A discusión

BARGAINING IN COMMITTEES OF REPRESENTATIVES: THE OPTIMAL VOTING RULE*

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ABSTRACT

Committees are often made up of representatives of different-sized groups of individuals, and make decisions by means of a voting rule which specifies what vote configurations can pass a decision. This raises the question of the choice of the optimal voting rule, given the different sizes of the groups that members represent. In this paper we take a new departure to address this problem, assuming that the committee is a bargaining scenario in which negotiations take place 'in the shadow of the voting rule' in search of unanimous consensus. That is, a general agreement is looked for, but any winning coalition can enforce an agreement.

Keywords: Voting rule, Bargaining, Nash solution.

1 INTRODUCTION

Committees are often made up of representatives of different-sized groups of individuals, and make decisions by means of a voting rule (often a weighted majority rule, but more generally any arbitrary voting rule) which specifies what vote configurations can pass a decision. Examples of committees of representatives of this type are provided by the councils ruling different kinds of organizations, including important political examples such as the Council of Ministers in the EU. This raises the question of the 'fair', 'optimal' or most adequate voting rule, given the different sizes of the groups that members represent, if a principle of equal representation is to be implemented. This issue has been approached so far by different authors by modelling the decision-making process as an idealized two-stage process, and assessing the 'decisiveness' (i.e., the probability of being crucial or pivotal) in making a decision that can be imputed to each individual in the different groups assuming that each representative follows the majority opinion in his/her 'constituency'. This allows for an assessment of the 'fairness' of the voting rule of the committee (see, e.g., Penrose (1946), Owen (1975), Laruelle and Widgrén (1998), Felsenthal and Machover (1998))[1].

Based either on the assessment of the likelihood of being 'decisive' or of being 'satisfied' or 'successful' (Rae (1969), Brams and Lake (1978), Barry (1980), Straffin, Davis, and Brams (1981), see also Laruelle and Valenciano (2005a), and, in a different framework, Barberà and Jackson (2004)), this approach makes sense in the case of a 'take-it-or-leave-it' committee. That is, a committee only entitled to accept or reject proposals submitted to it by some external agency.

In this paper we take a new departure to address the question of the optimal voting rule in a committee of representatives. We assume that the committee is a bargaining scenario in which negotiations take place 'in the shadow of' a voting rule. In the cooperative game theoretic literature on bargaining since Nash's (1950) seminal paper, bargaining is supposed 'by definition' to be a process that can be settled only by unanimity. Asymmetry between players' bargaining powers can only arise from the bargaining environment. In many contexts it is often the case in a committee which uses a (possibly nonsymmetric) voting rule to make decisions that the final vote is merely the formal settlement of a bargaining process in which the issue to be voted upon has been adjusted to gain the acceptance of all members. We base our approach on Laruelle and Valenciano (2005b), where the following question is addressed: What agreements can a rational agent expect to arise when faced with the prospect of engaging in such a situation? That is, when a general agreement is looked for, but any winning coalition can enforce a (possibly non unanimous) agreement. Based on this answer, and assuming that a principle of equal

[1]See Garrett and Tsebelis (1999) for a critique of this approach.

representation is to be implemented, here we answer this question: What is the optimal voting rule in a bargaining committee of representatives of different-sized groups?

The rest of the paper is organized as follows. In Section 2 we introduce some basic notation and briefly review the theoretical results that are required. In Section 3 we address the issue of the optimal voting rule in a bargaining committee of representatives. Section 4 examines some related work. Section 5 concludes by recapitulating the foundations of the recommendation implicit in the main result of the paper and pointing out some of its limitations and the main lines of further research.

2 A BARGAINING COMMITTEE

The first element in a *bargaining committee* is the voting rule. The set $N = \{1, ..., n\}$ labels the *seats* on the committee. As only yes/no voting is considered, a *vote configuration* can be represented by the set of 'yes'-voters. So, any $S \subseteq N$ represents the result of a vote in which only the members of the committee occupying seats in S voted 'yes'. An N-voting rule is specified by a set $W \subseteq 2^N$ of *winning* (i.e., which would lead to passing a decision) vote configurations such that (i) $N \in W$; (ii) $\emptyset \notin W$; (iii) If $S \in W$, then $T \in W$ for any T containing S; and (iv) If $S \in W$ then $N \backslash S \notin W$. $\mathcal{W}$ denotes the set of all such N-voting rules.

The n members or *players* of a committee which uses an N-voting rule are labelled by the seats in N that they occupy, and we refer to the subset of players denoted by $S \subseteq N$ as *coalition* S. We assume that a committee of n (N-labelled) members makes decisions by means of an N-voting rule W in the following sense. They can reach any alternative within a set A, as well as any lottery over them, as long as: (i) a winning coalition supports it, and (ii) no player is imposed upon an agreement worse than the *status quo,* where all players will remain if no winning coalition supports any agreement. It is also assumed that every player has expected utility (von Neumann and Morgenstern, 1944) (vNM) preferences, so that the relevant information concerning the players' preferences can be encoded à la Nash in utility terms by a feasible set of utility vectors $D \subseteq R^N$, together with the particular vector $d \in D$ associated with the disagreement or *status quo.* Thus, the pair (D, d) is a summary of the situation concerning the players' decision.

Accepting this simplification, the whole situation can be summarized by a pair (B, W), where $B = (D, d)$ is a classical n-person bargaining problem that represents the configuration of preferences in the committee, and W is the N-voting rule to enforce agreements. Thus, consistently with the interpretation that accompanied its introduction, any pair $(B, W) \in \mathcal{B} \times \mathcal{W}$ will be referred to as an N-*bargaining committee* (B, W)[2].

[2]Classical bargaining problems and simple transferable utility (TU) games can be seen as particular

What conditions can be imposed on a map $\Phi : \mathcal{B} \times \mathcal{W} \rightarrow R^N$ for vector $\Phi(B, W) \in R^N$ to be considered as a reasonable expectation of utility levels of a *general* agreement in a bargaining committee (B, W)? Reasonable prerequisites, if $B = (D, d)$, are: $\Phi(B, W) \in D$ (feasibility), and $\Phi(B, W) \geq d$ (individual rationality). In addition to these, in Laruelle and Valenciano (2005b) the following conditions, the result of adapting to this setting some conditions from Nash (1950) and Shapley (1953), are required: *Efficiency (Eff), Anonymity (An), Independence of irrelevant alternatives (IIA), Invariance w.r.t. positive affine transformations (IAT), Null player* (*NP*). Assuming standard conditions on B, we have the following generalization of Nash's characterization.

Theorem 1 *(Laruelle and Valenciano, 2005b) Let $\Phi : \mathcal{B} \times \mathcal{W} \rightarrow R^N$ be a solution that satisfies Eff, An, IIA, IAT and NP, then*

$$\Phi(B, W) = Nash^{\varphi(W)}(B), \tag{1}$$

for some $\varphi : \mathcal{W} \rightarrow R^N$ that satisfies efficiency, anonymity and null player.

Where $Nash^w(B)$ denotes the w-weighted asymmetric Nash bargaining solution (Kalai, 1977) of an n-person (pure) bargaining problem $B = (D, d)$, for a vector of nonnegative weights $w = (w_i)_{i \in N}$, given by

$$Nash_i^w(B) := \begin{cases} \arg_i \max_{x \in D_d} \prod_{j \in J} (x_j - d_j)^{w_j} & \text{if } i \in J, \\ d_i & \text{if } i \in N \setminus J, \end{cases}$$

with $J = \{i \in N : w_i > 0\}$. The 'weights' are usually interpreted as the 'bargaining power' of the players (see, e.g., Binmore (1998, 2005)). Classical Nash's (1950) solution to the bargaining problem corresponds to the case of equal weight for all players. That is,

$$Nash(B) = \arg_i \max_{x \in D_d} \prod_{j \in J} (x_j - d_j). \tag{2}$$

Therefore, any map $\varphi : \mathcal{W} \rightarrow R^N$ that satisfies efficiency, anonymity and null player would fit into formula (1) and yield a solution $\Phi(B, W)$ that satisfies the five rationality conditions. Although the main conclusions of this paper are presented in section 3 for any such map φ, a special case is worth distinguishing. By adding the condition of *Transfer (T)* (Dubey, 1975) the same authors prove the following:

Theorem 2 *(Laruelle and Valenciano, 2005b) There exists a unique solution/value $\Phi : \mathcal{B} \times \mathcal{W} \rightarrow R^N$ that satisfies Eff, An, IIA, IAT, NP and T, and it is given by*

$$\Phi(B, W) = Nash^{Sh(W)}(B). \tag{3}$$

cases of this model. The n-person classical bargaining problem corresponds to the case of a committee bargaining *under the unanimity rule,* that is $W = \{N\}$.

Where $Sh(W)$ denotes the Shapley-Shubik (1954) index of the rule W, given by

$$Sh_i(W) = \sum_{\substack{S:i \in S \in W \\ S \backslash i \notin W}} \frac{(n-s)!(s-1)!}{n!}.$$

Formulae (1) and (3) have a clear interpretation. As Binmore points out, the asymmetric Nash solutions can be justified as reflecting the different 'bargaining power' of the players *"determined by the strategic advantages conferred on players by the circumstances under which they bargain"* (1998, p. 78). In the case of this model of a bargaining committee the voting rule, possibly nonsymmetric, is the only source of differences in 'strategic advantages'. Thus, according to formulae (1) and (3), under the conditions assumed in either case, either vector $\varphi(W)$ or $Sh(W)$ gives the 'bargaining power' that the voting rule confers to each member of the committee.

3 A BARGAINING COMMITTEE OF REPRESENTATIVES

Assume that each member i of a committee of n members, labelled by N, represents a group M_i of size m_i. If these groups are disjoint and $M = \cup_{i \in N} M_i$, the cardinal of M is $m = \sum_{i \in N} m_i$. Let us denote by $\mathcal{M}$ the partition $\mathcal{M} = \{M_1, M_2, .., M_n\}$. And assume that it is a bargaining committee in the sense considered in the previous section. It seems clear that if the different groups are of different sizes a symmetric voting rule is not adequate for such committee, at least if a principle of equal representation is to be implemented. This raises the issue of the choice of the 'most adequate' voting rule under these conditions. The first and main job towards providing an answer is a precise specification of what is meant by 'adequate', 'fair', 'right', or, the term we have chosen here, 'optimal'. 'Optimal' in what sense and from which or whose point of view? The basic idea, which we further specify presently, is this: a voting rule is 'optimal' if any individual of any group is indifferent between bargaining directly and leaving it in the hands of a representative. Utopian as it may sound (and as it *is* in general), we will show that this is implementable if a certain level of symmetry (not uniformity!) of preferences within every group is assumed.

In general a bargaining committee of representatives will negotiate different issues over time under the same voting rule. In every case, depending on the particular issue, a different configuration of preferences will emerge in the population represented by the members of the committee. Thus it does not make sense to make the 'optimal' voting rule dependent on the preference profile, nor does it make sense to assume unanimous preferences within every constituency. On the other hand, if there is no relationship at all between the preferences of the members within each group it is not clear on what normative grounds to found the choice of a voting rule for the committee of representatives. In order to found an answer we assume that the configuration of preferences in the population represented is symmetric *within each group* in the following sense.

Assume that $B = (D, d)$ ($d \in D \subseteq R^M$) is the m-person bargaining problem representing the configuration of preferences of the m individuals in M. We say that a permutation $\pi : M \rightarrow M$ respects $\mathcal{M}$ if for all $i \in N$, $\pi(M_i) = M_i$. We say that B is $\mathcal{M}$-*symmetric* if for any permutation $\pi : M \rightarrow M$ that respects $\mathcal{M}$, it holds $\pi d = d$, and for all $x \in D$, $\pi x \in D$. In words, B is $\mathcal{M}$-symmetric if for any group (M_i) the disagreement payoff is the same for all its members ($d_k = d_l$, for all $k, l \in M_i$), and fixing in any way the payoffs of the other players in $M \backslash M_i$, the set of feasible payoffs for the players in that group (M_i) is symmetric[3]. Notice that this does not mean at all that all players within each group have the same preferences. In fact it includes all symmetric situations ranging from unanimous preferences to the 'zero-sum' case of strict competition within each group. But note that if the payoffs of all the players in $M \backslash M_i$ are fixed, the outcome of bargaining within M_i (under unanimity and assuming anonymity) would yield the same utility level for all players in M_i. Thus $\mathcal{M}$-symmetry in B entails the following consequences.

Let M, N, and $\mathcal{M}$, as above, and let $B = (D, d)$ an $\mathcal{M}$-symmetric M-configuration of preferences. Assuming (as a term of reference) the players in M negotiate directly under unanimity, according to Nash's bargaining model, the outcome would be $Nash(B)$. On the other hand, as B is $\mathcal{M}$-symmetric, it must be

$$Nash_k(B) = Nash_l(B) \qquad (\forall i \in N, \forall k, l \in M_i).$$

Namely, in each group all players would receive the same payoff according to Nash's bargaining solution. Therefore the optimal solution of the maximization problem (2) that yields $Nash(B)$ coincides with the optimal solution of the same maximization problem when the set of feasible payoff vectors is constrained to yield the same payoff for any two players in the same group. Formally, denote by B^N the N-bargaining problem $B^N = (D^N, d^N)$, where

$$D^N := \left\{(x_1, ..., x_n) \in R^N : \quad (\underbrace{x_1, .., x_1,}_{m_1 - times} \quad ...... \quad \underbrace{, x_n, .., x_n}_{m_n - times}) \quad \in D\right\},$$

and by d^N the vector in R^N whose i-component is, for each $i \in N$, equal to d_k (the same for all $k \in M_i$). Namely, B^N is the bargaining problem that would result by taking one individual from each constituency as representative for bargaining on behalf of it, under the commitment of later bargaining symmetrically within that constituency after the level

[3]This is equivalent to saying in Chae and Heidhues' (2004) terms that all groups are *homogeneous*.

of utility of the other constituencies has been settled. We have that, for all $i \in N$ and all $k \in M_i$,

$$Nash_k(B) = \arg_k \max_{x \in D_d} \prod_{l \in M} (x_l - d_l)$$

$$= \arg_i \max_{x \in D^N_{d^N}} \prod_{j \in N} (x_j - d_j)^{m_j} = Nash^{\bar{m}}_i(B^N). \tag{4}$$

where $\bar{m} = (m_1, ..., m_n)$. That is to say, for the configuration of preferences or M-bargaining problem B, a player k in M would obtain the same utility level by direct (m-player unanimous) bargaining, as a representative would obtain by bargaining on behalf of him/her (and of all the players in the same group) under the configuration of preferences B^N *if each representative were endowed with a bargaining power proportional to the size of the group.* The problem then is how to 'implement' a weighted Nash bargaining solution. In other words and more precisely, *how to implement a bargaining environment that confers the right bargaining power to each representative*[4].

In view of Theorem 1, if a 'power index' (i.e., an efficient, anonymous and ignoring null players map $\varphi : \mathcal{W} \to R^N$) is considered the right assessment of bargaining power, and for some N-voting rule W it holds

$$\frac{\varphi_i(W)}{m_i} = \frac{\varphi_j(W)}{m_j} \qquad (\forall i, j \in N),$$

then *this rule would exactly implement such environment.* In particular, if the index is the Shapley-Shubik index (Theorem 2), an optimal voting rule would be one for which

$$\frac{Sh_i(W)}{m_i} = \frac{Sh_j(W)}{m_j} \qquad (\forall i, j \in N).$$

Then, interpreting the term 'bargaining power' in the precise game-theoretic sense formerly specified, the above discussion can be summarized in the following

Theorem 3 *The optimal voting rule in a bargaining committee of representatives is one that gives each member a bargaining power proportional to the size of the group he/she represents.*

From the point of view of applications there are still some issues. There is the question of the 'right' power index (i.e., the right $\varphi(W)$) for assessing the bargaining power that the voting rule confers to each member of the committee. This issue is not settled but, as Laruelle and Valenciano (2005b) point out, this would require additional assumptions

[4]Here we use the term 'implementation' in a general sense, and not in the standard technical sense of providing a noncooperative game that yields the desired outcome as an equilibrium.

about the bargaining protocol in the committee, and possibly a noncooperative analysis. Also, in general, whatever the φ, no rule will yield exactly the required bargaining weights. Thus there is the technical problem of finding the voting rule closest to optimality[5]. There may also be a problem of multiplicity. For instance, if all the groups are of equal size any symmetric voting rule would be optimal according to Theorem 3. In a case such as this there is at least a second point of view to refine the choice: The ease of decision-making (Coleman (1971), Felsenthal and Machover (1998)).

In spite of these practical problems, we think it is worth stressing the clear message of Theorem 3, consistent with intuition and different from previous recommendations.

4 RELATED WORK

The answer provided by Theorem 3 to the optimal voting rule issue is a completely new departure from previous ones, but it is worth noting the formal similarity with some of them. First, with the naive proposal of a weighted majority rule with weights proportional to the size of the groups, which has long been criticized but is sometimes still used[6]. Also, in the 'take-it-or-leave-it' scenario, the two-stage idealization yields the 'square root' rule, which solves the normative problem of the fair distribution of 'decisiveness' in such a committee, assuming that each representative follows the majoritarian opinion in his/her constituency (see, e.g., Penrose (1946), Owen (1975), Laruelle and Widgrén (1998), Felsenthal and Machover (1998)). That is, assuming the group sizes' are big enough, the 'fair' rule is the one for which the Banzhaf (1965) index of each representative is proportional to the square root of the size of the group he/she represents. Curiously enough, in a somewhat inconsistent way, some US courts have accepted the Banzhaf index as a measure of the voting power of the members in a committee, but approved a voting system that made the Banzhaf index of each representative *proportional to his/her district's size* (Benoit and Kornhauser, 2002). Note also that unlike the traditional a priori voting power approach the voters' preferences are a crucial ingredient in the model considered here.

On the other hand, the case in which bargaining among groups occurs has often been considered in economic literature since Harsanyi (1977)[7]. In Harsanyi (1977), where Nash

[5]Whatever the size of the committee the number of voting rules is finite, while the set of possible combinations of group sizes is not. A similar problem occurs for the 'square root rule' that solves the normative problem of the fair distribution of decisiveness in a 'take-it-or-leave-it' committee as commented in the the next section.

[6]See Benoit and Kornhauser (2002, pp. 2252-2259) for an interesting account of the different criteria endorsed by U.S. courts on the issue of "fair and effective representation."

[7]See footnote 1 in Chae and Heidhues (2004) for an interesting quantification based on two leading journals. Usually for two-party negotiations, under different frameworks and with different goals, several

classical bargaining solution is extended to n players, the following 'joint-bargaining paradox' is discussed (cf. 10.7, pp. 203-211). Consider the three-person TU-like bargaining game B in which the set of feasible payoffs is defined by the inequalities $u_1+u_2+u_3 \leq 30$, and the disagreement payoff vector is $(0,0,0)$. If the three players bargain as different independent agents according to Nash's bargaining model, the solution will be $(10,10,10)$. *"But suppose players 2 and 3 decide to act as one player and agree that they will split equally the joint payoff that they obtain this way. Then the game will become a two-person game between coalition* $\{2,3\}$ *and player* 1*. Hence each side will obtain a payoff* $u_1 = u_{23} = 15$*. If players* 2 *and* 3 *later split their joint payoff* u_{23}*, then the final outcome will become* $(15, 7.5, 7.5)$*. Consequently the fact that players 2 and 3 have joined forces has actually decreased their payoffs from 10 to 7.5"* (Harsanyi, 1977, p. 203). He solves the paradox by analyzing the situation by means of Zeuthen's (1930) Principle, and offering two explanations. In both cases the explanation shows the *"weakening of the bargaining position"* of the player acting on behalf of the two-player coalition. Moreover, he points out that: *"any possible solution concept will show this behavior if it satisfies the symmetry and the joint-efficiency postulates."* But this is the critical point: symmetry (like anonymity) in the classical setting ignores the possibility of bargaining under asymmetric conditions, or under rules different from unanimity. In fact, the origin of the paradox is admitting for a moment that by bargaining as a single player "*players 2 and 3 have joined forces.*" This contradicts common sense views in real-world situations actually, where committees of representatives whose members represent groups of different sizes rarely bargain under unanimity. They often use nonsymmetric voting rules (even if usually chosen on no clear grounds) to bargain under. As Harsanyi put it in somewhat tautological terms: *"If two or more players form a coalition for bargaining purposes, this will tend to strengthen their bargaining position if this organizational change strengthens their determination to obtain better terms and weakens their reluctance to risk a conflict."* This is exactly what an optimal rule would implement, compensating the loss behind Harsanyi's paradox in terms of bargaining power.

It is also interesting to examine some recent work concerning 'group bargaining'. In two recent papers Chae and Heidhues (2004) and Chae and Moulin (2004) address this problem from an axiomatic point of view. Their model consists of a classical bargaining problem plus a partition of the set of players into subsets that represent the bargaining groups. Chae and Heidhues (2004) characterize axiomatically a 'group bargaining solution' for situations where different groups bargain with each other. It is an extension of Nash's solution to

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noncooperative models have been provided (see, for instance, Jun (1989), Perry and Samuelson (1994), Haller and Holden (1997), and Cai (2000)).

the bargaining problem within as well as across groups. They impose a condition of 'Representation of Homogeneous Groups', whose interpretation is the following. Every member of a homogeneous group obtains what he would obtain if he alone bargained on behalf of the group. They show that by adding this condition to Nash's axioms a unique solution is characterized. Namely, the *asymmetric Nash solution in which the weight of every representative is the reciprocal of the size of the group he belongs to.* That is, the reciprocal of what our Theorem 3 prescribes! The explanation is easy. They *impose* the indifference for any player between bargaining (under unanimity) directly or as a representative. But this can only be achieved by 'penalizing' representatives proportionally to the size of the group they represent. Again this is the effect of taking symmetric bargaining-under-unanimity as the only conceivable way of bargaining.

In Chae and Moulin (2004) the family of asymmetric Nash solutions where the bargaining power of an agent in a group of size m_i is m_i^{α}, with $\alpha \geq -1$, is characterized axiomatically. As they point out: *"One benchmark of the family F^{α} is the group-insensitive solution F^1: this solution is the ordinary symmetric Nash bargaining solution, ignoring the partition altogether".* In other words, these are exactly the weights for which (4) holds, and this is the solution that an optimal rule would implement[8].

Finally there is the relevant work of Baron and Ferejohn (1989). In particular their distinction between 'closed' and 'open' amendment rules in a legislature, parallels our distinction between a 'take-it-or-leave-it' committee and a bargaining committee in a different framework. But their analysis is non cooperative and has a descriptive/predictive purpose, while here we adopt a normative approach, cooperatively founded, in search of an answer to the question of the choice of voting rule. On the other hand, their analysis is constrained to the majority rule, and the case of a transferable utility configuration of preferences, while we use a more general setting: non necessarily symmetric voting rules, and non transferable utility preference profiles.

5 CONCLUSIONS

From the point of view of voting power theory and collective decision or 'constitutional' design, this paper contributes to some clarification. Namely, an alternative to the traditional a priori voting power approach to the issue of the optimal voting rule, only adequate for what we call a 'take-it-or-leave-it' committee, has been provided. A new approach consistent with the idea of a bargaining committee is the main contribution of this paper. In real world situations things may most often not be that black-and-white. Often the

[8]They show how this solution as well as those associated with $\alpha > 1$ are free from the 'joint-bargaining paradox'.

same committee acts at some times as a 'take-it-or-leave-it' committee, and at others as a bargaining committee. In any case the two clear-cut extreme cases are valuable terms of reference as benchmarks for more complex ones.

Let us examine critically the meaning and foundations of the normative recommendation for a pure bargaining committee implicit in Theorem 3 and point out some lines of further research. The cornerstones of Theorem 3 are, basically, Theorem 1 and a principle of equal representation implementable under certain conditions on the voters preferences. On the one hand, the classical Nash bargaining solution can be interpreted in positive terms as a prediction of the outcome of negotiations among ideally rational players in a perfectly transparent or complete information environment (e.g., Binmore (2005))[9]. While such an interpretation may be plausible for the case of two bargainers, it is not that credible for a larger number. In some cases a committee represents thousands or even millions of individuals (consider, e.g., the Council of Ministers of the EU). In such cases 'direct bargaining' is unthinkable in practical terms, but the Nash solution can still be used as an ideal term of reference for normative purposes. The same applies to its extension given by Theorem 1, which is at the base of Theorem 3. Theorem 1 is supported by rather general 'axioms' interpretable as ideal rationality conditions in a complete information environment in the same spirit as Nash's seminal paper.

No doubt it would be desirable to complement the cooperative/axiomatic foundation with a non cooperative analysis. As Binmore, (2005) puts it: "*Cooperative game theory sometimes provides simple characterizations of* what *agreement rational players will reach."* This is exactly what Theorem 1 provides for the situation specified. But this is not the end of the story. As the second part of the sentence just quoted goes "*but we need noncooperative game theory to understand* why." This points out the main line for further research. In fact Theorem 1 (and as a consequence Theorem 3) *does not* provide a single answer to the question raised. But we see no drawback here. It is our conjecture that a non cooperative model of a bargaining committee will support the results given by Theorems 1 and 3, and account for the different answers implicit in both depending on the specification of the bargaining protocol. In particular we expect the particular answer associated with the Shapley-Shubik index and Theorem 2 to appear as a special case with at the least one 'focal point' character within a range of bargaining protocols.

At the foundations of Theorem 3 an egalitarian principle of equal representation has also been assumed. This justifies the desideratum of a voting rule for the committee such that all people represented see as indifferent direct bargaining (ideal and unfeasible) and

[9]Some authors (e.g., Mariotti, 1999, 2000), favor a normative interpretation of Nash's bargaining solution.

leaving it in the hands of a committee of representatives. This in general is obviously utopian, but it has been proved implementable at least under some ideal symmetry conditions. In real world situations this condition may well fail to occur in most cases. Only in the ideal case of $\mathcal{M}$-symmetry is the principle of equal representation sufficient to determine an answer. But this idealization seems a reasonable term of reference if a voting rule is to be chosen[10].

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[10] There are also situations in which the members of a committee represent qualitatively different groups. For instance, in the council governing a department or a university there are usually representatives of different categories of faculty, students and administrative staff. In this case, in addition to number, there is a second possible source to justify asymmetry. In this case the recommendation of Theorem 3 should be adjusted, perhaps by first assessing the relative importance of the different groups by means of different multiplicative weights of their actual sizes.

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