Vertical Product Differentiation, Network Externalities, and Compatibility Decisions

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Abstract

We analyse the subgame perfect equilibrium of a four stage game in a model of vertical product differentiation, where the consumer's evaluation of a product depends on its inherent quality and on its network's size. First, two firms choose their product's inherent quality. Then they may mutually agree on providing an adapter before competing in prices. Finally, consumers buy. We find that, despite the high quality firm's preference for incompatibility, an adapter is always provided in equilibrium. Social welfare is greater than without an adapter and can be improved by regulating compatibility only in those cases where qualities are differentiated too much.

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1 Introduction

In industries with network externalities, different market shares of incompatible products represent different quality levels for consumers. Network externalities therefore create an element of vertical product differentiation. The quality of a product is, however, also often determined by other quality aspects. A personal computer with a Pentium microprocessor, for example, still has a higher quality than one with a 486 processor, even if both use the same operating system and have the same network externality from sharing the same pool of available software products. Although a firm has direct control over the inherent quality of its product, the magnitude of the network externality does not only depend on its own prices and quality decisions, it also depends on the price and quality decisions of its rivals.

This paper focuses on compatibility decisions of oligopolistic firms where network externalities interact with other quality dimensions which the firms can control. In contrast to the majority of the existing literature (see Bental, Spiegel (1995), de Palma, Leruth (1996), and Economides, Flyer (1997)) where network size is the only vertical dimension, we assume that consumers' willingness to pay increases in both the product's inherent quality and the size of its network. We analyze a four stage model where two firms first choose the inherent qualities of their products. Then they decide whether to install a two-way adapter in order to achieve compatibility. Neither firm can act unilaterally here because proprietary information or a licence from the other seller is needed for the adapter. Finally, firms set their prices and consumers decide whether and which of the two products to buy. Given this sequence of decisions, firms develop the main features of their products independently. They are, nevertheless, able to co-operate later on in order to increase the network externalities.[1]

It turns out that quality differentiation does not only reduce price competition, it also facilitates the co-ordination for the achievement of compatibility. Firms choose different qualities and compatibility is always achieved. While network effects without compatibility strengthen price competition, compatibility results in higher prices for both firms and generally higher profits for the low quality firm. On the other hand, the market share of the high quality firm will be larger with incompatibility and hence its profit may be higher

[1] An example could be the independent development of the operating systems for IBM-Dos and Macintosh Computers. Their incompatibility is partly bridged by software products which allow files to be saved in different formats. They serve as imperfect adapters. Software can only have this feature if the providers of the operating systems give the necessary information to the software producers.

than with compatibility. However, given a relatively low degree of quality differentiation, the negative effect of the more intense price competition outweighs the positive effect of the greater market share. Therefore, by choosing its own quality appropriately, the low quality firm can always induce the high quality firm to agree on compatibility.

De Palma, Leruth (1996), and Economides, Flyer (1997) analyse compatibility decisions under Cournot competition using the network size as the only vertical dimension of product differentiation. De Palma, Leruth (1996) use a duopoly setting and conclude that the firms would agree on compatibility in a preliminary stage of the game if they were sufficiently uncertain about which of them would become the large and, thus, high quality provider and which the small and, thus, low quality provider. In Economides, Flyer (1997) compatibility is achieved in a preliminary stage among the firms that join the same coalition, defined by technical standards. They show that a grand coalition with full compatibility, can only be an equilibrium if the network externality is rather weak. In our model, firms ensure by their quality choice that they agree on compatibility.

Farrell and Saloner (1992) show that, for horizontally differentiated products, compatibility of competing technologies might be achieved by converters, whereas Navon, Shy and Thisse (1995) analyse the effect of positive and negative network externalities without addressing compatibility issues. The latter show that for incompatible products, price competition becomes tougher when the magnitude of the positive network externality increases. Farrell and Saloner conclude from their duopoly model that a converter blunts price competition. We derive a corresponding result for vertically differentiated products.

Belleflamme (1998) analyses how firms decide which of two technologies to adopt if their marginal costs decrease in the number of firms adopting the same technology. Assuming Cournot competition and exogenously differentiated products, Belleflamme shows that an equilibrium where all firms adopt the same technology is more likely the more differentiated the products are. With more differentiated products competition is weak and the positive network effect exceeds the negative effect of the rivals' reduced costs. In our framework, product differentiation can induce the opposite effect. Small differentiation may ensure that both firms agree on the provision of an adapter because price competition is tougher without it.

In the next section we present our model. Then we derive the subgame perfect equilibrium for two alternative three stage games, where firms are either committed to compatibility or to incompatibility. In section 5 we fully characterize the subgame perfect equilibria of the game with an endogenized adapter decision and compare the social welfare of different equilibria. Finally, we discuss variations of our model and draw some conclusions regarding the regulation of network access and licensing.

2 The Model

We assume a mass of N consumers with $N = 1$. Every consumer has a unit demand and buys either one unit of the good or none at all. The consumers differ in their willingness to pay for the inherent quality of the product, but not in their willingness to pay for the network externality. The surplus of the consumer is given by:[2]

$$u(x, q_i, \tilde{z}_i, p_i) = \begin{cases} xq_i + \mu q_i \tilde{z}_i - p_i & \text{if she buys product } i, \\ 0 & \text{if she buys neither product,} \end{cases} \tag{1}$$
$$\text{with } i = 1, 2 \text{ and } 0 \leq \mu < \min[1, \hat{x}/2].$$

where $\tilde{z}_i$ is the network size, q_i is the inherent quality of product i and p_i is its price. The parameter μ represents the strength of the network externality. Consumers differ in x which is uniformly distributed on the interval $[0, \hat{x}]$ with $\hat{x} > 1$, but the parameter μ is identical for all consumers.[3] Note that (1) implies that $\partial^2 u(\cdot)/(\partial \tilde{z}_i \partial q_i) \geq 0$, i.e., the marginal utility of an increase in the network size is higher the greater the product's inherent quality. In addition, the marginal rate of substitution between q_i and $\tilde{z}_i$ is decreasing in x. The higher x the higher a consumer's relative valuation of quality.[4]

If no adapter is supplied, the network size is identical to z_i, the number of consumers who buy product i. With an adapter, the network size comprises the consumers of both products, i.e., $\tilde{z}_i = z_i + z_j$ with $i, j = 1, 2$ and $i \neq j$.

When consumers decide which product to purchase or not to buy at all, they maximise their surplus (1) taking as given the decisions of all other consumers. Thus, consumers play a Nash-equilibrium. If multiple equilibria exist, we assume that consumers select one equilibrium, as long as it is not Pareto dominated by another equilibrium which is then played.

[2]The demand side is modelled similarly to Mussa, Rosen (1978) and Tirole (1988), p. 296 ff., but they do not take into account network externalities.

[3]The restrictions on $\hat{x}$ and μ, together with our cost function, ensure a price equilibrium in pure strategies with two viable firms and an always uncovered market.

[4]A consumer with a higher x may, for example, use his computer for more advanced and specialized services implying that the additional utility from a larger network is lower.

There are two firms, each producing one version of the product, with identical cost functions:

$$c(q_i, z_i) = q_i z_i \text{ with } i = 1, 2.$$

Marginal costs are constant and linearly increasing in the inherent quality. Each firm i with $i = 1, 2$ can choose its product quality q_i from the interval $[0, \bar{q}]$ where $\bar{q}$ represents the technologically feasible maximum quality. In order to achieve compatibility the firms can agree to install an adapter. Co-operation is necessary because each firm needs proprietary information or a licence from its rival in order to provide an adapter. For the sake of simplicity we assume that an adapter is costless. We exclude side payments between the firms.

We assume that the preferences of the consumers and the cost functions of the firms are common knowledge and consider the subgame perfect equilibrium of the following four stage game with complete information:[5]

1. Firms 1 and 2 choose the inherent qualities q_1 and q_2 of their products from the interval $[0, \bar{q}]$ non co-operatively.
2. The firms decide whether to install a two-way adapter in their product or not. If they do, they need the consent of their rival.
3. The firms set their prices p_1 and p_2 non co-operatively.
4. The consumers decide on buying one, or none, of the two products non co-operatively.

The subgame perfect equilibrium of the complete game can easily be derived from the subgame perfect equilibria which arise for the game where the second stage is neglected and the reduced game is analysed for exogenously incompatible products on the one hand, and for exogenously compatible products on the other hand.

3 Prices and Qualities without an Adapter

Here we assume that the firms cannot install an adapter, and, without loss of generality, that firm 1 supplies a product with a higher quality than firm 2, i.e., $q_1 > q_2$.[6]

[5] In section 6 we discuss whether a different order of moves in the quality and the adapter decision would change our results.

[6] The analysis of the quality choice of the firms shows that the firms will not choose identical qualities.

3.1 The Decisions of the Consumers

In general there are four possible types of Nash equilibria: one where both firms attract a positive number of consumers, two where only one firm has a positive number of consumers, and one where both firms have no consumers:

$$\begin{array}{llll} \text{Type 1 NE:} & z_1 > 0, \quad z_2 > 0, & & \\ \text{Type 2 NE:} & z_1 > 0, \quad z_2 = 0, & \text{Type 3 NE:} & z_2 > 0, \quad z_1 = 0 \\ \text{Type 4 NE:} & z_1 = z_2 = 0. & & \end{array}$$

Due to the network externalities consumers may mainly evaluate the products in terms of the difference in their inherent qualities or their network externalities. In the latter case, the perceived quality of the products is mainly determined by the size of the groups into which the consumers split. Hence, multiple Nash equilibria of different types may exist. Note, however, that in any of these possible Nash equilibria consumers must group according to their type x.[7]

Consider now the Type 1 NE. Here, there must be consumers with $x \in [0, \underline{x})$ who do not buy from either of the two firms, consumers with $x \in (\underline{x}, \overline{x})$ who buy q_2, and those with $x \in (\overline{x}, \hat{x}]$ who buy q_1. Applying the following indifference conditions for the consumer with $\underline{x}$ and the one with $\overline{x}$:

$$\begin{aligned} u(\overline{x}, q_1, z_1, p_1) &= u(\overline{x}, q_2, z_2, p_2), & (2) \\ u(\underline{x}, q_2, z_2, p_2) &= 0, & (3) \\ \text{with } z_1 = \frac{\hat{x} - \overline{x}}{\hat{x}} \quad &\text{and} \quad z_2 = \frac{\overline{x} - \underline{x}}{\hat{x}}, \end{aligned}$$

we can compute $\underline{x}$ and $\overline{x}$, and conclude that a Type 1 NE exists if, and only if, $\underline{x}$ and $\overline{x}$ satisfy $0 < \underline{x} < \overline{x} < \hat{x}$.[8]

In order to characterize the Type 2 and Type 3 NE we define:

$$\begin{aligned} \underline{u}_i(x, q_i, p_i) &= xq_i - p_i, & (4) \\ \overline{u}_i(x, q_i, p_i) &= xq_i + \mu q_i \frac{1}{\hat{x}}(\hat{x} - \underline{x}_i) - p_i \text{ with } \underline{x}_i = \frac{(p_i - q_i\mu)\hat{x}}{q_i(\hat{x} - \mu)}. & (5) \end{aligned}$$

[7] This result follows from $\partial u(\cdot)/(\partial x) = q_i$ and $\partial^2 u(\cdot)/(\partial x \partial q_i) > 0$. See Jaskold Gabszewicz, Thisse (1979) and (1980), and Shaked, Sutton (1982) and (1983) for similar results without network effects.

[8] Since (2) and (3) are linear in $\underline{x}$ and $\overline{x}$, a unique solution for $\underline{x}$ and $\overline{x}$ exists. Furthermore, a Nash equilibrium where all consumers buy ($z_1 + z_2 = 1$) does not exist, because even if all consumers bought the same product at its marginal cost q_i, consumers with $x = 0$ would not buy: $u(0, q_i, 1, q_i) = (\mu - 1)q_i < 0$.

The function $\underline{u}_i(x, q_i, p_i)$ is the surplus of a consumer of type x, given that she is the only consumer who buys q_i, whereas $\overline{u}_i(x, q_i, p_i)$ represents the surplus of the same consumer if all the other consumers with a positive surplus from q_i also buy it. A Type 2 or Type 3 NE exists if $0 < \underline{x}_i < \hat{x}$ and:

$$\overline{u}_i(x, q_i, p_i) \geq \underline{u}_j(x, q_j, p_j) \text{ with } j \neq i \text{ for all } x \in [\underline{x}_i, \hat{x}]. \tag{6}$$

Finally, a Type 4 NE exists if, and only if,

$$\underline{u}_i(x, q_i, p_i) < 0 \text{ with } i = 1, 2 \text{ for all } x \in [0, \hat{x}]. \tag{7}$$

Defining $\overline{p}_1(p_2; q_1, q_2) = \{p_1 | \bar{x} = \hat{x}\}$ and $\underline{p}_1(p_2; q_1, q_2) = \{p_1 | \underline{x} = \overline{x}\}$ we can summarize the analysis of (2) to (7) in the following Lemma:[9]

Lemma 1 *A unique Nash equilibrium of the consumers' interaction always exists, if the qualities of the two firms satisfy:*

$$q_1 > \left(\frac{\hat{x}}{\hat{x} - \mu}\right)^2 q_2, \tag{8}$$

It is a Type 1 NE if $\overline{p}_1(\cdot) > p_1 > \underline{p}_1(\cdot)$ *holds. Otherwise the Nash equilibrium is either a Type 2 or a Type 3 or a Type 4 NE.*

Proof: The proof is outlined in the Appendix. ■

For all cases where (8) is violated, consumers evaluate the products mainly in terms of the perceived network externalities. Hence, multiple Nash equilibria of different types may exist.

3.2 Firms' Pricing Decision

Consider first the case in which the firms' qualities satisfy (8). Using Lemma 1 we can calculate the firms' profit functions $\pi_{1N}(p_1, p_2; q_1, q_2)$ and $\pi_{2N}(p_1, p_2; q_1, q_2)$. Analyzing these profit functions we obtain:

Lemma 2 *Assume that the firms' qualities satisfy (8). The equilibrium prices of the two firms* p_1^*, p_2^* *are given by:*

$$\begin{aligned} p_1^* &= q_1 \left[1 + \frac{(\hat{x} - 1)\left[2q_1(\hat{x} - \mu)^2 - q_2\hat{x}(2\hat{x} - \mu)\right]}{4q_1(\hat{x} - \mu)^2 - q_2\hat{x}^2}\right] > q_1, \\ p_2^* &= q_2 \left[1 + \frac{(\hat{x} - 1)\left[q_1(\hat{x} - \mu)(\hat{x} - 2\mu) - q_2\hat{x}^2\right]}{4q_1(\hat{x} - \mu)^2 - q_2\hat{x}^2}\right] > q_2, \end{aligned}$$

[9] In order to simplify notation we use $\overline{p}_1(\cdot)$ and $\underline{p}_1(\cdot)$ in the following. Only if the functions are considered at certain prices or qualities will we stick to the original notation.

if the following condition holds:

$$q_1 > \frac{q_2 \hat{x}^2}{(\hat{x}-\mu)(\hat{x}-2\mu)} \tag{9}$$

If (9) does not hold the equilibrium prices are:

$$p_1^* = \underline{p}_1(q_2; q_1, q_2) \geq q_1, \quad p_2^* = q_2,$$

Proof: The proof is outlined in the Appendix. ■

Given (9), both equilibrium prices decrease with μ and increase in $\hat{x}$:

$$\frac{\partial p_i^*}{\partial \mu} < 0, \quad \frac{\partial p_i^*}{\partial \hat{x}} > 0 \text{ with } i = 1, 2. \tag{10}$$

The stronger the network externality the more important is it to fight for market shares, because the latter increasingly determines the products' perceived qualities. Prices decrease because price competition becomes tougher.[10] An increase in $\hat{x}$ implies that consumers are dispersed over a larger space. This leads to a less tough price competition because the increase in demand that can be induced by a price reduction is smaller.

For all cases where (9) is violated, the network externality becomes relatively more important for the consumers' decision and monopolization is the optimal pricing strategy. Since the positive network effect in the consumer's surplus increases with the quality, and since all consumers prefer a high quality, firm 1 attracts all consumers.

The same argument applies if the quality differentiation is so small that (8) does not hold. Although consumers' decisions are almost completely determined by the networks' sizes, and multiple Nash equilibria of the consumers' interaction may exist, our assumption that consumers never play a Nash-equilibrium which is Pareto dominated by another equilibrium implies the following lemma:

Lemma 3 *If the firms' qualities do not satisfy (8), the equilibrium prices are $p_2^* = q_2$ and either:*

$$\begin{array}{ll} (i) & p_1^* = \min\{\underline{p}_1(q_2; q_1, q_2), \hat{p}_1(q_2; q_1, q_2)\} \geq q_1 \\ & \textit{with } \hat{p}_1(q_2; q_1, q_2) = \{p_1 \mid \overline{u}_1(\hat{x}, q_1, p_1) = \overline{u}_2(\hat{x}, q_2, q_2)\} \\ \textit{or } (ii) & p_1^* = \max\{\bar{p}_1(q_2; q_1, q_2), q_1\} \geq q_1 \end{array}$$

[10] Navon, Shy and Thisse (1995) derive this effect with horizontal product differentiation.

Proof: The proof is outlined in the Appendix. ■

Part (i) of Lemma 3 applies to those cases where consumers would select a Type 2 NE. Part (ii) applies when consumers would play a Type 1 or a Type 3 NE.

3.3 The Quality Decisions of the Firms

Lemmas 2 and 3 ensure that the low quality firm never chooses a quality that violates (9). Hence, the relevant reduced profit functions $\pi_{1N}(q_1, q_2)$ and $\pi_{2N}(q_1, q_2)$ are derived by substituting the prices given in Lemma 2 into $\pi_{1N}(p_1, p_2; q_1, q_2)$ and $\pi_{2N}(p_1, p_2;\ q_1, q_2)$. Solving for the optimal qualities yields:

Proposition 1 *Given that no adapter is provided, the three-stage game has two subgame perfect equilibria in pure strategies where the equilibrium qualities are given by:*

$$q_i^* = \bar{q}, \quad q_j^* = \frac{\bar{q}(\hat{x}-\mu)^2\left[11\hat{x} - 10\mu - \sqrt{3(3\hat{x}^2 + 28\hat{x}\mu - 20\mu^2)}\right]}{2\hat{x}^2(7\hat{x} - 5\mu)} < \bar{q}$$

with $i, j = 1, 2$ *and* $i \neq j$.

Proof: Solving $\partial\pi_{2N}(q_1, q_2)/\partial q_2 = 0$ yields the best response function $q_2^R(q_1)$. In addition, $\partial\pi_{1N}(q_1, q_2)/\partial q_1 \mid_{q_2 = q_2^R(q_1)} > 0$ holds for all $q_1 \in [q_2^R(q_1), \bar{q}]$ and $\pi_{1N}(\bar{q}, q_2^R(\bar{q})) > \pi_{2N}(q_2^R(\bar{q}), q_2)$ is satisfied for all $q_2 \in [0, q_2^R(\bar{q})]$. Two subgame perfect equilibria exist because it cannot be determined, which firm chooses the high or the low quality. ■

Since a higher quality implies a higher market share and reduces price competition, the profit of the high quality firm is monotonically increasing in its quality. Hence, the high quality firm will always choose the largest quality possible. For the low quality firm, an increase in its own quality implies a positive effect because of a greater market share, and a negative effect because of a more intense price competition. Equalizing the two effects yields the optimal quality q_j^*.

Defining the degree of quality differentiation as $|q_1^* - q_2^*|$, Corollary 1 follows directly from differentiating q_2^* with respect to μ and $\hat{x}$:

Corollary 1 *The degree of quality differentiation in the subgame perfect equilibrium is increasing in the extent of the network externality* μ *and decreasing in the parameter* $\hat{x}$ *measuring the heterogeneity of consumers.*

From (10) we know that price competition becomes tougher with a larger network externality, whereas it becomes weaker with a larger dispersion of consumers. Thus, the optimal quality for the low quality firm decreases with the network externality and increases with the dispersion of consumers.

4 Prices and Qualities with an Adapter

In the previous section we characterized the equilibrium of the truncated game with no adapter. But since our ultimate purpose is to analyse the complete game with endogenized adapter decisions, we now derive the subgame perfect equilibria for the game, given that an adapter is provided. Again we assume $q_1 > q_2$.

4.1 The Decisions of the Consumers and the Pricing Decisions of the Firms

The provision of an adapter implies that, if both firms realise a positive market share, the network size of product i, $i = 1, 2$, is given by $\tilde{z}_i = z_1 + z_2 = \tilde{z}$ and that it is identical for both products. Therefore, network externalities do not create an ambiguous incentive to group together. Multiple equilibria cannot occur in the consumers' interaction.

Analyzing the firms' profit functions $\pi_{1A}(p_1, p_2; q_1, q_2)$ and $\pi_{2A}(p_1, p_2; q_1, q_2)$, we can characterize the Bertrand Nash equilibrium as follows:

Lemma 4 *The price setting subgame has a unique Nash equilibrium, if an adapter is provided. The equilibrium prices with $q_1 > q_2$ are given by:*

$$\begin{aligned} p_1^{**} &= q_1 \left[1 + \frac{2\hat{x}(\hat{x}-1)(q_1 - q_2)}{(4\hat{x} - 3\mu)q_1 - \hat{x}q_2} \right] > q_1, \\ p_2^{**} &= q_2 \left[1 + \frac{\hat{x}(\hat{x}-1)(q_1 - q_2)}{(4\hat{x} - 3\mu)q_1 - \hat{x}q_2} \right] > q_2. \end{aligned}$$

Proof: The proof is outlined in the Appendix. ■

In contrast to the case without an adapter, both equilibrium prices increase in μ and $\hat{x}$ if perfect compatibility is achieved by an adapter:

$$\frac{\partial p_i^{**}}{\partial \mu} > 0, \quad \frac{\partial p_i^{**}}{\partial \hat{x}} > 0 \text{ with } i = 1, 2. \tag{11}$$

An increase in a firm's market share does not necessarily increase the firm's network size, and, if it does, the network size of the competing firm is increased in the same way. This decreases the incentive to fight for market shares by reducing prices. Moreover, greater network externalities increase the consumer's surplus the more the higher the product's quality. Thus, given different qualities, the difference in the evaluation of the two products increases in the network externality. This also relaxes price competition. Once again, an increase in the dispersion of the consumers, measured by $\hat{x}$, weakens price competition.

4.2 The Quality Decisions of the Firms

By substituting the equilibrium prices given in Lemma 4 into $\pi_{1A}(p_1, p_2; q_1, q_2)$ and $\pi_{2A}(p_1, p_2; q_1, q_2)$ we obtain the reduced profit functions $\pi_{1A}(q_1, q_2)$ and $\pi_{2A}(q_2, q_1)$. Solving for the optimal qualities yields:

Proposition 2 *Given that an adapter is provided, the three stage game has two subgame perfect equilibria where the equilibrium qualities are given by:*

$$q_i^{**} = \bar{q}, \quad q_j^{**} = \frac{\bar{q}(4\hat{x} - 3\mu)}{7\hat{x} - 6\mu} < \bar{q},$$

with $i, j = 1, 2$ *and* $i \neq j$.

Proof: The proof is analogous to the proof of Proposition 1. ■

Once again, the high quality firm always chooses the highest quality possible, whereas the low quality firm chooses its best response to $\bar{q}$. Note, that compared to the case of no adapter, price competition is less tough with an adapter. Hence, the optimal qualities for the low quality firm is higher when an adapter is supplied, $q_j^{**} > q_j^{*}$.

Defining the degree of quality differentiation as $|q_1^{**} - q_2^{**}|$, we obtain:

Corollary 2 *The degree of quality differentiation in the subgame perfect equilibrium is decreasing in the extent of the network externality* μ *and increasing in the parameter* $\hat{x}$ *measuring the heterogeneity of consumers.*

Since price competition becomes less tough with a larger network externality, the incentive for quality differentiation decreases with the network externality. Even though price competition becomes weaker when consumers' heterogeneity increases, the degree of quality differentiation also increases with the heterogeneity of consumers.

5 The Game with an Endogenous Adapter

We now turn to the analysis of the complete four stage game where firms can decide after their choice of qualities whether to install an adapter or not. An adapter will be installed if, and only if, both firms agree, i.e., if, and only if, the profits of both firms are higher with an adapter than without.

Comparing first the firms' profits in the equilibrium with exogenous incompatibility and with exogenous compatibility yields (we assume $q_1 > q_2$ for both equilibria):[11]

$$\pi_{1A}(q_1^{**}, q_2^{**}) < \pi_{1N}(q_1^*, q_2^*), \quad \pi_{2A}(q_2^{**}, q_1^{**}) > \pi_{2N}(q_2^*, q_1^*) \tag{12}$$

Without an adapter the high quality firm has a larger competitive advantage. Its higher quality attracts more consumers, thus increasing its network size and, by this effect, the quality perceived by consumers. Hence, the high quality firm would prefer no adapter to be provided, whereas the low quality firm would prefer an adapter.

Note that, no matter whether firm 1 expects the provision of an adapter or not, it will never deviate from $\bar{q}$. Furthermore, comparing firm 2's profits with an adapter and without yields:

$$\pi_{2A}(q_2, \bar{q}) > \pi_{2N}(q_2, \bar{q}) \text{ for all } q_2 < \bar{q}. \tag{13}$$

The low quality firm always prefers the provision of the adapter and it may want to force firm 1 to agree on this issue. Firm 1 can, however, always unilaterally prevent an adapter by refusing its consent. Hence, the low quality firm can only achieve an agreement if it chooses its q_2 such that the positive effect of a less intense price competition outweighs the negative effect of a lower market share for firm 1, i.e.,

$$\pi_{1A}(\bar{q}, q_2) \geq \pi_{1N}(\bar{q}, q_2).$$

For this strategy to be optimal for firm 2 its profit must be at least as high as without an adapter and the respective optimal quality q_2^*:

$$\pi_{2A}(q_2, \bar{q}) \geq \pi_{2N}(q_2^*, \bar{q}).$$

Let us now define the quality levels which make the high quality firm indifferent between agreeing and not agreeing on an adapter. Since the equilibrium

[11] We substitute the optimal quality levels from Proposition 1 and 2, respectively, into the reduced profit functions $\pi_{1N}(q_1, q_2)$ and $\pi_{1A}(q_1, q_2)$, and $\pi_{2N}(q_2, q_1)$ and $\pi_{2A}(q_2, q_1)$.

prices without an adapter depend on whether the chosen degree of quality differentiation is in the scope of Lemma 2 or of part (i) or (ii) of Lemma 3, we have to define two different quality levels:

$$\begin{aligned}\tilde{q}_2 &= \Big\{q_2 \mid \pi_{1A}(\bar{q}, q_2) = \pi_{1N}(\underline{p}_1, \bar{q}, q_2)\Big\} < q_2^{**} \\ \tilde{\tilde{q}}_2 &= \{q_2 \mid \pi_{1A}(\bar{q}, q_2) = \pi_{1N}(\overline{p}_1, \bar{q}, q_2)\} > q_2^{**}\end{aligned}$$

where $\pi_{1N}(\underline{p}_1, \bar{q}, q_2)$ denotes the high quality firm's profit with $p_1 = \underline{p}_1(q_2; \bar{q}, q_2)$ and $p_2 = q_2$ and $\pi_{1N}(\overline{p}_1, \bar{q}, q_2)$ its profit with $p_1 = \overline{p}_1(q_2; \bar{q}, q_2)$ and $p_2 = q_2$. From the comparisons of the firms' profits with and without an adapter, we can derive the following Proposition:

Proposition 3 *At the two subgame perfect equilibria of the four-stage game, both firms will always agree in the second stage of the game on the provision of an adapter. The high quality firm chooses $\bar{q}$ in the first stage of the game. The low quality firm chooses its optimal quality q_2^{**} for a small network externality μ (relative to $\hat{x}$) and $\tilde{q}_2$ for intermediate network externalities. For high network externalities it chooses $\tilde{q}_2$, if consumers select a Type 2 NE, and either $\tilde{\tilde{q}}_2$ or q_2^{**} if consumers select a Type 1 ore a Type 3 NE.*

Proof: The proof is outlined in the Appendix. ■

Given a rather small network externality, the low quality firm can induce the high quality firm to agree on the adapter by choosing its optimal quality with an adapter. For small network effects, the quality differentiation in the adapter case is rather high. Hence, compatibility yields a relative high market share for firm 1. The positive effect of higher equilibrium prices with compatibility dominates the negative effect of a reduced market share.

With an adapter, the optimal quality differentiation from firm 2's perspective is, however, monotonically decreasing in the network externality μ. With higher network externalities and smaller product differentiation, firm 1 would lose more consumers by agreeing on the provision of an adapter. Here, the negative effect of a smaller market share dominates the positive effect of higher equilibrium prices. Firm 1 would be better off with incompatibility. Therefore, in order to achieve compatibility firm 2 has to deviate from its optimal adapter quality by choosing the lower quality $\tilde{q}_2$ and thus a greater quality differentiation. Nevertheless, $\tilde{q}_2$ is also monotonically increasing in the network externality, which implies that the quality differentiation becomes so small that, without an adapter, multiple equilibria of the consumers' game may occur. This does not matter when consumers select a Type 2 NE because the relevant price equilibrium does not change (see Lemma 3). However, if consumers select a Type 3 or a Type 1 NE, the relevant price equilibrium without an adapter changes and becomes less favourable for firm 1. This results in an extra opportunity for firm 2 to make firm 1 indifferent by choosing the higher quality, $\widetilde{\widetilde{q}}_2$. For extremely high levels of the network externality, firm 2 may even return to its optimal quality strategy for the adapter case.

Finally, we can analyse the welfare implications of endogenous adapter decisions. For this purpose we define social welfare as the sum of consumers' surplus and the two firms' profits. Let $W_N(q_1, q_2)$ denote social welfare without an adapter and $W_A(q_1, q_2)$ social welfare with an adapter. Comparing $W_A(q_1, q_2)$, evaluated at the equilibrium qualities, with endogenous adapter decisions, with $W_N(q_1, q_2)$, evaluated at the equilibrium qualities if no adapter is provided, we obtain the following Proposition:

Proposition 4 *Social welfare with endogenous adapter decisions always exceeds social welfare which could be realised if no adapter is provided.*

Proof: From simple, but tedious, calculations it is possible to show that $W_A(\bar{q}, q_2^{**}) > W_N(\bar{q}, q_2^*)$, $W_A(\bar{q}, \tilde{q}_2) > W_N(\bar{q}, q_2^*)$, and $W_A(\bar{q}, \widetilde{\widetilde{q}}_2) > W_N(\bar{q}, q_2^*)$ hold in the relevant ranges of μ. ■

Although prices for given quality levels are higher with an adapter than without, this effect on social welfare is more than offset by the larger network externality that consumers enjoy and by the lower quality differentiation, i.e., the higher low quality.

Since smaller quality differentiation usually results in higher social welfare, it is not surprising that, in all those cases where the low quality firm chooses $\tilde{q}_2$, social welfare of the three stage game with an exogenous adapter exceeds that of the four stage game. In all those cases where the low quality firm chooses $\widetilde{\widetilde{q}}_2$, social welfare of the four stage game exceeds that of the three stage game with an exogenous adapter.

6 Variations of the model

With regard to the result that both firms agree on the provision of an adapter, it might be argued that we considered a very special case by (1) assuming a specific timing and (2) restricting the analyses to a two-way adapter and Bertrand competition.

With respect to the timing of our model, it could also be assumed that firms either choose their qualities and decide on an adapter simultaneously, or that firms decide on an adapter first and choose their qualities thereafter. In both variations the low quality firm would lose the possibility of forcing the high quality firm into compatibility. An equilibrium without the provision of an adapter would therefore always exist, whereas an adapter equilibrium exists only for small network externalities. However, in both modifications it has to be assumed that the development of an adapter is technologically impossible once the firms have determined the main design of their products. Otherwise the low quality firm can use its quality choice so that compatibility is achieved and we would be back in our model.

Finally, compatibility is also reached if each firm has the opportunity of installing a one-way adapter, or if we considered Cournot instead of Bertrand competition. By installing a one-way adapter, each firm creates greater network effects for its own product. Hence, both firms will install an adapter, full compatibility will be achieved, and firms choose the same qualities as in our framework. Similarly, Cournot competition would change the equilibrium qualities but not the result concerning compatibility. As in the model of vertical product differentiation without network externalities (see Bonnano (1986)) both firms would choose the highest possible quality level. In addition both firms would agree on compatibility, because it shifts the demand curves outward.

7 Conclusions

In our model, both firms agree on the provision of an adapter although the high quality firm would always prefer an equilibrium without an adapter. This is due to the fact that the low quality firm can successfully prevent the incompatibility equilibrium through its quality choice. While we assumed specific functional forms for the individual consumer's surplus and firms' costs, we expect the main results of our model to be maintained in a more general framework. The tougher price competition without an adapter should be one of them. But, since a high quality firm may agree on an adapter just because it wants to evade tough price competition, the low quality firm should always have the option of choosing its quality so that compatibility is achieved. Of course, there may also be costs of installing an adapter. It is obvious, however, that the two firms should always be able to agree on the provision of an adapter, as long as the additional profits of the two firms are higher than these extra costs. In order to ensure that this condition holds, the low quality firm might either adapt its quality choice or bear a greater share of the costs.

Our result concerning the higher social welfare with an adapter is not very surprising. The network externalities consumers enjoy are greater, and the low quality is higher with an adapter than without, whereas the high quality is identical. Since firms agree on an adapter in equilibrium anyway, our analysis supports the necessity of regulating the access to the network only in rare cases. Only if the low quality firm chooses a higher quality differentiation than it would if the adapter were provided exogenously, can social welfare be improved by a regulation which gives each firm the opportunity of providing an adapter without its rival's consent. An example of such a policy would be to make licensing compulsory for all those patented parts used in an adapter. If the quality differentiation in equilibrium is, however, smaller than with an exogenous adapter such a policy would even reduce social welfare.

Appendix[12]

Proof of Lemma 1

From the inspection of (2)-(7) the functions $\underline{p}_1(\cdot)$ and $\overline{p}_1(\cdot)$ describe the critical prices that determine which type of Nash equilibrium is played by the consumers. The Nash equilibrium is always unique, if no group of consumers has an incentive to deviate, meaning:

$$\frac{\partial u(x, q_1, \frac{\hat{x}-x}{\hat{x}}, p_1)}{\partial x} > \frac{\partial u(x, q_2, \frac{\hat{x}-x}{\hat{x}}, p_2)}{\partial x} \text{ with } \underline{x} = \frac{p_2\hat{x} - \mu q_2 x}{q_2(\hat{x}-\mu)}.$$

The latter is equivalent to (8).

Proof of Lemma 2

If the qualities satisfy (9) the firms' profit functions are (see Lemma 1):

$$\pi_{1N}(p_1, p_2; q_1, q_2) = \begin{cases} \frac{1}{\hat{x}}(p_1 - q_1)(\hat{x} - \underline{x}_1) & \text{for } p_1 \leq \min\{\overline{p}_1(\cdot),\, q_1\hat{x}\}, \\ \frac{1}{\hat{x}}(p_1 - q_1)(\hat{x} - \overline{x}) & \text{for } \overline{p}_1(\cdot) > p_1 > \underline{p}_1(\cdot), \\ 0 & \text{for } p_1 \geq \min\{\overline{p}_1(\cdot),\, q_1\hat{x}\}. \end{cases}$$

$$\pi_{2N}(p_2, p_1; q_2, q_1) = \begin{cases} \frac{1}{\hat{x}}(p_2 - q_2)(\hat{x} - \underline{x}_2) & \text{for } p_1 > \overline{p}_1(\cdot) \text{ and } p_2 < q_2\hat{x}, \\ \frac{1}{\hat{x}}(p_2 - q_2)(\overline{x} - \underline{x}) & \text{for } \overline{p}_1(\cdot) > p_1 > \underline{p}_1(\cdot), \\ 0 & \text{for } p_1 \leq \underline{p}_1(\cdot) \text{ or } p_2 \geq q_2\hat{x}. \end{cases}$$

Solving $\partial\pi_{1N}(\cdot)/\partial p_1 = 0$ and $\partial\pi_{2N}(\cdot)/\partial p_2 = 0$ for the best price response functions, shows that their intersection is unique and given by p_1^* and p_2^*.[13]

[12] For more detailed proofs see Baake, Boom (1997).

[13] The second order conditions are always fulfilled in our analysis.

Proof of Lemma 3

Consider first the case where consumers would play a Type 2 NE. With $p_2 = q_2$ the highest price p_1 which guarantees that a Type 2 NE exists and is not Pareto-dominated by another equilibrium is given by $\underline{p}_1(\cdot)$ for $q_1 \geq q_2\hat{x}/(\hat{x}-\mu)$ and $\hat{p}_1(\cdot)$ for $q_1 < q_2\, \hat{x}/\,(\hat{x}-\mu)$. If consumers would play a Type 3 NE, firm 1 can monopolize the market with either $\bar{p}_1(\cdot)$ or q_1, depending on the degree of quality differentiation. With $p_1 = q_1$ and $p_2 = q_2$ a Type 3 NE is Pareto dominated by a Type 2 NE. Similar arguments hold, if consumers play a Type 1 NE.

Proof of Lemma 4

One has to solve the analogous equations to (2)-(7) in order to derive when which type of Nash equilibrium is played by the consumers. For a Type 1 NE $0 < \underline{\underline{x}} < \overline{\overline{x}} < \hat{x}$ must hold where $\underline{\underline{x}}$ and $\overline{\overline{x}}$ are derived from the solution to:

$$u(\overline{\overline{x}}, q_1, \tilde{z}, p_1) \;=\; u(\overline{\overline{x}}, q_2, \tilde{z}, p_2) \tag{14}$$

$$u(\underline{\underline{x}}, q_2, \tilde{z}, p_2) \;=\; 0 \text{ with } \tilde{z} = \frac{\hat{x} - \underline{\underline{x}}}{\hat{x}} \tag{15}$$

Defining $\underline{\underline{p}}_1(p_2; q_1, q_2) = \{p_1 \mid \underline{\underline{x}} = \overline{\overline{x}}\} \equiv \underline{\underline{p}}_1(\cdot)$ and $\overline{\overline{p}}_1(p_2; q_1, q_2) = \{p_1 \mid \overline{\overline{x}} = \hat{x}\} \equiv \overline{\overline{p}}_1(\cdot)$ the profits of the two firms are:

$$\pi_{1A}(p_1, p_2) \;=\; \begin{cases} \frac{1}{\hat{x}}(p_1 - q_1)(\hat{x} - \underline{x}_1) & \text{for } p_1 \leq \underline{\underline{p}}_1(\cdot) \text{ and } p_1 < q_1\hat{x}, \\ \frac{1}{\hat{x}}(p_1 - q_1)(\hat{x} - \overline{\overline{x}}) & \text{for } \overline{\overline{p}}_1(\cdot) > p_1 > \underline{\underline{p}}_1, \\ 0 & \text{for } p_1 \geq \min\{\overline{\overline{p}}_1(\cdot), q_1\hat{x}\}, \end{cases}$$

$$\pi_{2A}(p_2, p_1) \;=\; \begin{cases} \frac{1}{\hat{x}}(p_2 - q_2)(\hat{x} - \underline{x}_2) & \text{for } p_1 \geq \overline{\overline{p}}_1(\cdot) \text{ and } p_2 < q_2\hat{x}, \\ \frac{1}{\hat{x}}(p_2 - q_2)(\overline{\overline{x}} - \underline{\underline{x}}) & \text{for } \overline{\overline{p}}_1(\cdot) > p_1 > \underline{\underline{p}}_1(\cdot), \\ 0 & \text{for } p_1 \leq \underline{\underline{p}}_1(\cdot) \text{ or } p_2 \geq q_2\hat{x}. \end{cases}$$

Solving $\partial\pi_{1A}(\cdot)/\partial p_1 = 0$ and $\partial\pi_{2A}(\cdot)/\partial p_2 = 0$ yields p_1^{**} and p_2^{**}.

Proof of Proposition 4

The two subgame perfect equilibria are due to the fact that which firm chooses the high or the low quality cannot be determined. The equilibrium quality of the low quality firm q_2 is given by[14]

$$\begin{array}{ll} q_2^{**} & \text{if: } 0 < \mu < 0.216\hat{x} \\ \tilde{q}_2 & \text{if: } \begin{cases} 0.216\hat{x} < \mu < 0.230\hat{x}. \\ 0.230\hat{x} < \mu < 0.5\hat{x} \text{ and consumers select a Type 2NE} \end{cases} \\ \tilde{\tilde{q}}_2 & \text{if: } 0.230\hat{x} < \mu < 0.242\hat{x}. \text{ and consumers select a Type 1 or a Type 3 NE} \\ q_2^{**} & \text{if: } 0.242\hat{x} < \mu < 0.5\hat{x} \text{ and consumers select a Type 1 or a Type 3 NE} \end{array}$$

[14]All critical values for μ are given approximately.

To proof these results let us define $\hat{q}_2$ as lowest quality level that violates (9) and $\widehat{\hat{q}}_2$ as the lowest quality that violates (8), given $q_1 = \bar{q}$.

Since $\mu < 0.155\hat{x} \Leftrightarrow \pi_{1A}(\bar{q}, q_2^{**}) > \pi_{1N}(\bar{q}, q_2^{**})$ and $q_2^{**} < \hat{q}_2$ as well as $\mu \leq 0.216\hat{x} \Leftrightarrow \pi_{1A}(\bar{q}, q_2^{**}) \geq \pi_{1N}(\underline{p}_1(q_2^{**}), \bar{q}, q_2^{**})$ and $\hat{q}_2 < q_2^{**} < \widehat{\hat{q}}_2$ firm 1 will agree on an adapter if firm 2 chooses its optimal quality q_2^{**}. For all $\mu > 0.216\hat{x}$ the low quality firm has to deviate from q_2^{**} in order to induce firm 1 to agree on an adapter.

Given that consumers play a Type 2 NE for $q_2 > \widehat{\hat{q}}_2$ firm 1's profit without an adapter is $\pi_{1N}(\underline{p}_1, \bar{q}, q_2)$ for $q_2 < (\hat{x} - \mu)\bar{q}/\hat{x}$ or $\pi_{1N}(\hat{p}_1, \bar{q}, q_2)$ otherwise. Since $\pi_{1N}(\hat{p}_1, \bar{q}, q_2) > \pi_{1A}(\bar{q}, q_2)$ and $\pi_{2A}(\tilde{q}_2, \bar{q}) > \pi_{2N}(q_2^{*}, \bar{q})$ for all $0.216\hat{x} < \mu \leq 0.5\hat{x}$ as well as $\tilde{q}_2 < (\hat{x} - \mu)\bar{q}/\hat{x}$ firm 2 chooses $\tilde{q}_2$.

If consumers play a Type 1 or a Type 3 NE for $q_2 > \widehat{\hat{q}}_2$ the profit of firm 1 without an adapter is $\pi_{1N}(\overline{p}_1, \bar{q}, q_2)$ for $\bar{q} > q_2\hat{x}/(\hat{x} - \mu)$ and 0 otherwise. Since $\mu < 0.242 \Rightarrow (\hat{x} - \mu)/\hat{x}\,\bar{q} > \tilde{\tilde{q}}_2 > \widehat{\hat{q}}_2$ as well as $\tilde{q}_2 < \widehat{\hat{q}}_2$ firm 2 can choose $\tilde{q}_2$ as well as $\tilde{\tilde{q}}_2$ for $\mu < 0.242$ in order to induce the agreement of firm 1. Furthermore, $\pi_{2A}(\tilde{q}_2, \bar{q}) \geq \pi_{2A}(\tilde{\tilde{q}}_2, \bar{q}) \Leftrightarrow \mu \leq 0.230\hat{x}$ implies that firm 2 chooses $\tilde{q}_2 < q_2^{**}$ for $0.216\hat{x} < \mu < 0.230\hat{x}$. In addition $\pi_{1A}(\bar{q}, q_2^{**}) \geq \pi_{1N}(\overline{p}_1, \bar{q}, q_2^{**}) \Leftrightarrow \mu > 0.242\hat{x}$. Thus, firm 2 chooses $\tilde{\tilde{q}}_2 > q_2^{**}$ for all $0.230\hat{x} < \mu < 0.242\hat{x}$ and q_2^{**} for all $0.242\hat{x} < \mu < 0.5\hat{x}$.

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