

# **THE BIAS FOR FORWARD EXCHANGE RATE AND THE RISK PREMIUM: AN EXPLANATION WITH A STOCHASTIC AND DYNAMIC GENERAL EQUILIBRIUM MODEL\***

**Juan A. Lafuente and Jesús Ruiz\*\***

WP-EC 2002-20

Correspondencia a: J. A. Lafuente, Unidad Predepartamental de Finanzas y Contabilidad, Universitat Jaume I, Campus Riu Sec 12071 – CASTELLON (SPAIN). E-Mail: [lafuen@cofin.uji.es](mailto:lafuen@cofin.uji.es) / Tf.: +34-964-729092 (Directo) / Fax: +34-964-728565.

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.

Primera Edición Agosto 2002

Depósito Legal: V-3098-2002

IVIE working papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

---

\* Financial support from Spanish Ministry of Education through grants PB98-0831 and BEC2000-1388-C04-03 is gratefully acknowledged. We have also benefited from seminar audience at CREB. The usual caveat applies.

\*\* J.A. Lafuente: Dpto. de Finanzas y Contabilidad, Universitat Jaume I; J. Ruiz: Dpto. Economía Cuantitativa, Universidad Complutense de Madrid.

# THE BIAS FOR FORWARD EXCHANGE RATE AND THE RISK PREMIUM: AN EXPLANATION WITH A STOCHASTIC AND DYNAMIC GENERAL EQUILIBRIUM MODEL

Juan A. Lafuente and Jesús Ruiz

## ABSTRACT

Forward exchange rate unbiasedness is rejected in test for international exchange markets. Such issue can be interpreted as evidence of a biased forward rate and/or time-varying risk premia. This paper proposes a stochastic general equilibrium model which generates substantial variability in the magnitude of predictable excess returns. Simulation exercises suggest that higher persistency in the monetary policy produces higher bias in the estimated slope coefficient in the regression of the change in the logarithm of the spot exchange rate on the forward premium. Also, our model suggest that the nature of the transmission between monetary shocks can explain the excess returns puzzle. Empirical evidence for the DM-USD rate that support our theoretical results is provided.

**Keywords:** Expectations theory, Risk premium, Forward exchange rates, Simulations.

**JEL classification:** C22, F31, F47.

## RESUMEN

La insesgidez del tipo *forward* ha sido ampliamente rechazada en los estudios empíricos sobre los mercados de tipo cambio internacionales. Este aspecto puede interpretarse como la existencia de un sesgo en la capacidad predictiva del tipo *forward* y/o la presencia de una prima de riesgo cambiante en el tiempo. Este trabajo propone un modelo dinámico y estocástico de equilibrio general que genera amplia volatilidad en la prima de riesgo. Los ejercicios de simulación llevados a cabo sugieren que una mayor persistencia de la política monetaria produce un mayor sesgo en la pendiente estimada de una regresión del cambio en el logaritmo del tipo *spot* sobre la prima de riesgo. Además, el modelo sugiere que la naturaleza de la transmisión de los *shocks* monetarios puede explicar dicho sesgo. Finalmente, el trabajo presenta evidencia empírica sobre el tipo de cambio entre el marco alemán y el dólar americano en línea con los resultados teóricos.

**Palabras clave:** Teoría de las expectativas, Prima de riesgo, Tipo de cambio *forward*, Simulación.

**JEL clasificación:** C22, F31, F47.

# 1 Introduction

There is systematic evidence in the literature which refers that the estimated slope coefficients in the regression of the change in the logarithm of the spot rate on the forward premium significantly departs from one (see, for example, Tauchen (2001), Baillie and Bollerslev (2000), McCallum (1994), and Fama (1984)). Such discrepancy from the underlying value in the uncovered interest rate parity implies that the forward rate is not an unbiased predictor of the future spot rate, suggesting the possibility of unexploited profit opportunities. Potential explanations of this excess returns puzzle generally are assigned to three kind of categories: a) the most popular is that such pattern arises as a consequence of a time-varying risk premia; b) a second explanation relies on the nature of expectations. Under no rational expectations agents do not efficiently use the available information set, incurring in systematic forecasting errors over a significant number of time periods ahead; and c) the peso problem, that is, market participants anticipate by rational learning process a future discrete shift in policy that is not performed within the sample period analyzed (see Lewis, (1995)).

Even though a great number of studies have examined the ability of general equilibrium models related to the Lucas (1982) model to explain the forward premium puzzle (see, for example, Hodrick (1989), Macklem (1991), Canova and Marrinan (1993), Bekaert (1994)), they unsuccessfully explained the substantial variability that occur in the magnitude of predictable excess returns. The bias of test for a risk premium in forward exchange rates is yet regarded as one of the most important unresolved puzzle in international finance.

In this paper we develop a theoretical general equilibrium model to explain *short* and *long*-run risk premium in forward markets for foreign exchange that, not only provide additional insights about the potential explaining factors of the forward risk premium, but also reproduce the forward premium anomaly under rational expectations. The model takes as benchmark the Dutton's model (1993) which is based on the general equilibrium models of Lucas (1982). Our model extend the first one in three ways: a) we consider two forward (one and two periods) exchange rates as a hedge instruments for spot exchange rate a more realistic approach to real markets in where different time to maturity can be traded. This enriches the analysis because of it would be possible to identify the effect of the time to maturity in the derivative contract on the forward market risk premia,

b) it is considered the possibility that domestic and foreign consumptions goods will be complementary or substitutes. Therefore, the model allows to estimate the impact of the nature of consumptions goods. If, for example, dollars are relatively risk, the uncertainty about the future spot exchange should affect differently on the forward risk premia under complementaries or substitutes consumption goods, and c) the weight of each, domestic and foreign, consumption good in the utility function is not necessary the same. Consequently, a broad set of scenarios can be simulated in order to explore for possibly explanation factors of the risk premium.

The solution of the model involves to evaluate expectations of nonlinear expressions. Therefore, numerical solutions are provided. Our solution method allows to solve jointly for both prices and positions in one and two-periods ahead forward contracts. This is an interesting extension relative to the Dutton's solution method.

Simulation exercises are carried out with a variety of parameter values, revealing the ability of the model to reproduce the bias for forward exchange rates to predict the future evolution of spot rates. Under rational expectations, the model suggest the key factors that generates high volatility for the risk premium. We find that the persistency of the monetary policy and the time to maturity in forward contracts is correlated with the size of the slope coefficient in the regression of the change in the logarithm of the spot exchange rate on the forward premium. Under high persistency the estimated slopes dramatically decreases below one. Also, the estimated slopes corresponding to the *long* time to maturity contract are relatively lower.

Empirical evidence for the US-Germany case that support our theoretical results is provided. Our findings are consistent with those reported by Baillie and Bollerslev (2000) from a structural model which is designed to represent the statistical properties of spot and forward exchange rates that underlie in the uncovered interest parity.

The rest of the paper is organized as follows: section 2 present the model. In section 3 simulations of risk premium are presented and theoretical results about the bias of forward premium are provided. Section 4 refers empirical evidence for the USD-DM exchange rate that is consistent with our theoretical results. Finally, section 5 summarizes and makes concluding remarks.

## 2 The Model

There are two countries with its own currency and a single consumer. In each country the representative firm receives an endowment of a single traded good. The only tradable financial assets are the money forward periods exchange contracts. Also, there is no contingent claims markets, so all possibilities to reduce risk are concerning the forward exchange market, where two maturity contracts are available.

The two consumers own titles to the firms in their respective countries. The timing of the model can be summarized as follows: 1) at the beginning of each period, both firms pay to the respective consumers in its country a dividend equal to all incomes achieved the previous period. Then, the consumer turns in its dividends for a new money, and the old money becomes worthless. This implies that all money will be spent; 2) after receiving the money supply, consumers liquidate their forward contracts traded in foreign exchange in the two previous periods, 3) consumers spend their money on the two goods. Domestic goods must be purchased with its own currency. All transactions take place at equilibrium prices. 4) At the end of each period, consumers make forward contracts to delivery of currency in the next two periods.

Endowments of goods and money supplies are stochastic, and its natural logarithm follow an autoregressive process with a Normal innovation. Let us to denote  $X_t$  and  $M_t$  for any good endowment or money supply, respectively:

$$\ln X_t = \mu_X(1 - \rho_X) + \rho_X \ln X_{t-1} + \xi_{X,t} , \quad \xi_{X,t} \sim N(0, \sigma_X^2) , \quad (1)$$

$$\ln X_t^* = \mu_{X^*}(1 - \rho_{X^*}) + \rho_{X^*} \ln X_{t-1}^* + \xi_{X^*,t}^* , \quad \xi_{X^*,t}^* \sim N(0, \sigma_{X^*}^2) , \quad (2)$$

$$\ln M_t = \mu_M(1 - \rho_M) + \rho_M \ln M_{t-1} + \xi_{M,t} , \quad \xi_{M,t} \sim N(0, \sigma_M^2) , \quad (3)$$

$$\ln M_t^* = \mu_{M^*}(1 - \rho_{M^*}) + \rho_{M^*} \ln M_{t-1}^* + \xi_{M^*,t}^* , \quad \xi_{M^*,t}^* \sim N(0, \sigma_{M^*}^2) , \quad (4)$$

where the asterisk denotes the foreign country. Correlations between any four shocks ( $\rho_{MM^*}$ ,  $\rho_{XX^*}$ ,  $\rho_{MX}$ ,  $\rho_{MX^*}$ ,  $\rho_{M^*X}$ ,  $\rho_{M^*X^*}$ ) are initially restricted to be zero.

### 2.1 The Consumer's problem

The utility function of the home consumer is a CES function:

$$U_t = \frac{1}{1-\gamma} [\phi (C_{D,t})^\epsilon + (1-\phi) (C_{F,t})^\epsilon]^{(1-\gamma)/\epsilon} , \quad (5)$$

where  $C_{D,t}$  and  $C_{F,t}$  are the consumption levels of domestic and foreign goods at time  $t$ ,  $\gamma$  is the coefficient of relative risk aversion, and  $\frac{1}{1-\epsilon}$  is the elasticity of substitution, and  $\phi$  is the weight for each consumption good. If  $\epsilon$  approaches to zero consumption goods becomes more complementarities, whereas perfect substitutability arises when  $\epsilon$  is equal to one. The parameter  $\phi$  measures the weight of each consumption good in the utility function. The optimization problem for the home consumer is:

$$\begin{aligned} \text{Max} \quad & E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} [\phi (C_{D,t})^\epsilon + (1-\phi) (C_{F,t})^\epsilon]^{(1-\gamma)/\epsilon} \right] \quad (6) \\ & \{C_{D,t}, C_{F,t}\} \\ \text{s.t.} \quad & \end{aligned}$$

$$\begin{aligned} & P_{D,t}C_{D,t} + S_t P_{F,t}C_{F,t} \leq Y_t , \\ Y_t = & M_t + T_{t-1,1} \frac{S_t - F_{t-1,1}}{F_{t-1,1}} + T_{t-2,2} \frac{S_t - F_{t-2,2}}{F_{t-2,2}} , \end{aligned}$$

where  $P_{D,t}$  and  $P_{F,t}$  are the prices of domestic and foreign goods at time  $t$ ,  $Y_t$  is the total income in period  $t$ ,  $S_t$  is the spot exchange rate,  $F_{t-1,1}$ ,  $F_{t-2,2}$  are the prices for the two maturity forward contracts available,  $T_{t-1,1}$  and  $T_{t-2,2}$  are the respective amount of its currency that the home country sold forward in the two previous periods. The money supply ( $M_t$ ) plus the profits on each forward currency trading in period  $t$  equals the total home income. A similar optimization problem can be pointed out for the foreign consumer, that is:

$$\begin{aligned} \text{Max} \quad & E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} [\phi (C_{D,t}^*)^\epsilon + (1-\phi) (C_{F,t}^*)^\epsilon]^{(1-\gamma)/\epsilon} \right] \quad (7) \\ & \{C_{D,t}^*, C_{F,t}^*\} \\ \text{s.t.} \quad & \end{aligned}$$

$$\begin{aligned} & P_{D,t}C_{D,t}^* + S_t P_{F,t}C_{F,t}^* \leq Y_t^* S_t , \\ Y_t^* = & M_t^* + T_{t-1,1}^* \frac{S_t - F_{t-1,1}}{F_{t-1,1} S_t} + T_{t-2,2}^* \frac{S_t - F_{t-2,2}}{F_{t-2,2} S_t} . \end{aligned}$$

### 2.1.1 Optimal good choices.

In any period  $t$  the home consumer chooses levels of  $C_{D,t}$  and  $C_{F,t}$  that maximize  $U_t$  subject to the level of total home income. First order conditions for choice of  $C_{D,t}$  and  $C_{F,t}$  are:

$$[\phi (C_{D,t})^\epsilon + (1 - \phi) (C_{F,t})^\epsilon]^{\frac{1-\sigma}{\epsilon}-1} (C_{D,t})^{\epsilon-1} - \lambda_t P_{D,t} = 0 \quad , \quad (8)$$

$$[\phi (C_{D,t})^\epsilon + (1 - \phi) (C_{F,t})^\epsilon]^{\frac{1-\sigma}{\epsilon}-1} (C_{F,t})^{\epsilon-1} - \lambda_t S_t P_{F,t} = 0 \quad , \quad (9)$$

$$Y_t - P_{D,t} C_{D,t} - P_{F,t} S_t C_{F,t} = 0 \quad . \quad (10)$$

From 8 and 9 yields the following relationships:

$$C_{F,t} = \left[ \frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t} \quad , \quad (11)$$

where  $\sigma = \frac{1}{1-\epsilon}$  is the elasticity of substitution. Using 11 and the budget constraint, the demand function for the domestic good is the following:

$$C_{D,t} = \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi}\right)^\sigma (S_t P_{F,t})^{1-\sigma}} \quad . \quad (12)$$

Substituting 12 into equation 11 the next demand function for the foreign good arises:

$$C_{F,t} = \left[ \frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi}\right)^\sigma (S_t P_{F,t})^{1-\sigma}} \quad (13)$$

Similar equation to (12) and (13) can be easily found for the foreign country:

$$C_{F,t}^* = \left[ \frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}^* \quad (14)$$

$$C_{D,t}^* = \frac{Y_t^* S_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi}\right)^\sigma (S_t P_{F,t})^{1-\sigma}} \quad (15)$$

Substituting 15 into 14 we obtain the analytical expression for  $C_{F,t}^*$ .

### 2.1.2 Forward Contracting

As well as the allocation of current resources between the two goods, the home consumer choose in period  $t$  the levels of the one and two periods forward contracting, that is  $T_{t,1}$  and  $T_{t,2}$ . The Euler conditions are:

$$E_t \left[ \lambda_{t+1} \beta^{t+1} \left( \frac{S_{t+1} - F_{t,1}}{F_{t,1}} \right) \right] = 0 \quad , \quad (16)$$

$$E_t \left[ \lambda_{t+2} \beta^{t+2} \left( \frac{S_{t+2} - F_{t,2}}{F_{t,2}} \right) \right] = 0 \quad , \quad (17)$$

where  $E_t$  denotes the conditional expectation to the information set available in period  $t$ . From 16:

$$E_t [\lambda_{t+1} S_{t+1}] = E_t [\lambda_{t+1} F_{t,1}] \quad ,$$

and taking into account 8 yields:

$$F_{t,1} = \frac{E_t \left[ \frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1}} \right]}{E_t \left[ \frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1} S_{t+1}} \right]} \quad . \quad (18)$$

Similar rearranging from 17 when taking into account 8 leads to the following expression for the two-periods forward price:

$$F_{t,2} = \frac{E_t \left[ \frac{\partial U_{t+2}}{\partial C_{F,t+2}} \frac{1}{P_{F,t+2}} \right]}{E_t \left[ \frac{\partial U_{t+2}}{\partial C_{F,t+2}} \frac{1}{P_{F,t+2} S_{t+2}} \right]} \quad . \quad (19)$$

Analogous expressions to (18) and (19) can be obtained when the foreign consumer chooses in period  $t$  the levels of the one and two periods forward contracting, that is  $T_{t,1}^*$  and  $T_{t,2}^*$ :

$$F_{t,1}^* = \frac{E_t \left[ \frac{\partial U_{t+1}^*}{\partial C_{F,t+1}^*} \frac{1}{P_{F,t+1}^*} \right]}{E_t \left[ \frac{\partial U_{t+1}^*}{\partial C_{F,t+1}^*} \frac{1}{P_{F,t+1}^* S_{t+1}^*} \right]} \quad , \quad (20)$$

$$F_{t,2}^* = \frac{E_t \left[ \frac{\partial U_{t+2}^*}{\partial C_{F,t+2}^*} \frac{1}{P_{F,t+2}^*} \right]}{E_t \left[ \frac{\partial U_{t+2}^*}{\partial C_{F,t+2}^*} \frac{1}{P_{F,t+2}^* S_{t+2}^*} \right]} \quad . \quad (21)$$



## 2.2 Market-Clearing

### 2.2.1 Equilibrium in the Goods Market.

The world constraints on consumptions of the two traded goods in both countries implies that the total endowment of the two goods must be equal the consumption of each good in the respective countries, that is:

$$C_{D,t} + C_{D,t}^* = X_{D,t} \quad , \quad (22)$$

$$C_{F,t} + C_{F,t}^* = X_{F,t} \quad . \quad (23)$$

Equilibrium prices of the two goods depend on the home and foreign money supplies as well as their total endowment in each country. Taking into account that a) money is worthless after each period and b) each country's good only can be purchased with the country's currency, the following cash-in-advance spending constraints must be hold:

$$P_{D,t}X_{D,t} = M_t \quad , \quad (24)$$

$$P_{F,t}X_{F,t} = M_t^* \quad . \quad (25)$$

Since goods endowments  $X_{D,t}$  and  $X_{F,t}$ , and money supplies  $M_t$  and  $M_t^*$  are exogenous, the two above equations determine prices of consumption goods.

The solution of the model requires the evaluation of expectations in equations 18 and 19, in where highly non-linear expressions appear. This avoids the possibility of an analytical solution. Appendix 1 provides detailed explanation about the solution method to obtain simulated equilibrium in spot and forward exchange markets. It allows the joint search of all variables (prices and positions) concerning the forward market. In equilibrium, the following relationships between home and foreign derivative positions holds

$$T_{t-l,l} = -T_{t-l,l}^* \quad , \quad l = 1, 2. \quad (26)$$

## 3 Simulation of forward prices and risk premiums

The equilibrium spot rates can be obtained as follows: using the budget constraints,  $P_{D,t} C_{D,t} + S_t P_{F,t} C_{F,t} = Y_t$  and  $P_{D,t} C_{D,t}^* + S_t P_{F,t} C_{F,t}^* = Y_t^* S_t$

and equations (11) and (14), we can solve analytically the spot exchange as a function of the exogenous stochastic variables  $X_{D,t}$ ,  $X_{F,t}$ ,  $M_t$ ,  $M_t^*$ :

$$S_t = \frac{1 - \phi}{\phi} \left( \frac{X_{F,t}}{X_{D,t}} \right)^\varepsilon \frac{M_t}{M_t^*}. \quad (27)$$

### 3.1 Definition of Risk Premium

To avoid the implications of Siegel's paradox we use the following definition of the risk premium in the forward market:

$$rp_{t,t+l} = f_{t,l} - E_t(s_{t+l}), \quad l = 1, 2. \quad (28)$$

where  $E_t(\cdot)$  denotes the mathematical expectation conditioned on the set of all relevant information at time  $t$ ,  $s_t$  is the logarithm of the domestic currency price of foreign currency at time  $t$  and  $f_{t,l}$  is the logarithm of the forward exchange rate with delivery at time  $t + l$ .

### 3.2 Parameter scenarios where the forward premium anomaly arises

#### 3.2.1 Testing the unbiasedness hypothesis

The main objective of the paper is to analyze the parameter set that could reproduce the forward premium bias. The central hypothesis that we analyze in this paper is the Uncovered Interest Rate Parity (UIP) condition, which states that:

$$E_t(\Delta s_{t+l}) = f_{t,l} - s_t = i_t - i_t^*, \quad (29)$$

where  $E_t$  denotes the conditional expectation to the information set available on time  $t$ ;  $i_t$  and  $i_t^*$  are the interest rates on domestic and foreign deposits, respectively, and  $\Delta$  denotes the first difference operator, that is,  $\Delta s_{t+l} \equiv s_{t+l} - s_{t+l-1}$ .

To test for unbiasedness hypothesis, the literature has widely focused on the following regression relating the change in the spot rate to the forward-spot spread:

$$\Delta s_{t+l} = \alpha_l + \beta_l(f_{t,l} - s_t) + u_{t+l,l}, \quad (30)$$

The estimation of equation (30) tries to test the ability of the forward-spot differential to forecast the direction of change in spot rate. Regardless the sampling frequency, the UIP condition implies that  $\alpha_l = 0$  and  $\beta_l = 1$ . However, empirical evidence has widely reported on estimated slopes that turn out to be below than one or even negatives<sup>1</sup>. This finding not only reject the UIP condition, but also is contradictory with either form of the expectations hypothesis.

The analytical expression for the OLS estimation of  $\beta_l$  is:

$$\beta^{ols} = \frac{Cov(f_{t,l} - s_t, s_{t+l} - s_t)}{Var(f_{t,l} - s_t)}, \quad (31)$$

where  $Var(\cdot)$  refers to variance, and  $Cov(\cdot)$  denotes the covariance. As pointed out in Engel (1995), if the estimator is consistent, under rational expectations it follows that:

$$p \lim(\beta^{ols}) = 1 - \beta_{rp} \quad (32)$$

where  $\beta_{rp} = \frac{Cov(E_t(s_{t+l}) - s_t, f_{t,l} - E_t(s_{t+l})) + Var(f_{t,l} - E_t(s_{t+l}))}{Var(f_{t,l} - s_t)}$ . From this expression it can be observed that low values of  $\beta^{ols}$  can be explained under rational expectations if  $Var(f_{t,l} - E_t(s_{t+l}))$  is enough large. The risk premium is widely considered the most likely source of the puzzle, but taking into account the regression results reported in the literature the required volatility are far larger than most researchers would accept. One of the major task in the literature concerns to explain why the risk premium has such a large variance. Our model provide some insights about this issue.

### 3.2.2 Theoretical Results

In all numerical simulations the discount factor  $\beta$  and the relative risk aversion  $\gamma$  are constant and equal to 0.99 and 1.50, respectively<sup>2</sup>. We consider a variety of scenarios than can be summarized as follows: a) we focus the analysis on the effects of the monetary policy (we leave further work the analysis of the effects of real shocks on risk premia in forward markets for

---

<sup>1</sup>A recent survey can be found in Engel (1995).

<sup>2</sup>Parameter values inside the interval [0.90; 0.99] and [1.10; 5.00] for  $\beta$  and  $\gamma$ , lead to similar results to those reported in the paper.

foreign exchange). Therefore, only a uncertainty source is considered: monetary shocks. This way we consider either one or two shocks; b) we distinguish between situations in where there is no persistency in the shocks of both countries from other ones in where only the home country have persistency in the monetary shock<sup>3</sup>. The nature of the interaction between monetary policies is also examined. When two shocks are considered we allow for three possibilities: uncorrelated, positive and negatively correlated monetary shocks. The considered absolute value for the correlation coefficient between domestic and foreign shocks is 0.9. Tables 1 to 8 (Appendix 2) summarize the theoretical results from estimating equation 30 using simulated spot and forward exchange rates with  $\varepsilon = 1$ <sup>4</sup>. Several interesting questions emerge from those tables:

1. The estimated slopes are systematically lower than one, a consistent finding with expression 30, as appointed out by Engel (1996). This means that  $\beta_{rp} > 0$ . This finding has been documented in many empirical studies (see, for example, Bilson (1981), Fama (1984), Bekaert and Hodrick (1993), Backus et al. (1993) and Mark et al. (1993)).
2. There is a negative relationship between the estimated slope coefficient and the time to maturity.
3. A relative higher persistency in the monetary policy of the domestic country produces lower estimated value for the slope. This finding is consistent with those reported in Baillie and Bollerslev (2000). Those authors simulate forward premiums according to a highly stylized UIP-FIGARCH model (Fractionally Integrated GARCH model), showing that a long memory in the forward premium produces wide dispersion in the slope coefficients. Tauchen (2001) simulates the sampling distribution of the slope coefficient in equation (30), showing that such to be the case when spot rates are generated with a near to non-stationary AR(1) process. This is not surprising when equation (27) is observed. Under high persistency in the monetary policy of the domestic country,

---

<sup>3</sup>Under no correlation between monetary shocks this situation can be interpreted as the home country behaves as a *leader* since it can update the forecasting of money supply. The considered autoregressive parameter is 0.9.

<sup>4</sup>Similar results are found with  $\varepsilon = 0$ , which are available from the authors upon request.

spot rate is very autocorrelated, and consequently the forward premium should have high persistency;

4. More interestingly, our model suggest that under a relative high persistency in the domestic/foreign monetary policy the volatility of the forward premium is greater. For example, comparing Table 1 and 2, it can be observed that the volatility under persistency is above five times the volatility that corresponds to the case where monetary policy forecast can not be updated using current information. Qualitative similar results arise when comparing Table 3 to 6, Table 4 to 7 and Table 5 to 8;
5. Also, the transmission of the monetary policy effects between both countries appears to be a significant factor to explain departures from the UIP. When monetary shocks are positively correlated the estimated slope show higher discrepancy with the unitary value. Indeed the maximum average anomaly appears when shocks are positively correlated and the domestic monetary policy is very persistent.
6. The UIP condition only holds when, under no persistency, monetary shocks are either uncorrelated or negatively correlated. However, this is a non-realistic scenario for most of the developed economies, which generally take as a benchmark the Fed ´s monetary policy.

In summary, our model suggest that the anomaly should appear when one country act as a leader and a high persistent monetary policy is applied. Such is the case in most of empirical analysis that concerns the dollar exchange rate. In the next section we provide empirical evidence by analyzing the ability of the model to reproduce the bias when actual monetary shocks are used.

## **4 Empirical evidence regarding the transmission: The US-Germany case.**

The US-German exchange market is specially interesting. Kim and Roubini (2000) suggest a identification scheme using VAR methodology where the money supply equation is assumed to be the reaction function of the monetary authority. Those authors provide a solution to the *forward discount*

*bias puzzle* for most G7 countries with monthly data along the period 1974:4 to 1992:2. However, the impulse response function of exchange rate for German Mark versus US dollar suggest that monetary shocks produce at some horizons (12-24 months) significant and persistent excess returns that would be inconsistent with UIP condition.

In this section we focus the analysis on a most recent period. Monthly data from 1988:12 to 2001:01 are used. As we have already mentioned, our model suggests that the nature of the correlation between monetary shocks can explain the variability of the slopes. Such issue is just we want to test for the US-German case. The two times to maturity that we consider are 1 and 2 months.

Figures 1 and 2 depict the corresponding confidence intervals at the 95% level of estimated slopes for both maturities from 5-year rolling regressions, with the first estimate obtained by beginning at 1992:06 and using a total of 60 observations through 1997:06. Even though based on the asymptotic two standard errors no significant discrepancies with the unitary value arise, a bias in the majority of the sample period can be observed. But also, and more importantly, the accuracy of the point estimates is extremely low, as reflecting the wide range for the confidence intervals. What about the ability of the model to reproduce such patterns when the correlation between the monetary policies is taken into account?. To answer this question we proceed as follows:

1. To simulate the model, stationary series for both monetary aggregates must be used. Then, we apply the Hodrick-Prescott filter to the monthly M1 series of US and German economy to decompose into trend and cyclical component the time evolution of this variable.
2. A bivariate VAR(12)<sup>5</sup> is estimated with the M1 cyclical components. Table 9 provides the parameter estimates.
3. We generate M1 cycles for both countries by using the estimated VAR coefficients and five hundred *i.i.d.* disturbances. We repeat one hundred times this step to generate such number of M1 cycles. Then, we remove the first 386 observations for each time series.

---

<sup>5</sup>The number of lags is identified using the following statistic:  $(T - c) |\ln |\Sigma_1| - \ln |\Sigma_2||$  where  $T$  denotes the sample size,  $\Sigma_1$  and  $\Sigma_2$  are the covariance matrix under the null and alternative hypothesis respectively, and  $c$  is the correction proposed in Sims (1980).

4. Finally we simulate the theoretical model by using the above M1 series as  $M_t$  and  $M_t^*$ .

Figures 3 and 4 show the rolling  $\beta_l$  coefficients that we estimate from simulated series and the actual ones for  $l=1$  month and  $l=2$  months, respectively, jointly with the rolling correlations coefficients between the cyclical components of M1 monetary aggregates. Even though the level of the actual and simulated rolling slope coefficients is different, interestingly enough the pattern is very similar, revealing that the model can reproduce the bias of tests for a risk premium for the DM-\$ forward exchange rate. The model explains the fluctuations of the values for estimated slopes rather than to exactly fit the actual pattern. However, the second issue should be, and really is, the main objective. To quantify the ability of the model to reproduce the already mentioned pattern we regress the actual slopes on the simulated ones, yielding a R-squared of 0.25 and 0.28 for the one and two periods-ahead, respectively, suggesting that our model partially reproduces the total variability of current slopes.

Also, as predicted by the model, Figures 3 and 4 suggest the existence of a relationship between the nature of the monetary shocks and the variability of the estimated slopes. To account for this statement, we perform the following regression:

$$\beta_{l,t} = \delta_0 + \delta_1 \rho_{MM^*,t} + u_{l,t} \quad l = 1, 2 \quad (33)$$

where  $\beta_{l,t}$  denotes the actual rolling slope and  $\rho_{MM^*,t}$  the correlation coefficient between the cyclical M1 components. According with our theoretical results, a R-squared of 0.15 and 0.74 is respectively obtained with  $l = 1$  and 2, revealing that, at least for the DM-USD exchange rate, the correlation between monetary shocks partially explain the bias for of tests for a risk premium in forward exchange rates.

## 5 Summary and concluding remarks

In this paper we examine the bias of tests for a risk premium in forward exchange rates which refers to significant discrepancies with the unitary value in the estimated slope coefficients from regressions of the change in the logarithm of the spot rate on the forward premium. We perform a theoretical analysis by extending the dynamic and stochastic general equilibrium model

with goods endowment proposed in Dutton (1993). Our contribution is the introduction of a two-period forward contract in the derivative market. Also, a solution method under rational expectations is provided.

Our main objective is to explore the effects of the monetary policy and their interactions between the domestic and foreign country on the behavior of the risk premium in order to explain the inconsistency with the UIP condition. Our simulation results suggest that a high persistency in the domestic monetary policy produces greater volatility in the forward premium, and consequently the estimated slope coefficients show greater deviations from one. Moreover, the nature of the transmission between monetary shocks is a potential explaining factor for excess returns puzzle. Under persistency, the estimated slopes dramatically decrease below one when monetary shocks are positively correlated. Finally, we find that the time to maturity of the derivative contract is positively related with the bias of risk premium in forward exchange rates. The UIP condition only holds in the absence of persistency when monetary shocks are uncorrelated or negatively correlated. However, this is an unlikely scenario for most of developed economies.

Empirical evidence for the German mark-US dollar exchange rate is provided, supporting the existence of a relationship between the nature of the monetary policies and the variability of the slopes. The model can reproduce the pattern of actual slope coefficients when simulations are carried out by using monetary shocks that we obtain from the estimation of a bivariate VAR on the M1 monetary aggregates.

While the focus of this paper is the effect of the monetary policy, a similar analysis can be made taking into account the presence of both monetary and real shocks. We leave further work under such scenarios for further research.

## References

- [1] Backus, D., Gregory A. and C. Telmer (1993), Accounting for forward rates in markets for foreign currency, *Journal of Finance* 48, 1887-1908.
- [2] Baillie, R.T. and T. Bollerslev (2000), The forward premium anomaly is not as bad as you think, *Journal of International Money and Finance*, 19, 471-488.
- [3] Bilson, J. (1981), The "speculative efficiency" hypothesis, *Journal of Business* 54:435-452.



- [4] Bekaert, G.(1994), Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model, *Journal of International Economics* 36, 29-52.
- [5] Bekaert, G. and R. J. Hodrick (1993), On biases in the measurement of foreign exchange risk premiums, *Journal of International Money and Finance*, 12, 115-138.
- [6] Canova F. and J. Marrinan (1993), Profits, risk and uncertainty in foreign exchange markets, *Journal of Monetary Economics* 32, 259-286.
- [7] Dutton, J. (1993), Real and monetary shocks and risk premia in forward markets for foreign exchange, *Journal of Money Credit and Banking* 25(4), 731-754.
- [8] Engle C. (1996), The forward discount anomaly and the risk premium: A survey of recent evidence, *Journal of Empirical Finance* 3, 123-192.
- [9] Fama, E.F. (1984), Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319-338.
- [10] Hodrick, R.J. (1989), Risk, uncertainty and exchange rates, *Journal of Monetary Economics* 23, 433-459.
- [11] Kim, S. and N. Roubini (2000), Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach, *Journal of Monetary Economics* 45, 561-586.
- [12] Lewis, K.K. (1995). Puzzles in international financial markets. In: Grossman, G. and K. Rogooff (Eds.). *Handbook of International Economics*, vol. 3. North Holland, Amsterdam, pp. 1913-1971.
- [13] Lucas, Robert (1982), Interest rates and currency prices in a two-country world, *Journal of Monetary Economics* 10, 335-59.
- [14] Mackelm, R.T. (1991), Forward exchange rates in artificial economies, *Journal of International Money and Finance*, 10, 365-391.
- [15] Nelson, M., Wu, Y. and W. Hai (1993), Understanding spot and forward exchange rate regressions (Ohio State University, Columbus, OH).
- [16] Sims, C. (1980), Macroeconomics and reality, *Econometrica* 48, 1-49.

- [17] Tauchen, G. (2001), The bias of tests for a risk premium in forward exchange rates, *Journal of Empirical Finance* 8, 695-704.

## Appendix 1. Solution Method

This appendix contains the step that we use in the solution method. As we pointed out in Section 3, the problem concerning the home and foreign consumer is highly non-linear, not allowing to achieve an analytical solution. Therefore a numerical approach must be used.

After providing numerical values for the structural parameters involved in the theoretical economy, that is,  $\{\beta, \gamma, \phi, \varepsilon, \sigma_X, \sigma_{X^*}, \sigma_M, \sigma_{M^*}, \mu_X, \mu_{X^*}, \mu_M, \mu_{M^*}, \rho_X, \rho_{X^*}, \rho_M, \rho_{M^*}\}$ , the next stages are:

1. We obtain one hundred realizations for the stochastic variables  $X_{D,t}, X_{F,t}, M_t, M_t^*$  in each time period  $t = 1, \dots, 100$ .
2. One hundred realizations of both home and foreign prices of the consumption goods are computed according to equations (24) and (25), in each time period. Let us to denote this numerical set as  $\{(P_{D,t,i}, P_{F,t,i}), i, t = 1, \dots, 100\}$ , where  $i$  and  $t$  denote the realization and the time period, respectively.
3. Similar numerical set to the previous one for  $P_D$  and  $P_F$  is computed for the spot exchange rate using equation (27), that is,  $\{S_{t,i}, i, t = 1, \dots, 100\}$ .

Computation of the forward prices and derivative positions for the one and two period ahead traded contracts  $[F_{t,1}, F_{t,2}, T_{t,1}, T_{t,2}]$ . From equations (11) and (14), substituting into equations (18) and (19) the following expressions can be obtained:

$$\begin{aligned}
 F_{t,1} &= \frac{E_t \left[ C_{D,t+1}^{\varepsilon-1} \left( \phi C_{D,t+1}^\varepsilon + (1-\phi) C_{F,t+1}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+1}} \right]}{E_t \left[ C_{D,t+1}^{\varepsilon-1} \left( \phi C_{D,t+1}^\varepsilon + (1-\phi) C_{F,t+1}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+1} S_{t+1}} \right]} \\
 &= \frac{E_t [W_{D,t+1}]}{E_t [W_{D,t+1} / S_{t+1}]} \quad , \tag{34}
 \end{aligned}$$

$$\begin{aligned}
F_{t,2} &= \frac{E_t \left[ C_{D,t+2}^{\varepsilon-1} \left( \phi C_{D,t+2}^\varepsilon + (1-\phi) C_{F,t+2}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+2}} \right]}{E_t \left[ C_{D,t+2}^{\varepsilon-1} \left( \phi C_{D,t+2}^\varepsilon + (1-\phi) C_{F,t+2}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+2} S_{t+2}} \right]} \\
&= \frac{E_t [W_{D,t+2}]}{E_t [W_{D,t+2}/S_{t+2}]} \quad , \tag{35}
\end{aligned}$$

$$\begin{aligned}
F_{t,1} &= \frac{E_t \left[ C_{F,t+1}^{*\varepsilon-1} \left( \phi C_{D,t+1}^{*\varepsilon} + (1-\phi) C_{F,t+1}^{*\varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+1}} \right]}{E_t \left[ C_{F,t+1}^{*\varepsilon-1} \left( \phi C_{D,t+1}^{*\varepsilon} + (1-\phi) C_{F,t+1}^{*\varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+1} S_{t+1}} \right]} \\
&= \frac{E_t [W_{F,t+1}]}{E_t [W_{F,t+1}/S_{t+1}]} \quad , \tag{36}
\end{aligned}$$

$$\begin{aligned}
F_{t,2} &= \frac{E_t \left[ C_{F,t+2}^{*\varepsilon-1} \left( \phi C_{D,t+2}^{*\varepsilon} + (1-\phi) C_{F,t+2}^{*\varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+2}} \right]}{E_t \left[ C_{F,t+2}^{*\varepsilon-1} \left( \phi C_{D,t+2}^{*\varepsilon} + (1-\phi) C_{F,t+2}^{*\varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+2} S_{t+2}} \right]} \\
&= \frac{E_t [W_{F,t+2}]}{E_t [W_{F,t+2}/S_{t+2}]} \quad . \tag{37}
\end{aligned}$$

We solve jointly  $F_{t,1}$ ,  $F_{t,2}$ ,  $T_{t,1}$  and  $T_{t,2}$  by searching values that satisfy the following approximations of the equations (34) to (37):

$$\begin{aligned}
F_{t,1} &= \frac{\sum_{i=1}^N \left[ C_{D,t+1,i}^{\varepsilon-1} \left( \phi C_{D,t+1,i}^\varepsilon + (1-\phi) C_{F,t+1,i}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+1,i}} \right]}{\sum_{i=1}^N \left[ C_{D,t+1,i}^{\varepsilon-1} \left( \phi C_{D,t+1,i}^\varepsilon + (1-\phi) C_{F,t+1,i}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+1,i} S_{t+1,i}} \right]} \\
&= \frac{\sum_{i=1}^N [W_{D,t+1}]}{\sum_{i=1}^N [W_{D,t+1}/S_{t+1}]} \quad , \tag{38}
\end{aligned}$$

$$\begin{aligned}
F_{t,2} &= \frac{\sum_{i=1}^N \left[ C_{D,t+2,i}^{\varepsilon-1} \left( \phi C_{D,t+2,i}^\varepsilon + (1-\phi) C_{F,t+2,i}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+2,i}} \right]}{\sum_{i=1}^N \left[ C_{D,t+2,i}^{\varepsilon-1} \left( \phi C_{D,t+2,i}^\varepsilon + (1-\phi) C_{F,t+2,i}^\varepsilon \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{D,t+2,i} S_{t+2,i}} \right]} \\
&= \frac{\sum_{i=1}^N [W_{D,t+2,i}]}{\sum_{i=1}^N [W_{D,t+2,i}/S_{t+2,i}]} \quad , \tag{39}
\end{aligned}$$

$$\begin{aligned}
F_{t,1} &= \frac{\sum_{i=1}^N \left[ C_{F,t+1,i}^{* \varepsilon-1} \left( \phi C_{D,t+1,i}^{* \varepsilon} + (1-\phi) C_{F,t+1,i}^{* \varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+1,i}} \right]}{\sum_{i=1}^N \left[ C_{F,t+1,i}^{* \varepsilon-1} \left( \phi C_{D,t+1,i}^{* \varepsilon} + (1-\phi) C_{F,t+1,i}^{* \varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+1,i} S_{t+1,i}} \right]} \\
&= \frac{\sum_{i=1}^N [W_{F,t+1,i}]}{\sum_{i=1}^N [W_{F,t+1,i}/S_{t+1,i}]}, \tag{40}
\end{aligned}$$

$$\begin{aligned}
F_{t,2} &= \frac{\sum_{i=1}^N \left[ C_{F,t+2,i}^{* \varepsilon-1} \left( \phi C_{D,t+2,i}^{* \varepsilon} + (1-\phi) C_{F,t+2,i}^{* \varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+2,i}} \right]}{\sum_{i=1}^N \left[ C_{F,t+2,i}^{* \varepsilon-1} \left( \phi C_{D,t+2,i}^{* \varepsilon} + (1-\phi) C_{F,t+2,i}^{* \varepsilon} \right)^{\frac{1-\gamma}{\varepsilon}-1} \frac{1}{P_{F,t+2,i} S_{t+2,i}} \right]} \\
&= \frac{\sum_{i=1}^N [W_{F,t+2,i}]}{\sum_{i=1}^N [W_{F,t+2,i}/S_{t+2,i}]}. \tag{41}
\end{aligned}$$

Taking into account that under rational expectations  $E_t[W_{t+1}] = \Psi_1 a_t + E_{t-1}[W_{t+1}]$ , where  $a_t$  is a white noise, the expression of the two period forward price in  $t-1$  is:

$$F_{t-1,2} = \frac{\sum_{i=1}^N [W_{D,t+1,i}] - \Psi_{D,1} \left( W_{D,t} - \sum_{i=1}^N [W_{D,t,i}] \right)}{\sum_{i=1}^N [W_{D,t+1,i}/S_{t+1,i}] - \tilde{\Psi}_{D,1} \left( W_{D,t}/S_t - \sum_{i=1}^N [W_{D,t,i}/S_{t,i}] \right)}, \tag{42}$$

or equivalently for the foreign consumer:

$$F_{t-1,2} = \frac{\sum_{i=1}^N [W_{F,t+1,i}] - \Psi_{F,1} \left( W_{F,t} - \sum_{i=1}^N [W_{F,t,i}] \right)}{\sum_{i=1}^N [W_{F,t+1,i}/S_{t+1,i}] - \tilde{\Psi}_{F,1} \left( W_{F,t}/S_t - \sum_{i=1}^N [W_{F,t,i}/S_{t,i}] \right)}. \tag{43}$$

Next, we proceed as follows:

i) We posit initial conditions for the parameters  $\{\Psi_{D,1}^{(0)}, \tilde{\Psi}_{D,1}^{(0)}, \Psi_{F,1}^{(0)}, \tilde{\Psi}_{F,1}^{(0)}\}$ .

ii) Also, we need an initial vector. Let us to denote it by  $\{F_{0,1}, F_{-1,2}, T_{0,1}, T_{-1,1}\}$ . Then, one hundred realizations of  $C_{D,1,i}, C_{F,1,i}, C_{D,1,i}^*, C_{F,1,i}^*, Y_{1,i}, Y_{1,i}^*$  in  $t=1$  trough equations (11), (14) and the following expressions:

$$\begin{aligned}
Y_{1,i} &= M_{1,i} + T_{0,1} \left( \frac{S_{1,i} - F_{0,1}}{F_{0,1}} \right) + T_{-1,2} \left( \frac{S_{1,i} - F_{-1,2}}{F_{-1,2}} \right), \\
Y_{1,i}^* &= M_{1,i}^* - T_{0,1} \left( \frac{S_{1,i} - F_{0,1}}{F_{0,1} S_{1,i}} \right) - T_{-1,2} \left( \frac{S_{1,i} - F_{-1,2}}{F_{-1,2} S_{1,i}} \right),
\end{aligned}$$

$$C_{D,1,i} = \frac{Y_{1,i} P_{D,1,i}^{-\sigma}}{P_{D,1,i}^{-\sigma} + \left(\frac{1-\phi}{\phi}\right)^\sigma (S_{1,i} P_{F,1,i})^{1-\sigma}}$$

$$C_{D,1,i}^* = \frac{Y_{1,i}^* S_{1,i} P_{F,1,i}^{-\sigma}}{P_{D,1,i}^{-\sigma} + \left(\frac{1-\phi}{\phi}\right)^\sigma (S_{1,i} P_{F,1,i})^{1-\sigma}}$$

*iii*) With the previous data set,  $\{C_{D,1,i}, C_{F,1,i}, C_{D,1,i}^*, C_{F,1,i}^*, Y_{1,i}, Y_{1,i}^*\}_{i=1}^{100}$ , we iterate using the Gauss-Newton algorithm in the system concerning equations (38), (39), (42) and (43). After achieving the fixed point in the space  $(F_{1,1}, F_{0,2}, T_{1,1}, T_{0,2})$  and evaluating in  $t = 1$  with the variables  $\{F_{1,1}, F_{0,2}, T_{1,1}, T_{0,2}\}$  the corresponding expressions, it is possible to compute values for  $C_{D,1}, C_{F,1}, C_{D,1}^*, C_{F,1}^*, Y_1, Y_1^*$ , independently of the realization values.

*iv*) The steps *ii*) and *iii*) are repeated recursively for each time period, allowing to obtain the numerical solutions for the remainder of the sample size, that is,  $\{C_{D,t}, C_{F,t}, C_{D,t}^*, C_{F,t}^*, Y_t, Y_t^*\}_{t=2}^{100}$ . However, this solution depends on the initial condition  $\{\Psi_{D,1}^{(0)}, \tilde{\Psi}_{D,1}^{(0)}, \Psi_{F,1}^{(0)}, \tilde{\Psi}_{F,1}^{(0)}\}$ . To filter this effect, we estimate an autoregressive process for the expressions of  $W_{D,t}, (W_{D,t}/S_t), W_{F,t}, (W_{F,t}/S_t)$  that can be computed with the simulated series of the previous solution. We use five lags in the AR specification, a robust structure in order to forecast the previous expressions. With the fitted autoregressive processes, estimation of  $\Psi^s$  are recovered to evaluate the discrepancy with  $(\Psi_{D,1}^{(0)}, \tilde{\Psi}_{D,1}^{(0)}, \Psi_{F,1}^{(0)}, \tilde{\Psi}_{F,1}^{(0)})$  using the euclidean norm. The used convergence criterion is  $10^{-6}$ . When the norm is lower,  $\{C_{D,t}, C_{F,t}, C_{D,t}^*, C_{F,t}^*, Y_t, Y_t^*\}_{t=1}^{100}$  is the final numerical solution, whereas the norm is higher we back to step *i*) to iterate with the new initial condition for the vector  $\{\Psi_{D,1}^{(0)}, \tilde{\Psi}_{D,1}^{(0)}, \Psi_{F,1}^{(0)}, \tilde{\Psi}_{F,1}^{(0)}\}$ .

## Appendix 2. Statistical Tables.

Table 1.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0005	0.0005	0.0005	0.0005
$dt(\hat{\alpha}_l)$	0.0005	0.0005	0.0005	0.0005
$\hat{\beta}_l$	0.7968	0.7959	0.6692	0.6677
$dt(\hat{\beta}_l)$	0.0984	0.0987	0.0830	0.0831
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0017	0.0017	0.0012	0.0012

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = 0.005, \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_{M^*}^2 = \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 2.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0008	0.0008	0.0008	0.0008
$dt(\hat{\alpha}_l)$	0.0006	0.0006	0.0006	0.0006
$\hat{\beta}_l$	0.1339	0.1354	0.0701	0.0706
$dt(\hat{\beta}_l)$	0.0701	0.0508	0.0266	0.0265
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0092	0.0092	0.0168	0.0169

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = 0.005, \rho_M = 0.9, \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_{M^*}^2 = \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 3.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0010	0.0010	0.0010	0.0010
$dt(\hat{\alpha}_l)$	0.0007	0.0007	0.0007	0.0007
$\hat{\beta}_l$	0.8207	0.8224	0.6226	0.6316
$dt(\hat{\beta}_l)$	0.0995	0.0992	0.0748	0.0757
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0019	0.0019	0.0023	0.0022

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = 0, \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 4.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0002	0.0003	0.0002	0.0002
$dt(\hat{\alpha}_l)$	0.0002	0.0002	0.0002	0.0002
$\hat{\beta}_l$	0.7630	0.7719	0.6642	0.6674
$dt(\hat{\beta}_l)$	0.0982	0.0983	0.0851	0.0850
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0008	0.0008	0.0006	0.0006

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = 0.9, \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).



Table 5.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0015	0.0015	0.0014	0.0015
$dt(\hat{\alpha}_l)$	0.0010	0.0010	0.0010	0.0010
$\hat{\beta}_l$	0.9682	0.9677	0.9254	0.9240
$dt(\hat{\beta}_l)$	0.1009	0.1005	0.0962	0.0959
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0020	0.0020	0.0020	0.0020

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = -0.9, \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 6.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0018	0.0018	0.0018	0.0018
$dt(\hat{\alpha}_l)$	0.0009	0.0009	0.0009	0.0009
$\hat{\beta}_l$	0.3094	0.3108	0.1634	0.1633
$dt(\hat{\beta}_l)$	0.0732	0.0736	0.0383	0.0382
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0074	0.0074	0.0153	0.0154

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = 0, \rho_M = 0.9, \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 7.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	-0.0004	-0.0004	-0.0004	-0.0004
$dt(\hat{\alpha}_l)$	0.0005	0.0005	0.0005	0.0005
$\hat{\beta}_l$	0.1121	0.1131	0.0567	0.0570
$dt(\hat{\beta}_l)$	0.0476	0.0478	0.0242	0.0242
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0094	0.0094	0.0187	0.0188

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = 0.9, \rho_M = 0.9, \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 8.

Theoretical result from the estimation of equation (30)				
	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\hat{\alpha}_l$	0.0040	0.0040	0.0040	0.0040
$dt(\hat{\alpha}_l)$	0.0013	0.0013	0.0013	0.0013
$\hat{\beta}_l$	0.4587	0.4607	0.2855	0.3143
$dt(\hat{\beta}_l)$	0.0842	0.0844	0.0519	0.0571
Risk premium volatility				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0070	0.0069	0.0127	0.0109

All statistics have been computed from one hundred replications of the model with a sample size equal to 100.  $\phi$  measures the degree of substitutability or complementarity. The parameters of exogenous processes are the following: ( $\sigma_M^2 = \sigma_{M^*}^2 = 0.005, \rho_{MM^*} = -0.9, \rho_M = 0.9, \rho_{M^*} = \rho_X = \rho_{X^*} = 0, \sigma_X^2 = \sigma_{X^*}^2 = 0$ ).

Table 9.

Parameter estimates in the bivariate VAR				
Dependent variable				
	German M1		US M1	
lag	German M1	US M1	German M1	US M1
1	0.88 (0.10)*	-0.07 (0.25)	-0.03 (0.04)	0.90 (0.10)
2	-0.034 (0.13)	-0.63 (0.36)	0.08 (0.06)	-0.33 (0.15)
3	-0.09 (0.13)	0.76 (0.36)	-0.18 (0.05)	0.28 (0.15)
4	-0.16 (0.12)	-0.57 (0.38)	0.14 (0.05)	0.19 (0.16)
5	0.32 (0.13)	0.06 (0.37)	-0.01 (0.06)	-0.43 (0.16)
6	-0.10 (0.14)	0.51 (0.36)	-0.08 (0.06)	0.61 (0.15)
7	0.05 (0.14)	-0.77 (0.34)	0.09 (0.06)	-0.59 (0.14)
8	-0.44 (0.13)	1.59 (0.31)	-0.16 (0.05)	0.56 (0.13)
9	0.35 (0.12)	-1.67(0.32)	0.10 (0.05)	-0.19 (0.13)
10	-0.28 (0.13)	0.91 (0.33)	-0.03 (0.05)	-0.44 (0.14)
11	0.27 (0.12)	0.09 (0.36)	0.05 (0.05)	0.46 (0.15)
12	-0.04 (0.10)	-0.25 (0.26)	0.02 (0.04)	-0.12 (0.11)

\*Asymptotic standard errors are in parentheses.

### Appendix 3. Figures

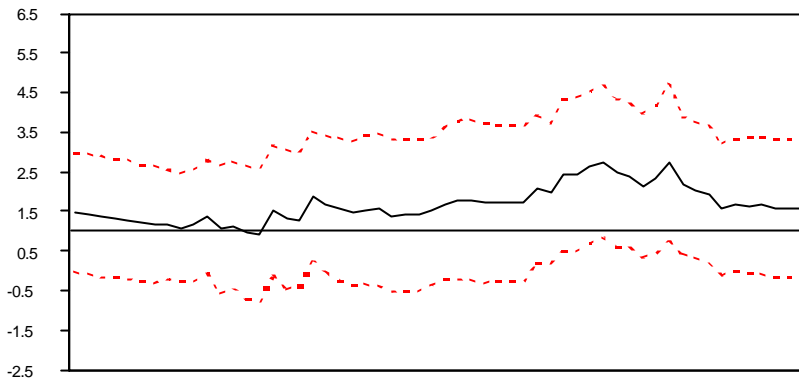


Figure 1. Rolling slopes from one-period ahead forward premium. The dashed lines are the conventional two OLS standard error confidence bands.

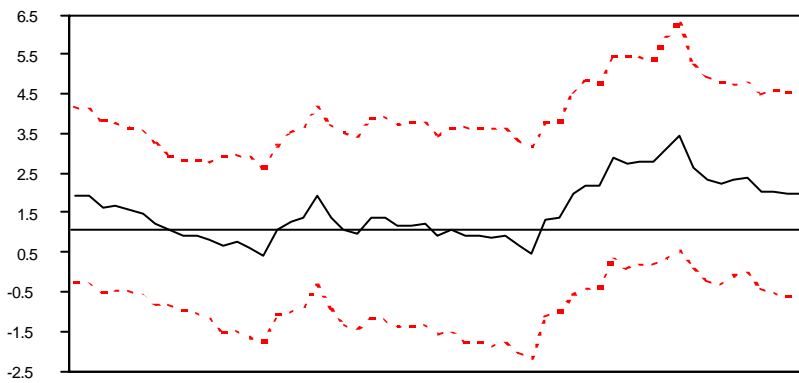


Figure 2. Rolling slopes from two-periods ahead forward premium. The dashed lines are the conventional two OLS standard error confidence bands.

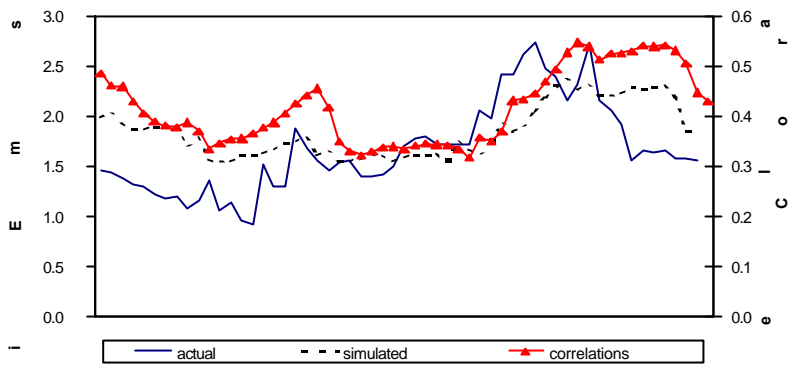


Figure 3. Rolling correlation between M1 cyclical components and rolling slopes from one-period ahead forward premium.

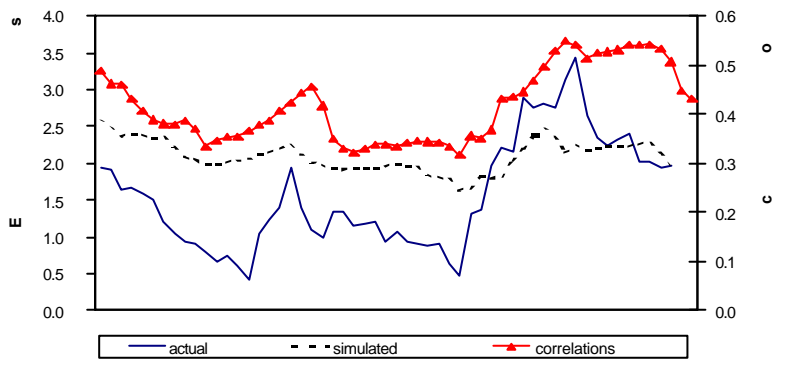


Figure 4. Rolling correlation between M1 cyclical components and rolling slopes from two-periods ahead forward premium.