OPTIMAL REGULATION AND GROWTH IN A NATURAL-RESOURCE-BASED ECONOMY*

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ABSTRACT

This paper develops a two-sector model for a renewable natural resource based economy. Pareto efficient results show the optimal harvesting rate that allows for sustained long-run optimal growth, which is upper-bounded by the biological rate of reproduction. Regulation prevents from resource over-exploitation and exhaustion which arise under open access. The Ramsey policy allowing the competitive economy to reach the first-best solution, leads the government to tax harvesting activity from firms and distribute the receipts among households. In the short-run the tax is variable. In the long-run, the lower the intrinsic rate of reproduction the higher the constant unit tax on the resource use.

Keywords: Natural Resources, Efficiency, Open Access, Ramsey Regulation, Growth.
JEL classification: C62, D90, O41, Q2, H2.

1 Introduction

Any introductory textbook places the abundance of resources as a fundamental determinant of long-run growth in an economy. However, different works, in particular those with an empirical approach, suggest that such abundance, when referred to natural resources, can become, in fact, a significant obstacle to the long-run growth (Gylfason et al., 1999; Gylfason, 2001b). This paradoxical result is not strictly due to the abundance of natural resources, but to management problems, frequently with political connotations, associated with the difficulty to define correct property rights and the mechanism for appropriation of generated rents, in a context where the extractive activity is the cornerstone of the economy.

In this work, we get away from a pure extractive economy and contextualize the analysis in a more modern capitalistic economy, where aside with the extractive sector there is a sector engaged in the production of a final consumption good. The basic trait of our economy is that natural resource is an essential input for the production in the other sector. In addition, we consider the natural resource as removable, given that renewable resources are also exhaustible and they allow, like non-renewable resources, to examine the critical aspects of the relation between resources and growth. These critical aspects refer to the following recurrent questions. What is the optimal rate at which the resource must be harvested? Can a natural-resource-based economy experience sustainable long-run growth? When will extinction occur? How do optimal and competitive behaviour differ? How can a competitive natural-resource-based economy be regulated in order to achieve Pareto optimal outcomes? At last, all these questions are intimately related with the problem of the efficient management of the natural resources, which when, as in our work, the natural resource is renewable, forces to consider the extraction rate of the resource in light of its intrinsic regeneration rate.

The topic of efficient management of natural resources has received a lot of attention, both for renewable resources and for non-renewable resources. It is well-known that under open-access conditions, property rights are not well-defined. Then, natural resource is equally harvested by all extractors and no one is willing to invest in the resource stock maintenance since the expected benefits associated with the resource units saved by an individual entrepreneur in one period are likely to be reaped by others in the next (Gordon, 1954; Hardin, 1968). In such a case, the resource stock will be severely depleted and, consequently, the open-access regime is far to guarantee an efficient resource use and a path of sustainable growth (Berck, 1979).

The solution of management resource problems requires overcoming the lack of a property rights system. Aznar and Ruiz-Tamarit (2005) show that, when the resource use is not under an open-access regime but under private property rights that are uniformly distributed among families, the competitive equilibrium following an efficient pattern of resource use is able to provide a sustainable long-run path for the economy. However, the scope of this work is limited since they ignore the relevance of open-access as starting point in real world and the specific practices that can be used to implement the attribution of property rights.

One way to ensure efficient resource use and sustainable long-run growth is to allocate property rights over the resource to a sole owner (i. e. a central planner), which decides about its extraction and use internalizing external effects. Optimal results provided in this case help to evaluate the efficiency reached by alternative allocations of property rights in private hands, which are of the great interest for us, because the capitalistic characterization of our economy. In practice, there are only two real options for allocation of private property rights over natural resource: harvesting transferable quotas, from one side, and taxes, from the other side. Although both systems, if appropriately chosen, give the same results in terms of efficiency, our attention will be devoted to the utilization of taxes. This approach could allow us to make pronouncements about the convenience of using ecotaxes in economies with productive sectors which depend on the exploitation of natural resources, opening the possibility to do an optimal fiscal policy that corrects market failures and carries competitive results to the social optimum.

In this work, we use an adaptation of the two-sector endogenous growth model formulated by Lucas (1988). Specifically, we consider a final consumption good sector which produces a final-single good through physical capital and the natural resource, and an extraction sector which provides the renewable natural resource to the first one. The paper is articulated upon the analysis of two different scenarios, the central planner case and the regulated competitive economy, with a brief description of the open-access regime. For each scenario, we formulate a intertemporal optimization problem and then solve it applying optimal control techniques through the maximum principle of Pontryagin. The first-order conditions of these dynamical optimization problems conform a non-linear dynamical Hamiltonian system governing the evolution of the variables in the model. In this context, the dimension reduction strategy has been widely applied. People define new variables as ratios between the variables of the original system in such a way that they transform the original system into another system with a lower dimension (see Mulligan and Sala i Martin, 1994; Caballé and Santos, 1993; and Benhabib and Perli, 1994) . Then, they study this transformed system and look for steady states which will be interpreted as balanced growth paths for the original system. However, the reduction of dimension entails a loss of information which may lead to mistakes and miss-interpretations and it cannot serve to fully characterize the dynamics of the original variables in level. For this reason, in this work we apply a new resolution technique recently developed by Boucekkine and Tamarit (2004b). Moreover, we will use Gauss hypergeometric functions to obtain an explicit representation of the equilibrium dynamics of the variables in level. The results obtained show that such functions might be most useful in the assessment of the transition dynamics and asymptotics of endogenous growth models.

Our first scenario, corresponding to the case of a central planner, allows us to explicit the conditions for both a socially optimal allocation and a sustained and sustainable long-run endogenous growth. The broad picture of the open-access regime helps us to send light over the problems of overexploitation and, eventually, exhaustion of natural resource. In the second scenario, the regulated competitive economy, we analyze the possibility of recovering the efficient results provided for the social planner case through taxes over the resource use. The study of this third scenario is approached by formulating a dynamic Ramsey problem, which will be solved through the primal approach based upon the choice of quantities. The results concerning the optimality of dynamic taxation could be easily applied in order to elucidate if market failures that usually accompany the resource management can be solved by public intervention.

The article is organized as follows. In section 2 we describe initially the economy and introduce the assumptions featuring a general equilibrium two-sector growth model. One sector is devoted to purely extractive activities concerning a renewable resource, while the other sector is devoted to produce a final consumption good using both physical capital as the renewable resource. Then, we solve the dynamical system provided by the first-order conditions and obtain explicitly the optimal paths for the implied variables. Section 3 starts with a brief description of the standard open-access model, in order to evidence the forces that may cause resource exhaustion. After this we present a model of a regulated economy, where the possibility to recover optimal solutions, in terms of resource use and sustained growth, is studied through the formulation of a dynamic Ramsey problem. Finally, section 4 presents some general conclusions of the work.

2 The socially optimal resources allocation model

2.1 The model

We consider a two-sector closed economy populated by many identical and infinitely lived rational agents. Population is assumed constant and denoted by N. Individual preferences are defined over the real per capita consumption of an aggregate single-good at date t, $c(t)$, and represented by a CIES function with $0 < \sigma^{-1} < +\infty$,

$$U\left(c\left(t\right)\right) = \frac{c(t)^{1-\sigma} - 1}{1 - \sigma}. \tag{1}$$

In this economy, as in Aznar and Ruiz-Tamarit (2005), there is a renewable natural resource or natural capital, denoted by $Q(t)$, which plays an essential role as input. The stock of this natural capital changes over time because of two flows having opposite effects. First, beyond any human intervention, the natural resource grows exponentially at a constant intrinsic rate $\delta > 0$. Second, the natural resource is subject to an extraction process, or harvesting activity, because it is required to produce the final good. We define $z(t)$ as the aggregate extraction rate with $z(t) \in [0,1]$, and $z_i(t)$ as the extraction rate of the individual firm. If we consider all firms as identical and sharing the same objectives, and for the sake of simplicity we assume the same number of firms as consumers, we get $z(t) = \sum_{i=1}^{N} z_i(t) = N z_i(t)$. Finally, we assume a linear harvesting function with the marginal product of effort being equal to the average one. This implies that the resource stock diminishes, each period, by the amount $z(t) Q(t)$.

The extraction rate is an endogenous variable, which has a direct effect on the opportunity set for present and future consumption. When the resource is harvested for too long at a rate exceeding its regeneration capability, the stock of natural capital will decrease over time. So, even if natural capital is a biologically renewable resource, this does not mean that it should be economically inexhaustible. In fact, there is a crucial difference between physical and natural capital: while the first one may be used repeatedly without any consequence on its available quantity, because only depreciation can reduce it, the second one disappears automatically from the stock as it is used for production.

Combining the two previous flows we obtain the law of motion[1]

$$\dot{Q}(t) = \delta (1 - z(t)) Q(t) - z(t) Q(t). \tag{2}$$

In the final single-good sector, production is ensured by many identical competitive firms with technology represented by a Cobb-Douglas function $Y_i = AK_i^{\beta} (z_i Q)^{1-\beta}$. Production, Y_i, depends positively on the stock of physical capital, K_i, as well as on the amount of natural resource that is harvested, destroyed, or transformed each period $z_i Q$.[2] Aggregating over firms, given that $Y(t) = \sum_{i=1}^{N} Y_i(t) = NY_i(t)$ and $K(t) = \sum_{i=1}^{N} K_i(t) =$

[1] This law of motion does not depart so dramatically from the logistic law, more commonly used in the literature. As Brander and Taylor (1997) points out "(in the logistic case) the proportional growth rate would be approximately equal to the intrinsic rate of growth of the resource if congestion effects are negligible in the sense that carrying capacity were very large relative to the current stock". Moreover, our law of motion helps to privilege economic concerns over pure biological considerations. Laws of motion similar to that used in our work may be found in Mourmouras (1991; 1993), López (1994) and Koskela et al. (2002).

[2] We abstract from labor as an explicit factor of production because it is assumed inelastically supplied.

$NK_i(t)$, we get

$$Y(t) = AK^{\beta}(t)(z(t)Q(t))^{1-\beta}. \tag{3}$$

The efficiency parameter A represents the constant technological level and β is the elasticity of output with respect to physical capital. Production function shows constant returns to scale over all factors, but diminishing returns to $K(t)$ and $Q(t)$ taken isolatedly. We ignore harvesting costs and, consequently, total output $Y(t)$ may be allocated either to aggregate consumption or to physical capital accumulation.[3] For the sake of simplicity, we assume that there is no physical capital depreciation. Hence, the current aggregate resources constraint is

$$\dot{K}(t) = AK^{\beta}(t)(z(t)Q(t))^{1-\beta} - Nc(t). \tag{4}$$

The socially efficient resource allocation problem for this economy, given a constant social intertemporal discount rate $\rho > 0$, consists in choosing the controls $c(t)$ and $z(t)$ $\forall t \geq 0$ which solve[4]

$$\max_{\{K,Q,c,z\}} \int_0^{\infty} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} Ne^{-\rho t} dt$$
$$\text{s.t.} \quad (2) \quad \text{and} \quad (4), \tag{P1}$$

for $K(0) = K_0 > 0$ and $Q(0) = Q_0 > 0$ given.

We assume that $\delta > \rho$ for positive long-run growth to arise, as we shall see below. Now, removing time subscripts from variables, the current value Hamiltonian associated

Each agent is endowed with a fixed quantity of work effort, which we normalize to one, and consequently it may be considered as included in the constant term of the production function.

[3] Stiglitz (1980) points out that natural resouces require human activity to extract and convert them into a useful form. Therefore, extraction costs are usually modelled as reducing the amount of output available for consumption and investment. However, according to Plourde (1970), we assume that harvesting costs are negligible, an assumption which is innocuous in the study of the efficient solution. As we show in the main text, it does not impose any limitation on the scope of the optimization problem, which retains its dynamic formulation and allows for well-defined interior solutions.

[4] We specify the optimization problem without the static control constraints $0 \leqslant z \leqslant 1$. Accordingly, we will obtain unconstrained trajectories but, later on, we identify sufficient conditions on parameters which ensure that optimal values are interior.

with the previous dynamic optimization problem may be written as

$$H^c(K, Q, \vartheta_1, \vartheta_2, c, z; \sigma, N, A, \beta, \delta, t \geq 0) =$$

$$= \frac{c^{1-\sigma} - 1}{1 - \sigma} N + \vartheta_1 \left[AK^\beta (zQ)^{1-\beta} - Nc \right] + \vartheta_2 \left[(\delta - (1 + \delta) z) Q \right],$$

where ϑ_1 and ϑ_2 are the co-state variables (social shadow prices) associated with K and Q, respectively. Then, the socially optimal solution arises from the set of first order necessary conditions supplied by the Pontryagin's maximum principle,

$$c^{-\sigma} = \vartheta_1, \tag{5}$$

$$\vartheta_1 (1 - \beta) AK^\beta z^{-\beta} Q^{-\beta} = \vartheta_2 (1 + \delta), \tag{6}$$

$$\dot{\vartheta}_1 = \rho \vartheta_1 - \vartheta_1 \beta AK^{\beta-1} z^{1-\beta} Q^{1-\beta}, \tag{7}$$

$$\dot{\vartheta}_2 = \rho \vartheta_2 - \vartheta_1 (1 - \beta) AK^\beta z^{1-\beta} Q^{-\beta} - \vartheta_2 \delta \left(1 - \left(\frac{1 + \delta}{\delta} \right) z \right), \tag{8}$$

$$\dot{K} = AK^\beta z^{1-\beta} Q^{1-\beta} - Nc, \tag{9}$$

$$\dot{Q} = \delta \left(1 - \left(\frac{1 + \delta}{\delta} \right) z \right) Q. \tag{10}$$

The boundary conditions are K_0 and Q_0, and

$$\lim_{t \to \infty} \vartheta_1 K \exp \{-\rho t\} = 0, \tag{11}$$

$$\lim_{t \to \infty} \vartheta_2 Q \exp \{-\rho t\} = 0. \tag{12}$$

Given concavity properties of the involved functions, the first order conditions are also sufficient for a maximum. According to (5), on the margin, final good has to be socially equal valued in its two uses: consumption and physical capital accumulation. Moreover, given that consumption cannot be infinite at a finite date, it implies that $\vartheta_1 \neq 0$ at any finite t. According to (6), at equilibrium, the social value of the marginal productivity of natural resource (when harvested) has to be equal to the social value of its marginal contribution to natural capital accumulation (when saved). This equation also implies that $\vartheta_1 \neq 0$ at any finite t, provided the economy starts with finite and strictly positive endowments of physical and natural capital. The Euler equation (7) states that the marginal productivity of physical capital (the benefit of delaying consumption) equals its social rental price, which in the absence of depreciation is $\rho - \frac{\dot{\vartheta}_1}{\vartheta_1}$. Moreover, after substituting (6) into (8), we find that the intrinsic rate of growth of natural resource (the social marginal benefit of no harvesting) has to be equal to its social marginal opportunity cost (the difference between the social discount rate and the rate of capital gains or losses), $\delta = \rho - \frac{\dot{\vartheta}_2}{\vartheta_2}$. This is a modified version of the Hotelling rule, which imposes intertemporal efficiency to the resource extraction activities.

Next, from (5) and (6) we get the control functions

$$c = \vartheta_1^{-\frac{1}{\sigma}}, \tag{13}$$

$$z = \left(\frac{(1-\beta)A}{1+\delta}\right)^{\frac{1}{\beta}} \left(\frac{\vartheta_1}{\vartheta_2}\right)^{\frac{1}{\beta}} \frac{K}{Q}. \tag{14}$$

Substituting these expressions in (7)-(10) we obtain the dynamic system

$$\dot{\vartheta}_1 = \rho\vartheta_1 - \xi\vartheta_2^{-\frac{1-\beta}{\beta}}\vartheta_1^{\frac{1}{\beta}}, \tag{15}$$

$$\dot{\vartheta}_2 = -(\delta-\rho)\vartheta_2, \tag{16}$$

$$\dot{K} = \frac{\xi}{\beta}\vartheta_1^{\frac{1-\beta}{\beta}}\vartheta_2^{-\frac{1-\beta}{\beta}}K - N\vartheta_1^{-\frac{1}{\sigma}}, \tag{17}$$

$$\dot{Q} = \delta Q - \left(\frac{1-\beta}{\beta}\right)\xi\vartheta_1^{\frac{1}{\beta}}\vartheta_2^{-\frac{1}{\beta}}K, \tag{18}$$

where $\xi \equiv \frac{\beta(1+\delta)}{1-\beta}\left(\frac{(1-\beta)A}{1+\delta}\right)^{\frac{1}{\beta}} > 0$. These equations, together with the initial conditions K_0 and Q_0, and the transversality conditions (11) and (12), make the equilibrium dynamics completely determined over time.

2.2 Closed-form solution

The complete closed-form solution for the variables appearing in the dynamic system (15)-(18), as well as for the controls of the model, may be found recursively according to the following procedure. First, take (16) and integrate directly to obtain

$$\vartheta_2 = \vartheta_2(0)\exp\{-(\delta-\rho)t\}, \tag{19}$$

where $\vartheta_2(0)$ has still to be determined. Second, substitute the result (19) into (15), which gives us a standard Bernoulli's differential equation in ϑ_1, which may be integrated as

$$\vartheta_1 = \left(\frac{\delta}{\xi}\right)^{\frac{\beta}{1-\beta}} \vartheta_2(0) \exp\{\rho t\} \left[-1 + \exp\left\{\frac{(1-\beta)\,\delta}{\beta} t\right\} + \frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{-\frac{\beta}{1-\beta}}, \quad (20)$$

where $\vartheta_1(0)$ and $\vartheta_2(0)$ have still to be determined. Third, substitute the results (19) and (20) into (17), which gives us a linear differential equation in K with variable time-dependent coefficients, $\overset{\bullet}{K} = \psi_1(t)K - \psi_2(t)$. This equation may be integrated and solved giving us the needed result for our fourth step. This consists in substituting all the previous results, including the solution for K, into (18) and solving the so obtained linear differential equation in Q with a variable non-homogeneous coefficient, $\overset{\bullet}{Q} = \delta Q - \psi_3(t)$. The solution to these two linear differential equations are explicitly derived in the Appendix by making use of the Gaussian Hypergeometric function ${}_2F_1(a, b; c; z)$, with complex arguments a, b, c and z.

According to Abramowitz and Stegun (1972) this special function is given by the series

$$ {}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(c)_n}\,\frac{z^n}{n!}\,, $$

where $(x)_n$ is the Pochhammer rising factorial symbol. The latter may also be defined in terms of the special function Gamma, Γ, as

$$ (x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \frac{\int_0^\infty t^{x+n-1}\,e^{-t}\,dt}{\int_0^\infty t^{x-1}\,e^{-t}\,dt}\,. $$

When $Re(c) > Re(b) > 0$ the Gauss hypergeometric function admits the Euler integral representation in the whole complex plane, cut along the real axis from 1 to ∞,

$$ {}_2F_1(a, b, c; z) = \frac{\Gamma(c)}{\Gamma(b)\,\Gamma(c-b)} \int_0^1 t^{b-1}\,(1-t)^{c-b-1}\,(1-tz)^{-a}\,dt, $$

which gives the analytic continuation of the series representation beyond the unit circle defined by $|z| < 1$. Notice that while the series representation is practical for numerical computations, the Euler integral representation is particularly suitable for our analytical study.

Next, we supply the socially optimal results for all the quantity-variables (states and controls) of the model.

Proposition 1 *Any particular non-explosive solution to the dynamic system (15)-(18) has to satisfy the initial conditions K_0 and Q_0, as well as the limiting conditions (11) and (12). These ones impose the constraints:*

$$\delta\beta - \delta\sigma - \beta\rho < 0, \tag{21}$$

$$\delta(1-\sigma) - \rho < 0, \tag{22}$$

$$\frac{K_0}{{}_2F_1(0)}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}} = \left(\frac{\delta}{\xi}\right)^{\frac{\sigma-\beta}{\sigma(1-\beta)}} \frac{\sigma\beta N\left(\vartheta_2(0)\right)^{-\frac{1}{\sigma}}}{-\left(\delta\beta - \delta\sigma - \beta\rho\right)}\ , \tag{23}$$

$$\frac{{}_2F_1(0)}{{}_2\widetilde{F}_1(0)} = \frac{(1-\beta)\,\xi\sigma}{-\left(\delta(1-\sigma)-\rho\right)\beta}\frac{K_0}{Q_0}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}}. \tag{24}$$

Proof. See Appendix. ■

Proposition 2 *Under the social optimality conditions, if (21)-(24) hold then:*

(i) it does exist a unique and positive path for the physical capital stock K, starting from K_0,

$$K = K_0\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}}\left(\frac{\xi}{\delta}\right)^{\frac{1}{1-\beta}}\frac{{}_2F_1(t)}{{}_2F_1(0)}\exp\left\{\frac{\delta\beta-\delta\sigma-\beta\rho}{\beta\sigma}t\right\}$$
$$\cdot\left[-1+\exp\left\{\frac{(1-\beta)\,\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}}; \tag{25}$$

(ii) this optimal path shows transitional dynamics, approaching asymptotically to the unique positive balanced growth path

$$\bar{K} = \frac{K_0}{{}_2F_1(0)}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}}\left(\frac{\xi}{\delta}\right)^{\frac{1}{1-\beta}}\exp\left\{\frac{\delta-\rho}{\sigma}t\right\}, \tag{26}$$

along which, given $\delta > \rho$, K grows permanently at a positive constant rate $\bar{g}_K = \frac{\delta-\rho}{\sigma}$.

Proof. See Appendix. ■

Proposition 3 *Under the social optimality conditions, if (21)-(24) hold then:*

(i) it does exist a unique and positive path for the renewable natural resource stock Q*, starting from* Q_0*,*

$$Q = Q_0 \frac{{}_2\widetilde{F}_1(t)}{{}_2\widetilde{F}_1(0)} \exp\left\{\frac{\delta-\rho}{\sigma}t\right\}; \tag{27}$$

(ii) this optimal path shows transitional dynamics, approaching asymptotically to the unique positive balanced growth path

$$\bar{Q} = \frac{Q_0}{{}_2\widetilde{F}_1(0)} \exp\left\{\frac{\delta-\rho}{\sigma}t\right\}, \tag{28}$$

along which, given $\delta > \rho$*,* Q *grows permanently at a positive constant rate* $\bar{g}_Q = \frac{\delta-\rho}{\sigma}$*.*

Proof. See Appendix. ■

Proposition 4 *Under the social optimality conditions, if (21)-(24) hold and* $\delta > \rho$ *then:*

(i) it does exist a unique, interior and monotonous path for the extraction or harvesting rate

$$0 < z = -\frac{\delta(1-\sigma)-\rho}{\sigma(1+\delta)} \frac{{}_2F_1(t)}{{}_2\widetilde{F}_1(t)} < 1; \tag{29}$$

(ii) this optimal path, which starts from $z(0) = -\frac{\delta(1-\sigma)-\rho}{\sigma(1+\delta)} \frac{{}_2F_1(0)}{{}_2\widetilde{F}_1(0)}$*, shows transitional dynamics converging asymptotically to the interior constant value*

$$0 < \bar{z} = -\frac{\delta(1-\sigma)-\rho}{\sigma(1+\delta)} < 1; \tag{30}$$

(iii) the initial extraction rate satisfies the constraint $0 < z(0) < 1$ *if and only if* $\frac{K_0}{Q_0}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}} < \frac{(1+\delta)\beta}{(1-\beta)\xi}$*.*

Proof. From (14), given (19), (20), (25) and (27) we get (29). Then, taking the limit as t tends to infinity we obtain (30). Uniqueness is a property of these trajectories which is inherited from the previous Propositions. Monotonicity of z comes from the monotonicity of the ratio ${}_2F_1(t)/{}_2\widetilde{F}_1(t)$, which has been proved in Boucekkine and Ruiz-Tamarit (2005), *Lemma* 1. Moreover, $\bar{z}$ is strictly interior for $\delta > \rho$ because of (22) and $\sigma > 0$. The interiority of $z(0)$ comes from (22), which ensures the lower bound, and the *iff* condition in *(iii)*, which under (24) guarantees the upper bound. Finally, given that z follows a monotonous convergent trajectory, the interiority of this one is a direct consequence of the interiority of both $z(0)$ and $\bar{z}$. ■

Proposition 5 *Under the social optimality conditions, if (21)-(24) hold then:*

(i) it does exist a unique and positive path for consumption per capita c, starting from $c(0) = (\vartheta_1(0))^{-\frac{1}{\sigma}}$*,*

$$c = (\vartheta_2(0))^{-\frac{1}{\sigma}} \left(\frac{\xi}{\delta}\right)^{\frac{\beta}{\sigma(1-\beta)}} \exp\left\{-\frac{\rho}{\sigma}t\right\} \left[-1 + \exp\left\{\frac{(1-\beta)\,\delta}{\beta}t\right\} + \frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{\sigma(1-\beta)}}; \tag{31}$$

(ii) this optimal path shows transitional dynamics, approaching asymptotically to the unique positive balanced growth path

$$\bar{c} = (\vartheta_2(0))^{-\frac{1}{\sigma}} \left(\frac{\xi}{\delta}\right)^{\frac{\beta}{\sigma(1-\beta)}} \exp\left\{\frac{\delta-\rho}{\sigma}t\right\}, \tag{32}$$

along which, given $\delta > \rho$*, c grows permanently at a positive constant rate* $\bar{g}_c = \frac{\delta-\rho}{\sigma}$*.*

Proof. Given (13) and (20) we get (31), and taking the limit as t tends to infinity we obtain (32). Uniqueness and positivity are two properties of these trajectories because of the parameter constraints of the model. ■

From previous Propositions,[5] we can directly deduce the following corollaries, which translate in terms of the aggregate production level, the relative shadow prices and the ratio between capital stocks, the results just proved.

[5] It must be emphasized that all these results are absolutely general in the sense that they encompass three different subcases arising from the relationship between the two parameters representing the inverse of the intertemporal elasticity of substitution, σ, and the physical capital share, β. These subcases have drawn great attention in growth literature because they cause different patterns of dynamic behaviour. However, what we supply here is a compact general solution for all of them based on the Gauss Hypergeometric function, with arguments $a > 1$, $\tilde{a} > 0$ and $c > 2$ because of the parameter constraints (21) and (22) implied by transversality conditions and $b \gtreqless 0$ depending on $\sigma \gtreqless \beta$.

Corollary 1 *Under the social optimality conditions, if (21)-(24) hold then:*

(i) it does exist a unique and positive path for production Y, starting from $Y(0) = AK_0 \left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1-\beta}{\beta}} \left(\frac{(1-\beta)\xi}{(1+\delta)\beta}\right)^{1-\beta}$,

$$Y = Y(0)\frac{\vartheta_1(0)}{\vartheta_2(0)}\left(\frac{\xi}{\delta}\right)^{\frac{\beta}{1-\beta}}\frac{{}_2F_1(t)}{{}_2F_1(0)}\exp\left\{\frac{\delta(1-\sigma)-\rho}{\sigma}t\right\}$$

$$\cdot\left[-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{1-\beta}}; \tag{33}$$

(ii) this optimal path shows transitional dynamics, approaching asymptotically to the unique positive balanced growth path

$$\bar{Y} = \frac{Y(0)}{{}_2F_1(0)}\frac{\vartheta_1(0)}{\vartheta_2(0)}\left(\frac{\xi}{\delta}\right)^{\frac{\beta}{1-\beta}}\exp\left\{\frac{\delta-\rho}{\sigma}t\right\}, \tag{34}$$

along which, given $\delta > \rho$, Y grows permanently at a positive constant rate $\bar{g}_Y = \frac{\delta-\rho}{\sigma}$.

Proof. Given that $Y = AK^{\beta}z^{1-\beta}Q^{1-\beta}$, Propositions 2, 3 and 4 and some additional algebra suffice to prove this corollary. ■

Corollary 2 *Under the social optimality conditions, if (21)-(24) hold then:*

(i) it does exist a unique and positive path for the relative prices $\frac{\vartheta_1}{\vartheta_2}$ and a unique, positive path for the ratio $\frac{K}{Q}$, starting from $\frac{\vartheta_1(0)}{\vartheta_2(0)}$ and $\frac{K_0}{Q_0}$ respectively,

$$\frac{\vartheta_1}{\vartheta_2} = \left(\frac{\delta}{\xi}\right)^{\frac{\beta}{1-\beta}}\exp\{\delta t\}\left[-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{-\frac{\beta}{1-\beta}}, \tag{35}$$

$$\frac{K}{Q} = \frac{K_0}{Q_0}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}}\left(\frac{\xi}{\delta}\right)^{\frac{1}{1-\beta}}\frac{{}_2F_1(t)}{{}_2\widetilde{F}_1(t)}\frac{{}_2\widetilde{F}_1(0)}{{}_2F_1(0)}\exp\left\{-\frac{\delta}{\beta}t\right\}$$

$$\cdot\left[-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}}; \tag{36}$$

(ii) these optimal paths move asymptotically showing transitional dynamics, and converge respectively to the unique and positive constant values, i.e. balanced paths,

$$\left(\frac{\bar{\vartheta}_1}{\vartheta_2}\right) = \left(\frac{\delta}{\xi}\right)^{\frac{\beta}{1-\beta}} > 0, \tag{37}$$

$$\left(\frac{\bar{K}}{Q}\right) = -\frac{\delta\left(1-\sigma\right)-\rho}{\sigma\left(1+\delta\right)}\left(\frac{\beta A}{\delta}\right)^{\frac{1}{1-\beta}} > 0. \tag{38}$$

Proof. Taking (19) and (20) on the one hand, (25) and (27) on the other, and dividing between them we get (35) and (36), respectively. Then, taking respectively the limit as t tends to infinity we derive (37) and, using (24), also (38). Finally, given the parameter constraints of the model, uniqueness and positivity are both properties characterizing these trajectories. ■

In short, we find that the optimal behavior places the economy on the unique non-explosive path, which in turn converges to a unique balanced growth path. The short-run optimal levels for all the endogenous variables show transitional dynamics with convergence to their corresponding long-run optimal levels. This feature is also shared by the short-run rates of growth, which along the transition converge to a constant long-run rate of growth, $\frac{\delta-\rho}{\sigma}$ for K, Q, Y and c and zero for z, $\frac{\vartheta_1}{\vartheta_2}$ and $\frac{K}{Q}$. The difference between long-run and short-run trajectories may be checked by comparing their corresponding initial values, which are substantially different. The initial value of the long-run trajectories of K, Q, Y and c are related to the initial value of the short-run ones, i.e. the initial conditions in the case of K and Q, by a multiplicative term which depends simultaneously on both K_0 and Q_0, as well as on the structural parameters of the model. The initial value of the long-run trajectories of z, $\frac{\vartheta_1}{\vartheta_2}$ and $\frac{K}{Q}$ are independent of K_0 and Q_0.

Along the balanced growth path, the optimal long-run rate of growth of K, Q, Y and c is determined endogenously and depends positively on the intrinsic rate of growth of the natural resource out of harvesting activities, δ. Moreover, the lower the social discount rate ρ and the higher the patience of agents σ^{-1}, the higher the long-run rate of growth. Nevertheless, this one appears upper-bounded because of the transversality condition which introduces the constraint $\frac{\delta-\rho}{\sigma} < \delta$. Consequently, although the model predicts an endogenously determined positive long-run rate of growth, this one appears limited by the biological rate of reproduction (regeneration), which is given exogenously.

The balanced growth path also implies an optimal rate of harvesting $\bar{z}$, which is constant and depends positively on both the social discount rate and the inverse of the intertemporal elasticity of substitution. However, it depends positively on the intrinsic rate of growth of the natural resource if and only if $\sigma > 1 + \rho$, otherwise the relationship is non-positive. Another endogenous variable that is constant along the balanced growth path is $\frac{K}{Q}$, which depends positively on ρ, σ and A, but shows an ambiguous link with δ.

There is also an interesting long-run relationship among endogenous variables. The optimal value of the long-run rate of growth is directly proportional to the difference between *(i)* the maximum harvesting rate which is compatible with a non-decreasing stock of natural resource, $z^s \equiv \frac{\delta}{1+\delta}$, and *(ii)* the optimal value of the long-run harvesting rate, $\bar{z}$. According to this, positive long-run growth is sustainable as long as $\bar{z} < z^S$. Namely, condition $\delta > \rho$, which ensures a positive rate of growth, also implies that the harvesting rate is lower than $\frac{\delta}{1+\delta}$. Moreover, given that z^S is increasing with the intrinsic rate of growth δ, exogenous elements affecting this one, such as government ecologically-based interventions, may have an important positive impact on the margins for growth. In particular, we find that an increase in δ that expands the margins for sustainability because increases z^S and reduces $\bar{z}$,[6] has an additional positive effect increasing the long-run rate of growth.

In previous paragraphs, including Propositions and Corollaries, we have assumed $\delta > \rho$. However, as Clark (1973) and Clark and Munro (1978) point out, if the own rate of interest of the resource stock is less than the discount rate, which corresponds here to $\rho > \delta$, then the extinction of the natural resource becomes an optimal policy.[7] In such a

[6] As we have shown, if the intertemporal elasticity of substitution is low enough, an increase in δ increases $\bar{z}$, but less than proportionally to the increase in z^S.

[7] Beddington et al. (1975) shows that a large social discount rate means that the central planner is less willing to sacrifice current profits in order to conserve his resources, and in the limit he is not at all interested in conservation. His harvesting strategy is then the same as that of the individual agent in an open access regime and, acting on his own, he has no power to conserve the resource whether he wants to or not. In general, Smith (1969), Beddington et al. (1975) or Leung and Wang (1976) consider that the danger of intentional extinction depends on the evolution of the market value (selling price), the operating costs and the efficiency of harvesting technology, as well as on the size of the natural resource

case, the long-run optimal harvesting rate given in (30) is strictly interior if $\sigma > \rho - \delta > 0$, and the long-run value of Q given in (28) tends to zero.

After having characterized the first-best solution, showing that an optimal exploitation of the renewable natural resource in both senses intra and inter-temporally, can produce sustained positive long-run growth, we will now study the behavior of the economy in a decentralized competitive environment.

3 The resources allocation in the decentralized competitive economy

3.1 The standard open access model

First of all, we shall remind the lessons from the well-known model for a decentralized economy without governmental regulation and open access to the renewable natural resource. Under open access there is a lack of any kind of property rights or ownership of the resource, which is a free good for individuals but scarce for society. Thus, no one can prevent another from using the natural resource and appropriating a share of its returns. In the open access context, there is an incentive for potential entrants to join the industry, and for already installed firms to expand their capacity in order to capture increased rents. Instead, no one has particular incentive to take into account the dynamics of the natural resource at the aggregate level. Individual harvesters find not a motive for investing (saving) in stock maintenance since they fail to take account of its full user cost. Therefore, they do not consider the impact of their current harvesting decisions on the size of the future stock.

According to the literature, two main results can be expected. First, rivalry in harvesting, which plays the role of a negative externality, will cause exhaustion of the resource, as in the famous article by Hardin (1968). Second, resource extraction will continue until all economic rents are dissipated, as Gordon (1954) shows in his open access fishery model. In

stock.

the competitive model with resource exploitation under open access, harvesting costs may be relevant because, as Smith (1968) shows, the presence of increasing extraction costs may prevent the exhaustion of the resource. Namely, the existence of negative stock externalities provides a mechanism that, as long as harvesting depletes the stock and rises harvesting costs, ceteris paribus discourages harvesting itself. However, Smith (1969), Gould (1972), Berck (1979), and Brander and Taylor (1998) show that if the minimum size of the natural resource at which exploitation is profitable is lower than the minimum biologically viable size, the open access regime implies over-exploitation and (eventually) the exhaustion or extinction of the natural resource.[8]

The impossibility for long-run growth and socially efficient resource management in the open access system are both consequence of a market failure. The market sends out incorrect signals to the harvesters of the resource and, consequently, it results in inefficient outcomes.[9] Incompleteness or total absence of well-defined property rights lead to market failures and create economic as well as biological problems. In order to solve such market failures, other than the political process which give birth to a property rights system by implementing individual transferable quotas (ITQ), there are two main regulatory options. First, input or effort controls which address to the problem by making it difficult for harvesters to respond to the incorrect market signals. Usually, these take

[8] Some pioneering work [Munro (1979), Levhari and Mirman (1980)], assuming that the interaction of agents has the structure of a repeated game, shown that it is possible for cooperation to emerge in the open access context. The idea is interesting because if harvesters cooperate the tendency towards resource depletion can be avoided. Unfortunately, repeated games do not capture the structure of most natural resource harvesting in real life, where payoffs to players depend critically on the size of the relevant resource stock. In addition to the existence of stock externalities, the mapping of strategies to payoffs changes from period to period, which places this kind of games in the category of dynamic games. Until now, however, the few attempts which have been made to design realistic dynamic models for natural resource exploitation under open access regimes seem little fruitful (Brown, 2000).

[9] The open access system is clearly different from the common property regime in which a group of owners control for the resorce and exclude others from using it. Although the collective self-management by individuals may not result in fully efficient outcomes, it could preserve the resource from exhaustion by enhancing its rental value [Bulte et al. (1995), Brown (2000)]

the form of constraints on the size and number of harvesters as well as of limits over time and areas for harvesting. Second, output controls which face up to the problem by changing the market signals themselves. Wide evidence demonstrates that input controls are essentially inadequate because they cause overcapitalization and do not avoid *fishing races*. Hence, the only real option is output controls which may adopt two basic forms: taxes and harvesting quotas (TAC). Along the next section we analyze agents behavior in a decentralized competitive economy when a tax on the resource use is introduced.[10]

3.2 The regulated competitive economy

From now on, the renewable natural resource will be considered as an essential input for production which is treated as a private good because it shares the properties of rivalry and excludability with the remaining inputs. Although for the sake of simplicity the model considers costless harvesting, there is not open access to the natural capital stock because firms have to pay for the use of such a resource. There are N fixed firms, as many as households, which freely adjust their K_i and z_i values trying to capture quasi-rents. Each competitive firm pays a unit price q and ignores the dynamics of Q. We assume, as before, that there are no depreciation charges. Under these assumptions, each firm solves the static optimization problem[11]

$$\max_{\{K_i,z_i\}} \pi_i = F(K_i, z_iQ) - rK_i - qz_iQ = AK_i^{\beta}\left(z_iQ\right)^{1-\beta} - rK_i - qz_iQ, \tag{F}$$

where r represents the market interest rate and the opportunity cost for physical capital. The positive price q represents a unit tax that individual firms pay to the government, revealing that natural resource is scarce. The rental prices for both types of capital services satisfy the first order necessary conditions

$$r = F_K(K_i, z_iQ) = \beta AK_i^{\beta-1}\left(z_iQ\right)^{1-\beta}, \tag{39}$$

[10] Clark (1980) proves that, if harvesting quotas are freely transferable, the quota system will have the same effects in efficiency terms as taxes.

[11] The absence of extraction costs has not negative consequences for a well-defined interior solution. Consequently, we specify the optimization problem without the static control constraints $0 \leqslant Nz_i \leqslant 1$, which are obviously satisfied by unconstrained solution trajectories.

$$q = F_2(K_i, z_i Q) = (1-\beta) A K_i^{\beta} (z_i Q)^{-\beta}. \tag{40}$$

Given the assumption of constant returns to scale, output exactly exhausts by paying inputs according to their marginal productivities. Hence, $rK_i + qz_iQ = Y_i$ and economic rents π_i are zero.

The problem facing the representative price-taker household, given a constant intertemporal discount rate $\rho > 0$, consists in choosing the control c $\forall t \geq 0$, which solves the dynamic optimization problem

$$\begin{aligned} &\max_{\{K_i, c\}} \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \\ &\text{s. t.} \quad \dot{K}_i = rK_i + T_i - c, \\ &K_i(0) = K_{i0} > 0. \end{aligned} \tag{H}$$

In the dynamic budget constraint, the term T_i represents transfers from the government that are treated as lump-sum by households. The first order necessary conditions are

$$c^{-\sigma} = \mu_1, \tag{41}$$

$$\dot{\mu}_1 = (\rho - r)\,\mu_1, \tag{42}$$

$$\dot{K}_i = rK_i + T_i - c. \tag{43}$$

The boundary conditions are K_{i0} and

$$\lim_{t \to \infty} \mu_1 K_i \exp\{-\rho t\} = 0. \tag{44}$$

On the other hand, the amount of tax receipts are assumed to be equal to the amount of lump-sum transfers to households, in such a way that government is constrained with a period-by-period balanced budget

$$\sum_{i=1}^{N} T_i = NT_i = T = qzQ = qNz_iQ = q\left(\sum_{i=1}^{N} z_i\right) Q. \tag{45}$$

We consider a benevolent government, choosing q and T such that the induced allocations maximize agent's utility, subject to the constraint that final allocations must be consistent with the (decentralized) competitive equilibrium under regulation.[12] The government takes now into account the explicit dynamics of Q, as given in (2). The dynamic Ramsey problem for this economy consists in a policymaker maximizing the aggregate welfare subject to: (i) the first order conditions from both the household and the firm maximization problems, (ii) the government budget constraint, and (iii) the economy-wide resource constraints. Formally, the government solves the intertemporal optimization problem

$$\max_{\{K,Q,c,z\}} \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} N e^{-\rho t} dt$$
$$\text{s. t.} \quad (39)\text{-}(45),\ (2),\ (4) \text{ and } K_0,\ Q_0 > 0. \tag{G}$$

Constraints (39)-(45), after some substitutions and the solution of a linear differential equation in the product $\mu_1 K$, under the transversality condition (44), may be summarized in the implementability condition

$$\int_0^\infty \left[N c^{1-\sigma} - (1-\beta) A K^\beta z^{1-\beta} Q^{1-\beta} c^{-\sigma} \right] e^{-\rho t} dt = \mu_1(0) K_0, \tag{46}$$

where $\mu_1(0)$ comes from the solution to the household problem (H). According to Chari and Kehoe (1999), the implementability condition represents the intertemporal budget constraint of either the consumer or the government, where the consumer and firm first order conditions have been used to substitute out the prices and policies. Consequently, we can use the primal approach for solving this Ramsey problem where, instead of choosing the tax paths, the government will choose the set of allocations and shadow prices that maximize consumers' utility subject to the implementability and feasibility constraints; namely, the allocations that can be implemented as a competitive equilibrium with distortionary taxes. The current value Hamiltonian associated with this dynamic optimization problem is

$$H^c(K, Q, \theta_1, \theta_2, \eta, c, z; \sigma, N, A, \beta, \delta, t \geq 0) =$$

[12] The government makes its policy choice under full commitment: policy rules are announced and, then, the government is not allowed to revising the path of fiscal instruments over time. According to this, the problem of time inconsistency is eliminated.

$$\frac{c^{1-\sigma}-1}{1-\sigma}N+\eta\left[Nc^{1-\sigma}-(1-\beta)AK^{\beta}z^{1-\beta}Q^{1-\beta}c^{-\sigma}\right]$$
$$+\theta_1\left[AK^{\beta}z^{1-\beta}Q^{1-\beta}-Nc\right]+\theta_2\left[(\delta-(1+\delta)z)Q\right]-\eta\mu_1(0)K_0,$$

where θ_1 and θ_2 are the co-state variables (shadow prices) associated with K and Q, and η is a constant over time multiplier associated with the integral constraint. The set of equations arising as first order necessary conditions are

$$\left[1+\eta(1-\sigma)+\frac{\eta\sigma(1-\beta)A}{Nc}K^{\beta}z^{1-\beta}Q^{1-\beta}\right]c^{-\sigma}=\theta_1, \tag{47}$$

$$\theta_1(1-\beta)AK^{\beta}z^{-\beta}Q^{-\beta}-\eta(1-\beta)^2AK^{\beta}z^{-\beta}Q^{-\beta}c^{-\sigma}=\theta_2(1+\delta), \tag{48}$$

$$\dot{\theta}_1=\rho\theta_1-\theta_1\beta AK^{\beta-1}z^{1-\beta}Q^{1-\beta}+\eta\beta(1-\beta)AK^{\beta-1}z^{1-\beta}Q^{1-\beta}c^{-\sigma}, \tag{49}$$

$$\dot{\theta}_2=\rho\theta_2-\theta_1(1-\beta)AK^{\beta}z^{1-\beta}Q^{-\beta}-\theta_2\delta\left(1-\left(\frac{1+\delta}{\delta}\right)z\right)$$
$$+\eta(1-\beta)^2AK^{\beta}z^{1-\beta}Q^{-\beta}c^{-\sigma}, \tag{50}$$

$$\dot{K}=AK^{\beta}z^{1-\beta}Q^{1-\beta}-Nc, \tag{51}$$

$$\dot{Q}=\delta\left(1-\left(\frac{1+\delta}{\delta}\right)z\right)Q, \tag{52}$$

together with the static constraint (46) or implementability condition, the initial conditions K_0 and Q_0, and the transversality conditions

$$\lim_{t\to\infty}\theta_1K\exp\{-\rho t\}=0, \tag{53}$$

$$\lim_{t\to\infty}\theta_2Q\exp\{-\rho t\}=0. \tag{54}$$

Equations (47)-(54), plus (46) and initial conditions, determine a second-best Ramsey allocation. However, we will not study existence and uniqueness issues, nor will we try to get the closed-form solution to the resulting nonlinear dynamic system, as we did before with the previous models. Instead, here we assume that such a solution exists and focus on the problem of the existence of an optimal tax structure, allowing to implement the Pareto optimal allocation as a taxed competitive equilibrium.

Comparing the above first order necessary conditions with (5)-(12), we observe that they are equal except for the presence of η, which comes from the pseudo-utility function that replaces the utility function in the primal approach to the Ramsey problem. Therefore, as in Yuen (1991), the second-best problem reduces to the socially efficient first-best problem when the constant multiplier associated with the implementability condition is zero. Under $\eta = 0$, the shadow prices of both the physical capital stock and the natural resource stock appearing in the Ramsey problem, are equal to their corresponding social shadow prices: $\theta_1 = \vartheta_1$ and $\theta_2 = \vartheta_2$.[13]

Now, we can derive the particular solution trajectories for the tax-price associated with the use of the natural resource and the lump-sum transfer, which represent the Ramsey policy consistent with the first-best solution to the Ramsey problem (G).

Proposition 6 *In the decentralized competitive economy, the first-best allocations may be implemented under government regulation by setting the tax-price q and the lump-sum transfer T according to the following trajectories:*

$$q = q(0)\frac{\vartheta_1(0)}{\vartheta_2(0)}\left(\frac{\xi}{\delta}\right)^{\frac{\beta}{1-\beta}} \exp\{-\delta t\}\left[-1+\exp\left\{\frac{(1-\beta)\,\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{1-\beta}}, \quad (55)$$

$$T = T(0)\frac{\vartheta_1(0)}{\vartheta_2(0)}\left(\frac{\xi}{\delta}\right)^{\frac{\beta}{1-\beta}} \frac{{}_2F_1(t)}{{}_2F_1(0)}\exp\left\{\frac{\delta(1-\sigma)-\rho}{\sigma}t\right\}$$
$$\cdot\left[-1+\exp\left\{\frac{(1-\beta)\,\delta}{\beta}t\right\}+\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{1-\beta}}, \quad (56)$$

where $q(0) = (1-\beta)\,A\left(\frac{(1+\delta)\beta}{(1-\beta)\xi}\right)^{\beta}\frac{\vartheta_2(0)}{\vartheta_1(0)} > 0$
and $T(0) = (1-\beta)\,AK_0\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1-\beta}{\beta}}\left(\frac{(1-\beta)\xi}{(1+\delta)\beta}\right)^{1-\beta} > 0.$

[13]Taking the first order conditions for controls (5)-(6) and (47)-(48) we can stablish the following relationships:

$$\theta_1 = \vartheta_1\left[1+\eta\left(1-\sigma+\frac{\sigma\left((1-\beta)\,A\right)^{\frac{1}{\beta}}}{N\,(1+\delta)^{\frac{1-\beta}{\beta}}}\frac{\vartheta_1^{\frac{1-\beta}{\beta}+\frac{1}{\sigma}}}{\vartheta_2^{\frac{1-\beta}{\beta}}}K\right)\right],$$
$$\frac{\theta_1}{\theta_2} = \frac{\vartheta_1}{\vartheta_2}+\eta\,(1-\beta)\,\frac{\vartheta_1}{\theta_2},$$

which tell us that the bigger is η the larger the divergence between (regulated) competitive and social shadow prices.

Proof. From (40) and (45), simply by substituting (25), (27) and (29), which show the first-best trajectories for involved variables, we get (55) and (56), respectively. To derive $q(0)$ and $T(0)$ we also use (24). ■

Corollary 3 *Trajectories for q and T show transitional dynamics. The tax-price q converges asymptotically to the constant value*

$$\bar{q} = (1-\beta)\, A \left(\frac{(1+\delta)\,\beta}{(1-\beta)\,\xi} \right)^{\beta} \left(\frac{\xi}{\delta} \right)^{\frac{\beta}{1-\beta}} > 0, \tag{57}$$

and the lump-sum transfer T approaches asymptotically to the exponential path

$$\bar{T} = \frac{T(0)}{{}_2F_1(0)} \frac{\vartheta_1(0)}{\vartheta_2(0)} \left(\frac{\xi}{\delta} \right)^{\frac{\beta}{1-\beta}} \exp\left\{ \frac{\delta-\rho}{\sigma} t \right\}, \tag{58}$$

along which, given $\delta > \rho$, it grows permanently at a positive constant rate $\bar{g}_T = \bar{g}_Y = \frac{\delta-\rho}{\sigma}$.

Proof. Taking the limit as t tends to infinity in (55) and (56) leads to (57) and (58), respectively. Moreover, the second one may also be proved using the relationship $\bar{T} = (1-\beta)\,\bar{Y}$ and (34). ■

Initially $T(0) = q(0)z(0)Q_0$ but, as the economy develops, q tends to (57) as long as z converges to (30), even if Q moves approaching (28). The optimal dynamic tax continuously changes as the state of the economy evolves toward the balanced growth path. In the long-run, the unit tax-price q tends to a constant value, though the aggregate lump-sum transfer T converges to a growing path because of the endogenous optimal long-run growth. The aggregate lump-sum transfer always represents a fixed proportion of total output, $T = (1-\beta)\,Y$, given by the natural resource share. Over time, the different values of q make private agents to correctly evaluate the scarcity of the natural resource stock and decide according to the true social costs, while T plays a subsidiary role giving levied resources back to the economy in a way that implies a redistribution from firms to consumers. Both together contribute to correct the bad effects from the absence of a usual market for the natural resource as a factor of production, i.e. they internalize all benefits and costs associated with the harvesting activity. Moreover, the first-best Ramsey policy is unique given the uniqueness of trajectories (55) and (56), which in turn arise from the uniqueness of the first-best allocations. Finally, taking (57) and substituting for ξ we get $\bar{q} = (1-\beta) A^{\frac{1}{1-\beta}} \left(\frac{\beta}{\delta}\right)^{\frac{\beta}{1-\beta}}$, which means that in the long-run the government has to fix a higher unit tax-price as smaller is the intrinsic growth rate of the natural resource, and as greater is the production efficiency level in the final-good sector. Along the transition, q converges to $\bar{q}$ from above or below, depending on whether $\frac{\vartheta_1(0)}{\vartheta_2(0)} \lessgtr \left(\frac{\delta}{\xi}\right)^{\frac{\beta}{1-\beta}} = \left(\overline{\frac{\vartheta_1}{\vartheta_2}}\right)$.

4 Conclusions

In this paper we have shown that an economy which produces a final good using physical capital and some harvested quantity of a renewable natural resource may experience sustained long-run growth. Using the Gauss Hypergeometric special function we got the short- and long-run closed-form trajectories for all the variables in levels. We identified uniqueness and convergence, although not necessarily in a monotonous way, as the main properties of such trajectories. Pareto efficient results reveal that under efficient management of the natural resource there exists an interior optimal harvesting rate which allows for positive long-run optimal growth. However, this rate of growth appears upper-bounded by the intrinsic rate of growth of the natural resource. Namely, endogenous economic growth is limited by exogenous biological reproduction. Moreover, intentional exhaustion of the natural resource and long-run economic collapse could also be optimal for a high enough social discount rate. In such a case, a very impatient society is not willing to sacrifice current profits and shows no interest in conservation.

When we study the behavior of the economy organized in a decentralized competitive way, the open access regime in which there are rivalry in harvesting and no excludability from the use of the natural resource leads to over-exploitation and exhaustion. The absence of well-defined property rights represents a market failure which causes the above inefficient outcomes making sustained long-run growth unfeasible. Then, choosing among different instruments for government intervention (ITQ's, input or effort controls, TAC's and taxes) we decided to analyze regulation by means of a unit tax that firms have to pay for the use of the resource. In this case, natural resource is rival and excludable (private good) and government distributes all its receipts to households under the form of lump-sum transfers. An example of this may be found in Alaska where the government collects taxes from oil extraction (a non-renewable resource) and then they are equally distributed to citizens.

We solved a dynamic Ramsey problem, which usually determines second-best allocations, and chose among its solutions the unique Ramsey policy which allows the competitive economy to reach the first-best solution. This is an uncommon result arising from the fact that the original distortion consists in the absence of a well-defined market for the natural resource harvesting activity, which is replenished by a benevolent government with the suitable fiscal policy. We have shown that regulation may prevent from resource over-exploitation and exhaustion. Moreover, one important lesson from this paper is that in the short-run the unit tax on the resource use must be variable according to the evolution of the economy' state towards the balanced growth path. In the long-run, the corresponding constant unit tax has to be higher as lower is the intrinsic rate of reproduction in the natural resource sector and as higher is the productivity (efficiency level) in the final-good sector.

Finally, we want to point out that the optimal unit tax studied in this paper is not implemented, stricto sensu, as an eco-tax because the lump-sum compensations are not ecologically oriented. In general, eco-taxes are intended to achieve a specific ecological effect and, consequently, the revenue derived from the eco-tax should not become part of the general budget but should be specifically targeted. In our context, this would mean that the above revenues should finance governmental ecologically-based interventions aimed at increasing the intrinsic rate of growth of the natural resource. Given the existence of a biological constraint on the long-run rate of economic growth, this is very important because any increase in reproduction or regeneration rates will have a significant positive impact on the margins for sustained growth as well as on the long-run rate of growth itself. However, this matter is beyond the scope of the paper and it has been left to one side for future research.

5 Appendix: proof of Propositions 1-3

Substitute (19) and (20) into (17) and get the linear differential equation $\dot{K} = \psi_1(t)K - \psi_2(t)$, where

$$\psi_1(t) \equiv \frac{\xi}{\beta}\vartheta_1^{\frac{1-\beta}{\beta}}\vartheta_2^{-\frac{1-\beta}{\beta}} = \frac{1}{\frac{\beta}{\delta} + \left(\frac{\beta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}} - \frac{\beta}{\delta}\right)\exp\left\{-\frac{(1-\beta)\delta}{\beta}t\right\}}$$

and

$$\psi_2(t) \equiv N\vartheta_1^{-\frac{1}{\sigma}}$$

$$= N\left(\frac{\xi}{\beta\delta}\right)^{\frac{\beta}{\sigma(1-\beta)}}(\vartheta_2(0))^{-\frac{1}{\sigma}}\exp\left\{-\frac{\rho}{\sigma}t\right\}\left[\left(-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}\right)\beta + \frac{\beta\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{\beta}{\sigma(1-\beta)}}.$$

The general solution is

$$K = K_0\exp\left\{\int_0^t \psi_1(s)ds\right\} - \int_0^t \psi_2(r)\exp\left\{\int_r^t \psi_1(z)dz\right\}dr$$

$$= \frac{K_0\left[\left(-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}\right)\beta + \frac{\beta\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}}}{\left(\frac{\beta\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right)^{\frac{1}{1-\beta}}}$$

$$-N\frac{\left(\frac{\xi}{\beta\delta}\right)^{\frac{\beta}{\sigma(1-\beta)}}}{\vartheta_2(0)^{\frac{1}{\sigma}}}\left[\left(-1+\exp\left\{\frac{(1-\beta)\delta}{\beta}t\right\}\right)\beta + \frac{\beta\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}}\int_0^t \Phi(r)\,dr,$$

where

$$\Phi(r) = \left[\left(\frac{\beta\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}} - \beta\right)\exp\left\{-\frac{(1-\beta)\rho}{\beta-\sigma}r\right\} + \beta\exp\left\{\frac{(1-\beta)(\delta\beta-\delta\sigma-\beta\rho)}{\beta(\beta-\sigma)}r\right\}\right]^{\frac{\beta-\sigma}{\sigma(1-\beta)}}.$$

The value of the integral in the solution trajectory for K, using the Gauss Hypergeometric function according to Boucekkine and Ruiz-Tamarit (2005), is

$$\int_0^t \Phi(r)\,dr = \frac{\sigma\beta^{\frac{\beta-\sigma}{\sigma(1-\beta)}+1}}{\delta\beta-\delta\sigma-\beta\rho}\left({}_2F_1(t)\exp\left\{\frac{\delta\beta-\delta\sigma-\beta\rho}{\beta\sigma}t\right\} - {}_2F_1(0)\right),$$

where

$${}_2F_1(t) = {}_2F_1\left(a,b;c;\left(1-\frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right)\exp\left\{-\frac{(1-\beta)\delta}{\beta}t\right\}\right),$$

$$ {}_2F_1(0) ={}_2 F_1 \left(a, b; c; 1 - \frac{\delta}{\xi} \left(\frac{\vartheta_1(0)}{\vartheta_2(0)} \right)^{-\frac{1-\beta}{\beta}} \right), $$

$$ a = -\frac{\delta\beta - \delta\sigma - \beta\rho}{\sigma\delta(1-\beta)}, \quad b = -\frac{\beta - \sigma}{\sigma(1-\beta)}, \quad c = a + 1 = 1 - \frac{\delta\beta - \delta\sigma - \beta\rho}{\sigma\delta(1-\beta)}. $$

Now, looking for a particular solution trajectory, we consider the transversality condition $\lim_{t\to\infty} \vartheta_1 K \exp\{-\rho t\} = 0$. Using (20) and the solution for K just obtained, for any $\vartheta_1(0)$ and $\vartheta_2(0)$ finite and different from zero and given that ${}_2F_1(0)$ is constant and in the limit as t tends to infinity ${}_2F_1(\infty) ={}_2 F_1(a, b; c; 0) = 1$, this boundary condition holds if and only if

$$ \beta\rho + \delta\sigma - \delta\beta > \sigma\delta(1-\beta) > 0 $$

and

$$ K_0 \left(\frac{\xi}{\beta\delta} \right)^{\frac{1}{1-\beta}} \left(\frac{\vartheta_1(0)}{\vartheta_2(0)} \right)^{\frac{1}{\beta}} + \frac{{}_2F_1(0) N \left(\frac{\xi}{\beta\delta} \right)^{\frac{\beta}{\sigma(1-\beta)}}}{\vartheta_2(0)^{\frac{1}{\sigma}}} \frac{\sigma\beta^{\frac{\beta-\sigma}{\sigma(1-\beta)}+1}}{\delta\beta - \delta\sigma - \beta\rho} = 0, $$

which correspond to (21) and (23) respectively. Finally, substituting these ones in the above general solution for K we get (25) and, taking the limit as t tends to infinity, (26).

On other hand, substituting (19), (20) and the previous solution for K into (18), we get the linear differential equation $\overset{\bullet}{Q} = \delta Q - \psi_3(t)$, where

$$ \psi_3(t) \equiv \frac{1-\beta}{\beta} \xi \vartheta_1^{\frac{1}{\beta}} \vartheta_2^{-\frac{1}{\beta}} K = \frac{(1-\beta)\xi K_0}{\beta} \left(\frac{\vartheta_1(0)}{\vartheta_2(0)} \right)^{\frac{1}{\beta}} \frac{{}_2F_1(t)}{{}_2F_1(0)} \exp\left\{ \frac{\delta - \rho}{\sigma} t \right\}. $$

The general solution is

$$ Q = Q_0 \exp\{\delta t\} - \int_0^t \psi_3(r) \exp\{\delta(t-r)\} dr $$

$$ = \exp\{\delta t\} \left[Q_0 - \frac{(1-\beta)\xi K_0}{{}_2F_1(0)\beta} \left(\frac{\vartheta_1(0)}{\vartheta_2(0)} \right)^{\frac{1}{\beta}} \int_0^t {}_2F_1(r) \exp\left\{ \frac{\delta(1-\sigma) - \rho}{\sigma} r \right\} dr \right]. $$

According to Boucekkine and Ruiz-Tamarit (2005), the integral at the end of the previous expression has the following solution in terms of the Gauss Hypergeometric function

$$ \int_0^t {}_2F_1(r) \exp\left\{ \frac{\delta(1-\sigma) - \rho}{\sigma} r \right\} dr = \frac{\sigma \left({}_2\widetilde{F}_1(t) \exp\left\{ \frac{\delta(1-\sigma)-\rho}{\sigma} t \right\} - {}_2\widetilde{F}_1(0) \right)}{\delta(1-\sigma) - \rho}, $$

where

$$
\begin{aligned}
{}_2\widetilde{F}_1(t) &= {}_2F_1\left(\widetilde{a}, b; c; \left(1 - \frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right)\exp\left\{-\frac{(1-\beta)\,\delta}{\beta}t\right\}\right), \\
{}_2\widetilde{F}_1(0) &= {}_2F_1\left(\widetilde{a}, b; c; 1 - \frac{\delta}{\xi}\left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{-\frac{1-\beta}{\beta}}\right), \\
\widetilde{a} &= a - 1 = -\frac{\delta\beta\,(1-\sigma) - \beta\rho}{\sigma\delta\,(1-\beta)}.
\end{aligned}
$$

Once again, to find a particular solution trajectory we have to consider the transversality condition, $\lim_{t\to\infty} \vartheta_2 Q \exp\{-\rho t\} = 0$. Using (19) and the previous solution for Q, for any $\vartheta_1(0)$ and $\vartheta_2(0)$ finite and different from zero and given that ${}_2\widetilde{F}_1(0)$ is constant and ${}_2\widetilde{F}_1(\infty) = {}_2F_1\left(\widetilde{a}, b; c; 0\right) = 1$, this boundary condition holds if and only if

$$
\rho > \delta\,(1-\sigma)
$$

and

$$
Q_0 = \frac{(1-\beta)\,\xi\sigma}{\beta\rho - \delta\beta\,(1-\sigma)} K_0 \left(\frac{\vartheta_1(0)}{\vartheta_2(0)}\right)^{\frac{1}{\beta}} \frac{{}_2\widetilde{F}_1(0)}{{}_2F_1(0)},
$$

which correspond to (22) and (24) respectively. Finally, substituting these ones in the general solution for Q we get (27) and, taking the limit as t tends to infinity, (28).

Conditions (23) and (24) make up a system of two equations with two unknowns, $\vartheta_1(0)$ and $\vartheta_2(0)$. Their values are determined in the following way: (24) determines a unique value for the ratio $\frac{\vartheta_1(0)}{\vartheta_2(0)}$, then (23) determines the value of $\vartheta_2(0)$, which after multiplying by the value of the ratio itself gives the value of $\vartheta_1(0)$. In Boucekkine and Ruiz-Tamarit (2005) it is shown, with a nonlinear system very close to (23)-(24) with the inequality constraints (21) and (22), that solutions for $\vartheta_1(0)$ and $\vartheta_2(0)$ are unique and positive. Therefore, trajectories for K, $\bar{K}$, Q and $\bar{Q}$ as given in (25)-(28) are also unique and positive.

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