A MODEL OF VOTING WITH INCOMPLETE INFORMATION AND OPINION POLLS *

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WP-AD 99-25

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ABSTRACT

A one-dimensional model of spatial political competition with incomplete information is developed. It is assumed that voters care about the distribution of votes among the two candidates. Voters have an incomplete information about the distribution of voters' types. We provide conditions for which the publication of opinion polls may solve the informational problem voters face. The main result states that only when the distribution of voters is polarized we could expect that voters act as if they were fully informed.

Key Words: Spatial competition, Incomplete Information, Opinion Polls.

JEL Classification Number: D72.

1   Introduction

The first models on spatial political competition used to assume complete information about all the relevant parameters for the determination of the equilibrium strategies. Namely, parties and agents know the distribution of agents' types and also the policy implemented after the election. In the two party case, complete information (with single peakedness) implies that both parties will adopt the ideal policy of the median voter. In more recent models, however, it is standard to assume that parties do not know the exact distribution of agents. This relaxation of the information requirements has important implications: if the two parties are ideological the equilibrium policies don't (in general) converge to the median (see Wittman[13]). In those models, whether agents know the aggregate distribution of agents' types or not it is not relevant since, at least in the two-party case, voting for the "closest" party is always a dominant strategy. The reason for this is quite simple. Suppose that an agent prefers the policy proposed by party L rather than the policy proposed by party R. If he votes for R either he doesn't change the outcome or he brings R's victory. In a similar way voting for L may have two consequences either the outcome is not affected or L wins. Clearly, such an agent will always vote for L. Notice that a key assumption in the previous argument was that the implemented policy coincides with the one proposed by the winning party. Thus, the only relevant fact after the election is which party gets more than 50 per cent of the votes. It doesn't make any difference if the winner gets, say, 51 or 90 per cent of the votes.

Recently, several authors have challenged this view of the political process (see Alesina and Rosenthal [1], [2], Austen-Smith and Banks [3] , Gerber and Ortuno [7],Grossman and Helpman [8], and Ortuno [11]). They develop models of political spatial competition in which the implemented policy depends on the whole distribution of votes. The intuition behind these models is that in many democratic societies the implemented policy is a compromise between the proposals put forward by the different parties. The higher the number of votes obtained by a party the closer the achieved compromise is to its proposal. In this case, agents may have preferences over the distribution of votes. As a consequence of this assumption voting for the closest party will not always be a dominant strategy. Hence, agents may want to know what other agents will do in the election day.

In this paper we also adopt the mentioned "compromise" approach to political competition to analyze a model in which **agents don't have complete**

information on the distribution of other agents' types. This seems to be a very realistic assumption for elections with a large number of voters. In most countries, however, many election polls are regularly conducted and made public before the election. Thus, these polls may reduce the informational problem agents face. In order to decide for which party to vote an agent needs to know the (expected) distribution of votes and this is what election polls provide. The problem, however, is that this same information from the polls may be expected to modify the behavior of voters (see Bowden [4]). We will try to give an answer to the following question: under which circumstances can a sequence of polls induce a voting behavior that coincides (or it is very close) to the one we would observe under complete information ? To answer this question we will assume that all agents vote for one of the two competing parties (so no abstention). However, when an agent responds to a public opinion poll , before election day, she can be "undecided". She will announce to which party her vote will go only if she is "reasonable" sure about that.

The main result in this paper suggests that polls are only efficient mechanisms to "converge" to the complete information case if the distribution of agents types is "polarizated". I.e., when the distribution of agents' ideal policies is U-shaped, (and the parties proposal are not on the same side of the distribution) polls will converge and the actual outcome will converge to the complete information case. When the distribution is Λ-shaped this will not happen in general.

The model in Myerson and Weber [10] is related to the model herein, in that they consider the informational role of opinion polls. There each agent can be pivotal and this is an essential feature of the model. Simon [12] postulates the existence of a bandwagon effect arising from the publication of opinion polls. McKelvey and Ordeshook [9] consider a model with uninformed and informed voters. The first ones don't know the policy positions of the candidates. It is shown that in equilibrium polls provide the relevant information to the uninformed voters and the voting outcome coincides with the complete information voting outcome. Cukierman [6] also considers a model with informed and uninformed voters. Here candidates are evaluated by their policy positions and by a "general ability position". Informed voters have better information about candidates' general abilities than uninformed voters. Polls provide information to the uninformed about candidates' abilities. Thus some voters may interpret a candidate's high approval in the polls as a signal that he is more able. However, this high approval may be

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a consequence of a change in the distribution of voters preferences on the policy space. As a result "the candidate wins with a margin higher than the margin he would have obtained either in the absence of polls or under perfect information" (pg. 182).

In all these models the publication of opinion polls generate a bandwagon or momentum effect. In our model, however, the dynamic is quite different: a candidate's high approval in a poll may induce many undecided agents to vote for the other candidate. [1]

2   The Model

We study a two party election. Parties are denoted by $j \in \{L, R\}$. Party j proposes policy $t_j \in \Re$. We will assume here that parties have already chosen their proposals and they are not able to change them. Thus, $t_j, j \in \{L, R\}$ are fixed numbers in our model. Without loss of generality we will assume that $t_L = 0$ and $t_R = 1$. The set of agents' types is $\Re$. An agent of type $x \in \Re$ has utility function $v(.; x) : \Re \to \Re$. The function $v(.; x)$ attains a unique maximum at x and $\forall y, z \in \Re$ if $\mid y - x \mid \geq \mid z - x \mid$ then $v(z; x) \geq v(y; x)$. Thus, x is the "ideal policy" for agent of type x and the utility is a decreasing function of the distance to x. The distribution of types is given by the CDF $F : \Re \to [0, 1]$ with continuous density function f. Notice that we are assuming a continuum of agents.

At election day all agents vote for one of the two competing parties. Throughout the paper we will assume that there is no uncertainty about the two proposal, i.e. $(t_L, t_R) = (0, 1)$ is common knowledge. Let Ψ be the percentage of votes obtained by party L. We now departure from most models in spatial political competition and assume, in a similar fashion as in Alesina and Rosenthal[1],[2][2], Grossman and Helpman[8], Ortuno[11] and Gerber-Ortuno[11], that the implemented policy, $x(t_L, t_R)$, is a compromise between t_L and t_R, namely,

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[1]Recent elections in Austria, Spain and Finland seem to confirm this possibility. It seems that many moderate people believed that according to the last polls the outcome would be given too much power to one side of the political spectrum. To compensate for that, they decided, in the last moment, to vote in the other direction. The result was a big failure in the polls' predictions.

[2]Alesina and Rosenthal consider that the implemented policy is a convex combination of the policy put forward by the presidency and the policy put forward by the Congress.

$$x(t_L, t_R) = \Psi\, t_L\, +\, (1 - \Psi)\, t_R. \tag{1}$$

Since we take $(t_L, t_R) = (0, 1)$ we can write the implemented policy as $x(\Psi) = 1 - \Psi$.

Thus, the greater the number of votes obtained by L the closer the implemented policy to 0. The linearity of equation (1) is, clearly, a strong restriction. We will comment on the implications of this for our results later on. Given that there is a continuum of agents no single agent can affect the implemented policy. We assume, however, that agents want to behave in such a way that after the election nobody "regrets" her vote. Thus, if an agent $x \geq x(\Psi)$ voted for L, after the outcome is revealed, she will regret the way she voted. Even though her vote couldn't have changed anything she voted in the wrong direction. With this intuitions in mind we can define our equilibrium concept (see Alesina-Rosenthal[1][2] and Gerber-Ortuno[7] for a similar equilibrium concept).

Let s_x denote the vote or "strategy" chosen by agent x. We assume that $s_x \in \{s_x^0, s_x^1, s_x^{0.5}\}$. The interpretation is that s_x^0 (s_x^1) means a vote for L (R) and $s_x^{0.5}$ means that with probability 0.5 he votes for L. Let $s = (s_x)_{x \in \Re}$ be a list of strategies one for each type of agent and let $\Psi(s)$ be the associated percentage of votes obtained by party L.

Definition 1 *A list of strategies* $s = (s_x)_{x \in \Re}$ *forms a **Voting Equilibrium** (VE) if* $\forall x \in \Re$

$$
\begin{aligned}
s_x = s_x^0 \quad & implies \quad x\left(\Psi(s)\right) > x \\
s_x = s_x^1 \quad & implies \quad x\left(\Psi(s)\right) < x \\
s_x = s_x^{0.5} \quad & implies \quad x\left(\Psi(s)\right) = x.
\end{aligned}
$$

It is not difficult to see that our VE is a standard Strong Nash Equilibrium. An agent votes to move the implemented policy toward her ideal policy. Even though she cannot do it on her own she might believe that people with types close to hers will behave in the same way. At equilibrium there is no set of agents (with positive measure) that can deviate from their strategies and change the outcome in a beneficial way for all of them.

Lemma 1 *There always exists a unique Voting Equilibrium s*.*

The proof follows easily from continuity of the implemented policy on Ψ and the continuity and monotonicity of F.

Given the (unique) Voting Equilibrium s^* we write the associated implemented policy as $x^* = x\left(\Psi(s^*)\right)$. Notice that at equilibrium all agents to the left (right) of the implemented policy x^* vote for L (R), and $x^* = 1 - F(x^*)$. The formal definition of a VE doesn't require common knowledge of the distribution of types. It is clear, however, that this is only a meaningful concept if F is common knowledge. Thus, we will denote x^* as the **full information policy** and $\epsilon^* = F(x^*) = \Psi(s^*)$ the **full information voting outcome**.

3   Incomplete Information

It is natural to assume that in many real situations agents don't have complete information on F. For some agents this will not be a problem. For example, an agent of type $x < 0$ will always vote for L regardless what other agents do. Thus, it is clear that all agents with type outside the interval $[0, 1]$ have a dominant strategy. However, "moderate" agents, i.e. agents with types in between the two political proposals, have no dominant strategies. Think, for example, in agents with ideal policy around point 0.4. If they believe that more than 60 percent of the voters will vote L they will prefer to vote for R (even though they are closer to the proposal of party L). If they think, on the contrary, that, say, no more than 50 percent will vote for L, they will vote for L. This implies some type of "underdog" effect among moderate voters. (See Ceci and Kain [5] for empirical evidence supporting that this effect is more likely to happen among moderate voters).

Notice that the only relevant information those agents need (to calculate their optimal strategy) is the share of votes that each party will get. It can be argued, then, that polls can provide this information. In this case agents would vote as if they were under complete information. The problem is, however, that "Voting intentions may change from day to day as the voters' perceptions evolve during a campaign, so that a poll, when published, may invalidate itself."(Myerson and Weber [10, page 102]). The question then is to find under which circumstances the election under incomplete information and polls lead to the same outcome as the election outcome when the distribution of agents' types is common knowledge. The answer to this question depends very much on the assumptions on voters' beliefs and the way they use the information provided by the polls. Here we will adopt a

very simple model to deal with those issues that will yield clear results. We believe, however, that the basic results are robust to more general cases and more "rational" types of agents.

We suppose that there is a sequence of polls at periods $t = 0, 1, 2, ..., T-1$. The election takes place at period T. For $t < T$ we write $p_{x,t}^i$, $i \in \{0, 1, \emptyset\}$ to denote the voting intention that agent x (if asked) declares in period t. Thus, $p_{x,t}^0$ $(p_{x,t}^1)$ denotes that x will vote for L (R) and $p_{x,t}^\emptyset$ denotes that x doesn't know yet. This possibility of "indecision" is essential in our model. This is also a very common feature in many real elections.

A poll at the end of period t is a vector $P_t = (\epsilon_t, u_t)$ such that ϵ_t is the percentage of people that, during period t, announced the intention to vote for L and u_t the percentage of people who announced not to know yet which party they will vote for. Thus we see a poll as providing information on current intentions (in this sense a poll is not a prediction of the election outcome). We assume that P_t is a statistically perfect poll and appropiately samples and measures voting intentions at that period. Since we impose the condition that in the election day everybody votes for one of the two parties we may write the outcome of the election as $\epsilon_T \equiv \{$percentage of people who vote for $L\}$.

At each period $t = 0, 1, ..., T-1$ agent x has subjective beliefs on the outcome ϵ_T given by $\sigma_{x,t}$. These beliefs are CDF on $[0, 1]$, i.e. $\sigma_{x,t}(y) =$Probability $\{\epsilon_T \leq y\}$. In general $\sigma_{x,t}$ will depend on the original beliefs $\sigma_{x,0}$ and the information provided by the polls $P_{t'}$, $t' < t$. Define

$$
\begin{aligned}
m_{x,t} &= \sup\{y : \sigma_{x,t}(y) = 0\} \\
M_{x,t} &= \inf\{y : \sigma_{x,t}(y) = 1\}
\end{aligned}
$$

We suppose that at period $t = 0, 1, ..., T-1$, agent x responds to a poll in the following way

$$
p_{x,t} = \begin{cases} p_{x,t}^1 & \text{if } m_{x,t} 0 + (1 - m_{x,t}) 1 < x \\ p_{x,t}^0 & \text{if } M_{x,t} 0 + (1 - M_{x,t}) 1 \geq x \\ p_{x,t}^\emptyset & \text{otherwise} \end{cases} \tag{2}
$$

The expression (2) just says that an agent answers that her vote will be for, say, R if the implemented policy can never be, according to her beliefs, to the right of her ideal policy. This is equivalent to saying that agents want to rule out the possibility of regret, i.e., the possibility that later on they do something different from what they had stated. Thus, agents only announce

their vote intention when they are quite sure about it.

This may be seen as a very ad hoc feature of our model. We believe, however, that there is some intuitive justification for it. An agent has nothing to gain by announcing that her vote will be for, say, L if she will finally vote for R. This seems to be consistent with the fact that in many elections the percentage of undecided voters is quite high [3]. Moreover, the results in this paper are robust to changes that allow for more "flexible" rules. Thus, suppose that agents announce their vote intentions eventough they are not completely sure about their final vote. The results would be similar to the ones provided here as long as agents believe that with a high enough probability they will vote for the same party they said they were going to vote[4].

Now we have to describe the way people vote at period T. There are several ways to model this decision problem. Here it will be assumed that the relevant variable is the expected value of the implemented policy. Thus, at period T agent x votes in the following way

$$
s_x = \begin{cases} s_x^0 & \text{if } \int_0^1 (x - (1 - \epsilon))\, d\sigma_{x,T}(\epsilon) < 0 \\ s_x^1 & \text{if } \int_0^1 (x - (1 - \epsilon))\, d\sigma_{x,T}(\epsilon) > 0 \\ s_x^{0.5} & \text{otherwise} \end{cases} \tag{3}
$$

A type x agent will vote for L only if the expected implemented policy –according to her subjective beliefs– is to the right of x. In the case the expected implemented policy coincides with the ideal policy an agent will vote with probability 0.5 for L (this is not essential and we could, alternatively, assume that she doesn't vote). One might object against this type of behavior because (3) doesn't depend on the utility function. It turns out that this is not a problem in our model. Agents who were not undecided at period $T-1$ will not be undecided at period T either, so their behavior wouldn't change if we introduce the expected utility in (3). The problem arises with people who at period T –the election day– don't know yet if the outcome will be to the right or the left of their ideal policies. In principle, it seems more reasonable to assume that they consider the expected utility rather than the expected

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[3]This, however, can also be explained by different reasons. Thus, if voters are "regret averse" or not becomes an empirical question.

[4]An alternative approach would assume that an agent announces her vote for the party that, for example, in expected terms is more likely to be the one she finally votes for. This, however, would rule out the possibility of undecided agents. Moreover, this approach requires more information than the one we assume here about the way agents change their beliefs $\sigma_{x,t}$.

outcome. This alternative, however, wouldn't change the type of qualitative results provided in this paper and so we stick to (3) for simplicity.

Now we must specify the way beliefs may change through time. Let $P_{t-1} = (\epsilon_{t-1}, u_{t-1})$ be the information provided by the poll at the end of period $t-1$. Then the beliefs at period t are such that for all x

$$
\begin{aligned}
M_{x,t} &= \epsilon_{t-1} + u_{t-1} \\
m_{x,t} &= \epsilon_{t-1}
\end{aligned}
\tag{4}
$$

Here we are saying that agents believe that the number of votes L (R) will get cannot be less than ϵ_{t-1} (more than $\epsilon_{t-1} + u_{t-1}$). Notice that this restriction is compatible with the possibility that the ratio of votes in favor of L over votes in favor of R changes over time. The equalities in (4) also imply that, at period t, agents believe that with some positive probability (almost) all undecided agents will vote L and with some positive probability (almost) all undecided agents will vote R. In other words, this assumption captures the idea of a very imperfect information on the distribution of types.

We must now specify the beliefs agents have at period $t = 0$, when no poll has been conducted yet. We will assume –consistent with the idea that agents have a very imperfect information on F– that first period beliefs, $\epsilon_{x,0}$, are such that only agents outside the interval $(0, 1)$ will be sure about the party they will vote for. Therefore, we have that $P_0 = (\epsilon_0, u_0)$, where $\epsilon_0 = F(0)$ and $u_0 = F(1) - F(0)$.

4   Results

We are interested in the sequence of polls and election outcomes

$$
P_0 = (\epsilon_0, u_0), P_1 = (\epsilon_1, u_1), ..., P_{T-1} = (\epsilon_{T-1}, u_{T-1}), \epsilon_T
$$

If polls are good agregators of information we should observe $\epsilon_T = \epsilon^*$ or at least that $\lim_{T \to \infty} \epsilon_T = \epsilon^*$. (By $T \to \infty$ we mean that the number of polls, before election day, goes to infinity). A sufficient condition for this to happen is that

$$
\lim_{T \to \infty} (\epsilon_{T-1}, u_{T-1}) = (\epsilon^*, 0)
\tag{5}
$$

At period $t = 0$ we have $P_0 = (\epsilon_0, u_0)$. Now it is quite straightforward to calculate X_L^1, the set of types who at period $t = 1$ announce to vote for L.

$$
X_L^1 = \{x \le x_L^1, \text{where } x_L^1 = 1 - F(1)\}
\tag{6}
$$

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In a similar way the set of types who announce to vote for R is

$$X_R^1 = \{x \geq x_R^1, \text{where } x_R^1 = 1 - F(0)\} \tag{7}$$

Notice that (6) and (7) follow from the behavior described by (2). Agents at period $t = 1$ believe that there is some positive probability that almost all the undecided agent at $t = 0$ vote for L. Were that the case the implemented policy would be x_L^1 and then all agents with ideal points to the left of such number should vote for L (a similar argument works for x_R^1). For the rest of periods we can define –in a similar way to X_L^1 and X_R^1– the set of types announcing their vote for L and the set of types announcing their vote for R.

$$X_L^t = \{x \leq x_L^t, \text{where } x_L^t = 1 - F(x_R^{t-1})\} \tag{8}$$
$$X_R^t = \{x \geq x_R^t, \text{where } x_R^t = 1 - F(x_L^{t-1})\} \tag{9}$$

Then the variables we need to analyze are x_L^t and x_R^t: at period t agents with type in the interval $[x_L^t, x_R^t]$ are undecided and agents with type $x < x_L^t$ ($x > x_L^t$) will vote for L (R). Thus, given initial conditions $x_L^0 = 0, x_R^0 = 1$ the dynamic process to consider is given by the following system of equations

$$\begin{array}{llll} i) & x_L^t & = & 1 - F(x_R^{t-1}) \\ ii) & x_R^t & = & 1 - F(x_L^{t-1}) \end{array} \tag{10}$$

If $\lim_{T \to \infty} x_L^{T-1} = \lim_{T \to \infty} x_R^{T-1} = x^*$ then (5) is satisfied. Given that the function F doesn't need to be linear, in principle the system in (10) may have many fixed points.

To simplify the analysis and the exposition of our results we will consider distributions of agents with density functions which are symmetric with respect to point 0.5.

$$\Delta \equiv \{f : \forall x, \ f(0.5 - x) = f(0.5 + x)\}$$

Furthermore, we will suppose that the distribution of types is either U-shaped or U-inverted-shaped on the interval $[0, 1]$

$$\Theta \equiv \left\{ f : \begin{array}{ll} \text{either} & f \text{ is increasing in} [0, 0.5] \text{and decreasing in } [0.5, 1] \\ \text{or} & f \text{ is decreasing in} [0, 0.5] \text{and increasing in} [0.5, 1] \end{array} \right\}$$

Thus, we will restrict ourselfes to the set of density functions $\Gamma \equiv \Theta \cap \Delta$. The restriction on symmetric distributions is not needed at all and it is just

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adopted for simplicity. The fact that the density function has to be in Θ is, on the contrary, very much needed for our results.[5] In the case f weren't in Θ the functions i) and ii) in (10) could cross each other in many different ways and it would be difficult to obtain general results.

We say that $f \in \Gamma$ is **polarized** (**centered**) if $f(0.5) \leq f(x), \forall x \in [0, 1]$ $(f(0.5) > f(x), \forall x \in [0, 1])$.

Theorem 2 *Let $f \in \Gamma$ and $F(0) > 0$. Then*

i) *if f is polarized then*

$$\lim_{T \to \infty} x_L^{T-1} = \lim_{T \to \infty} x_R^{T-1} = x^* = 0.5$$

ii) *if f is centered and there exits $\bar{x}_L \in (0, 0.5),$ and $\bar{x}_R \in (0.5, 1)$ such that $F(\bar{x}_L) = \bar{x}_L$ and $F(\bar{x}_R) = \bar{x}_R$ then*

$$\lim_{T \to \infty} x_L^{T-1} = \bar{x}_L < x^* = 0.5 < \lim_{T \to \infty} x_R^{T-1} = \bar{x}_R$$

Proof. i) Notice that $x = 0.5$ is an equilibrium point of (10). Moreover, the full information policy is $x^* = 0.5$. We can write (10) as

$$\begin{array}{llll} a) & x_R^{t-1} & = & F^{-1}(1 - x_L^t) \\ b) & x_R^t & = & 1 - F(x_L^{t-1}) \end{array} \qquad (11)$$

Clearly $F^{-1}(1 - x_L^t)$ and $1 - F(x_L^{t-1})$ are both decreasing functions in $[0, 0.5]$ (see figure 1) and for $x^* = 0.5$ we have $x^* = F^{-1}(1 - x^*)$ and $x^* = 1 - F(x^*)$. Thus we only need to show that

$$F^{-1}(1 - x) > 1 - F(x) \ \forall x \in [0, 0.5] \qquad (12)$$

Inequality (12) is equivalent to

$$x > 1 - F(1 - F(x)) \ \forall x \in [0, 0.5] \qquad (13)$$

Given that $F(0) > 0$ and f is symmetric and polarized we have $F(x) > x \, \forall x \in [0, 0.5)$, $F(x) < x \, \forall x \in (0.5, 1]$ and $1 - x < 1 - F(x) \forall x \in (0.5, 1]$. Then (13) follows easily from these inequalities. Thus, the dynamic process given by (10) converges to the fixed point $x_L = x_R = 0.5 = x^*$ (see figure 1).

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[5] Notice that f may be neither U-shaped nor U-inverted-shaped on the whole domain

ii) First we show that there is not $\hat{x} \in (0, 0.5)\, \hat{x} \neq \bar{x}_L$ such that $\hat{x} = F(\hat{x})$. But this follows from the fact that $F(0) > 0$, $F(0.5) = 0.5$, $F(\bar{x}_L) = \bar{x}_L$ and, given that f is centered, convexity of F in the interval $[0, 0.5]$. A similar argument works for $x \in (0.5, 1)$. It now has to be proven that the process in (10) converges to $(\bar{x}_L, \bar{x}_R)$ (see figure 2). We have that $F(x) > x\, \forall x \in [0, \bar{x}_L)$ and $F(x) < x\, \forall x \in (\bar{x}_R, 1]$. Then, an argument similar to the one in i) shows that $x > 1 - F(1 - F(x))\, \forall x \in [0, \bar{x}_L)$.(It can also be shown that $x > 1 - F(1 - F(x))\, \forall x \in (\bar{x}_L, 0.5]$. Hence $(\bar{x}_L, \bar{x}_R)$ is stable). **Q.E.D.**.

The theorem shows that in the case of a polarized distribution of types if the number of polls is large enough the process converges to the complete information outcome. In the case of a centered distribution this doesn't need to happen. In particular, if f is enough concentrated around the center, the curvature of F will satisfy the conditions of the Theorem and agents in $[\bar{x}_L, \bar{x}_R]$ are always undecided. Notice that the Theorem says nothing about what undecided agents will do in election day. They decide their vote according to the inequalities in (3), which are functions of their beliefs $\sigma_{x,T}$. But these beliefs do not need to be consistent with the complete information case. Therefore, the outcome of the election will be, in general, different from the complete information outcome.

It is important to notice again that the symmetry assumption has been adopted for simplicity. In the case f were not symmetric the situation would basically be the same (although x^* will not necessarily be equal to 0.5). The main different would be that for some f –which according to the definition are polarized– the process wouldn't converge to x^*. This may happen if the distribution is very much concentrated on just one side of the interval $[0, 1]$. (The opposite situation could happen for centered distributions).

5   Final Remarks

We have analyzed a very simple theoretical model of voting with incomplete information. It has been shown that opinion polls not always solve the informational problem voters face. Some strong assumptions adopted in the model require further comments.

a) Agents are supposed to be honest when they answer the question about their voting intentions. Without this assumption it would be quite difficult to model the informational role of polls and the problem would become in-

tractable.

b) The implemented policy (6) is a convex combination of the two proposals. In a more general model the weight associated with the proposal of the winning party may be greater than its percentage of votes. This would create the possibility of more equilibria in system (10). However, if the distribution of types is concentrated, it is still true that the process doesn't converge to the full information case.

c) It is clear that our agents are extremely "conservative"in their answers to the polls. In a more general model an agent may answer that she will vote for L even tough she is not completely sure that the implemented policy will be to her right. Our qualitative results would also hold in this case as long as agents are enough (although not completely) "conservative."

d) Abstention is not allowed in the model. In real elections, however, abstention is a very important problem. One could introduce abstention in our model by assuming that undecided agents in period $T - 1$ (in the last poll) don't vote. In this way uncertainty would be resolved by the end of period $T - 1$. This, however, creates a new problem since those undecided agents, knowing now the way the other agents will vote, shouldn't be undecided anymore. Thus, incorporating abstention in a coherent way is not an easy task. This alternative is left for further research.

e) One may argue that agents in the model are not rational in the sense that they don't take advantage of all the information provided by the polls. For example, if the number of undecided voters reminds constant for many periods some agents may be able to learn about the shape of the distribution function(in the paper, on the contrary, we assume that in such a case an agent still assigns a positive probability that almost all undecided agents vote for the same party). Thus, a more rational approach might yield quite different results from the ones we provide here. This alternative is also left for future research.

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