# Strategic Pricing, Signalling, and Costly Information Acquisition*

Helmut Bester[†] and Klaus Ritzberger[‡]

**Abstract**

Consider a market where an informed monopolist sets the price for a good or asset with a value unknown to potential buyers. Upon observing the price, buyers may pay some cost for information about the value before deciding on purchases. To restrict buyer beliefs we generalize the idea of the Cho–Kreps "intuitive criterion". Then there is no separating equilibrium with fully revealing prices. Yet, as the cost of information acquisition becomes small, the equilibrium approaches the full information outcome and prices become perfectly revealing.

*Keywords:* quality uncertainty, price signalling, information acquisition

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[†]Free University of Berlin, Department of Economics, Boltzmannstr. 20, D–14195 Berlin, Germany, and CEPR; email: hbester@wiwiss.fu-berlin.de .

[‡]Institute for Advanced Studies, Stumpergasse 56, A–1060 Vienna, Austria; email: ritzbe@ihs.ac.at .

# 1 Introduction

It has long been felt that prices perform a dual role in markets with unknown product quality. They clear markets *and* may signal product quality. This occurs in particular when informed traders are present (Wolinsky (1983), Cooper and Ross (1984), Riordan, (1986)) and/or when other quality signals are absent or noisy (Jones and Hudson (1996)). For instance, the marketing literature has long argued that consumers infer quality from price (Monroe (1973)). For a wide variety of consumption goods there is empirical evidence on a positive price-quality correlation (Gerstner (1985), Tellis and Wernerfeld (1987), Curry and Riesz (1988)), even when advertising does not serve as an effective quality signal (Caves and Greene (1996)). For the introduction of new and better quality products, models predict an upward price distortion for signaling purposes, with and without cost asymmetries across qualities (see Wilson (1980), Milgrom and Roberts (1986), Bagwell and Riordan (1991), Ellingsen (1997) and Judd and Riordan (1994), respectively). For insurance markets, model predictions of a negative relation between price and the insurer's default risk get supported empirically (Cummins and Danzon (1997)).

For financial markets there is an even stronger variant. When quality is the return (distribution) of an asset, the *efficient market hypothesis* holds that prices aggregate all relevant private information, thus revealing it to all market participants (see Fama (1970), Grossman (1976, 1978, 1981)). Yet, in this context also a potential conflict between the two roles which prices play has been pointed out. That prices reflect private information requires the presence of informed traders. But, if information acquisition is costly, perfectly informative prices eliminate the incentive to collect information. But then there is no information which prices can reflect. At completely uninformative prices it, however, becomes profitable to acquire information. This has become known as the *Grossman–Stiglitz paradox* (Grossman and

Stiglitz (1980)). It effectively points to the non-existence of an equilibrium in *pure* strategies.

The model by Grossman and Stiglitz keeps the price formation implicit by employing the competitive paradigm, where traders are price-takers and prices are determined from a market-clearing condition. Hence, the way information is passed from individual trades on to price and then on to what is publicly known is collapsed into a simultaneous determination of a market-clearing price and an information structure. This simultaneity of pricing and information processing prevents a strategic analysis of pricing decisions on the one, and information acquisition on the other hand.

This paper, therefore, considers a market where an informed monopolist sets the price and uninformed buyers may infer quality from the price or pay for access to an external source of information (or both). The pricing side of the model is thus in the tradition of monopoly pricing models with unknown quality (Wilson (1980), Wolinsky (1983), Cooper and Ross (1985), Milgrom and Roberts (1986), Riordan (1986), Bagwell and Riordan (1991), Judd and Riordan (1994), Ellingsen (1997)). This structure not only applies to commodity markets, as indicated above, but also to asset markets. An example may be a new investment fund issuing shares. The fund's quality or value will depend on the fund managers' abilities, but those are unknown to investors at the time of the share issue.

In contrast to the above literature, we do not assume the presence of informed traders, whose reaction a high-quality seller may exploit to separate himself. Rather all buyers observe the price, infer whatever information it may contain, and then decide whether or not to buy information. The buyers' final purchasing decisions are thus based on the price observation plus, possibly, costly outside information.

Also we do not assume production cost asymmetries across suppliers of different qualities. In the absence of such asymmetries informative prices arise solely from the high-quality seller's attempt to separate himself. High prices serve as signals from which uninformed buyers infer high quality, granted there are informed agents who buy at high prices.

Since the price does potentially serve as a signal of quality, the multiplicity of equilibria familiar from the signalling-games literature arises (see Mailath, Okuno-Fujiwara and Postlewaite (1993)). Here we extend the idea of the "intuitive criterion" of Cho and Kreps (1987) to select among equilibria. It rules out counter-intuitive equilibria driven by overly pessimistic beliefs.

Under this refinement we establish a version of the Grossman–Stiglitz paradox. For small costs of information acquisition there is no separating equilibrium, i.e., prices cannot be perfectly informative. If it were, no buyer would pay for information acquisition and a low-quality seller would mimic high-quality ones. But there is also no pooling equilibrium. If there were, some fraction of buyers would become informed, thus making it profitable for the high-quality seller to separate himself by excessively high prices. Hence, the equilibrium price does reveal some information, though imperfectly. Indeed, we show that there is a unique equilibrium consistent with our refinement, which involves mixed pricing strategies to resolve the paradox.

The important insight concerns the case of arbitrarily small costs of information acquisition. We show that as the information cost vanishes, the price becomes perfectly revealing. Moreover, the sellers' profits and their pricing policies approach their full information levels. Hence, while the Grossman–Stiglitz paradox holds true (for pure strategies), allowing sellers to randomize resolves the paradox in a favorable way: Sufficiently small costs of information acquisition induce equilibrium outcomes almost in line with the efficient market hypothesis.

In the financial markets literature there are other approaches to to information acquisition under potentially informative prices. Verrecchia (1982), in a competitive asset market, adds an extra source of uncertainty ("noise"), which prevents the price from being a sufficient statistic. Thus traders have an incentive to invest in information acquisition. If the noise goes to zero faster than the cost of information, the induced equilibrium price becomes almost perfectly informative. Hellwig (1982) allows uninformed traders only to condition on past prices, but not on the present price. Again there is an incentive to buy advance information. When the time interval between trading dates becomes arbitrarily small, the price process becomes almost perfectly informative. Both these models remain within the realm of the competitive (price-taking) paradigm.

The paper is organized as follows. Section 2 describes the model. Section 3 studies the buyers' decision problem and derives demand. In Section 4 we define the equilibrium and motivate our belief refinement. Section 5 presents the analysis of equilibrium and its limiting properties when information costs become arbitrarily small. Section 6 contains concluding remarks.

# 2 The Model

There is a single seller of some good or asset who knows its quality $q > 0$. Buyers only know that the seller supplies quality $q_H$ with probability $\lambda \in (0,1)$ and quality $q_L < q_H$ with probability $1 - \lambda$. Hence, there are two types $i \in \{H, L\}$ of the seller. The seller's valuation of the good or his production cost is zero.

Buyers do not interact strategically with each other. This allows us to consider each buyer in isolation independently of whether there is just a single buyer or a set of many buyers. Each buyer purchases at most one unit of the good. His utility from purchasing quality $q$ at the price $p$ is $q - p$. His utility from not buying is $u$. The seller does not observe the buyer's outside option payoff, he only knows that $u$ is uniformly distributed on $[0, \hat{u}]$, with $q_H \leq \hat{u}$. Without loss of generality, we normalize $\hat{u} \equiv 1$.

Upon observing the price, each buyer may test for the quality of the good by paying a fixed cost $k > 0$. For simplicity we assume that this test fully reveals the true quality. Further, by assuming that the test is not publicly observable, we rule out that the seller can condition his price on the buyer's decision to become informed. Similarly, the outcome of the test cannot be credibly communicated, which precludes any payments that are contingent on the buyer's posterior information about $q$. The assumption that testing is not contractible distinguishes our model from the literature on auditing and monitoring (see, e.g., Border and Sobel (1987) and Mookherjee and Png (1989)).

In summary, we consider the following sequence of events:

1. The seller commits to a price offer $p$.

2. Each buyer decides about whether to become informed about $q$ by paying $k$.

3. Each buyer decides whether to purchase the good or not.

In what follows we study the Perfect Bayesian Equilibria of this game. In particular, we are interested in the question of whether the equilibrium outcomes for small values of $k$ are similar to the full information equilibrium. If the buyer were perfectly informed about $q$, he would purchase the good whenever $q - p \geq u$. Thus the type-$i$ seller would maximize his profit $p(q_i - p)$ by charging $\hat{p}_i = q_i/2$ which earns him $q_i^2/4$.

# 3 Information Acquisition and Demand

After observing the seller's price, the buyer updates his beliefs as to which type of the seller he faces. Denote by $\mu(p)$ his conditional probability of seller type $H$ given the price $p$. Thus the buyer's expected payoff from not testing for the quality is

$$\max\left[\mu(p)q_H + (1-\mu(p))q_L - p, u\right]. \tag{1}$$

The informed buyer purchases the good if $q - p \geq u$. Therefore, the expected payoff from becoming informed is

$$\mu(p)\max\left[q_H - p, u\right] + (1-\mu(p))\max\left[q_L - p, u\right] - k. \tag{2}$$

Obviously, for all buyers with $u > q_H - p$ the optimal action is not to buy and also to refrain from investing $k$. Similarly, all buyers with $u < q_L - p$ will optimally purchase the good without testing its quality. The following two Lemmas characterize the equilibrium behaviour of the remaining buyers.

**Lemma 1** *Upon observing $p$, a buyer with outside option $u$ optimally invests $k$ if and only if*

$$\underline{u} \equiv q_L - p + \frac{k}{1-\mu(p)} \leq u \leq q_H - p - \frac{k}{\mu(p)} \equiv \overline{u}.$$

*Moreover, he purchases the good only if the test reveals quality $q_H$.*

**Proof:** Performing the test is optimal if and only if the expression in (2) is at least as large as the expression in (1). If this is the case, one must have $q_L - p < u < q_H - p$, which proves the last statement of the Lemma. Thus investing $k$ is optimal if and only if

$$\mu(p)[q_H - p] + (1-\mu(p))u - k \geq \max\left[\mu(p)q_H + (1-\mu(p))q_L - p, u\right]. \tag{3}$$

This condition is equivalent to the first statement of the Lemma. Q.E.D.

Notice that the interval $[\underline{u}, \overline{u}]$ is non-empty only if

$$k \leq (q_H - q_L)\mu(p)(1-\mu(p)) \equiv \bar{k} \tag{4}$$

Thus, even for small values of $k$ no buyer will test for the quality if $\mu(p)$ is either close to zero or close to unity. The buyer purchases information only when his beliefs are sufficiently diffuse.

**Lemma 2** *Let $k \leq \bar{k}$. Then, upon observing $p$, a buyer with outside option $u$ optimally purchases the good without paying to become informed if and only if*

$$u \leq \underline{u} = q_L - p + \frac{k}{1 - \mu(p)}.$$

**Proof:** Purchasing the good without a test is optimal if and only if

$$\mu(p)q_H + (1 - \mu(p))q_L - p \geq \max\left[\mu(p)(q_H - p) + (1 - \mu(p))u - k,\, u\right]. \quad (5)$$

For $k \leq \bar{k}$ this condition is equivalent to the statement in the Lemma. Q.E.D.

The two Lemmas identify three types of demand behavior: Buyers with low outside options purchase the good unconditionally without testing. Buyers with intermediate outside options become informed and purchase the good only if the test confirms high quality. Finally, buyers with high values of $u$ are not interested in purchasing the good. As long as $k < \bar{k}$, it follows from the Lemmas that all buyers with $u \leq \overline{u}$ purchase high quality while only buyers with $u \leq \underline{u}$ purchase low quality. When $k \geq \bar{k}$, testing plays no role and all buyers with $u \leq \mu(p)q_H + (1 - \mu(p))q_L - p$ purchase the good. Equilibrium demand is, therefore, characterized by the coefficients

$$\begin{aligned} a_L(\mu) &\equiv \min\left[\mu q_H + (1-\mu)q_L,\, q_L + \frac{k}{1-\mu}\right], \\ a_H(\mu) &\equiv \max\left[\mu q_H + (1-\mu)q_L,\, q_H - \frac{k}{\mu}\right]. \end{aligned} \quad (6)$$

These coefficients represent the intercepts of the (linear) demand functions facing the two types of sellers. For all $\mu$ it is the case that $a_H(\mu) \geq a_L(\mu)$ so that the high-quality seller's demand is at least as high as the low-quality seller's demand. Notice also that both $a_L(\mu)$ and $a_H(\mu)$ are strictly increasing in $\mu$, which means that more optimistic beliefs generate higher demand for either type of the seller. Moreover, $a_L(0) = a_H(0) = q_L$ and $a_L(1) = a_H(1) = q_H$.

For seller type $i \in \{H, L\}$ the profit function at the belief $\mu$ and price $p$ is defined by

$$\Pi_i(p, \mu) \equiv p \max \left[a_i(\mu) - p, 0\right]. \qquad (7)$$

By (6) one has $\partial \Pi_H / \partial k \leq 0$ and $\partial \Pi_L / \partial k \geq 0$. For a given $(p, \mu)$, an increase in the buyer's cost of becoming informed typically benefits the low-quality seller and hurts the high-quality seller.

# 4 Pricing and Beliefs

Denote by $\sigma_i(p)$ the probability that the type-$i$ seller charges the price $p \geq 0$.[1] The functions $(\sigma_H^*, \sigma_L^*)$ constitute an equilibrium if there is $\mu^*(\cdot)$ such that for $i = H, L$

$$\Pi_i(p, \mu^*(p)) \geq \Pi_i(p', \mu^*(p')) \text{ for all } p' \geq 0, \text{ whenever } \sigma_i^*(p) > 0; \qquad (8)$$

and

$$\mu^*(p) = \frac{\lambda \sigma_H^*(p)}{\lambda \sigma_H^*(p) + (1 - \lambda) \sigma_L^*(p)}, \text{ whenever } \sigma_H^*(p) + \sigma_L^*(p) > 0. \qquad (9)$$

Equilibrium condition (8) states that, for given beliefs $\mu^*(\cdot)$, the seller assigns positive probability only to those prices that maximize his profit. The second equilibrium requirement (9) is that the buyers' beliefs are consistent with Bayes' rule whenever possible. For a given equilibrium $(\sigma_H^*, \sigma_L^*, \mu^*)$ denote by $\Pi_i^*$ the equilibrium profit of seller type $i \in \{H, L\}$. Say that a price $p$ is out-of-equilibrium if $\sigma_L^*(p) + \sigma_H^*(p) = 0$.

By (8), the seller takes into account the effect of his pricing decision on the buyers' expectations. Yet, condition (9) does not impose any restriction on

[1] As our analysis shows, each seller will randomly choose a finite number of prices. Therefore, we can avoid measure theoretic complications by restricting the definition of equilibrium to this kind of pricing strategies.

out-of-equilibrium beliefs. As in other signaling games, beliefs are arbitrary out-of-equilibrium, which may lead to a multiplicity of equilibria. This is so because the profitability of a deviation depends on the buyers' interpretation of it. Suppose, for instance, that the high-quality seller reduces his price. If the buyers interpret the new price as a signal of low quality, then the reduced price may actually lower the seller's demand.

To avoid this problem, typically one employs restrictions on out-of-equilibrium beliefs. A prominent example is the "intuitive criterion" of Cho and Kreps (1987), which has been successfully applied to a large number of signalling games, including market environments where prices signal quality (see, e.g., Bagwell and Riordan (1991) and Bester (1993)). The intuitive criterion requires that, for any out-of-equilibrium price $p$, $\mu^*(p) = 1$ if $\Pi_H(p, 1) > \Pi_H^*$ and $\Pi_L(p, 1) < \Pi_L^*$. The idea is that the price $p$ should be considered as a signal of high quality if – given this belief – only the high-quality seller has an incentive to deviate to $p$. Unfortunately, as the following examples show, in our context the intuitive criterion is not sufficient to rule out counter-intuitive outcomes.

*Example 1:* Let $p' \equiv q_L/2$ and $p'' \equiv [q_h + \sqrt{q_H^2 - q_L^2}]/2$. Then $\sigma_L^*(p') = 1$, $\sigma_H^*(p'') = 1$ constitutes a separating equilibrium which is supported by $\mu^*(p) = 0$ if $p < p''$, and $\mu^*(p) = 1$ if $p \geq p''$. In this equilibrium $\Pi_H^* = \Pi_L^* = 0.25\, q_L^2$. It is easily verified that $\mu^*(p)$ satisfies the intuitive criterion because $\Pi_H(p, 1) = \Pi_L(p, 1)$ for all $p$.

In this example, the seller's pricing behavior reveals the quality of the good and so there is no information acquisition in equilibrium. Yet, even for small values of $k$, the high-quality seller earns the same profit as the low-quality seller. This happens because he is unable to set a price below $p''$ that would induce some buyers to invest $k$. Since buyers have deterministic beliefs, condition (4) cannot hold for any $k > 0$.

*Example 2:* Let $p' \equiv q_L/2$. Then $\sigma_L^*(p') = \sigma_H^*(p') = 1$ constitutes a pooling equilibrium which is supported by $\mu^*(p) = 0$ if $p \neq p'$, and $\mu^*(p) = \lambda$ if $p = p'$. Again, $\mu^*(p)$ satisfies the intuitive criterion because $\Pi_H(p, 1) = \Pi_L(p, 1)$ for all $p$.

In the second example, the equilibrium price reveals no information and is again independent of $k$. Although, for $k$ small enough, some buyers inspect the quality of the good, the high-quality seller cannot distinguish himself from the low-quality seller by setting a price above $p'$.

Both examples rely on beliefs with the property that no buyer decides to become informed after observing an out-of-equilibrium price. To provide a more effective role for information acquisition, we refine the intuitive criterion by the following assumption:

**Assumption** For any $\delta \in [0, 1]$ and for any out-of-equilibrium price $p$,

$$\Pi_H(p, \delta) > \Pi_H^* \text{ and } \Pi_L(p, \delta) < \Pi_L^*$$

implies $\mu^*(p) \geq \delta$.

Our assumption contains the intuitive criterion as the special case $\delta = 1$. It extends the idea of this criterion to a situation where a deviation to $p$ is profitable only for the $H$-type when the buyer believes that the deviation originates from the $H$-type with probability $\delta$. This belief is already rather pessimistic, because it actually gives no incentive to the $L$-type to deviate to $p$. We require that in such a case the buyer's belief should not be even more pessimistic.[2]

# 5 Equilibrium

In the remainder we maintain our assumption on out-of-equilibrium beliefs to characterize the equilibrium outcome for small information costs. The next

[2]If, instead of requiring the implication for all $\delta \in [0, 1]$, we would only require it for the prior $\delta = \lambda$, all our results, except for uniqueness of equilibrium, would still go through.

Lemma establishes a lower bound on the high-quality seller's equilibrium payoff.

**Lemma 3** *In any equilibrium,* $\Pi_H^* \geq \frac{1}{4}(q_H - k/\lambda)^2$ *whenever* $k$ *is sufficiently small.*

**Proof:** Suppose to the contrary that $\Pi_H^* < 0.25\,(q_H - k/\lambda)^2$. Let $p' \equiv (q_H - k/\lambda)/2$. Then, by (6) and (7), $p'$ maximizes $\Pi_H(p, \lambda)$ and

$$\Pi_H(p', \lambda) = \frac{1}{4}\left(q_H - \frac{k}{\lambda}\right)^2 > \Pi_H^* \tag{10}$$

for $k$ sufficiently small. Moreover

$$\Pi_L(p', \lambda) < \max_p \Pi_L(p, \lambda) = \frac{1}{4}\left(q_L + \frac{k}{1-\lambda}\right)^2 \tag{11}$$

Thus, for $k$ sufficiently small, $\Pi_L(p', \lambda) < q_L^2/4$. Since the low-quality seller can always get the same payoff as under full information, $\Pi_L(p', \lambda) < q_L^2/4 \leq \Pi_L^*$. By our assumption, this together with (10) implies $\mu^*(p') \geq \lambda$. Therefore, $\Pi_H(p', \mu^*(p')) \geq \Pi_H(p', \lambda) > \Pi_H^*$, a contradiction to equilibrium condition (8). Q.E.D.

As an immediate consequence of Lemma 3, for small values of $k$, our restriction on beliefs eliminates the example of a separating equilibrium in the previous Section. More generally, $(\sigma_H^*, \sigma_L^*)$ is a *separating equilibrium* if $\sigma_H^*(p)\sigma_L^*(p) = 0$ for all $p$. In a separating equilibrium the two seller types never charge the same price and so the equilibrium price reveals the seller's type. The next result shows that this cannot happen in a market with small information costs.

**Proposition 1** *For* $k$ *sufficiently small, there is no separating equilibrium.*

**Proof:** In a separating equilibrium there is a $p'$ and a $p''$ such that $\sigma_H^*(p') > 0, \sigma_L^*(p') = 0$ and $\sigma_H^*(p'') = 0, \sigma_L^*(p'') > 0$. Therefore, $\mu^*(p') = 1$ and $\mu^*(p'') = 0$. By equilibrium condition (8),

$$\begin{aligned} \Pi_H^* &= \Pi_H(p',1) \geq \Pi_H(p'',0) &= \Pi_L(p'',0) = \Pi_L^*, \\ \Pi_L^* &= \Pi_L(p'',0) \geq \Pi_L(p',1) &= \Pi_H(p',1) = \Pi_H^*, \end{aligned} \tag{12}$$

so that $\Pi_L^* = \Pi_H^*$. Since $\Pi_L^* = \Pi_L(p'',0) \leq \max_p \Pi_L(p,0) = 0.25\, q_L^2$, this implies $\Pi_H^* = q_L^2/4$. Since $q_L < q_H$ this yields a contradiction to Lemma 3 when $k$ is sufficiently small. Q.E.D.

Proposition 1 states the impossibility of fully revealing prices reminiscent from the Grossmann–Stiglitz paradox: If the equilibrium price reveals the true quality of the good, then no buyer will test for the quality, even when information costs are arbitrarily small. But, if no buyer becomes informed, then prices cannot reflect quality information. The main difference to the Grossman–Stiglitz paradox is that in our model prices are chosen by the (informed) seller rather than by a fictitious Walrasian auctioneer. The restriction on beliefs implies that the seller will not use his price as a perfect signal of quality when the buyers can acquire quality information at a low cost.

Our next result shows that there is no pooling equilibrium, in which equilibrium prices are completely non-informative as in the second example of the previous Section. Pooling would make it profitable for the high-quality seller to separate himself by charging a higher price, which would induce some fraction of buyers to become informed. Rather, it turns out that the market outcome for small information costs must exhibit *partial* pooling: The low-quality seller imitates with positive probability any price that the high-quality seller might quote. When this happens, quality remains uncertain and some buyers invest in information acquisition. Yet, it also happens with positive probability that the low-quality seller reveals himself by quoting the same price as under full information.

**Proposition 2** *Let $k$ be sufficiently small. Then, in any equilibrium $\sigma_H^*(p')$ $> 0$ implies $\sigma_L^*(p') > 0$. Moreover, $\sigma_H^*(q_L/2) = 0$ and $\sigma_L^*(q_L/2) > 0$.*

**Proof:** Suppose there is a $p'$ such that $\sigma_H^*(p') > 0$ and $\sigma_L^*(p') = 0$. Then $\mu^*(p') = 1$ and $\Pi_H^* = \Pi_H(p', 1) = \Pi_L(p', 1) \leq \Pi_L^*$, because type $L$ can imitate the $H-$type's behavior. Therefore, by Lemma 3,

$$\Pi_L^* = \Pi_L(p'', \mu^*(p'')) \geq \Pi_H^* \geq 0.25\ (q_H - k/\lambda)^2 \tag{13}$$

for all $p''$ such that $\sigma_L^*(p'') > 0$. For $k$ small enough, (6) and (7) then imply that $\mu^*(p'') \geq \lambda$ whenever $\sigma_L^*(p'') > 0$. Therefore, by (9), $\sigma_H^*(p'') \geq \sigma_L^*(p'')$ whenever $\sigma_L^*(p'') > 0$. Adding up over the support of $\sigma_L^*(\cdot)$ implies $\sigma_H^*(p'') = \sigma_L^*(p'')$ whenever $\sigma_L^*(p'') + \sigma_H^*(p'') > 0$. Thus $\sigma_H^*(p'') > 0$ implies $\sigma_L^*(p'') > 0$ for all $p''$, a contradiction.

Now suppose $\sigma_L^*(p') > 0$ if and only if $\sigma_H^*(p') > 0$. As $\Pi_H^* = \Pi_H(p', \mu^*(p'))$ for $\sigma_H^*(p') > 0$, Lemma 3 then implies that $\mu^*(p') \geq \lambda$ whenever $\sigma_H^*(p') > 0$. By the same argument as above, this implies $\mu^*(p') = \lambda$ whenever $\sigma_H^*(p') > 0$. Since the low-quality seller can always guarantee himself the profit $q_L^2/4$ by charging $p = q_L/2$, one has for $k$ sufficiently small that

$$\Pi_L^* = \Pi_L(p', \lambda) = p' \left(q_L - p' + \frac{k}{1-\lambda}\right) \geq \frac{1}{4} q_L^2. \tag{14}$$

By Lemma 3, however,

$$\Pi_H^* = \Pi_H(p', \lambda) = p' \left(q_H - p' - \frac{k}{\lambda}\right) \geq \frac{1}{4} \left(q_H - \frac{k}{\lambda}\right)^2, \tag{15}$$

which implies $p' = (q_H - k/\lambda)/2$. For $k$ sufficiently small, this yields a contradiction to (14). Therefore, there is a $p''$ such that $\sigma_L^*(p'') > 0$, $\sigma_H^*(p'') = 0$ and $\mu^*(p'') = 0$. Since $\Pi_L(p, 0) < q_L^2/4$ for all $p \neq q_L/2$, this implies that $\sigma_L^*(q_L/2) > 0$ and $\sigma_H^*(q_L/2) = 0$. Q.E.D.

Key to uniqueness of equilibrium under the belief restriction is the following Lemma:

**Lemma 4** *In any equilibrium, $\sigma_H(p') > 0$ implies that $\Pi_H(p', \mu^*(p')) \geq \Pi_H(p, \mu)$ for all $(p, \mu)$ such that $\Pi_L(p, \mu) \leq \Pi_L^*$.*

**Proof:** Suppose that there is a $(p, \mu)$ such that $\Pi_H(p', \mu^*(p')) < \Pi_H(p, \mu)$ and $\Pi_L(p, \mu) \leq \Pi_L^*$. Note that this implies $\mu > 0$, because $\Pi_H^* = \Pi_H(p', \mu^*(p')) \geq q_L^2/4 \geq \Pi_H(p, 0)$. Then, for $\epsilon > 0$ small enough, one has $\Pi_L(p, \mu - \epsilon) < \Pi_L^*$ and $\Pi_H(p', \mu^*(p')) < \Pi_H(p, \mu - \epsilon)$. By our assumption on beliefs this implies $\mu^*(p) \geq \mu - \epsilon$. Therefore, $\Pi_H(p, \mu^*(p)) \geq \Pi_H(p, \mu - \epsilon) > \Pi_H(p', \mu^*(p'))$, a contradiction to equilibrium condition (8). Q.E.D.

Now we can prove existence and uniqueness of equilibrium under the belief restriction for small values of $k$. In this equilibrium, the high-quality seller adopts a pure strategy by setting a price $p^* > q_L/2$. The low quality seller randomizes between imitating the high-quality seller's price and revealing low quality by charging $q_L/2$ :

**Theorem 1** *For $k$ sufficiently small, there is a unique equilibrium. In this equilibrium, $\sigma_H^*(p^*) = 1$ for some $p^* > q_L/2$. Moreover, one has $\sigma_L^*(p^*) > 0$, $\sigma_L^*(q_L/2) > 0$ and $\sigma_L^*(p^*) + \sigma_L^*(q_L/2) = 1$.*

**Proof:** By Lemma 4, $\sigma_H(p') > 0$ implies that $(p', \mu^*(p'))$ maximizes $\Pi_H(p, \mu)$ subject to $\Pi_L(p, \mu) \leq \Pi_L^*$. By Proposition 2, $\Pi_L^* = q_L^2/4$ for $k$ small enough. Also, by Lemma 3, $k < (q_H - q_L)\mu^*(p')(1 - \mu^*(p'))$. Therefore, $(p', \mu^*(p'))$ maximizes $\Pi_H(p, \mu)$ subject to $p(q_L - p + k/(1 - \mu) \leq q_L^2/4$. Using Lemma 3, it is easy to show that the solution of this maximization problem must satisfy $p' \geq q_L/2$ if $k$ is small. Therefore, the constraint $p(q_L - p + k/(1 - \mu)) \leq q_L^2/4$ can be rewritten as

$$p \geq \varphi(\mu) \equiv \frac{q_L}{2} + \frac{k + \sqrt{k^2 + 2\,k\,q_L(1 - \mu)}}{2(1 - \mu)}. \tag{16}$$

One can show that $\varphi'(\mu) > 0$ and $\varphi''(\mu) > 0$. Therefore, the set of all $(p, \mu)$ that satisfy (16) is convex. Because $\Pi_H(\cdot, \cdot)$ is strictly quasi-concave, there is a unique $(p^*, \mu^*(p^*))$ which maximizes $\Pi_H(p, \mu)$ subject to (16). By uniqueness of $(p^*, \mu^*(p^*))$, one has $\sigma_H^*(p^*) = 1$. By construction, the belief $\mu^*(p) = 0$ for $p \neq p^*$ and $\mu^*(p) = \mu^*(p^*)$ for $p = p^*$ supports $\sigma_H^*(p^*) = 1$ and is consistent with our assumption on beliefs.

As the constraint (16) is binding and $\sigma_L^*(q_L/2) > 0$, $\Pi_L(p^*, \mu^*(p^*)) = \Pi_L(q_L/2, 0) = q_L^2/4 = \Pi_L^*$. Suppose there is a $p'' \notin \{q_L/2, p^*\}$ with $\sigma_L^*(p'') > 0$. As $\sigma_H^*(p'') = 0$, this implies $\mu^*(p'') = 0$ and so $\Pi_L(p'', \mu^*(p'')) < \Pi_L^*$, which is inconsistent with profit maximization. Thus $\sigma_L^*(p^*) + \sigma_L^*(q_L/2) = 1$. Note that $\varphi(\mu) \to \infty$ as $\mu \to 1$, which implies $\mu^*(p^*) < 1$. Also, by Lemma 3 one must have $\mu^*(p^*) > \lambda$. Therefore equilibrium condition (9), which requires that

$$\mu^*(p^*) = \frac{\lambda}{\lambda + (1-\lambda)\sigma_L^*(p^*)}, \tag{17}$$

uniquely defines $\sigma_L^*(p^*) \in (0, 1)$. Thus the low-quality seller's strategy is uniquely defined and it satisfies $\sigma_L^*(p^*) > 0$, $\sigma_L^*(q_L/2) > 0$ and $\sigma_L^*(p^*) + \sigma_L^*(q_L/2) = 1$. Q.E.D.

Finally, we show that as $k$ becomes very small the equilibrium outcome approaches the full information outcome. Indeed, it already follows from Proposition 2 that the low-quality seller gets the same profit as under full information, because he reveals low quality with positive probability by charging the price $q_L/2$. Similarly, Lemma 3 implies that the high quality seller's profit becomes identical to his profit under full information as $k$ tends to zero.

**Proposition 3** *As $k \to 0$, the equilibrium price $p^*$ charged by the high quality seller converges to $q_H/2$. Moreover the probability $\sigma_L^*(q_L/2)$ that the low-quality seller charges $q_L/2$ converges to one.*

**Proof:** It follows from Lemma 3 that $\Pi_H^* \to q_H^2/4$. Since this is the highest payoff possible, one must have $p^* \to q_H/2$ as $k \to 0$. From $\Pi_L^* = p^*(q_L -$

$p^* + k/(1 - \mu^*(p^*)) = q_L^2/4$, it follows that $\mu^*(p^*) \to 1$ as $k \to 0$. By (17) this implies $\sigma_L^*(p^*) \to 0$ so that $\sigma_L^*(q_L/2) \to 1$. Q.E.D.

Thus, in the limit as $k$ tends to zero each seller type charges the same price as under full information. This means that with small information costs prices almost reveal the quality and that the market outcome approximates the equilibrium under full information. Nonetheless, for all $k > 0$ the information revealed by prices remains noisy. This has to be the case since the seller's pricing behavior is informative only because some buyers invest $k$. As the following result shows, a positive fraction of buyers becomes informed even in the limit $k \to 0$, where prices become perfectly informative.

**Proposition 4** *In the limit $k \to 0$ a buyer with outside option $u$ becomes informed after observing $p^*$ if*

$$\frac{q_L^2}{2\,q_H} < u < \frac{q_H}{2}\,.$$

**Proof:** For $k$ sufficiently small, one has

$$\Pi_L^* = p^* \left( q_L - p^* + \frac{k}{1 - \mu^*(p^*)} \right) = \frac{q_L^2}{4}. \tag{18}$$

In the limit $k \to 0$ this implies

$$\frac{k}{1 - \mu^*(p^*)} \to \frac{(q_H - q_L)^2}{2\,q_H}, \tag{19}$$

because $p^* \to q_H/2$. This together with Lemma 1 proves the Proposition. Q.E.D.

Hence, that prices become revealing as $k \to 0$ does *not* imply vanishing information acquisition. Rather, it is *because* of a non-vanishing fraction of informed buyers that prices almost certainly reveal quality.

Unfortunately, it does not seem possible to solve explicitly for the equilibrium described in Theorem 1. For this reason, we resort to a numerical

| $k$ | 0.1 | 0.075 | 0.05 | 0.025 | 0.01 | 0.0075 | 0.005 | 0.0025 |
|---|---|---|---|---|---|---|---|---|
| $p^*$ | 0.6088 | 0.5622 | 0.5214 | 0.4964 | 0.4960 | 0.4968 | 0.4977 | 0.4988 |
| $\sigma_L^*(p^*)$ | 0.8969 | 0.7622 | 0.5476 | 0.2569 | 0.0893 | 0.0652 | 0.0423 | 0.0206 |

Table 1: *Equilibrium for* $q_H = 1, q_L = 1/2, \lambda = 1/2$

example with $q_H = 1, q_L = 1/2$ and $\lambda = 1/2$. Table 1 reports the equilibrium solution for $p^*$ and $\sigma_L^*(p^*)$ for various values of the parameter $k$. As the table shows, the probability that the low-quality seller sets $p^*$ decreases with $k$. Surprisingly, however, $p^*$ does not monotonically depend on $k$ when it approaches $q_H/2 = 1/2$ in the limit $k \to 0$. Thus, depending on $k$, the high-quality seller's price can be higher as well as lower than under full information.

# 6 Conclusion

This paper considers a market with asymmetrically informed traders. Specifically, prices are set by an informed monopolist supplier. Buyers, who are originally uninformed, may decide to become informed at a cost, after having observed the price offer. Thus, whether prices reveal information depends on how the supplier's pricing policy reacts to the buyers' interpretation of price signals.

For arbitrary out-of-equilibrium beliefs this constitutes a situation where virtually anything can happen. If beliefs are restricted by our generalization of the "intuitive criterion", the equilibrium is unique. It is a partial pooling equilibrium in mixed strategies. Prices only imperfectly reveal private information and some buyers always invest in information acquisition. As the information cost becomes negligible, the equilibrium approaches the full information outcome and prices become perfectly informative.

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