

Entrepreneurial Financing, Advice, and Agency Costs

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Abstract

This paper studies the interplay between advice and agency costs in entrepreneurial financing. Advice exacerbates agency problems, because the agent may use it at the investor's disadvantage. Depending on the magnitude of the agency problem, optimal financing relationships may induce full, partial, or no advice. Because the trade-off is delicate, investors need to control the information flow carefully. This explains the dual role of financing and consulting by investors in entrepreneurial financing.

Keywords: optimal advice, agency costs, informed investors, entrepreneurial financing

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1 Introduction

Investors in entrepreneurial ventures often play a dual role. First, they have to control important agency problems that typically arise between the entrepreneur and the investor. Second, they provide advice and guidance to help entrepreneurs turn their innovative projects into commercial success. The dual role of investors explains the demand for active investors, such as venture capitalists, who are specialists in fulfilling both roles (e.g., Shalman (1990), Kaplan and Strömberg (2004)). Indeed, theory confirms that an active, hands-on approach is helpful in controlling agency problems that are typical of entrepreneurial ventures (e.g., Admati and Pfleiderer (1994), Bergemann and Hege (1997)). It has also shown that many features of financial contracts which we observe in practise, may result from an entrepreneur's need for help and advice from an active, knowledgeable investor (e.g., Habib and Johnsen (2000), Casamatta (2003), Schmidt (2003), Repullo and Suarez (2004)).

Taking the dual role of entrepreneurial investors seriously, this paper takes a closer look at the interplay between advice and agency problems and identifies a potential conflict. Advice may exacerbate agency problems, because the entrepreneur may use it to the investor's disadvantage. Optimal financial arrangements will take this negative effect into account. Indeed, we show how the interplay between advice and agency costs leads to a theory of optimal advice. Moreover, in order to implement the optimal degree of advice, it is important that the investor plays the dual role of both financier and advisor, because the entrepreneur suffers from a time-inconsistency problem. From an *ex ante* perspective, he wants to obtain too much information *ex post*. The investor has the correct incentives to control this time-inconsistency problem. Hence, by focusing on the informational aspect of advice, we offer an explanation of the dual role of consulting and investing in entrepreneurial financing.¹

We study the problem in a standard Jensen and Meckling (1976) agency model of outside finance: After the investor provides the initial investment,

¹For an alternative explanation, see Casamatta (2003) who shows that a double moral hazard problem makes the dual role of venture capitalists optimal.

the entrepreneur takes some unobservable action that influences the outcome of the project. We extend this classical setup by private information on part of the investor. More specifically, we assume that the investor has superior knowledge about the commercial potential of the entrepreneur's project.² This knowledge is relevant to the entrepreneur and the investor may reveal her private information in the form of advice.

In this setup, we show that the investor's advice has two contradicting effects. If the advice entails good news about the project, it makes effort more worthwhile and the entrepreneur responds by increasing his effort. This represents a positive effect, because in the agency problem of Jensen and Meckling (1976), the entrepreneur's effort is inefficiently low. Yet, if the advice provides bad news, the entrepreneur reduces his effort, thereby exacerbating the under-supply of effort. We show that this negative effect may outweigh the positive effect of advice. In general, the outcome of the resulting trade-off depends on the magnitude of the underlying agency problem and leads to a theory of optimal advice.

Related to the current paper is Habib and Johnsen (2000), who were the first to consider a setup in which an entrepreneur may actively try to obtain information from knowledgeable investors to guide his entrepreneurial activities. However, Habib and Johnsen (2000) abstract from any ex post agency problems between the entrepreneur and the investor. As a result, information is always beneficial and full revelation is optimal. From this perspective, the new insight of the current paper is that agency problems may overturn these results: when the ex post agency problem is severe, obtaining information from a knowledgeable investor is harmful.

In addition, this paper complements the literature on inside investors (e.g. Admati and Pfleiderer (1994), Bergemann and Hege (1997), Casamatta (2003), Schmidt (2003) and Repullo and Suarez (2004)). Also this literature assumes a more active role for the investor, but her role relates to some

²E.g., Kaplan and Strömberg (p.2204, 2004) explicitly argue that investors are often better at determining the commercial success of projects due to a superior knowledge about expected demand, marketability, and consumer adaptation.

activity. In contrast, the current paper emphasizes the role of private information. The paper is also related to Manove et. al. (2001) and Inderst and Müller (2006), who examine an investor’s superior screening technology to distinguish between good and bad projects. Screening represents a natural source of an investor’s private information, because the fact that the investor is willing to invest reveals some, but not all her information. In particular, when the entrepreneur gets financed, he will know that his project is “good”, but not how good it actually is. It is this remaining degree of asymmetric information on which this paper focuses.

In order to solve for the optimal degree of advice we phrase the problem as one of optimal mechanism design. From a technical perspective this paper provides the innovation that we analyze an adverse selection framework in which we cannot employ the revelation principle. The problem which arises is that the entrepreneur, as contract designer, chooses an unobservable action that depends on his belief. Because a revelation of information affect these beliefs, one cannot apply the classical revelation principle. We therefore use a modified revelation principle developed in Bester and Strausz (2001) to compute the optimal mechanism.

The rest of the paper is organized as follows. The next section introduces the principal–agent setup. Section 3 derives the first best solution and analyzes the finance problem when there is only moral hazard. Section 4 studies the problem of optimal advice. It first shows that contracts without advice may be superior to contracts with advice. Subsequently, it derives the optimal amount of advice. Section 5 concludes and discusses the implications of our findings. All formal proves are relegated to Appendix A. Appendix B demonstrates that our results do not depend on our focus on deterministic finance contracts.

2 The Setup

Consider an entrepreneur who has a non–scalable project that requires an initial investment of $I > 0$. If the project is successful, it yields a value of $x \equiv 1$. An unsuccessful project yields zero. The probability of success,

$p(e, \theta)$, depends on the entrepreneur's effort e and the project's commercial potential, θ . The probability of success increases with the entrepreneur's effort, e , and the project's commercial potential, θ . In particular, we assume $p(e, \theta) \equiv e\theta$ so that the project's potential and effort are complements; the higher the project potential θ , the larger the marginal effect of effort and vice versa.³ The entrepreneur's cost of effort is $c(e) = e^2/2$.

The entrepreneur is aware that the project's commercial potential is high, θ_h , with probability ν and low, $\theta_l < \theta_h$, with probability $1 - \nu$. The project's expected potential is therefore $\bar{\theta} = \nu\theta_h + (1 - \nu)\theta_l$.

Because the entrepreneur has no wealth, he must raise the required investment I from the outside investor. The outside investor has experience with financing similar projects and is therefore better at judging the project's commercial potential; she observes the parameter θ perfectly. The entrepreneur is aware of the investor's superior knowledge.

The entrepreneur has no wealth and he can repay the investor only if his project succeeds. We assume that the outcome of the project is verifiable. Hence, a finance contract specifies a repayment, $R \in [0, 1]$, contingent on the project being successful.⁴

The entrepreneur and the investor are risk neutral. In particular, the entrepreneur's payoff is

$$V(e, R|\theta) = \theta e(1 - R) - c(e).$$

Similarly, the investor's payoff is

$$U(e, R|\theta) = \theta eR - I.$$

Outside options and interest rates are normalized to zero.

³The multiplicative specification yields a tractable framework in which we may work out the trade-off between advice and agency problem analytically.

⁴For convenience, we focus on deterministic finance contracts and demonstrate in Appendix B that deterministic contracts are optimal for ν small enough.

To circumvent signalling issues, we assume that the entrepreneur makes a take-or-leave-it offer R to the privately informed investor.⁵ We study the following sequence of events:

t=0: Nature chooses the project's potential $\theta \in \{\theta_h, \theta_l\}$ and informs the investor.

t=1: The entrepreneur offers a repayment schedule $R(\cdot)$ to the investor.

t=2: The investor accepts or rejects. If she rejects, the game ends.

t=3: The entrepreneur chooses his effort $e \geq 0$.

t=4: Nature determines whether the project succeeds or fails.

Note that the investor's private information is relevant to the entrepreneur for his effort choice at stage 3. We are interested whether under optimal finance relationships the investor reveals her information to the entrepreneur.

3 Two Benchmarks

3.1 Full Information

In order to develop some intuition about the model, we start with the full information case in which both effort and the state of demand are publicly observable.

First, suppose the entrepreneur can finance the project himself and observes the project's potential, θ , perfectly. In this case, the entrepreneur's payoff from the project is $V(e|\theta_i) = e\theta_i - c(e) - I$. First order conditions yield the optimal, first best effort level

$$e_i^* \equiv \theta_i.$$

This effort level yields the entrepreneur a payoff of $\theta_i^2/2 - I$. Hence, in the state θ_i the entrepreneur executes his project if and only if $I \leq I_i^* \equiv \theta_i^2/2$.

⁵By giving all bargaining power to the entrepreneur, we also ensure that if full advice is suboptimal, it is so from the entrepreneur's perspective.

Now suppose there is still complete information, but the entrepreneur must, due to a lack of private funds, raise the required investment I from the investor. Since effort is observable, a general finance contract is a pair (e, R) , dictating an effort level e and a repayment $R \in [0, 1]$ conditional on the project being successful. The investor accepts a contract (e, R) whenever

$$\theta_i e R - I \geq 0. \quad (1)$$

It follows that the entrepreneur proposes a contract (e, R) that solves the following maximization problem

$$\max_{e, R} \theta_i e (1 - R) - e^2/2 \quad \text{s.t. (1)}. \quad (2)$$

The solution is $(e, R) = (\theta_i, I/\theta_i^2)$ and yields the entrepreneur a payoff $V(\theta_i, I/\theta_i^2) = \theta_i^2/2 - I$. As before, the entrepreneur executes the project exactly when $I \leq I_i^*$. Despite the need for outside finance, the first best solution is still attainable, because all information is shared symmetrically.

3.2 A Pure Agency Problem

In the second benchmark we analyze our setup as a standard agency model of outside finance. In particular, let the project's commercial potential, θ , be observable and the entrepreneur's effort, e , be unobservable. In this case, the contract can condition repayments directly on the project's potential θ_i , but not on the entrepreneur's choice of effort; the effort choice underlies a moral hazard problem which causes the agency problem. It follows that the contract has the form (R_l, R_h) and dictates a repayment R_i contingent on the actual state being θ_i .

In the state θ_i the entrepreneur's utility from an effort level e is

$$V(e_i, R_i | \theta_i) = \theta_i e_i (1 - R_i) - e_i^2/2.$$

His optimal choice of effort satisfies the first order condition

$$e = \hat{e}_i \equiv \theta_i (1 - R_i). \quad (3)$$

The effort level \hat{e}_i is smaller than the respective first best level e_i^* , because the entrepreneur receives only a share $1 - R_i$ of the project's return. The investor's claim R_i mutes the entrepreneur's incentive for effort.

Anticipating the effort level \hat{e}_i , an investor θ_i accepts a repayment R_i whenever

$$\theta_i \hat{e}_i R_i = R_i \theta_i^2 (1 - R_i) \geq I. \quad (4)$$

Hence, under moral hazard the entrepreneur's optimal contract, R_i^m , solves the problem

$$\max_{e_i, R_i} \theta_i e_i (1 - R_i) - e_i^2 / 2 \quad \text{s.t. (3) and (4)}.$$

The problem has a solution only when the required investment I is small enough. In particular, for $I \leq \bar{I}_i \equiv \theta_i^2 / 4$ the solution is

$$R_i^m \equiv \left(1 - \sqrt{1 - 4I / \theta_i^2} \right) / 2.$$

For $I > \bar{I}_i$, the program does not admit a solution, because there does not exist a contract that enables the investor to recoup her investment I .

Proposition 1 *Assume effort is unobservable and the state of demand is public information. For $I \leq \bar{I}_l$ the optimal contract is (R_l^m, R_h^m) and the project is executed in both states. For $I \in (\bar{I}_l, \bar{I}_h]$ the project is executed only in state θ_h under the contract R_h^m . For $I > \bar{I}_h$ the project is not executed in either state.*

The proposition shows that the agency problem causes two types of inefficiencies. First, it leads to an *undersupply* of effort, because the entrepreneur receives only a share of the return from his effort while incurring its entire cost. Second, *underinvestment* occurs for $I \in (\bar{I}_i, I_i^*)$; in the first best the project gets financed for any $I \leq I_i^*$, whereas with moral hazard the project is only executed for $I \leq \bar{I}_i$. The underinvestment effect is related to the undersupply of effort, because the low effort level lowers the project's net value. As a result, it is only profitable for a smaller range of investments I .

4 Advice

We now return to the original setup and study how the investor's private information affects the investment problem. Since the investor's private information is relevant to the entrepreneur, the entrepreneur may select a contract that induces the investor to reveal the project's commercial potential to

him. We interpret the revelation of such information as advice. Depending on the amount of information revelation, a contract may lead to full, no, or partial advice. This section studies the optimality of such contracts.

4.1 Full Advice

In this subsection we first study *full-advice* contracts that induce the investor to reveal all her information to the entrepreneur. In the spirit of the revelation principle, we may capture these contracts by pairs (R_h, R_l) which give the investor an incentive to pick the contract R_h if she has learned that the project's commerciability is high and, otherwise, picks the contract R_l .

By the investor's choice of contract, the entrepreneur learns the project's potential θ . As a consequence, the entrepreneur chooses his effort under full information and, as established in the previous section, picks the effort level \hat{e}_h when the project has a high commercial potential and \hat{e}_l when its potential is low.

Anticipating the entrepreneur's choice of effort, the investor has an incentive to reveal a high commercial potential truthfully, whenever $\theta_h \hat{e}_h R_h - I \geq \theta_h \hat{e}_l R_l - I$. Likewise, she honestly reveals the project's low potential if $\theta_l \hat{e}_l R_l - I \geq \theta_l \hat{e}_h R_h - I$. The two constraints are equivalent to the single requirement

$$\hat{e}_h R_h = \hat{e}_l R_l. \tag{5}$$

In order for the investor to participate, she must at least recoup her initial investment I . Anticipating that the entrepreneur picks the effort level \hat{e}_i , the participation constraints are

$$\theta_l \hat{e}_l R_l \geq I \tag{6}$$

for an investor who knows that the project's potential is low and

$$\theta_h \hat{e}_h R_h \geq I \tag{7}$$

for an investor with information that the potential is high.

Intuitively, the participation constraint is stricter in the low state θ_l than in the state θ_h . Hence, by his choice of contracts, the entrepreneur may guarantee the participation of the investor in both states or only when her information is favorable. In the following, we concentrate on *full participation* contracts that induce a participation irrespective of the project's commercial potential.⁶

The optimal finance contract with full advice and full participation is a solution of the following maximization problem

$$\begin{aligned} \max_{R_l, R_h} \quad & V^r = \nu(\theta_h \hat{e}_h(1 - R_h) - \hat{e}_h^2/2) + (1 - \nu)(\theta_l \hat{e}_l(1 - R_l) - \hat{e}_l^2/2) \\ \text{s.t.} \quad & (3), (5), (6), (7) \end{aligned}$$

The following proposition derives its solution.

Proposition 2 *Full advice and full participation is implementable only if $I \leq \bar{I}_l$. The optimal contract that induces full advice is*

$$R_l^r = \left(1 - \sqrt{1 - 4I/\theta_l^2}\right) / 2 \text{ and } R_h^r = \left(1 - \sqrt{1 - 4I/(\theta_l\theta_h)}\right) / 2.$$

It yields the entrepreneur

$$V^r = \left[\nu\theta_h^2 \left(1 + \sqrt{1 - 4I/(\theta_l\theta_h)}\right)^2 + (1 - \nu)\theta_l^2 \left(1 + \sqrt{1 - 4I/\theta_l^2}\right)^2 \right] / 8.$$

The proposition first notes that full advice may not be implementable. Indeed, when the required investment is large, the project has a negative net present value so that the entrepreneur cannot convince the investor to lend him her money. Since the entrepreneur chooses his effort under full information, this cut-off value is identical to the cut-off value, \bar{I}_l , that we obtained in Subsection 3.2.

The proposition further shows that the optimal repayment, R_h , in case of good news, $\theta = \theta_h$, is lower than the optimal repayment, R_l , in case of bad news. This follows because the investor's repayment is more likely when

⁶Proposition 7 demonstrates the optimality of such contracts when ν is low.

the project's commercial potential is high due to two reasons. First, a high potential implies that the project is more likely to succeed. Second, because the entrepreneur learns the project's potential, he chooses a higher effort level in the state θ_h . This increases the likelihood of repayment even more.

4.2 No Advice

To show that the investor's advice may actually harm the entrepreneur, this subsection analyzes contracts that do not reveal any information to the entrepreneur and shows that such *no-advice* contracts may be superior to the full-advice contract of the previous subsection.

Because the entrepreneur does not learn anything from a no-advice contract, his beliefs remain unaffected. Hence, given a finance contract R , the entrepreneur expects that an effort level e results in the expected payoff

$$V^n(e, R) = \nu(\theta_h e(1 - R)) + (1 - \nu)(\theta_l e(1 - R)) - e^2/2.$$

Maximizing his payoff, the entrepreneur picks the effort level

$$e = \hat{e} \equiv (\nu\theta_h + (1 - \nu)\theta_l)(1 - R) = \bar{\theta}(1 - R). \quad (8)$$

Because the entrepreneur should not learn anything from the investor's behavior, a no-advice contract requires that the investor accepts it irrespective of her private information. An investor, who has learned that the project's potential is low, expects to recoup her initial investment whenever $\theta_l \hat{e} R \geq I$. Anticipating the entrepreneur's effort choice, \hat{e} , she accepts a contract R if

$$\theta_l \bar{\theta}(1 - R) R \geq I. \quad (9)$$

Likewise, an investor, who has learned that potential is high ($\theta = \theta_h$), expects to recoup her investment whenever $\theta_h \hat{e} R \geq I$. Given the choice of effort \hat{e} , she accepts a contract R if

$$\theta_h \bar{\theta}(1 - R) R \geq I. \quad (10)$$

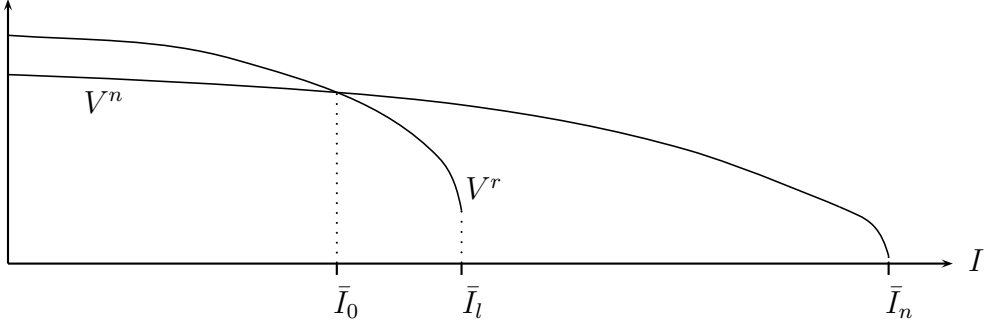


Figure 1: Entrepreneur's payoffs.

It follows that the following program yields the optimal no-advice contract:

$$\max_{e,R} V^n(e, R) \text{ s.t. (8), (9), (10)} \quad (11)$$

The next proposition derives a solution to this problem.

Proposition 3 *An optimal no-advice contract, R^n , exists only if $I \leq \bar{I}_n \equiv \theta_l \bar{\theta} / 4$ and exhibits*

$$R^n = \left(1 - \sqrt{1 - 4I / (\theta_l \bar{\theta})} \right) / 2.$$

It yields the entrepreneur the payoff

$$V^n = \left(\bar{\theta} + \bar{\theta} \sqrt{1 - 4I / (\theta_l \bar{\theta})} \right)^2 / 8.$$

We may now compare the entrepreneur's payoff from an optimal full-advice contract to the optimal no-advice contract.

Proposition 4 *There exists an $\bar{I}_0 \in (0, \bar{I}_l]$ such that a contract with no advice is superior to a contract with advice if and only if $I \in (\bar{I}_0, \bar{I}_n]$. We have $\bar{I}_0 = \bar{I}_l \equiv \theta_l^2 / 4$ if $\nu \geq \bar{\nu}$ and $\bar{I}_0 \in (0, \bar{I}_l)$ when $\nu < \bar{\nu}$, where*

$$\bar{\nu} = \frac{\theta_l(\theta_l^2 + \theta_h \theta_l + 2\theta_h^{3/2} \sqrt{\theta_h - \theta_l} - 2\theta_h^2)}{(\theta_h - \theta_l)^2(3\theta_h + \theta_l)}.$$

The proposition shows that whether advice is optimal depends on the investment level I . Only for smaller investments ($I < \bar{I}_0$) advice is worthwhile

to the entrepreneur. Figure 1 illustrates this result graphically for the case $\nu < \bar{\nu}$. Due to $\bar{I}_0 < \bar{I}_1$, we may identify two different reasons why no-advice contracts outperform full-advice contracts. The first reason is that full-advice contracts may simply not exist. This occurs when the required investment I is relatively large ($I > \bar{I}_l$). Second, even when full-advice contracts do exist, they may be inferior to no-advice contracts. This occurs when the required investment I is intermediate ($I \in (\bar{I}_0, \bar{I}_1)$). Hence, only if the required investment is small ($I \leq I_0$) the entrepreneur benefits from advice.

We now address the question why advice may be bad for the entrepreneur when the level of investment, I , is high. The result follows from the investor's participation constraint. In order for the investor to recoup her initial investment, I , the entrepreneur must provide an adequate level of effort. As I increases, this requirement becomes more difficult to fulfill. The agency problem leads to an undersupply of effort, which makes it even harder to satisfy the requirement. The problem exacerbates further, when the investor's advice entails bad news to the entrepreneur, because he then responds with an even lower effort level. Indeed, the effort level \hat{e}_l that is induced under a full-advice contract is smaller than the effort level \hat{e} which results without any advice. Hence, no-advice contracts have the advantage that the entrepreneur exerts a higher effort when the project's potential is low. This advantage becomes important when the required investment, I , is high so that it is difficult to fulfill the investor's participation constraint.

4.3 Optimal Advice

The previous subsections contrasted the two extremes of full versus no advice and showed that optimality depends on the underlying agency problem. The result raises the question whether these extremes are really optimal or whether the entrepreneur may gain from inducing partial advice. This subsection addresses this question and derives the optimal amount of advice. It shows that, depending on the magnitude of the agency problem, partial advice may indeed be superior to the two extremes.

In order to deduce the optimal amount of advice, we will treat it as

being induced by the financial contract in some optimal way. This raises the question what types of contract induce a partial revelation of information and what kind of mechanisms we need to consider in order to find the optimal one.

Clearly, we cannot use the revelation principle and restrict attention to direct mechanisms that induce truthful revelation, because such mechanisms represent exactly those contracts that reveal all information. This demonstrates that the classical revelation principle fails to hold and truthful direct mechanisms may not be optimal.

Formally, the failure of the revelation principle is due to the limited commitment of the entrepreneur. As a result we use a modified revelation principle as developed in Bester and Strausz (2001). This modified principle implies for the current framework that there is still no loss of generality by focusing on direct menus (R_l, R_h) which gives the investor an incentive to report truthfully. However, the optimal direct mechanism may require the investor to misreport her type with positive probability, despite her (weak) incentive to report truthfully. Such lying represents a partial revelation of information and expresses the idea behind the modified revelation principle that any optimal outcome can be replicated by a direct mechanism with lying.⁷

Restricting our attention to menus (R_l, R_h) the subsequent behavior of the investor and the entrepreneur can be described by a combination $\Gamma = (\alpha_l, \alpha_h, \nu_l, \nu_h, e_l, e_h)$. The variable α_i describes the probability that the investor θ_i reports her type truthfully. The variable ν_i represents the entrepreneur's updated belief that the investor is of type θ_h given that she claimed type θ_i . Finally, e_i describes the entrepreneur's choice of effort when the investor made the claim θ_i . For a given contract (R_l, R_h) we look for behavior that constitutes a Perfect Bayesian Equilibrium. This implies that the combination Γ has to satisfy four restrictions.

First, the investor must have a weak incentive to report her type truth-

⁷For instance, the outcome of a non-revelation contract is replicated when both types of investors lie with a probability $1/2$.

fully. Hence, given the effort levels (e_l, e_h) , it must hold for type θ_h that $\theta_h e_h R_h - I \geq \theta_h e_l R_l - I$, whereas for type θ_l it must hold $\theta_l e_l R_l - I \geq \theta_l e_h R_h - I$. Taken together these inequalities are equivalent to the condition

$$e_l R_l = e_h R_h. \quad (12)$$

As before, the requirement that the investor must have a weak incentive to report her type truthfully implies that she is indifferent between the repayment schedules R_h and R_l . Note that the condition guarantees that any reporting strategy $\alpha_i < 1$, which involves some positive probability of lying, is also optimal. Since constraint (12) originates from the investor's private information, we refer to it as the *adverse selection constraint*.

Second, the entrepreneur's beliefs must be Bayes' consistent with the investor's reporting strategy (α_l, α_h) . That is, the beliefs ν_i satisfy Bayes' Law:

$$\nu_l = \nu_l(\alpha) \equiv \frac{\nu(1 - \alpha_h)}{\nu(1 - \alpha_h) + (1 - \nu)\alpha_l}; \quad \nu_h = \nu_h(\alpha) \equiv \frac{\nu\alpha_h}{\nu\alpha_h + (1 - \nu)(1 - \alpha_l)}. \quad (13)$$

Third, given the entrepreneur's beliefs his effort choice must be optimal. Because the entrepreneur's expected payoff from an effort level e is

$$V(e|R, \tilde{\nu}) = (1 - \tilde{\nu})\theta_l e(1 - R) + \tilde{\nu}\theta_h e(1 - R) - e^2/2,$$

his optimal effort level is

$$e(\tilde{\nu}, R) \equiv [\tilde{\nu}(\theta_h - \theta_l) + \theta_l](1 - R).$$

It follows that the effort choices (e_l, e_h) satisfy

$$e_l = e(\nu_l, R_l); \quad e_h = e(\nu_h, R_h). \quad (14)$$

The equations in (14) represent the *moral hazard constraints*. They describe the entrepreneur's unobservable behavior in response to the repayment scheme R and his beliefs $\tilde{\nu}$. Intuitively, the entrepreneur's effort is increasing in his belief $\tilde{\nu}$ and decreasing in the repayment R .

Finally, the combination Γ must guarantee the investor her reservation utility, since otherwise she would reject to participate. This condition translates to the participation constraints

$$\theta_l e_l R_l \geq I; \quad \theta_h e_h R_h \geq I. \quad (15)$$

Summarizing, the combination Γ constitutes a Perfect Bayesian Equilibrium if and only if it satisfies the conditions (12) to (15). Our task is to derive a Perfect Bayesian Equilibrium that yields the entrepreneur the largest payoff. Given an equilibrium Γ , this payoff is

$$\begin{aligned} V(\Gamma) \equiv & (1 - \nu)[\alpha_l(\theta_l e_l(1 - R_l) - e_l^2/2) + (1 - \alpha_l)(\theta_l e_h(1 - R_h) - e_h^2/2)] \\ & + \nu[\alpha_h(\theta_h e_h(1 - R_h) - e_h^2/2) + (1 - \alpha_h)(\theta_h e_l(1 - R_l) - e_l^2/2)]. \end{aligned}$$

Consequently, we consider the maximization problem:

$$\max_{R_l, R_h, \Gamma} V(\Gamma) \text{ subject to (12) - (15).}$$

In order to solve this problem, we first substitute the moral hazard constraints (14) into the adverse selection constraints (12) and obtain a quadratic relationship between R_l and R_h :

$$(\theta_l + \nu_l(\theta_h - \theta_l))(1 - R_l)R_l = (\theta_l + \nu_h(\theta_h - \theta_l))(1 - R_h)R_h. \quad (16)$$

Moreover, since $\theta_l < \theta_h$, the adverse selection constraints (12) imply that the relevant participation constraint in (15) is $\theta_l e_l R_l \geq I$. Substitution of the respective moral hazard constraint in (14) transforms this participation constraint into

$$\theta_l(\theta_l + \nu_l(\theta_h - \theta_l))(1 - R_l)R_l \geq I. \quad (17)$$

The constraints (16) and (17) play a crucial role in the analysis. Figure 2 displays, for a given reporting behavior α , the constraints graphically. The two parabola represent the adverse selection constraints (16). The vertical lines reflect the participation constraint (17). The dashed curves illustrate

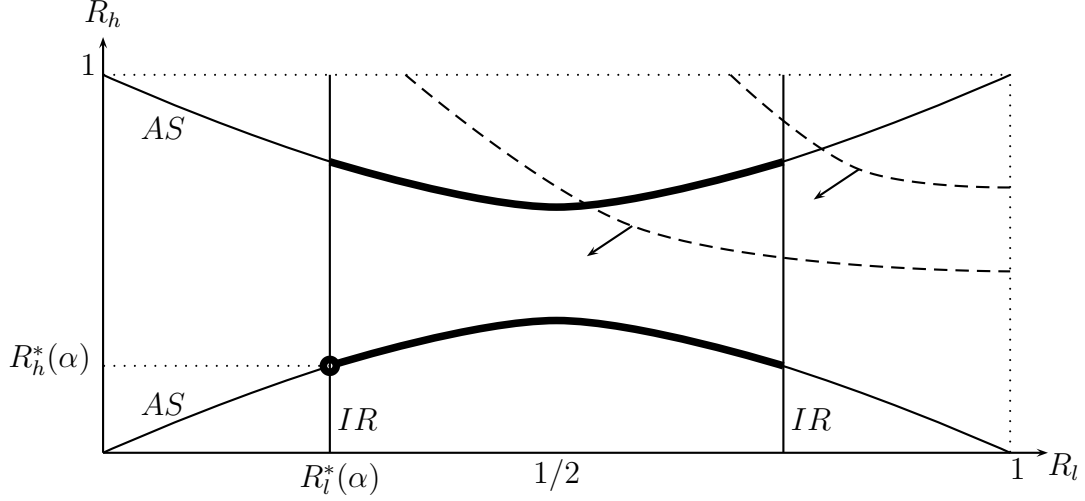


Figure 2: Adverse Selection and Participation Constraints.

two iso-utility levels of the entrepreneur. As may be expected, the arrows indicate that his utility levels increase towards the origin.

Figure 2 reveals the main idea behind the subsequent analysis. The thickened parts of the parabola describe all the combinations (R_l, R_h) that satisfy the adverse selection (16) and the participation constraints (17). Since the entrepreneur's utility increases towards the origin, the optimal repayment schedule is located at $(R_l^*(\alpha), R_h^*(\alpha))$. However, the figure does not reveal that a specific reporting behavior α is only implementable if the required investment I is small enough. The following proposition addresses this issue and derives the optimal repayment schedule $(R_l^*(\alpha), R_h^*(\alpha))$ analytically.

Proposition 5 *A reporting behavior α is implementable if and only if*

$$I \leq \bar{I}(\alpha) \equiv \theta_l(\theta_l + \nu_l(\alpha)(\theta_h - \theta_l))/4. \quad (18)$$

The optimal repayment schedule $(R_l^(\alpha), R_h^*(\alpha))$ that induces an implementable α is*

$$R_l^*(\alpha) \equiv \frac{1}{2} - \frac{\sqrt{\nu_l(\alpha)\theta_h + (1 - \nu_l(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\nu_l(\alpha)\theta_h + (1 - \nu_l(\alpha))\theta_l}}$$

and

$$R_h^*(\alpha) \equiv \frac{1}{2} - \frac{\sqrt{\nu_h(\alpha)\theta_h + (1 - \nu_h(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\theta_h\nu_h(\alpha) + (1 - \nu_h(\alpha))\theta_l}}.$$

Proposition 5 shows that a reporting strategy α is implementable if and only if the required investment I is small relative to the equilibrium belief $\nu_l(\alpha)$. For full advice we have $\alpha = (\alpha_l, \alpha_h) = (1, 1)$ so that $\nu_l(1, 1) = 0$ and $\bar{I}(1, 1) = \bar{I}_l$. This confirms the findings of Proposition 2 that the entrepreneur cannot induce the investor to reveal all her information if $I > \bar{I}_l$.

In order to determine the optimal reporting strategy among all implementable reporting strategies, we define

$$\bar{I}_1 \equiv \frac{2\theta_h\theta_l^2(2\theta_h - \theta_l)}{(4\theta_h - \theta_l)^2} \text{ and } \bar{I}_2 \equiv \frac{2\theta_h\theta_l\bar{\theta}(2\theta_h - \bar{\theta})}{(4\theta_h - \bar{\theta})^2}.$$

The following proposition derives our main result, the optimal degree of advice.

Proposition 6 *Suppose it is optimal to ensure the investor's participation in both states θ_h and θ_l . Then for $I \leq \bar{I}_1$ the optimal contract is fully revealing with $\alpha_h = \alpha_l = 1$. For $I \in [\bar{I}_2, \bar{I}_n]$ the optimal contract is non-revealing with $\alpha_h = 0$ and $\alpha_l = 1$. For $I \in (\bar{I}_1, \bar{I}_2)$ the optimal contract is partially revealing with*

$$\alpha_l = 1 \text{ and } \alpha_h = \frac{(1 - \hat{R}_l)\hat{R}_l\theta_l\bar{\theta} - I}{\nu(1 - \hat{R}_l)\hat{R}_l\theta_h\theta_l} \in (0, 1).$$

where

$$\hat{R}_l = \frac{1}{4} \left(1 + \sqrt{1 - 4I/(\theta_h\theta_l)} \right).$$

Figure 3 illustrates the proposition graphically. It contrasts the entrepreneurs payoff V^* from an optimal degree of information revelation to the extremes of full and no advice of Figure 1. It shows that, for intermediate values $I \in (\bar{I}_1, \bar{I}_2)$, partial revelation is superior to either full or no revelation.

Proposition 6 shows that the optimal contract does not switch abruptly from full to no advice; the optimal degree of advice changes smoothly with

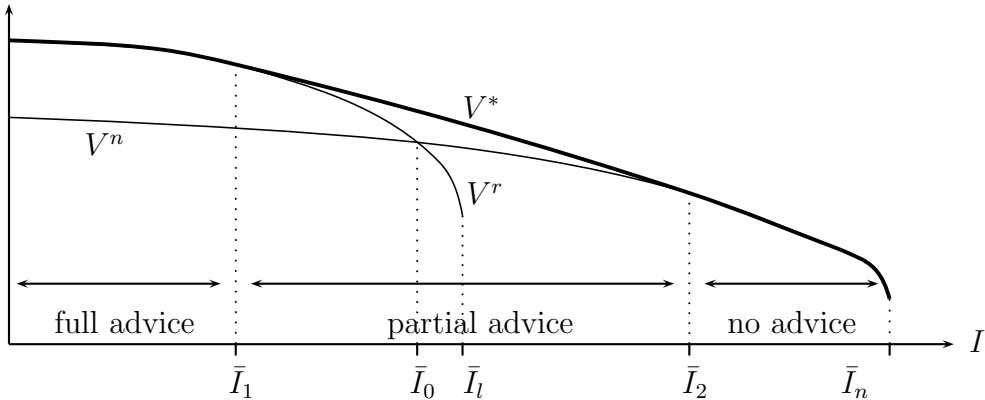


Figure 3: Optimal reporting α_h^* and the entrepreneur's payoffs.

the project's profitability as expressed by the investment level I . As I rises and the project becomes less profitable, the optimal contract reveals less and less information.

The proposition reinforces the intuition provided in Subsection 4.2. Optimal advice reveals as much information as possible but ensures that the entrepreneur has enough incentives to provide an adequate level of effort when the project's commercial potential is low. This observation follows from the fact that the optimal contract makes only the investor's message θ_l less informative. If the entrepreneur receives the message θ_l , this might even occur when $\theta = \theta_h$. Consequently, the entrepreneur's belief exhibits $\nu_l(\alpha) > 0$, which gives the entrepreneur enough incentives to provide an adequate effort level. In contrast, the message θ_h is always fully informative. Hence, the entrepreneur correctly anticipates that a message θ_h only comes from an investor with information $\theta = \theta_h$. This maximizes his incentives for effort after receiving the message θ_h .

Finally, we address the qualifier of Proposition 6 that it is optimal to ensure the investor's participation in both states. The following proposition confirms the intuition that this is the case when the ex ante probability, ν , is small enough. In this case, it is relatively unlikely that the commerciality is high so that a contract that is only accepted in this state yields the entrepreneur rather little in expected terms.

Proposition 7 *There exists some $\hat{\nu} > 0$ such that for $\nu < \bar{\nu}$ the optimal contract induces participation of the investor in both states θ_l and θ_h .⁸*

Hence, for ν small enough, the contracts of Proposition 6 are indeed optimal finance contracts.

5 Discussion and Conclusion

When investors possess superior information, the question arises whether they should reveal their information by playing the role of consultants. This paper showed that giving advice to entrepreneurs may not be optimal, because it exacerbates agency problems. In particular, if the investor reveals bad information, it exacerbates the undersupply of effort problem of Jensen and Meckling (1976). In general, optimal finance contracts carefully calibrate the amount of information revelation that they induce to mitigate agency problems. Depending on the magnitude of the agency problem, optimal contracts induce partial, full, or no advice.

The need for a careful revelation of information offers an explanation for the dual role of investors as financiers and consultants which has been observed in, for instance, venture capital financing (cf. Cassamatta 2003). In principle these two roles could be provided by two different economic agents: a knowledgeable party could provide the advice as a consultant, whilst a third party provides the financing. Our results provide an argument why we often see these two roles provided by only one economic agent. We have shown that, in general, an investor and entrepreneur have diverging interests in the amount of information that the entrepreneur receives. Consequently, it is important to give the investor precise control over the amount of information that flows to the entrepreneur. When the investor and the consultant are one and the same, this control is perfect. In contrast, if the information is provided by some third party, then the flow of information must be governed by complicated contracts which may lead to additional inefficiencies. In

⁸For the range $I \in (\bar{I}_1, \bar{I}_2)$ the cutoff value $\hat{\nu}$ may be obtained analytically as
$$\frac{\theta_l \theta_h [(4\theta_h - 3\theta_l) \sqrt{1 - 4I / (\theta_h \theta_l)} - 4\theta_h + 5\theta_l - 2I / \theta_h]}{\theta_l \theta_h [\sqrt{\theta_h^2 - 4I - 4\theta_h + 5\theta_l + 3(\theta_h - \theta_l) \sqrt{1 - 4I / (\theta_h \theta_l)}}] + 2I(\theta_h - 2\theta_l)}.$$

particular, when information flows are non-contractible, then the dual role of the investor overcomes this non-contractibility naturally.

The results of this paper are also relevant for discussions about mandatory disclosure rules. Mostly this discussion focuses on the fiduciary duties of the firm's management to its financiers. Yet when investors have private information, similar questions arise for the financier's side. In this respect, the current paper shows that when the agency problem is severe such mandatory disclosure rules have negative effects and exacerbate the underinvestment problem. As illustrated in Figure 3, investors would not find it profitable to finance investment projects of intermediate profitability, $I \in (I_l, I_n]$, when full disclosure is mandatory. Without such rules such investments projects would be financed and the underinvestment does not occur.

Our results may also shed a new light on the puzzling but persistent observations that entrepreneurs seem overly and more optimistic about their projects than financiers (e.g. Cooper et al. 1988). First, this finding supports our initial idea that financiers are indeed better informed than entrepreneurs. Second, our results point out that in a Jensen&Meckling model of outside finance overconfidence corrects an undersupply of effort. Hence, investors have no interest in correcting the beliefs of overconfident entrepreneurs. That is, although our framework cannot explain the existence of overconfident entrepreneurs, it can explain its persistence; keeping the entrepreneur overly optimistic about the project corrects the undersupply of effort.

Appendix A: Proofs

Proof of Proposition 2:

From (5) it follows that (6) implies (7). Hence, (6) is the relevant constraint. By (5) we may rewrite (6) as

$$R_l \in \left[\left(1 - \sqrt{1 - 4I/\theta_l^2}\right) / 2, \left(1 + \sqrt{1 - 4I/\theta_l^2}\right) / 2 \right],$$

which can only be satisfied if $I \leq \bar{I}$.

Note that $V^r = \nu\theta_h^2(1 - R_h)^2/2 + (1 - \nu)\theta_l^2(1 - R_l)^2/2$ is decreasing in both R_h and R_l . For any given $R_l \in [0, 1]$ the incentive constraint (5) specifies two possible values for R_h . In particular, $R_h = \left(1 \pm \sqrt{1 - 4(1 - R_l)R_l\theta_l/\theta_h}\right) / 2$. The larger value cannot be optimal, because V^r is decreasing in R_h . This implies that the optimal value for R_h must be smaller than $1/2$. Likewise, for any given $R_h \in [0, 1]$ the incentive constraint specifies two possible values for R_l . In particular, $R_l = \left(1 \pm \sqrt{1 - 4(1 - R_h)R_h\theta_h/\theta_l}\right) / 2$. Again, the larger value cannot be optimal, because V^r is decreasing in R_l . This implies that the optimal value for R_l must be smaller than $1/2$.

Hence, at an optimal solution we have $R_l < 1/2$ and $R_h = \tilde{R}_h(R_l) \equiv (1 - \sqrt{1 - 4(1 - R_l)R_l\theta_l/\theta_h})/2$. Note that

$$\partial\tilde{R}_h/\partial R_l = \frac{(1 - 2R_l)\theta_l}{\sqrt{\theta_h^2 - 4(1 - R_l)R_l\theta_l\theta_h}}$$

is positive for $R_l < 1/2$. Therefore, $dV^r/dR_l = \partial V^r/\partial R_l + \partial V^r/\partial R_h * \partial\tilde{R}_h/\partial R_l < 0$. Hence, whenever $R_l < 1/2$ and $R_h = \tilde{R}_h(R_l)$, the entrepreneur's payoff V^r is decreasing in R_l . As a consequence, the participation constraint

$$R_l \in \left[\left(1 - \sqrt{1 - 4I/\theta_l^2}\right) / 2, \left(1 + \sqrt{1 - 4I/\theta_l^2}\right) / 2 \right].$$

binds at the lower value and the optimal contract is

$$R_l^r \equiv \left(1 - \sqrt{1 - 4I/\theta_l^2}\right) / 2 \text{ and } R_h^r \equiv \tilde{R}_h(R_l^r) = \left(1 - \sqrt{1 - 4I/(\theta_l\theta_h)}\right) / 2.$$

Q.E.D.

Proof of Proposition 3: Since $\theta_h > \theta_l$ it follows that (9) implies (10). As a consequence, we may disregard (10). Rewriting (9) yields

$$R \in \left[\left(1 - \sqrt{1 - 4I/(\theta_l\bar{\theta})}\right) / 2, \left(1 + \sqrt{1 - 4I/(\theta_l\bar{\theta})}\right) / 2 \right], \quad (19)$$

which can only be satisfied if $I \leq \theta_l \bar{\theta}/4$.

Using (8) to substitute out e we obtain $V^n = V^n(\hat{e}, R) = \bar{\theta}^2(1 - R)^2/2$. Hence, we must maximize V^n under (19). Since V^n is decreasing in R for $R < 1$, the smallest value in (19) is optimal. Q.E.D.

Proof of Proposition 4: The difference $V^r - V^n$ evaluated at $I = 0$ is $(\theta_h - \theta_l)^2(1 - \nu)\nu$ and therefore strictly positive. The difference $V^r - V^n$ evaluated at $I = \bar{I}_l$ is negative, whenever

$$\nu < \bar{\nu} = \frac{(2\theta_h^{3/2}\sqrt{\theta_h - \theta_l} - 2\theta_h^2 + \theta_l^2 + \theta_h\theta_l)\theta_l}{(\theta_h - \theta_l)^2(3\theta_h + \theta_l)}.$$

Due to the continuity of V^r and V^n , we find some $\bar{I}_0 \in [0, \bar{I}_l)$ such that $V^r = V^n$. Due to $dV^r/dI < dV^n/dI < 0$, the curve V^r cuts the curve V^n at most once and \bar{I}_0 is unique and has the property that $V^r > V^n$ if and only if $I < \bar{I}_0$.

Finally we show that $\bar{\nu} > 0$: Because $\theta_h > \theta_l$, we have

$$4\theta_h^3(\theta_h - \theta_l) - ((\theta_h^2 - \theta_l^2) + \theta_h(\theta_h - \theta_l))^2 = \theta_l^2(\theta_h^2 - \theta_l^2 + 2(\theta_h^2 - \theta_l^2)) > 0$$

Therefore

$$4\theta_h^3(\theta_h - \theta_l) > ((\theta_h^2 - \theta_l^2) + \theta_h(\theta_h - \theta_l))^2$$

Since both sides are positive, we may take square roots and obtain

$$2\theta_h^{3/2}\sqrt{\theta_h - \theta_l} > (\theta_h^2 - \theta_l^2) + \theta_h(\theta_h - \theta_l)$$

From this it follows that the numerator of $\bar{\nu}$ is positive. Since also the denominator is positive, $\bar{\nu}$ itself is positive. Q.E.D.

Proof of Proposition 5: Implementability of α is equivalent to the existence a combination (R_l, R_h) that satisfies (16) and (17). We show that condition (18) is both necessary and sufficient for the existence of such a pair. From (17) it follows $I \leq (1 - R_l)R_l\theta_l(\theta_h\nu_l(\alpha) + (1 - \nu_l)\theta_l) \leq (\theta_l(\theta_h\nu_l(\alpha) + (1 - \nu_l)\theta_l))/4$. Hence, whenever (18) is violated, then (17) is violated for any R_l . Consequently, (18) represents a necessary condition for implementation.

Sufficiency follows from the observation that when (18) holds, then for $R_l = 1/2$ inequality (17) holds. Moreover, since $\nu_h(\alpha) \geq \nu_l(\alpha)$ it follows that for $R_l = 1/2$ one may find an $R_h \in [0, 1/2]$ such that (16) holds.

To derive the optimal combination (R_l^*, R_h^*) that implements α we first establish that, given some fixed R_l , the entrepreneur's utility is decreasing in R_h . This follows from a substitution of (14) and (13) into $V(\Gamma)$, as this yields

$$\frac{dV(\Gamma)}{dR_h} = -\frac{(1-R_h)(\alpha_h\theta_h\nu + (1-\nu)(1-\alpha_l)\theta_l)^2}{\alpha_h\nu + (1-\alpha_l)(1-\nu)} \leq 0 \quad (20)$$

Moreover, since

$$\frac{dV(\Gamma)}{dR_l} = -\frac{(1-R_l)((1-\alpha_h)\theta_h\nu + (1-\nu)\alpha_l\theta_l)^2}{(1-\alpha_h)\nu + \alpha_l(1-\nu)} \leq 0, \quad (21)$$

it follows that, given some R_h , the entrepreneur's utility is also decreasing in R_l .

From (20) it follows after solving (17) with respect to R_h that, whenever R_l^* is optimal then $R_h^* = \tilde{R}_h(R_l^*)$ is optimal, where

$$\tilde{R}_h(R_l) \equiv \frac{1}{2} - \sqrt{\frac{(1-\nu_h(\alpha))\theta_l + \nu_h(\alpha)\theta_h - 4(1-R_l)R_l((1-\nu_l(\alpha))\theta_l + \nu_l(\alpha)\theta_h)}{4(\nu_h(\alpha)\theta_h + (1-\nu_h(\alpha))\theta_l)}}.$$

Now suppose $R_l^* \in (1/2, 1]$ is optimal, then $R_h^* = \tilde{R}_h(R_l^*)$ is optimal. However, since $\tilde{R}_h(R_l) = \tilde{R}_h(1-R_l)$, also the combination (\hat{R}_l, R_h^*) , with $\hat{R}_l \equiv 1-R_l^*$, satisfies the adverse selection constraint (16). Moreover, \hat{R}_l satisfies the participation constraint (17) whenever R_l^* does. Hence, also (\hat{R}_l, R_h^*) implements the reporting strategy α . But since $\hat{R}_l < R_l^*$ it follows from (21) that (\hat{R}_l, R_h^*) yields a higher utility such that $R_l^* > 1/2$ cannot be optimal.

Hence, $R_l^* \leq 1/2$. But for $R_l \leq 1/2$, the function $\tilde{R}_h(R_l)$ is increasing, since

$$\frac{\partial \tilde{R}_h}{\partial R_l} = \frac{1-2R_l}{\sqrt{\nu_h(\alpha)\theta_h + (1-\nu_h(\alpha))\theta_l}} \times \frac{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l}{\sqrt{(1-\nu_h(\alpha))\theta_l + \nu_h(\alpha)\theta_h - 4(1-R_l)R_l((1-\nu_l(\alpha))\theta_l + \nu_l(\alpha)\theta_h)}}$$

is non-negative for $R_l \leq 1/2$. Hence, as R_l decreases also $\tilde{R}_h(R_l)$ decreases and from (20) and (21) it follows that the entrepreneur's utility increases.

Consequently, the optimal combination $(R_l, \tilde{R}_h(R_l))$ is the lowest value R_l such that the participation constraint (17) is still satisfied. That is,

$$R_l^* = \frac{1}{2} - \frac{\sqrt{\nu_l(\alpha)\theta_h + (1 - \nu_l(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\nu_l(\alpha)\theta_h + (1 - \nu_l(\alpha))\theta_l}}$$

and

$$R_h^* = \frac{1}{2} - \frac{\sqrt{\nu_h(\alpha)\theta_h + (1 - \nu_h(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\theta_h\nu_h(\alpha) + (1 - \nu_h(\alpha))\theta_l}}.$$

. Q.E.D.

Proof of Proposition 6: Solving $R_h^*(\alpha)$ and $R_l^*(\alpha)$ with respect to α_1 and α_2 yields

$$\alpha_h^*(R_h, R_l) = \frac{((1 - R_h)R_h\theta_l^2 - I)((1 - R_l)R_l\theta_l\bar{\theta} - I)}{\nu(R_h - R_l)(1 - R_h - R_l)(\theta_h - \theta_l)\theta_l I} \quad (22)$$

$$\alpha_l^*(R_h, R_l) = \frac{((1 - R_l)R_l\theta_l\theta_h - I)((1 - R_h)R_h\theta_l\bar{\theta} - I)}{(1 - \nu)(R_h - R_l)(1 - R_h - R_l)(\theta_h - \theta_l)\theta_l I} \quad (23)$$

Substitution into $V(\Gamma)$ yields

$$\hat{V}(R_h, R_l) \equiv \frac{((1 - R_h)(1 - R_l)(R_h + R_l)\theta_l\bar{\theta} - I)I}{2R_h R_l (1 - R_h - R_l)\theta_l^2}. \quad (24)$$

Hence, the optimal contract is found by maximizing $V(\Gamma)$ over the domains

$$R_h \in D_h \equiv [\underline{D}_h, \overline{D}_h] \equiv \left[\frac{1}{2} \left(1 - \sqrt{\frac{\theta_h\theta_l - 4I}{\theta_h\theta_l}} \right), \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\theta}\theta_l - 4I}{\bar{\theta}\theta_l}} \right) \right]$$

and

$$R_l \in D_l \equiv [\underline{D}_l, \overline{D}_l] \equiv \left[\frac{1}{2} \left(1 - \sqrt{\frac{\bar{\theta}\theta_l - 4I}{\bar{\theta}\theta_l}} \right), \frac{1}{2} \left(1 - \sqrt{\frac{\theta_l^2 - 4I}{\theta_l^2}} \right) \right].$$

The second order derivative is

$$\frac{d^2\hat{V}(R_h, R_l)}{dR_h^2} = \frac{((3/2R_h - (1 - R_l))^2 + 3R_h^2/4)((1 - R_l)R_l\theta_l\bar{\theta} - I)I}{(R_h^3 R_l (1 - R_h - R_l)^3 \theta_l^2)} \geq 0$$

where the inequality follows, because $(1 - R_l)R_l\theta_l\bar{\theta} \geq I$ for all $R_l \in D_l$. Consequently, $\hat{V}(R_h, R_l)$ is convex in R_h so that it does not have an interior maximum. I.e., the optimal value of R_h is either \underline{D}_h or \bar{D}_h . Note that, by (22) and (23), the candidate $R_h = \bar{D}_h$ implies the full pooling solution $a_h = 1$ and $a_l = 0$. Yet, since $R_h = \underline{D}_h$ and $R_l = \underline{D}_l$ also implies the full pooling solution (with $a_h = 0$ and $a_l = 1$), any payoff attainable with $R_h = R_2^*$ is also attainable under $R_h = R_1^*$. Consequently, we may discard the candidate \bar{D}_h and concentrate on \underline{D}_h .

Taking the first order condition of $\hat{V}(\underline{D}_h, R_l)$ with respect to R_l yields

$$R_l^* = \frac{1}{4} \left(1 + \sqrt{1 - 4I/(\theta_h\theta_l)} \right).$$

It satisfies the second order condition, as

$$\frac{\partial^2 V}{\partial R_l^2}(\underline{D}_h, R_l^*) = -\frac{512(1 - \nu)(\theta_h - \theta_l)I^2}{\theta_l\theta_h \left(1 - \sqrt{1 - 4I/(\theta_h\theta_l)}\right) \left(1 + \sqrt{1 - 4I/(\theta_h\theta_l)}\right)^4} < 0.$$

Hence, R_l^* is optimal whenever, it lies in the domain D_l . Straightforward calculations yield

$$R_l^* \geq \underline{D}_l \Leftrightarrow I \geq \bar{I}_1 \text{ and } R_l^* \leq \bar{D}_l \Leftrightarrow I \leq \bar{I}_2.$$

(To see that $\bar{I}_2 > \bar{I}_1$ note that $Sign[\bar{I}_2 - \bar{I}_1] = Sign[8(2 - \nu)\theta_h^2 + 3(1 - \nu)\theta_l^2 - \theta_h\theta_l(5 + 11(1 - \nu))]$. The sign of the last expression is positive if and only if $\nu < 1 + (\theta_h(8\theta_h - 5\theta_l))/((8\theta_h - 3\theta_l)(\theta_h - \theta_l))$ which holds for any $\nu \in [0, 1]$.) Q.E.D.

Proof of Proposition 7

For $I \in [\bar{I}_1, \bar{I}_2]$ it follows from Proposition 6 that the entrepreneur's optimal payoff from ensuring the participation of both types of investors is

$$V^* = \theta_h(\theta_h - 5(1 - \nu)(\theta_h - \theta_l)) + \frac{1}{4}(\theta_h + 3(1 - \nu)(\theta_h - \theta_l))\sqrt{1 - 4I/(\theta_h\theta_l)} - \frac{I\bar{\theta}}{2\theta_l}.$$

For $I > [\bar{I}_2, \theta_l\bar{\theta}/4]$ it follows from Proposition 6 that the entrepreneur's optimal payoff from ensuring the participation of both types of investors is

$$V^* = \frac{1}{4} \left(\bar{\theta}^2 + \bar{\theta}\sqrt{\bar{\theta}^2 - 4I\bar{\theta}/\theta_l} - 2I\bar{\theta}/\theta_l \right).$$

The optimal contract when there is only participation of the θ_h investor coincides with the optimal contract in Proposition 1, because in any such Perfect Bayesian equilibrium the Bayes' consistent belief, ν^e , of the entrepreneur after an acceptance of the contract is 1. Consequently, the payoff associated with this contract is

$$V^h \equiv \frac{1}{4} \left(\theta_h^2 + \theta_h \sqrt{\theta_h^2 - 4I} - 2I \right) \nu.$$

For $\nu = 0$ it holds $V^h = 0 < V^*$. Since V^h and V^* are continuous in ν it follows that $V^* > V^h$ for $\nu > 0$ small enough.

Q.E.D.

Appendix B: Stochastic Contracts

This appendix shows that the suboptimality of full advice does not depend on the absence of stochastic investment contracts (R, π) that, in addition to the repayment R , specify a probability π with which investment takes place. Although less intuitive and notional more cumbersome, such stochastic contracts are more general than deterministic ones.

With stochastic contracts the two incentive constraints that ensure full advice are

$$\pi_h(\theta_h \hat{e}_h R_h - I) \geq \pi_l(\theta_h \hat{e}_l R_l - I) \quad (25)$$

and

$$\pi_l(\theta_l \hat{e}_l R_l - I) \geq \pi_h(\theta_l \hat{e}_h R_h - I), \quad (26)$$

where the effort levels \hat{e}_h and \hat{e}_l are defined as in (3). Note that the incentive constraints (25) and (26) only imply the constraint (5) for $\pi_h = \pi_l$. Hence, they are weaker than the incentive constraints under deterministic contracts (5).

The optimal stochastic investment contract with full advice and full participation solves the following maximization problem

$$\begin{aligned} P^s : \quad & \max_{R_i^*, \pi_i^*, \hat{e}_i^*} \quad \nu \pi_h(\theta_h \hat{e}_h(1 - R_h) - \hat{e}_h^2/2) + (1 - \nu) \pi_l(\theta_l \hat{e}_l(1 - R_l) - \hat{e}_l^2/2) \\ & \text{s.t. } (3), (6), (7), (25), (26). \end{aligned}$$

In a series of lemmas we show that the results of Proposition 2 also hold when we consider stochastic contracts. In particular, the next lemma shows that the first observation of Proposition 2 carries over directly to stochastic contracts.

Lemma 1 *With stochastic contracts full advice is implementable only if $I \leq \bar{I}_l$.*

Proof: Combining (3) and (6) yields the requirement that $\theta_l^2(1-R_l)R_l \geq I$. The expression $\theta_l^2(1-R_l)R_l$ attains the maximum $\theta_l^2/4$ for $R_l = 1/2$. Hence, we have $\theta_l^2(1-R_l)R_l < \theta_l^2/4 = \bar{I}_l$ so that the requirement is violated whenever $I > \bar{I}_l$. Q.E.D.

The remainder of this appendix shows that when ν is small enough, we may also obtain the second result of Proposition 2, because for small ν optimal contracts are deterministic, i.e., $\pi_h = \pi_l = 1$.

Due to symmetry the following lemma shows that any optimal contract has repayments that are smaller than one half.

Lemma 2 *For any solution $(R_h^*, \pi_h^*, \hat{e}_h^*, R_l^*, \pi_l^*, \hat{e}_l^*)$ to program P^s we have $R_h \leq 1/2$ and $R_l \leq 1/2$.*

Proof: Suppose $R_h^* > 1/2$. Consider the combination $(R'_h, \pi_h^*, \hat{e}'_h, R_l^*, \pi_l^*, \hat{e}_l^*)$ with $R'_h = 1 - R_h^* < 1/2$ and $\hat{e}'_h = \theta_h(1 - R'_h) < \hat{e}_h^*$. The combination satisfies all the constraints of program P^s , because $\hat{e}_h^* R_h^* = \theta_h(1 - R_h^*) R_h^* = \hat{e}'_h R'_h$. It yields the entrepreneur a higher utility, because it attains the same success probability of the project at a lower effort level. Optimality of $R_l^* > 1/2$ can be similarly refuted. Q.E.D.

The next lemma shows that, just as with deterministic contracts, the individual rationality constraint of the investor with a high signal θ_h is not binding at the optimum.

Lemma 3 *The incentive constraint (25) and the individual rationality constraint (6) imply the individual rationality constraint (7).*

Proof: It follows

$$\pi_h(\theta_h \hat{e}_h R_h - I) \geq \pi_l(\theta_h \hat{e}_l R_l - I) \geq \pi_l(\theta_l \hat{e}_l R_l - I) \geq 0,$$

where the first inequality uses (25), the second follows due to $\theta_h > \theta_l$ and the third is implied by (6). Hence, whenever $\pi_h > 0$, we have $\theta_h \hat{e}_h R_h - I \geq 0$ Q.E.D.

We may use the previous result to identify the binding incentive constraint.

Lemma 4 *The incentive constraint (25) binds at the optimum.*

Proof: Suppose that for a solution $(R_h^*, \pi_h^*, \hat{e}_h^*, R_l^*, \pi_l^*, \hat{e}_l^*)$ to program P^s the incentive constraint (25) is slack. Hence, we may lower R_h by $\varepsilon > 0$ such that (25) remains satisfied. By Lemma 3 the individual rationality constraint (7) remains satisfied. In response to a decrease in R_h the effort level e_h changes from $\hat{e}_h^* = \theta_h(1 - R_h^*)$ to $\hat{e}'_h = \theta_h(1 - R_h^* + \varepsilon)$. By Lemma 2 we have $R_h^* < 1/2$ so that $\hat{e}_h R_h = \theta_h(1 - R_h)R_h$ is falling in R_h . Hence, the reduction of R_h^* by ε leaves the incentive constraint (26) satisfied. Moreover, by a revealed preference argument we have

$$\theta_h \hat{e}'_h (1 - R_h^* + \varepsilon) - \hat{e}'_h{}^2/2 \geq \theta_h \hat{e}_h^* (1 - R_h^* + \varepsilon) - (\hat{e}_h^*)^2/2 \geq \theta_h \hat{e}_h^* (1 - R_h^*) - (\hat{e}_h^*)^2/2$$

so that the objective function rises. This contradicts the optimality of $(R_h^*, \pi_h^*, \hat{e}_h^*, R_l^*, \pi_l^*, \hat{e}_l^*)$. Q.E.D.

In general, incentive compatibility requires that contracts exhibit a monotonicity in types. This is expressed by the following lemma.

Lemma 5 *If a solution to program P^s exists, it exhibits $\pi_h \geq \pi_l$ and the individual rationality constraint (6) is binding.*

Proof: By Lemma 4 the incentive constraint (25) is satisfied in equality. This implies

$$\pi_h \hat{e}_h R_h - \pi_l \hat{e}_l R_l = (\pi_h - \pi_l)I/\theta_h.$$

Hence, we may express the incentive constraint (26) as

$$(\pi_h - \pi_l)(\theta_h - \theta_l) \geq 0.$$

It implies $\pi_h \geq \pi_l$. Moreover, whenever $\pi_h \geq \pi_l$ the incentive constraint (26) is automatically satisfied.

In order to show that the individual rationality constraint (6) is binding first consider $\pi_h > \pi_l$. In this case, the incentive constraint (26) is slack. Hence, if also the individual rationality constraint (6) is slack, we may raise the entrepreneur's utility by reducing R_l by some $\varepsilon > 0$ small enough.

For the case $\pi_h = \pi_l$ the two incentive constraints (25) and (26) simplify to the single constraint (3) so that the original analysis in the body text applies. Q.E.D.

For a fixed, monotone pair of investment probability (π_h, π_l) the following lemma derives the optimal repayment schedule.

Lemma 6 *For a given (π_h, π_l) with $\pi_h \geq \pi_l$, the optimal repayments are*

$$R_h^s = \frac{1}{2} - \frac{\sqrt{\pi_h^2 \theta_h^2 \theta_l^2 - 4\pi_h \theta_l (\pi_l (\theta_h - \theta_l) + \pi_h \theta_l) I}}{2\pi_h \theta_h \theta_l} \quad \text{and} \quad R_l^s = \frac{1}{2} - \frac{\sqrt{\theta_l^2 - 4I}}{2\theta_l}.$$

Proof: From the previous two lemmas it follows that the optimal repayments R_h and R_l satisfy

$$\pi_h (\theta_h^2 (1 - R_h) R_h - I) = \pi_l (\theta_h \theta_l (1 - R_l) R_l - I) \quad \text{and} \quad \theta_l^2 (1 - R_l) R_l = I.$$

Solving these two equalities with respect to R_h and R_l yield four solutions, where only the one mentioned in the lemma satisfies $R_h \leq 1/2$ and $R_l \leq 1/2$, as required by Lemma 2. Q.E.D.

Substitution of R_h^s and R_l^s into the objective function of program P^s yields

$$\begin{aligned} V^s(\pi_h, \pi_l) &= \frac{1}{2} (1 - \nu) \pi_l \theta_l^2 \left(\frac{1}{2} + \frac{\sqrt{\theta_l^4 - 4\theta_l^2 I}}{2\theta_l^2} \right)^2 + \\ &\quad \frac{1}{2} \nu \pi_h \theta_h^2 \left(\frac{1}{2} + \frac{\sqrt{\pi_h^2 \theta_h^4 \theta_l^2 - 4\pi_h \theta_h^2 \theta_l (\pi_l (\theta_h - \theta_l) + \pi_h \theta_l) I}}{2\pi_h \theta_h^2 \theta_l} \right)^2 \end{aligned}$$

Using this expression we may show that stochastic contracts are suboptimal when the likelihood of a high commercial success, ν , is small.

Lemma 7 *There exists a $\tilde{\nu} > 0$ such that for all $\nu < \tilde{\nu}$ the optimal stochastic contract is degenerated and exhibits $\pi_h = \pi_l = 1$.*

Proof: Taking the derivative with respect to π_h yields

$$\frac{\partial V^s}{\partial \pi_h} = \frac{1}{4}\nu \left(\frac{\theta_h^2(\pi_h\theta_l(\theta_h^2 - 4I) - 2\pi_l(\theta_h - \theta_l)I)}{\sqrt{\pi_h\theta_h^2\theta_l(\pi_h\theta_l(\theta_h^2 - 4I) - 4\pi_l(\theta_h - \theta_l)I)}} + \theta_h^2 - 2I \right),$$

which is strictly positive, whenever the root in the denominator exists. Therefore, $\partial V^s/\partial \pi_h > 0$ so that $\pi_h = 1$ is optimal.

Moreover, we have

$$\frac{\partial^2 V^s}{\partial \pi_l^2} = -\frac{\nu\pi_h^2\theta_h^4(\theta_h - \theta_l)^2\theta_l I}{(\pi_h\theta_h^2\theta_l(\pi_h\theta_h^2\theta_l - 4(\pi_l(\theta_h - \theta_l) + \pi_h\theta_l)I)^{3/2}} < 0$$

so that V^s is concave in π_l .

Evaluating the derivative of V^s with respect to π_l at $\pi_l = \pi_h = 1$ yields

$$\frac{\partial V^s}{\partial \pi_l}(1, 1) = (1 - \nu)(\theta_l^2 + \sqrt{\theta_l^4 - 4\theta_l^2 I})/4 - \frac{\nu\theta^2(\theta_h - \theta_l)I}{2\sqrt{\theta_h^3\theta_l(\theta_l\theta_h - 4I)}} - \frac{(\theta_l + \theta_h\nu - 2\theta_l\nu)I}{2\theta_l},$$

which is linear in ν . For $\nu = 0$ we have

$$\frac{\partial V^s}{\partial \pi_l}(1, 1) |_{\nu=0} = \frac{1}{4} \left(\theta_l^2 - 2I + \sqrt{\theta_l^4 - 4\theta_l^2 I} \right),$$

which is positive. The concavity of V^s with respect to π_l then implies that $\pi_l = 1$ is optimal. From the linearity of $\partial V^s/\partial \pi_l$ it further follows that there exists a $\tilde{\nu} > 0$ so that for all $\nu < \tilde{\nu}$ the derivative is positive and $\pi_l = 1$ is optimal. Q.E.D.

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