ADVERSE SELECTION AND MANAGERIAL INCENTIVES[*]

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A B S T R A C T

We analyze managerial contracts (i.e. incentive schemes based on a linear combination of profits and sales) under asymmetric information about costs. In the competitive setting with ex ante symmetric information, standard strategic effects appear. Under adverse selection in both, monopolistic and competitive settings, we show that, in order to decrease the manager's expected informational rents, the owner will optimally pay the manager to keep sales low or, on the contrary, keep them high. Moreover, the interactions between the strategic and the informational rent effects have a non-additive nature, implying non-standard results. Unlike the monopolistic framework, we show that, in the competitive framework, the manager may become aggressive under ex ante symmetric information than under adverse selection. Unlike the setting with ex ante symmetric information, we show that, under adverse selection, the manager may become more aggressive in the monopolistic framework than in the competitive one.

Key words: Managerial incentives, Adverse selection, Quantity competition.

1 Introduction

Managerial contracts (i.e., incentive schemes based on a linear combination of profits and sales) have been used in many theoretical analyses of relevant issues regarding Industrial Organization. Under Cournot competition, Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987) show that the owner of a firm can obtain greater profits if he distorts managerial incentives away from profit maximization. Considering a mixed duopoly, Barros (1995) proves that delegation through managerial contracts can improve welfare. Faulí-Oller and Motta (1996) show that a manager might undertake unprofitable takeovers when he decides on the production, under a managerial contract, and can also make takeover decisions. Moreover, the approach of managerial contracts is consistent with empirical evidence. The empirical analysis carried out by Murphy (1985), Jensen and Murphy (1990), and Conyon (1997), emphasizes the widely observed fact that managerial compensation is linked to both profits and sales.

To the best of our knowledge, the current analyses of managerial contracts consider only situations with complete information or with ex ante symmetric information.[1] In fact, in the standard settings that employ managerial contracts, it is commonly assumed that managers know the actual values of the relevant parameters (as costs and productivity parameters) after contracts are signed. The aim of this paper is, therefore, to understand how results on managerial contracts are affected by the presence of adverse selection in situations where managers realize the actual values of the parameters before contracts are signed.

In this paper, we analyze managerial contracts in several settings with different degrees of competition and asymmetry of information about costs. We suppose a general continuous distribution for the marginal cost, and quantity competition. It is assumed that, at the time of contracting a manager, the owner of a firm is ignorant of the actual value of the marginal cost, which is observed by managers before making production decisions. We consider two different kinds of information structures. We assume that, in the setting with ex ante symmetric information, the manager knows the marginal cost after the contract is signed, and, in the setting under adverse selection, he knows it before the contract is signed. We also consider two different frameworks that differ in their levels of competition in the market. In the monopolistic framework only one managerial firm is assumed (i.e., a firm with an owner that proposes a contract to a manager). In the competitive framework, to

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[1]Managerial incentives have also been formulated as direct mechanisms based on outputs in adverse selection contexts. See, for instance, Barros (1997) and Caillaud, Jullien, and Picard (1995).

simplify the analysis, we suppose a duopoly, formed by one managerial firm and one entrepreneurial firm (i.e., a profit-maximizing firm) with identical and constant marginal costs.

Just like other complete information models or ex ante symmetric information models that have been presented in the literature [see, for instance, Fershtman and Judd (1987)], our model exhibits a strategic effect: A firm induces its manager to be more aggressive (i.e., increases the weight of sales against that of profits in the managerial contract) in order to push out the manager's reaction function to increase the firm's payoff. The present model, however, shows the existence of an informational rent effect under adverse selection.

We prove that, in the monopolistic adverse selection setting, the owner makes the manager more aggressive in order to decrease the manager's expected informational rents if the market demand level (the intercept of the inverse demand) is high enough. If it is not, the owner will pay the manager to keep sales low, if the expected marginal cost is high enough and the variance of the marginal cost is low enough. This is a consequence of the degree of aggressiveness that maximizes the manager's informational rent compared with the optimal degree under ex ante symmetric information.

On the other hand, we show that, in the monopolistic adverse selection setting, the optimal aggressiveness level, which is maximal relative to the marginal cost distribution, implies no incentives to make higher profits. However, competition pushes this aggressiveness level upwards and yields different results. First, in the competitive adverse selection setting, the owner should increase the aggressiveness level as a consequence of both, the strategic effect and the informational rent effect, when the market demand level is high enough. Secondly, when the expected marginal cost is low enough or high enough, the informational rent effect is very weak, relative to the strategic effect, and the owner will induce the manager to be aggressive. Thirdly, for intermediate values of the expected marginal cost, and if the demand level is low enough, the owner will pay the manager to keep sales low if the variance of marginal cost is sufficiently low.

Finally, we show that the interactions between the strategic and the informational rent effects have a non-additive nature, implying non-standard results. We first prove that, unlike the monopolistic framework, in the competitive setting the manager may become more aggressive under ex ante symmetric information than under adverse selection, if the market demand level is sufficiently high or when the expectation and the variance of the marginal cost are sufficiently low. Next, we show that, unlike the setting with ex ante symmetric information, under adverse selection the manager may become more aggressive in the monopolistic framework than in the competitive one

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if the expected marginal cost is sufficiently low.

The rest of this paper is organized as follows. In Section 2 we analyze the monopolistic framework under ex ante symmetric information and under adverse selection. In Section 3 we study the competitive framework under ex ante symmetric information and under adverse selection. In Section 4 we compare the managers' aggressiveness levels obtained in previous sections. Finally, our conclusions are presented in Section 5.

2   The monopolistic framework

In this section, we analyze managerial incentives when there is only one firm in the market. Demand is given by $P = a - Q$, where Q is the total quantity and P is the price. We assume a constant marginal cost given by $c \in [0, 1]$. To guarantee a positive monopolistic output, we suppose $a > 1$. The game that we consider in this section has three stages: At Stage 1, the owner of firm chooses the parameters $\alpha \in \mathcal{R}$ and $h \in \mathcal{R}$ that determine the contract $T = \alpha\pi + (1 - \alpha)S - h$ proposed to the manager, where π and S stand respectively for the profits and sales of the firm, and h is a constant. This approach is consistent with the widely accepted fact that a manager's compensation is linked to both profits and sales.[2] At Stage 2, the manager either accepts or rejects the contract and, in the last case, he obtains his reservation utility that is normalized to zero. Finally, at Stage 3, if the manager has accepted the contract, he decides on the production and he is paid in accordance with the contract.

In the act of contracting, the owner of the firm does not know the actual value of the marginal cost $C = c$. To guarantee that, at Stage 3, the manager chooses a positive output, we assume $\alpha \leq a$. The owner merely knows that C is distributed with expectation $E(C) = e$ and variance $Var(C) = v > 0$. Given $E(C^2) = d$, since $v = d - e^2 > 0$, we consider the parameter space $\mathcal{P} = \{(e, d) \mid 0 < e^2 < d < e < 1\}$. Finally, we assume that, in the final stage, the manager knows the actual value of c before deciding on the production.

Given a contract, it is easy to verify that the quantity, the owner's gross profit, and the corresponding payoffs at Stage 3 of the game, are, respectively:

$$Q(c, \alpha) = \frac{1}{2}(a - c\alpha),$$

$$\pi(c, \alpha) = \frac{1}{4}(a + c(\alpha - 2))(a - c\alpha),$$

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[2]See, for instance, the empirical analysis by Murphy (1985), Jensen and Murphy (1990) and, more recently, by Conyon (1997)

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and

$$T_M(c,\alpha,h) = \frac{1}{4}(a - c\alpha)^2 - h,$$

$$U(c,\alpha,h) = \pi(c,\alpha) - T_M(c,\alpha,h) = \frac{1}{2}c(\alpha - 1)(a - c\alpha) + h.$$

For this monopolistic framework, we consider two different classes of information structures. In the *ex ante symmetric information* setting, we assume that the manager knows the actual value of c after the contract is signed and, in the *adverse selection* setting, we suppose that the manager knows the actual value of c before the contract is signed. We will study the optimal value of α in both settings.

Previously, let us consider the *complete information* setting (i.e., when c is verifiable). Since the individual rationality constraint implies $T_M(c,\alpha,h) \geq 0$, the owner optimally sets $T_M(c,\alpha,h) = 0$ and, therefore, he extracts all the manager's surplus. The optimal value of α must maximize $\pi(c,\alpha)$ for each c. It is easy to prove that this optimal value is $\alpha = 1$. This is not surprising, since the principal (owner) and the agent (manager) are, both, risk neutral and, in this setting, there is no hidden information. If there is no danger of adverse selection or of moral hazard on the part of the principal, therefore, the agency does not matter if the principal can make the agent a residual claimant $(T = \pi - h)$.[3] This result can by easily extended to the ex ante symmetric information setting.

2.1   Monopoly under ex ante symmetric information

Under ex ante symmetric information, the manager will accept the contract only if his expected utility $E[T_M(c,\alpha,h)]$ is greater than his reservation utility. For any α, the owner will optimally set h so that $E[T_M(c,\alpha,h)] = 0$. Thus, in this setting, the owner will maximize

$$E[\pi(c,\alpha)] = \frac{1}{4}(a^2 - 2ae + d(\alpha - 2)\alpha)$$

with respect to α.

It is easy to show that the previous function reaches the maximum value when α is equal to $\alpha_{SI}^M = 1$ and the optimal contract is profit-maximizing. The intuition here is similar to that of the complete information setting. Under ex ante symmetric information, the manager does not possess hidden

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[3]See Katz (1991) for an analysis in a competitive setting with unobservable contracts. In our monopoly setting, contract observability does not matter.

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information at the time of contracting and the owner can extract all the manager's surplus by adjusting the value of h. This leads to a profit-maximizing optimal contract. The conclusion changes under adverse selection.

2.2 Monopoly under adverse selection

In this setting, the manager will only accept the contract if $T_M(c,\alpha,h) \geq 0$ for any $c \in [0, 1]$. Thus, the owner will solve the following program

$$\begin{cases} \max_{\alpha,h} E[U(c,\alpha,h)] \\ \text{s.t.} \\ T_M(c,\alpha,h) \geq 0, \quad \forall c \in [0,\, 1]. \end{cases} \tag{1}$$

Note that (1) is the reduced form of an adverse selection problem. Given the contract (α, h), the previous function $Q(\cdot, \alpha)$ is the only incentive compatible action profile that corresponds to the agent's utility function

$$V_M(x,c;\alpha,h) = \alpha(P(x)x - cx) + (1-\alpha)P(x)x - h =$$
$$= P(x)x - c\alpha x - h,$$

where x and c are the action and the agent's type respectively. It is known that, under some assumptions, an action profile is implementable if and only if the action profile is monotonic in types [see, for instance, Guesnerie and Laffont (1984)]. The assumptions include the Spence-Mirrlees condition, which indicates that $\partial_{xc}V_M$ has a constant sign. Moreover, the monotonicity of an implementable action profile depends on that sign. In the present monopolistic adverse selection setting, the sign depends on managerial contracts, because $\partial_{xc}V_M = -\alpha$, and, since V_M is monotonic in c, the managers's payoff T_M is also monotonic in c. Therefore, $\alpha = 0$ is a threshold for which the manager obtains the constant rent $a^2/4 - h$ for any c. For $\alpha < 0$, the minimum of $T_M(c,\alpha,h)$ with respect to c is reached at $c = 0$ with a minimal rent equal to $a^2/4 - h$. For $\alpha > 0$, the minimal rent is $(a-\alpha)^2/4 - h$ and it is reached at $c = 1$. In consequence, it is optimal for the owner to set h equal to

$$h_M(\alpha) = \begin{cases} a^2/4 & \text{if } \alpha \leq 0, \\ (a-\alpha)^2/4 & \text{if } 0 \leq \alpha \leq a. \end{cases} \tag{2}$$

and, given the contract $(\alpha, h(\alpha))$, the c-manager's informational rent is

$$T_M(c,\alpha,h_M(\alpha)) = \frac{1}{4}(a-c\alpha)^2 - h_M(\alpha). \tag{3}$$

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and program (1) is equivalent to maximize

$$E[U(c,\alpha,h_M(\alpha))] = E[\pi(c,\alpha)] - E[T_M(c,\alpha,h_M(\alpha))] = \tag{4}$$
$$= \frac{1}{2}(\alpha - 1)(ae - d\alpha) + h_M(\alpha).$$

Expression (3) follows the standard properties of the agent's utility under an incentive compatible and individually rational contract in adverse selection principal-agent models. Below the threshold ($\alpha < 0$), the "worst" type of manager is $c = 0$. He will receive no rent and the others will obtain rents that are increasing in c. Over the threshold ($\alpha > 0$), the "worst" type is $c = 1$ and rents are decreasing in c. If $\alpha = 0$, the manager obtains no rent for any c. This explains the properties of the optimal value of α in this monopolistic setting under adverse selection. First, note that the owner's (expected) gross profit is maximized at $\alpha = 1$, which represents the profit-maximizing contract under complete information or under ex ante symmetric information. On the other hand, the manager behaves as a monopolist with the distorted marginal cost $c\alpha$, when α is non-negative, and his profit increases when $c\alpha$ decreases. Under adverse selection, the owner will distort the manager's aggressiveness taking into account the expected informational rent perceived by the manager according to (4).

The first property is that it is not optimal for the owner to set α at a negative value, as this would rank the manager's types in the opposite direction to the manager's efficiency (here the "worst" type corresponds to the most efficient). Specifically, since informational rents decrease in α when it is negative (because in this case $c\alpha$ will come close to a) and the owner's gross profit is maximized at $\alpha = 1$, the owner's payoff increases with α when α is negative.

The second property is related to the manager's aggressiveness, which is inversely correlated to α. At first sight, it would seem that an aggressive manager would obtain less informational rents than an unaggressive one since the decision of the former depends less on cost considerations than that of the latter. This argument, however, is mistaken. The informational rent function (3) is concave and has one relative maximum on $[0, a]$ (see Figure 1). The reason is that the individual rationality constraints are less demanding when $\alpha \in [0, a]$ is close to 0 or a. This follows from the fact that the manager's payoff $T_M(\cdot)$ at Stage 3 corresponds to the optimal profit (minus a constant) of a monopolist with a distorted marginal cost equal to $c\alpha$. On the one hand, if α approaches a, the distorted marginal cost increases and the optimal profit decreases. On the other hand, if α approaches 0, the distorted marginal cost goes to 0 and the optimal profit increases and becomes flatter relative to c. In both cases, it is cheaper for the owner to induce the acceptance of
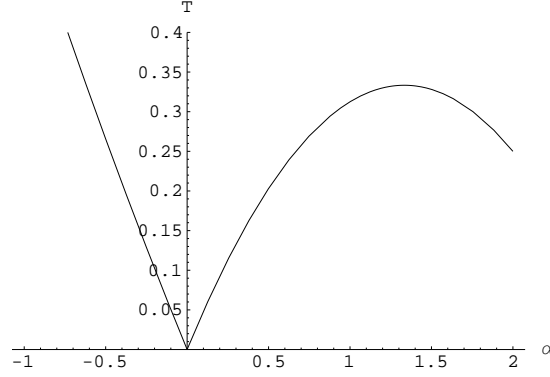
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Figure 1: Informational rents in the monopoly under adverse selection for $a = 2$ and $c = 0.5$.

the contract. The value $\alpha^* \in [0, a]$ that maximizes informational rents is, therefore, important to the analysis. In this setting it is $\alpha^* = a/(1 + c)$ that is increasing in a and decreasing in c. Therefore, for high market demand levels (i.e., for a sufficiently high a), that value is greater than 1 for any c and the owner will induce the manager to be more aggressive than in the ex ante symmetric information setting: $\alpha = 1$ is not optimal for the owner under adverse selection because a slightly lower value implies not only a second-order decrease in the owner's gross profits, but also a first-order decrease in the manager's informational rents. In markets with low demand levels, α^* may be lower than 1 for high realizations of the marginal cost. This suggests that if the market demand level is sufficiently low, the expectation of the marginal cost is high enough and its variance is low enough, the owner may pay the manager to keep sales low. These assertions are stated in the following proposition:

Proposition 1 *In the monopoly under adverse selection $(a > 1)$, the optimal contract satisfies the following properties:*

(1) The optimal value of α satisfies:

    (a) $\alpha_{AS}^{M} = \frac{d - a(1-e)}{2d-1} \in (0, a)$ if $d > a(1 \Leftrightarrow e)$ and $d > \frac{ae}{2a-1}$.

    (b) $\alpha_{AS}^{M} = 0$ if $d \leq a(1 \Leftrightarrow e)$ and $d > \frac{a(2e-1)}{2(a-1)}$.

    (c) $\alpha_{AS}^{M} = a$ if $d \leq \frac{ae}{2a-1}$ and $d < \frac{a(2e-1)}{2(a-1)}$.

    (d) If $d = \frac{ae}{2a-1}$ and $e \leq 1 \Leftrightarrow \frac{1}{2a}$ then, indifferently, the owner optimally chooses $\alpha_{AS}^{M} = 0$ or $\alpha_{AS}^{M} = a$.

(2) If $a \geq 2$ then $\alpha_{AS}^{M} < 1$.

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(3) In the case (1a), $\alpha_{AS}^M > 1$ if and only if $d < 1 \Leftrightarrow a(1 \Leftrightarrow e)$, when $a < 2$.

Proof: See Appendix.

Note that, when $a \geq 1 + \frac{1}{\sqrt{2}}$, the optimal value of α in the monopoly under adverse selection becomes $\alpha_{AS}^M = \frac{d - a(1-e)}{2d-1} \in (0, a)$ if $d > a(1 \Leftrightarrow e)$, otherwise $\alpha_{AS}^M = 0$. Figure 2 represents the relevant regions for α_{AS}^M, in the expectation–variance space when a is equal to any value between $1 + \frac{1}{\sqrt{2}}$ and 2 (note that $d = v + e^2$). It is obvious that the owner optimally pays the manager to keep
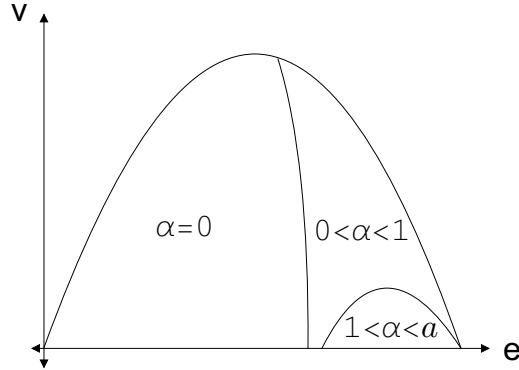
Figure 2: α_{AS}^M for $1 + \frac{1}{\sqrt{2}} \leq a < 2$.

sales low, for intermediate market demand levels, only if the expectation of the marginal cost is high enough and its variance is low enough. The same conclusion holds for low market demand levels $(a < 1 + \frac{1}{\sqrt{2}})$. Figure 3 represents the relevant regions for $1 < a < 1 + \frac{1}{\sqrt{2}}$.
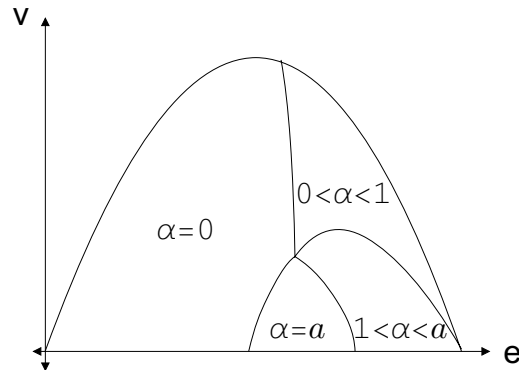

Figure 3: α_{AS}^M for $1 < a < 1 + \frac{1}{\sqrt{2}}$.

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3 The competitive framework

In this section we consider a competitive framework with two firms in a homogeneous product market. In the *managerial* firm (A), the output decision is made by a manager, as in the previous section. To simplify the analysis, we suppose that the other firm (B) is an *entrepreneurial* firm, i.e., a standard profit-maximizing firm. Both firms have identical constant marginal costs.

The timing of the game is now as follows. At Stage 1, the owner of firm A chooses, as before, the contract $T = \alpha\pi + (1-\alpha)S - h$ as proposed to the manager and this is publicly announced. At Stage 2, the manager accepts or rejects the contract. Finally, at Stage 3, if the manager has accepted the contract, firm B and the manager decide simultaneously on their respective outputs. We assume that the owner always proposes a contract that the manager will accept.

As in the previous section, we assume that, in the act of contracting, the owner of firm A is ignorant of the actual value of the marginal cost $C = c$. We assume $c \in [0, 1]$, and demand is given by $P(Q) = a - Q$ with $a > 1$. As before, the owner of firm A knows only that C is distributed such that $E(C) = e$ and $E(C^2) = d$, with $(e, d) \in \mathcal{P}$. In the last stage, both the manager and firm B know the actual value of c before deciding on their productions. To guarantee positive outputs for any c, in the third stage we assume $\alpha \in [2 - a, (1 + a)/2]$. Given a contract, it is easy to verify that the quantities, profits and payoffs in the third stage of the game are:

$$q_A(c, \alpha) = \frac{1}{3}(a + c - 2c\alpha),$$

$$q_B(c, \alpha) = \frac{1}{3}(a - 2c + c\alpha),$$

$$\pi_A(c, \alpha) = \frac{1}{9}(a - 2c + c\alpha)(a + c - 2c\alpha),$$

$$\pi_B(\alpha, c) = \frac{1}{9}(a - 2c + c\alpha)^2,$$

$$T_C(c, \alpha, h) = \frac{1}{9}(a + c - 2c\alpha)^2 - h,$$

$$U_A(c, \alpha, h) = \pi_A(c, \alpha) - T_C(c, \alpha, h) = \frac{1}{3}c(\alpha - 1)(a + c - 2c\alpha) + h.$$

As in the previous section, we consider two different classes of information structures. In the ex ante symmetric information setting, the manager realizes the actual value of c after the time of contracting, and in the adverse selection setting, the manager knows the actual value of c before the contract is signed.

Under complete information (i.e., when c is verifiable), the owner extracts all the manager's surplus by setting the value of h such that $T_C(c, \alpha, h) = 0$ for any c and any α. The optimal value of α must, therefore, maximize $\pi_A(c, \alpha)$ for each c. It is easy to show that this optimal value is $\alpha = \max(2 - a, \frac{5}{4} - \frac{a}{4c})$, which is always lower than 1. The intuition of this result agrees with the work of Sklivas (1987), and Fershtman and Judd (1987). By inducing the manager to be more aggressive (i.e., decreasing α with respect to 1), the owner of firm A pushes the manager's reaction function out at the last stage of the game and, therefore, the output of firm A increases and the output of firm B decreases. This strategic behavior leads to an increase in the net profits of firm A. This result also holds in the setting with ex ante symmetric information.

3.1 The competitive setting under ex ante symmetric information

As before, under ex ante symmetric information, the owner of firm A can extract all the surplus in his relationship with the manager, if the owner sets h such that $E[T_C(c, \alpha, h)] = 0$ for any α. The optimal contract must, therefore, maximize

$$E[\pi_A(c, \alpha)] = \frac{1}{9}[a^2 - ae(1 + \alpha) + d(-2 + 5\alpha - 2\alpha^2)].$$

It easy to show that this function is strictly concave and has a unique maximum point at $\alpha = \frac{5}{4} - \frac{ae}{4d}$ for any $(e, d) \in \mathcal{P}$. In the competitive setting under ex ante symmetric information, therefore, the optimal value of α is

$$\alpha_{SI}^C = \max\left(2 - a, \frac{5}{4} - \frac{ae}{4d}\right),$$

which is always lower than 1. Note that, depending on the probability distribution of costs, this strategic effect may be so strong that α_{SI}^C becomes equal to $2 - a$.

3.2 The competitive setting under adverse selection

Under adverse selection, the actual value of c is the manager's private information at the time of contracting. So, the manager will only accept the contract if $T_C(c, \alpha, h) \geq 0$ holds for any $c \in [0, 1]$. The owner will, thus, solve the following program:

$$\begin{cases} \max_{\alpha, h} E[U_A(c, \alpha, h)] \\ \text{s.t.} \\ T_C(c, \alpha, h) \geq 0, \quad \forall c \in [0, 1]. \end{cases} \tag{5}$$

Program (5) is the reduced form of an adverse selection problem. Given the contract (α, h), in this competitive setting, the function $q_A(\cdot, \alpha)$ can be considered as the only incentive compatible action profile that corresponds to the agent's utility function

$$V_C(x, c; \alpha, h) =$$
$$= \alpha(P(x + q_B(c, \alpha))x - cx) + (1 - \alpha)P(x + q_B(c, \alpha))x - h =$$
$$= P(x + q_B(c, \alpha))x - c\alpha x - h,$$

where x and c are, respectively, the action and the manager's type, and $q_B(c, \alpha)$ is the equilibrium output of firm B.

In this setting, the Spence-Mirrlees condition holds, since $\partial_{xc}V_C = -2(2\alpha - 1)/3$, and the monotonicity of both $q_A(\cdot, \alpha)$ and the manager's payoff T_C depends on the sign of $\alpha - 1/2$. Therefore, under competition, the threshold is $\alpha = 1/2$. The intuition here is as follows: The threshold under which the manager will obtain no rent cannot be $\alpha = 0$ as it is in the monopolistic framework. When the contract and the manager's reaction function at Stage 3 are independent of costs ($\alpha = 0$), the manager obtains informational rents because his equilibrium payoff and his output decision at Stage 3 depend on costs via the reaction function of firm B, which always depends on costs. Only if $\alpha = 1/2$, the manager's reaction function depends on cost in such a way that, in the corresponding equilibrium, the manager's decision and his payoff are independent of costs. When $\alpha = 1/2$, the manager obtains the constant rent $a^2/9 - h$. When $\alpha > 1/2$, the function T_C is decreasing in c and its minimal value is $(1 + a - 2\alpha)^2 - h$, which is reached at $c = 1$. However, when $\alpha < 1/2$, the manager's payoff T_C is increasing in c and its minimal value is $a^2/9 - h$, which is reached at $c = 0$. For any $\alpha \in [2 - a, (1 + a)/2]$, therefore, the owner optimally sets $h = h_C(\alpha)$ where

$$h_C(\alpha) = \begin{cases} a^2/9 & \text{if} \quad \alpha \leq 1/2, \\ (1 + a - 2\alpha)^2/9 & \text{if} \quad 1/2 \leq \alpha. \end{cases} \quad (6)$$

Note that if $a > 3/2$ then $1/2 < 2 - a$ and $h_C(\alpha) = (1 + a - 2\alpha)^2/9$ for any $\alpha \in [2 - a, (1 + a)/2]$. When $a \leq 3/2$, the function $h_C(\cdot)$ consists of the two sides in expression (6). In this competitive framework, therefore, the manager's informational rent is

$$T_C(c, \alpha, h_C(\alpha)) = (a + c - 2c\alpha)^2 - h_C(\alpha). \quad (7)$$

From (7), program (5) is equivalent to maximize, on $\alpha \in [2 - a, (1 + a)/2]$, the following function:

$$E[U_A(c, \alpha, h_C(\alpha))] = E[\pi_A(c, \alpha)] - E[T_C(c, \alpha, h_C(\alpha))] = \quad (8)$$
$$= \frac{1}{3}(\alpha - 1)(ae - d(2\alpha - 1)) + h_C(\alpha).$$

As in the case of the monopolistic framework, expression (7) follows the standard properties of the agent's utility under an incentive compatible and individually rational contract. But now the properties depend on α being relative to a higher threshold (1/2 instead of 0).

In this competitive framework with $a > 3/2$, the optimal value of α is always greater than, or equal to, the threshold of 1/2. The explanation for this is similar to that for the monopolistic framework. A value of α that is lower than 1/2 ranks the manager's types in the opposite direction to that indicated by the manager's efficiency, which implies that the manager obtains too much informational rents relative to the owner's expected gross profit. When $a < 3/2$, it is obvious that the optimal value of α is greater than 1/2, since, in this case, we assume $\alpha \in [2 \Leftrightarrow a, (1 + a)/2]$ to guarantee positive outputs at Stage 3, and $2 \Leftrightarrow a > 1/2$.

The second property is that, as in the case of the monopolistic framework, the optimal value of α may be higher or lower than 1 in this competitive setting. Several explanations must now be added, due to the presence of competition.

First, the previous section shows that, if the market demand level is sufficiently high, a monopolistic owner should make his manager more aggressive to decrease the manager's informational rents. Moreover, in the competitive framework under ex ante symmetric information, the owner strategically decreases α to obtain more profits. In the competitive adverse selection setting, therefore, the owner should decrease α as a consequence of both the strategic effect and the informational rent effect, when the demand level is high enough.

Secondly, when the expectation of the marginal cost is close to 0 or 1 (its variance will be close to 0 in these cases) the informational rent effect is very weak relative to the strategic effect that leads to an aggressive manager. This suggests that if the expected marginal cost is sufficiently close to 0 or 1, the owner should induce the manager to be aggressive even under adverse selection.

Thirdly, for intermediate values of the expected marginal cost, and if the market demand level is low enough, the situation resembles that of the monopolistic setting. The value of α that maximizes the expected informational rent is now important in this analysis. As suggested by the monopolistic framework, a low variance of the marginal cost may imply that the informational rent effect leads to increasing α. On the other hand, a low market demand level implies that the strategic effect is insignificant. These arguments suggest that the owner may pay the manager to keep sales low, if both the market demand level and the variance of marginal cost are sufficiently low, and if the value of the expected marginal cost is intermediate. The

following proposition formalizes these ideas.

Proposition 2 *In the competitive setting under adverse selection ($a > 1$), the optimal contract satisfies the following properties:*

(1) *When $1 < a \leq 3/2$:*

    (a) $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-8} \in \left(2 \Leftrightarrow a, \frac{1+a}{2}\right)$ *if* $d > \frac{ae}{2a-1}$ *and* $e > \frac{4(a-1)+(5-4a)d}{a}$.

    (b) $\alpha_{AS}^C = 2 \Leftrightarrow a$ *if* $e \leq \frac{4(a-1)+(5-4a)d}{a}$ *and* $d > \frac{ae-2(a-1)}{2-a}$.

    (c) $\alpha_{AS}^C = \frac{1+a}{2}$ *if* $d \leq \frac{ae}{2a-1}$ *and* $d < \frac{ae-2(a-1)}{2-a}$.

    (d) *If* $d = \frac{ae-2(a-1)}{2-a}$ *and* $e \leq \frac{4a-2}{3-a}$*, the owner of firm A indifferently chooses* $\alpha_{AS}^C = 2 \Leftrightarrow a$ *or* $\alpha_{AS}^C = \frac{1+a}{2}$.

(2) *When $a > 3/2$: $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-8} \in \left(1/2, \frac{1+a}{2}\right)$ if $d > a(4 \Leftrightarrow 3e)/3$ and $\alpha_{AS}^C = 1/2$ otherwise.*

(3) *When $a \geq (4 + \sqrt{6})/5$: $\alpha_{AS}^C < 1$.*

(4) *When $1 < a < (4 + \sqrt{6})/5$: In the case (1a), $\alpha_{AS}^C > 1$ if and only if $d < \frac{4}{3} + a\left(e \Leftrightarrow \frac{4}{3}\right)$.*

Proof: See Appendix.

Note that, when $(4 + \sqrt{6})/5 \leq a \leq 3/2$, the optimal contract satisfies $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-8} \in \left(2 \Leftrightarrow a, \frac{1+a}{2}\right)$ if $d > \frac{4+a(e-4)}{5-4a}$ and $\alpha_{AS}^C = 2 \Leftrightarrow a$ otherwise. Figure 4 represents the relevant regions for this case, in which we have $\alpha_{AS}^C < 1$. This conclusion holds also when $a > 3/2$ (see Figure 5). This implies that,
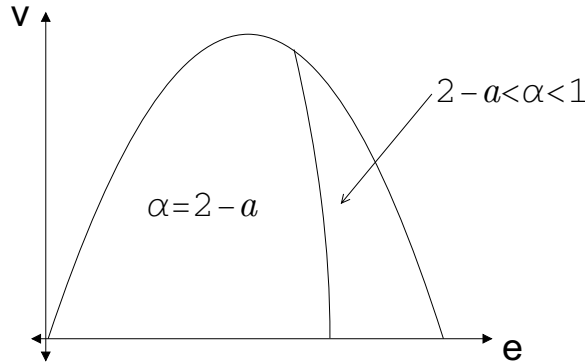

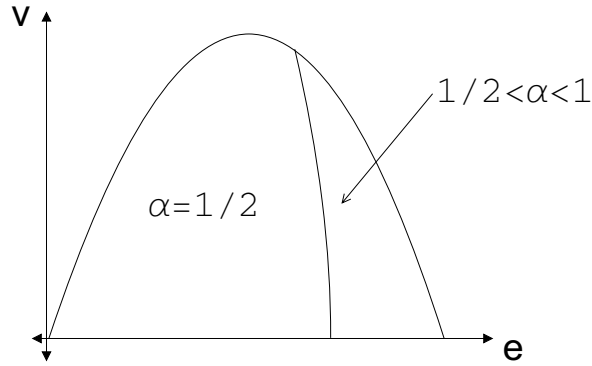Figure 4: α_{AS}^C for $(4 + \sqrt{6})/5 \leq a \leq 3/2$.

Figure 5: α_{AS}^C for $a > 3/2$.

for a market demand level that is sufficiently high, the owner of firm A induces the manager to be aggressive, as under ex ante symmetric information.

Nevertheless, if the market demand level is sufficiently low, the owner of firm A may optimally pay the manager to keep sales low. Figure 6 represents the relevant regions for $1 < a < (4 + \sqrt{6})/5$. We observe that this happens when the variance of the marginal cost is sufficiently low and its expectation belongs to an intermediate region in $[0, 1]$.
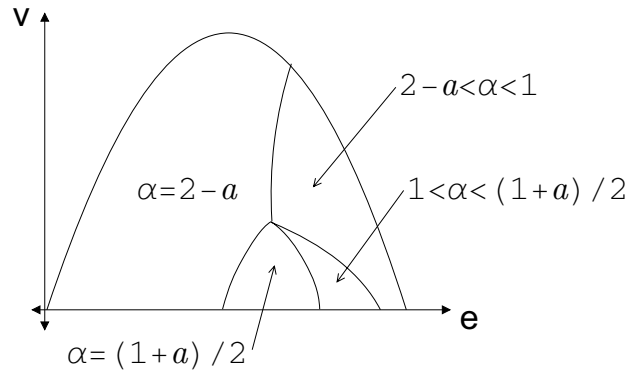
Figure 6: α_{AS}^C for $1 < a < (4 + \sqrt{6})/5$.

4   The comparison of settings

In this section we compare the optimal levels of the manager's aggressiveness, which depends inversely on the value of α, regarding the level of competition and the class of information structure considered. First, for the competitive

framework, we compare the manager's aggressiveness under ex ante symmetric information with the corresponding one under adverse selection. Secondly, we consider the adverse selection setting and we compare his aggressiveness in the monopolistic and the competitive frameworks.

Section 2 shows that in the monopolistic framework, the manager is more aggressive under adverse selection than he is under ex ante symmetric information ($\alpha_{AS}^M < 1 = \alpha_{SI}^M$) if the market demand level is high enough. This conclusion does not hold in the competitive framework, since, on the one hand, competition implies an optimal value of α that is higher than the threshold $1/2$ in the adverse selection setting, and on the other hand, under ex ante symmetric information a high strategic effect may imply an optimal value of α that is lower than the threshold $1/2$. The following proposition, therefore, shows that the interaction between the strategic effect and the informational rent effect has a non-additive nature.

Proposition 3 *Let α_{SI}^C and α_{AS}^C be, respectively, the optimal values of α under ex ante symmetric information and under adverse selection in the competitive framework. We then have:*

(a) If $3/2 < a < 3$ then $\alpha_{SI}^C \gtrless \alpha_{AS}^C \iff d \gtrless ae/3$.

(b) If $a \geq 3$ then $\alpha_{SI}^C < \alpha_{AS}^C$.

Proof: See Appendix.

Part (b) of the previous proposition show that, for markets with sufficiently high demand levels, the manager is always more aggressive under ex ante symmetric information than he is under adverse selection in the competitive framework. From $d = v + e^2$, the previous proposition shows that, in the competitive framework with a moderate level of market demand, the manager is more aggressive under ex ante symmetric information than he is in under adverse selection only for sufficiently low expected costs and variances (see Figure 7).

Previous sections have shown that, in the setting with ex ante symmetric information, the manager is always more aggressive under competition than he is under monopoly ($\alpha_{SI}^C < 1 = \alpha_{SI}^M$) due to a strategic effect. This conclusion does not hold in the adverse selection setting. The reason is the existence of a non-additive interaction between the strategic effect and the informational rent effect. The following proposition shows that, in the adverse selection setting, the manager is more (resp. less) aggressive under competition than he is under monopoly, if and only if the expected cost is higher (resp. lower) than $1 \iff \frac{1}{2a}$.
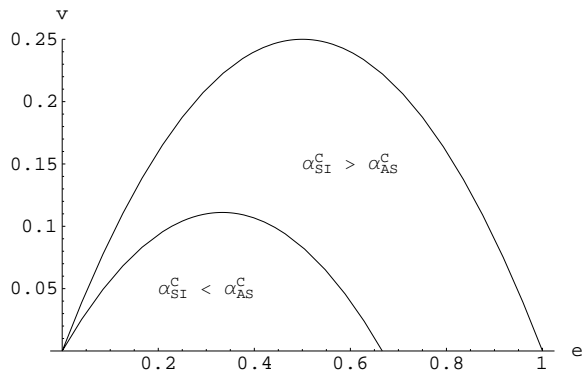
Figure 7: Optimal values of α in the competitive framework with $a = 2$.

Proposition 4 *Let α_{AS}^M and α_{AS}^C be, respectively, the optimal values of α in the monopolistic and competitive frameworks under adverse selection with $a \geq 1 + \frac{1}{\sqrt{2}}$. Then, it follows*

$$\alpha_{AS}^M \gtrless \alpha_{AS}^C \quad \Leftrightarrow \quad e \gtrless 1 \Leftrightarrow \frac{1}{2a}.$$

Proof: See Appendix.

5   Conclusions

In this paper, we have analyzed managerial contracts in the presence of monopoly or competition and assuming ex ante symmetric information or adverse selection relative to the marginal cost. It is assumed that, at the time of contracting a manager, the owner of a firm is ignorant of the actual value of the constant marginal cost. The marginal cost is observed by managers before making production decisions. As empirical evidence confirms that managerial contracts are linked to both profits and sales, it is assumed that managerial compensation is a linear combination of these two factors.

We consider two different classes of information structures. In the setting with ex ante symmetric information, we assume that the manager realizes the actual value of the marginal cost after the contract is signed and, in the setting under adverse selection, that he knows this marginal cost before the contract is signed. We also consider two frameworks that differ in the competitive level of the market. In the monopolistic framework, only one managerial firm is assumed (i.e., a firm with an owner that proposes a contract to a manager). In the competitive framework, to simplify the analysis, we suppose a duopoly, formed by one managerial firm and one entrepreneurial firm (i.e.,

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profit-maximizing firm) with identical constant marginal costs. Moreover, we assume a linear inverse demand function and quantity competition, and we consider a general continuous probabilistic distribution for the marginal cost.

Just like several other models with complete information or ex ante symmetric information presented in the literature, the present model exhibits the presence of strategic effects: A firm makes its manager more aggressive (i.e., increases the weight of sales against that of profits in the managerial contract) to push the manager's reaction function out since this increases the firm's net payoff. The present model, however, shows the existence of an informational rent effect.

We prove that, in the monopolistic adverse selection setting, the owner makes the manager more aggressive to decrease the manager's expected informational rents when the market demand level is high enough. If it is not, however, the owner will pay the manager to keep sales low if the expected marginal cost is high enough and the variance of the marginal cost is low enough. This is a consequence of the aggressiveness level that maximizes the manager's informational rent compared to the optimal one under ex ante symmetric information.

In the monopolistic adverse selection setting, the optimal aggressiveness level, which is maximal, relative to the marginal cost distributions, implies no incentives for profits. Competition, however, pushes this aggressiveness level upwards and several conclusions must be added. First, in the competitive adverse selection setting, the owner should increase the aggressiveness level as a consequence of both, the strategic effect and the informational rent effect, when the market demand level is high enough. Secondly, when the expectation of the marginal cost is low enough or high enough (its variance will be close to zero in such cases) the informational rent effect is very weak relative to the strategic effect, and the owner should induce the manager to be aggressive even under adverse selection. Thirdly, for intermediate values of the expected marginal cost and if the market demand level is low enough, the situation resembles that of the monopolistic setting, and the owner will pay the manager to keep sales low if the variance of the marginal cost is sufficiently low.

Finally, we show that the interactions between the strategic and the informational rent effects have a non-additive nature. We first prove that, unlike the monopolistic framework, the manager may become more aggressive in the competitive setting, under ex ante symmetric information, than under adverse selection, if the market demand level is high enough or when the expectation and the variance of the marginal cost are sufficiently low. We then show that, unlike the setting with ex ante symmetric information,

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the manager may become more aggressive in the adverse selection setting under monopoly than under competition, if the expected marginal cost is sufficiently low.

A Proof of Proposition 1

For $\alpha \leq 0$, consider $U_1(\alpha) = E[\pi(c,\alpha) - T_M(c,\alpha,h_M(\alpha))]$. Simple calculations show that
$$U_1(\alpha) = \frac{1}{2}(\alpha - 1)(ae - d\alpha) + \frac{a^2}{4}.$$
It follows that $U_1(\cdot)$ is strictly concave with a maximum point at $\alpha = \frac{d+ae}{2d}$ for any $(e,d) \in \mathcal{P}$. The maximum of $U_1(\cdot)$ on $(-\infty, 0]$ is, therefore, reached at $\alpha = 0$ with a value equal to $U_1(0) = a(a-2e)/4$.

For $\alpha \in [0, a]$ consider $U_2(\alpha) = E[\pi(c,\alpha) - T_M(c,\alpha,h_M(\alpha))]$. We now have
$$U_2(\alpha) = \frac{1}{2}(\alpha - 1)(ae - d\alpha) + \frac{1}{4}(a - \alpha)^2.$$
Simple calculations show that

$$U_2''(\alpha) = \frac{1}{2} - d,$$

$$U_2'(\alpha) = 0 \Leftrightarrow \alpha = \tilde{\alpha} = \frac{d - a(1 - e)}{2d - 1}.$$

Moreover, we have $a - \tilde{\alpha} = \frac{(2a-1)d - ae}{2d-1}$. As we assume $a > 1$, to obtain the maximum of $U_2(\cdot)$ given $(e,d) \in \mathcal{P}$ we can divide the analysis into several cases considering the functions $d_0(e) = \frac{a(2e-1)}{2(a-1)}$, $d_1(e) = a(1-e)$ and $d_2(e) = \frac{ae}{2a-1}$. Some cases will be impossible and some conditions will be superfluous for $a \geq 1 + \frac{1}{\sqrt{2}}$. Figures 8 and 9 represent, respectively, the relevant cases for $1 < a < 1 + \frac{1}{\sqrt{2}}$ and $a \geq 1 + \frac{1}{\sqrt{2}}$.

Case 1: $d > d_1(e)$, $d > d_2(e)$ (the last condition is superfluous if $a \geq 1 + \frac{1}{\sqrt{2}}$). It follows $U_2''(\cdot) < 0$ and $\tilde{\alpha} \in (0, a)$. The maximum of $U_2(\cdot)$ on $[0, a]$ is, therefore, reached at $\alpha = \tilde{\alpha}$ and
$$U_2(\tilde{\alpha}) = \frac{d(2(a-1)a + d) - 2a(a + d - 1)e + a^2e^2}{8d - 4}.$$

Case 2: $1/2 < d \leq d_1(e)$. We then have $U_2''(\cdot) < 0$ and $\tilde{\alpha} \leq 0$. Thus, the maximum of $U_2(\cdot)$ is reached at $\alpha = 0$, with $U_2(0) = a(a-2e)/4$.

Case 3: $1/2 < d \leq d_2(e)$ (impossible when $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, $U_2''(\cdot) < 0$ and $a \leq \tilde{\alpha}$ hold. Thus, the maximum of $U_2(\cdot)$ is reached at $\alpha = a$ with $U_2(a) = a(a-1)(e-d)/2$.
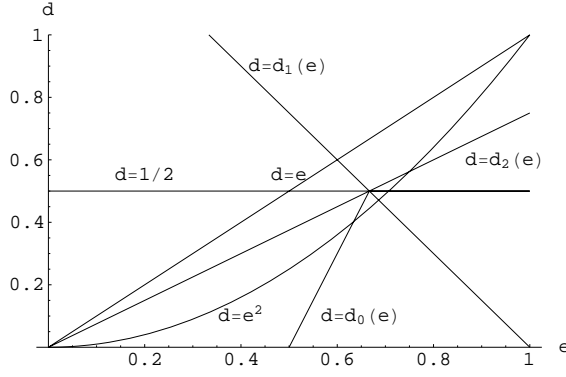
Figure 8: Cases for Proposition 1 with $a = 1.5$.
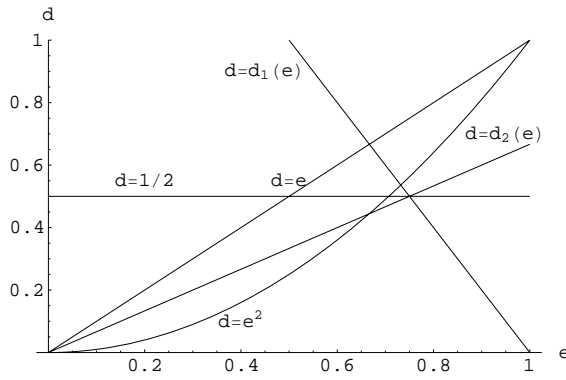

Figure 9: Cases for Proposition 1 with $a = 2$.

Case 4: $d = 1/2$. It follows $U_2''(\cdot) = 0$ and $U_2'(\alpha) = (1 + 2a(e - 1))/4$. Since $d = d_1(e)$ intersects $d = d_2(e)$ at $d = 1/2$, $e = 1 - \frac{a}{2a}$, we can divide this case into three occurrences.

(4.1) $e < 1 - \frac{1}{2a}$ (this always holds if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, $U_2'(\cdot) < 0$ and the maximum of $U_2(\cdot)$ is reached at $\alpha = 0$.

(4.2) $e = 1 - \frac{1}{2a}$ (impossible if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, $U_2(\cdot)$ is constantly equal to $(a - 1)^2/4$.

(4.3) $e > 1 - \frac{1}{2a}$ (impossible if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, $U_2'(\cdot) > 0$ and the maximum of $U_2(\cdot)$ is reached at $\alpha = a$.

Case 5: $d_2(e) \leq d < 1/2$. It follows $U_2''(\cdot) > 0$ and $\tilde{\alpha} \geq a$ and, in this case, the maximum is reached at $\alpha = 0$.

Case 6: $d < d_1(e)$, $d < d_2(e)$. Here, we have $\tilde{\alpha} \in (0, a)$ and $U_2''(\cdot) > 0$. The maximum is, therefore, reached at $\alpha = 0$ or at $\alpha = a$. Simple computations show that $U_2(0) - U_2(a) = a(a - 2d + 2ad - 2ae)/4$, which is positive if and only if $d > d_0(e)$. Since $d = d_0(e)$ intersects $d = d_1(e)$ (and

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$d = d_2(e)$) at (e, d) with $d = 1/2$ and $e = 1 - \frac{1}{2a}$, and, moreover, the slope of $d = d_0(e)$ is steeper than that of $d = d_2(e)$, we can divide this case into three occurrences.

   (6.1) $d < d_0(e)$ (impossible if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, the maximum of $U_2(\cdot)$ is reached at $\alpha = a$.

   (6.2) $d = d_0(e)$ (impossible if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, the maximum of $U_2(\cdot)$ is indifferently reached at $\alpha = 0$ or $\alpha = a$.

   (6.3) $d > d_0(e)$ (unique possibility if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, the maximum of $U_2(\cdot)$ is reached at $\alpha = 0$.

   Case 7: $d_1(e) \leq d < 1/2$ (impossible if $a \geq 1 + \frac{1}{\sqrt{2}}$). Here, $U_2''(\cdot) > 0$ and $\tilde{\alpha} \leq 0$ hold. The maximum of $U_2(\cdot)$ is reached at $\alpha = a$ in this case.

   Since $U_1(0) = U_2(0)$, the previous cases show that the optimal value of α, for $(e, d) \in \mathcal{P}$, is given by part (1) of proposition 1.

   Consider now the previous Case 1. It follows that $1 \gtrless \alpha_{AS}^M$ if and only if

$d \gtrless 1 - a(1 - e)$. If $a \geq 2$ the slope of $d = 1 - a(1 - e)$ at $e = 1$ is $a \geq 2$ while the slope of $d = e^2$ is 2. Therefore, $a \geq 2$ implies $d > 1 - a(1 - e)$ at any $(e, d) \in \mathcal{P}$ and $\alpha_{AS}^M < 1$. This proves part (2).

   Finally, $a < 2$ implies that $d = 1 - e(1 - e)$ intersects $d = e^2$ and part (3) holds. **Q.E.D.**

B   Proof of Proposition 2

Part (1). Assume $1 < a \leq 3/2$. It follows $1/2 \leq 2 - a$ and the owner's expected payoff given by (8) is

$$B_2(\alpha) = \frac{1}{3}(\alpha - 1)\big(ae - d(2\alpha - 1)\big) + \frac{1}{9}(1 + a - 2\alpha)^2,$$

for $\alpha \in [2 - a, (1 + a)/2]$. It follows

$$B_2''(\alpha) = \frac{-4\,(-2 + 3d)}{9},$$

and $B_2'(\alpha) = 0$ if and only if $\alpha = \hat{\alpha}$ where

$$\hat{\alpha} = \frac{9d - 4 - a(4 - 3e)}{12d - 8}.$$

Moreover, we have $\hat{\alpha} - (2 - a) = \frac{3(ae - 4(a-1) - (5 - 4a)d)}{4(3d-2)}$ and $\frac{1+a}{2} - \hat{\alpha} = \frac{3(d(2a-1) - ae)}{4(3d-2)}$. To obtain the maximum of $B_2(\cdot)$ in this case, we divide the analysis into

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several cases, considering the functions $e_1(d) = \frac{4(a-1)+(5-4a)d}{a}$, $d_2(e) = \frac{ae}{2a-1}$ and $d_3(e) = \frac{ae-2(a-1)}{2-a}$. Some cases will be impossible and some conditions will be superfluous when $a \geq (4 + \sqrt{6})/5$. Figures 10 and 11 represent, respectively, the relevant cases for $1 < a < (4+\sqrt{6})/5$ and $(4+\sqrt{6})/5 \leq a \leq 3/2$.
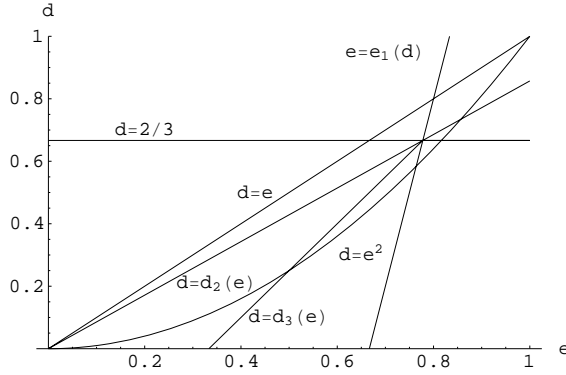

Figure 10: Cases for Proposition 2 with $a = 1.2$.
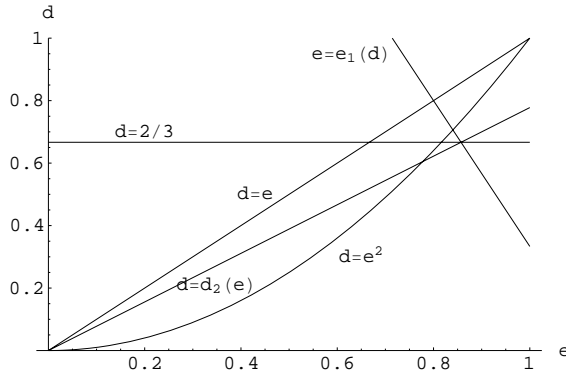
Figure 11: Cases for Proposition 2 with $a = 1.4$.

Case 1: $d > d_2(e)$ and $e > e_1(d)$ (the first condition is superfluous if $(4+\sqrt{6})/5 \leq a$). Here, we have $2 \Leftrightarrow a < \hat{\alpha} < (1+a)/2$ and $B_2''(\cdot) < 0$ because $d > 2/3$. Therefore, the maximum of $B_2(\cdot)$ is reached at $\alpha = \hat{\alpha}$.

Case 2: $2/3 < d$ and $e \leq e_1(d)$. Now $B_2''(\cdot) < 0$ but $\hat{\alpha} \leq 2 \Leftrightarrow a$. The maximum is reached at $\alpha = 2 \Leftrightarrow a$.

Case 3: $2/3 < d \leq d_2(e)$ (impossible if $a \geq (4 + \sqrt{6})/5$). We have $B_2''(\cdot) < 0$ and, since $e > e_1(d)$ holds in this case, it follows that $\hat{\alpha} \geq (1+a)/2$. The maximum of $B_2(\cdot)$ is, therefore, reached at $\alpha = (1 + a)/2$.

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Case 4: $d = 2/3$. In this case $B_2(\cdot)$ is linear and $B_2'(\alpha) = (2 + a(3e - 4))/9$, which is positive if and only if $e > \frac{4}{3} - \frac{2}{3a}$. We divide this case into several occurrences.

(4.1) $e < \frac{4}{3} - \frac{2}{3a}$ (unique possibility if $a \geq (4 + \sqrt{6})/5$). This implies that the maximum of $B_2(\cdot)$ is reached at $\alpha = 2 - a$.

(4.2) $e = \frac{4}{3} - \frac{2}{3a}$ (impossible if $a \geq (4 + \sqrt{6})/5$). Here, the maximum is reached at any value between $2 - a$ and $(1 + a)/2$.

(4.3) $e > \frac{4}{3} - \frac{2}{3a}$ (impossible if $a \geq (4 + \sqrt{6})/5$). The maximum of $B_2(\cdot)$ is reached at $\alpha = (1 + a)/2$.

Case 5: $d_2(e) \leq d < 2/3$. Here, $B_2''(\cdot) > 0$ and $\hat{\alpha} \geq (1 + a)/2$. The maximum is reached at $\alpha = 2 - a$ in this case.

Case 6: $d < d_2(e)$ and $e < e_1(d)$. We now have $B_2''(\cdot) > 0$ and $2 - a < \hat{\alpha} < (1 + a)/2$. The maximum is $\alpha = 2 - a$ or $\alpha = (1 + a)/2$. Simple computations show that

$$B_2(2 - a) - B_2(\frac{1 + a}{2}) = \frac{1}{2}(a - 1)(2d - 2 + a(2 - d - e)),$$

which is positive if and only if $d > d_3(e)$. On the other hand, the straight lines $d = d_2(e)$, $e = e_1(d)$ and $d = d_3(e)$ intersect at $e = \frac{4}{3} - \frac{2}{3a}$, $d = 2/3$. Moreover, the slope of $e = e_1(d)$ is higher than the slope of $d = d_3(e)$ that, in turn, is higher than the slope of $d = d_2(e)$. Therefore, we can divide this case into several occurrences.

(6.1) $d < d_3(e)$ (impossible if $a \geq (4 + \sqrt{6})/5$). Here, the maximum of $B_2(\cdot)$ is reached at $\alpha = (1 + a)/2$.

(6.2) $d = d_3(e)$ (impossible if $a \geq (4 + \sqrt{6})/5$). Here, the maximum of $B_2(\cdot)$ is indifferently reached at $\alpha = (1 + a)/2$ or $\alpha = 2 - a$.

(6.3) $d > d_3(e)$ (unique possibility if $a \geq (4 + \sqrt{6})/5$). Here, the maximum of $B_2(\cdot)$ is reached at $\alpha = 2 - a$.

Case 7: $d < 2/3$ and $e \geq e_1(d)$ (impossible if $a \geq (4 + \sqrt{6})/5$). We now have $B_2''(\cdot) > 0$ and $\hat{\alpha} \leq 2 - a$. It follows that the maximum is reached at $\alpha = (1 + a)/2$ in this case.

These cases prove part (1) of the proposition.

Part (2). Assume $a > 3/2$. It follows $2 - a < 1/2 < \frac{1+a}{2}$.

For $\alpha \leq 1/2$ consider $B_1(\alpha) = E[\pi_A(c, \alpha) - T_C(c, \alpha, h(\alpha))]$. With some calculations, it follows:

$$B_1(\alpha) = \frac{1}{3}(\alpha - 1)(ae - d(2\alpha - 1)) + \frac{a^2}{9}.$$

We have $B_1''(\alpha) = -4d/3 < 0$ and $B_1'(\alpha) = 0$ if and only if $\alpha = \frac{3}{4} + \frac{ae}{4d}$. Therefore, the maximum of $B_1(\cdot)$ on $[2 - a, 1/2]$ is reached at $\alpha = 1/2$ with a maximal value $B_1(1/2) = a(2a - 3e)/18$.
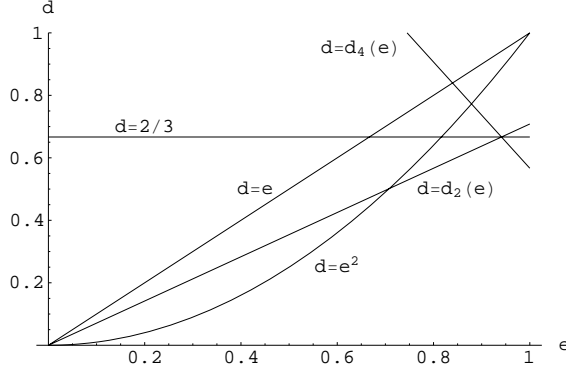
Figure 12: Cases for Proposition 2 with $a = 1.7$.

For $1/2 \leq \alpha \leq (1 + a)/2$ consider the function $B_2(\cdot)$ defined in part (1), but now for $\alpha \in [1/2, (1 + a)/2]$. In this case we have $\hat{\alpha} - 1/2 = \frac{3d - a(4 - 3e)}{4(3d - 2)}$ and $\frac{1+a}{2} - \hat{\alpha} = \frac{3(d(2a-1) - ae)}{4(3d-2)}$. To obtain the maximum of $B_2(\cdot)$ we can consider the following cases (see Figure 12) in which $d_2(e) = \frac{ae}{2a-1}$, $d_4(e) = a(4 - 3e)/3$ and, of course, $0 < e^2 < d < e < 1$.

Case 1: $d > d_4(e)$. Here, we have $B_2''(\cdot) < 0$ and $1/2 \leq \hat{\alpha} < (1 + a)/2$. Thus, the maximum of $B_2(\cdot)$ on $[1/2, (1 + a)/2]$ is reached at $\alpha = \hat{\alpha}$ with a maximal value equal to

$$B_2(\hat{\alpha}) = \frac{d(8(a - 1)a + 3d) - 2ae(4(a - 1) + 3d) + 3a^2 e^2}{24(3d - 2)}.$$

Case 2: $2/3 < d \leq d_4(e)$. It follows $B_2''(\cdot) < 0$ and $\hat{\alpha} \leq 1/2$. Therefore, the maximum of $B_2(\cdot)$ is reached at $\alpha = 1/2$ with a value $B_2(1/2) = a(2a - 3e)/18$.

Case 3: $d = 2/3$. In this case $B_2(\cdot)$ is linear and $B_2'(\alpha) = (2 + a(3e - 4))/9$. Since $2/3 < e < \sqrt{2/3}$ holds in this case, the previous expression is lower than $(2 + (\sqrt{6} - 4)a)/9 \leq 0$. Therefore, $B_2(\cdot)$ is decreasing, and the maximum is reached at $\alpha = 1/2$.

Case 4: $d_2(e) \leq d < 2/3$. Here, $U_2''(\cdot) > 0$ and $\hat{\alpha} \geq (1 + a)/2$. This implies that the maximum is reached at $\alpha = 1/2$.

Case 5: $d < d_2(e)$. Here, we have $B_2''(\cdot) > 0$ and $1/2 < \hat{\alpha} < (1 + a)/2$. It follows that the maximum is reached at $\alpha = 1/2$ or at $\alpha = (1 + a)/2$. In this case we have:

$$B_2(1/2) - B_2(\frac{1 + a}{2}) = a(- 3d + a(2 + 3d - 3e))/18,$$

which is positive if and only if $- 3d + a(2 + 3d - 3e) > 0$. Since $d > e^2$, this expression is greater than $- 3e^2 + a(2 + 3(e - 1)e)$ which, in turn, is greater

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than $3(2-e)(1-e)/2 > 0$ because we assume $a > 3/2$. As a consequence, the maximum of $B_2(\cdot)$ on $[1/2, (1+a)/2]$ is reached at $\alpha = 1/2$ in this case.

These cases show that, for any $(e,d) \in \mathcal{P}$, the maximal value of $B_2(\cdot)$ on $[1/2, (1+a)/2]$ is $B_2(\hat{\alpha})$ if $d > d_4(e)$ and it is $B_2(1/2)$ if $d \le d_4(e)$.

Since $B_1(1/2) = B_2(1/2)$, the maximum of $E[\pi_A(c, \alpha) - T_C(c, \alpha, h(\alpha))]$ is reached at $\alpha = \hat{\alpha}$ if $d > d_4(e)$. It is reached at $\alpha = 1/2$ if $d \le d_4(e)$. This proves part (2).

Part (3). To show this part we consider two cases:

Case 1: $(4+\sqrt{6})/5 \le a \le 3/2$. From part (1), we have $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-8} \in (2-a, (1+a)/2)$ if $e > e_1(d)$, and $\alpha_{AS}^C = 2-a$ otherwise. If $e \le e_1(d)$ then $\alpha_{AS}^C = 2-a < 1$. If $e > e_1(d)$, we have $\alpha_{AC}^C < 1$ if and only if $d > d_5(e)$, where $d_5(e) = (4 + a(3e-4))/3$. The intersection of $e = e_1(d)$ with $d = d_5(e)$ is $e = \frac{4}{3} - \frac{2}{3a}$ and $d = 2/3$. Therefore, if there exists $(e,d) \in \mathcal{P}$ such that $e > e_1(d)$ and $d \le d_5(e)$, we have $e > \frac{4}{3} - \frac{2}{3a} \ge \sqrt{2/3}$. On the other hand, the expression $e^2 - d_5(e)$ is greater than or equal to $(4(\sqrt{6}-1)-3(4+\sqrt{6})e + 15e^2)/15$ because we suppose $a \ge (4+\sqrt{6})/5$. This last function is convex with a minimal value at $e = (4 + \sqrt{6})/10 < \sqrt{2/3}$. It follows that, for $e > \sqrt{2/3}$, the last function is positive. In consequence, if $e > e_1(d)$ and $d \le d_5(e)$ hold then $e^2 > d_5(e) \ge d$ which contradicts $(e,d) \in \mathcal{P}$. Thus, $e > e_1(d)$ implies $d > d_5(e)$ and $\alpha_{AC}^C < 1$.

Case 2: $a > 3/2$. From part (2), if $d \le d_4(e)$ we have $\alpha_{AS}^C = 1/2$. If $d > d_4(e)$, we have $\alpha_{AC}^C < 1$ if and only if $d > d_5(e)$. Since $a > 3/2$, it follows $e^2 - d_5(e) > \frac{2}{3} - \frac{3e}{2} + e^2 > 0$. Therefore, $d \le d_5(e)$ implies $d < e^2$. In this case, $d > d_5(e)$ holds for any $(e,d) \in \mathcal{P}$ and, consequently, $\alpha_{AS}^C < 1$.

This proves part (3).

Part (4). Assume $1 < a < (4 + \sqrt{6})/5$ and consider case 1 of part (1). As in case 1 of part (3), we have $\alpha_{AC}^C > 1$ if and only if $d < d_5(e)$, where $d_5(e) = (4 + a(3e-4))/3$. **Q.E.D.**

C   Proof of Proposition 3

Let α_{SI}^C and α_{AS}^C be the values described in Section 3. Note that $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d}$ if $d > \frac{ae}{4a-3}$ and $\alpha_{SI}^C = 2-a$ otherwise. On the other hand, $\frac{5}{4} - \frac{ae}{4d} > 1/2$ if and only if $d > ae/3$.

Part (a). Assume $3/2 < a < 3$. We divide the proof into several cases. Here, we have $\frac{ae}{4a-3} < \frac{ae}{3} < \frac{a(4-3e)}{3}$.

Case 1: $d > a(4-3e)/3$. Here, we have $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-8}$ and $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d}$. Therefore, $\alpha_{SI}^C > \alpha_{AS}^C$ if and only if $3d^2 + d(-3 + a(2-3e)) + ae > 0$.

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This expression is a function that decreases in e because $d > 2/3$ holds for the case considered. Since $e < \sqrt{d}$, it follows that the expression is greater than $3(d-1)d + a(\sqrt{d} + 2d - 3d^{3/2})$ which, in turn, is higher than $3(\sqrt{d} - 3d^{3/2} + 2d^2)/2 > 0$ because $a > 3/2$. In consequence, we have $\alpha_{SI}^C > \alpha_{AS}^C$ in this case.

 Case 2: $ae/3 < d \le a(4-3e)/3$. It follows $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d} > \alpha_{AS}^C = 1/2$.
 Case 3: $d = ae/3$. Here, $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d} = \alpha_{AS}^C = 1/2$ holds.
 Case 4: $\frac{ae}{4a-3} < d < ae/3$. Now, it follows $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d} < \alpha_{AS}^C = 1/2$.
 Case 5: $d \le \frac{ae}{4a-3}$. As $a > 3/2$, we have $\alpha_{SI}^C = 2 - a < \alpha_{AS}^C = 1/2$.
 This proves Part (a).

 Part (b). Suppose $a \ge 3$. In this case $a(4-3e)/3 > 1$ holds for any $(e,d) \in \mathcal{P}$. Therefore, we always have $\alpha_{AS}^C = 1/2$. On the other hand, it follows that $d < e \le ae/3$ and $\frac{5}{4} - \frac{ae}{4d} < 1/2$. We consider two cases.

 Case 1: $d > \frac{ae}{4a-e}$. It follows that $\alpha_{SI}^C = \frac{5}{4} - \frac{ae}{4d} > \alpha_{AS}^C = 1/2$.
 Case 2: $d \le \frac{ae}{4a-e}$. We have $\alpha_{SI}^C = 2 - a < \alpha_{AS}^C = 1/2$.
 This proves part (b). **Q.E.D.**

D Proof of Proposition 4

Let α_{AS}^M and α_{AS}^C be the values defined in Propositions 1 and 2. We divide the proof into several cases. Note that $1 + \frac{1}{\sqrt{2}} > 3/2$.

 Case 1: $1 + \frac{1}{\sqrt{2}} \le a < 2$. In this case, $1 \gtrless \alpha_{AS}^M$ if and only if $d \gtrless 1 - a(1-e)$. However we always have $\alpha_{AS}^C < 1$. We divide this case into several occurrences.

 (1.1) $d > a(4-3e)/3$ with $d \le 1 - a(1-e)$. Here, we have $\alpha_{AS}^M \ge 1 > \alpha_{AS}^C$.
 (1.2) $d > a(4-3e)/3$ with $d > 1 - a(1-e)$. We now have $\alpha_{AS}^M < 1$ and $\alpha_{AS}^C < 1$ with $\alpha_{AS}^M = \frac{d-a(1-e)}{2d-1}$ and $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-4}$. Since $d > 2/3$ holds under (1.2), we have $\alpha_{AS}^M > \alpha_{AS}^C$ if and only if $\phi(d) > 0$, where $\phi(d) = -4 + 9d - 6d^2 + a(4 - 5e + d(6e - 4))$. On the one hand, as the intersection point between $d = a(4-3e)/3$ and $d = 1 - a(1-e)$ is given by $e = \frac{7}{6} - \frac{1}{2a}$ and $d = (3 + a)/6$, (1.2) implies $d > (3 + a)/6$. On the other hand, as the intersection point between $d = a(4-3e)/3$ and $d = e$ is given by $d = e = \frac{4a}{3(1+a)}$, we have $e > \frac{4a}{3(1+a)}$ under (1.2). The function $\phi(d)$ is concave and, therefore, $\phi(d)$ must be greater than the minimum between $\phi((3+a)/6)$ and $\phi(1)$. The value $\phi((3+a)/6) = -1 - \frac{5}{6}(a-3)a + (a-2)ae$ is higher than $(a(3 + a) - 6)/6 > 0$ because we are assuming $1 + \frac{1}{\sqrt{2}} \le a < 2$ and $e < 1$. The value $\phi(1) = ae - 1$ is greater than $\frac{4a^2}{3+3a} - 1 > 0$, because $e > \frac{4a}{3(1+a)}$

holds and $a \geq 1 + \frac{1}{\sqrt{2}}$. Therefore, $\phi(d) > 0$ holds under (1.2) and we have $\alpha_{AS}^M > \alpha_{AS}^C$.

(1.3) $a(1-e) < d \leq a(4-3e)/3$. We now have $\alpha_{AS}^C = 1/2$ and $\alpha_{AS}^M = \frac{d-a(1-e)}{2d-1}$. As (1.3) implies $d > 1/2$, it follows $\alpha_{AS}^M \gtreqless 1/2 = \alpha_{AS}^C$ if and only if $e \gtreqless 1 - \frac{1}{2a}$. To show that $e = 1 - \frac{1}{2a}$ is feasible under (1.3) and does not intersect the other occurrences, note first that the intersection point between $d = a(1-e)$ and $d = e^2$ has $e = (-a + \sqrt{a}\sqrt{4+a})/2 \in (0, 1)$. Secondly, the intersection point between $d = a(4-3e)/3$ and $d = e$ has $e = \frac{4a}{3(1+a)} \in (0, 1)$. Since we have $(-a + \sqrt{a}\sqrt{4+a})/2 < 1 - \frac{1}{2a} < \frac{4a}{3(1+a)}$, it follows that, under (1.3), $\alpha_{AS}^M \gtreqless \alpha_{AS}^C$ if and only if $e \gtreqless 1 - \frac{1}{2a}$.

(1.4) $d \leq a(1-e)$. Here, $\alpha_{AS}^M = 0 < \alpha_{AS}^C = 1/2$ holds.

This proves the proposition for Case 1.

Case 2: $2 \leq a < 3$. We divide the proof into several occurrences.

(2.1) $d > a(4-3e)/3$. Here, we have $\alpha_{AS}^M = \frac{d-a(1-e)}{2d-1} < 1$ and $\alpha_{AS}^C = \frac{9d-4-a(4-3e)}{12d-4} < 1$. Since (2.1) implies $d > 2/3$, it follows that $\alpha_{AS}^M > \alpha_{AS}^C$ if and only if $\phi(d) > 0$, where $\phi(d) = -4 + 9d - 6d^2 + a(4 - 5e + d(6e - 4))$. The intersection between $d = a(4-3e)/3$ and $d = e^2$ is a point (e, d) with $e = (-3a + \sqrt{3}\sqrt{a}\sqrt{16+3a})/6$. Thus, (2.1) implies $d > (-3a + \sqrt{3}\sqrt{a}\sqrt{16+3a})^2/36 \geq 2(7 - \sqrt{33})/3 \in (0, 1)$. The intersection between $d = a(4-3e)/3$ and $d = e$ is a point with $e = \frac{4a}{3(1+a)}$. Thus, (2.1) implies $e > \frac{4a}{3(1+a)} \geq 8/9$. The function $\phi(\cdot)$ is concave and, under (2.1), it is greater than the minimum between $\phi(2(7-\sqrt{33})/3)$ and $\phi(1)$. The expression $\phi(2(7-\sqrt{33})/3) = (-542 - 44a + 69ae - 2\sqrt{33}(-47 + a(-4 + 6e)))/3$ is greater than $2(-813 + 26a - \sqrt{33}(4a - 141))/9 > 0$ because the expression is increasing in e and $e > 8/9$. The expression $\phi(1) = ae - 1$ is higher than $-1 + 8a/9 > 0$ because $e > 8/9$. This proves that $\phi(d) > 0$, under (2.1) and we have $\alpha_{AS}^M > \alpha_{AS}^C$.

(2.2) $a(1-e) < d \leq a(4-3e)/3$. Similarly to (1.3), we have $\alpha_{AS}^M \gtreqless \alpha_{AS}^C$ if and only if $e \gtreqless 1 - \frac{1}{2a}$.

(2.3) $d \leq a(1-e)$. Here, $\alpha_{AS}^M = 0 < \alpha_{AS}^C = 1/2$ holds.

This proves the proposition for Case 2.

Case 3: $a \geq 3$. Now $d < a(4-3e)/3$ holds for any $(e, d) \in \mathcal{P}$ and, in consequence, $\alpha_{AS}^C = 1/2$. We divide the proof into two occurrences.

(3.1) $d > e(1-e)$. As (3.1) implies $d > 1/2$, it follows $\alpha_{AS}^M \gtreqless 1/2 = \alpha_{AS}^C$ if and only if $e \gtreqless 1 - \frac{1}{2a}$. To show that $e = 1 - \frac{1}{2a}$ is feasible under (3.1) and does not intersect the other occurrences, note that the intersection point

between $d = a(1-e)$ and $d = e^2$ has $e = (-a + \sqrt{a}\sqrt{4+a})/2 \in (0,\, 1)$. Since we have $(-a + \sqrt{a}\sqrt{4+a})/2 < 1 - \frac{1}{2a}$ for any $a \geq 3$, it follows that, under (3.1), $\alpha_{AS}^M \gtrless \alpha_{AS}^C$ if and only if $e \gtrless 1 - \frac{1}{2a}$.

(3.2) $d \leq a(1-e)$. Here, $\alpha_{AS}^M = 0 < \alpha_{AS}^C = 1/2$ holds.

This proves the proposition for Case 3, and consequently the three cases together prove the original proposition. **Q.E.D.**

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