FIRM-SPECIFIC TEMPORAL VARIATION IN TECHNICAL EFFICIENCY: RESULTS OF A STOCHASTIC OUTPUT DISTANCE FUNCTION

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A B S T R A C T

The aim of this paper is to test the temporal variation of technical efficiency of the Spanish Savings Banks during the period 1985-1994. Furthermore, a stochastic output distance function (Shephard, 1970) is employed to accommodate multiple output technology. The distance function provides the advantage that it does not need information about prices, so it can accommodate the multiproduct nature of the financial sector only using the quantities as data. The temporal variation of efficiency is modeled using an extension of Battese and Coelli (1992), allowing for firm-specific patterns of temporal change.

Key words: Time-varying Technical Efficiency, Stochastic Distance Functions, Panel Data.

R E S U M E N

El objetivo de este trabajo es contrastar la variación temporal de la eficiencia técnica de las Cajas de Ahorros españolas durante el periodo 1985-1994. Para ello la tecnología se modeliza a través de la función de distancia (Shephard, 1970) y el término de ineficiencia se especifica mediante una generalización del modelo propuesto por Battese y Coelli (1992). La función de distancia tiene la ventaja de que puede recoger tecnologías multiproducto sin precisar de información acerca de precios. Esta ventaja es mayor en un sector como el bancario en el que, generalmente, los precios se construyen a partir de gastos, lo cual podría suponer un problema. Por último, el modelo propuesto permite que el término de ineficiencia varíe con el tiempo de forma particular para cada empresa.

Palabras Clave: Eficiencia técnica variante en el tiempo, Funciones de distancia estocásticas, Datos de panel.

1. INTRODUCTION

In the latest years, applications using distance functions [Shephard (1970)] have begun to be very usual [Färe et al. (1993), Lovell et al. (1994), Coelli and Perelman (1996) or Grosskopf et al. (1997)]. The principal advantage of distance functions is that they allow the possibility of specifying a multiple-input, multiple-output technology without price information so it can accommodate the multiproduct nature of the financial sector only using data on quantities.

On the other hand, the use of panel data to estimate frontier functions avoids restrictive assumptions about the distribution of the error terms, and solves some econometric problems over the cross-sectional data. In turn, it is only required to assume that technical efficiency be time-invariant (Schmidt and Sickles, 1984). However, some researchers have focused on relaxing this last assumption at the cost of imposing some structure on the model. Cornwell, Schmidt and Sickles (1990), Kumbhakar (1990), Battese and Coelli (1992) and Lee and Schmidt (1993) have proposed time-varying technical efficiency panel data models. The first of these models allows for firm-specific patterns of temporal change in technical efficiency and it models technical efficiency through the intercept of the production frontier[1]. The rest of these models adopt a different approach in that they model technical efficiency through an error component but assume that efficiency change is the same for all firms.

In this paper, we propose a model that allows for firm-specific patterns of temporal change in technical efficiency using an error component model. The greater flexibility of this model captures effects not visible in models that assume a common pattern of efficiency change. Moreover, we apply this model using a stochastic output distance function to represent the technology.

The paper unfolds as follows. In Section 2 we define output distance functions. The model is presented in Section 3, and in the subsequent section the likelihood function and estimation issues are discussed. An empirical example is presented in Section 5. Finally, Section 6 concludes.

[1] See Cornwell, Schmidt and Sickles (1990).

2. OUTPUT DISTANCE FUNCTIONS

A production technology transforming inputs $\mathbf{x} = (x_1, x_2,..., x_N) \in \Re_+^N$ into outputs $\mathbf{y} = (y_1, y_2,..., y_M) \in \Re_+^M$ can be represented using the technology set[2]:

$$T = \{ (x, y) : x \in \Re_+^N, y \in \Re_+^M, x \text{ can produce } y \} \quad [1]$$

The output set, P(x), denotes the set of all output vectors, $\mathbf{y} \in \Re_+^M$, that are obtainable from the input vector $\mathbf{x} \in \Re_+^N$. That is:

$$P(x) = \{ y : (x, y) \in T \} \quad [2]$$

The output Distance Function can be defined in terms of the output set as:

$$D_o(x, y) = \min . \{ \Psi > 0 : (\frac{y}{\Psi}) \in P(x) \} \quad [3]$$

The Output Distance Function is defined as the maximum feasible expansion of the output vector with the input vector held fixed. That is, given an input vector, **x**, the value of the output distance function, $D_O(\mathbf{x}, \mathbf{y})$, places $\mathbf{y}/D_O(\mathbf{x}, \mathbf{y})$ on the outer boundary of P(**x**) and on the ray through **y**. The preceding discussion suggests that the distance function will take a value which is less than or equal to one if the output vector, **y**, is an element of the feasible production set, P(**x**). That is, $D_O(\mathbf{x}, \mathbf{y}) \leq 1$ if $\mathbf{y} \in P(\mathbf{x})$. However, this conclusion is only valid under the assumption of weak disposability of outputs[3].

The Output Distance Function is non-decreasing, positively linearly homogeneous and convex in **y**, and decreasing in **x** (Färe and Primont, 1995). The homogeneity condition is for our purposes one of the most important, due to the role played in the estimation process. To see this, note that for any scalar $\theta > 0$,

[2] We assume that the technology satisfies the axioms listed in Färe and Primont (1995).

[3] Thus the assumption of weak disposability of outputs is the "price" that must be paid if the technology is to be characterized by the Output Distance Function (Färe and Primont, 1995).

$$D_o(x, \theta y) = \min.\{\Psi > 0 : (\frac{\theta y}{\Psi}) \in P(x)\} =$$

$$= \min.\{(\frac{\Psi \theta}{\theta}) > 0 : (\frac{y}{\Psi / \theta}) \in P(x)\} = \quad [4]$$

$$= \theta \min.\{(\frac{\Psi}{\theta}) > 0 : (\frac{y}{\Psi / \theta}) \in P(x)\} = \theta D_o(x, y)$$

On the other hand, distance functions are closely related with efficiency measurement. More specifically, the output-oriented Farrell measure of technical efficiency is the maximal feasible radial expansion of output vector:

$$DF_O = \max.\{\Omega : \Omega . y \in P(x)\} \quad [5]$$

Thus, the output distance function is the inverse of the output-oriented Farrell measure of technical efficiency, defined in [5]. This measure lies between 1 and $+\infty$, and the greater this measure, the smaller the efficiency. However, in order to be consistent with the most parametric efficiency studies, we will use a measure that lies between 0 and 1. In other words, we will use directly the value of the output distance function as a measure of efficiency. This measure will take a value of unity if **y** is located on the outer boundary of the production possibility set, and a value less than one if **y** is in the interior of the production possibility set[4].

[4] This is the same solution adopted by Coelli and Perelman (1996).

3. A PANEL DATA MODEL WITH TIME-VARYING TECHNICAL EFFICIENCY

As Färe and Primont (1996) demonstrate, recalling the duality between the revenue function and the output distance function would lead to a typical additive-error regression model[5]. So, we write the general form of the stochastic output distance function as:

$$1 = D_o(y_{it}, x_{it}; \beta) h(\varepsilon_{it}) \quad ; \quad h(\varepsilon_{it}) = \exp(u_i + v_{it}) \qquad [6]$$

in which deviations from 1 are accommodated in the specification of h(.), β is a vector of parameters, u_i is a one-sided efficiency component and v_{it} is a standard noise component that is assumed to follow a normal distribution with zero mean and variance σ_v^2.

In this model, technical efficiency is firm-specific but time-invariant. The simplest way to generalize [6] is to allow the error component representing technical efficiency to be time-varying, and to make some assumptions concerning its structure (Lovell, 1996). Battese and Coelli (1992) proposed to replace u_i with:

$$u_{it} = \{\exp[-\eta(t-T)]\} u_i \qquad [7]$$

The parametrization in [7] implies that, although each producer has his own level of technical efficiency in the last period, $\exp\{-u_i\}$, the pattern of change on technical efficiency is common to all the producers. In this paper we generalize function [7] to allow for more flexibility in the way that technical efficiency changes over time. Specifically, we propose a model with firm-specific patterns of temporal change. The efficiency term (u_{it}) is now defined as:

$$u_{it} = \{\exp[-\xi_i(t-T)]\} u_i \qquad [8]$$

where ξ_i are firm-specific parameters allowing for different patterns of temporal variation among different firms, and u_i is the positive truncation of a $N(0,\sigma^2)$ distribution. This is a relaxation of Battese and Coelli's model.

5 Note that to apply the duality between stochastic output distance functions and costs functions constant returns to scale must be imposed (Färe and Primont, 1996). In our case, we prefer to relax the returns to scale instead of resorting to duality.

4. THE LIKELIHOOD FUNCTION AND ESTIMATION OF INEFFICIENCY

In log terms, the stochastic distance function can be written as:

$$0 = \ln D_o(y_{it}, x_{it}; \beta) + \varepsilon_{it} \quad [9]$$

$$\varepsilon_{it} = v_{it} + \{ \exp [-\xi_i (t - T)] \} u_i \quad [10]$$

It is assumed that the v_{it}´s are independent and identically distributed (i.i.d.) $N(0,\sigma_v^2)$ random variables. The u_i´s are assumed to be non-negative truncations of the $N(\mu,\sigma_u^2)$ distribution and the u_i´s and the v_{it}´s are independent[6]. The next step is to find the distribution of u conditional on ε. The density functions for v and u are:

$$f(v) = \frac{1}{(2\pi)^{1/2}\sigma_v} \exp [-\frac{1}{2}(\frac{v}{\sigma_v})^2] \quad [11]$$

$$f(u) = \frac{1}{[1 - F(-\frac{\mu}{\sigma})] (2\pi)^{1/2} \sigma_u} \exp [-\frac{1}{2}(\frac{u-\mu}{\sigma_u})^2] \quad [12]$$

where F(·) represents the cumulative probability density function of a standard normal random variable. Since they are independent, the joint probability density function (pdf) of v and u is the product of their individual densities:

$$f(u, v) = \frac{1}{[1 - F(-\frac{\mu}{\sigma})] (2\pi) \sigma_u \sigma_v} \exp [-\frac{1}{2}(\frac{u-\mu}{\sigma_u})^2 - \frac{1}{2}(\frac{v}{\sigma_v})^2] \quad [13]$$

In vectorial notation, let v_i be the (T_ix1) vector of the v_{it}'s associated with the T_i observations for the i-th firm. Using results from the multivariate normal distribution when the T_i observations are independent, we obtain:

[6] Without the restrictions implied by linear homogeneity in outputs, the parameters of the model cannot be estimated. This problem is obviously due to the fact that the regressand is a constant. However, as long as the homogeneity restrictions are imposed, least squares will provide estimates of all the regression parameters.

$$f(u_i, v_i) = \frac{1}{[1-F(-\frac{\mu}{\sigma})](2\pi)^{(T_i+1)/2}\sigma_u \sigma_v^{T_i}} \exp[-\frac{1}{2}(\frac{u_i - \mu}{\sigma_u})^2 - \frac{1}{2}(\frac{v_i' v_i}{\sigma_v^2})] \quad [14]$$

Using $\varepsilon_{it} = v_{it} + \{\exp[-\xi_i (t-T)]\} u_i$, and being ε_i the (T_ix1) vector of the ε_{it}'s associated with the T_i observations for the i-th firm, the joint pdf of (u_i,ε_i) is:

$$f(u_i, \varepsilon_i) = \frac{1}{[1-F(-\frac{\mu}{\sigma_u})](2\pi)^{T_i+1/2}\sigma_u \sigma_v^{T_i}} \exp\{-\frac{1}{2}[(\frac{u_i - \mu}{\sigma_u})^2 +$$

[15]

$$+\frac{(\varepsilon_i - \exp[-\xi_i(t-T)]u_i)'(\varepsilon_i - \exp[-\xi_i(t-T)]u_i)}{\sigma_v^2}]\}$$

This expression is quite similar to [A.7] in Battese and Coelli (1992) except for the specification of ε_i. The density function of ε_i, obtained by integrating f (u_i, ε_i) over the range of u_i, namely $u_i \geq 0$, is:

$$f(\varepsilon_i) = \frac{1}{[1-F(-\frac{\mu}{\sigma_u})]} \frac{\sigma_i^*}{\sigma_u \sigma_v} [1-F(-\frac{\mu_i^*}{\sigma_i^*})] f[\frac{(\varepsilon_{it} - \mu \exp[-\xi_i(t-T)])\sigma_i^*}{\sigma_u \sigma_v}] \quad [16]$$

where f (·) represents the pdf of a standard normal random variable and:

$$\mu_i^* = \frac{\mu \sigma_v^2 + \exp\{-\xi_i(t-T)\}\varepsilon_i \sigma_u^2}{\sigma_v^2 + [\exp\{-\xi_i(t-T)\}]'[\exp\{-\xi_i(t-T)\}]\sigma_u^2} \quad [17]$$

$$\sigma_i^* = \frac{\sigma_u^2 \sigma_v^2}{\sigma_v^2 + [\exp\{-\xi_i(t-T)\}]'[\exp\{-\xi_i(t-T)\}]\sigma_u^2} \quad [18]$$

Therefore, the conditional density of u_i given ε_i is the ratio of [15] to [16]:

$$f(u_i / \varepsilon_i) = \frac{1}{[1-F(-\frac{\mu^*}{\sigma^*})](2\pi)^{1/2}\sigma_i^*} \exp[-\frac{1}{2}(\frac{u_i - \mu_i^*}{\sigma_i^*})^2] \quad [19]$$

This is the density function of the positive truncation of a $N(\mu^*,\sigma^{*2})$ distribution. Expression [19] is identical to [A.11] of Battese and Coelli (1992), though the definition of μ_i^* and σ_i^* in [17] and [18] differs from their paper.

The likelihood function can be written as the product of density functions in [16]. For computational purposes it is convenient to rewrite [16] as:

$$f(\varepsilon_i)=\frac{\exp\{-\frac{1}{2}[(\frac{\varepsilon_i'\varepsilon_i}{\sigma_u^2})+(\frac{\mu}{\sigma})^2-(\frac{\mu_i^*}{\sigma_i^*})^2]\}[1-F(-\frac{\mu_i^*}{\sigma_i^*})]}{(\sigma_V^2+[\exp\{-\xi_i(t-T)\}]'[\exp\{-\xi_i(t-T)\}]\sigma_u^2)^{1/2}[1-F(-\frac{\mu}{\sigma_u})](2\pi)^{T_i/2}\sigma_v^{T_i-1}} \qquad [20]$$

The log-likelihood function of the output distance function model [9] and [10] is:

$$\ln(\eta^*;y)=-\frac{1}{2}(\Sigma T_i)\ln(2\pi)-\frac{1}{2}\Sigma(T_i-1)\ln(\sigma_v^2)-$$

$$-\frac{1}{2}\Sigma\ln(\sigma_v^2+[\exp\{-\xi_i(t-T)\}]'[\exp\{-\xi_i(t-T)\}]\sigma_u^2)-$$

$$-N\ln[1-F(-\frac{\mu}{\sigma_u})]+\Sigma\ln[1-F(-\frac{\mu^*}{\sigma^*})]-$$

$$-\frac{1}{2}\Sigma[\frac{(-\ln D_O(y,x,\beta))'(-\ln D_O(y,x,\beta))}{\sigma_v^2}]-\frac{1}{2}N(\frac{\mu}{\sigma})^2+\frac{1}{2}\Sigma(\frac{\mu_i^*}{\sigma_i^*})^2 \qquad [21]$$

where $\eta^* = (\beta', \sigma_v^2, \sigma_u^2, \mu, \xi_i)'$.

As we said before, the homogeneity restrictions must be imposed to estimate the model. We have two alternatives, and equivalent, ways to do it. First, we can estimate the output distance function (the regressand being a constant) with the homogeneity restrictions on the parameters. Second, linear homogeneity in the output quantities implies a regression of the general form:

$$\frac{1}{y_{Nit}} = D_o(y^*_{it}, x_{it}; \beta) h(\varepsilon_{it}) \quad [22]$$

where $y^*_{it} = (y_{1it}/y_{Nit}, y_{2it}/y_{Nit}, ..., y_{N-1it}/y_{Nit})$. In practice, we can choose one output and re-write the constant regressand and the other outputs using the selected output as a numeraire (see Grosskopf et al., 1997). The choice of the output is arbitrary and the resulting estimates will be invariant to the normalization. While such a scaling is not necessary for estimation purposes, we follow this strategy to impose the homogeneity restrictions[7].

In addition, we adopt the standard flexible translog functional form to represent the technology, including dummy variables (D_t) to account for temporal effects. So, the generic model, defined in [9] and [10], can be written in logs terms as (using [22]):

$$-\ln y_{Nit} = \alpha_0 + \sum_{t=2}^{T} \delta_t D_t + \sum_{k=1}^{M} \alpha_k \ln x_{kit} + \sum_{j=1}^{N-1} \beta_j \ln y^*_{jit} + \frac{1}{2} \sum_{k=1}^{M} \sum_{h=1}^{M} \alpha_{kh} \ln x_{kit} \ln x_{hit} +$$

$$+ \frac{1}{2} \sum_{j=1}^{N-1} \sum_{h=1}^{N-1} \beta_{jh} \ln y^*_{jit} \ln y^*_{hit} + \sum_{k=1}^{M} \sum_{j=1}^{N-1} \gamma_{kj} \ln x_{kit} \ln y^*_{jit} + v_{it} + \{ \exp[-\xi_i (t-T)] \} u_i \quad [23]$$

where

$$y^*_{jit} = \frac{y_{jit}}{y_{Nit}} \quad [24]$$

Technical efficiency indexes are obtained directly from the value of the stochastic distance function. In other words, the technical efficiency indexes are obtained from the following expression:

$$TE_{it} = D_O(y_{it}, x_{it}; \beta) \exp(v_{it}) = \exp\{-u_i \exp[-\xi_i (t-T)]\} \quad [25]$$

On the other hand, both inputs and outputs appear as regressors in distance functions, so we should be explicit about the possibility of simultaneous equation bias. When we are working with output (input) distance functions, the inputs (outputs) should be treated as exogenous and the outputs (inputs) will be endogenous. However, Coelli and Perelman (1996) argue that when the

7 An application using this scaling is Coelli and Perelman (1996).

normalization in [22] is used, only output ratios appear as regressors and these ratios may be assumed to be exogenous since the output distance function is defined for radial expansion of all outputs given the input levels, and hence by definition the output ratios are held constant for each firm[8].

5. EMPIRICAL ILLUSTRATION: THE SPANISH SAVINGS BANKS

The model developed above is illustrated with an empirical application to the Spanish Savings Banks. The data are yearly data (1985-1994) from the Confederación Española de Cajas de Ahorros (CECA). A considerable number of mergers and acquisitions have occurred in this period, raising questions about the best way to treat this issue. The approach followed in this paper is the use of an unbalanced panel. The merged entities disappear from the sample and appear as a different entity (result of that merger). Thereby, the number of firms declines from 77 the first year to 47 the last year, being 642 the total number of observations.

Nevertheless, the use of an unbalanced panel also has its limitations, since the problem is not simply related with the loss of data, but also with the possibility that the disappearance maybe lead to attrition problems. If the probability that the observations disappear from the sample is correlated with the phenomenon being modeled, then traditional statistical methods will result in biased and inconsistent estimates (Hsiao, 1986). We will assume that such correlation does not exist, since the correction of the possible bias would vindicate for itself a new investigation.

Three outputs and three inputs are included. The outputs are: (y_1) Bonds, cash and other assets different from loans; (y_2) Loans to non-banks; and (y_3) Deposits from non-banks and banks. The inputs are: (x_1) Physical Capital, measured by the value of fixed assets in the balance; (x_2) Labor, measured by the wage expenses, implicitly assuming equal input prices for all banks[9]; and (x_3) Other Expenses.

[8] The same argument can be used with input distance functions.

[9] An alternative would be to use a variable measured in physical units, vg. the number of workers. However, even in this case we need the implicit assumption of homogeneous staff composition among the firms. In principle, this last option seems to introduce more important problems that the one chosen because prices do not vary much from one firm to another due to the great similarities in all the wage agreements. The same approach was used by Berg et al. (1991) for the Norwegian Banks.

The empirical results for the estimated model are presented in Tables 1 and 2[10]. All the coefficients are highly significant and the elasticities possess the expected signs at the geometric mean. Therefore, at this point, the estimated distance function fulfills the property of monotonicity (non-decreasing in outputs and decreasing in inputs).

TABLE 1: Maximum-likelihood estimates (frontier parameters) (1)(2)

VARIABLE	PARAMETER	ESTIMATES	VARIABLE	PARAMETER	ESTIMATES
Constant	α_0	-0.0996** (0.0164)	C x K	γ_{24}	0.1323 (0.0921)
Other Assets (A)	β_1	0.0993** (0.0420)	C x E	γ_{25}	-0.4200** (0.1166)
Credits (C)	β_2	0.5839** (0.0450)	L x K	α_{34}	0.1468** (0.0577)
Labor (L)	α_3	-0.5587** (0.0262)	L x E	α_{35}	0.3576** (0.1128)
Capital (K)	α_4	-0.0773** (0.0184)	K x E	α_{45}	-0.1162* (0.0552)
O. Expenses (E)	α_5	-0.2722** (0.0236)	D_{86}	δ_{86}	-0.0287** (0.0089)
A^2	β_{11}	0.2345 (0.2797)	D_{87}	δ_{87}	-0.0967** (0.0105)
C^2	β_{22}	0.9323** (0.2683)	D_{88}	δ_{88}	-0.2014** (0.0125)
L^2	α_{33}	-0.4774** (0.1240)	D_{89}	δ_{89}	-0.2117** (0.0147)
K^2	α_{44}	-0.0372 (0.0497)	D_{90}	δ_{90}	-0.2001** (0.0172)
E^2	α_{55}	-0.3434** (0.1227)	D_{91}	δ_{91}	-0.2771** (0.0197)
A x C	β_{12}	0.3456 (0.2931)	D_{92}	δ_{92}	-0.3221** (0.0223)
A x L	γ_{13}	0.3698** (0.1281)	D_{93}	δ_{93}	-0.3293** (0.0254)
A x K	γ_{14}	0.1244 (0.0815)	D_{94}	δ_{94}	-0.4259** (0.0296)
A x E	γ_{15}	-0.5674** (0.1264)	----	$\sigma^2 = \sigma_u^2 + \sigma_v^2$	0.2642** (0.0533)
C x L	γ_{23}	0.2785* (0.1252)	----	lambda = σ_u/σ_v	0.0093** (0.0020)

(1) Standard Errors in parentheses.

(2) ** (*) Parameter significant at 99% (95%) confidence level.

TABLE 2: Maximum -likelihood estimates (efficiency parameters)

[10] The value of the parameters that maximize the log-likelihood function [21] were solved using GAUSS.

PARAMETER	ESTIMATES	PARAMETER	ESTIMATES
ξ_1	-0.0479* (0.0223)	ξ_{23}	0.0523* (0.0317)
ξ_2	-0.1377* (0.0646)	ξ_{24}	-0.0441 (0.0300)
ξ_3	-0.1230** (0.0388)	ξ_{25}	0.1105* (.00477)
ξ_4	0.0524 (0.0436)	ξ_{26}	-0.0702** (0.0194)
ξ_5	0.0330 (0.0337)	ξ_{27}	-0.0001 (0.0100)
ξ_6	-0.0183 (0.0240)	ξ_{28}	0.1051** (0.0354)
ξ_7	-0.0381* (0.0175)	ξ_{29}	-0.0006 (0.0116)
ξ_8	0.1108 (0.1114)	ξ_{30}	-0.0161 (0.0255)
ξ_9	0.0349 (0.0515)	ξ_{31}	-1.0890 (1.4544)
ξ_{10}	-0.2413* (0.1174)	ξ_{32}	-0.0994** (0.0321)
ξ_{11}	-0.5821* (0.3469)	ξ_{33}	-3.7497 (71.555)
ξ_{12}	-1.8213 (11.5778)	ξ_{34}	0.1400** (0.0315)
ξ_{13}	-0.1639** (0.0621)	ξ_{35}	0.0102 (0.0141)
ξ_{14}	-2.8765 (4.0301)	ξ_{36}	-0.1017** (0.0214)
ξ_{15}	-0.1079** (0.0361)	ξ_{37}	-0.0347 (0.0232)
ξ_{16}	-0.0694* (0.0364)	ξ_{38}	-0.0276 (0.0306)
ξ_{17}	-0.0489** (0.0123)	ξ_{39}	-0.0256** (0.0100)
ξ_{18}	0.1230** (0.0518)	ξ_{40}	-0.0972** (0.0188)
ξ_{19}	0.0918* (0.0432)	ξ_{41}	-0.0316 (0.0590)
ξ_{20}	0.0881** (0.0275)	ξ_{42}	-0.0268 (0.0693)
ξ_{21}	0.0232 (0.0196)	ξ_{43}	-0.0536** (0.0208)
ξ_{22}	-4.1441 (11.6593)	ξ_{44}	-0.0099 (0.0140)

TABLE 2: Maximum -likelihood estimates (efficiency parameters) ***(Continuation)***

PARAMETER	ESTIMATES	PARAMETER	ESTIMATES
ξ_{45}	-0.0707** (0.0174)	ξ_{67}	-0.0548 (0.0522)
ξ_{46}	-0.0952* (0.0525)	ξ_{68}	-0.0728** (0.0235)
ξ_{47}	-0.0655* (0.0381)	ξ_{69}	-0.0186 (0.0417)
ξ_{48}	0.0126 (0.0085)	ξ_{70}	-0.1208** (0.0374)
ξ_{49}	-0.0062 (0.0199)	ξ_{71}	-0.1999* (0.0963)
ξ_{50}	0.0750* (0.0405)	ξ_{72}	-2.1362 (2.5956)
ξ_{51}	-0.0382 (0.0270)	ξ_{73}	0.3056** (0.0908)
ξ_{52}	-0.0126 (0.0180)	ξ_{74}	-2.0533 (10.6507)
ξ_{53}	-0.1387 (0.1109)	ξ_{75}	-0.0496 (0.0308)
ξ_{54}	-0.0745 (0.1331)	ξ_{76}	-0.1319** (0.0323)
ξ_{55}	-0.3081 (0.1912)	ξ_{77}	-0.0163 (0.0304)
ξ_{56}	-0.0152 (0.0144)	ξ_{78}	0.0361 (0.0364)
ξ_{57}	-0.0446* (0.0198)	ξ_{79}	-0.3537* (0.1750)
ξ_{58}	-0.0957** (0.0312)	ξ_{80}	-0.0244 (0.0354)
ξ_{59}	-0.0338 (0.0230)	ξ_{81}	0.0240 (0.0349)
ξ_{60}	0.0701* (0.0417)	ξ_{82}	-0.2623** (0.0694)
ξ_{61}	-0.0344 (0.0256)	ξ_{83}	-0.0973** (0.0393)
ξ_{62}	-0.3078** (0.0664)	ξ_{84}	-0.0226 (0.0877)
ξ_{63}	0.0418 (0.0282)	ξ_{85}	-0.3834 (0.2588)
ξ_{64}	-0.0883** (0.0264)	ξ_{86}	-0.4506** (0.1086)
ξ_{65}	-0.1393** (0.0309)	ξ_{87}	-0.1097* (0.0540)
ξ_{66}	0.0468 (0.0377)	ξ_{88}	-0.0175 (0.0727)

Following Färe and Primont (1996), the scale elasticity is given by:

$$e_O\ (x, y) = \frac{\partial\ \ln\theta}{\partial\ \ln\lambda} \qquad [26]$$

where:

$$D_o\ (\lambda x, \theta y) = 1 \qquad [27]$$

The scale elasticity is the proportional increase in all outputs caused by an increase of the same proportion in all inputs. Applying the implicit function rule to [26] and the homogeneity condition in the output vector, the scale elasticity is:

$$EE\ (x, y) = -\Delta_x D_o\ (x, y)\, x \qquad [28]$$

In words, the scale elasticity is the negative of the sum of the input elasticities. The scale elasticity, in the approximation point, is 0.9082, indicating the presence of decreasing returns to scale at the mean.

The parameters relating to the temporal variation of technical efficiency are presented in Table 2. Results show that there are 45 ξ parameters not statistically different from zero, so we cannot reject the time-invariant technical efficiency hypotheses for those 45 firms. There are 10 firms that improve their technical efficiency level during the period, and 33 that are getting worse. All these results are obtained using the 90% confidence level. If we use the 95% criterion, there are 62 ξ parameters not statistically different from zero with only 5 firms improving their technical efficiency levels.

Since the proposed method is a slight modification of the BC model, results are compared with those obtained from BC's application (imposing the restriction of equal ξ across firms). Table 3 lists the results of the application of the BC model to the same data.

As can be seen, the frontier parameters are quite similar to those obtained with our model. The ξ parameter (η in their notation) is negative and not statistically different from zero at the 95% confidence level. If we compare this result with the outcome in Table 2, we can conclude that our model captures effects not visible in the Battese and Coelli (1992) model, and, by extension, not visible in all those models in which a common pattern of efficiency change is assumed[11].

[11] For example, the models by Kumbhakar (1990) and Lee and Schmidt (1993).

TABLE 3: Maximum-likelihood estimates (Battese and Coelli, 1992) (1)(2)

VARIABLE	PARAMETER	ESTIMATES	VARIABLE	PARAMETER	ESTIMATES
Constant	α_0	-0.2008** (0.0296)	C x E	γ_{25}	-0.3238** (0.1259)
Other Assets (A)	β_1	0.1980** (0.0464)	L x K	α_{34}	0.1785** (0.0671)
Credits (C)	β_2	0.5478** (0.0462)	L x E	α_{35}	0.3492** (0.1182)
Labor (L)	α_3	-0.5762** (0.0333)	K x E	α_{45}	-0.1928** (0.0567)
Capital (K)	α_4	-0.0492** (0.0199)	D_{86}	δ_{86}	-0.0248* (0.0111)
O. Expenses (E)	α_5	-0.2475** (0.0236)	D_{87}	δ_{87}	-0.0793** (0.0125)
A^2	β_{11}	0.2212 (0.2548)	D_{88}	δ_{88}	-0.1759** (0.0141)
C^2	β_{22}	0.3046 (0.2559)	D_{89}	δ_{89}	-0.1836** (0.0158)
L^2	α_{33}	-0.5786** (0.1300)	D_{90}	δ_{90}	-0.1526** (0.0179)
K^2	α_{44}	0.0230 (0.0557)	D_{91}	δ_{91}	-0.2122** (0.0202)
E^2	α_{55}	-0.2095 (0.1315)	D_{92}	δ_{92}	-0.2395** (0.0219)
A x C	β_{12}	-0.1899 (0.2715)	D_{93}	δ_{93}	-0.2321** (0.0233)
A x L	γ_{13}	0.2149 (0.1444)	D_{94}	δ_{94}	-0.2985** (0.0250)
A x K	γ_{14}	0.1494* (0.0883)	----	$\sigma^2 = \sigma_u^2 + \sigma_v^2$	0.2219** (0.0478)
A x E	γ_{15}	-0.4271** (0.1384)	----	lambda = σ_u/σ_v	0.0189** (0.0046)
C x L	γ_{23}	0.0992 (0.1282)	----	ξ	-0.0001 (0.0060)
C x K	γ_{24}	0.2197** (0.0917)			

(1) Standard Errors in parentheses.

(2) ** (*) Parameter significant at 99% (95%) confidence level.

The BC model is nested in our model and we can therefore impose the restrictions $\xi_1=\xi_2=...=\xi_i=\xi$ in order to test the hypothesis of common pattern of efficiency change across firms. This hypothesis is strongly rejected by the likelihood ratio statistic of 290.04. In other words, the new model is shown to be a preferable specification for these data.

To obtain the technical efficiency indexes, we reestimate the model imposing the restriction of $\xi_i=0$ on all those firms whose ξ parameter is not statistically different from zero at the 90% confidence level, in order to improve the precision of the technical efficiency estimates[12]. The results of the estimation of this restricted model are shown in Table 4 and the parameters relating to the temporal variation of technical efficiency are presented in Table 5. The restricted model is not rejected by the likelihood ratio statistic of 46.42 (the ratio is distributed χ^2 with degrees of freedom equal to the number of restrictions under the null, so the critical value is 61.37). The parameters are quite similar to those obtained with the unrestricted model, they are significant and again the elasticities possess the expected signs.

With this new estimation, we obtain the technical efficiency indexes by applying equation [25]. Summary statistics of the predicted technical efficiency indexes appear in Table 6 (firms with the maximum and minimum value in each year are in parentheses). Mean technical efficiency is decreasing over time from 0.6853 in 1985 to 0.6656 in 1994.

The efficiency ranking changes each year: firm number 33 is the most efficient in the period 1985-89 and firm number 87 is the most efficient from 1990 to 1994. Firm number 48 is the least efficient in the period 1985-89. Firm 68 is the least efficient in 1990 and firm number 17 is the least efficient firm from 1991 to 1994. The changes in the rankings are due to the flexibility of the model, with the largest change occurring in firm 88. This firm goes from number 7 in the ranking in 1991 ($TE^{88}_{1991}=0.8709$) to number 26 ($TE^{88}_{1994}=0.6488$). In only four years (this firm is the result of a merger in 1991) it loss 23 points in its efficiency level.

[12] Stronger assumptions generate stronger results, but they strain one´s conscience more. In general, the choice of maintained assumptions can only be determined by a careful consideration of the data and the characteristics of the industry under study (Bauer, 1990, p. 41). In this case, we impose those restrictions due to the fact that we cannot reject that they are different from zero.

TABLE 4: Maximum-likelihood estimates. Restricted model (frontier parameters) (1)(2)

VARIABLE	PARAMETER	ESTIMATES	VARIABLE	PARAMETER	ESTIMATES
Constant	α_0	-0.2253** (0.0308)	C x K	γ_{24}	0.1470* (0.0802)
Other Assets (A)	β_1	0.1210** (0.0398)	C x E	γ_{25}	-0.3864** (0.1227)
Credits (C)	β_2	0.5799** (0.0396)	L x K	α_{34}	0.1551** (0.0548)
Labor (L)	α_3	-0.5877** (0.0309)	L x E	α_{35}	0.2693** (0.1110)
Capital (K)	α_4	-0.0478** (0.0162)	K x E	α_{45}	-0.0825* (0.0486)
O. Expenses (E)	α_5	-0.2713** (0.0207)	D_{86}	δ_{86}	-0.0231** (0.0085)
A^2	β_{11}	0.2624 (0.2483)	D_{87}	δ_{87}	-0.0853** (0.0098)
C^2	β_{22}	0.9160** (0.2328)	D_{88}	δ_{88}	-0.1862** (0.0112)
L^2	α_{33}	-0.4139** (0.1286)	D_{89}	δ_{89}	-0.1927** (0.0125)
K^2	α_{44}	-0.0406 (0.0439)	D_{90}	δ_{90}	-0.1761** (0.0141)
E^2	α_{55}	-0.2868** (0.1199)	D_{91}	δ_{91}	-0.2470** (0.0156)
A x C	β_{12}	0.3972* (0.2660)	D_{92}	δ_{92}	-0.2864** (0.0170)
A x L	γ_{13}	0.2743* (0.1318)	D_{93}	δ_{93}	-0.2890** (0.0178)
A x K	γ_{14}	0.1861** (0.0730)	D_{94}	δ_{94}	-0.3733** (0.0189)
A x E	γ_{15}	-0.5459** (0.1229)	----	$\sigma^2 = \sigma_u^2 + \sigma_v^2$	0.3001** (0.0571)
C x L	γ_{23}	0.2513* (0.1294)	----	lambda = σ_u/σ_v	0.0080** (0.0017)

(1) Standard Errors in parentheses.

(2) ** (*) Parameter significant at 99% (95%) confidence level.

TABLE 5: Maximum -likelihood estimates. Restricted model (efficiency parameters) (1)(2)

PARAMETER	ESTIMATES	PARAMETER	ESTIMATES	PARAMETER	ESTIMATES
ξ_1	-0.0336* (0.0169)	ξ_{26}	-0.0613** (0.0167)	ξ_{62}	-0.2501** (0.0580)
ξ_2	-0.0497 (0.0526)	ξ_{28}	0.0897** (0.0278)	ξ_{64}	-0.0409** (0.0170)
ξ_3	-0.0918** (0.0290)	ξ_{32}	-0.0657** (0.0206)	ξ_{65}	-0.0980** (0.0229)
ξ_7	-0.0194* (0.0119)	ξ_{34}	0.1101** (0.0240)	ξ_{68}	-0.0879** (0.0220)
ξ_{10}	-0.0590 (0.0597)	ξ_{36}	-0.0665** (0.0150)	ξ_{70}	-0.0943** (0.0304)
ξ_{11}	-0.1609* (0.1024)	ξ_{39}	-0.0241** (0.0076)	ξ_{71}	-0.0679* (0.0420)
ξ_{13}	-0.0976* (0.0426)	ξ_{40}	-0.0662** (0.0138)	ξ_{73}	0.2290** (0.0600)
ξ_{15}	-0.0915** (0.0296)	ξ_{43}	-0.0263* (0.0146)	ξ_{76}	-0.0807** (0.0237)
ξ_{16}	-0.0486* (0.0261)	ξ_{45}	-0.0485** (0.0130)	ξ_{81}	-0.2663* (0.1476)
ξ_{17}	-0.0453** (0.0107)	ξ_{46}	-0.0334* (0.0250)	ξ_{84}	-0.1762** (0.0550)
ξ_{18}	0.0900** (0.0346)	ξ_{47}	-0.0409* (0.0299)	ξ_{85}	-0.0566* (0.0398)
ξ_{19}	0.0688* (0.0324)	ξ_{50}	0.0608* (0.0298)	ξ_{88}	-0.3808** (0.0913)
ξ_{20}	0.0712** (0.0168)	ξ_{57}	-0.0215* (0.0133)	ξ_{89}	-0.0861* (0.0516)
ξ_{23}	0.0423* (0.0225)	ξ_{58}	-0.1001** (0.0309)		
ξ_{25}	0.0989** (0.0319)	ξ_{60}	0.0537* (0.0317)		

(1) Standard Errors in parentheses.

(2) ** (*) Parameter significant at 99% (95%) confidence level.

TABLE 6: Technical efficiency indexes. Descriptive Statistics

YEAR	MAX (FIRM)	MIN (FIRM)	MEAN
1985	0.9896 (33)	0.3623 (48)	0.6853
1986	0.9896 (33)	0.3623 (48)	0.6833
1987	0.9896 (33)	0.3623 (48)	0.6808
1988	0.9896 (33)	0.3623 (48)	0.6777
1989	0.9896 (33)	0.3623 (48)	0.6774
1990	0.9643 (87)	0.3310 (68)	0.6682
1991	0.9643 (87)	0.3489 (17)	0.6771
1992	0.9643 (87)	0.3322 (17)	0.6756
1993	0.9643 (87)	0.3157 (17)	0.6744
1994	0.9643 (87)	0.2992 (17)	0.6656

6. CONCLUDING REMARKS

In this paper a model for efficiency measurement with panel data has been proposed and applied in an Output Distance Function framework. The distance function has the advantage that it does not need information about prices, so it can accommodate multioutput technologies using only the quantities as data. Unlike Kumbhakar (1990), Battese and Coelli (1992), and Lee and Schmidt (1993), the model allows for firm-specific patterns of efficiency temporal change. The background is similar to the paper of Cornwell, Schmidt and Sickles (1990) but here we model technical efficiency through an error component whereas they model it through the intercept of the production frontier. This is a plausible extension of previous models, and it is supported by the data to which we apply it.

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