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<th>Properties of Spectral Shape Parameter k and its Approximate Expression</th>
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<td>YAMAGUCHI, Masataka; NONAKA, Hirokazu</td>
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Properties of Spectral Shape Parameter $\kappa$ and its Approximate Expression

Masataka YAMAGUCHI* and Hirokazu NONAKA**

Using each of the corrected versions of the generalized TMA and Thornton spectra proposed for a universal spectrum of wind waves in finite-depth water conditions, this study investigates properties of the spectral shape parameter $\kappa$ expressed by a complex number with the absolute number $|\kappa|$ and the argument $\psi$, and the relation between $|\kappa|$ and $\psi$, and predicts their approximate expressions with high accuracy both in infinite-depth conditions and in finite-depth conditions. One finding is that $|\kappa|$ and $\psi$ are uniquely related in infinite-depth conditions. Another finding is that the approximate expressions yield reasonable estimates of $|\kappa|$ and $\psi$, in cases where the expression gives the maximum relative error of 0.99% for $|\kappa|$ and that of 0.50% for $\psi$ in a full range of finite-depth conditions.

Key Words: generalized JONSWAP spectrum, infinite-depth conditions, generalized TMA and Thornton spectra, finite-depth conditions, spectral shape parameter with a complex number form $\kappa$, approximate expressions

1. Introduction

Various parameters for representing the shape characteristics of the frequency spectrum of wind-generated waves have been proposed. These are 1) the spectral width parameter $\varepsilon$ using the 0th, 2nd and 4th order spectral moments, 2) the spectral width parameter $\nu$ using the 0th, 1st and 2nd order spectral moments, 3) the spectral peakedness parameter $Q_p$ using the 0th order spectral moment and the 1st order moment of the squared spectrum, 4) the spectral shape parameter $\kappa$ related to the autocorrelation of wave envelopes which was named by Goda and Kudaka [1]. The definitions and general characteristics of these parameters are described in the monograph by Goda [2] and in the report by Kitano [3].

Yamaguchi et al. [4][5] gave the corrected version of each of the generalized TMA(G-TMA) and Thornton(G-Thornton) spectra for a universal spectrum of wind waves in finite-depth water conditions by the introduction of a non-trivial term, which results in the peak value of the spectrum at the supposed peak frequency $f_p$, and then proposed rather highly accurate approximate expressions for the spectral moments-based integral quantities including the spectral width parameter $\nu$ and the spectral peakedness parameter $Q_p$. It may be said that making an approximate expression of the spectral width parameter $\varepsilon$ is unnecessary, because it is not a proper parameter to describe the spectral width due to its strong dependence on the higher frequency tail of the spectrum.

In this study, we investigate the characteristics of the spectral shape parameter $\kappa$ for the universal spectrum in infinite-depth conditions (JONSWAP spectrum) and that in finite-depth conditions (TMA spectrum). Then we construct approximate expressions of the spectral shape parameter $\kappa$ using various types of the

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universal spectra both in infinite-depth conditions and in finite-depth conditions and make their error range clear. As the spectral shape parameter $\kappa$ is expressed as a complex number, we take the property into account in the development of the approximate expression.

2. Universal Spectrum of Wind Waves in Finite-Depth Water

Both the G-TMA spectrum and the G-Thornton spectrum with a separate correction term $\text{Cor}$ are written by the same form of expression as

$$E(f) = \alpha_n(2\pi)g^{-3}u^2(2\pi\mu)^{n+}f_{-}^{m+}\exp\left(-\frac{m}{n}f_{-}^{m+}(1-\text{Cor})\right)\gamma^\frac{(1-i\gamma)}{2\sigma^2} \Phi(kh)$$

(1)

$$\mu = f_p u / g, \ f_{-} = f / f_p$$

(2)

where $E(f)$ is the energy density spectrum (the frequency spectrum), $f$ the frequency, $m, n$ the spectral tail parameters, $\alpha_n$ the equilibrium constant, $g$ the acceleration of gravity, $\gamma$ the peak enhancement parameter, $\sigma$ the peak width parameter ($\sigma = \sigma_p; f \leq f_p, \sigma = \sigma_v; f > f_p$), $u$ the friction velocity of the wind, $\mu$ the dimensionless peak frequency, and $k$ the wave number corresponding to the frequency $f$ and the water depth $h$. For the corrected G-TMA spectrum, the $\text{Cor}$ term and the finite-depth effect term $\Phi(kh)$ are indicated as

$$\text{Cor} = \frac{2k_p h}{m(\sinh k_p h + 2k_p h)} \left(\frac{m-3}{m} + \frac{4k_p h (\cosh k_p h + 1)}{\sinh 2k_p h + 2k_p h}\right)$$

(3)

$$\Phi(kh) = \left(\frac{\tanh kh}{1 + 2k_p / \sinh 2k_p h}\right)^{m-3}$$

(4)

where $k_p$ is the wave number corresponding to the peak frequency $f_p$ and the water depth $h$. Similarly, for the G-Thornton spectrum, the two terms are

$$\text{Cor} = \frac{2(m-3)}{m} \frac{2k_p h}{\sinh 2k_p h + 2k_p h}$$

(5)

$$\Phi(kh) = \left(\tanh kh\right)^{m-3}$$

(6)

In infinite-depth conditions, $\text{Cor} = 0$ and $\Phi(kh) = 1$ in both spectra, and then Eq.(1) reduces to the generalized JONSWAP(G-JONSWAP) spectrum which is expressed as

$$E(f) = \alpha_n(2\pi)g^{-3}u^2(2\pi\mu)^{n+}f_{-}^{m+}\exp\left(-\frac{m}{n}f_{-}^{m+}\right)\gamma^\frac{(1-i\gamma)}{2\sigma^2}$$

(7)

3. Definition of Spectral Shape Parameter $\kappa$ and Its Properties

3.1 Definition of $\kappa$

The expression defining the spectral shape parameter $\kappa$ proposed by Rice $^{(6)}$ is given as

$$\kappa = \frac{1}{m_0} \int_0^\infty E(f) \cos2\pi f T df + \frac{i}{m_0} \int_0^\infty E(f) \sin2\pi f T df = \text{Re}(\kappa) + i \text{Im}(\kappa)$$

(8)

where $m_0$ is the 0th order spectral moment. In Eq.(8), the form re-written by Goda $^{(1)}$ is used and the form of a complex number is adopted according to the suggestion by Kitano $^{(2)}$ and Kitano et al. $^{(7)}$. The spectral shape
parameter $\kappa$ is expressed by using the absolute value $|\kappa|$ and the argument $\psi$ as follows.

$$\kappa = |\kappa|e^{i\psi} = |\kappa|\cos\psi + i|\kappa|\sin\psi, \ \ \psi = \arg\kappa = \tan^{-1}\left\{\text{Im}(\kappa)/\text{Re}(\kappa)\right\} \quad (9)$$

For the averaged wave period $\tilde{T}$ in Eq.(8), the following wave period $T_{n01}$, which is defined by a ratio of the 0th order spectral moment $m_0$ to the 1st order spectral moment $m_1$, is employed in order to retain a consistent expression based on the spectrum and to make the dependency on a higher frequency tail of the spectrum weak.

$$\tilde{T} = T_{n01} = m_0/m_1, \ \ m_n = \int_{a}^{b} f^nE(f)df; \ \ n = 0, 1 \quad (10)$$

3.2 Properties of $\kappa$

A numerical integration using an adaptive Simpson's method is carried out to obtain the data of the spectral shape parameter $\kappa$, because it is impossible to do the analytical integration of Eq.(8). The integrations of $\text{Re}(\kappa)$ and $\text{Im}(\kappa)$ in Eq.(8) are separately conducted in each range of dimensionless frequency $f$ from 0 to 4, 4 to 10, 10 to 20, 20 to 40, 40 to 70, 70 to 100 and 100 to 150 and summed up. The numerical solution for a range of dimensionless frequency from 0 to 10 is about the same as that with a higher upper limit of the integration. This means that the numerical solution with the upper frequency limit of 150 in the integration may correspond to the quasi-exact solution.

First, let us see the characteristics of the spectral shape parameter in infinite-depth condition $\kappa_{\text{deep}}$, where the subscript 'deep' means the infinite-depth condition. Under the spectral parameter conditions with constant values for $m$, $n$, $\sigma_a$ and $\sigma_b$, $\kappa_{\text{deep}}$ is a function of only the peak enhancement parameter $\gamma$. The numerical integrations are made for the 14 cases of $\gamma=1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 8, 9$ and 10. The left panel of Fig. 1 shows the relation between either the absolute value $|\kappa|_{\text{deep}}$ or the argument $\psi_{\text{deep}}$ of the spectral shape parameter $\kappa$ and the peak enhancement parameter $\gamma$ in the case of the JONSWAP spectrum ($m = 5, n = 4, \sigma_a = 0.07, \sigma_b = 0.09$). The $|\kappa|_{\text{deep}}$ increases monotonically from 0.38942 at $\gamma=1$ to 0.71462 at $\gamma=10$ and the $\psi_{\text{deep}}$ does from 5.2547 rad. (301.07 degs.) at $\gamma=1$ to 5.7449 rad. (329.16 degs.) at $\gamma=10$. The right panel of Fig. 1 indicates the relation between $[|\kappa|\left|1 - \cos(\psi/2)\right|]_{\text{deep}} = [|\kappa|\tilde{\psi}_{\text{deep}}]$ and $\gamma$, where $\tilde{\psi}$ is defined as,

$$\tilde{\psi} = 1 - \alpha - \cos(\psi/2) \quad (11)$$

The quantity $[|\kappa|\tilde{\psi}_{\text{deep}}]$ is used as the reference value when the approximate expression of $\kappa$ in finite-depth conditions is constructed. The parameter $\alpha$ in Eq.(11) changes every universal spectrum and $\alpha=0.42404$ holds in both cases of the JONSWAP spectrum in infinite-depth conditions and the TMA spectrum in deep conditions.

![Fig. 1 Change of $|\kappa|_{\text{deep}}$, $\psi_{\text{deep}}$ and $[|\kappa|\tilde{\psi}]_{\text{deep}}$ with increase of $\gamma$.](https://example.com/fig1.png)
finite-depth conditions. The \( |k\bar{\psi}|_{\text{deep}} \) gives a monotonical increase with \( \gamma \) as well as \( |k|_{\text{deep}} \) and \( \psi_{\text{deep}} \). The reason why \( |k\bar{\psi}|_{\text{deep}} \) instead of \( |k|_{\text{deep}} \) is used, will be illustrated later.

Fig. 2 gives the relation between the argument \( \psi_{\text{deep}} \) (degs.) and the absolute value \( |k|_{\text{deep}} \) in the case of the JONSWAP spectrum, where the \( \psi_{\text{deep}} \) data to the minimum value of \( |k|_{\text{deep}} \) on the horizontal axis corresponds to the case of \( \gamma=1 \) and that to the maximum value of \( |k|_{\text{deep}} \) corresponds to the case of \( \gamma=10 \). We can see a uniquely dependent relation between \( \psi_{\text{deep}} \) and \( |k|_{\text{deep}} \). Therefore, either \( |k|_{\text{deep}} \) or \( \psi_{\text{deep}} \) can be estimated from the other using a regression relation between \( |k|_{\text{deep}} \) and \( \psi_{\text{deep}} \). It should be noted here that Kitano et al.\(^{[7]}\) developed a theory in which the ratio between significant wave period \( T_m \) and mean wave period \( T_{\text{m}} \) is expressed as a function of the argument \( \psi_{\text{deep}} \). A uniquely dependent relation between \( \psi_{\text{deep}} \) and \( |k|_{\text{deep}} \) is also observed in the case of another type of universal spectrum with tail parameters such as \( m=4, n=4 \).

Next let us investigate the behavior of the spectral shape parameter \( \kappa \) in finite-depth conditions. Under the fixed conditions of the spectral parameters such as \( m, n, \sigma_v, \sigma_s \), both \( |k| \) and \( \psi \) depend not only on \( \gamma \) but also on the finite-depth effect parameter \( k_p h (=2\pi h/L_p) \), where \( L_p \) is the wave length corresponding to the peak frequency \( f_p \) at the water depth of \( h \). The numerical integration of \( |k| \) and \( \psi \) for the range of dimensionless frequency \( *f \) from 0 to 150 is conducted for \( \gamma \) changing from 1 to 10(1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 8, 9, 10; 14 cases) as well as in the case of infinite-depth conditions and for \( T_p \sqrt{g/h} \) changing from 1 to 10(with an increment of 0.1) and from 11 to 50(with an increment of 1), where \( T_p \) (=1/f \( _p \)) is the peak period. The number of cases in finite-depth conditions for a fixed \( \gamma \) is 131. The total number of runs in finite-depth conditions reaches 14x131=1,834. The parameter \( T_p \sqrt{g/h} \) is transformed into \( h/L_p \) using the following dispersion relationship.

\[
\left(\frac{2\pi}{T_p \sqrt{g/h}}\right)^2 = k_p h \cdot \tanh k_p h
\]

\( h/L_p \) denotes the relation between either the absolute value \( |k| \) or the argument \( \psi \) and the relative water depth \( h/L_p \) for every peak enhancement parameter \( \gamma \) in the case of the TMA spectrum \( m=5, n=4, \sigma_v=0.07, \sigma_s=0.09 \). In the case of \( \gamma=1 \), a decreasing \( h/L_p \) yields the absolute value \( |k| \) which has the maximum value followed by the minimum value and then an increasing trend. In the case of larger \( \gamma \), a decreasing \( h/L_p \) yields the maximum of the absolute value \( |k| \) but does not give the minimum. On the other hand, a decreasing \( h/L_p \) yields the argument \( \psi \) which has the maximum followed by a consistently decreasing trend. The values of \( |k| \) and \( \psi \) become greater with increasing \( \gamma \) for the same \( h/L_p \) respectively.
Fig. 3 Change of either $|k|$ or $\psi$ with decrease of $h/L_p$.

Fig. 4 shows the relation between the argument $\psi$ (degs.) and the absolute value $|k|$ with a parameter of either $T_p \sqrt{g/h}$ or $\gamma$ in the case of the TMA spectrum. In the left panel, the data of $\psi$ to the minimum value of $|k|$ on the horizontal axis for every $T_p \sqrt{g/h}$ corresponds to the case of $\gamma = 1$ and the data of $\psi$ to the maximum value of $|k|$ corresponds to the case of $\gamma = 10$. A uniquely dependent relation between $\psi$ and $|k|$ is observed for a fixed $T_p \sqrt{g/h}$ and $\psi$ increases monotonically with $|k|$. In the right panel, the minimum value of $\psi$ on the vertical axis for every $\gamma$ corresponds to the case of $T_p \sqrt{g/h} = 50$ and the initial value of the opposite side on the same $\psi$ curve corresponds to the case of $T_p \sqrt{g/h} = 1$. In this panel, the relation between $\psi$ and $|k|$ gives a hook (bird-beak)-like variation, and both the region where $\psi$ is a two-valued function of $|k|$ and the region where $|k|$ is a two-valued function of $\psi$ exist on the $\psi - |k|$ curve for a fixed $\gamma$. As a result, the magnitude relation of the $\psi - |k|$ curve for a fixed $T_p \sqrt{g/h}$ changes among 1, 4 and 4.5 of $T_p \sqrt{g/h}$, as can be seen in the left panel. Thus, the $\psi - |k|$ relation in finite-depth conditions gives a complicated behavior with respect to the finite-depth effect parameter such as $T_p \sqrt{g/h}$.

Fig. 4 Relation between $\psi$ and $|k|$ with a parameter of either $T_p \sqrt{g/h}$ or $\gamma$. 

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4. Approximate Expression of Spectral Shape Parameter $\kappa$

4.1 Spectral Conditions

The spectral conditions used for constructing approximate expressions of the absolute value $|\kappa|$ and the argument $\psi$ of the spectral shape parameter $\kappa$ both in infinite-depth conditions and in finite-depth conditions are the 3 cases indicated in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>$m$</th>
<th>$n$</th>
<th>$\sigma_m$</th>
<th>$\sigma_n$</th>
<th>Cor</th>
<th>$\Phi(kh)$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>0.07</td>
<td>0.09</td>
<td>Eq.(3)</td>
<td>Eq.(4)</td>
<td>Corrected TMA spectrum</td>
</tr>
<tr>
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<td>4</td>
<td>4</td>
<td>0.115</td>
<td>0.114</td>
<td>Eq.(3)</td>
<td>Eq.(4)</td>
<td>Corrected FRF spectrum</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
<td>0.07</td>
<td>0.09</td>
<td>Eq.(5)</td>
<td>Eq.(6)</td>
<td>Corrected Thornton spectrum</td>
</tr>
</tbody>
</table>

4.2 Approximate Expressions in Infinite-Depth Conditions

According to Yamaguchi et al.\cite{5}, a 3-point-fixed 4th order polynomial is used to construct each of the approximate expressions of the spectral shape parameters such as $|\kappa|_{\text{deep}}$, $\psi_{\text{deep}}$ and $[|\kappa|\psi]_{\text{deep}}$ as well as the spectral width parameter $\nu_{\text{deep}}$ and the spectral peakedness parameter $(Q_{\gamma})_{\text{deep}}$.

\[
(|\kappa|_{\text{deep}}, \psi_{\text{deep}}, [|\kappa|\psi]_{\text{deep}}) = a\gamma^4 + b\gamma^3 + c\gamma^2 + d\gamma + e
\]  

(13)

Imposing the conditions that the polynomial passes through each of the 3 data points prescribed at $\gamma=1$, $\gamma=10$ and a value of $\gamma$ between 1 and 10, reduces the number of the coefficients to be determined to 2. The coefficients are obtained by applying Eq.(13) to each of the 14 data samples of $|\kappa|_{\text{deep}}$, $\psi_{\text{deep}}$ and $[|\kappa|\psi]_{\text{deep}}$ corresponding to $\gamma$ between 1 and 10 by use of the least-squares-method.

The curve of Eq.(13) fitted to each sample of $|\kappa|_{\text{deep}}$, $\psi_{\text{deep}}$ and $[|\kappa|\psi]_{\text{deep}}$ in the case of the JONSWAP spectrum is given in the above Fig. 1. The relative error $\delta$ for each of $|\kappa|_{\text{deep}}$ and $\psi_{\text{deep}}$ ranges from -0.26 to 0.30 % and from -0.07 to 0.06 %, which suggests an excellent goodness-of-fit respectively. Also, the relative error of Eq.(13) fitted to the data sample of $[|\kappa|\psi]_{\text{deep}}$ is between -0.19 and 0.22 % and the accuracy of Eq.(13) is high. Quantity such as $[|\kappa|\psi]_{\text{deep}}$ is introduced as an alternative of $|\kappa|_{\text{deep}}$ for the sake of a substantial impossibility of properly approximating the complicated behavior of $|\kappa|_{\text{deep}}$ in finite-depth conditions. We can say that Eq.(13)-based approximate expression has a satisfactory applicability in infinite-depth conditions.

Table 2 summarizes the coefficients($a$, $b$, $c$, $d$, $e$) of the approximate expression for each of $|\kappa|_{\text{deep}}$, $\psi_{\text{deep}}$ and $[|\kappa|\psi]_{\text{deep}}$, and a range of the relative error $\delta_{\text{deep}}$ in the 3 cases given in Table 1. The table includes the value of $\alpha$ for each case. The maximum absolute value of the relative error $\delta_{\text{deep}}$ is 0.30 % for $|\kappa|_{\text{deep}}$, 0.15 % for $\psi_{\text{deep}}$ and 0.25 % for $[|\kappa|\psi]_{\text{deep}}$ respectively. This suggests that Eq.(13) provides a good fit when calculating any of the given spectral conditions.

4.3 Approximate Expressions in Finite-Depth Conditions

Yamaguchi et al.\cite{4,5} conducted a trial-fitting of the following expression with 6 unknown parameters($p$, $q$, $r$, $s$, $u$, $w$) to each of the spectrum moment-based integral quantities in finite-depth non-dimensionalized conditions, with that in infinite-depth conditions in the cases of the corrected G-TMA and G-Thornton spectra.

\[
f(k,h) = \left(1 + \frac{r(k,h)^u}{\sinh r(k,h)^u}\right)^p \cdot \left\{\tanh s(k,h)^u\right\}^q
\]  

(14)
and estimated the 6 parameters (p, q, r, s, u, w) in Eq.(14). Eq.(14) reduces to 1 in infinite-depth condition \((k, h \to \infty)\) and is asymptotic to 0 in nearly zero-water depth condition \((k, h \to 0)\). Simultaneous determination of the 6 parameters is very difficult. Therefore, the least-squares-method-based solution of the 3 parameters \((p, q, r)\) is obtained from the log-transformed Eq.(14) under prescribed combining for the remaining 3 parameters \((s, u, w)\), and then a quasi-optimum solution for the 6 parameters is selected from the solution of the parameters \((p, q, r)\) for a huge number of combinations of the prescribed parameters \((s, u, w)\) in the sense of least-squares-errors. In this study, Eq.(14) is fitted to each of the following quantities.

\[
\Psi = \left\{ \psi, \psi_{\text{deep}} - g(\gamma) \right\}/\left\{ 1 - g(\gamma) \right\}, \quad K\Psi = \left\{ |k| \bar{\psi}/\left[ |k| \psi_{\text{deep}} - \beta h(\gamma) \right]/\left\{ 1 - \beta h(\gamma) \right\} \right\}
\]

(15)

where each of \( g(\gamma) \) and \( h(\gamma) \) is a function of \( \gamma \) and a constant value is given to \( \beta \). The same method as Yamaguchi et al.\[3,4,5\] is used for estimating the 6 parameters \((p, q, r, s, u, w)\). The number of combinations of the parameters \((s, u, w)\) is 27,000, in cases where each parameter changes from 0.1 to 3.0 with an increment of 0.1.

The history (progress) in which \( \psi, \Psi \) and \( K\Psi \) instead of \( \psi \) and \( |k| \psi \) come to be used in constructing the approximate expressions related to \(|k|\) in finite-depth conditions is described as follows.

1) Change of \( \psi/\psi_{\text{deep}} \) with \( k/h \) is similar to Eq.(14) and may be approximated by use of Eq.(14).

2) As \( |k|/|k|_{\text{deep}} \) has the local maximum and minimum with respect to \( k/h \), as can be observed in Fig. 3, the behavior may not be approximated well with Eq.(14). For that reason, \( k/h \)-dependence of the quantity combined \(|k|\) with \( \psi \) such as \(|k|\cos\psi/\left[ |k| \cos\psi \right]_{\text{deep}} \) or \(|k|\sin\psi/\left[ |k| \sin\psi \right]_{\text{deep}} \) was investigated. The results were that the latter quantity is improper for the approximation by use of Eq.(14) because of the behavior similar to \(|k|/|k|_{\text{deep}}\) and that the former quantity is also not available for the approximation by use of Eq.(14) because of a significant error magnification at the stage of \(|k|\) estimation, although \(|k|\cos\psi/\left[ |k| \cos\psi \right]_{\text{deep}}\) itself can be approximated well with Eq. (14) as can \( \psi/\psi_{\text{deep}} \).

3) \( \Psi \) and \( K\Psi \) defined by Eq.(15) are used in order to make a highly accurate approximation by use of Eq.(14) possible.

4) Under the fixed conditions of the spectral parameters \((m, n, \sigma_s, \sigma_s)\), each of \( \psi/\psi_{\text{deep}} \) and \(|k|\bar{\psi}/\left[ |k| \bar{\psi} \right]_{\text{deep}} \) is asymptotic to a different constant value as \( k/h \to 0 \) for every \( \gamma \). The \( \gamma \)-dependence of

<table>
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<tr>
<th>Case</th>
<th>parameter</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
<th>( \delta_{\text{deep}} % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( k_{\text{deep}} )</td>
<td>-0.76988 \times 10^{-4}</td>
<td>0.21753 \times 10^{-2}</td>
<td>-0.24037 \times 10^{-1}</td>
<td>0.14461</td>
<td>0.26675</td>
<td>-0.26 \to 0.30</td>
</tr>
<tr>
<td>0.42404</td>
<td>( \psi_{\text{deep}} )</td>
<td>0.74345 \times 10^{-4}</td>
<td>-0.15506 \times 10^{-2}</td>
<td>0.66685 \times 10^{-2}</td>
<td>0.070634</td>
<td>5.1789</td>
<td>-0.07 \to 0.06</td>
</tr>
<tr>
<td>2</td>
<td>( k_{\text{deep}} )</td>
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<td>0.20384 \times 10^{-2}</td>
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<td>0.23169</td>
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</tr>
<tr>
<td>0.09741</td>
<td>( \psi_{\text{deep}} )</td>
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<td>3</td>
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<td>0.21753 \times 10^{-2}</td>
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<td>5.1789</td>
<td>-0.07 \to 0.06</td>
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</tbody>
</table>

Table 2 Coefficients of approximate expression and range of relative error in infinite-depth conditions.
the asymptote can be approximated with high accuracy by use of a 3-point-fixed 4th order polynomial respectively as

\[ g(y) = a_2y^4 + b_2y^3 + c_2y^2 + d_2y + e_2, \quad h(y) = a_3y^4 + b_3y^3 + c_3y^2 + d_3y + e_3 \]

(16)

Moreover, \( \alpha \) in Eq.(11) is an asymptote of \( \cos(y/2) \) associated with a zero-decreasing \( k_jh \) in \( y = 1 \) under the fixed conditions of the spectral parameters \( m, \ n, \ \sigma_u, \ \sigma_h \). In the case of TMA spectrum \( (m = 5, \ n = 4, \ \sigma_u = 0.07, \ \sigma_h = 0.09) \), \( \alpha = 0.42404 \) holds. The \( \alpha \) value for the investigated spectral condition is given in the above Table 2.

5) Usage of the quantities such as \( \Psi' \) and \( K^{\Psi'} \) defined by Eq.(15) yields a significant improvement of accuracy of approximation by use of Eq.(14), because each quantity increases slight from 1 followed by the maximum value and decreases monotonically to 0 with a diminishing \( k_jh \), as can be described by Eq.(14). An introduction of \( \beta \) aims to make the change of \( K^{\Psi'} \) associated with \( k_jh \) gentle.

6) Usage of \( \cos(y/2) \) instead of \( \cos y \) in Eq.(11) aims to retain a negative value of \( \cos(y/2) \) through a whole range of \( y \) actually appearing. The value of \( \tilde{\varphi} \) is between 1 and 2, and in particular, changes between 1.0 and 1.7 in the case of the TMA spectrum \( (\alpha = 0.42404) \). Thus, the error is hardly magnified when \( |k| \) is obtained through a division of the estimated \( k \tilde{\varphi} \) by \( \tilde{\varphi} \) changing between 1 and 2.

The maximum relative error for \( |k|\tilde{\varphi} \) is 3.4\% when a single expression of Eq.(14) is fitted to the \( K^{\Psi'} \) data in a range of \( T_p\sqrt{g/h} \) from 1 to 50. This slightly-discouraging result led us to make use of a two-fold expression of Eq.(14) for more exactly approximating a \( k_jh \)-behavior of the \( K^{\Psi'} \) data. A trial and error approach yielded the separate fitting of Eq.(14) to the \( K^{\Psi'} \) data for each of a range of \( T_p\sqrt{g/h} \) from 1 to 7.2 and a range of \( T_p\sqrt{g/h} \) from 7.2 to 50. A limiting value of \( T_p\sqrt{g/h} \) for the application of the approximate expression is determined as 7.4, taking into account a balance of errors associated with both the expressions.

Next, each of the parameters \( (p, \ q, \ r) \) obtained by the fitting of Eq.(14) has a uniquely \( y \)-dependent variation. It is approximated by using a 3-point-fixed 4th order polynomial as well as the infinite-depth case. For example, it is written for the parameter \( r \) as

\[ r = a_5y^4 + b_5y^3 + c_5y^2 + d_5y + e_5 \]

(17)

, where estimation of two in the 5 coefficients is due to the application of the least-squares-method.

As a summary, the approximate expression for estimating the argument \( \Psi' \) and the absolute value \( |k| \) in the case of the TMA spectrum \( (m = 5, \ n = 4, \ \sigma_u = 0.07, \ \sigma_h = 0.09) \) is exemplified with 5 digits as

\[ \Psi' = \left\{ \frac{\Psi/\Psi_{\text{deep}} - g(y)}{1 - g(y)} \right\} = \left\{ 1 + \frac{r_1(k_jh)^{\eta_1}}{\sinh r_1(k_jh)^{\eta_1}} \cdot \tanh s_1(k_jh)^{\eta_1} \right\}^{\eta_1} \]

(18)

\[ \Psi_{\text{deep}} = 0.74345 \times 10^{-4}y^4 - 0.15506 \times 10^{-2}y^3 + 0.66685 \times 10^{-2}y^2 + 0.70634 \times 10^{-1}y + 5.1789 \]

(19)

\[ g(y) = 0.13815 \times 10^{-5}y^4 - 0.37681 \times 10^{-3}y^3 + 0.35338 \times 10^{-2}y^2 - 0.57585 \times 10^{-1}y + 0.73245 \]

(20)

\[ s_1 = 1.0, \ u_1 = 1.0, \ w_1 = 2.0 \]

(21)

\[ p_1 = 0.72387 \times 10^{-4}y^4 - 0.18276 \times 10^{-2}y^3 + 0.16232 \times 10^{-1}y^2 - 0.58298 \times 10^{-1}y + 0.21301 \]

(22)

\[ q_1 = -0.34384 \times 10^{-4}y^4 + 0.90413 \times 10^{-3}y^3 - 0.83739 \times 10^{-2}y^2 + 0.28366 \times 10^{-1}y + 1.0175 \]

(23)

\[ r_1 = 0.39980 \times 10^{-3}y^4 - 0.10070 \times 10^{-1}y^3 + 0.89147 \times 10^{-1}y^2 - 0.31509y + 8.1834 \]

(24)
$$K\Psi = \frac{[k\tilde{\Psi}]_{\text{exp}} - \beta h(y)}{1 - \beta h(y)} = \left\{ 1 + \frac{r_2 (k, h) w_2}{\sinh r_2 (k, h) w_2} \right\} \cdot \left\{ \tanh s_2 (k, h) w_2 \right\}$$

(25)

$$[k\tilde{\Psi}]_{\text{exp}} = -0.95377 \times 10^4 \gamma^4 + 0.27786 \times 10^2 \gamma^3 - 0.32390 \times 10^{-1} \gamma^2 + 0.21351 \gamma + 0.37954$$

(26)

$$h(y) = 0.10052 \times 10^{-3} \gamma^4 - 0.26206 \times 10^{-2} \gamma^3 + 0.23708 \times 10^{-1} \gamma^2 - 0.67997 \times 10^{-1} \gamma + 0.61910$$

(27)

1) $i=1$: $T_p \sqrt{g/h} = 1 \sim 7.4$ or $h/L_p = 0.15366 \sim 6.2832$

$$s_{21} = 0.3, \quad u_{21} = 2.1, \quad w_{21} = 1.4$$

(28)

$$p_{21} = -0.70343 \times 10^{-4} \gamma^4 + 0.17715 \times 10^{-2} \gamma^3 - 0.15303 \times 10^{-1} \gamma^2 + 0.47181 \times 10^{-1} \gamma + 0.33330 \times 10^{-1}$$

(29)

$$q_{21} = -0.31867 \times 10^{-5} \gamma^4 - 0.35583 \times 10^{-4} \gamma^3 + 0.27728 \times 10^{-2} \gamma^2 - 0.40597 \times 10^{-3} \gamma + 0.32350$$

(30)

$$r_{21} = -0.41671 \times 10^{-3} \gamma^4 + 0.10447 \times 10^{-1} \gamma^3 - 0.91777 \times 10^{-1} \gamma^2 + 0.32697 \gamma + 0.24177$$

(31)

2) $i=2$: $T_p \sqrt{g/h} = 7.4 \sim 50$ or $h/L_p = 0.020053 \sim 0.15366$

$$s_{22} = 0.3, \quad u_{22} = 0.8, \quad w_{22} = 1.4$$

(32)

$$p_{22} = -0.21919 \times 10^{-3} \gamma^4 + 0.59050 \times 10^{-2} \gamma^3 - 0.58640 \times 10^{-1} \gamma^2 + 0.27467 \gamma - 0.80962$$

(33)

$$q_{22} = -0.12360 \times 10^{-5} \gamma^4 + 0.32290 \times 10^{-2} \gamma^3 - 0.29822 \times 10^{-1} \gamma^2 + 0.10786 \gamma + 0.31218 \times 10^{-2}$$

(34)

$$r_{22} = -0.47993 \times 10^{-3} \gamma^4 + 0.15178 \times 10^{-1} \gamma^3 - 0.19086 \gamma^2 + 0.11879 \times 10^{-1} \gamma + 0.14560 \times 10^{-1}$$

(35)

, where the asymptote of $\cos(\Psi/2)$ is $\alpha = 0.42404$ and an arbitrarily-selected value of $\beta$ is 0.25. These expressions extending from Eq.(18) to Eq.(35) with Eq.(11) constitute a coupled system for estimating $\Psi$ and $[k\tilde{\Psi}]$ of the spectral shape parameter $N$. At the first step, $\Psi$ is calculated from Eq.(18) by using Eq.(19) - Eq.(24) for the given $J$. At the second step, $[k\tilde{\Psi}]$ is calculated by giving Eq.(25) the results obtained from Eq.(26), Eq.(27) and Eq.(28) - Eq.(31) or Eq.(32) - Eq.(35), and then $[k\tilde{\Psi}]$ is finally estimated by using $\tilde{\Psi}$ obtained from Eq.(11).

Fig. 5 shows the comparison between the results computed by the approximate expression and the data for
each of $\psi$ (rad.) and $|\kappa|$ and the corresponding relative error $\delta$ in the case of the TMA spectrum ($m = 5$, $n = 4$, $\sigma_x = 0.07$, $\sigma_y = 0.09$). As for $\psi$, the approximate expression reproduces the behavior of the input data well and the accuracy is very high, as suggested by the relative error $\delta$ ranging from -0.26 to 0.26%. As for $|\kappa|$, the approximate expression also yields a good correspondence with the input data in a whole range.

**Table 3 Coefficients of approximate expression and parameters in finite-depth conditions.**

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<tr>
<td>$g(y)$</td>
<td>0.13815×10^{-4} &amp; -0.37681×10^{-1} &amp; 0.35338×10^{-2} &amp; -0.57585×10^{-2} &amp; 0.73245 &amp; 1.0 &amp; 1.0 &amp; 2.0</td>
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<tr>
<td>$p_1$</td>
<td>0.72387×10^{-4} &amp; -0.18276×10^{-1} &amp; 0.16232×10^{-1} &amp; -0.58298×10^{-1} &amp; 0.21301 &amp; 0.10175×10 &amp; 0.10175×10</td>
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<tr>
<td>$q_1$</td>
<td>-0.34384×10^{-3} &amp; 0.90413×10^{-3} &amp; -0.83739×10^{-2} &amp; 0.28366×10^{-1} &amp; 0.81834×10</td>
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<tr>
<td>$r_1$</td>
<td>0.39980×10^{-3} &amp; -0.10070×10^{-1} &amp; 0.89147×10^{-1} &amp; -0.31509</td>
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<tr>
<td>$h(y)$</td>
<td>0.10052×10^{-1} &amp; -0.26206×10^{-1} &amp; 0.23708×10^{-1} &amp; -0.67997×10^{-1} &amp; 0.61910</td>
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<td>$p_{21}$</td>
<td>-0.70343×10^{-2} &amp; 0.17715×10^{-2} &amp; -0.15303×10^{-1} &amp; 0.47181×10^{-1} &amp; 0.33330×10^{-1} &amp; 0.32350</td>
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<td>$q_{21}$</td>
<td>-0.31867×10^{-1} &amp; 0.90413×10^{-3} &amp; -0.83739×10^{-2} &amp; 0.28366×10^{-1} &amp; 0.81834×10</td>
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<td>$r_{21}$</td>
<td>-0.34384×10^{-3}</td>
<td>0.90413×10^{-2} &amp; -0.83739×10^{-2} &amp; 0.28366×10^{-1} &amp; 0.81834×10</td>
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<td>$p_{22}$</td>
<td>-0.21919×10^{-1} &amp; 0.59050×10^{-3} &amp; -0.56401×10^{-1} &amp; 0.27467 &amp; -0.80962</td>
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<td>$q_{22}$</td>
<td>-0.12360×10^{-1} &amp; 0.32290×10^{-3} &amp; -0.29822×10^{-1} &amp; 0.10786</td>
<td>0.31218×10^{-2}</td>
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<tr>
<td>$g(y)$</td>
<td>0.19956×10^{-4} &amp; -0.57388×10^{-1} &amp; 0.59145×10^{-2} &amp; -0.16681×10^{-1} &amp; 0.63760 &amp; 0.8 &amp; 1.4 &amp; 0.5</td>
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<td>$p_1$</td>
<td>-0.18008×10^{-2} &amp; 0.46645×10^{-1} &amp; -0.42299 &amp; 0.15179×10^{-1} &amp; 0.61905×10^{-1}</td>
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<tr>
<td>$q_1$</td>
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<td>$r_1$</td>
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<td>$h(y)$</td>
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<tr>
<td>$p_{21}$</td>
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<tr>
<td>$q_{21}$</td>
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<td>0.31218×10^{-2}</td>
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<td>$r_{21}$</td>
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<td>$p_{22}$</td>
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<td>$q_{22}$</td>
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<td>$r_{22}$</td>
<td>-0.92460×10^{-4} &amp; 0.32454×10^{-2} &amp; -0.63741×10^{-1}</td>
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<td>Case3</td>
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<tr>
<td>$g(y)$</td>
<td>0.13815×10^{-1} &amp; -0.37681×10^{-1} &amp; 0.35338×10^{-2} &amp; -0.57585×10^{-2} &amp; 0.73245 &amp; 0.6 &amp; 1.7 &amp; 1.0</td>
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<td>$p_1$</td>
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<td>$q_1$</td>
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<td>$r_1$</td>
<td>-0.18113×10^{-3} &amp; 0.61854×10^{-2} &amp; -0.81533×10^{-1} &amp; 0.51547</td>
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<tr>
<td>$h(y)$</td>
<td>0.15589×10^{-1} &amp; -0.41455×10^{-1} &amp; 0.39564×10^{-1} &amp; -0.15171 &amp; 0.97379</td>
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<tr>
<td>$p_{21}$</td>
<td>0.30582×10^{-3} &amp; -0.82050×10^{-2} &amp; 0.80416×10^{-1} &amp; -0.35125 &amp; 0.64298</td>
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<td>$q_{21}$</td>
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<td>0.14462×10^{-1}</td>
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<td>$p_{22}$</td>
<td>-0.27753×10^{-3} &amp; 0.75364×10^{-2} &amp; -0.75425×10^{-1} &amp; 0.34884 &amp; -0.87409</td>
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<td>$q_{22}$</td>
<td>-0.11689×10^{-3} &amp; 0.30542×10^{-2} &amp; -0.28158×10^{-1} &amp; 0.10050</td>
<td>0.17880×10^{-1}</td>
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<td>$r_{22}$</td>
<td>0.45684×10^{-3} &amp; -0.88223×10^{-2} &amp; 0.25289×10^{-1} &amp; 0.38764</td>
<td>0.24595×10^{-1}</td>
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of \(h/L_p\). The relative error \(\delta\) extends from -0.53 to 0.57% in the first range of \(h/L_p\) from 0.154 to 6.28 and from -0.73 to 0.48% in the second range of \(h/L_p\) from 0.020 to 0.154.

Table 3 gives a list of the coefficients with 5 digits in the approximate expression of \(\psi\) or \(K\psi\) and the parameters \((s, u, w)\) in the quasi-optimum solution of Eq. (14) for each of the 3 universal spectral cases in finite-depth conditions. Also, Table 4 lists up a range of the relative error \(\delta\) for the estimate of each of the argument \(\psi\) and the absolute value \(|\kappa|\) in the 3 universal spectral cases in finite-depth conditions. The relative errors for the estimates of both \(\psi\) and \(|\kappa|\) are significantly small in any of the 3 cases.

5. Conclusions

This study investigated the characteristics of the spectral shape parameter \(\kappa\) expressed as a complex number with the absolute value \(|\kappa|\) and the argument \(\psi\) both in infinite-depth conditions and in finite-depth conditions, using each of the corrected versions of G-TMA and G-Thornton spectra proposed for a universal spectrum of wind waves and constructed the approximate expressions by which \(|\kappa|\) and \(\psi\) can be estimated for the 3 cases of universal spectrum. Another finding is that the relation between \(|\kappa|\) and \(\psi\) in infinite-depth conditions is uniquely-determined and that it varies in a complicated manner, accompanied by augmentation of finite-depth water effect (decrease of \(k_p h\)). Another finding is that the approximate expressions for estimating \(|\kappa|\) and \(\psi\) in infinite-depth conditions have very high accuracy respectively and that those in finite-depth conditions also have reasonable accuracy, in cases where the expression gives the maximum relative error of 0.99% for \(|\kappa|\) and that of 0.50% for \(\psi\) in a whole range of finite-depth water conditions. The maximum relative errors suggest the practical applicability of the approximate expressions for estimating \(|\kappa|\) and \(\psi\).

References