information is equal to $\frac{t_i+E[Q|(T_I+Q)/2=p]}{2}$, which can be simplified to $\frac{t_i+p}{2}$. Such outsider finds it profitable to win at any price below this value since all the remaining bidders have no private information about the common value. Consequently, it is weakly dominant for our outsider to remain in the auction until the price reaches $\frac{t_i+p}{2}$.

Consider now the information sets in which the insider is still active. Our outsider can only win if the insider quits before she does. In this case, two things may happen: either the auction finishes, in which case our outsider wins and pays the price at which the insider quit, say b; or the game moves to the information sets we discussed in the previous paragraph. In both cases, an outsider with type t_i find it profitable to remain in the auction at b if and only if $\frac{t_i+E[Q|(Q+T_I)/2=b]}{2} - b$ is positive. It is easy to see this holds true if and only if $t_i \geq b$. Hence, it is weakly dominant for the outsider to remain in the auction until the price reaches t_i.

The next proposition sums up the previous results:

Proposition 1. *In our proposed solution of the game, the insider remains in the auction until the price reaches her true value, whereas the outsiders play the following* *strategies:*

- *Symmetric information structure (SIS): an outsider with type t_i leaves the auction at price $\frac{t_i+q}{2}$, where q is the realization of the common value.*

- *Hidden insider structure (HIS): an outsider with type t_i leaves the auction at price $\frac{t_i+E[Q]}{2}$ (note that $E[Q] = 1/2$.)*

- *Asymmetric information structure (AIS): an outsider with type t_i leaves the auction at price $\frac{t_i+p}{2}$, if the insider has left the auction at price p; and at price t_i, if the insider has not left the auction yet.*

Figure 1 compares the outsiders' bid function in the asymmetric information structure with those that correspond to the symmetric information and the hidden insider structures. Note that we cannot represent the bid function of the symmetric information structure directly since it depends on the realization of Q. What we do is to plot it taking averages with respect to Q. Note that the expected bid function in the symmetric information structure equals the bid function of the hidden insider structure. Finally, we only plot the bid function that corresponds to information paths in which the incumbent is still active in the asymmetric information structure. This is the only part of the bid function that affects the incumbent's expected utility.

Figure 1 shows that outsiders with high types keep bidding longer (on average) in the asymmetric information structure than in the symmetric information structure (or than in the hidden insider structure) if the insider remains in the auction. The opposite happens to outsiders with low type. To understand why, we shall look at the information that an outsider infers in the asymmetric information structure in the event that she wins at a price p at which the insider quits. Recall that this information determines whether the outsider finds it profitable to remain in the auction at price p when the insider is still active in the auction. The outsider learns from the former event that: (i) the insider finds it unprofitable to win at prices above p, and (ii) the insider finds it profitable to win at lower prices.

In the asymmetric information structure, the outsider may have less incentives to remain active if she thinks that the cause of (i) is that the insider knows that the

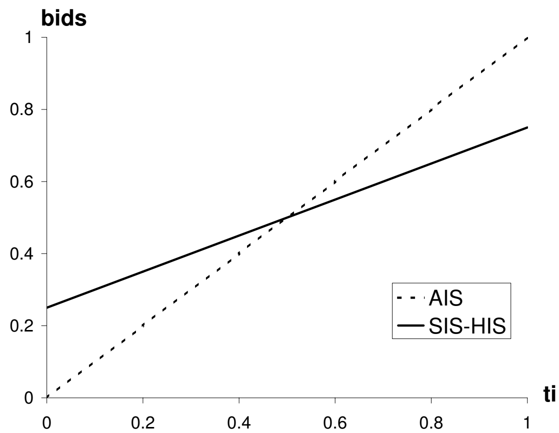

Figure 1: Equilibrium bid functions for the symmetric information structure (SIS), the asymmetric information structure (AIS) and the hidden insider structure (HIS).

common value is less than what the outsider thought. We call this effect the *winner's curse*, as ignoring this information may lead the outsider to win when it is unprofitable. However, the outsider may think that the cause of (i) is that the insider has a lower private value than her.

Similarly, the outsider may have more incentives to remain active if she thinks that the cause of (ii) is that the insider knows that the common value is larger than what the outsider thought. We call this effect the *loser's curse*, as ignoring this information may lead the outsider to lose when it is profitable to win. However, the outsider may also think that the cause of (ii) is that the insider has a greater private value.

If the outsider has a relatively large private value, the probability of the winner's curse will be low and that of the loser's curse will be large, and thus, the overall effect will be an increase in the outsider's incentives to remain active. Moreover, we would expect the opposite effect if the outsider has a relatively low private value. However, if the common value component is common knowledge, or simply if the outsiders are unaware of the presence of the insider, none of these effects exist. This explains the difference in the bid behavior shown in Figure 1.

The different shape of the bid functions in Figure 1 suggests that if there is a sufficient number of outsiders, the insider may be better off in the hidden insider structure or in the symmetric information structure than in the asymmetric information structure. To see why, note that the insider can win if and only if she bids higher than the highest type of the outsiders, and that this type will be close to one when there is a sufficient number of outsiders.

To check the above conjecture, note that in both auction formats the insider wins if and only if her bid is higher than the highest bid of the outsiders. By substituting the equilibrium bid function it is easy to see that the insider's expected utility in the symmetric information structure is equal to:

$$E\left[\max\left\{\frac{T_I+Q}{2}-\frac{T_{(1)}+Q}{2},0\right\}\right]=\int_0^1\int_t^1\frac{x-t}{2}\,dx\,dt^n, \tag{1}$$

where $T_{(1)}$ denotes the highest type of the outsiders.

Similarly, the insider's expected utility in the asymmetric information structure is equal to:

$$E\left[\max\left\{\frac{T_I+Q}{2}-T_{(1)},0\right\}\right]=\int_0^1\int_t^1(x-t)\,\hat{f}(x)\,dx\,dt^n, \tag{2}$$

where $\hat{f}$ denotes the distribution function of $\frac{T_I+Q}{2}$, this is $\hat{f}(x)=4x$, if $x\in[0,1/2]$, and $\hat{f}(x)=4(1-x)$, if $x\in[1/2,1]$.

If we compare the above two equations, we may distinguish two forces operating in opposite directions, at least for large n. The first one is related to the existence of the winner's and loser's curse in the asymmetric information structure but not in the symmetric structure. Formally, it corresponds to the fact that we integrate x with respect to the density of $\frac{T_I+Q}{2}$, this is $\hat{f}$, in Equation (2), but with respect to the density of T_I, which is equal to one, in Equation (1).

In the asymmetric information structure, high type outsiders bid as if they had good news about the common value to protect themselves from the loser's curse. This means that an insider can beat in the auction a high type outsider if and only if the insider has a high private value and the common value is also high. On the other hand, in the symmetric information structure, an insider beats a high private value outsider if and only if the insider has a high private value. In particular, the event that the insider wins is independent of the common value, since the realization of the common value shifts the insider's and the outsiders' bids in the same way.

The event "high private and common value" has lower probability than the event "high private value". This is reflected in the fact that $\hat{f}(x)$ is less than one for x sufficiently close to one. But, if the number of outsiders is sufficiently large, the distribution of $T_{(1)}$ in the equations above put most of its probability on values of x close to one. We can then conclude that the first force operates in the direction of making more advantageous for the insider the symmetric information structure if n sufficiently large.

The second force makes it less profitable for the insider to win the auction in the symmetric information than in the asymmetric information structure. Formally, it corresponds to the fact that in Equation (1) we integrate $x - t$ and in Equation (2) we integrate $\frac{x-t}{2}$. The intuition of this effect is that for a fixed realization of the bidders' private values, and conditional on the insider winning the auction, an increase in the common value increases the profitability of winning in the asymmetric information structure but not in the symmetric information structure. The reason is that in the symmetric information structure, the increase in the common value increases the insider's value and the price paid, whereas in the asymmetric information structure it only increases the insider's value. We refer to this effect as the *loss of informational rents*, since it is due to the fact that the common value is no longer private information of the insider.

The first effect dominates the latter if n is sufficiently large. To see why, note that in this case, the integral with respect to t puts most of its probability measure on values close to one, which means that x must also be close to 1. But, if x is sufficiently close to one, $(x - t)\hat{f}(x) < \frac{x-t}{2}$, and hence the first effect dominates. Consequently, the insider gets higher expected utility in the symmetric information structure than in the asymmetric information structure.

The next proposition sums up these arguments:

Proposition 2. *If n is large enough, the insider's expected utility in the symmetric information structure is larger than in the asymmetric information structure, i.e. there is an insider's curse.*

We now turn to the hidden insider structure. As in the symmetric information structure, there is neither a winner's curse nor a loser's curse. Actually, the outsiders' equilibrium bid functions coincide in both information structures when we take averages with respect to the common value. Nevertheless, they differ in one important aspect. In the hidden insider structure, the outsiders' bids do not depend on the the common value. This means that the insider gets the informational rents from the common value. Since this is not the case in the symmetric information structure, it suggests that the insider may get greater expected utility in the hidden insider structure. To demonstrate this, note that the insider's expected utility in the hidden insider structure is equal to:

$$E\left[\max\left\{\frac{T_I + Q}{2} - \frac{T_{(1)} + E[Q]}{2}, 0\right\}\right], \tag{3}$$

which is greater than:[13]

$$E\left[\left(\frac{T_I + Q}{2} - \frac{T_{(1)} + E[Q]}{2}\right)\mathbf{1}_{\{T_I \geq T_{(1)}\}}\right]. \tag{4}$$

This can be shown to be equal to the incumbent's expected utility in the symmetric information structure, see Equation (1).

Combining this result with Proposition 2 we get the following result:

Proposition 3. *If n is large enough, the insider's expected utility in the hidden insider structure is larger than in the symmetric information structure and also than in the asymmetric information structure, i.e. the insider has incentives to hide.*

Figure 2 illustrates Propositions 2 and 3.

[13] We write $\mathbf{1}_X$ for the indicative function of the condition X. It takes value one if X is true and zero otherwise.

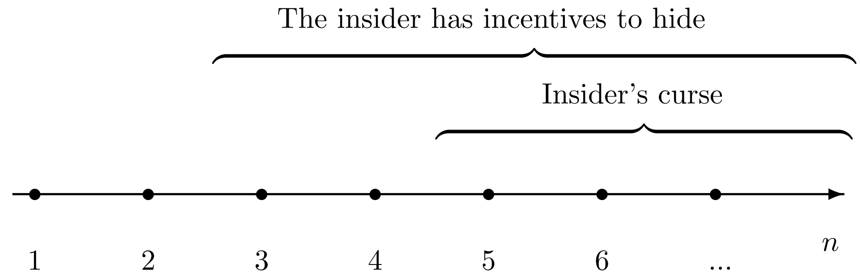

Figure 2: Region in which there is an insider's curse and region in which the insider has incentives to hide.

4 A General Model of the Insider's Curse

We demonstrated in the previous section, and by means of an example, that there may be an insider's curse in auctions. A careful reader may suspect that this counterintuitive result was an artifact of our special assumptions. This section shows that this is not the case. In fact, we shall prove that there may also be an insider's curse when outsiders have private information about the common value, when there are ex ante asymmetries in the distribution of the private value, when bidders are risk averse or risk lovers, and when the private and the common value present general complementarities or substitutabilities. We shall also consider in this section the effect of information disclosure on the allocative efficiency of the auction and on the auctioneer's expected revenue. Finally, we shall relate the insider's curse to the weight of the private value component by showing that when it tends to zero, the insider's curse disappears.

Basically, we assume the same set of players, the same auction set-up, and the same structure of private information as in the previous section. However, we assume neither additive separability of the private and common value nor risk neutrality. We assume that each bidder gets a von Neuman-Morgestern utility $u(v(Q, T_i) - p)$ $(i \in \mathcal{I})$ from winning the auction at price p, and a von Neuman-Morgestern utility $u(0)$ from losing the auction that we normalize to zero. We assume u and v to be continuous, bounded, strictly increasing and such that there exists a $\mu > 0$ that satisfies:

$$\frac{1}{\mu} \leq \frac{|u(x) - u(x')|}{|x - x'|}, \frac{|v(q,t) - v(q,t')|}{|t - t'|}, \frac{|v(q,t) - v(q',t)|}{|q - q'|} \leq \mu,$$

for any $x \neq x'$, any $q \neq q'$ and any $t \neq t'$.

We also allow the T_i's to be correlated with Q, this is, we allow the signals T_i to be informative of the other bidders T_i's and of Q, although with one restriction. We assume that there exists a random variable W such that the T_i's and Q are independent conditional on W, i.e. that each T_i is informative of Q and the other T_i's only up to W. This assumption together with the assumptions in the next paragraph implies that an outsider that observes the price at which all the other outsiders have quit, and hence knows the other outsiders' types in equilibrium, still has uncertainty about Q, and that this uncertainty does not vanish as n tends to infinity. Otherwise, the revelation of Q could be meaningless.

We denote by H the distribution of W and by $\mathcal{W}$ its support. We assume the distribution of Q conditional on $\{W = w\}$, denoted by $G(.|w)$, to have a density $g(.|w)$ and support $[\underline{q}, \overline{q}]$. We also denote by $F(.|w)$ the distribution of the insider's private value, say T_I, conditional on $\{W = w\}$, and assume it to have a density $f(.|w)$ and support $[\underline{t}, \overline{t}]$. Let $G(.)$ and $F(.)$ be the marginal distribution of Q and T_I respectively, and $g(.)$ and $f(.)$ their densities.

Finally, we assume that each T_i $(i \neq I)$ follows the same distribution, but we allow this distribution to differ from that of T_I. For simplicity, we assume that the conditional distribution of each T_i's $(i \neq I)$ is the same as that of T_I but shifted to the left a parameter[14] $\gamma \in (-\Delta, \Delta)$, where $\Delta \equiv \overline{t} - \underline{t}$, this is[15] $\Pr\{T_i \leq t | W = w\} = F(t + \gamma | w)$. The parameter γ measures the strength of the private value advantage (or disadvantage, if negative) of the insider. For instance, if γ tends to Δ the probability that the insider's private value is larger than the outsiders' tends to one. Similarly, if γ tends to $-\Delta$, the former probability tends to zero. Finally, we assume that there

[14] All the results of the paper still hold if the distribution of each T_i conditional on $Q = q$ parameterized by γ, say F_γ, satisfies two conditions: (i) $F_0 = F$, and (ii) $\lim_{\gamma \to -\Delta} \min[\text{support}(F_\gamma)] = \overline{t}$.

[15] We denote with $\mathbf{Pr}\{.|.\}$, the probability of the event at the left hand side of the vertical bar conditional on the event at the right hand side.

exists an $\eta > 0$ such that

$$\frac{1}{\eta} \leq g(q|w), f(t|w) \leq \eta,$$

for any $t \in [\underline{t}, \overline{t}]$, $q \in [\underline{q}, \overline{q}]$ and $w \in \mathcal{W}$.

Note that although outsiders may have private information about the common value, we are assuming that their information is redundant to the insider. This assumption, reasonable in many real-life examples, makes our model tractable.[16] We can apply the same arguments as in Section 3 to show that the insider has a unique weakly dominant strategy, to remain in the auction until $v(Q, T_I)$ is reached, independent of the information that outsiders may have. We shall assume that the insider follows this strategy so that we do not consider her as a strategic player. Similarly, the symmetric information structure has a very simple analysis as, again, outsiders have a unique weakly dominant strategy, to bid $v(Q, T_i)$. We shall assume they follow this strategy.

Nevertheless, the equilibrium analysis of the asymmetric information structure under our general assumptions presents one complication: a separating equilibria does not always exist. We may construct partially pooling equilibria, but they are complicated to characterize. Instead, we shall present our main result without assuming equilibrium behavior. More precisely, we only assume that outsiders do not use weakly dominated strategies.[17] We later provide natural conditions under which a separating equilibrium exists, and we characterize this equilibrium.

Proposition 4. *Suppose that the outsiders do not use weakly dominated strategies. The insider gets greater expected utility in the symmetric information structure than in the asymmetric information structure, i.e. there is an insider's curse, if either:*

- *n is sufficiently large and $\gamma = 0$,*
- *γ is sufficiently small.*

[16]In general, auction models in which bidders have information of different precision with respect to a common value component present problems of existence of equilibrium, see Athey (2000). Thus, we may expect these problems had we not made the latter assumption.

[17]We mean weakly dominated strategies in the reduced game that results from assuming that the insider follows her unique weakly dominant strategy. In this sense, it is like applying two steps of elimination of weakly dominated strategies

See the proof in the Appendix.

The first point of the above proposition generalizes in some sense Proposition 2. The second point says that there may be an insider's curse if the outsiders have a sufficiently strong private value advantage compare to the insider. As we explain in the Introduction, these are the two cases in which we would expect to find an insider's curse.

Clearly, if there exists an equilibrium in which outsiders do not use weakly dominated strategies, it will have an insider's curse in the same conditions as in Proposition 4. We, next, prove the existence of such equilibrium under some additional assumptions. To make things simpler we assume in what follows that the bidders' types are independent, i.e. that the random variables $T_I, ..., T_n, Q$ are stochastically independent. The analysis may be extended to correlated types, but some new difficulties will arise. For instance, we could not use an equilibrium concept based on weakly dominant strategies. We also make the following assumption:

Assumption (SCC): For any $t \in [\underline{t}, \overline{t}]$, $E[u(v(Q,t) - b)|v(Q,T_I) = b]$ crosses zero at most once for $b \in [v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$.

This assumption is related to the single crossing condition as defined by Krishna (2002), although it differs in that the insider's signal is a two dimensional vector and we do not assume risk neutrality. This assumption guarantees that the outsider's expected utility of winning when the price, say b, equals the outsider's bid (i.e. $E[u(v(Q,t) - b)|v(Q,T_I) = b]$) is lower at higher prices. This condition is not only intuitive, but it is also satisfied with some generality. For instance, if the private and the common components are perfect substitutes in u, a sufficient condition is that $E[T_I|Q + T_I = b]$ is increasing in b. This last condition holds if the density of G is log-concave.[18] Note that log-concavity of the density is a common feature of many distribution functions, for instance the uniform, the beta with parameters no less than one (a particular case

[18]This result may be proved checking that the random variables $X(t) \equiv v(Q,T_I) - v(Q,t)$ and $\hat{V} \equiv v(Q,T_I)$ are affiliated for any t.

is $F(q) = q^r$, where $r \geq 1$) and any truncated exponential, normal, logistic, extreme-value, chi-square, chi, and Laplace distributions.[19]

We now characterize a separating equilibrium for the asymmetric information structure using an auxiliary function that we introduce in the next lemma.

Lemma 1. *Under assumption (SCC), there exists a unique function* $\beta : [\underline{t}, \overline{t}] \rightarrow [u(v(\underline{q}, \underline{t})), u(v(\overline{q}, \overline{t}))]$ *that satisfies* $E[u(v(Q,t) - \beta(t))|v(Q, T_I) = \beta(t)] = 0$*. This function is continuous and strictly increasing.*

See the proof in the Appendix.

Proposition 5. *Suppose that the Assumption (SCC) holds, and that the insider is playing her unique weakly dominant strategy. Then, it is weakly dominant for the outsiders to play the following bid functions:*

- *If the insider is still active,*

$$b_{AIS}(t) = \begin{cases} u(v(\underline{q}, t)) & \text{if } t < \underline{t} \\ \beta(t) & \text{if } t \in [\underline{t}, \overline{t}], \\ u(v(\overline{q}, t)) & \text{if } t > \overline{t}. \end{cases}$$

- *If the insider left the auction at price* p*, to use the bid function* $b_{AIS}(t|p)$ *implicitly defined*[20] *by the equation* $E[u(v(Q,t) - b_{AIS}(t|p))|v(Q, T_I) = p] = 0$.

Proof. In an open ascending auction, a bidder's strategy determines at which prices the bidder wins. A strategy is, thus, optimal if it assures that the bidder wins at any price at which it is profitable to win. Our proof will show that this is actually the case of our proposed strategy, if the insider follows her weakly dominant strategy, and independently of the strategies followed by the other outsiders. An outsider i can win the auction in two cases: (i) when the last bidder to quit the auction is the insider, and (ii) when the last bidder to quit the auction is another outsider. In case (i), the price

[19] A detailed list of distribution functions with log-concave densities can be found in Bagnoli and Bergstrom (1989).

[20] Note that Assumption (SCC) implies that $b_{AIS}(t|p)$ is uniquely defined.

that our outsider pays is equal to the price at which the insider quit, say p, and in that case our outsider's expected utility if she wins is equal to $E[u(v(Q,t_i)-p)|v(Q,T_I)=p]$, where t_i denotes outsider i's private type. By Assumption (SCC), this expected utility is positive if and only if $\beta(t_i) \geq p$, i.e. exactly in the cases in which our outsider wins if she follows our proposed strategy. Consider now that the outsider wins in case (ii). The fact that an outsider fixes the price, say at a level b, means that the insider must have left the auction at a lower price, say p. Our outsider's expected utility of winning is in this case $E[u(v(Q,t_i)-b)|v(Q,T_I)=p]$, which is positive if and only if $b_{AIS}(t_i|p) \geq b$, exactly the cases in which our outsider wins if she follows our proposed strategy. ■

Jackson (1999) and Pesendorfer and Swinkels (2000) have argued that auction models with private and common value components and multidimensional types (i.e. private and common value components privately known by the agents) may have problems of existence of equilibria with some generality. Proposition 5 shows that our model does not present these problems under some mild assumptions.

We next explore the consequences of the insider's curse on the efficiency and optimality of the auction. To do so, we compare the symmetric information structure with the asymmetric information structure. From this comparison we can study whether a social planner or a revenue-maximizing auctioneer has incentives to allow the insider to reveal her private information about the common value, or even to make it easier, for instance by making verifiable the insider's revelation.

The following proposition follows trivially and hence no proof is provided.

Proposition 6. *The good is allocated to the bidder with highest private value in the symmetric information structure, but not in the asymmetric information structure. Consequently, the symmetric information structure implements a more efficient allocation than the asymmetric information structure.*

For the next proposition, we shall assume that types are statistically independent, bidders are risk neutral, F has an increasing hazard rate, i.e. that $\frac{f(x)}{1-F(x)}$ is increasing in x, and $\gamma = 0$ (i.e. the outsiders and the insider preferences are ex ante symmetric).

We refer to these assumptions as the *regular case*.

Proposition 7. *In the regular case, the auctioneer gets higher expected revenue in the symmetric information structure than in the asymmetric information structure. In fact, the open auction under the symmetric information structure gives higher expected revenue than any auction mechanism that always sells the good, either under the symmetric or the asymmetric information structure.*

See the proof in the Appendix.

To understand the intuition of the last proposition, note that the auctioneer's expected revenue in the asymmetric information structure cannot exceed the maximum expected revenue in the hypothetical case in which outsiders do not know the common value, the insider knows it, and the auctioneer also knows it. It can be shown that in this case the auctioneer can maximize his expected revenue by revealing the common value to the outsiders. Intuitively, since bidders do not have any ex ante asymmetry with respect to the private types, the revelation of the common value levels the playing field and makes it more competitive. The proposition follows since the open ascending auction is an optimal mechanism in the symmetric information structure.

Nevertheless, it can be shown that there are cases in which the auctioneer gets higher expected revenue in the asymmetric information structure than in the symmetric information structure. For instance, this may happen when the outsiders have an ex ante private value advantage sufficiently strong, (in our model, γ sufficiently smaller than 0). In this case, giving additional private information to the disadvantaged bidder may increase the competition and hence, the auctioneer's revenue.

Finally, to understand more clearly the role played by the private value component, we study the possibility of an insider's curse as the private value component vanishes. For this purpose, we shall analyze a variation of the model studied in this section. Basically, we maintain all the assumptions of this section except the bidder's utility function. We assume that a bidder $i \in \mathcal{I}$ gets a von Neumann-Morgenstern utility $u(v(Q, \alpha T_i) - p)$, where $\alpha \in (0, 1)$ from winning the auction at price p, and a utility of $u(0) = 0$ from losing the auction. However, we maintain the assumptions on u and

v that we made at the beginning of the section. Note that for a fixed value of α this new model is simply a relabelling of the model studied in the rest of the section.

Proposition 8. *Suppose that outsiders do not play weakly dominated strategies, that* $\gamma = 0$ *and that* n *is fixed. The insider gets greater expected utility in the asymmetric information structure than in the symmetric information structure if* α *is sufficiently small.*

See the proof in the Appendix.

To understand why the Proposition holds it is useful to think of the limit game in which there is no private value, this is when $\alpha = 0$. In this case, it is easy to see that the insider gets zero expected utility in the symmetric information structure in equilibrium. However, in the asymmetric information structure, there are equilibria in which the insider gets positive expected utility. Proposition 8 holds because in the proposed limit, one of these equilibria is selected. It is, however, important to remark that if $\gamma < 0$, it may be the case that, in the limit, an equilibrium of the asymmetric information structure in which the insider gets zero expected utility can be selected. Thus, in general, the above proof cannot be used for $\gamma < 0$, and in fact, it is unclear if a version of the above proposition would hold.

5 A General Model with a Hidden Insider

In this section, we reconsider the insider's incentive to hide. Recall that one important drawback of the analysis in Section 3 was that we assumed that outsiders behave as if there were no insider when, in fact, the insider was present in the auction. We fix this problem in this section and also study the validity of the argument with a more general model of uncertainty, general complementarities and substitutabilities between the common and the private value, and when bidders are not risk neutral.

For this purpose, we shall redefine the hidden insider structure studied in Section 3, compute the insider's expected utility and compare it with the insider's expected utility in the symmetric and asymmetric information structure studied in the last section.

In this section, we assume the same auction set-up, the same set of bidders, and the same utility function, with its restrictions, as in the former section. We also assume that each bidder knows privately her private value. However, we assume that with some probability $1-\rho \in (0,1)$ none of the bidders knows the common value, whereas with the complementary probability, ρ, one of the bidders randomly drawn from the pool of bidders, and only her, knows the common value. We again call this bidder, the *insider*, and the other bidders, *outsiders*. We assume that all the bidders have the same probability of being an insider.

In particular, we assume that if there is an insider, she knows the common value component, whereas outsiders know neither the common value, nor the identity of the insider, nor the presence of the insider in the auction. They only know the probability distribution of these events. Technically, this implies a complication as bidders have to make equilibrium conjectures not only about the common value, but also about the presence of the insider, and the identity of the insider. This is even more complicated due to the dynamic structure of our game.

We shall make the same assumptions as in the former section with respect to the distribution of the insider's and outsiders' private value, and the common value. Note that these assumptions imply that the insider not only differs from outsiders in the fact that she has better information, but also in the distribution of the private value. Additionally, we shall assume, for the sake of tractability, that all these random variables are stochastically independent.

Although our model does not include the case in which outsiders know that there is an insider but they do not know her identity, this is $\rho = 1$, we cover cases arbitrarily close. Moreover, it seems reasonable to conjecture similar results. We think so because the analysis of the continuation game after one bidder leaves the auction is the same as in our game, at least for the case in which, in equilibrium, the quitting of the first bidder does not perfectly reveal if this bidder is the insider.

Our study, however, does not cover situations in which bidders do not have ex ante the same probability of being the insider. A particular case is when there is only one single bidder, say the *suspected insider*, who can be the insider. Our arguments do not extend easily to this case. The main difficulty seems to be that in the cases in which revealing helps the insider, i.e. large n or small γ, the outsiders may learn anyway if there is an insider in the most relevant cases. Consider, for instance, the case in which the number of outsiders is large. A suspected insider wins mostly with high bids since other bids are outbid with high probability. But, sufficiently high bids of the suspected insider must reveal in equilibrium that she does know the common value, i.e. that she is an insider. This is because the maximum bid that a suspected insider is willing to submit is higher when she knows the common value than when she does not. Note that in the model of this section, this does not happen because all outsiders suspect all the other bidders as being potential insiders, and these suspicions make them bid above their maximum expected value when another bidder does so. But, since all outsiders do so, they cannot distinguish if there is really an insider or not.

In our comparison, we assume that the insider plays her unique weakly dominant strategy to stay in the auction until the price reaches $v(Q, T_I)$. Consequently, we shall consider that the outsiders are the only strategic players. We state our results without ensuring existence of an equilibrium. In the last section of the Appendix we provide a characterization of an equilibrium in terms of a partial differential equation.

Proposition 9. *Suppose that the private types are independent, and that the bidders are risk neutral, i.e. u is linear. If the outsiders play a symmetric equilibrium in increasing strategies, the insider gets greater expected utility in the hidden insider structure than in the asymmetric information structure and also than in the symmetric information structure if either γ is sufficiently small, or n is sufficiently large for $\gamma = 0$.*

See the proof in the Appendix.

To understand the proposition note that in both, the symmetric and the asymmetric information structure, and when we take either of the limits, the insider's expected utility converges to zero. It is straightforward to see that this holds in the case of the symmetric information structure, and thus, because of Proposition 4, that it also holds in the asymmetric information structure.

However, this cannot happen in the hidden insider structure. The reason is that for this to be the case, the price in the auction should converge to the insider's value or above when the insider is in the auction. In particular, this means that if the insider is present in the auction and her private component and the common component are maximum, the price must converge to the insider's maximum value or above. In this case, outsiders with the same private component as the insider would get at most zero expected utility. However, outsiders cannot distinguish during the auction if the insider is present or not before the insider quits. This means that if the insider were not present when the price reaches the former maximum value, outsiders would have negative expected value. They pay a price that corresponds to the maximum realization of the common value, but since there is no insider, outsiders can only expect an average common value. The conclusion is that if this were the case, outsiders would have incentives to deviate by leaving the auction earlier to protect from the negative expected utility of winning at those prices.

6 Conclusions

We study an auction model that shows that strong competition may create bidder's incentives to disclose private information about a common component of the value of the good for sale. We provide an intuitive explanation based on the interplay of two forces, the winner's and the loser's curse, that affect in equilibrium the bids of the other bidders.

Although our model displays incentives to reveal information, an important issue is whether this information can be revealed credibly or not. This question is specially relevant because in our model, the insider has incentives to mislead the outsiders, for instance, by understating the value of the good. This is a problem that also arises in other results about information disclosure, for instance those of Milgrom and Weber (1982a). It is usually argued that this should not be a concern if either the informed party cares about her reputation or the information disclosed can be verified.

Another important feature of our results is that we only consider the case in which the insider can either reveal no information or reveal it fully. For instance, we may suspect that in real auctions, the insider will disclose information only when she has bad news. Note, however, that in this case, the outsiders would infer in equilibrium that if the insider does not give any news, it must be because she has got good news. This argument has been explored by Postlewaite, Okuno-Fujiwara, and Suzumura (1990) in an abstract model of strategic information revelation, and by Benoit and Dubra (2003) in an auction set-up similar to ours. Their results imply that when the insider's private information about the common value is verifiable, there must be full information disclosure in equilibrium if the information revelation game takes place before the insider learns her private value. Otherwise, Benoit and Dubra's results applied to our model show that although we can always find an equilibrium of the game of information revelation in which there is full information revelation, this may not be the case in other equilibria.

We may interpret the insider's difficulties to reveal her private information from a normative point of view. To see why, it is important to remark, first, that our results also show that insider's information revelation improves social welfare, and under some regularity conditions, the auctioneer's expected revenue. Hence, we may conclude from our analysis that it is in the interest of the auctioneer or the regulator to develop the mechanisms that may allow the insider to reveal her private information. Two such mechanisms may be introducing stronger penalties for false information disclosure or developing institutions like more transparent accounting that help verifiability.

It is interesting to note another normative implication of our analysis. Our results show that under strong competitive pressure, the main dangers of insider trading may come from the fact that an insider can hide. Otherwise, we have shown that there exists an insider's curse, and hence, either the insider is willing to disclose her private information, or she does not have any incentives to acquire it in the first place.

Another key element of our analysis is that the bidders' value of the good has private components. In fact, we show that the incentives to disclose private information disappear as the weight of the private value component vanishes. Our results also suggest some kind of strategic complementarities between additional private information about a common value component and ex ante private value advantages. The returns for additional private information are negative for a bidder with a private value disadvantage sufficiently strong and positive for a bidder with a private value advantage. This point, however, requires further analysis.

Appendix

Proof of Proposition 4

To compare the insider's expected utility under the two information set-ups, the symmetric information structure and the asymmetric information structure, it is useful to condition on a realization of the highest type of the outsiders, say $T_{(1)} = t_{(1)}$. We shall refer to these expected values as $u^*_{SIS}(t_{(1)})$ and $u^*_{AIS}(t_{(1)})$ respectively.

In the symmetric information structure, the insider wins if and only if her private value is greater than $t_{(1)}$, and in that case the insider pays the bid of $t_{(1)}$ that equals $v(Q, t_{(1)})$. Hence,

$$u^*_{SIS}(t_{(1)}) = \int_{\mathcal{W}} \int_{\underline{q}}^{\overline{q}} \int_{t_{(1)}}^{\bar{t}} U\left(v(q, t_I) - v(q, t_{(1)})\right) f(t_I|w)\, dt_I\, dG(q|w)\, dH(w)$$
$$\geq \int_{t_{(1)}}^{\bar{t}} \frac{t_I - t_{(1)}}{\eta\, \mu^2}\, dt_I. \quad (5)$$

We next compute an upper bound for U^*_{AIS}. To do so, we start by providing a lower bound to the outsiders' bids:

Lemma 2. *In the asymmetric information structure model, it is weakly dominated for an outsider with type* $t_i > \bar{t} - \frac{v(\overline{q},\bar{t}) - v(\overline{q},\underline{t})}{2\eta^4\mu^5}$ *to leave the auction at a price* $b < \underline{b}(t_i) \equiv v(\overline{q}, \bar{t}) - 2\eta^4\, \mu^5\, (\bar{t} - t_i)$*, if the insider is still active in the auction at price* b*.*

Proof. Suppose that outsider i uses the above strategy, then we argue that she can weakly increase her expected utility when she has type t_i if she remains in the auction when the insider is active until either price $\underline{b}(t_i)$ is reached or the insider quits, instead of quitting at price b. This change only affects the outsider's payoffs if the insider leaves the auction at a price, say b', between b and $\underline{b}(t_i)$ and the outsider wins at that price. We, next, show that the outsider's expected utility is strictly positive in these cases, or similarly, that:

$$E[u(v(Q,t_i)-b')|v(Q,T_I)=b', \vec{T}=\vec{t}] > 0,$$

for any vector of outsider's private values $\vec{t} \in [\underline{t}, \overline{t}]^n$ such that its i-th component is equal to t_i.

The claim of the lemma for bids $b' < v(\underline{q}, \underline{t})$ is straightforward since the conditions of the lemma, and in particular that $t_i > \overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}$, imply that $t_i \geq \underline{t}$. Suppose next that $b' \geq v(\underline{q}, \underline{t})$, and denote by $q^*(t;b')$ the function implicitly defined by $v(q^*(t;b'),t) = b'$, this is the insider's indifference curve that corresponds to a price b' without uncertainty. Moreover, let $\underline{t}^*(b') \equiv \min\{t \in [\underline{t}, \overline{t}] : v(\overline{q}, t) \geq b'\}$ and $\overline{t}^*(b') \equiv \max\{t \in [\underline{t}, \overline{t}] : v(\underline{q}, t) \leq b'\}$, i.e. the minimum and the maximum insider's private value associated to the indifference curve $v(q,t)=b'$. Thus, we have:

$$
\begin{aligned}
&E[u(v(Q,t_i)-b')|v(Q,T_I)=b',\vec{T}=\vec{t}] = \\
&\quad E[u(v(Q,t_i)-v(Q,T_I))|v(Q,T_I)=b',\vec{T}=\vec{t}] \geq \\
&\quad E[u(v(Q,\bar{t})-\mu(\bar{t}-t_i)-(v(Q,\bar{t})-\frac{1}{\mu}(\bar{t}-T_I)))|v(Q,T_I)=b',\vec{T}=\vec{t}] \geq \\
&\quad E[u(-\mu(\bar{t}-t_i)+\frac{1}{\mu}(\bar{t}-T_I))|v(Q,T_I)=b',\vec{T}=\vec{t}] \geq \\
&\quad E[u(-\mu(\bar{t}-t_i))+\frac{1}{\mu^2}(\bar{t}-T_I)|v(Q,T_I)=b',\vec{T}=\vec{t}] \geq \\
&\quad E[-\mu^2(\bar{t}-t_i)+\frac{1}{\mu^2}(\bar{t}-T_I)|v(Q,T_I)=b',\vec{T}=\vec{t}] = \\
&\quad -\mu^2(\bar{t}-t_i)+\frac{1}{\mu^2}E[\bar{t}-T_I|v(Q,T_I)=b',\vec{T}=\vec{t}] = \\
&-\mu^2(\bar{t}-t_i)+\frac{1}{\mu^2}\frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')}\int_{\mathcal{W}}(\bar{t}-t_I)f(t_I|w)\Pi_{j\neq I}f(t_j-\gamma|w)g(q^*(t_I;b')|w)\,dH(w)dt_I}{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')}\int_{\mathcal{W}}f(t_I|w)\Pi_{j\neq I}f(t_j-\gamma|w)g(q^*(t_I;b')|w)\,dH(w)dt_I} \geq \\
&\quad -\mu^2(\bar{t}-t_i)+\frac{1}{\eta^4\,\mu^2}\frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')}\int_{\mathcal{W}}(\bar{t}-t_I)\Pi_{j\neq I}f(t_j-\gamma|w)\,dH(w)dt_I}{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')}\int_{\mathcal{W}}\Pi_{j\neq I}f(t_j-\gamma|w)\,dH(w)dt_I} = \\
&\quad -\mu^2(\bar{t}-t_i)+\frac{1}{\eta^4\,\mu^2}\frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')}(\bar{t}-t_I)dt_I}{\bar{t}^*(b')-\underline{t}^*(b')} = \\
&\quad -\mu^2(\bar{t}-t_i)+\frac{1}{\eta^4\,\mu^2}(\bar{t}-\frac{\bar{t}^*(b')+\underline{t}^*(b')}{2}) \geq \\
&\quad -\mu^2(\bar{t}-t_i)+\frac{\bar{t}-\underline{t}^*(b')}{2\,\eta^4\,\mu^2} \geq -\mu^2(\bar{t}-t_i)+\frac{v(\bar{q},\bar{t})-v(\bar{q},\underline{t}^*(b'))}{2\,\eta^4\,\mu^3},
\end{aligned}
$$

which is strictly positive under the conditions of the Lemma. To see why, we can distinguish two cases, when $b' > v(\bar{q},\underline{t})$ and when $b' \leq v(\bar{q},\underline{t})$. In the first case, $v(\bar{q},\underline{t}^*(b'))=b'$, and thus, we can prove our claim using the fact that $b' < \underline{b}(t_i)$. In the second case, $v(\bar{q},\underline{t}^*(b'))=v(\bar{q},\underline{t})$, and thus, the claim can be proved using the fact that we restrict to $t_i > \bar{t}-\frac{v(\bar{q},\bar{t})-v(\bar{q},\underline{t})}{2\eta^4\mu^5}$. ■

This lemma implies an upper bound to the insiders' expected utility in the asymmetric information structure[21] for $t_{(1)} > \bar{t}-\frac{v(\bar{q},\bar{t})-v(\bar{q},\underline{t})}{2\eta^4\mu^5}$:

$$u^*_{AIS}(t_{(1)}) \leq E\left[\left[u(v(Q,T_I)-\underline{b}(t_{(1)}))\right]1_{\{v(Q,T_I)\geq\underline{b}(t_{(1)})\}}|T_{(1)}=t_{(1)}\right].$$

[21]We denote by $H(.|T_{(1)}=t_{(1)})$ the distribution function of W conditional on $T_{(1)}=t_{(1)}$.

To operate on this expression, it is easier to integrate the insider's utility with respect to the random variable $\hat{V} = v(Q, T_I)$, whose distribution conditional on $W = w$ we denote by $\hat{F}(.|w)$. This is,

$$u^*_{AIS}(t_{(1)}) \leq \int_{\mathcal{W}} \int_{\underline{b}(t_{(1)})}^{v(\bar{t},\bar{t})} u(\hat{v} - \underline{b}(t_{(1)}))\, d\hat{F}(\hat{v}|w)\, dH(w|T_{(1)} = t_{(1)}).$$

Lemma 3. *For any, $w \in \mathcal{W}$, the distribution function $\hat{F}(\hat{v}|w)$ has support $[v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$ and a density, say $\hat{f}(\hat{v}|w)$, which is bounded above by $\eta^2\mu^2(v(\overline{q}, \overline{t}) - \hat{v})$, for any $\hat{v}$ in the support.*

Proof. That the support of $\hat{F}(\hat{v}|w)$ is $[v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$ follows directly from the fact that Q and T_I conditional on $W = w$, for $w \in \mathcal{W}$, have support $[\underline{q}, \overline{q}]$ and $[\underline{t}, \overline{t}]$ respectively. Next, note that using the functions $\underline{t}^*$, $\overline{t}^*$ and q^* introduced in the proof of Lemma 2,

$$\hat{F}(\hat{v}|w) = F(\underline{t}^*(\hat{v})|w) + \int_{\underline{t}^*(\hat{v})}^{\overline{t}^*(\hat{v})} G(q^*(t;\hat{v})|w) f(t|w)\, dt.$$

This distribution function has a density. To see why we only need to differentiate it. Note that this can be done using the implicit function theorem to compute $\frac{\partial q^*(t;\hat{v})}{\partial \hat{v}}$, and $\frac{\partial \overline{t}^*(\hat{v})}{\partial \hat{v}}$ if $\hat{v} < v(\underline{q}, \overline{t})$, and $\frac{\partial \underline{t}^*(\hat{v})}{\partial \hat{v}}$ if $\hat{v} > v(\overline{q}, \underline{t})$. Note also that if $\hat{v} \leq v(\underline{q}, \overline{t})$, then $q^*(\overline{t}^*(\hat{v}); \hat{v}) = \underline{q}$, and if $\hat{v} \geq v(\overline{q}, \underline{t})$, then $q^*(\underline{t}^*(\hat{v}); \hat{v}) = \overline{q}$. Finally, note that if $\hat{v} \geq v(\underline{q}, \overline{t})$, then $\overline{t}^*(\hat{v}) = \overline{t}$, and that if $\hat{v} \leq v(\overline{q}, \underline{t})$, then $\underline{t}^*(\hat{v}) = \underline{t}$. We may use the above remarks to show after some straightforward simplifications on the differential of $\hat{F}(\hat{v}|W)$ that:

$$\hat{f}(\hat{v}|w) = \int_{\underline{t}^*(\hat{v})}^{\overline{t}^*(\hat{v})} \frac{g(q^*(t;\hat{v})|w) f(t|w)}{\frac{\partial v(q^*(t;\hat{v}),t)}{\partial q}}\, dt$$

which is bounded above by $\eta^2\mu(\overline{t}^*(\hat{v}) - \underline{t}^*(\hat{v})) \leq \eta^2\mu(\overline{t} - \underline{t}^*(\hat{v})) \leq \eta^2\mu^2(v(\overline{q}, \overline{t}) - v(\overline{q}, \underline{t}^*(\hat{v})))$ which implies the lemma since $v(\overline{q}, \underline{t}^*(\hat{v})) = \max\{\hat{v}, v(\overline{q}, \underline{t})\}$. ■

If $t_{(1)} \in (\overline{t} - \frac{v(\overline{q},\overline{t}) - v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \overline{t}]$, the last two lemmas can be used to show:

$$u^*_{AIS}(t_{(1)}) \le \int_{\underline{b}(t_{(1)})}^{v(\overline{q},\overline{t})} u(\hat{v} - \underline{b}(t_{(1)}))\eta^2\mu^2(v(\overline{q},\overline{t}) - \hat{v})\, d\hat{v} \le$$

$$\int_{\underline{b}(t_{(1)})}^{v(\overline{q},\overline{t})} (\hat{v} - \underline{b}(t_{(1)}))\eta^2\mu^3(v(\overline{q},\overline{t}) - \hat{v})\, d\hat{v} \le$$

$$\text{Change of variable: } \left\{ \begin{array}{c} \hat{v} = \underline{b}(t_I) \\ d\hat{v} = 2\eta^4\,\mu^5 dt_I \end{array} \right\} =$$

$$2\eta^6\mu^8 \int_{t_{(1)}}^{\overline{t}} (\underline{b}(t_I) - \underline{b}(t_{(1)}))(v(\overline{q},\overline{t}) - \underline{b}(t_I))\, dt_I =$$

$$8\eta^{16}\mu^{16} \int_{t_{(1)}}^{\overline{t}} (t_I - t_{(1)})(\overline{t} - t_I) dt_I. \quad (6)$$

As a consequence:

$$u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \ge \int_{t_{(1)}}^{\overline{t}} (t_I - t_{(1)}) \frac{1 - 8\eta^{17}\mu^{18}(\overline{t} - t_I)}{\eta\,\mu^2}\, dt_I,$$

for $t_{(1)} \in (\overline{t} - \frac{v(\overline{q},\overline{t}) - v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \overline{t}]$. From which we can derive the following result:

Lemma 4. *If* $t_{(1)} \in \left(\overline{t} - \min\left\{\frac{v(\overline{q},\overline{t}) - v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \frac{1}{8\eta^{17}\mu^{18}}\right\}, \overline{t}\right)$, *then* $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) > 0$.

The ex ante difference of the insider's expected utility in the symmetric information structure and the asymmetric information structure equals:

$$\int_{\mathcal{W}} \int_{\underline{t}-\gamma}^{\overline{t}-\gamma} \int_{t_{(1)}}^{\overline{t}} \left(u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)})\right) dt_I\, dF(t_{(1)} - \gamma|w)^n\, dH(w).$$

Thus, Proposition 4 (ii) follows directly from Lemma 4. To prove Proposition 4 (i) note that for $\gamma = 0$, and by a continuity argument on Lemma 4, for an $\epsilon > 0$ sufficiently small there must exist a partition $[\underline{t}, a) \cup [a, b] \cup (b, \overline{t}]$ of the support of $T_{(1)}$ such that $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \ge \epsilon$ for $t_{(1)} \in [a, b]$, and $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \ge 0$ for $t_{(1)} \in [b, \overline{t}]$. Thus, we have the following lower bound for the ex ante difference of the insider's expected utility:

$$\int_{\mathcal{W}} F(b|w)^n \left\{ \left[\frac{F(a|w)}{F(b|w)}\right]^n (-M) + \left(1 - \left[\frac{F(a|w)}{F(b|w)}\right]^n\right) \epsilon \right\} dH(w),$$

where $-M \equiv \min_{t_{(1)} \in [\underline{t}, a]} \{u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)})\}$. Clearly, the above integral is strictly positive for n sufficiently large.

Proof of Lemma 1

The equation that defines β is continuous in t and β, the left hand side is positive at $\beta = u(v(\underline{q}, \underline{t}))$, and negative at $\beta = u(v(\overline{q}, \overline{t}))$, hence, there exists a β that solves the equation of the lemma for any $t \in (\underline{t}, \overline{t})$. Moreover, Assumption (SCC) implies that it is uniquely defined, and hence, continuous. By inspection, $\beta(\underline{t}) = u(v(\underline{q}, \underline{t}))$ and $\beta(\overline{t}) = u(v(\overline{q}, \overline{t}))$. Finally, to see that β is strictly increasing suppose that $\beta(t) \geq \beta(t')$ for $t < t'$. From the definition of β applied to t and the assumption that u and v are strictly increasing, we have that $E[u(v(Q, t') - \beta(t))|v(Q, T_I) = \beta(t)] > 0$. At the same time, the definition of β at t' plus Assumption (SCC) imply $E[u(v(Q, t') - \beta(t))|v(Q, T_I) = \beta(t)] \leq 0$, which is a contradiction.

Proof of Proposition 7

We provide an indirect proof. We, first, compute the optimal mechanism in an auxiliary problem in which bidders' private information is as in the asymmetric information structure but in which we assume that the auctioneer knows the common value Q. This last assumption means that the auctioneer's mechanism can depend directly on the common value without the need of including the corresponding insider's incentive compatibility constraint. In order to characterize the optimal mechanism in this auxiliary problem we can restrict without loss of generality to direct mechanisms. In our set-up, these are mechanisms in which: (i) each bidder's strategy space is equal to her space of private values $[\underline{t}, \overline{t}]$; (ii) the mechanism is characterized by two functions $P : [\underline{t}, \overline{t}]^{n+1} \times [\underline{q}, \overline{q}] \to [0, 1]^{n+1}$ and $X : [\underline{t}, \overline{t}]^{n+1} \times [\underline{q}, \overline{q}] \to \mathbb{R}^{n+1}$, where $P_i(t_i, t_{-i}, q)$ and $X_i(t_i, t_{-i}, q)$ are respectively Bidder i's probability of winning the auction and Bidder i's transfers to the auctioneer when i announces t_i, and all the other bidders announce the vector of private types t_{-i}, and the common value equals q; and (iii), there is an equilibrium in which all bidders report their private value truthfully. This last condition can be expressed formally in terms of the following incentive compatibility constraints:

- For $i \neq I$,

$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} [(t_i + q)P_i(t_i, t_{-i}, q) - X_i(t_i, t_{-i}, q)]\, g(q) \prod_{j \neq i} f(t_j)\, dq\, dt_{-i} \geq$$

$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} \left[(t_i + q)P_i(\tilde{t}_i, t_{-i}, q) - X_i(\tilde{t}_i, t_{-i}, q)\right] g(q) \prod_{j \neq i} f(t_j)\, dq\, dt_{-i},$$

  for any $t_i, \tilde{t}_i \in [\underline{t}, \overline{t}]$.

- For $i = I$,

$$\int_{[\underline{t},\overline{t}]^n} [(t_I + q)P_I(t_I, t_{-I}, q) - X_I(t_I, t_{-I}, q)] \prod_{j \neq I} f(t_j)\, dt_{-I} \geq$$

$$\int_{[\underline{t},\overline{t}]^n} \left[(t_I + q)P_I(\tilde{t}_I, t_{-I}, q) - X_I(\tilde{t}_I, t_{-I}, q)\right] \prod_{j \neq I} f(t_j)\, dt_{-I},$$

  for any $t_i, \tilde{t}_i \in [\underline{t}, \overline{t}]$, and $q \in [\underline{q}, \overline{q}]$.

And the following individual rationality constraints:

- For $i \neq I$,

$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} [(t_i + q)P_i(t_i, t_{-i}, q) - X_i(t_i, t_{-i}, q)]\, g(q) \prod_{j \neq I} f(t_j)\, dq\, dt_{-i} \geq 0,$$

  for any $t_i \in [\underline{t}, \overline{t}]$.

- For $i = I$,

$$\int_{[\underline{t},\overline{t}]^n} [(t_I + q)P_I(t_I, t_{-I}, q) - X_I(t_I, t_{-I}, q)] \prod_{j \neq I} f(t_j)\, dt_{-I} \geq 0,$$

  for any $t_I \in [\underline{t}, \overline{t}]$, and $q \in [\underline{q}, \overline{q}]$.

Note that since the insider knows the common value, her incentive compatibility and participation constraints hold for any value of Q. On the other hand, since the outsiders do not know the common value, their incentive compatibility and participation constraints hold only on average with respect to Q.

If we apply Myerson's (1981) machinery to this problem, we find that a mechanism is optimal in our auxiliary problem if and only if: the good is allocated to the bidder with highest private value, any outsider with a private value $\underline{t}$ gets zero expected utility and an insider with a private value $\underline{t}$ and conditional on any realization of the common value $q \in [\underline{q}, \overline{q}]$ gets also zero expected utility.

One possible implementation of the above optimal mechanism of our auxiliary problem is that the auctioneer announces, first, the common value component and, afterwards, runs an open ascending auction. The revenue of this mechanism is the same as the revenue that the auctioneer gets with an open ascending auction in the symmetric information structure. Hence, the proposition follows because our auxiliary problem include as an special case the open ascending auction in the asymmetric information structure. The reason is that the auctioneer can always commit to forget the common value.

Proof of Proposition 8

We first show that the insider's expected utility in the symmetric information structure converges to zero as α tends to zero. For this purpose, note that under this assumption, the model is a pure private value auction, and as such, any bidder i bids $v(Q, T_i)$, in an equilibrium in non weakly dominated strategies. As a consequence, the insider wins if and only if $v(Q, \alpha T_I)$ is greater than $v(Q, \alpha T_{(1)}))$, and in that case, pays $v(Q, \alpha T_{(1)}))$. This means that the limit of the insider's expected utility is equal to:

$$\lim_{\alpha \to 0} E\left[\max\{u(v(Q, \alpha T_I) - v(Q, \alpha T_{(1)})), 0\}\right] = 0. \tag{7}$$

We complete the proof by showing that this is not the case for the asymmetric information structure if outsiders do not use weakly dominated strategies. Note that the insider bids $v(Q, T_I)$ and hence, conditional on winning she gets non negative utility. This means that the insider gets strictly positive probability if there exists a set of outsiders' types with probability bounded away from zero that quit before a certain bound strictly lower than $v(\overline{q}, \alpha\overline{t})$ whenever the insider is still in the auction.

To prove this, we shall show that prices above $v(\underline{q}, \alpha\overline{t})$ are weakly dominated for outsiders with a type below $\underline{t} + \frac{1}{\eta^4\mu^4}\frac{\overline{t}-\underline{t}}{2}$, if the insider is still active. This may be proved by showing that for an outsider i, the expected utility of winning at a price $p \geq v(\underline{q}, \alpha\bar{t})$ when the insider quits at $b \in [v(\underline{q}, \alpha\bar{t}), p]$ and conditional on any vector of outsiders' types $\vec{t}$, is negative. We show this below with the help of the functions $\underline{t}^*(b) \equiv \min\{t \in [\underline{t}, \bar{t}] : v(\bar{q}, \alpha t) \geq b\}$, and $q^*(t; b)$ implicitly defined by $v(q^*(t;b), \alpha t) = b$ which mimic the same functions that we used in the proof of Lemma 2 but for the indifference curve $v(q, \alpha t) = b$. Recall that since $b \geq v(\underline{q}, \alpha\bar{t})$, the maximum value of t in the indifference curve of b is $\bar{t}$, this is $\max\{t \in [\underline{t}, \bar{t}] : v(\underline{q}, \alpha t) \leq b\} = \bar{t}$.

$$
\begin{aligned}
&E[u(v(Q, \alpha t_i) - p)|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] \leq \\
&\quad E[u(v(Q, \alpha t_i) - v(Q, \alpha T_I))|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] = \\
&E[u(v(Q, \alpha t_i) - v(Q, \alpha \underline{t}) - (v(Q, \alpha T_I) - u(v(Q, \alpha \underline{t}))))|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] \leq \\
&E[u(\alpha\mu(t_i - \underline{t}) - \frac{\alpha}{\mu}(T_I - \underline{t}))|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] \leq \\
&E[u(\alpha\mu(t_i - \underline{t})) - \frac{\alpha}{\mu^2}(T_I - \underline{t})|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] \leq \\
&E[\alpha\mu^2(t_i - \underline{t}) - \frac{\alpha}{\mu^2}(T_I - \underline{t})|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}] = \\
&\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\mu^2}E[(T_I - \underline{t})|v(Q, \alpha T_I) = b, \vec{T} = \vec{t}]\right) = \\
&\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\mu^2}\frac{\int_{\underline{t}^*(b)}^{\bar{t}}\int_{\mathcal{W}}(t_I - \underline{t})f(t_I|w)\Pi_{j\neq I}f(t_j|w)g(q^*(t_I;b)|w)\,dH(w)dt_I}{\int_{\underline{t}^*(b)}^{\bar{t}}\int_{\mathcal{W}}f(t_I|w)\Pi_{j\neq I}f(t_j|w)g(q^*(t_I;b)|w)\,dH(w)dt_I}\right) \leq \\
&\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\eta^4\mu^2}\frac{\int_{\underline{t}^*(b)}^{\bar{t}}\int_{\mathcal{W}}(t_I - \underline{t})\Pi_{j\neq I}f(t_j|w)\,dH(w)dt_I}{\int_{\underline{t}^*(b)}^{\bar{t}}\int_{\mathcal{W}}\Pi_{j\neq I}f(t_j|w)\,dH(w)dt_I}\right) = \\
&\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\eta^4\mu^2}\frac{\int_{\underline{t}^*(b)}^{\bar{t}}(t_I - \underline{t})dt_I}{\bar{t} - \underline{t}^*(b)}\right) = \\
&\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\eta^4\mu^2}\left(\frac{\bar{t} + \underline{t}^*(b)}{2} - \underline{t}\right)\right) \leq \\
&\qquad\alpha\left(\mu^2(t_i - \underline{t}) - \frac{1}{\eta^4\mu^2}\frac{\bar{t} - \underline{t}}{2}\right) < 0.
\end{aligned}
$$

Proof of Proposition 9

Proposition 4 says that the insider gets higher expected utility in the symmetric information structure than in the asymmetric information structure in the same limits as in the current proposition. Consequently, in order to prove the Proposition, we only need to show that in these limits the insider gets higher expected utility in the hidden insider structure than in the symmetric information structure. The strategy of the proof is to show that in either limit considered in the proposition, the insider's expected utility in a symmetric equilibrium converges to zero in the symmetric information structure but remains bounded away from zero in the hidden insider structure.

We start computing an upper bound to the insider's expected utility in the symmetric information structure. A similar argument to that in Equation (5) in the Appendix may be used to show that:

$$u^*_{SIS}(t_{(1)}) \leq \int_{t_{(1)}}^{\bar{t}} \mu\eta(t_I - t_{(1)})\, dt_I.$$

Thus, taking expectations with respect to $T_{(1)}$ it is easy to see that the insider's expected utility in the symmetric information structure goes to zero as either γ goes to $-\Delta$ or n goes to infinity for $\gamma = 0$.

We next complete the proof by contradiction. We assume that the insider's expected utility in an equilibrium of the hidden insider structure converges to zero (in either of the above limits), and we show that if the outsiders use the same strictly increasing strategies, they have incentives to deviate. We shall focus on our exposition in the limit $\gamma \to -\Delta$. We explain at the end of the proof how to adapt the proof for the limit $n \to \infty$.

If the insider's expected utility tends to zero, it must also tend to zero conditional on the insider's maximum type $(Q, T_I) = (\bar{q}, \bar{t})$. This insider bids $v(\bar{q}, \bar{t})$ and wins with a positive profit whenever the final price in the auction is bounded away from this value. As a consequence, for any $p < v(\bar{q}, \bar{t})$, the probability that the price in the auction is strictly less than p conditional on the event that there is an insider with type $(Q, T_I) = (\bar{q}, \bar{t})$, must tend to zero.

The set of equilibrium paths that correspond to the event that there is an insider with a given type, for instance $(\overline{q}, \overline{t})$, and she wins at a price below p is the same as the set of equilibrium paths that correspond to the event that there is no insider and an outsider i with maximum type, i.e. $T_i = \overline{t} - \gamma$, wins at a price below p. These equilibrium paths are those generated by the bids of n outsiders with type in $[\underline{t}-\gamma, \overline{t}-\gamma]$ whenever they fix the price below p. Consequently, the probability that the price in the auction is strictly less than p conditional on the event that there is no insider and that an outsider i with a type $T_i = \overline{t} - \gamma$ wins must tend to zero when γ tends to $-\Delta$.

We next claim that, for any $\tilde{t} \in (\overline{t}, \overline{t} + \Delta)$, the same limit as above must hold if we condition on $T_i = \tilde{t}$ instead of on $T_i = \overline{t} - \gamma$. Formally that,

$$\lim_{\gamma \to -\Delta} \Pr\{P < p | NI, i \text{ wins}, T_i = \tilde{t}\} = 0, \tag{8}$$

where P denotes the price in the auction, "NI" the event that there is no insider, and "i wins" the event that the outsider i wins. To see why note three things. First, from the conclusions of the former paragraph we can deduce that the probability that the price in the auction is strictly less than p conditional on the events that there is no insider, that an outsider i with a type $T_i = \overline{t} - \gamma$ wins, and that the largest type of the losing outsiders is less than or equal to $\tilde{t}$, also tends to zero when γ tends to $-\Delta$. The reason is that we are conditioning on a smaller set whose probability conditional on the original set is bounded away from zero as γ tends to $-\Delta$.

Second, if there is no insider, the winner in the auction is the outsider with highest type, and the price is determined by the bids of the other outsiders. As a consequence, and because of the independency of the outsiders' types, the type of the outsider that wins is informative of the price only up to the point that it establishes an upper bound for the distribution of the types of the losing bidders. Hence, if we condition on a lower upper bound, for instance $\tilde{t}$, the type of the outsider that wins is not informative of the price. A consequence is that the distribution of the price in the auction when we condition on the event that there is no insider, and that the types of the losing outsiders are not more than $\tilde{t}$, is the same whether we condition additionally on the type of the winning bidder is equal to $\overline{t} - \gamma$ or on the type of the winning bidder is equal to $\tilde{t}$.

Third and last, the event that the winning outsider has a type equal to $\tilde{t}$ implies the event that the types of the losing outsiders are no more than $\tilde{t}$.

Clearly, the event that there is no insider and an outsider with type $T_i = \tilde{t}$ wins the auction has probability bounded away from zero, as γ tends to $-\Delta$. We can thus conclude from the limit in Equation (8) that an outsider with type $T_i = \tilde{t}$ wins the auction at prices above p with probability bounded away from zero as γ tends to $-\Delta$. But the expected utility of winning the auction at such prices is bounded from above by:

$$\left(1 - \Pr\left\{NI|P \geq p, i \text{ wins}, T_i = \tilde{t}\right\}\right) v(\overline{q}, \tilde{t}) + \\ \Pr\left\{NI|P \geq p, i \text{ wins}, T_i = \tilde{t}\right\} E\left[v(Q, \tilde{t})\right] - p. \quad (9)$$

The reason is that if there is an insider, the most optimistic case in which the outsider may win is when the common value is $\overline{q}$, but when there is no insider, the expected value of the good is equal to $E\left[v(Q, \tilde{t})\right]$. Moreover, the price paid would be no less than p.

We can use the limit in Equation (8) to show that the bound for the outsider's expected utility in Equation (9) is bounded from above by $v(\overline{q}, \tilde{t}) + (1-\rho)E[v(Q, \tilde{t})] - p$, as γ tends to $-\Delta$. Suppose that $\tilde{t}$ is such that the latter expression is negative. Then, we can conclude that an outsider with such a type $\tilde{t}$ gets negative expected utility if she wins the auction at a price above p if γ is sufficiently close to $-\Delta$. This implies that such outsiders may prove with a deviation in which they quit before the price reaches p.

The proof for the limit $n \to \infty$ is similar. The only difference is that instead of taking a generic $\tilde{t}$ we take $\overline{t}$. Note that this proof is simpler as some of the former steps become unnecessary.

Characterization of an Equilibrium of the Hidden Insider Structure

In this section of the appendix we provide a characterization of a symmetric separating equilibrium of the model in Section 5 assuming additionally: risk neutrality ($u(x) = x$), additive separability of the private and the common value component ($v(q,t) = q+t$), and Assumption (SCC). We also restrict to the case $\gamma = 0$, this is to the case in which the distribution of the insider's private value coincides with the outsiders' one. Our equilibrium can be extended to the case $\gamma \neq 0$, however, the proofs become much longer and more complex, mostly because we have to consider different cases to analyze the partial differential equation in Equation (11).

To construct our equilibrium we shall use some auxiliary functions. Let

$$\Psi(b_2, t_2|\rho_2, q_2) \equiv \frac{(1-\rho_2)f(t_2)}{\rho_2 \hat{f}(b_2)} \frac{t_2 + q_2 - b_2}{b_2 - (t_2 + E[Q|T_I + Q = b_2])},$$

where $b_2 \in [\underline{q} + \underline{t}, \overline{q} + \overline{t}]$, $t_2 \in [\underline{t}, \overline{t}]$, $\rho_2 \in (0,1)$, and $q_2 \in [\underline{q}, \overline{q}]$. The function Ψ is such that the following condition is satisfied if $b_2' = \Psi(b_2, t_2|\rho_2, q_2)$:

$$t_2 + \frac{\rho_2 \hat{f}(b_2)}{\rho_2 \hat{f}(b_2) + (1-\rho_2)f(t_2)/b_2'} E[Q|T_I + Q = b_2] + \frac{(1-\rho_2)f(t_2)/b_2'}{\rho_2 \hat{f}(b_2) + (1-\rho_2)f(t_2)/b_2'} q_2 = b_2. \quad (10)$$

This condition will assure that in our proposed equilibrium, the expected value of the good for an outsider with type t_2 conditional on winning at price b_2 is equal to b_2 in information sets with two bidders active. To understand why, note that the terms in the left hand side and after t_2 indicate the expected common value conditional on the last bidder leaving the auction at price b_2, given that the other outsiders use a bid function that equals b_2 and has slope b_2' at t_2, given that the probability that the other bidder still active is the insider is equal to ρ_2 and given that the expected common value conditional on the event that the other remaining bidder in the auction is not the insider[22] is equal to q_2.

[22] Recall that the insider may not be active in a given information sets because either all the bidders

Let also,

$$\rho^u(t,b,b',\rho) \equiv \frac{\rho f(t)/b'}{\rho f(t)/b' + \rho\hat{f}(b) + (1-\rho)f(t)/b'},$$

and,

$$q^u(t,b,b',q,\rho) \equiv$$

$$\frac{\rho\hat{f}(b)}{\rho\hat{f}(b) + (1-\rho)f(t)/b'}E[Q|Q+T_I=b] + \frac{(1-\rho)f(t)/b'}{\rho\hat{f}(b) + (1-\rho)f(t)/b'}q.$$

The function ρ^u will be used to update the probability that the insider is among the active bidders when the game moves to a new information set because one bidder has left the auction at a price b. Similarly, the function q^u will be used to update the expected common value conditional on the event that the insider is not among the remaining bidders, when the game moves to a new information set because one bidder has left the auction at a price b. To avoid long expressions, we shall refer to both functions simply as $\rho^u(.)$ and $q^u(.)$ when there is no risk of misunderstanding.

Finally, let $t^*(q)$ be such that $\beta(t^*(q)) = t^*(q)+q$, and $b^*(q) = \beta(t^*(q)) = t^*(q)+q$ for any $q \in (\underline{q}, \overline{q})$, where recall that the function β was already defined in Section 4. We shall assume for simplicity that the function t^* is uniquely defined, and thus $b^*(q)$.[23] Thus, if $t \in [\underline{t}, t^*(q))$, then $\beta(t) < t+q$, and otherwise $\beta(t) \geq t+q$, see Figure 3. Note that $t+q$ and $\beta(t)$ are, respectively, the price at which an outsider with type t is indifferent between winning and losing if the last bidder in leaving the auction is an outsider or an insider. Thus, depending on whether $t > t^*(q)$ or $t < t^*(q)$, an outsider with type t has, respectively, more or less incentives to remain in the auction if there is an insider than if there is not.

For any $\rho_3 \in (0,1)$, $q_3 \in (\underline{q}, \overline{q})$, and $t_4 \in [\underline{t}, t^*(q_0)]$ we define the following partial differential equation:

$$(PDE) \equiv \frac{\partial b_2(t_2,t_3)}{\partial t_2} = \Psi(b_2(t_2,t_3), t_2|\rho^u(.), q^u(.)), \tag{11}$$

in the domain $\{(t_2,t_3) \in [t_4, \overline{t}]^2 : t_2 \geq t_3\}$, and with boundary conditions $b_2(t_3,t_3) = b_3(t_3)$ and $b_2(\overline{t},\overline{t}) = \overline{t}+\overline{q}$, and where, stands for $\rho^u(t_3, b_3(t_3), b_3'(t_3), \rho_3)$, and $q^u(.)$

were outsiders or because one bidder was an insider but she has already left the auction.

[23]A sufficient condition is that the densities of f and g are log-concave.

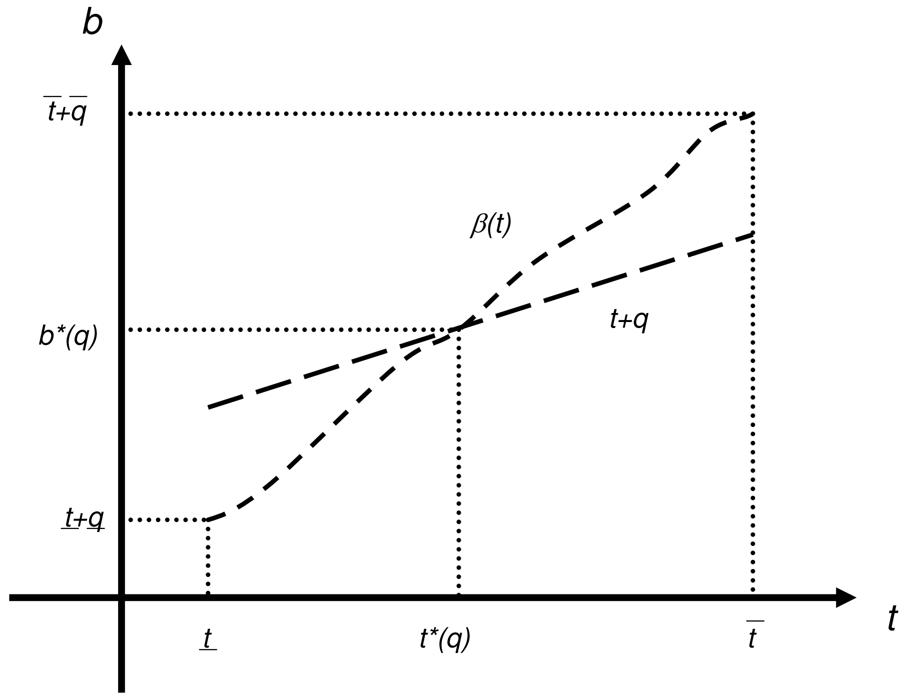

Figure 3: Functions $\beta(t)$, $t+q$, t^* and b^*.

for $q^u(t_3, b_3(t_3), b_3'(t_3), q_3, \rho_3)$. Figure 4 displays the domain of the partial differential equation and the boundary conditions.

The function $b_3(t_3)$ will be used to define the equilibrium bid of an outsider with type t_3 when there are three bidders active, and the function $b_2(t_2; t_3)$, the bid of an outsider with type t_2 when there are two bidders active and where t_3 is the outsider's type that corresponds in equilibrium to the price at which the last bidder quit. Equation (11) says that b_2 as a function of t_2 satisfies the condition (10) given the updated values ρ^u and q^u. The boundary conditions specify some kind of continuity in the transition from bid function b_3 to bid function b_2, see also the comments immediately before Proposition 10, and that b_2 finishes at a level equal to the maximum possible value of the insider, i.e. $\overline{q} + \overline{t}$.

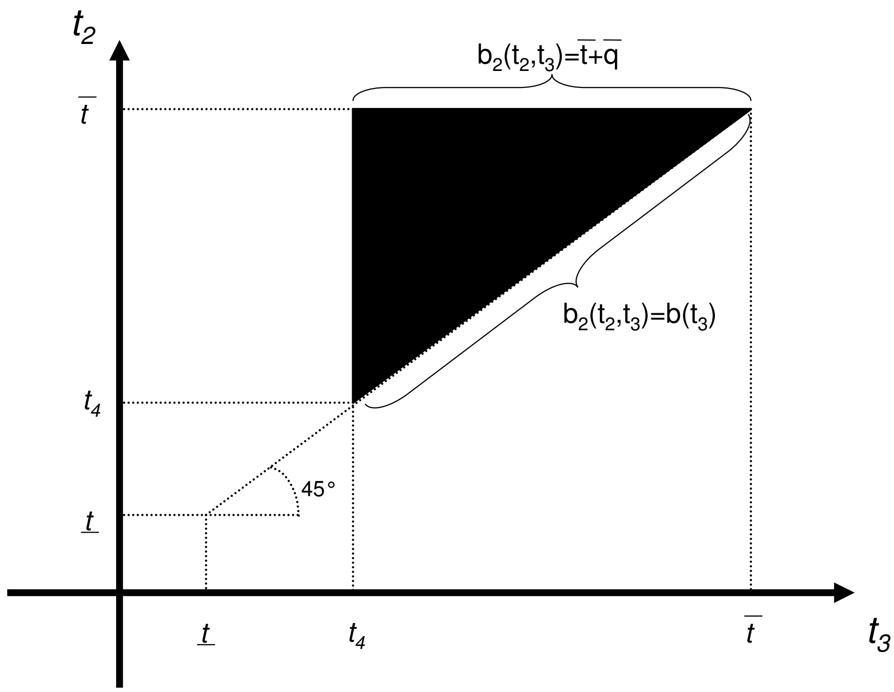

Figure 4: Domain and boundary conditions of (PDE).

In the next lines, we assume that there exists a pair of functions strictly increasing a.e. in the first argument that are solution to (PDE). We shall refer to these functions as $b_2^*(t_2; t_3|\rho_3, q_3, t_4)$ and $b_3^*(t_3|\rho_3, q_3, t_4)$. We use these functions to define a strategy that we later show it is in fact an equilibrium strategy of the hidden insider structure.

To simplify the exposition, we shall assume that the number of bidders is strictly greater than three. We use two auxiliary state variables, ρ_i and q_i, $i = 2, 3, ..., n, n+1$, where n will refer to the number of active bidders. These two variables will carry information inferred from the history that we use to define the strategy. Let $\rho_{n+1} \equiv \frac{\rho}{(n-1)(1-\rho)+\rho}$ and $q_{n+1} \equiv E[Q]$.

We define our strategy for the case in which no bidder has left the auction yet as the minimum of $\beta(t)$ and $t + q_{n+1}$. We next proceed recursively. We assume that the bid function has been defined for any information set with $i+1$ bidders or more and define the bid function for information sets with i bidders.

First, we consider the case of information sets in which the price at which the last bidder quit, that we denote by b_{i+1}, does not correspond to the bid of any outsider's type in the information set immediately before the one under consideration. In that case, the strategy specifies a quitting price $t + E[Q|Q + T_I = b_{i+1}]$ in all the remaining information sets.

Finally, we consider the remaining information sets. Then, we denote by t_{i+1} the outsiders' type that corresponds to the price at which the last bidder quit, b_{i+1}, and by b'_{i+1} the slope of the bid function in the information set before the current one and evaluated at type t_{i+1}. We also let $\rho_i \equiv \rho^u(t_{i+1}, b_{i+1}, b'_{i+1}, \rho_i)$ and $q_i \equiv q^u(t_{i+1}, b_{i+1}, b'_{i+1}, \rho_{i+1}, q_{i+1})$. Then, the bid function is equal to:

- $\min\{\beta(t), t + q_i\}$, if $i > 3$.
- $b^*_3(t|\rho_3, q_3, t_4)$, if $i = 3$.
- $b^*_2(t; t_3|\rho_3, q_3, t_4)$, if $i = 2$.

We provide next an intuitive description of the proposed bid function. We start with the bid function at information sets with more than three bidders active. In these information sets we have some freedom to determine equilibrium bids. The reason is that the auction does not finish there, and hence, the bids do not determine the price directly. The only restriction is that the quitting prices are not too high, as otherwise, it could rise the prices in information sets in which the price is finally set, this is, when there are two bidders active, to too high levels. Our proposed bid function takes into account this restriction.

The situation in information sets with three bidders active is similar up to one point. The strategy must be continuous in the sense that if an outsider with type t quits at p when three bidders are active, an outsider with type t must also quit at p in the continuation game with two bidders active. This means that after one bidder quits at p, there is no interval of prices with lower bound p in which the probability that an outsider quits is zero. This condition is included as a boundary condition of the partial differential equation (PDE). Note that if this condition were not satisfied, and the insider had a type in that interval, the final price in the auction would be equal to the insider's bid. In that case, it may be shown that the outsider that wins the auction either gets strictly positive or strictly negative expected utility with generality. The anticipation of these gains or losses may create outsiders' incentives to deviate in the information set with three bidders active.

Finally, the intuitive explanation of the strategy in information sets in which there are only two bidders active and in information sets after a bid that does not correspond to the outsider's strategy is similar. In both cases, the bid function is such that the outsider is indifferent between winning and losing at the price at which she quits, given the information that can be derived from the observable history and under the assumption that all the outsiders has followed the proposed strategy in the former information sets. The explanation for this in information sets after a bid that does not correspond to the outsider's strategy is straightforward. In the other information sets, this is true because because b_2^* satisfies the indifference condition (10), since it solves the partial differential equation (PDE).

Proposition 10. *Suppose that (PDE) has a solution that it is strictly increasing a.e., then the strategy defined above is an equilibrium of the hidden insider structure.*

Proof. We compute the expected utility of winning of a given outsider of type t assuming that all the other bidders follow the proposed strategy. We distinguish two possible histories in which our outsider may win the auction. In the first one, there is an information set in which one bidder quits at a price, say p, that does not correspond to any type of the outsiders according to the proposed strategy. Note that by definition of our strategy this may only happen in information sets in which there are at least three active bidders. In such histories, the bidder that left the auction at price p must be the insider and hence, the expected value of the good for our outsider is equal to $t + E[Q|Q + T_I = p]$. Moreover, since there must be at least one other outsider in the auction after the insider's quit and our proposed strategies are strictly increasing, if our outsider wins, the price must be equal to $t_2 + E[Q|Q + T_I = p]$, where t_2 denotes the highest type of the other outsiders. Consequently, our outsider finds profitable to win the auction if and only if $t \geq t_2$. This is achieved with the proposed strategy.

Consider now histories in which all the prices observed in the auction belong to the range of our bid function. Denote by $\{b_j\}_{j=2,\ldots,n+1}$ the sequence of prices where b_j corresponds to the price at which a bidder quit in the information set with i bidders active. Denote also by $\{t_j\}_{j=2,\ldots,n+1}$ the types of the outsiders that corresponds to the prices $\{b_j\}_{j=2,\ldots,n+1}$ according to our proposed strategy. Let also $\{b'_j\}_{j=2,\ldots,n+1}$ be the slope (with respect to the bidder's type) of our bid function evaluated at the corresponding type and information set. In this case, the expected value of the good for an outsider of type t that wins the auction equals:

$$t + \frac{(1-\rho)\, n! \prod_{i=2}^{n+1} f(t_i)/b'_i \cdot q + \sum_{i=2}^{n+1} \rho\,(n-1)!\, \hat{f}(b_j) \prod_{i=2,i\neq j}^{n+1} f(t_i)/b'_i \cdot E[Q|Q+T_I=b_j]}{(1-\rho)\, n! \prod_{i=2}^{n+1} f(t_i)/b'_i + \sum_{j=2}^{n+1} \rho\,(n-1)!\, \hat{f}(b_j) \prod_{i=2,i\neq j}^{n+1} f(t_i)/b'_i}$$

The price that this bidder pays if she wins is equal to b_2, that corresponds to the bid of an outsider's type t_2 in an information set with only two bidders active. The value of this bid can can be computed after some tedious substitution of the state variables ρ_i and q_i as:

$$t_2 + \frac{(1-\rho)\, n \prod_{i=2}^{n+1} f(t_i)/b'_i \cdot q + \sum_{j=2}^{n+1} \rho\, \hat{f}(b_j) \prod_{i=2,i\neq j}^{n+1} f(t_i)/b'_i \cdot E[Q|Q+T_I=b_j]}{(1-\rho)\, n \prod_{i=2}^{n+1} f(t_i)/b'_i + \sum_{j=2}^{n+1} \rho\, \hat{f}(b_j) \prod_{i=2,i\neq j}^{n+1} f(t_i)/b'_i}$$

It can be shown by simple substraction and simplification of the two last equations that our outsider finds profitable to win the auction if and only if $t \geq t_2$, and this is exactly what the proposed strategy does. ■

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