

ESTIMATION WITH PRICE AND OUTPUT UNCERTAINTY

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Submitted September 2003; accepted November 2004

This paper extends the existing estimation methods to allow estimation under simultaneous price and output uncertainty. In contrast with the previous literature, our approach is applicable to the direct and indirect utility functions and does not require specification and estimation of the production function. We derive estimating equations for the two most common forms of output risk (additive and multiplicative risks) and empirically determine which form is appropriate. Moreover, our estimation method can be utilized by future empirical studies in several ways. First, our method can be extended to include multiple sources of uncertainty. Second, it is applicable to other specifications of output uncertainty. Third, it can be used to conduct hypothesis tests regarding the functional forms and distributions. Furthermore, it enables the future empirical researcher to empirically verify/refute the theoretical comparative statics results.

JEL classification codes: D8, D2

Key words: estimating equations, output uncertainty, price uncertainty, utility

I. Introduction

Empirical studies in the presence of hedging (usually agricultural commodities) are abundant. They derive estimating equations under output price uncertainty by applying uncertainty analogues of Hotelling's lemma and Roy's identity to the indirect expected utility function (see Pope 1980, and Dalal 1990). However, their method is not directly applicable to the models with price and output uncertainty. While focusing on hedging, few empirical studies include both price and output uncertainty using a computational approach.

The literature can be divided into three main categories: Literature that deals with theoretical estimating methods, empirical literature that includes hedging, and empirical literature in the absence of hedging.

* I am grateful to the referees and the co-editor Mariana Conte Grand for their helpful comments. Correspondence should be addressed to: malghalith@yahoo.com.

The first category includes Pope (1980) and Dalal (1990). They used duality theory to derive estimating equations based on approximation of the indirect utility function. These equations use the decision variable(s) as the dependent variable(s) and the moments of the distribution as the independent variables.

The second category focuses on hedging of agricultural commodities. Arshanapalli and Gupta (1996) and Antonovitz and Roe (1986) derived estimating equations under output price uncertainty by adopting duality theory; that is, applying uncertainty analogues of Hotelling's lemma and Roy's identity to the indirect expected utility function (Pope and Dalal's approach). Other studies did not use duality theory. For example, Lapan and Moschini (1994) and Rolfo (1980) employed a computational approach. Rolfo computed the ratio of hedging to expected output for cocoa producers. Lapan and Moschini calculated the same ratio for soya bean farmers. Li and Vukina (1998) showed that dual hedging under price and output uncertainty reduces the variance of the income of corn farmers.

The third category does not include hedging. Relying on duality theory, Appelbaum and Ullah (1997) provided a non-parametric estimation of the moments for some manufacturing industries facing output price uncertainty. Here too, some studies did not utilize duality theory. Chavas and Holt (1996) dealt with estimation under technological uncertainty. They devised methods for generating data series for the moments of the distributions. Relying on Taylor approximations of the direct utility function, Kumbhakar (2002a, 2002b, and 2001) provided empirical analysis using samples of Norwegian salmon farmers. Kumbhakar (2002a) assumed quadratic utility and production functions under output price uncertainty. Kumbhakar (2002b and 2001) adopted Just and Pope's specification of production uncertainty. Kumbhakar (2002b) dealt with estimation under production risk and technical inefficiency. Kumbhakar (2001) included output price, input prices, and production uncertainty.

There is a shortage of empirical studies under uncertainty in the absence of hedging (especially studies that deal with output uncertainty). Kumbhakar (2002b and 2001) provided empirical analysis under output uncertainty. Our approach differs from Kumbhakar's approach in three respects. First, Kumbhakar relied on the direct utility function, whereas we use a combination of the direct and indirect utility functions; that is, we modify the duality approach to accommodate output uncertainty. Second, Kumbhakar adopted a different specification of output uncertainty. Third, our approach does not require specification or estimation of the production function.

Consequently, this paper provides three main contributions. First, it extends

existing estimation methods to allow empirical estimation under simultaneous price and output uncertainty. Second, it derives estimating equations for the most two common forms of output risk: additive risk and multiplicative risk (see Honda 1987, Grant 1985, Lapan and Moschini 1994). Third, it empirically determines which form is appropriate. When modeling output uncertainty, the multiplicative specification is consistently chosen over the additive form, despite the latter being arguably intuitively more obvious. The rationale for this seems to be that when production risk is the only source of uncertainty, additive uncertainty does not reduce output below the certainty level, while multiplicative uncertainty does. We empirically show this need not be always the case and thus additive uncertainty is indeed a reasonable a priori method of modeling production uncertainty.

II. Additive output uncertainty

A competitive firm (sector) faces an uncertain output price given by $p = \bar{p} + \sigma\varepsilon$, where ε is random with $E[\varepsilon] = 0$ and $\text{Var}(\varepsilon) = 1$, so that $E[p] = \bar{p}$ and $\text{Var}(p) = \sigma^2$. The level of output realized at the end of the production process is not known ex ante. Output has both a random and a nonrandom component and is given by q , where q is random and defined as $q = y + \theta\eta$ (additive risk), where η is random with $E[\eta] = 0$ and $\text{Var}(\eta) = 1$, so that $\text{Var}(q) = \theta^2$ and the expected value of output is $E(q) = y$. Both σ and θ are shift parameters with initial values equal to 1. We assume that ε and η are statistically independent and thus $\text{Cov}(\varepsilon, \eta) = 0$.¹ Costs are known with certainty and are given by a cost function, $c(y, w)$, which displays positive and increasing marginal costs so that $c_y(y, w) > 0$ and $c_{yy}(y, w) > 0$. While y represents expected output, it may usefully be thought of as the level of output, which would prevail in the absence of any random shocks to output. The firm may be thought of as having its target level of output and committing inputs that would generate this level in the absence of any random shocks. The cost function is then the minimum cost of producing any arbitrary output level y given the input price vector w . Thus, profit is $\pi = pq - c(y, w)$. The firm is risk-averse and seeks to maximize the expected utility of the profit. It therefore seeks to solve the problem

$$\max_y E[U(\pi)] = E[U(pq - c(y, w))].$$

The maximization problem implies the existence of an indirect expected utility function V , such that

¹ This assumption is empirically verified in Section IV.

$$V(\bar{p}, \sigma, \theta, w) = E[U(p(y^* + \theta\eta) - c(y^*, w))] \quad (1)$$

where y^* is the optimal value of y . Let π^* represent the value of π corresponding to y^* . The envelope theorem applied to (1) implies

$$\frac{\partial V}{\partial \sigma} \equiv V_\sigma = y^* E[U'(\pi^*)\varepsilon] + \theta E[U'(\pi^*)\eta\varepsilon]. \quad (2)$$

Consider the following approximation

$$U'(\pi^*) \approx U'(\hat{\pi}) + U''(\hat{\pi})(\pi^* - \hat{\pi}).$$

multiplying through by ε and taking expectations of both sides,

$$\begin{aligned} E[U'(\pi^*)\varepsilon] &\approx U'(\hat{\pi})E[\varepsilon] + U''(\hat{\pi})\{E[\pi^*\varepsilon] - \hat{\pi}E[\varepsilon]\} = U''(\hat{\pi})E[\pi^*\varepsilon] \\ &= U''(\hat{\pi})E[(\bar{p}y^* + \bar{p}\theta\eta + y^*\sigma\varepsilon + \sigma\varepsilon\theta\eta - c)\varepsilon] = y^*U''(\hat{\pi})\sigma. \end{aligned} \quad (3)$$

Similarly,

$$E[U'(\pi^*)\eta\varepsilon] \approx U''(\hat{\pi})E[\pi^*\eta\varepsilon] = U''(\hat{\pi})\sigma\theta. \quad (4)$$

Now, since $\hat{\pi}$ is a constant, $U''(\hat{\pi})$ is a parameter which can be estimated. Letting $\beta \equiv U''(\hat{\pi})$, and substituting the approximations for $E[U'(\pi^*)\varepsilon]$ and $E[U'(\pi^*)\eta\varepsilon]$ into (2) we obtain

$$y^{*2} = \frac{V_\sigma}{\beta\sigma} - \theta^2. \quad (5)$$

In order to get an expression for V_σ we need to have an expression for $V(\bar{p}, \sigma, \theta, w)$. Since the form of the indirect expected utility function is not known, we approximate it by a second-order Taylor series expansion about the arbitrary point $A(\hat{p}, \hat{\sigma}, \hat{\theta}, \hat{w})$ (for a detailed approximation, see Satyanarayan 1999 and Arshanapalli and Gupta 1996). Letting subscripts denote partial derivatives, we obtain

$$V_\sigma \approx V_\sigma(A) + V_{\bar{p}\sigma}\tilde{p} + \sum_i V_{\sigma_i}\tilde{w}_i + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}, \quad (6)$$

where tildes denote deviations from the point of expansion and all the partial

derivatives on the right-hand side of (6) are evaluated at the point of expansion. Substituting (6) into (5) yields

$$y^{*2} = \frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + \sum_i V_{\sigma i}\tilde{w}_i + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}}{\beta\sigma} - \theta^2. \quad (7)$$

The parameters (to be estimated) are $V_{\sigma}(A), V_{\bar{p}\sigma}, V_{\sigma i}, V_{\sigma\sigma}, V_{\sigma\theta}$ and β . Note that y^{*2} is homogeneous of degree 0 in all the parameters, and thus we need some normalization.² The established procedure in the literature is to set β equal to -1 (see Appelbaum et al. 1997 and Dalal 1990).³

Hence, our final estimating form is

$$y^{*2} = -\frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + \sum_i V_{\sigma i}\tilde{w}_i + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}}{\sigma} - \theta^2. \quad (8)$$

III. Multiplicative output uncertainty

In this section we derive an estimating equation for the multiplicative output uncertainty model. If output risk is multiplicative, $q = \nu y$, where $\nu = 1 + \theta\eta$ and thus $E[\nu] = 1$. This estimating equation will be comparable to the additive uncertainty equation. The objective function is

$$\max_y E[U(p\nu y - c(y, w))],$$

and as before, the maximization problem implies the existence of an indirect utility function V such that

$$V(\bar{p}, \sigma, \theta, w) = E[U(p\nu y^* - c(y^*, w))], \quad (9)$$

The envelope theorem applied to (9) implies

² It is homogeneous of degree 0 because, for instance, doubling the values of all the parameters will have no impact on the value of y^* . Thus normalization is needed to make y^* sensitive to the proportional change in the value of all the parameters.

³ Note that we used -1 rather than 1 because β denotes $U''(\hat{\pi})$, which must be negative (by the concavity of the utility function).

$$V_{\sigma} = y^* E[U'(\pi^*)v\varepsilon] = y^* E[u'(\pi^*)\varepsilon] + y^* \theta E[U'(\pi^*)\eta\varepsilon]. \quad (10)$$

We need to approximate $E[U'(\pi^*)\varepsilon]$ and $E[U'(\pi^*)\eta\varepsilon]$. Proceeding as before,

$$E[U'(\pi^*)\varepsilon] \approx U''(\hat{\pi})y^*\sigma \equiv \beta y^*\sigma, \quad (11)$$

where β is defined as before. Similarly,

$$E[U'(\pi^*)\eta\varepsilon] \approx U''(\hat{\pi})y^*\sigma\theta \equiv \beta y^*\sigma\theta. \quad (12)$$

Substituting (11) and (12) into (10) and rearranging, we obtain

$$y^{*2} = \frac{V_{\sigma}}{\beta\sigma(1+\theta^2)}. \quad (13)$$

Using (6), yields

$$y^{*2} = \frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + \sum_i V_{\sigma i}\tilde{w}_i + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}}{\beta\sigma(1+\theta^2)}, \quad (14)$$

where all the partial derivatives are evaluated at the point of expansion. Once again, y^* is homogeneous of degree 0 in all the parameters and thus normalization is required. As before we normalize β equal to -1 so that the estimating equation is

$$y^{*2} = -\frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + \sum_i V_{\sigma i}\tilde{w}_i + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}}{\sigma(1+\theta^2)}. \quad (15)$$

IV. Empirical exercise

The data required for estimation of (8) and (15) include the mean and standard deviation of output and its price. Since these are not directly observable, we have to generate these values from observable data. There is some arbitrariness in the method chosen to do so, since there is no unambiguously "best" approach. Some empirical studies have adopted an extremely simple approach, such as Arshanapalli and Gupta (1996), who used a simple moving average process, while others use much more complex methods.

In order to generate a series of expected prices, we have chosen to use the method developed by Chavas and Holt (1996), where the price at time t is considered as a random walk with a drift. Thus,

$$p_t = \delta + \alpha p_{t-1} + \varepsilon_t,$$

where p_t is the price at time t , p_{t-1} is the previous year's market price, δ is a drift parameter, and ε_t is a random variable with $E[\varepsilon_t] = 0$. Hence

$$E[p_t] = \delta + \alpha p_{t-1}.$$

Similarly, to generate a series for y^* , we model output at time t by

$$q_t = \phi + \varphi q_{t-1} + u_t,$$

where q_t is the output at time t , q_{t-1} is the previous year's output, and u_t is an error term with $E[u_t] = 0$. Hence,

$$E[q_t] = y_t^* = \phi + \varphi q_{t-1}.$$

To generate series for σ we will also use Chavas and Holt's method:

$$\sigma_t^2 = \sum_{j=1}^3 w_j (p_{t-j} - E_{t-j} p_{t-j})^2,$$

where the weights are 0.5, 0.33, and 0.17 (the same weights used by Chavas and Holt). This is done to reflect the idea of declining weights. The price variance is thus measured as the weighted sum of squared deviations of the previous prices from their expected values.

Similarly, the variance of output is

$$\theta_t^2 = \sum_{j=1}^3 w_j (q_{t-j} - y_{t-j}^*)^2.$$

We can implement our estimating methods using manufacturing data; uncertainty in the manufacturing sector is as common as it is in the agricultural sector.

Natural catastrophes, strikes, legal suits and blackouts are some sources of

uncertainty in the manufacturing sector. Demand shocks are the main cause of price uncertainty. For example, Appelbaum and Ullah (1997) identified output price uncertainty in several manufacturing industries. We used U.S. manufacturing time series data. The manufacturing output (q) is produced using four inputs: materials (m), energy (e), capital (k), and labor (l), with prices given, respectively, by w_m , w_e , w_k , and w_l . Gross output price and quantity data are taken directly from the worksheets of the U.S. Department of Commerce, Bureau of Economic Analysis. The quantity and the price of each input are derived or taken from Department of Census, Bureau of Economic Analysis.

Rewriting the estimating equations to explicitly introduce the four input prices we will be using, (8) becomes

$$y^{*2} = - \frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + V_{\sigma e}\tilde{w}_e + V_{\sigma l}\tilde{w}_l + V_{\sigma m}\tilde{w}_m + V_{\sigma k}\tilde{w}_k + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}^2}{\sigma} - \theta^2, \quad (16)$$

and equation (15) becomes

$$y^{*2} = - \frac{V_{\sigma}(A) + V_{\bar{p}\sigma}\tilde{p} + V_{\sigma e}\tilde{w}_e + V_{\sigma l}\tilde{w}_l + V_{\sigma m}\tilde{w}_m + V_{\sigma k}\tilde{w}_k + V_{\sigma\sigma}\tilde{\sigma} + V_{\sigma\theta}\tilde{\theta}^2}{\sigma(1+\theta^2)}. \quad (17)$$

For all the equations, the point of expansion is the mid-point in the data series.

First, we generated data series for ε and η using Chavas and Holt's method to test the independence assumption, using a standard test that relies on the correlation factor and the t-ratio. We strongly (at 1% significance level) accepted the null hypothesis that ε and η are independent. We used nonlinear least square regressions to estimate our estimating equations. We estimated (16) and (17).

V. Results and conclusions

The results are reported in Table 1. While the additive uncertainty model has an excellent fit, with $F = 21.99$ and α (the probability that all the parameters equal zero) tends to 0; this is not the case for the multiplicative uncertainty model. The latter has a very poor fit, where $F = .71$ and $\alpha = .664$. Hence, we reject the model altogether and conclude that the data is more consistent with the additive output uncertainty model.

Table 1. Results for multiplicative and additive uncertainty

Coefficients	Additive Uncertainty	Multiplicative Uncertainty
$V_{\sigma}(A)$	-21775.55 (1265.95)	-25.39 (3.983)
$V_{\sigma\bar{p}}$	-46913.08 (14471.17)	-34.05 (33.23)
$V_{\sigma\sigma}$	-579104.15 (47583.14)	5.412 (16.59)
$V_{\sigma m}$	8409.72 (22262.99)	28.47 (19.95)
$V_{\sigma e}$	-2037.61 (3466.67)	-257.55 (56.977)
$V_{\sigma k}$	818.16 (9201.01)	.4462 (.1308)
$V_{\sigma\theta}$	48.53 (59.35)	4.15 (4.911)
$V_{\sigma t}$	-57105.02 (15350.58)	-57.48 (30.40)
	F = 21.99	F = .71

Note: Standard errors are in parentheses.

The estimates in Table 1 can be used to show the marginal impact of each of the moments on y^* . To show this, partially differentiating (16) with respect to σ yields

$$2y^* \frac{\partial y^*}{\partial \sigma} = -\frac{\sigma V_{\sigma\sigma} - N}{\sigma^2},$$

where N is the numerator in (16): at the point of approximation $N = V_{\sigma}(A)$ and thus at the point of approximation

$$\frac{\partial y^*}{\partial \sigma} = -\frac{\hat{\sigma} V_{\sigma\sigma} - V_{\sigma}(A)}{2\hat{y}^* \hat{\sigma}^2},$$

the values $V_{\sigma\sigma}$ and $V_{\sigma}(A)$ can be obtained from Table 1 and the values $\hat{\sigma}$ and \hat{y}^*

are known values in the data series. Hence the value $\frac{\partial y^*}{\partial \sigma}$ can be obtained. Similarly,

$$\frac{\partial y^*}{\partial \theta} = -\frac{1}{2\hat{y}^*} \left(\frac{V_{\sigma\theta}}{\hat{\sigma}} - 2\hat{\theta} \right),$$

$$\frac{\partial y^*}{\partial \bar{p}} = -\frac{V_{\bar{p}\sigma}}{2\hat{y}^* \hat{\sigma}}.$$

An increase in σ and θ means an increase in price riskiness and output riskiness, respectively. We found $\frac{\partial y^*}{\partial \sigma} < 0$, $\frac{\partial y^*}{\partial \theta} < 0$ and $\frac{\partial y^*}{\partial \bar{p}} > 0$. Thus, an increase in price (output) riskiness reduces optimal output. The increase in the price mean increases optimal output. These results are intuitive and consistent with the theory.

To conclude, in at least this instance it appears that additive output uncertainty might be the more appropriate method of modeling output uncertainty. But this does not imply that, in general, multiplicative risk should be ruled out. Since some microeconomic theorists prefer the multiplicative form, the results are illustrative that additive risk shouldn't be ruled out without empirical evidence.

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