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Snow Removal Auctions in Montreal: Costs, Informational Rents, and Procurement Management *

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Abstract:
Using nonparametric estimation techniques adapted from Guerre et al. [2000], we infer cost distributions and informational rents, from 666 snow removal contracts offered for tender by the City of Montreal. Our results are compatible with standard received theory of competitive auctions: there is a positive correlation between costs and bids; rents increase with the variance of costs and decrease with the number of bidders. Bids and costs have decreased over the sample period, while informational rents remained stable. The City deserves credit for these results, as it has succeeded in exploiting economies of scale while maintaining competition.

Keywords:
Procurement auction, nonparametric estimation, informational rents, task design, municipal contracts.

JEL classification: D44, H400

1 Introduction

Snow removal is an important activity for the City of Montreal. Approximately 2,000 kilometers of streets and 3,500 kilometers of sidewalks have to be cleared after each snowstorm. Every year, on average 7,500,000 cubic meters of snow are removed and carried to snow dumps. The budget for snow removal was $52 million in 1998, accounting for about 3% of the total budget of the City. Considering the high cost of snow removal, the City wants to make sure that it buys outside services at minimum cost; it needs to know fairly accurately the contractors' actual costs. One way to get this information is for the City to carry out some of the work itself, as it does, while contracting out the rest to private suppliers. This kind of benchmarking has its limits, however, because a municipality is often less efficient than the private sector in providing public services.

It is well-known that, under conditions which include the absence of collusive behavior among suppliers, appropriate auctions insure that the most efficient supplier is selected and that the rent left in the hands of that supplier by the auctioneer is minimized, given the number of bidders. One may wonder then, why it is desirable to know more about contractors' costs.

One important reason is that the definition of the service to be contracted out may substantially affect its cost. For instance, the mapping and the size of the territories specified in the contracts affect the scale of snow-clearing and the distance to the snow dumping site. Knowledge of such repercussions may help the municipality streamline its snow removal and transportation operations.

A second important reason to seek knowledge of contractors' costs and rents is that rationalization of the work being auctioned out may have an impact on competition in the auctioning process. The City may for instance redefine its territory subdivisions in such a way as to reap economies of scale. An extreme case would be to have one single territory and delegate the activity to a single supplier. Such a practice, however, is likely to affect market structure. When comes the time to renew the contract (say every five years), fewer firms would be big enough to participate, but there would also be fewer territories to compete for. Therefore the level of competition, and the rent that the winning firm could reap from the contract, would be affected[1]. This suggests a potential trade-off between competition and economies of scale.

We propose to infer the contractors' costs, and the rents they obtain, using an econometric model of auction bidding applied to data on private bids for snow removal contracts auctioned off by the City of Montreal on 90 procurements between 1986 and 1998. Existing theoretical results on bidding strategies, bidding rents and optimal procurement (such as by Riley and Samuelson [1981] or by McAfee and McMillan [1987]) have been criticized for relying on the unknown distribution of private values (private costs in the present context). Empirical work is therefore needed to evaluate the performance of such a procurement mechanism and to provide insights into ways to improve upon it. Our work is a contribution in this direction. From a theoretical model, adapted from Riley and Samuelson [1981], we derive a structural econometric model from which the contractors' costs, cost distribution and rents can be computed.

The estimation raises several econometric issues, which have been examined by Florens, Hugo and Richard [1997] and Paarsch [1992] in the context of procurements and

[1] We do not discuss here the possibility of regulating a sole supplier (a monopoly) on the basis of its cost, without re-auctioning the contract regularly, because municipal law requires that any services, above a given expenditure, be put up for auction.

by Donald and Paarsch [1993, 1996], Laffont, Ossard and Vuong [1995], Laffont and Vuong [1993], Elyakime, Laffont, Loisel and Vuong [1994] and Guerre, Perrigne and Vuong [2000] in the context of sale auctions. We adapt the Guerre, Perrigne and Vuong [2000] procedure to a procurement mechanism. This method has the advantage of being fully nonparametric, so that it does not impose any functional form on the unknown distributions. We also introduce a variable reduction technique to model the heterogeneity of the auctioned contracts instead of keeping to the main explanatory variable for characterizing the contracts as is usually done.

The paper is organized as follows. Section 2 describes the institutional features and our assumptions for the present study. A game-theoretic model of procurement auctions, where firms compete on price, is developed in section 3. Section 4 is devoted to the identification of the structural elements and to the description of the estimation procedure. The results of the estimations are reported and analyzed in section 5. Finally, we summarize and conclude the paper in section 6.

2 A Description of the Auctioning of Snow Removal Contracts in Montreal

Every year, the City of Montreal publishes an invitation to tender for several snow removal contracts, corresponding to different territories of the City. The contracts are standardized and differ only with respect to the characteristics of the territories. Firms interested in submitting bids request specifications from the City. For each contract, the City provides a map, a description of the territory (length and distance to the snow dump) as well as (after 1990) the reserve price. On the day of the auction, the sealed bids are opened and the identity of all bidders and their bids are announced to those present. The contracts under auction have five year terms and put the winner in charge of cleaning up snow from the streets and sidewalks between November 15 and March 15 ("the snow season") during these five years, at the agreed price.

Participants bid on the price, in dollars per meter of street length per year, based on a "normal" snowfall of 200 centimeters per year. The lowest bid is accepted for each contract provided that the specified qualifications are met: the candidate must have the required equipment and must provide adequate financial warranties. In the course of contract execution, the price may be adjusted to allow for abnormal snowfalls. The price paid to the firm is increased by 0.4% for each centimeter above 200 centimeters; similarly, the price is reduced by 0.4% for each centimeter of snowfall below 200 centimeters down to 100 centimeters. Consequently, for a winning bid p_{il} by firm i in contract auction l, the yearly amount a_{il} received in dollar per meter of street length, is[2]:

$$a_{il} = .6p_{il}I_{\{q\leq 100\}} + [1 + .004(q - 200)]\, p_{il}I_{\{q>100\}} \tag{1}$$

where I is the indicator function, q is the actual snowfall during the year under consideration. As a result, the supplier is certain to receive at least 60% of the revenues corresponding to a normal snowfall of 200 centimeters at the bidded price, but shares with the City the risk associated with yearly fluctuations above 100 centimeters.

Total revenue to the firm is the product of total street length in the territory (in meters) by yearly amount a_{il}. Although prices are quoted in the same units from one contract to the other, each bid can be different, not only because auctions on different

[2] Usually it snows well over 100 centimeters a year (the average over the last 20 years was 206.6 centimeters). For the last five years the precipitations were between 179 and 327 centimeters. The lowest level of snowfall ever was 87.5 centimeters in the winter of 1979-1980 while the lowest level in our dataset was 131 centimeters.

territories are independent, but also because territories have different characteristics that affect the cost per meter. The City of Montreal requires a list of pieces of equipment for each territory. The supplier has to prove that he has the required capital. If he does not have all of it before the auction, he has to commit himself to buy the other pieces before the beginning of the contract.

We have data on the winning and losing bids, together with contract specifications, for 90 procurements of snow removal tendered between 1986 and 1998 by the City of Montreal, for a total of 666 bids. The number of bidders varies across auctions from two to fourteen.

Table 1 gives an overview of the main trends. Winning bids for snow removal contracts with the City of Montreal have gone down by 40% between 1986 and 1998. One possible explanation for this decline, as we will see in section five, is the returns to scale associated with the increase in territory size. Since the contracts have five year terms, the policy of increasing territory size could only be introduced progressively. On a given year, any new territory division can apply only to territories under renewal and can be offered only to the cohort of contractors whose current contracts are expiring. Transition technicalities and other historical features account for the fact that, over our 1986-98 sample period, one-quarter of the contracts are up for renewal each year for four years while, the fifth year, no procurement auction is organized (1989 and 1994). This illustrates some of the constraints applying to changes in territory size shown in Table 1.

Another explanation for the decline in bids might be increased competition. If the pool of potential suppliers remains constant as territory size increases and the number of territories diminishes, then the number of participants has to increase at each auction. However, changes in territory size may induce some suppliers into seeking other work. We shall examine which of the two explanations is most likely to prevail.

In Table 1, we also present the chronological evolution of the number of contracts, the number of bids and the resulting average number of bids per contract. We notice a reduction of 3 contracts in 1991 for the cohort of contracts awarded in 1986 and renewed in 1991 and a reduction of 2 contracts in 1993 for the cohort of contracts awarded in 1988 and renewed in 1993. There is no systematic pattern in the number of bidders for each successive contract within a given cohort. It increased in 1991 compared to 1986 but then it dropped sharply in 1996. It decreased from 8.50 in 1987 to 6.67 in 1992 but shooted up to 10.83 in 1997. It increased from 6.60 to 6.67 in 1993 and remained at that level in 1998, and it increased in 1995 compared to 1990. As a result, the average number of bidders per contract is wobbly. It tends to be higher for one cohort (the one with contracts starting in 1987, 1992 and 1997). In Table 2, we present some descriptive statistics for the sample period (1986-1998).

3 The Procurement Auction Model

We shall set up a model of procurement auctions based on assumptions which we consider to be realistic in the institutional context under study. The observed data on bids and winning bids are supposed to be generated by this model. Together with the characteristics of the contracts they will allow us to infer the contractors' private costs and informational rents. The model will also help us assess the effects on costs and rents of increases in territory size and other management decisions. We first turn to the assumptions underlying our model.

3.1 Institutional Details and Model Assumptions

Knowledge structure. We assume that the set of bidders for any particular contract is common knowledge (I_l for auction l). Firm i knows its own cost but only the distribution of the cost of its competitors. Agents' private costs are assumed to be independently drawn from a common distribution on $[\underline{c_l}, \overline{c_l}]$ with density function $f_l(\cdot)$. This stochastic structure is common knowledge. It is likely that individual costs differ across bidders because of differences in capital stock (number, age and type of machines), expertise, preferences for a territory (firms specialize in different types of territories or may have affinities with the municipal team), capacity utilization (which depends on the other commitments of the firm) and location (a firm must rent a warehouse if the territory is too far away). We assume that the bidders know from personal experience and from visiting the auctioned territory how much it would cost them to realize a particular contract. To the extent that individual cost differences are more important than uncertainties about the task which affect all firms symmetrically, the bidding process is best modeled as an independent private-values (IPV) procurement auction.

Risk, symmetry, independence. Because firms typically obtain numerous contracts from several municipalities, carry out other activities or can rent their equipment[3], we may assume them to be risk neutral at the level of a single auction. Discrimination, e.g. in favor of Montreal suppliers, in the award of procurement contracts, is prohibited; we assume that all bidders are otherwise indistinguishable, so that each firm is treated

[3]Some firms buy extra equipment on purpose, usually when these machines can be used in the out-of-snow season, to rent them to townships because of the very good return associated with this practice.

alike.

A firm's cost for performing a given task may change over time because of new capital acquisition, new experience, differences in auctioned territories or differences in alternative opportunities. Costs are assumed to be independently distributed over time. Moreover, we assume that the shape of the distribution does not change over time, only the interval over which it is defined. Consequently, the auctions are treated as a succession of independently repeated games.

We may also reasonably consider that the bids are independent across auctions in a given year. Indeed, although a firm may bid on several contracts, it cannot do so with prior knowledge of any auction outcome; thus it cannot bid conditionally on the results of other auctions. The maximum number of contracts it can win depends on its capital stock and on the number of financial warranties it must provide with its bids.[4] If a contractor bids on, and wins, more contracts than he can ultimately deliver, the City decides which ones he will eventually retain given his capacity. Since such *ex post* assignment is done in such a way as to minimize the City's cost rather than to maximize the bidder's rent, the latter is not likely to win by using, in a specific auction, any strategy involving other auctions. Also, a firm is not able to benefit from economies of scale or scope from contracts with neighboring territories: removing snow on one territory cannot reduce the cost on neighboring territory because the terms of the contract require that a firm which has two (or more) contracts must get all the required equipment simultaneously on each of the territories it is responsible for. Therefore, there is no gain, and no risk, from bidding at several auctions.

[4] Each bid must be accompanied by a deposit and by a warranty issued by a Canadian insurance company of an amount corresponding to 60% of the value of the contract.

Finally, the City of Montreal is not the only town which solicits bids for snow removal, and snow removal is not the sole activity of the firms involved. Over the year there are numerous invitations for tender, firms often bid simultaneously with several towns for many contracts, and they enter other types of contracts with other customers. Globally, firms win the number of contracts they wish (with one or several municipalities). Consequently, we shall consider that the procurement auctions for snow removal are individually independent auctions.

Competition. The demand for snow removal is relatively price insensitive, as public opinion is in favor of clearing the streets. For instance, initiatives by the City of Montreal to reduce overtime expenditures by suspending snow removal on weekends has raised public criticism. However, as is plain from the above discussion on independence, there are many buyers, many suppliers, many products, and the rules do not facilitate the control of any market or auction procedure.

Preventing bidders from using the same equipment on neighbouring territories, as does the City of Montreal, limits the benefit of a collusion for territory assignments. Barriers to entry are low: firms who handle snow removal also carry out other activities, like landscaping, general construction or excavation. Their equipment can have many uses, especially their trucks which can be equipped with removable plows. As a matter of fact, we have identified 67 different firms bidding for snow removal in Montreal between 1986 and 1998, some of them entering and others leaving the market. Obviously, differences among firms, the sheer number of potential entrants, and heterogeneity in the territories would complicate cartel coordination. For all these reasons, the bidding behavior is modeled as a noncooperative game under incomplete information.

Reserve Price. Before 1990, no maximum (reserve) price was used by the City of Montreal whereas, after 1990, a public reserve price p_{0l} for contract l was introduced. Since no bids were rejected before 1990, which would have served as evidence of an implicit reserve price, we assume that there was truly no restriction before 1990. The 1990-1998 period does not look different from the pre-1990 period. In particular, we observe no drop in the number of bidders after 1990 (Table 1). The mean number of participants, during the period without a maximum price, was 7.54 with a maximum of ten, whereas, during the period with a reserve price, the mean was 8.26, with a maximum of fourteen participants. Moreover, several suppliers indicated in interviews that the introduction of the reserve price did not affect their decision to participate. Consequently, we do not make any use of reserve price data in the empirical work reported below.

3.2 Bidding Strategies

We begin our presentation of the model with an informal discussion of the bidding game. We then determine its outcome and interpret it.

Consider a "buyer" (the City in our case) who auctions, in a first-price sealed-bid auction, several fixed-price contracts to I potential firms ($I \geq 2$). Although the contracts are relatively homogeneous, they are not identical. Therefore for each contract l, we allow the distribution of costs to depend on the characteristics of the contract z_l (to be defined in the next section). Let us denote the cumulative distribution function $F_l \equiv F(\cdot|z_l)$, the corresponding density function $f_l \equiv f(\cdot|z_l)$, the survival function $S_l = 1-F_l(\cdot) \equiv S(\cdot|z_l)$, and the interval over which the cost distribution is defined as $\left[\underline{c_l}, \overline{c_l}\right] \equiv [\underline{c}(z_l), \overline{c}(z_l)]$.

Total revenue and total cost depend on the actual snowfall, q. Total cost increases with the level of snow removal as firms have to operate their equipment and pay their employees for each extra hour worked. According to the snow-removers that we have interviewed, the costs increase linearly with the level of snowfall; thus we assume that the cost per centimeter of snowfall is constant. Snow removers also bear a fixed cost, which may differ from one territory to another, if only because equipment requirements differ between territories. Thus we write the total cost for firm i in contract l as

$$k_{il} + \frac{v_{il}}{200} q$$

where k_{il} is the fixed cost and v_{il} is the variable cost corresponding to a snowfall of 200 centimeters. We also define $c_{il} = k_{il} + v_{il}$ as the total cost corresponding to a snowfall of 200 centimeters. Given the per meter revenue defined by (1), the expected profit for the winner i of auction l is:

$$\begin{aligned} \phi_{il} = & \int_0^{100} \left[.6p_{il} - k_{il} - \frac{v_{il}}{200} * q \right] \gamma(q) dq \\ & + \int_{100}^{\infty} \left[(1 + .004(q - 200))p_{il} - k_{il} - \frac{v_{il}}{200} * q \right] \gamma(q) dq \end{aligned}$$

where the expectation is taken over snowfall, whose density is $\gamma(q)$. If we assume that $E(q) = 200$, the expected profit, conditional on winning the contract, can be rewritten as:

$$\phi_{il} = Ap_{il} - c_{il}$$

where A is defined as:

$$A = \left[.6 \int_0^{100} \gamma(q)dq + \int_{100}^{\infty} (1 + .004(q - 200))\gamma(q)dq \right]$$

Because observing less than 100 centimeters of snowfall a year is a rare event, we can assume that $\int_0^{100} \gamma(q)dq = \Pr(q \leq 100) \sim 0$.[5] It follows that $A \sim 1$ and then that:

$$\phi_{il} \sim p_{il} - c_{il}$$

According to whether it gets the contract or not, the profit of firm i bidding for contract l is thus equal to:

$$\pi_{il} = [p_{il} - c_{il}] * 1_{\{p_i < p_j, j \neq i\}}$$

Assuming that any two bidders with the same cost would submit the same bid, we restrict the analysis to equilibria in which all firms have the same strictly increasing and differentiable strategy $b_l(x)$. Although bidders have the same equilibrium strategy function, they differ by the argument x at which the function is evaluated. Riley and Samuelson [1981] have solved the Bayesian Nash equilibrium of such a game in the context of a sale auction.

Let us compute the Bayesian Nash equilibrium of this game. Bidder i with bidding price $b_l(x)$ wins if and only if all his rivals have a cost above $b_l^{-1}(b_l(x)) = x$. Therefore

[5] We decided to make this assumption rather than estimate the density $\gamma(q)$ because the error due to the omission of climate change in the estimation might have outweighed the gain obtained from a more precise specification of snow conditions.

the probability of winning when bidding the amount $b_l(x)$ is:

$$\begin{aligned}\Pr(i \text{ wins } l) &= \Pr(b_l(x) < b_l(c_{jl}), \forall j \neq i \,|\, z_l) \\ &= \Pr(x < c_{jl}, \forall j \neq i \,|\, z_l) \\ &= S_l^{I_l - 1}(x)\end{aligned}$$

where I_l is the number of players for contract l. This result holds because of the independence of costs and because of the monotonicity of $b_l(\cdot)$. In equilibrium, each player i would want to choose the strategy $b_l(x)$ that maximizes his expected payoff:

$$E\pi(c_{il}, x) = [b_l(x) - c_{il}] * S_l^{I_l - 1}(x)$$

The first-order condition for maximization is:

$$\frac{d}{dx}\left[b_l(x) S_l^{I_l - 1}(x)\right] = c_{il}\frac{d}{dx}\left[S_l^{I_l - 1}(x)\right]$$

By requiring the observations to correspond to a Nash equilibrium, we must have $x = c_{il}$, hence

$$\frac{d}{dc_{il}}\left[b_l(c_{il}) S_l^{I_l - 1}(c_{il})\right] = c_{il}\frac{d}{dc_{il}}\left[S_l^{I_l - 1}(c_{il})\right]. \tag{2}$$

Condition (2) is just one of the conditions necessary for equilibrium. Another necessary condition is that $b_l(\overline{c}_l) - \overline{c}_l$ be nonnegative. Otherwise, a bidder endowed with a cost $\overline{c}_l$ could do better by not participating in the auction. It is also necessary that $b_l(\overline{c}_l) - \overline{c}_l$ be non-positive. Otherwise, when $c_{il} = \overline{c}_l$ a small decrease in the bid from the common strategy $b_l(\overline{c}_l)$ to $b_l(\overline{c}_l) - \varepsilon$, such that $b_l(\overline{c}_l) - \overline{c}_l - \varepsilon > 0$, would raise bidder i's expected payoff from zero (because $S_l^{I_l - 1}(\overline{c}_l) = 0$) to some small positive number (because $S_l^{I_l - 1}(b_l^{-1}(b_l(\overline{c}_l) - \varepsilon)) \neq 0$), and consequently $b_l(\cdot)$ would not be the best strategy. These last two restrictions determine the boundary condition $b_l(\overline{c}_l) = \overline{c}_l$ which implies that the least efficient firm earns zero rent. This condition is not very restrictive insofar as the interval $[\underline{c}_l, \overline{c}_l]$ is common knowledge. Solving the differential equation (2) under this boundary condition, an optimally chosen bid $b_l(c_{il})$ must satisfy:

$$b_l(c_{il}) = c_{il} + \frac{\int_{c_{il}}^{\overline{c}_l} S_l^{I_l-1}(c)dc}{S_l^{I_l-1}(c_{il})}, i = 1, ..., I_l \tag{3}$$

The second-order condition is satisfied because $b_l(\cdot)$ is assumed to be strictly increasing (the proof is in the appendix). Note that this decision rule satisfies the original assumption of an increasing bid function: the lower a contractor's cost, the lower his bid.

The winning bidder is the contractor with the lowest cost $c_{l(1)}$. In choosing his bid, each agent assumes he has the lowest cost. We can show that $b_l(c_{il})$ as defined in (3) is equal to the expected second-lowest cost $c_{l(2)}$ conditional on the bidder's information that his own cost is $c_{l(1)}$. The bidder estimates how far on average the next cost is above his own cost. He then submits a bid that exceeds his own cost by precisely that amount. Hence, on average, the price reached in a first-price sealed-bid procurement auction is the second lowest cost. This is a variant of the well-known result on the revenue equivalence theorem. In a second-price procurement auction, the price exactly equals the cost of the bidder with the second-lowest cost $c_{l(2)}$; in a first-price procurement auction, the same holds in expectation. On average, the two prices in the two auctions are equal. This result is known since Vickrey's (1961) work.

The second term in (3) can be interpreted as the informational rent that accrues to the winning bidder. What is the effect of an increase in the number of competitors on the rent and the winning bid? The more bidders there are, the lower is the informational rent and hence the cost to the municipality. This was first shown by Holt [1979]. What is the effect of an increase in the variance of the cost distribution? The larger the variance, the larger the difference between the lowest cost and the second lowest cost. The economic rent to the winning bidder tends to increase with the variance of the distribution as shown in McAfee and McMillan [1986, 1987]. To illustrate this property, suppose that the costs follow an exponential distribution (with parameter λ_l and variance $\frac{1}{\lambda_l^2}$). The rent is then equal to $\frac{1}{\lambda_l(I_l-1)}$ and is clearly an increasing function of the variance of the distribution.

4 Identification and Estimation of the Structural Model

In this section, we explain how we estimate the theoretical model of section 3, using the method developed for a sale auction by Guerre, Perrigne and Vuong [2000]. The basic idea underlying the structural estimation is the following. Because bids are related to private costs, which are random and distributed as $F_l(\cdot) = F(\cdot|z_l)$, by equation (3) bids are also random and have a distribution, say $G_l(\cdot) = G(\cdot|z_l, i_l)$,where i_l is the actual number of bidders in auction l. Our strategy is to estimate $G_l(\cdot)$ nonparametrically and to retrieve $F_l(\cdot)$. We can then construct a pseudo-sample of bidders' costs, knowing the bid distribution and the observed bids, derive the cost distribution, and compute the informational rents enjoyed by the winning bidders.

As we observe the number of potential bidders I_l (since there is no reserve price before 1990 and the latter appears to be non-binding afterwards), the only unknown structural element of the model is the latent cost distribution $F_l(\cdot)$. Identification of the cost distribution from the bid distribution is not as straightforward: bids are related to costs via the equilibrium strategy (3); both the costs and the equilibrium strategy are linked to the cost distribution. The first theorem in Guerre, Perrigne and Vuong [2000] provides a solution to the identification problem. We adapt their result to the context of a procurement auction.

The result relies upon the fact that the strategy derivative $b_l'(\cdot)$, the cost distribution $F_l(\cdot)$ and the cost density $f_l(\cdot)$ can be eliminated simultaneously from the first-order-condition by introducing the bid distribution $G_l(\cdot)$ and the bid density $g_l(\cdot)$ as follows.

Rewrite the first-order condition (2) so as to obtain:

$$\frac{b'(c_{il})}{f_l(c_{il})}\frac{1-F_l(c_{il})}{(I_l-1)} = b_l(c_{il}) - c_{il}. \tag{4}$$

Introduce the bid distribution $G_l(\cdot)$ given by:

$$\begin{aligned} G_l(p) &= \Pr(b_l(c) \leq p|z_l, i_l) = \Pr(c \leq b_l^{-1}(p)|z_l, i_l) \\ &= F(b_l^{-1}(p)|z_l, i_l) = F(c|z_l) \end{aligned} \tag{5}$$

for all $p \in \left[\underline{p}_l, \overline{c}_l\right]$, where the upper bound comes from the boundary condition. As a matter of fact, the economic model assumes that the private values and the actual number of bidders are independent conditionally on z so that $F(c|z_l, i_l) = F(c|z_l)$. We also introduce the bid density $g_l(\cdot)$ defined by:

$$\begin{aligned} g_l(p) &= \frac{d}{dp}\left[G_l(p)\right] = \frac{d}{dp}\left[F_l(b_l^{-1}(p))\right] \\ &= \frac{1}{b_l'(c)}f_l(b^{-1}(p)) = \frac{1}{b_l'(c)}f_l(c) \end{aligned} \tag{6}$$

for all $p \in \left[\underline{p}_l, \overline{c}_l\right]$.

Introducing the bid distribution $G_l(\cdot)$ and the bid density $g_l(\cdot)$ in (4) using (5) and

(6), we obtain:

$$\frac{1}{(I_l-1)}\frac{1-G_l(p_{il})}{g_l(p_{il})}=p_{il}-c_{il}$$

where $p_{il} = b_l(c_{il})$ is the equilibrium strategy. The strategy derivative $b_l'(\cdot)$, the cost distribution $F_l(\cdot)$ and the cost density $f_l(\cdot)$ have been eliminated simultaneously. The unknown cost is now defined as a function of observable variables: the number of bidders, the bid, and the bid hazard rate $\frac{g_l(p_{il})}{1-G_l(p_{il})}$.

We use, hereafter, the notation $\xi_l(p_{il})$ for the inverse of the bid function defined by:

$$\xi_l(p_{il})=p_{il}-\frac{1}{(I_l-1)}\frac{1-G_l(p_{il})}{g_l(p_{il})} \tag{7}$$

for $p_{il}\in\left[\underline{p}_l,\overline{c}_l\right]$.

Proposition 1 *Let $I \geq 2$. Let $G_l(\cdot)$ be an absolutely continuous distribution defined on the interval $\left[\underline{p}_l,\overline{c}_l\right]$. Then there exists a distribution of bidders' private costs such that $F_l(\cdot)$ is the corresponding distribution of equilibrium bids in a first-price sealed-bid auction with independent private values and a non-binding reserve price (or no reserve price) if and only if: C1: The bids are independent and identically distributed as $G_l(\cdot)$. C2: The function $\xi_l(\cdot)$ defined in (7) is strictly increasing on $\left[\underline{p}_l,\overline{c}_l\right]$ and its inverse is differentiable on $[\underline{c}_l,\overline{c}_l]\equiv\left[\xi_l(\underline{p}_l),\xi_l(\overline{p}_l)\right]$ (with $\overline{p}_l=\overline{c}_l$). Moreover, when $F_l(\cdot)$ exists it is unique with support $[\underline{c}_l,\overline{c}_l]$ and satisfies $F_l(c_{il})=G_l\left(\xi_l^{-1}(c_{il})\right)$ for all $i\in[1,...,I_l]$, $l\in[1,...,L]$ (where L is the number of auctions). In addition, $\xi_l(\cdot)$ is the inverse of the equilibrium strategy $b_l(\cdot):\xi_l(\cdot)=b_l^{-1}(\cdot)$.*

P roof. Essentially adapt the proof of theorem 1 in Guerre, Perrigne and Vuong [2000] to our case. Details of the proof can be provided to the reader upon request. ■

Assuming that the firms behave as predicted by the theory (section 3), proposition 1 establishes that the latent cost distribution $F_l(c_{il})$ is identified from the distribution of the observed bids. To estimate the latter we use a nonparametric statistical method, which avoids picking an arbitrary functional form to describe the distribution. Our estimator is based on the kernel method (see Härdle [1990, 1991], Simonoff [1996] or Yatchew [1998]).

To recover the cost distribution from (7), we first need to estimate the density and cumulative conditional distribution functions of the observed bids. Guerre, Perrigne and Vuong [2000] have shown that the following nonparametric estimator of the conditional density function is optimal for their proposed two-stage estimation method

$$\widehat{g}(p|z_b) = \frac{\frac{1}{Lh_{gzb}h_{gp}} \sum_{l=1}^{L} \frac{1}{I_l} \sum_{i=1}^{I_l} K\left(\frac{z_b - Z_{bl}}{h_{gzb}}\right) K\left(\frac{p - P_{il}}{h_{gp}}\right)}{\frac{1}{Lh_{gzb}} \sum_{l=1}^{L} K\left(\frac{z_b - Z_{bl}}{h_{gzb}}\right)} \tag{8}$$

where $\{(P_{il}, Z_{bl})\}_{l=1,\dots L}^{i=1,\dots,I_l}$ is the sample of independent observations from the distribution of (P, Z_b), L is the number of auctions, I_l is the number of potential bidders in auction l, $K(\cdot)$ is a kernel, and h_{gzb} and h_{gp} are the bandwidths. The characteristics of the auction that we use to estimate the bid density are denoted z_b. This estimator can be rewritten as:

$$\widehat{g}(p|z_b) = \frac{1}{h_{gp}} \sum_{l=1}^{L} \sum_{i=1}^{I_l} w_{il}(z_b) K\left(\frac{p - P_{il}}{h_{gp}}\right)$$

where

$$w_{il}(z_b) = \frac{\frac{1}{I_l} K\left(\frac{z_b - Z_{bl}}{h_{gzb}}\right)}{\sum_{l=1}^{L} K\left(\frac{z_b - Z_{bl}}{h_{gzb}}\right)}.$$

We use the "quartic" or "biweight" Kernel:

$$K(u) = \frac{15}{16}(1 - u^2)^2 I_{\{|u| \leq 1\}}$$

where $I_{\{\cdot\}}$ is the indicator function. As the estimate inherits the properties of the kernel, we have chosen our kernel such that it is positive, defined on a bounded support and differentiable everywhere on the support. Our estimate is therefore a positive function, bounded and differentiable. The choice of a kernel does not really matter for the global accuracy of the estimation so it "should be chosen based on other issues, such as ease of computation or properties of the estimate" (Simonoff [1996]). The bandwidths are determined by the so-called "rule of thumb[6]". They are of order $-1/6$, that is they converge to 0 with a rate $n^{-\frac{1}{6}}$ (where n is the number of observations). We find $h_{gp} = 2.70$ and $h_{gzb} = 1.68$.

The conditional distribution $G(\cdot|\cdot)$ is estimated with the following estimator:

$$\widehat{G}(p|z_b) = \frac{\frac{1}{Lh_G}\sum_{l=1}^{L}\frac{1}{I_l}\sum_{i=1}^{I_l} I_{\{p_{il}\leq p\}} K\left(\frac{z_b - Z_{bl}}{h_G}\right)}{\frac{1}{Lh_G}\sum_{l=1}^{L} K\left(\frac{z_b - Z_{bl}}{h_G}\right)}.$$

This can be rewritten as:

$$\widehat{G}(p|z_b) = \sum_{l=1}^{L}\sum_{i=1}^{I_l} w_{il}(z_b)\, I_{\{p_{il}\leq p\}}$$

$$w_{il}(z_b) = \frac{\frac{1}{I_l}K\left(\frac{z_b - Z_{bl}}{h_G}\right)}{\sum_{l=1}^{L} K\left(\frac{z_b - Z_{bl}}{h_G}\right)}.$$

The bandwidth, set by the rule of thumb, is equal to $h_G = 1.45$. It is of order $-1/5$.

We can now compute the estimated hazard rates,

$$\widehat{\lambda}_l(p_{il}) = \frac{\widehat{g}(p_{il}|z_{bl})}{1 - \widehat{G}(p_{il}|z_{bl})}$$

and inserting those into equation (7) we get the pseudo-costs:

$$\widehat{c}_{il} = p_{il} - \frac{1}{(I_l - 1)}\frac{1}{\widehat{\lambda}_l(p_{il})}. \tag{9}$$

[6] The original rule of thumb was derived assuming that the underlying density was gaussian. But this gaussian reference rule can easily be converted to a rule based on a quartic kernel function: the constant is then 2.78 instead of 1.06. See Simonoff [1996, pages 45-46].

The potential rent for firm i in auction l is given by:

$$\widehat{rent}_{il} = \frac{1}{(I_l - 1)} \frac{1}{\widehat{\lambda}_l(p_{il})}. \tag{10}$$

We evaluate the rent for the winning bids only. As shown by expression (10), the rent decreases with the number of bidders and with the hazard rate. Because the kernel density estimator $\widehat{g}(p|z_b)$ is biased at the boundaries of the support, we have trimmed the estimated private costs that are near the border of the hypercube $\left[\underline{p}(z_{bl}), \overline{p}(z_{bl})\right]$ where $\underline{p}(z_{bl})$ (respectively $\overline{p}(z_{bl})$) is the minimum (respectively maximum) bid for all the values in the bin z_{bl} along a grid of values of z_b. In doing so, we have trimmed 20% of the observations.

The contracts for tender in different parts of the City differ by various characteristics: traffic, road width, territory size, distance to the dump site, and, over time, by the state of the arts in snow removal. The characteristics z_{bl} and z_{cl}, respectively the conditioning variables of the bid hazard rate and the cost distribution, should account for this heterogeneity. As we are restricted regarding the number of variables we can use in the estimation, we have to resort to a variable reduction technique.[7] A principal component analysis is performed to construct the variable z_b. It is a form of projection pursuit where the index is the proportion of total variance accounted for by the projected data. We perform a principal component analysis based on the territory size, the year, and the number of bidders. The territory size captures a possible scale effect, the year is a proxy for the state of technology, and the number of bidders captures the effect of competition, which appears directly in (7). The first principal component, which explains 55% of the

[7] "Although many estimation schemes, including kernel [...] directly generalize to higher dimensions, practical implementation lags behind this theoretical fact. [...] It seems likely that the most useful approach for higher dimensional data is dimension reduction of some sort." (Simonoff [1996]).

total variance, is :

$$z_b = 0.694 v_S + 0.688 v_T + 0.212 v_I \tag{11}$$

where v_S, v_T, v_I are the standardized size, year and number of bidders. As reserve prices generally vary with the characteristics, we do not introduce them explicitly in the vector of contract characteristics. Indeed, the computed correlation for 1990-1998 between z_b and the reserve price turns out to be -0.71. As shown in Figure 1, the average of the characteristic variable exhibits an upward trend over time.

A nonparametric estimate of the cost distribution can be obtained using the pseudo-sample $(\widehat{c}_{il}, z_{cl}), i = 1, ..., I_{Tl}, l = 1, ..., L_T$, where z_{cl} represents the characteristics of auction l that we use to estimate the cost density. If $f(c, z_c)$ denotes the joint density of (C, Z_c) and $f_z(z_c)$ denotes the marginal density of Z_c, then the conditional cost density estimator is given by:

$$\widehat{f}(c|z_c) = \frac{\frac{1}{Lh_{fzc}h_{fc}} \sum_{l=1}^{L_T} \frac{1}{I_l} \sum_{i=1}^{I_{lT}} K\left(\frac{z_c - Z_{cl}}{h_{fzc}}\right) K\left(\frac{c - c_{il}}{h_{fc}}\right)}{\frac{1}{Lh_{fzc}} \sum_{l=1}^{L_T} K\left(\frac{z_c - Z_{cl}}{h_{fzc}}\right)} \tag{12}$$

where L_T and I_{Tl} represent respectively the number of remaining auctions and bids after trimming. Using the previously defined "quartic" kernel and the same 'rule of thumb' as above, we find $h_{fc} = 2.78$ and $h_{fzc} = 1.70$.

The characteristics affecting the distribution of costs are the same as the characteristics affecting the price distribution, except that the number of bidders does not affect costs. Thus, for the estimation of the conditional cost density function, we use the first principal component:

$$z_c = .71(v_S + v_T) \tag{13}$$

which explains 81% of the total variance of size and trend. Its evolution is shown in

Figure 1.

5 Results

We apply the nonparametric estimation method outlined in the preceding section to our auction model and the 666 observed bids for snow removal contracts with the City of Montreal. First, we present the estimated conditional bid density functions and the inferred conditional cost distributions of the private contractors. Second, we discuss economies of scale and their exploitation by the City. Finally, we turn to competition and the implicit informational rents earned by the private contractors. In each case, we confront our results with the implications of the theoretical model, and we evaluate the actions taken by the City from that perspective.

5.1 Estimated Bid Density and Cost Density Functions

We begin with the presentation of the estimated bid densities conditional on various levels of the characteristic z_b. Figure 2 shows a selected number of estimated bid densities $g(\cdot|z_b)$ plotted side by side for a range of equidistant values of z_b[8]. The conditional density is unimodal and the mode appears to decrease monotonically with z_b. This result indicates that bids decrease with territory size, technological progress, and competition, since each of these variables is positively related to z_b. In addition, it is apparent that the variance of the bids decreases as z_b increases.

The preceding results allow us to construct the pseudo-sample of costs, using (9). We find that, for a given z_{cl}, the estimated cost schedule $\xi_l(\cdot)$ is indeed an increasing

[8] The values of z_b are chosen along a grid of 100 values constructed on $[\underline{z_b}, \overline{z_b}]$. We have chosen the: 5th (Q5), 20th (Q20), 35th (Q35), 50th (Q50), 65th (Q65), 80th (Q80) and 95th (Q95) values which correspond respectively to the following values of z_b : -2.16, -1.35, -0.54, 0.27, 1.09, 1.90 and 2.71.

function of the bids, as required by proposition 1[9]. The conditional cost density functions presented in Figure 3 have the same shape as the conditional bid density functions. Except for the fact that it is not affected by the number of bidders, the characteristic z_c[10] has the same interpretation as z_b: it increases with time (the state of technology) and with territory size. Thus, Figure 3 shows that an increase in territory size (everything else equal) leads to a reduction in the mean and the variance of the cost of snow removal by private contractors. This may signal the presence of unexploited economies of scale.

In Figure 4, we present a scatter plot of the costs for the different values of z_c, as well as a polynomial fit which highlights how the derivative of costs with respect to z_c varies with z_c. Since high values of z_c correspond to more recent auctions and larger territory sizes, it appears that unexploited economies of scale were present at the beginning of the sample period, but may be insignificant at the end.

5.2 Economies of Scale: Further Evidence from a Parametric Price Equation

The polynomial fit of Figure 4 is a short step away from an estimated parametric model. Despite the rationale for our use of nonparametric estimation, a parametric model is more readily interpretable quantitatively. We present such a model as a way to further document the above evidence.

To avoid the econometric difficulties associated with the regression of an estimated variable (unit cost) on explanatory variables, we estimate a bid equation rather than a

[9] Graphs of the $\xi_l(\cdot)$ functions are available upon request.

[10] The values of z_c are chosen along a grid of 100 values constructed on $[\underline{z_c}, \overline{z_c}]$. We have chosen the: 5th (Q5), 20th (Q20), 35th (Q35), 50th (Q50), 65th (Q65), 80th (Q80) and 95th (Q95) values which correspond respectively to the following values of z_c : -2.10, -1.39, -0.68, .03, 0.73, 1.44 and 2.15.

cost equation (standard deviations are between parentheses):

$$p = \underset{(0.98)}{13.46} \quad \underset{(0.08)}{-1.97 z_b} \quad \underset{(0.04)}{+0.4 z_b^2} \quad \underset{(0.28)}{-1.77 less25} \quad +b_5 dum \tag{14}$$

$$n = 666,\ DF = 596,\ \overline{R}^2 = 0.77.$$

Besides z_b, the explanatory variables include $less25$, a dummy variable which equals unity when the territory put up for tender is shorter than 25 kilometers long, and zero otherwise; and a vector of 66 firm dummies, dum, which controls for unobserved firm-idiosyncratic influences on the bids. Only three dummies are significant at the 5% level, which is consistent with the fact that (at least) the means of the conditional bid distributions $g(\cdot|z_{bl})$ from which the bids are drawn are the same from one observation to the next.[11]

Using the decomposition of z_b into the length of the territory, the year and the number of bidders we can compute the impact of each of these variables on the bid. We find that, everything else equal, each additional kilometer of territory reduces the bid by 1.5%, that the impact of technological progress is 3% a year and that each additional bidder lowers the bid by 1.3%. We get similar results if we run the regression on the winning bids only. Considering that the mean territory size increased by 8 kilometers between 1986-88 and 1997-98, and that the smallest territory (auctioned in 1986) was 27 kilometers shorter than the largest one (auctioned in 1996), the price impact of changes in territory size has been considerable.

However, the evidence also suggests that economies of scale were completely exploited by the end of the sample period. At the maximum value of z_b, 2.98, $\partial p/\partial z_b$ is in fact

[11] This particular parametric equation was chosen for its consistency (in terms of explanatory variables) with the nonparametric model. We have also run some regressions where the components of z_b are included as independent variables. The results are qualitatively similar.

positive.

5.3 Competition and Rents

By increasing the territory size, the City has decreased the number of contracts put up for auction. Such a policy may have reduced the number of bidders per contract, if it became harder to qualify, or increased that number, if many suppliers had to fight for fewer contracts.

It may have been unwise to increase the territory size if cost reductions were offset by increased rents due to lower competition. Such a possibility would not be incompatible with the combined evidence from Figure 4, Figure 5 and equation (14): for high values of z cost may decrease and yet price goes up.

It is, however, difficult to corroborate such an interpretation. First, if we regress the number of bidders on territory size and time, we find that the number of bidders tends to increase with territory size. The low R^2 of 0.07 suggests though that other factors explain most of the variation in the number of bidders.[12]

Second, as shown in Figure 5, the informational rent[13] earned by a contract winner is very stable at around 70 cents per meter of snow cleared over the winter on average, or about 7% of the winner's cost. Therefore, the cost of asymmetric information turns out not to be excessive and its evolution is no cause for concern.

In fact, the winner's rent is not only affected by competition, but also by differences between participants. Theory predicts that the rent decreases with the number of bidders

[12] In particular, the average number of bidders per contract tends to be higher for contracts starting in 1987, 1992 and 1997. Since contracts have a five-year duration, this can be explained by cohort effects, as the firms that won contracts in 1987 appeared again in 1992 and 1997.

[13] Figure 5 shows average winner's bids and estimated winner's costs over the sample period. The informational rent is the area between these two curves.

and increases with the bidders' cost variance. The correlation between the estimated rent and the variance of cost is positive (0.21) and the correlation between the estimated rent and the number of bidders is negative (-0.54); both signs are consistent with the theoretical model.

Table 3 gives the average of the winner's informational rent in different periods, along with a number of auction characteristics. The data has been grouped into four periods of three years each, except for the last period which includes only two years because it coincides with the end of the sample.[14] The average informational rent was slightly larger for the period 1993-1996, when the average number of bidders was lower and the mean territory size was higher. However, in 1997-1998 territory size was hardly lower and the number of bidders more than recovered, so that the rent was back to its stable level of .7$/m.

The variance of cost experiences a substantial drop over the period. Although several factor may explain this change, City officials explain that there is a threshold at about 25 kilometers: territories smaller than 25 km can be serviced with old, less specialized, equipment whereas larger territories are typically serviced with more standardized equipment. They claim convincingly that the elimination of small territories has created a more homogeneous pool of bidders. Indeed, Table 3 indicates that the variance was much lower once territories of less than 25 km no longer existed, and equation (14) indicates that this dummy variable had a significantly negative effect on price (i.e. on cost, rent, or both).

[14] As already mentioned, for historical reasons, no contracts were offered in 1994, which explains why the 1993-1996 period is indeed a three year period contract.

6 Conclusion

We have evaluated the performance of snow removal procurement from data on bids, the number of bidders and some of the characteristics of 666 contracts offered for tender by the City of Montreal between 1986 and 1998. Using nonparametric estimation techniques adapted from Guerre *et al.* [2000], we were able to compute bid density functions conditional on contract characteristics, and to infer the cost distributions of the firms that do contractual work for the City, as well as the informational rents they earn. Both the bid and the cost densities are unimodal, and their means and variances depend on contract characteristics.

Bids and costs have decreased over the sample period, while informational rents remained stable. Our results were shown to be compatible with standard received theory of competitive auctions: there is a positive correlation between costs and bids; rents increase with the variance of costs and decrease with the number of bidders.

Both the model and the techniques used to obtain the results rely on the assumption of perfect competition. We did not adopt this assumption without some strong *a priori* indications that it was a reasonable approximation of reality. We discussed the issue with City officials and private contractors who, respectively, congratulated themselves and deplored that competition was strong. Moreover, the City of Montréal (with a population of one million) is only one of several municipalities in an urban community of three million people, that use the services of private entrepreneurs for snow removal. The reservoir of entrepreneurs is even larger as their activities are not limited to snow removal. Firms as different as paving contractors, landscape contractors, lawn mowers, excavating and building firms can provide snow removal services during the winter season by making minor additions to their equipment. Finally, the data indicates that the number of bidders was consistently at around seven or eight, in most auctions. Our results tend to confirm this *a priori* information: rents left in the hands of the winning bidders were found to be consistently low over the sample period.

Although working hypotheses can always be challenged, we feel confident that our investigation of policy issues is well-grounded. The evidence is that the City has effectively exploited the opportunities it had to reduce the cost of snow removal services. The bids for, and the underlying costs of, snow removal have declined over the period 1986 to 1998. It appears that the City was not passive in that process. Within the framework of competitive supply auctions, there are at least three basic fashions in which it could reduce the amount the taxpayer had to pay for snow removal services. First, it could organize the work in such a way as to reduce the cost of performing the service. The evidence suggests that unexploited economies of scale were present at the beginning of our sample period, and that the territory size was increased so that no significant opportunities remained at the end of the sample period. Second, it could define the contracts so as to make the pool of bidders more homogeneous, thus reducing cost variance and cutting the rent left in the hands of the winner. There is some evidence that the cost variance was reduced over the sample period by making the territories more homogeneous. Third, it had to ensure that competition was maintained or enhanced. We have investigated potential adverse effects of the deliberate policy to increase the size of the territories, only to find that no threatening effect on competition could be identified.

Several other aspects of the City's procurement policies would be worth investigating. There is anecdotal evidence that territory location might be an important characteristic.

The distances between snow dumps and territories, and between the contractor's main location and its contract territory might be sufficiently different in various parts of the city to affect procurement outcomes. It is also possible that the individual costs of private contractors are not entirely independent but might have a common component due to meteorological forecasts or other factors. By stipulating how weather risks are shared by the City and the contractors, the City might influence the bidding. Finally, the actual and potential role of the reserve price might deserve further investigation.

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References

Donald, S.G., Paarsch, H.J., 1993. Piecewise Maximum Likelihood Estimation in Empirical Models of Auctions. International Economic Review 34, 121-148.

Donald, S.G., Paarsch, H.J., 1996. Identification, Estimation, and Testing in Parametric Empirical Models of Auctions within the Independent Private Value Paradigm. Econometric Theory 12, 517-567.

Elyakime, B., Laffont, J.J., Loisel, P., Vuong, Q., 1994. First-Price Sealed-Bid Auctions with Secret Reservation Prices. Annales d'Economie et de Statistique 34, 115-141.

Florens, J.P., Hugo, M.A., Richard, J.F., 1997. Game Theory Econometric Models: Application to Procurements in the Space Industry. European Economic Review 41, 951-959.

Guerre, E., Perrigne, I., Vuong, Q., 2000. Optimal Nonparametric Estimation of First-Price Auctions, Econometrica 68(3), 525-574.

Härdle, W., 1991. Smoothing Techniques with Implementation in S, Springer Series in Statistics, New York, Springer-Verlag.

Härdle, W., 1990. Applied Nonparametric Regression, Econometric Society Monographs, Cambridge, Cambridge University Press.

Hassani, S., Sarda, P. Vieu, P., 1986. Approche non paramétrique en théorie de la fiabilité: revue bibliographique. Revue de Statistiques Appliquées 35(4), 27-41.

Holt, C., 1979. Uncertainty and the Bidding for Incentive Contracts. American Economic Review 69(4), 697-705.

Laffont, J.J., Ossard, H., Vuong, Q., 1995. Econometrics of First-Price Auctions. Econometrica 63, 953-980.

Laffont, J.J. Vuong, Q., 1993. Structural Econometric Analysis of Descending Auctions. European Economic Review 37, 329-341.

McAfee, R. P., McMillan, J., 1986. Bidding for Contracts: A Principal-Agent Analysis. Rand Journal of Economics 17(3), 326-338.

McAfee, R. P., McMillan, J., 1987. Auctions and Bidding. Journal of Economic Literature 25, 699-738.

Riley, J. G., Samuelson, W. F., 1981. Optimal Auctions. American Economic Review 71(3), 381-92.

Simonoff, J. S., 1996. Smoothing Methods in Statistics. Springer Series in Statistics, New York, Springer-Verlag.

Vickrey, W., 1961. Counterspeculation, Auctions, and Competitive Sealed Tenders. Journal of Finance 16(1), 8-37.

Yatchew, A., 1998. Nonparametric Regression Techniques in Economics. Journal of Economic Literature 36, 669-721.

Appendix

Second-Order Condition

We prove that the second-order condition for the maximization of profit holds. The expected profit of a firm i is:

$$E\pi_l(c_{il}, x) = [b_l(x) - c_{il}]\, S_l^{I_l - 1}(x).$$

We define: $H_l(x) = S_l^{I_l - 1}(x)$. The first-order condition is:

$$\frac{\partial E\pi_l(c_{il}, x)}{\partial x} = \frac{d}{dx}\left[b_l(x) H_l(x)\right] - c_{il} H_l'(x) = 0.$$

To prove the second-order condition we have to show that:

$$\frac{\partial^2 E\pi_l(c_{il}, x)}{\partial x^2} = \frac{d^2}{dx^2}\left[b_l(x) H_l(x)\right] - c_{il} H_l''(x) \leq 0. \tag{15}$$

By the property of a Nash equilibrium, $x = c_{il}$, and hence

$$\frac{d}{dc_{il}}\left[b_l(c_{il}) H_l(c_{il})\right] - c_{il} H_l'(c_{il}) = 0. \tag{16}$$

Let us take the derivative of (16):

$$\frac{d^2}{dc_{il}^2}\left[b_l(c_{il}) H_l(c_{il})\right] - c_{il} H_l''(c_{il}) = H_l'(c_{il}).$$

The local sufficient condition (15) implies, at the equilibrium where $x = c_{il}$:

$$\frac{d^2}{dc_{il}^2}\left[b_l(c_{il}) H_l(c_{il})\right] - c_{il} H_l''(c_{il}) \leq 0.$$

Therefore we need to show that $H_l'(c_{il})$ is negative. We must have:

$$H_l'(c_{il}) = (I_l - 1) S_l^{I_l - 2}(c_{il}) s_l(c_{il}) \tag{17}$$

$$= -(I_l - 1)\left[1 - F_l(c_{il})\right]^{I_l - 2} f_l(c_{il}) \leq 0$$

Relation (17) holds because $1 - F_l(c_{il})$ and $f_l(c_{il})$ are positive. The local sufficiency condition is therefore proved. As the equilibrium strategy admits only one solution, the global sufficiency is verified.

Table 1: Auction Data Summary

Year	Average of All Bids ($/m)	Average Winning Bid ($/m)	Average Territory Size (m)	Number of Contracts	Number of Bidders	Average Number of Bidders
1986	17.06	15.73	24,502	18	125	6.94
1987	15.84	13.90	32,552	6	51	8.50
1988	14.61	13.13	29,994	5	33	6.60
1989	–[15]	–	–	–	–	–
1990	13.52	12.11	31,920	8	42	5.25
1991	12.64	11.57	31,114	15	133	8.87
1992	11.62	11.05	34,507	6	49	8.17
1993	11.51	10.13	32,477	3	20	6.67
1994	–[15]	–	–	–	–	–
1995	11.53	10.32	38,940	8	56	7.00
1996	10.49	10.01	37,534	12	72	6.00
1997	10.12	9.37	36,173	6	65	10.83
1998	10.07	9.46	32,137	3	20	6.67

Table 2: Overall Statistics

Variables	Mean	Std Error	Min.	Max.	# of Obs.
Bids (in $/m)	13.11	2.87	8.40	25.59	666
Winning Bids (in $/m)	11.93	2.40	8.40	18.92	90
# of Bidders per Auction	8	2.24	2	14	90

[15] No auction was organized in 1989 and 1994.

Table 3: Chronological Evolution of The Variables Under Study

	1986-1988	1990-1992	1993-1996	1997-1998
Mean of Winner's Bid ($\$/m$)	15.0	11.7	10.2	9.6
Mean of Winner's Cost ($\$/m$)	14.3	11.0	9.4	8.9
Mean of Winner's Rent ($\$/m$)	0.7	0.7	0.8	0.7
Mean of Number of Bidders	7.3	7.8	6.4	9.5
Variance of Costs	5.6	1.9	1.2	0.5
Mean of Size of Territory (km)	27.1	32.0	37.4	34.8
Contracts ($\lesseqgtr 25km$)	$\lesseqgtr 25km$	$\lesseqgtr 25km$	$> 25km$	$> 25km$

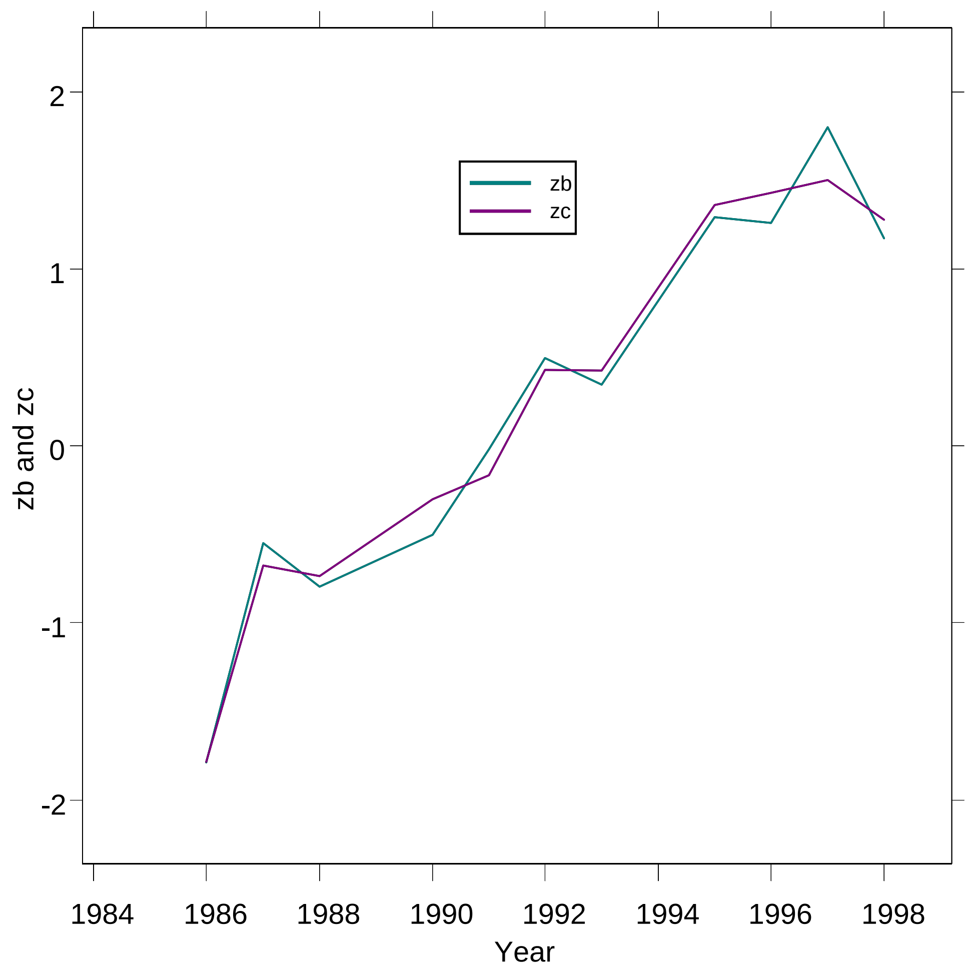

Figure 1: Evolution of the Characteristics z_b and z_c Used to Estimate, Respectively, the Conditional Bid Functions and the Conditional Cost Functions

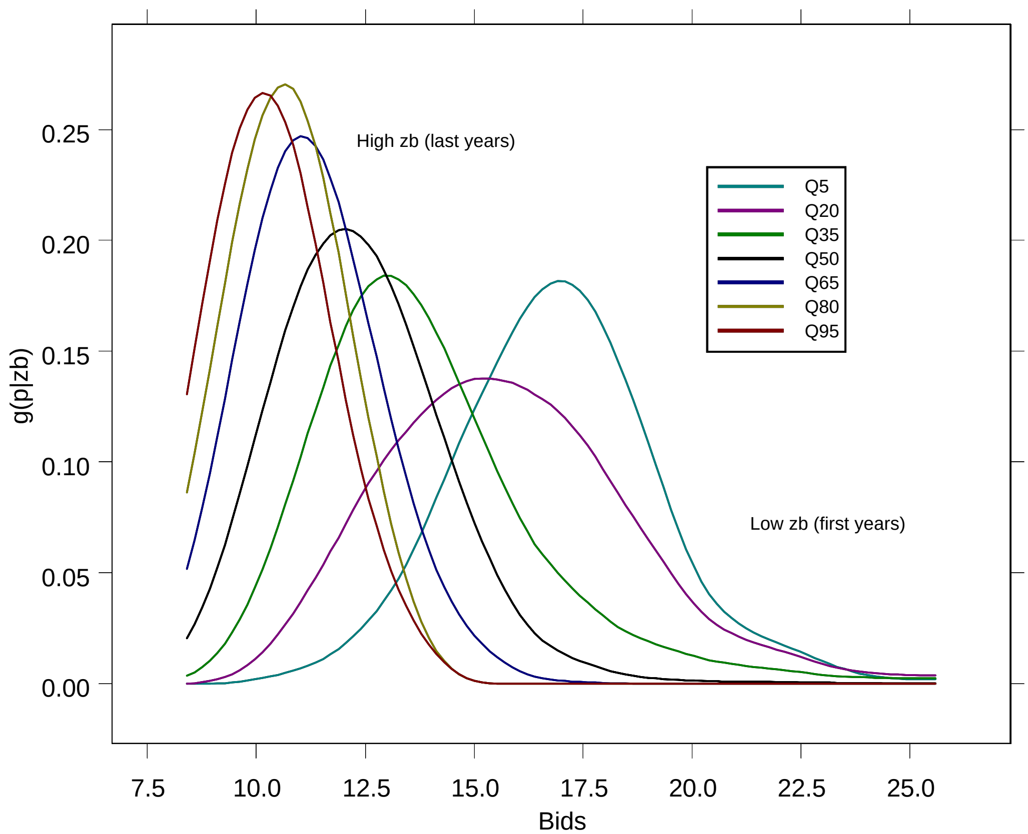

Figure 2: Bid Density Functions Conditional on Size, Year, and Number of Bidders

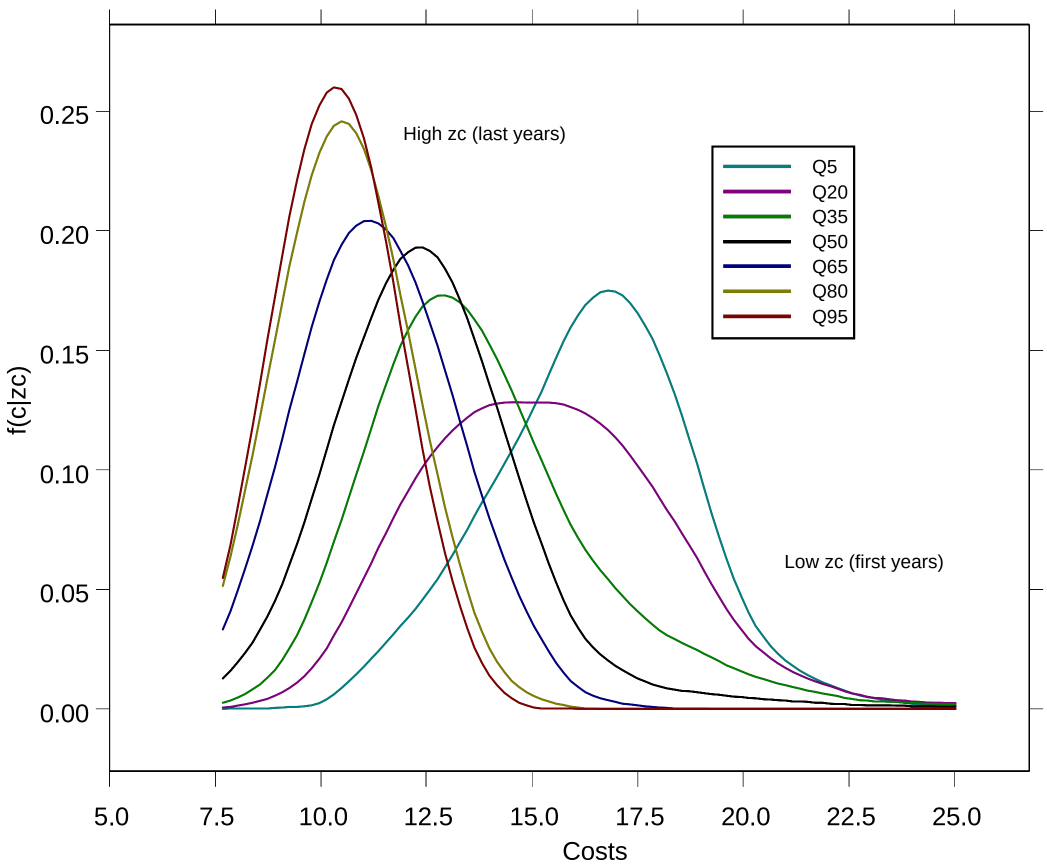

Figure 3: Cost Density Functions Conditional on Size and Year

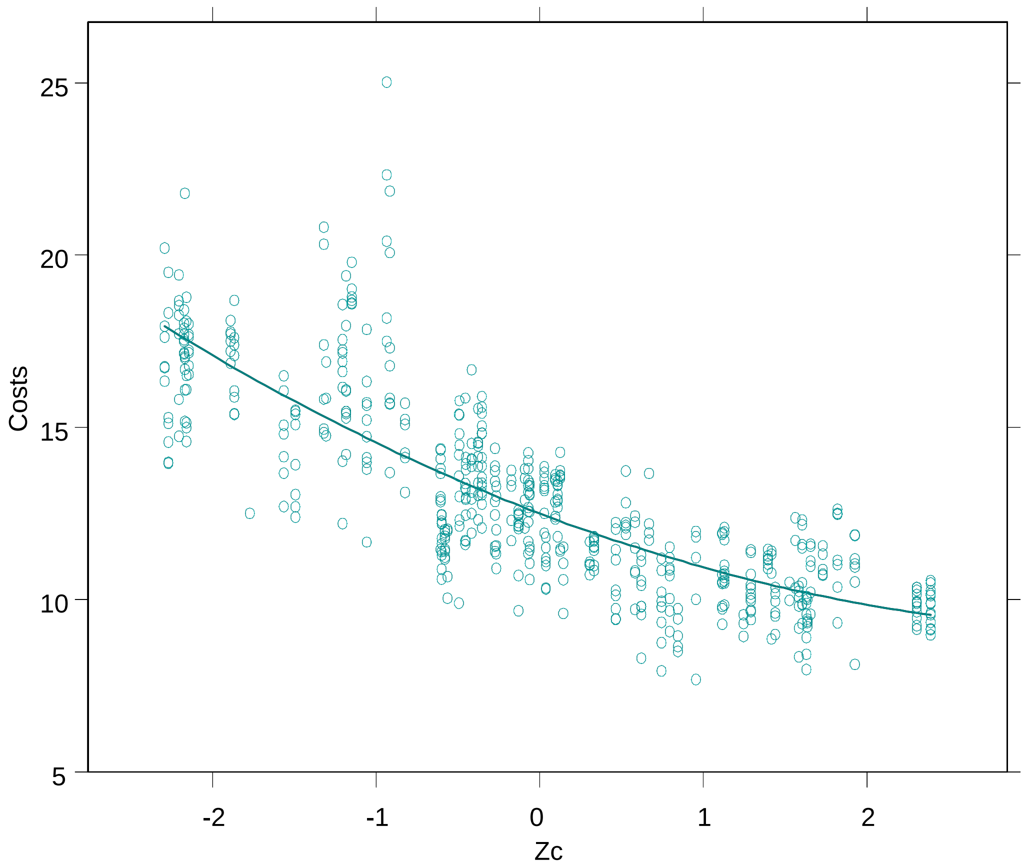

Figure 4: Unit Costs and Contract Characteristics: Unexploited Economies of Scale Diminish

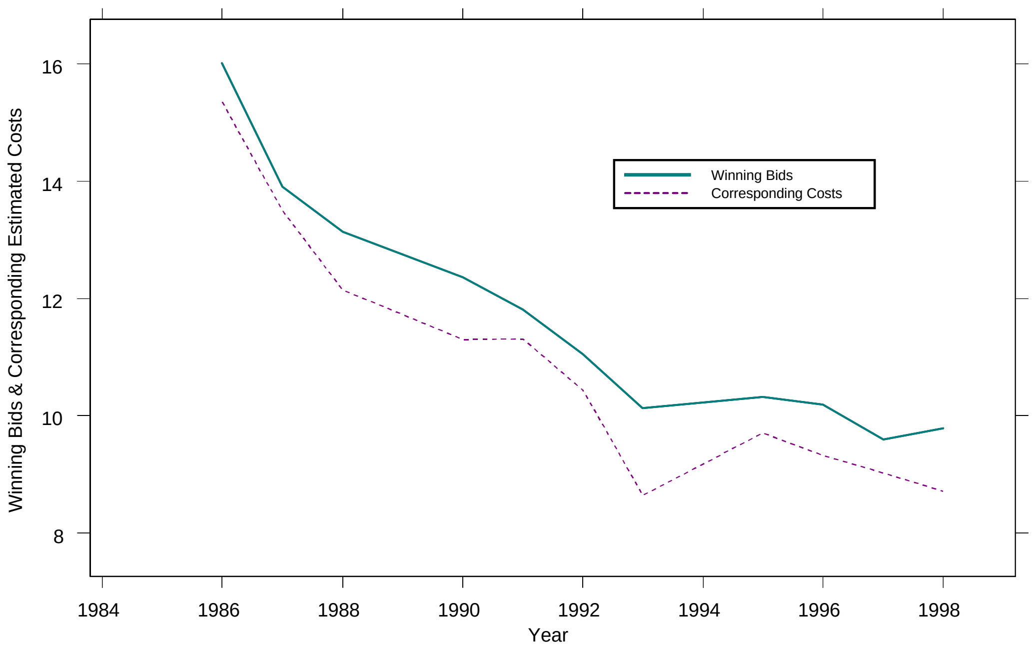

Figure 5: Winner's Average Bids and Estimated Private Cost