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ON THE VALUATION OF COMPANIES WITH GROWTH OPPORTUNITIES

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Each company faces day to day investment opportunities. Just by staying in business the company is taking a decision of reinvesting capital. These opportunities have to be fairly valued to overcome misallocation of resources. A project with high growth opportunities requires high reinvestments to take full advantage of them until it reaches its mature stage. These investments can be seen as a succession of call options on future growth. When a company with such prospects is valued using the discounted cash flow technique and growth is taken implicitly in the growing cash flows and the residual value, the value thus obtained will be higher than the true one (under certain circumstances). Technology advances and the effects of globalization create enormous growth opportunities, and so misvaluation risks are higher.

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I. Introduction

For decades there has been a fruitful use of the method of Discounted Cash Flow and Net Present Value (henceforth DCF and NPV respectively) to value and evaluate business projects and investment opportunities.[1] They have become standard tools that any financial analyst and manager should manage

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[1] For a more detailed analysis see the initial chapters of "Corporate Finance," by Stephen Ross.

and master to value investment prospects. The DCF works by discounting the expected stream of cash flows using a risk adjusted rate of return.[2] Even though this form of DCF became of great utility, it could not be used to value assets whose payoff are asymmetrical, like options and other derivatives. The breakthrough to the correct valuation of such contracts was made by the contributions of Black and Sholes (1973) and Merton (1973) with the derivation of the valuation formula under certain assumptions, and followed by Cox, Ross and Rubinstein (1979) and the development of the risk neutral approach to valuation.[3]

Since these developments practitioners in finance found themselves equipped with two powerful tools to value streams of cash flows, the standard discounted cash flow technique and the option pricing methodology.[4] Myers (1977) was the first to note that the value of a firm is composed of a stream of cash flows for whom both tools can be used to reflect its value. He showed that the value of any firm is composed of two building blocks, the value of assets in place and the value of growth opportunities. Dixit and Pindyck (1994) showed in a comprehensive book how uncertainty can modify investment rules taken for granted, and how the rule of "invest in projects with positive NPV" does not strictly obtain (in a sense that projects with negative NPV are nevertheless undertaken) for some cases. More recently, the work of Trigeorgis (1988, 1997) and Kulatilaka (1992, 1995) showed how traditional DCF analysis fails to take into account the value of options embedded in projects, prompting undervaluation, and providing rationality to the fact that projects with negative NPV are nevertheless undertaken by companies.[5] The basic idea developed is that the use of traditional DCF to obtain NPV does not consider the flexibility inherent in some projects the management has to react

[2] See next section for the analysis.

[3] See also Mason and Merton (1985).

[4] Although the option pricing technique is a particular form of discounting cash flows, we shall use the term traditional DCF to refer to discounting the expected stream of cash flows using a risk adjusted rate of return.

[5] The negative net present value is outwheigthed by the positive value of the options embedded.

to either favorable or unfavorable conditions, and hence does not include the value of such flexibility. This evidence has prompted a lot of academic work to show undervaluation of projects due to neglected embedded options.[6]

It is the purpose of this paper to show the other side of the coin, those situations where the growth on the cash flows of the project are subject to reinvesting, which in turn is contingent on favorable events. In this case, the cash flow is valued using the traditional DCF technique, and as it is the objective of this paper to show that, under certain conditions, the valuation thus obtained tend to overvalue the true value of the stream of cash flows.

II. Valuation Techniques[7]

On this section we shall state the basic assumptions governing our world, and a brief revision of the conditions underlying the different valuation methods.[8]

A. Assumptions[9]

The following assumptions will be made to make the world more tractable: (a) the typical investor is risk averse, which means she requires a premium to hold assets with uncertain payoffs, (b) capital markets are complete, which means there is a price to be paid to obtain insurance against any state of the nature, (c) the information set is the same for all investors, meaning information is symmetric, (d) growth options embedded in projects take the form of European derivatives, where early exercise is not allowed, where this assumption will help structure the problem in a simple way, (e) the risk free rate is non-stochastic and given, which is a derivation of the assumption of

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[6] See for example Kulatilaka (1992).

[7] We do not consider other methods like relative valuation (comparables) though we acknowledge their existence.

[8] The description of the two methods is adapted from the work mentioned in the introduction.

[9] These assumptions are not far more restrictive than those of the Capital Asset Pricing Model or the Contingent Claim Analysis.

complete capital markets, (f) the value of the company is unaffected by the capital structure, so there is no opportunity of creating value by changing the capital structure (in other words, the Modigliani-Miller theorem holds true), (g) the value of the business in each state of the nature is known, which in turn means there is no risk in assessing the payoffs in each state of the nature, (h) there is an appropriate way of obtaining the risk-adjusted rate of return properly reflecting risk preferences of investors,[10] (i) the probabilities of each state of the nature are known, and (j) in a binomial world when moving the value of probabilities, volatility changes. We shall ignore this effect on the risk-adjusted rate of return.

B. The Traditional Methodology

The traditional method accounts for the calculation of the expected value of future cash flows, discounting it using a risk adjusted rate of return,[11] intended to show the preferences towards risk of the average investor. In terms of a discrete distribution of probabilities, the present value of a one period project can be shown to be

$$V_t = \Sigma \frac{\Sigma \; p_{i,\; t+1} \; V_{i,\; t+1}}{(1+k)^{t+1}} \tag{1}$$

where $V_{i,\ t+1}$ represents the values the project or the firm can undertake in each state of the nature i at date t + 1 (from the cash flows it generates), p_i accounts for the likelihood of each state of the nature, k is the equilibrium risk adjusted rate of return from t to t +1.

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[10] For example, the assumptions of the Capital Asset Pricing Model hold true.

[11] To the purpose of obtaining the appropriate rate for equity, a standard Capital Asset Pricing Model (CAPM) of the form, $k = E(R_i) = r_f + b_i(E(R_m) - r_f))$, can be used, where the left hand side represents the expected return the project has to earn, and the right hand side accounts for two terms, r_f for the risk free rate, and a risk premium. According to the model, in equilibrium the investor pays only for the risk he cannot diversify by himself. It is assumed that value is independent of the capital structure, so there is no point on differentiating between equity and debt.

The value thus obtained is the value of the stream of cash flows, which is then compared with the required initial outlay in order to decide whether the opportunity is worth to be undertaken. If the difference between both (value minus cost of investment) is positive, the project is pursued.[12]

C. Contingent Claim Analysis

Alternatively, in a complete capital market an investor can pay a price π_i at time t to obtain a pure asset, which pays a dollar at t + 1 should state i of the nature happen and zero otherwise.[13] Investors wanting to ensure one dollar in every state of the nature will have to buy a complete set of pure assets paying for it the sum of the prices of each pure asset ($\Sigma\,\pi_i$). The portfolio thus obtained will have the property of being riskless (the payoff of such a portfolio is the same regardless of the state of the nature), hence in equilibrium and to rule out arbitrage opportunities, the return of such a portfolio has to be equal to the risk free return. We label the risk free rate by r, thus $\Sigma\,\pi_i = 1/(1+r)$. Therefore, in equilibrium an asset that pays or has a value of V_i dollars in the state of the nature i and zero otherwise has to be worth $\pi_i V_i$. We have that the value V of such a project or firm is shown to be:[14]

$$V_t = \Sigma\,\tilde{p}_i \;\; V_{i,t+1} \;\; \frac{1}{(1+r)} \tag{2}$$

In other words, the value is the expected value of the payoffs using a synthetic probability distribution, discounted at the risk free rate. It can be easily seen that this new probability distribution satisfies all the requirements of any probability distribution: non-negative values, the sum of all at a certain time adding up to one, etc. We have valued the project using the risk free rate in the discount factor, just as if the investor was risk neutral. Nevertheless, it is shown that the value of the project V_t obtained is the same under the two alternatives.

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[12] This is the NPV methodology.

[13] See for example the description by Varian (1992), chapter 20, pp. 448-452.

[14] See Appendix 1.

III. Growth Options

A. Flexibility on Decisions

Allocating resources in a company does not imply a rigid plan of activities, but a set of decisions conditional upon new information arriving, so decisions are sequential and cannot be fully planned in advance. This means decisions have to be taken as uncertainty unfolds, at the right moment. In these situations the manager needs not to take a decision until she counts with more information. As long as this flexibility does not cause a loss to the company, it has a positive value. These decisions the manager faces when allocating resources can be grouped into the following broad categories:[15] growth decisions, contraction (or even abandonment) decisions, and delay decisions. In all cases the company faces options that can be exercised only if events turn out to be favorable.[16] This reflects the right (not the obligation) the management has. This flexibility (or the options it implies) has value, and it is non-trivial for the value of the company.[17] In this paper I shall focus the analysis on reinvestment as a growth option, its structure and valuation.

B. Growth Decisions

A company can face a project which allows, in case events turn out to be good and circumstances are appropriate, to expand further. Even though this decision is not taken at the outset, the current value of the firm should reflect this option. Growth decisions that a manager can face are: expand business vertically (buy out or set up business within the value chain), expand business laterally (buy out or set up business not directly related with the core business), and expand the business (gain market share) by means of scope or scale.

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[15] Adapted from Kulatilaka (1992).

[16] Otherwise the company can let the option expire and not exercise it.

[17] For example, two companies identical in everything but with a particular customer portfolio each, which allows one company to cross sell more products or services should market conditions turn favorable, cannot be worth the same.

Continuing with the valuation structure described above, we assume that in a particular state of the nature j at t + 1, the investor has the opportunity to undertake further investments with expected cash flows of n times the value of the project or firm at this moment (n $V_{i,\,t+1}$) by paying a cost K. This means the investor will pay the cost K only if n $V_{j,\,t+1} \geq$ K, or n $V_{j,\,t+1}$ - K ≥ 0.[18] If this inequality does not hold, the investor would be paying more than what the asset is worth. It can be seen that the investor would buy the asset (exercise her option to expand) only in those states of the nature where V_{t+1} is sufficiently high. In formula, the payoff or value of business in each state of the nature becomes

$$V_{i,\,t+1} + Max\ (n\ V_{i,\,t+1} - K,\ 0) \tag{3}$$

and the current value of business is thus (we shall label the current value of this asset $V_{t,\,A}$),

$$\mathrm{V}_{\mathrm{t,\ A}} = \frac{\Sigma\ p_i\ \ (V_{i,\ t+1} + Max\ \ (n\ V_{i,\ t+1} - K,\ 0))}{1+k} \tag{4}$$

This value (as shown before), can also be obtained using the contingent claim analysis or risk neutral valuation from (2). Now we shall label the value obtained by this method $V_{t,\,B}$

$$\mathrm{V}_{\mathrm{t,\ B}} = \frac{\Sigma\ \tilde{p}_i\ \ (V_{i,\ t+1} + Max\ \ (n\ V_{i,\ t+1} - K,\ 0))}{1+r} \tag{5}$$

where synthetic (or risk neutral) probabilities derived previously are used.

Throughout this paper we shall demonstrate that growing cash flows for business with growth opportunities require investing needs until they reach their mature stage, and this investment needs are growth options which must be correctly valued. The mature stage, used to value the business, implies exercising a succession of call options (the reinvesting) which must be valued according to their nature, and hence we will see that (4) overvalues the true

[18] We avoid the analysis of agency problems between managers and shareholders.

value of the business. Should this hypothesis be verified, it would mean that for some cases traditional valuation methodology has to be adjusted to reflect the overestimation.

Proposition: If the growing cash flows of a project are used to value it using DCF, and the growth on the cash flows involves reinvesting to attain them and to achieve a mature stage, the value of the project thus obtained will include the results of growth options already exercised through reinvesting, and the result will be an overvaluation of the true value of the project. The result is valid as long as the expected rate of return is greater than the risk free rate (the risk premium is positive).

Proof: (Two States of the Nature, One Period Model) Consider the simplest case, where we have two states of the nature at t+1, and the project value V can adopt two possible values, one for each state i. Assume there exists a risk free asset which pays a return of r. The likelihood of state 1 is given by p, while likelihood of state 2 is the complement 1 - p. According to the traditional method of valuation showed in (1), an asset of such features would be worth

$$V_t = (p_1 V_{1,\ t+1} + p_2 V_{2,\ t+1}) \frac{1}{1+k}$$

where k is a representative risk adjusted rate of return. Consistently with Appendix 2, we can find a synthetic probability $\tilde{p}$ based on the values V_1 and V_2, through which we obtain an expected value of V at t + 1. Discounting this expected value by using the risk free rate, the same value V_t derived by traditional methodology obtains.

This probability distribution based on $\tilde{p}$ comes out from setting the return of the asset equal to the risk free return, and changing the density mass of the probability distribution at each point of the possible values V at t + 1. The probabilities thus obtained are consistent with the current or spot value of the asset.

Armed with this synthetic probability, V_t is obtained by taking the expected value and discounting it to the risk free rate of return. As it was shown, the value V_t remains the same under the two methodologies, but in the second case the value is obtained as if the investor was neutral to risk.

We now capture the random structure of V from the parameters V_1,V_2 and (1 + r), which in turn are used to obtain the set of synthetic probabilities $\tilde{p}$ consistent with V_t.

Suppose that the future value involves growth through reinvesting, so there is a growth option embedded. As it was put as example before, the investor has the right to pay a cost of K to seize n times the value of V at t + 1 (we shall assume that in state 1 (nV) is greater than K, while in state 2 it is smaller), to make the manager exercise his option only in one state of the nature.[19] The asset's payoff then becomes

$$V_{i,t+1} + Max\ (n\ V_{i,t+1} - K, 0) \qquad \text{for i = 1, 2.} \tag{6}$$

In state 1 we have, $V_{1,t+1} + (n\ V_{1,t+1} - K)$, while in state 2 the payoff is $V_{2,t+1}$. Given that the payoff in state 2 is the same under the two methods of valuation, for the sake of the comparison we can leave it aside and concentrate on the payoff in state 1. Under the traditional method of valuation, the value of the project including the expansion options would be

$$V_{t,\ A} = \frac{\Sigma\ p_i\ \ (V_{i,\ t+1} + Max\ \ (n\ V_{i,\ t+1} - K,\ 0))}{1+k} \tag{7}$$

which for two states of the nature is

$$V_{t,\ A} = [p\ \ (V_{1,\ t+1} + n\ V_{i,\ t+1} - K) + (1-p)\ \ V_{2,\ t+1}]\ \ \frac{1}{1+k} \tag{8}$$

rearranging terms we get

$$V_{t,\ A} = \frac{p}{1+k}\ \ (V_{1,\ t+1} + n\ V_{i,\ t+1} - K)\ \ +\ \ \frac{(1-p)}{1+k}\ \ V_{2,\ t+1} \qquad \text{and,}$$

[19] Otherwise would not be an option given it is exercised anyway.

$$V_{t,\ A} = \frac{p}{1+k}\ V_{1,\ t+1}\ +\ \frac{p}{1+k}\ (n\ V_{i,\ t+1} - K)\ +\ \frac{(1-p)}{1+k}\ V_{2,\ t+1}$$

making use of what we know about the value V_t, we notice that the structure of value is equal to the original value of the business plus the expansion option

$$V_{t,\ A} = \frac{p}{1+k}\ V_{1,\ t+1}\ +\ \frac{(1-p)}{1+k} V_{2,\ t+1}\ +\ \frac{p}{1+k}\ (n\ V_{i,\ t+1} - K)$$

being the first two terms equal to V_t

$$V_{t,\ A} = V_t + \frac{p}{1+k}\ (n\ V_{i,\ t+1} - K) \tag{9}$$

On the other hand, by using the risk neutral or contingent claim valuation method derived previously, we have

$$V_{t,\ B} = \Sigma\, \tilde{p}_i (V_{i,\ t} + 1 + Max\ (n\ V_{i,\ t+1} - K))\ \frac{1}{(1+r)} \tag{10}$$

which for the case of two states of the nature is given by

$$V_{t,\ B} = [\tilde{p}\ \ (V_{1,\ t+1} + n\ V_{1,\ t+1} - K)\ +\ (1-\tilde{p})\ V_{2,\ t+1}]\ \frac{1}{(1+r)} \tag{11}$$

following the same procedure of rearrangements of terms we have

$$V_{t,\ B}\ =\ \frac{\tilde{p}}{(1+r)}\ V_{1,\ t+1}\ +\ \frac{\tilde{p}}{(1+r)}\ (n\ V_{1,\ t+1} - K)\ +\ \frac{(1-\tilde{p})}{(1+r)}\ V_{2,\ t+1}$$

$$V_{t,\ B}\ =\ \frac{\tilde{p}}{(1+r)}\ (V_{1,\ t+1} + n\ V_{1,\ t+1} - K)\ +\ \frac{(1-\tilde{p})}{(1+r)}\ V_{2,\ t+1}$$

which according to our initial results can be written as

$$V_{t,\ B}\ =\ \frac{\tilde{p}}{(1+r)}\ V_{1,\ t+1}\ +\ \frac{(1-\tilde{p})}{(1+r)}\ V_{2,\ t+1}\ +\ \frac{\tilde{p}}{(1+r)}\ (n\ V_{1,\ t+1} - K)$$

We observe that again the value of the business is equal to the original value plus the growth or expansion option

$$V_{t,\ B} \ = \ V_t \ + \ \frac{\tilde{p}}{(1+r)} \ (n\ V_{1,\ t+1} - K) \tag{12}$$

comparing values for business obtained from each method ((9) and (12)), and simplifying for those terms equal in both derivations, we are left with the following simplified formula for traditional or DCF valuation

$$\frac{p}{(1+k)} \ (n\ V_{1,\ t+1} - K) \tag{13}$$

while the corresponding for risk neutral valuation is

$$\frac{\tilde{p}}{(1+r)} \ (n\ V_{1,\ t+1} - K) \tag{14}$$

given that the second factor of the multiplication is the same for both, we can drop it off for comparison purposes and concentrate on the first. If a univocal relationship is established between both, we are done. To this purpose, we make use of the components of any risk adjusted discount rate coefficient (1 + k). It is formed by the risk free factor (1 + r) times a risk premium (1 + θ)

$$(1 + k) = (1 + r)\ (1 + \theta) \tag{15}$$

Now we are allowed to make the last simplification. The risk free coefficient is present in both terms, so it can be dropped, then the comparison becomes

$$p \,/\, (1 + \theta) \ \ vs. \ \ \tilde{p} \quad \text{or rearranging} \quad p \ \ vs. \ \tilde{p} \ \ (1 + \theta) \tag{16}$$

if the first term in (16) is greater, it would mean that valuation of growth options by traditional DCF method overestimates the true value of the expansion opportunity. To prove this we use a basic axiom of the probabilistic theory, which says “...the probability is a non-negative number non-greater than 1."[20] Given that there is nothing in our derivation that can violate the axiom (the synthetic probability distribution comes out from a redistribution of mass at each point), and assuming the risk premium θ is positive[21] (being a parameter we can take it for given), $\tilde{p}$ can never be greater than p (if it was the case, and provided that we do not specify a specific value for this probability, we can always choose a value for $\tilde{p}$ to get a p greater than one, which in turn violates the axiom, so the relation must hold for every p and $\tilde{p}$. Hence, if the risk premium is positive for the underlying asset, the first term is always greater than the second, and the traditional method of valuation overestimates the true value of the growth option.

C. Extension of the Analysis from two States to n States of the Nature and to Continuous Time.

Having demonstrated the existence of overvaluation for the simple case of two states of the nature, we extend the framework to n states of the nature, where the random behavior of the variable is assumed to follow a binomial distribution with probability of success (upward movement) p, and n states of the nature. The maximum value that V can reach will have a probability of p^n associated, while the probability associated with the lowest value will be $(1 - p)^n$. For any value of V which requires j upward movements out of n possible, the probability associated will be $B(n; j; p) = C_j^n p^j (1 - p)^{n-j}$ where B denotes the binomial distribution.

Under the risk neutral valuation, the set of values V can adopt does not change, only does the density associated to each value, changing the mean of the distribution and adjusting it to the risk free return. As we saw in Section II, both methods give the same valuation for the underlying variable. The probability distribution thus obtained is of much help to value the options embedded in the project. We have to multiply each option payoff by its corresponding risk neutral probability, to obtain its expected, and then discount it to the risk free rate, obtaining the correct expected value. If we assume growth options are exercised when things go well, and we know that the true

[20] Mendenhall, Beaver and Beaver (1998), chapter 2, pp. 27-28.

[21] From our assumptions about risk preferences of the typical investor.

probabilities are greater for these states than their risk neutral counterpart, their complement for low value states will be smaller,[22] hence the inequality is reversed for low state values of the project. The demonstration is given by taking the upper bound, so that j = n, the true probability of this state or value would be

$$B\,(n; j; p) = C_n^n \; p^n \,(1 - p)^{n-n} \;=\; p^n \tag{17}$$

while the risk neutral would be

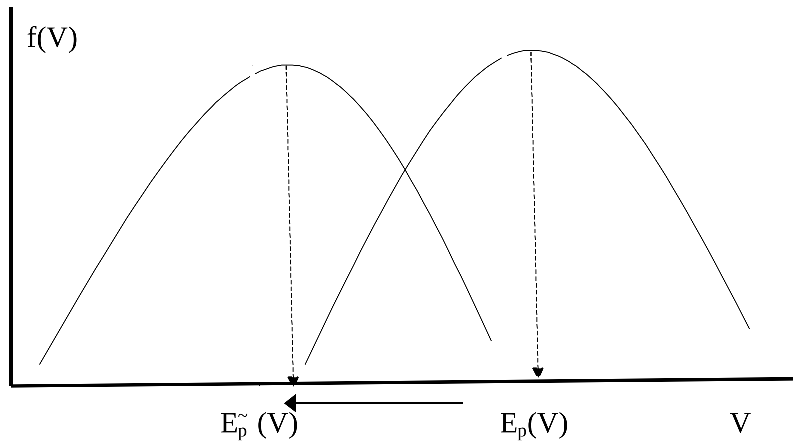
$$B\,(n; n; \tilde{p}) = C_n^n \; \tilde{p}^n \,(1 - \;p)^{n-n} \;= \tilde{p}^n \tag{18}$$

knowing that $\tilde{p}$ is smaller than p, any increasing monotonic transformation has to respect the inequality, so it can be said the following inequality holds, if $p \geq \tilde{p}$, then $p^n \geq \tilde{p}^n$. Both probability distributions have to integrate to one, so the excess in the upper side has to be offset by a diminution on the value of probabilities for low values of the underlying variable, so the inequality is reversed for such values $p \geq \tilde{p}$, then $(1 - p)^n \leq (1 - \tilde{p})^n$. When extending the framework to a continuous distribution, the binomial approximates the normal distribution as n $\rightarrow \infty$, where the effect can be seen better on Figure 1, where V is the value of the company, f(V) is the density function (assumed normal), and it is seen that there is a redistribution of mass to change the expected value, which is less for the risk neutral distribution under a positive risk premium.

High values tend to have lower probabilities now. It can be seen clearly the effect of changing from the true distribution to a synthetic distribution when the risk premium is positive. It can be observed there is a redistribution of mass in the probability distribution to change and reduce the first moment of the random variable (move the risk adjusted rate of return to the risk free, which is lower by assumed risk aversion). It is clearly seen that for high values of V the mass associated is lower under the risk neutral distribution, hence if the real distribution is used to value option it would be overvaluing its true value. This insight confirms our previous derivations. In the same

[22] Otherwise they will not add up to one.

Figure 1. Change in Drift and Redistribution of Density Mass for a Positive Risk Premium on a Normal Distribution



tense, for a low value of V the mass associated is lower, but this change does not affect the value of the option, which has positive value only for high realizations of V (otherwise is zero, never negative).

Remark: If the risk premium is negative, as would be the case if under the CAPM world the underlying asset happens to have a negative covariance with the market return, and hence a negative premium, the problem arising will be of undervaluation.

IV. Results

Due to result obtained, though the valuation for the underlying asset is the same under both mechanisms, when it comes to evaluate growing cash flows (horizontal, vertical or within the same market) embedded in the project, the traditional DCF overvalues the true option value. Although the discounting rate is smaller (and hence the discount coefficient is greater, which leads to increase the value of the option calculated by risk neutral valuation) this effect cannot offset the decrease in expected value due to the application of the new probability distribution.

As it was shown, the use of the true distribution and a risk adjusted rate when the applicable distribution is the risk neutral (or synthetic) with the risk free rate lead to overvaluation due to the asymmetry of the payoffs. Consider for instance a start up project. If for valuation purposes we forecast growing cash flows and a residual value consistent with them, and growth has to be supported by periodical investments until it reaches its mature stage, the value thus obtained will imply exercising successive growth options. Given that the value at the mature stage includes exercised growth options, there would be a tendency to overstate the true value of the start up. The degree of overvaluation will depend upon the values adopted by the following parameters: r (risk free rate), k (risk adjusted rate), p (probability of high values for the project), V_u (the value of the project in a good state) and V_d (the value of the project if things do not go too well).

A. Comparative Statics

A simulation model can provide more insights. Assume the two possible values the company can take are 135 in one scenario (with probability 43%) and 95 in the other (with probability 57%). The risk-adjusted discount rate is assumed to be 10%. Under the traditional DCF methodology, the value of the project would be 100. Now assume that at the following period the company is able to expand further by paying a cost of 200 to obtain an expected value of two times the value of the company at t + 1. This growth opportunity will be exercised only if the market proves to be good for the company (scenario 1). For the purposes of comparative static we change one parameter at a time, keeping the others constant. In Table 1 we can observe the results of our changes in the values of the parameters;[23] V_u is the value of the company in the good state of the nature, V_d in the bad state, r is the risk free rate, k is the risk adjusted discount rate, p is the true probability of the good state of the nature, and the expansion payoff (growth in cash flows) is the function $Max\ (2\ V_i - K)$.

We first change the upper value of V, then the lower value of V, we continue by changing the risk free rate and the risk adjusted rate of return, and finally we change the value of the true probability p. The results are the following:

[23] The results are based upon movements of Vu to 140, Vd to 85, r to 7%, k to 12% and p to 50%. In the last row the degree of arising overvaluation can be seen.

Table 1. Simulation Parameters and Results for Comparative Statics

	Initial value	Vu =140	Vd = 85	r = 7%	k = 12%	p = 50%
Present value of the asset	100	103.9	94.8	100	98.2	102.3
Risk neutral probability (p)	29%	31%	32%	34%	23%	35%
Growth option value under DCF	23.4	31.2	23.4	23.4	23.0	27.3
Growth opt. val. under risk neutral valuation	16.3	23.9	18.5	19.2	13.3	20.2
Extent of overvaluation	44%	31%	27%	22%	73%	35%

Notes: The initial values for the parameters are the following: upside value, V_u = 130; downside value, V_d = 95; riskfree rate, r = 5%; discount rate, k = 10%; probability of upside scenario, p = 43%; expansion payoff, 2 times the current value $V_i = 2\ V_i$; cost of investment of expansion, K = 200; and net payoff of expansion, Max (2 V_i - K, 0) = 260.

(a) an increase on the upper possible value V_u reduces the excess of overvaluation, (b) a decrease on the lower possible value V_d reduces the extent of overvaluation, (c) an increase on the risk free rate r reduces the excess of overvaluation, (d) an increase on the risk adjusted discount rate k increases the excess of overvaluation, and finally, (e) an increase on the real probability p of upward movements reduces the degree of overvaluation.

Now we shall explain the intuition underlying these effects from the formula for calculating risk neutral probabilities in our simple model; the probability $\tilde{p}$ comes from[24] the following formula:

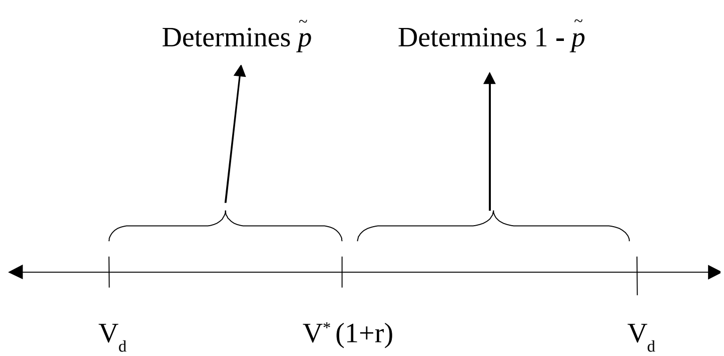
$$\tilde{p} = \frac{V\ (1+r) - V_2}{V_1 - V_2}$$

[24] See Appendix 2.

This can be better appreciated with the help of Figure 2, where it can be seen how the value V_u and V_d, together with the initial value V and the risk free rate r give rise to the risk neutral probability in a binomial world. An increase on the upper value V_u increases the expected value of the underlying asset. Given the methodology of calculation of the risk neutral probability $\tilde{p}$, we would expect the probability to diminish, however, this effect is more than offset by the move in the expected value of the asset (used together with the risk free rate of return to determine the risk neutral probabilities), which moves the division line between probabilities to the right. This effect overcomes the other, hence increasing $\tilde{p}$. This situation drives the risk neutral probability closer to its real counterpart (which is assumed to be constant here), reducing the extent of overvaluation.The decrease on V_d leads to the same effect. The changes on this extreme value are exactly the opposite as those described previously (the upper value going up is equivalent to the lower going down). In both cases the expected value of the underlying asset is affected, though in the opposite sense, impacting on the divisory line between risk neutral probabilities. An increase on V_u or a decrease on V_d broadens the range between the extreme values, affecting in an opposite way the expected value of the underlying asset but affecting in the same way the risk neutral probability, bringing it closer to the real counterpart, therefore reducing the degree of overvaluation.

Both an upward movement on the risk free rate r, or a reduction on the risk

Figure 2. Determination of the Risk Neutral Value for p from the Parameters of the Simulation



adjusted rate k, can be synthesized in a change on the risk premium of the asset (the risk adjusted rate can be decomposed into two components, the risk free component and the risk premium).

An increase of r (keeping k constant) as well as a decrease on k (given r), can be assimilated to a decrease on the equilibrium risk premium. However, the effects on the dependent values are not exactly the same.[25] An increase of r does not change the expected value of the asset, but affects the line dividing the risk neutral probabilities. Given how this probability $\tilde{p}$ is calculated, the division line is moved to the right, increasing it. This drives the risk neutral probability closer to the real probability, therefore reducing the extent of overvaluation.

The effect of an increase of k affects the expected value of the underlying asset moving the division line to the left, thereby reducing the risk neutral probability $\tilde{p}$ and broadening the gap between the synthetic and the real probability.

Finally, an increase of p increases the expected value of the underlying asset. This moves the division line to the right, therefore increasing $\tilde{p}$ and reducing the degree of overvaluation.

It stems from these explanations that the analysis mainly passes through the study of the movements of the division line that makes up the values of the risk neutral probabilities $\tilde{p}$ y 1-$\tilde{p}$. It is not complicated to find out from a visual inspection the consequences of movements on the value of the parameters.

B. An Application

In recent works[26] a methodology has been suggested to value internet[27] and technology companies (and by extension applicable to any start up project). This methodology is also used in the "venture capital valuation"

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[25] In fact the effects are the opposite.

[26] See Desmet, Francis, Hu, Koller, and Riedel (2000).

[27] The case study used is Amazon.com; roughly speaking, it is calculated the expected value of Amazon in 2010, using estimates of market share in different segments of business. The value is then discounted by means of a risk adjusted rate to the present to obtain the current value.

model.[28] The method works backwards, starting by obtaining the would-be value, which can be thought as the expected value, of the company at some point in the future, when it is consolidated and making profits. This expected value is then discounted using a risk-adjusted discount rate to obtain the value of the company today, being this methodology consistent with (1) and (4). Here our analysis starts to be applied; consider the value of the company in the future, in some years time; this value is reached after several investments outlays are made. Each of the installments is contingent on previous growth attained, so as long as nature shows up favorable for the project, new investment takes place to keep the growth rate. We are able then to say that the value in the future is contingent on nature showing favorable[29] until it reaches such a point.

If we then value the business by DCF, we would be falling into the overvaluation problem previously described. Our analysis suggests that by valuing contingent (on growth) streams of cash flows[30] using the discounted cash flow methodology, the value of the business will tend to be overestimated. The situation previously described is shown in Figure 3, where it is clearly seen that there is a reinvestment pattern (which is contingent on previous events) needed to attain the growth of cash flows and the value of the project at the mature stage. By directly discounting growing cash flows and residual value (methodology widely used to value high risk long-term projects, like the ones we deal with) will be falling under the problem described.

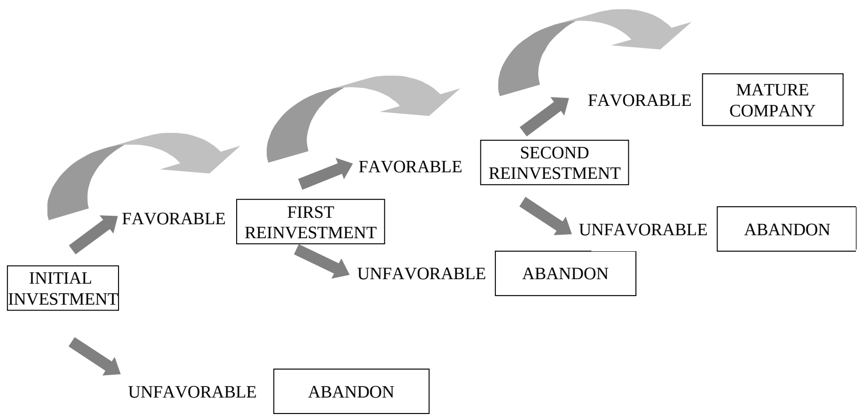
To the purpose of solving the problem of overvaluation detected and exposed previously, the following methodology is proposed to correctly evaluate the growth opportunities: (a) separate the outcomes of contingent decisions from the current value of the company, (b) analyze the random structure of events the company faces, (c) define a variation range for the possible values of the business, without including results of options, (d) calculate the present value using the DCF method, to determine the value of the underlying asset, and with this in hand, determine the risk neutral

[28] See Sahlman and Scherlis (1989).

[29] With the help of the management as well.

[30] We are able now to see how important were contingent payoffs just by taking a look ate the current economic and financial situation of the company.

Figure 3. Contingent Investment Sequence Needed to Maintain the Pattern of Growth for High Growth Companies



probability distribution, (e) use these probabilities to value the options, discounting the expected value to the risk free rate, and (f) add the value thus determined to the value of the company.

We know it is not an easy task, and that we have worked with a simplified model. However, the fact of thinking about contingent situations and possible outcomes represents a great advance to the company and manager's strategic thinking.

V. Conclusions

A now growing literature on real options is taking advantage of the theory and practice of financial options. It starts to be thought that options are everywhere within the company, and given that flexibility has value, the real option framework is the appropriate method to capture it. Throughout this paper it has been demonstrated that growth patterns in cash flows of high growth companies or projects embed growth options through successive investments and reinvestments, which if valued using through straight traditional DCF may give rise to an overvaluation problem.

The intention of this paper was to show that valuation of projects and business with growth opportunities must take into account the overvaluation effect they are exposed to, given that future value is contingent on favorable events. The present value of a business is composed of two elements: the present value of assets in place and the growth opportunities.

The weight of each component will be affected by the industry and the firm's own characteristics. To the extent that the company is in a mature industry, and the possibility of growing has been fully exploited and reflected in the current value of the firm assets, the growth component will tend to be relatively not significant with respect to the full value, so reinvestment needs will not be significant. On the other hand, for companies and industries in expansion or in newly created industries, the most of the value will be captured by growth options due to the need of reinvesting heavily, weighing more significantly in the full value. This contingent growth will have associated a high volatility, due primarily to the uncertainty surrounding the market, the product or service, competitors and substitutes. Being more significant the option component for this kind of industries, the use of the traditional DCF model for valuation purposes will offer more problems, prompting overvaluation.

The most significative and illustrative example can be captured by the impact of technology and globalization on growth opportunities of companies and industries. This affects industries asymmetrically and to different extents. For those companies that are affected the most, technology creates a complete new world of opportunities, and also creates risk of overvaluing business due to the problems described, under the assumption that investors use the DCF model as a valuation tool. Options must be valued as their nature claims.

However, it has been shown throughout this paper that both methods are complements rather than substitutes. Risk neutral probabilities cannot be obtained without figuring out the current value of the underlying asset, for which DCF is appropriate; so they work together towards the same goal. Nevertheless, each method has to be applied for the right situation to a proper analysis of the allocation of resources.

Our results are derived based upon a set of assumptions, so results are conditioned and the model developed is not very complicated. However, these assumptions are not more restrictive than those involved in the derivations of models like the Capital Asset Pricing Model or the Black Scholes formula. Nevertheless, this fact should not stop us from relaxing assumptions and searching for new results. This is a very attractive topic for future research.

Appendix 1

It follows that at t + 1 an asset with payoffs of V_i in each state of the nature i is worth,

$V_t = \Sigma\, \pi_i\; V_{i,t+1}$ at t.

Working on this formula, multiplying and dividing by $\Sigma\, \pi_i$ and redistributing, we obtain,

$$V_t \;=\; \Sigma \pi_i V_{i,\ t+1} \frac{\Sigma \pi_i}{\Sigma \pi_i} \;=\; \Sigma \frac{\pi_i}{\Sigma \pi_i}\; V_{i,\ t+1}\; \Sigma \pi_i$$

and making, $\tilde{p}_i = \dfrac{\pi_i}{\Sigma \pi_i}$, and $\Sigma \pi_i = \dfrac{1}{(1+r)}$, we obtain

$$V_t \;=\; \Sigma\, \tilde{p}_i\;\; V_{i,\ t+1}\;\; \frac{1}{(1+r)}$$

Appendix 2

In short, the changes introduced are: (a) take the current value of the asset, (b) set its return equal to the risk free return, (c) find the probabilities associated to this new expected value by changing the probability mass at each point of the possible values of V. In formula,

$$V \;=\; [\tilde{p}\;\; V_1 \;+\; (1-\tilde{p})\;\; V_2\,]\;\; \frac{1}{(1+r)} \qquad \text{rearranging terms,}$$

$$(1+r)\;\; V \;=\; \tilde{p}\;\; V_1 \;+\; (1-\tilde{p})\;\; V_2 \qquad \text{can be easily solved for}$$

$$\tilde{p} \;=\; \frac{V\;(1+r)\;-\;V_2}{V_1\;-\;V_2} \qquad \text{and} \qquad (1-\tilde{p}) \;=\; \frac{V_1\;-\;(1+r)\;\; V}{V_1\;-\;V_2}$$

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