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Market Integration, Matching, and Wages*

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Résumé:
Lorsqu'il est coûteux pour les agents économiques de trouver un partenaire d'échange, le fait d'intégrer de petits marchés dans un plus grand augmente les difficultés d'appariement. Nous examinons dans quelle mesure les nombre d'appariements dépend de la taille du marché en modélisant explicitement les tentatives de la firme d'attirer des travailleurs en affichant des salaires. Nous montrons que l'intégration réduit le pouvoir de marché des agents sur le côté le moins saturé du marché. Ainsi, s'il y a au moins autant de travailleurs que de postes, l'intégration des marchés augmente les salaires; s'il y en a beaucoup moins, l'intégration réduit les salaires. Ceci est le cas même si le ratio travailleurs/postes reste inchangé. Indépendamment de la réaction des salaires, l'intégration des marchés réduit le bien-être social lorsque chacun est pondéré uniformément et lorsque d'autres bienfaits de l'intégration comme la meilleure qualité des appariements sont absents. Nous caractérisons la limite supérieure des pertes de bien-être résultant de difficulte's plus élevées d'appariement et montrons que la perte marginale de bien-être décroît lorsque le marché s'intègre de plus en plus.

Abstract:
When it is costly for agents to find a match, integrating small markets into a large one increases the matching difficulty. We examine such dependence of the number of matches on the market size by explicitly modelling firms' attempt to attract workers by posting wages. It is shown that integration reduces the relative market power of agents on the much shorter side of the market. Thus, if there are at least as many workers as jobs, integrating markets increases wages; if there are much fewer workers than jobs, integration reduces wages. This is the case even though integration does not change the worker/job ratio in the market. Regardless of the wage response, market integration reduces social welfare when everyone is weighted equally and when other benefits of integration such as improved match qualities are absent. We characterize the upper bound on the welfare loss from increased matching difficulty and show that the marginal welfare loss shrinks as the market becomes increasingly integrated.

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1 Introduction

Market integration can bring many benefits to market participants. This is particularly true where participants have private information regarding their own valuations or costs, or when workers differ in productivity. For these markets integration can improve match qualities and increase welfare. Such benefits are well known. What is not carefully examined is that market integration calls for extensive coordination among agents and, when such coordination is lacking, market integration increases the matching difficulty between the two sides of the market. In this paper we characterize the strategic interactions among agents that lead to the matching difficulty and examine how market integration affects wages and welfare through this endogenous matching process. For this focus, we will assume that match qualities are homogeneous and perfectly observable.

The importance of the matching difficulty arises in a number of situations. For instance, when a planned economy makes a transition to a market economy, workers are given the freedom to look for jobs in a larger labour market than before. Even though the transition allows workers to switch from initially less productive matches to more productive ones, it is observed that the transition often leads to increased unemployment. To measure the welfare gain from the transition, it is then necessary to know how the transition changes unemployment and wages in addition to the effect on matching quality and productivity.

To analyze how the number of matches and wages depend on the size of the market, one needs a theory that explicitly models how matches are formed and affected by firms' wage decisions. For this purpose the Walrasian theory where markets are cleared at every instant is of little help. The search theory by Diamond (1984), Mortensen (1982), and Pissarides (1990) is capable of generating persistent unemployment but the exogenous matching function in that theory makes it difficult to address how agents might try to set prices to change the number of matches. In this framework, whatever effect the market size has on the number of matches is exogenously determined the moment the matching function is specified. In particular, Diamond (1984) uses a matching function that exhibits increasing returns to scale to justify a positive "thick market externality". It is not clear whether agents' strategic play could deliver a matching function that has this feature.

To capture the endogenous response of matches to market integration, we adopt the framework of Peters (1991) and Montgomery (1991). This framework is described for a labour market in the next section. The distinct feature is that firms can post wages to direct the search decision of workers, who make a trade-off between the wage and the probability of getting a match.[1] Besides the difference in focus, our contribution to this framework is to examine the equilibrium in finite sized markets. In contrast, Peters focuses on infinite markets and Montgomery approximates the finite equilibrium (discussed later). An exact characterization of the finite market equilibrium is necessary for examining how the equilibrium changes with the size of the market.

It is shown that market integration unambiguously reduces the aggregate number of matches by endogenously increasing the match uncertainty that each firm faces. Although firms do compete in posting wages to attract workers, wages do not always rise after integration. If there are as many workers as jobs, integration raises wages, but if there are much fewer workers than jobs, integration reduces wages. This is the case even though integration does not change the worker/job ratio in the market. Depending on the wage response, one side or both sides of the market may be worse off after integration. Regardless of the wage response, market integration reduces social welfare if all agents are weighted equally.

Despite this welfare result, it is not our intention to claim that market integration is always bad for the society: The special setup abstracts from any possible match quality improvements generated by market integration. We merely want to bring attention to the much neglected fact that a larger market increases the search cost, even though agents on one side of the market can try to direct the search decisions of agents on the other side of the market. Whatever

[1] Carlton (1978) seems to be the first one to formally analyze the trade-off in the goods market between price and service probability. Rather than generating this relationship endogenously by agents' strategic behavior, he exogenously assumes that each buyer has a smooth preference ordering over the pair of price and service probability. As discussed before, the endogeneity of such relationship is critical for the issue examined here.

benefits that market integration might have must outweigh this negative effect in order to be socially desirable. With this in mind, we calculate the upper bound on the social welfare loss from increased matching difficulty and show that the marginal welfare loss falls as markets get increasingly integrated.

This paper is related to but different from the search literature surveyed by McMillan and Rothschild (1994). An important feature of the model here is that firms can direct workers' search by posting wages. In contrast, models in the search literature typically assume that workers know only the distribution of wages before search and learn about a particular firm's wage after visiting the firm. This setup is not capable of capturing the trade-off between a wage and the match probability. Another related work is by Julien, et al. (1998), who allow firms to use reserve wages rather than actual wages to attract workers. Although they also analyze the coordination problem, they do not focus on the effects of market integration.

The remainder of this paper is organized as follows. Section 2 gives two examples to illustrate why an integrated market might have an increased matching difficulty. Section 3 extends the analysis. Section 4 derives the limit result where the market gets arbitrarily large and provides the upper bound on the welfare cost of market integration. Section 5 concludes the analysis and the appendix provides necessary proofs.

2 Examples

2.1 Example 1: Two Workers and Two Firms

Consider a labour market with two workers and two firms. Workers and firms are both risk-neutral. Throughout this paper, the generic index is i for workers and j for firms. The workers, termed worker 1 and worker 2, are identical in all aspects, each wanting to work for one job for an indivisible amount of time. The utility cost of time is normalized to zero. The firms, termed firm A and firm B, are also identical, each having one job to offer. The output of a worker is normalized to one.

Like a typical search model (e.g., Mortensen (1982)), the market here differs from a Walrasian market in two aspects. First, it is more costly for a worker to seek for offers from multiple firms than from a single firm. We use the extreme form of this assumption that each worker can seek for only one job at a time.[2] Second, agents cannot coordinate their decisions. This assumption may not be reasonable for the current example with only four agents but our intention is to use this example to illustrate the forces that are important for a large market, where the assumption of uncoordinated decisions is reasonable. These assumptions imply that agents may face uncertainty: it is possible that both workers end up with the same firm, in which case one of the workers fails to obtain the job and one of the firms fails to hire a worker.

In contrast to a standard search model, we do not assume that matches are dictated by an exogenous matching function. Instead, each firm actively posts a wage to attract workers, taking the other firm's wage offer as given, and each worker decides which firm to apply to after observing all posted wages. Assuming that agents can affect their own matches by posting wages seems realistic. To focus on the central issue of how firms use wages to mitigate their matching difficulty, we abstract from all other uncertainty and other transaction costs in the market.

To simplify the analysis further, we assume that the time horizon is one period. In Section 5, we will argue that the qualitative results are valid also for an infinite horizon, as long as the turnover is large in the market.

To begin, suppose that the market is originally separated into two sub-markets, each having one worker and one firm. Agents in one sub-market cannot transact in the other sub-market. Let w_j be the wage posted by firm j $(= A, B)$. Since there is only one firm in each sub-market and firms move first, it is clear that each firm posts the bottom wage, i.e., $w_A = w_B = 0$, and each worker applies to the only firm in his sub-market with probability one.[3] Firms' expected profit, denoted F, is the highest, $F_A = F_B = 1$; workers' expected surplus, denoted U, is the lowest, $U_1 = U_2 = 0$. Let us measure social welfare by the ex ante measure V, which gives the same

[2]Lang (1991) allows workers to have two job offers at a time. This generates wage dispersion among homogeneous firms, which is not the focus here.

[3]Workers applies to the corresponding firm with probability one despite a zero wage because the firms can post a positive wage arbitrarily close to 0 to induce a positive surplus for the workers. The equilibrium is the limit outcome when the wage approaches zero.

weight for each agent in the economy. Then the social welfare level is $V = 1/2$.

Now the two sub-markets are integrated into one so that each worker can apply to either of the two firms. The agents' actions are as follows. At the beginning of the period, each firm j posts a wage w_j, taking the other's wage as given. Observing all wages, each worker i $(= 1, 2)$ chooses a probability α_{Ai} to apply to firm A and a probability $\alpha_{Bi} = 1 - \alpha_{Ai}$ to apply to firm B, taking the other worker's strategy as given. If both workers end up with the same firm, the firm selects one of the two for the job, each with probability $1/2$, at the posted wage.[4]

Let us examine the strategy of worker i $(= 1, 2)$. If he applies to firm j, he does not get the job if and only if the other worker i' $(\neq i)$ also applies to the same firm and is chosen by the firm, the probability of which is $\alpha_{ji'}/2$. Thus, worker i's expected utility from applying to firm j is $(1 - \alpha_{ji'}/2)w_j$. Denote $W \equiv (w_A, w_B)$. Worker i's strategy is

$$\alpha_{Ai}(W) \begin{cases} = 1, & \text{if } (1 - \frac{\alpha_{Ai'}}{2})w_A > (1 - \frac{\alpha_{Bi'}}{2})w_B; \\ = 0, & \text{if } (1 - \frac{\alpha_{Ai'}}{2})w_A < (1 - \frac{\alpha_{Bi'}}{2})w_B; \\ \in [0, 1], & \text{if } (1 - \frac{\alpha_{Ai'}}{2})w_A = (1 - \frac{\alpha_{Bi'}}{2})w_B. \end{cases} \tag{1}$$

Now consider the firms. For firm j $(= A, B)$, the vacancy is filled if there is at least one worker applying to the firm, i.e., if the two workers do not both apply to the other firm j' $(\neq j)$. Since the probability that both workers apply to firm j' is $\alpha_{j'1}\alpha_{j'2}$, the expected profit of firm j is

$$F_j = (1 - \alpha_{j'1}\alpha_{j'2})(1 - w_j), \qquad j = A, B;\ j' \neq j.$$

An equilibrium consists of wages $W = (w_A, w_B)$ and workers' strategies $(\alpha_{j1}(W), \alpha_{j2}(W))_{j=A,B}$, with $\alpha_{j'i}(W) = 1 - \alpha_{ji}(W)$, such that (i) given wages and the other worker's strategy, each worker i's strategy $(\alpha_{ji}(W))_{j=1,2}$ maximizes his expected utility, and (ii) given $(\alpha_{j1}(W), \alpha_{j2}(W))_{j=A,B}$ and the other firm's wage, each firm posts a wage to maximize his expected profit. The important feature of the equilibrium is that each firm can choose a wage to influence workers' strategies and hence changes the number of matches he/she gets.

[4]The assumption that a firm can commit to the posted wage even when there are two workers applying to him is debatable, but it is not essential. One can instead allow firms to announce reserve wages and hold an auction after workers have applied, as in Julien, et al. (1998). This alternative mechanism generates dispersion in actual wages, which only complicates the analysis.

Let us examine the equilibrium where both workers use mixed strategies, i.e., $\alpha_{ji}(W) \in (0,1)$ for $j = A, B$ and $i = 1, 2$, leaving the discussion on pure strategies to the end of this sub-section. The following lemma can be established:

Lemma 1 *If* $\alpha_{ji}(W) \in (0,1)$ *for* $j = A, B$ *and* $i = 1, 2$, *then* $w_A > 0$, $w_B > 0$, $\alpha_{A1} = \alpha_{A2}$ *and* $\alpha_{B1} = \alpha_{B2}$.

Proof. First, let us show $w_A > 0$ and $w_B > 0$. Suppose, to the contrary, that at least one firm posts the bottom wage $w = 0$. Let this firm be firm B. If $w_A > 0$, both workers will choose firm A with probability one, since applying to firm B obtains zero surplus while applying to firm A obtains an expected surplus no less than $w_A/2 > 0$. In this case, workers will not mix between the two firms, contradicting the assumption of mixed strategies. If $w_A = 0$, it is profitable for firm A to increase the wage to $w_A = \varepsilon$, where ε is a sufficiently small positive number. Since $w_B = 0$, the wage increase will induce both workers to apply to firm A with probability one and so firm A's expected profit is $1 - \varepsilon$. The expected profit before the wage increase is $1 - \alpha_{A1}\alpha_{A2}$. By choosing $\varepsilon < \alpha_{A1}\alpha_{A2}$, which can be done since $\alpha_{A1}, \alpha_{A2} > 0$, firm A increases his expected profit. A contradiction.

Let $w_A > 0$ and $w_B > 0$. We show $\alpha_{A1} = \alpha_{A2}$. Using $\alpha_{Bi}(W) = 1 - \alpha_{Ai}(W)$ in (1) yields:

$$\left(1 - \frac{\alpha_{A2}}{2}\right) w_A = \frac{1 + \alpha_{A2}}{2} w_B;$$
$$\left(1 - \frac{\alpha_{A1}}{2}\right) w_A = \frac{1 + \alpha_{A1}}{2} w_B.$$

Subtracting the two equations yields: $(\alpha_{A1} - \alpha_{A2})(w_A + w_B) = 0$. Since $w_A + w_B > 0$, $\alpha_{A1} = \alpha_{A2}$ and so $\alpha_{B1} = \alpha_{B2}$. ■

The above lemma shows that if both workers mix between both firms, then the two workers must use the same strategy and the two firms must post wages above the bottom wage. Denote $\alpha_A = \alpha_{A1} = \alpha_{A2}$. Then, $\alpha_{B1} = \alpha_{B2} = 1 - \alpha_A$ and (1) yields:

$$\alpha_A(W) = \frac{2w_A - w_B}{w_A + w_B}. \tag{2}$$

Taking w_B as given, firm A solves:

$$\max_{w_A} \left[1-(1-\alpha_A(W))^2\right](1-w_A).$$

Similarly, taking w_A as given, firm B solves:

$$\max_{w_B} \left[1-(\alpha_A(W))^2\right](1-w_B).$$

Solving the above maximization problems establishes the following proposition:

Proposition 2 *In the integrated market with 2 workers and 2 firms, there is a unique mixed-strategy equilibrium that is characterized by:*

$$w_A = w_B = \frac{1}{2}, \ \alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}. \tag{3}$$

Firms' expected profit, workers' expected surplus and the social welfare level are:

$$F_A = F_B = \frac{3}{8}; \quad U_1 = U_2 = \frac{3}{8}; \quad V = \frac{3}{8}.$$

Market integration increases wage dramatically (from 0 to 1/2) and benefits workers, but firms are worse off and social welfare is reduced. With suitable transfers between firms and workers, both workers and firms prefer separated markets. Welfare is lower after integration because the matching difficulty increases with the market size. When markets are separated all agents are matched, since the worker in each sub-market does not have any choice but to apply to the only firm there. In contrast, with an integrated market each worker can choose from two firms and their uncoordinated decisions induce employment uncertainty. Each firm faces a positive probability of failing to get a worker, $(1-\frac{1}{2})^2 = \frac{1}{4}$.

The above equilibrium described is not the only one for the integrated market: The strategies in the separated markets also form a (pure-strategy) equilibrium in the integrated market.[5] To verify, note that, when worker 1 applies to firm A with probability 1 and worker 2 applies to firm B with probability 1, each firm gets one worker with probability one and so the expected profit

[5]The other pure-strategy equilibrium is such that worker 1 applies to firm B with probability one, worker 2 applies to firm A with probability one and both firms post $w = 0$.

is the highest level that a firm can possibly obtain. No firm has incentive to increase wage above 0, which would only reduce the expected profit. Given that both firms post $w = 0$, workers get zero surplus everywhere and so the decisions $(\alpha_{A1}, \alpha_{B2}) = (1, 1)$ are rational.

However, this pure-strategy equilibrium is not trembling-hand perfect in the integrated market. To see why, note that firms do not have incentive to increase wage in the pure-strategy equilibrium because they do not face any uncertainty and the expected payoff is at the highest possible level. But if there were any trembling by workers to mix between firms there would be uncertainty for firms, in which case it would be worthwhile for the firm to trade off between wage and the expected number of workers: Given that the other firm maintains the wage $w = 0$, a firm can increase his wage offer marginally, which would attract all workers and eliminate the firm's uncertainty. For this reason, the equilibrium with trembling hand will not converge to the pure-strategy equilibrium even when the amount of trembling shrinks to zero.

For general environments where there are more than two workers and firms in each sub-market before integration, the argument of trembling-hand perfection is much more difficult to be made. This is because the equilibrium before integration can be mixed strategies. To check whether it can survive trembling-hand perfection in the integrated market, one must examine how workers in one sub-group in the integrated market modify their strategies in response to a wage deviation by a firm in another sub-group. This is a difficult task even when there are only four workers and four firms in the economy.[6] Despite this difficulty, we will restrict attention throughout this paper to equilibria where all workers in the market (or sub-market) mix among all available firms in the market (or sub-market).

This restriction to completely mixed strategy equilibria reflects the premise that coordination, including the kind implied by partially mixed strategies, is difficult to be achieved in decentralized markets. Even for very small markets, experimental evidence in Ochs (1990) indicates significant coordination failure among agents. For example, when there are only four locations and nine

[6]This is not a problem in the case with only two workers and two firms, because the equilibrium in the separated markets is pure strategies and wages are zero. Starting from this situation in an integrated market, a wage increase by one firm draws the worker from the other group with probability one.

buyers with nine units of goods as the total supply, the matching failure rate can be as high as 2/9. Such a high failure rate for such a small "economy" is unlikely to occur if buyers only partially mix among the firms or use pure strategies. In fact, with each firm's price being exogenously set, Ochs has found that buyers mix among all locations, even when some firms have much more units of goods than do other firms and when prices vary. Moreover, when all firms have the same stock of goods, buyers buy from each firm with roughly the same probability, a result consistent with symmetric strategies.

2.2 Example 2: Four Workers and Four Firms

In Example 1 workers are better off from market integration, although aggregate welfare is lower. The increased workers' expected utility arises from the special feature that wages in the sub-market are bottom wages and so market integration increases wages drastically. If wages in the sub-markets were already high, market integration would increase wages only in small magnitudes and workers could be worse off. To illustrate this, consider a market with four identical workers and four identical firms. As before, workers are numbered with arabic numbers $i = 1, 2, 3, 4$. Firms now are also numbered with arabic numbers $j = 1, 2, 3, 4$. If the market is separated into two sub-markets, where each sub-market has two workers and two firms, then the wage in each sub-market is the one calculated in Example 1. That is, each firm posts $w = 1/2$ and the two workers in each sub-market apply to each of the firms with probability 1/2, yielding $F = U = V = 3/8 = 0.375$.

Now suppose that the sub-markets are integrated into one. The wage posted by firm j is w_j and the probability that worker i applies to firm j is α_{ji}, with $\sum_{j=1}^{4} \alpha_{ji} = 1$, for all i. As discussed in the last example, we focus on the equilibrium where all workers mix among all firms. One can extend Lemma 1 to show that all wages should be above the bottom wage and all workers use the same strategy. Since firms are identical with each other, all firms post the same wage, w (> 0), and workers apply to each firm with the same probability: $\alpha_{ji} = 1/4$ for all j and i.

To find equilibrium wage w, let us analyze a single firm's deviation to $w^d \neq w$, given that all other firms continue to post w. Since $w^d = 0$ will attract no worker at all, it is not profitable. Thus let $w^d > 0$. Responding to this deviation, each worker applies to the deviator with a revised probability α^d and applies to each of the other firms with $\hat{\alpha} = (1 - \alpha^d)/3$. If a worker applies to the deviator, he/she is expected to be hired with the following probability:

$$\sum_{t=0}^{3} \frac{1}{t+1} C_3^t (\alpha^d)^t (1 - \alpha^d)^{3-t} = \frac{1 - (1 - \alpha^d)^4}{4\alpha^d},$$

where $C_3^t = 3!/[t!(3 - t)!]$. Similarly, if a worker applies to a firm who posts w, a non-deviator, the probability that he will be hired by the firm is $\left[1 - (1 - \hat{\alpha})^4\right]/(4\hat{\alpha})$. For the worker to be indifferent between the deviator and a non-deviator, i.e., for $\alpha^d \in (0, 1)$, the worker must obtain the same expected surplus from the two types of firms. That is,

$$\frac{1 - (1 - \alpha^d)^4}{4\alpha^d} \cdot w^d = \frac{1 - \left(1 - \frac{1-\alpha^d}{3}\right)^4}{4(1 - \alpha^d)/3} \cdot w.$$

This indifference relation solves for a smooth function $\alpha^d = \alpha^d(w^d, w)$. Since the left-hand side of the relation is a decreasing function of α^d and the right-hand side is an increasing function of α^d, $\alpha^d(w^d, w)$ is an increasing function of w^d. That is, by increasing wage the deviator can attract workers to apply to him with a higher probability.

When each worker applies to the deviator with probability α^d, the deviator gets at least one worker with probability $1 - (1 - \alpha^d)^4$. Taking w as given, the deviator chooses w^d to solve:

$$\max_{w^d} \; F^d \equiv (1 - w^d)\left[1 - \left(1 - \alpha^d\right)^4\right] \quad \text{s.t. } \alpha^d = \alpha^d(w^d, w).$$

If w is the equilibrium wage in a mixed strategy equilibrium, then the deviation w^d cannot improve the firm's profit and so $w^d = w$ must be the solution to the above maximization problem. In this case, $\alpha^d = 1/4$. Setting $w^d = w$ and $\alpha^d = 1/4$ in the first-order condition of the maximization problem yields $w = 81/148$. In this equilibrium each firm's profit F, each worker's expected surplus U and the social welfare level V are: $F \approx 0.309$, $U \approx 0.374$, $V \approx 0.342$.

As in Example 1, market integration leads to a higher wage, a lower expected profit for firms and lower social welfare. The fundamental reason why market integration reduces welfare is the same as in Example 1, i.e., a larger market experiences a more severe matching difficulty and so the expected number of matches falls. The expected number of matches for each firm (or for each worker) is $1-(1-1/4)^4 \approx 0.684$ when the market is integrated and is $1-(1-1/2)^2 = 0.75$ when the market is separated into two.

In contrast to Example 1, market integration increases the wage by only a small amount: the wage increases by only $\frac{81}{148} - \frac{1}{2} \approx 0.047$. Since the number of matches per worker falls by roughly 0.066, the wage increase is not sufficient to offset the increased matching difficulty. Even without transfers, both workers and firms prefer separating the market into two.

3 A Large Market

The above examples share the feature that the total demand for workers is equal to the total supply. This is restrictive and cannot provide information on how the wage response to integration depends on the "tightness" of the market. In particular, it is not clear whether the positive wage response is a general feature. To generalize, let us now consider an economy with N workers and M firms, where N, $M \geq 4$ and N is not necessarily equal to M. Each firm has on vacancy to be filled. Denote $r = N/M$ as the worker/job ratio, sometimes referred to as the market tightness.

Suppose first that the market is separated into k sub-markets, with $n \equiv N/k$ workers and $m \equiv M/k$ firms in each sub-market. For simplicity let us assume that n and m are both integers. Denote $x = 1/m$ $(= k/M)$. Since x is increasing in k, a larger x corresponds to less integrated markets and the completely integrated market corresponds to $x = 1/M$. Thus we can refer to x as the degree of market separation. Note that the tightness in each sub-market is the same as in the integrated market. Since the case where each sub-market has only one firm or one worker is straightforward, let us exclude it by assuming $m \geq 2$ and $n \geq 2$. That is,

$$\frac{1}{M} \leq x \leq \bar{x} \equiv \frac{1}{2} \cdot \min\{1, r\}. \tag{4}$$

Let us now examine a sub-market and, as before, focus on the symmetric equilibrium where all workers mix among all firms in the sub-market. In this equilibrium each worker applies to each firm with probability $1/m = x$ and each firm posts a wage, which is denoted $w(x)$ to emphasize its dependence on x. To find the equilibrium wage w, again consider a single firm's deviation to a wage $w^d > 0$, while every other firm continues to post w. Observing the deviation and other wages, each worker applies to the deviator with probability α^d and applies to each of the non-deviators with probability $\hat{\alpha} = (1 - \alpha^d)/(m - 1)$.[7]

If a worker applies to the deviator, the probability that he will be chosen is:[8]

$$\sum_{t=0}^{n-1} \frac{1}{t+1} C_{n-1}^{t} \left(\alpha^d\right)^t \left(1 - \alpha^d\right)^{n-1-t} = \left[1 - (1 - \alpha^d)^n\right] / (n\alpha^d).$$

Similarly, if a worker applies to a non-deviator, he gets the job with probability $\left[1 - (1 - \hat{\alpha})^n\right] / (n\hat{\alpha})$. For the worker to be indifferent between the deviator and non-deviators, the following must hold:

$$\frac{1 - (1 - \alpha^d)^n}{n\alpha^d} \cdot w^d = \frac{1 - \left(1 - \frac{1-\alpha^d}{m-1}\right)^n}{n(1 - \alpha^d)/(m - 1)} \cdot w. \tag{5}$$

This defines a smooth function $\alpha^d = \alpha^d(w^d, w)$, where the dependence on w^d is positive. The smoothness implies that a marginal wage increase will not attract all workers: If workers chose probability one to apply to the deviator, each of them would be chosen with a very low probability.

Note that the right-hand side of (5) is an increasing function of α^d. Since α^d is an increasing function of w^d, the right hand side of (5) is an increasing function of w^d. That is, a wage increase by the deviator raises the expected payoff to workers who apply to non-deviators. This is because the wage increase attracts more workers to the deviator, reduces the congestion of workers applying to the non-deviators and so each worker applying to a non-deviator gets a job

[7] The way the model works does not literally require each worker to observe all posted wages. For example, if each worker observes only two wages randomly drawn from the posted ones, the essential results should continue to hold, but the exercise would be messy.

[8] To compute, define

$$A(y) = \sum_{t=0}^{n-1} \frac{1}{t+1} C_{n-1}^{t} \left(y\alpha^d\right)^t \left(1 - \alpha^d\right)^{n-1-t}.$$

Clearly $A(0) = 0$ and the probability to be computed is $A(1)$. Since

$$\frac{d}{dy}[yA(y)] = \sum_{t=0}^{n-1} C_{n-1}^{t} \left(y\alpha^d\right)^t \left(1 - \alpha^d\right)^{n-1-t} = (y\alpha^d + 1 - \alpha^d)^{n-1},$$

integration yields:

$$A(1) = \int_0^1 (y\alpha^d + 1 - \alpha^d)^{n-1} dy = \frac{1 - (1 - \alpha^d)^n}{n\alpha^d}.$$

with a higher probability than before. This is an indirect cost to the wage increasing firm, because the firm must match up with the increased workers' surplus from elsewhere. The indirect cost and the wage increase itself are both compensated by the increased number of applicants.[9]

When each worker applies to the deviator with probability α^d, the deviator successfully hires a worker with probability $1-(1-\alpha^d)^n$. Taking other firms' wages w as given, the deviator chooses w^d to solve:

$$\max_{w^d} (1-w^d)\left[1-\left(1-\alpha^d\right)^n\right] \quad \text{s.t. } \alpha^d = \alpha^d(w^d, w).$$

Again in the symmetric mixed-strategy equilibrium the deviation cannot be profitable and so $w^d = w(x)$ solves the above maximization problem, which in turn implies $\alpha^d = \alpha = 1/m = x$. Substituting $(w^d, \alpha^d) = (w, x)$ and $n = r/x$ into the first-order condition of the maximization problem yields:

$$w(x) = \left[1 + \frac{(1-x)^{-r/x} - 1}{r} - \frac{1}{1-x}\right]^{-1}. \tag{6}$$

Each firm's expected profit, each worker's expected surplus and the social welfare level can also be expressed as functions of x:

$$F(x) = (1-w)\left[1-(1-\alpha)^n\right] = [1-w(x)]\left[1-(1-x)^{r/x}\right]; \tag{7}$$

$$U(x) = w \cdot \frac{1-(1-\alpha)^n}{n\alpha} = w(x) \cdot \frac{1-(1-x)^{r/x}}{r}; \tag{8}$$

$$V(x) = \frac{M \cdot F(x) + N \cdot U(x)}{M+N} = \frac{1-(1-x)^{r/x}}{1+r}. \tag{9}$$

The expected number of matches per firm, denoted $H(x)$, is

$$H(x) = 1-(1-x)^{r/x}. \tag{10}$$

The social welfare level is proportional to the number of matches per firm. Recalling that a larger x corresponds to less integrated markets, we have:

[9] Montgomery (1991) assumes that a worker's expected payoff from the market is exogenous to each firm. This is true only when there are infinitely many agents on each side of the market.

Proposition 3 *Increasing market integration reduces the number of matches and reduces social welfare. Therefore, when suitable transfers between firms and workers are available, both workers and firms prefer market separation.*

Proof. Since $V(x) = H(x)/(1+r)$, it suffices to show that $H(x)$ is an increasing function of x. Define

$$g(x) = -\frac{1}{x}\ln(1-x). \tag{11}$$

Then $H(x) = 1 - e^{-rg(x)}$ and so $H'(x) > 0$ is equivalent to $g'(x) > 0$. Compute:

$$g'(x) = \frac{1}{x}\left(\frac{1}{1-x} - g(x)\right) = \frac{1}{x^2}\left[\frac{x}{1-x} + \ln(1-x)\right].$$

The function $\frac{x}{1-x} + \ln(1-x)$ has a value 0 at $x = 0$, a positive derivative for all $x \in (0,1)$ and so it is positive for all $x > 0$, yielding $g'(x) > 0$. ■

The welfare response is similar to that in Examples 1 and 2, but the wage response is more complicated. Noting that $\ln(1-x) < 0$ for all $x \in (0,1)$, it can be directly verified from (6) that $w'(x) > 0$ if and only if

$$r < f(x) \equiv 2x - \frac{1}{g(x)}\ln g'(x), \tag{12}$$

where $g(\cdot)$ is defined by (11). The following proposition is proved in Appendix A:

Proposition 4 *Let x be in the range specified by (4).*

(i) If $r \leq f(\frac{1}{M})$, then $w'(x) > 0$ for all x;

(ii) If $r \geq f(f(\frac{1}{2})/2) \approx 0.829$, then $w'(x) < 0$ for all x;

(iii) If $f(\frac{1}{M}) < r < f(f(\frac{1}{2})/2)$, then there exists $x_0 \in \left(\frac{1}{M}, \bar{x}\right)$ such that $w'(x) < 0$ for $x \in [\frac{1}{M}, x_0]$ and $w'(x) > 0$ for $x \in (x_0, \bar{x}]$.

The wage response to increased market size is ambiguous. If the number of workers relative to jobs in the economy is not too low, integrating markets (reducing x) increases wages (case (ii)). On the other hand, if the number of jobs exceeds the number of workers by a large margin (case (i)), integrating markets reduces wages. The simplest example for the latter case is when there are two workers and four firms. When the market is initially separated into two sub-markets, each sub-market has two firms and only one worker and so Bertrand competition drives wages to one. Wages can only be lower when the sub-markets are integrated. More generally, when there are many more jobs than workers in each sub-market, workers have a strong market power that supports high wages. Integrating the sub-markets in this case allows each firm to have access to a larger group of workers than before. Although the integration also allows each worker to have access to a larger group of firms, such a benefit to workers is relatively small at the margin as workers started with an already strong market power. In this case, wages decreases with integration, even though the integration does not change the market tightness.

Phrased differently, market integration increases the relative market power of the side of the market that is much longer. Although this may not seem controversial, the critical level of the market tightness for a positive wage response is roughly 0.83 rather than one. That is, even when the supply of workers is lower than but close to the demand, market integration increases wages. This is because of the asymmetric treatment of workers and firms in our model – firms can set wages to exploit the market but workers can only respond to these wages. This asymmetry gives firms a relatively higher market power even when $r = 1$, which is reduced by market integration.

It should be emphasized that the wage response is not a result of changes in the market tightness but rather of changes in the extent of coordination. In fact, the market tightness is constant before and after integration. But a more integrated market requires more extensive coordination. The cost of coordination is unevenly shared by the two sides of the market and the firms bear a larger part of the cost as they are the ones who competitively organize the market by posting wages.

From Propositions 3 and 4 it is clear that at least one group, firms or workers, is worse off after market integration. Workers are worse off when the worker/job ratio is low (i.e., when $r < f(x_0)$) and firms are worse off when the worker/job ratio is not too low (i.e., when $r \geq f(f(\frac{1}{2})/2) \approx$

0.829). One would like to know whether workers and firms can both be worse off (without transfers between the two groups), as in Example 2. Unfortunately this cannot be determined analytically since the wage response depends on the worker/job ratio in a non-monotonic fashion. The following three examples illustrate the patterns of the responses, with $M = 100$.

Example 3. $r = 1$. In this case Proposition 4 tells us that wages increase with market integration and so firms' expected profit falls, as depicted by Figure 1.1. Workers' expected surplus, $U(x)$, may rise of fall, depending on the initial degree of market integration. Let us start from the situation where the market is so severely separated that each sub-market has only two firms and two workers (i.e., $x = 1/2$) and then gradually increase the degree of market integration (i.e., reduce x). Initially, workers are better off slightly (not very discernible in Figure 1.1) when markets become more integrated, but further integration makes workers worse off. This non-monotonic pattern of workers' utility arises from the fact that wage increases generated by market integration diminish when markets become more and more integrated. When markets are severely separated, wage increases resulted from market integration are large enough to dominate the increased matching difficulty and to increase workers' utility. When markets are already integrated to some degree, however, the increased matching difficulty dominates wage increases and so workers are worse off. It is worthwhile noting that workers are worse off when markets are fully integrated ($x = 1/M$) than when markets are severely separated ($x = 1/2$).

Example 4. $r = 1.5$ (Figure 1.2). As in Example 3, wages rise with market integration and firms' expected profit falls. In contrast with Example 3, workers are better off with market integration.

Example 5. $r = 0.5$ (Figure 1.3). This case is opposite to Example 4. With increasing market integration, wages fall, firms are better off and workers are worse off.

These three examples indicate that market integration is most likely to reduce both workers' and firms' surpluses when the total supply of workers is close to the number of jobs. When one side of the market is much shorter than the other side, the longer side of the market benefits from market integration and the shorter side loses.

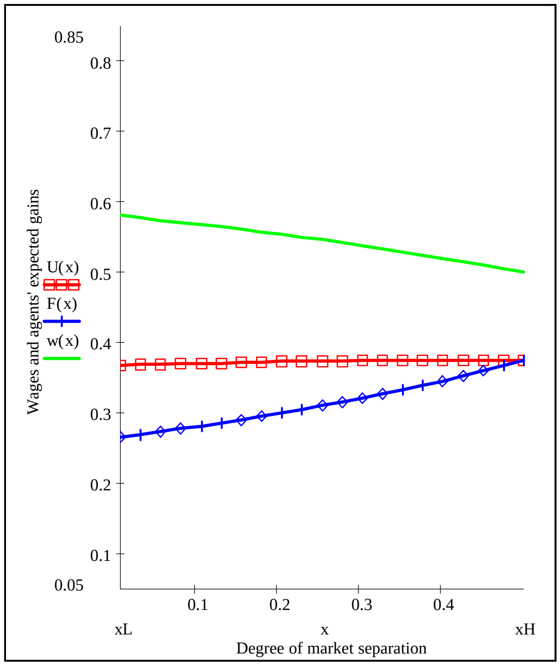

Figure 1.1. The case $r = 1$

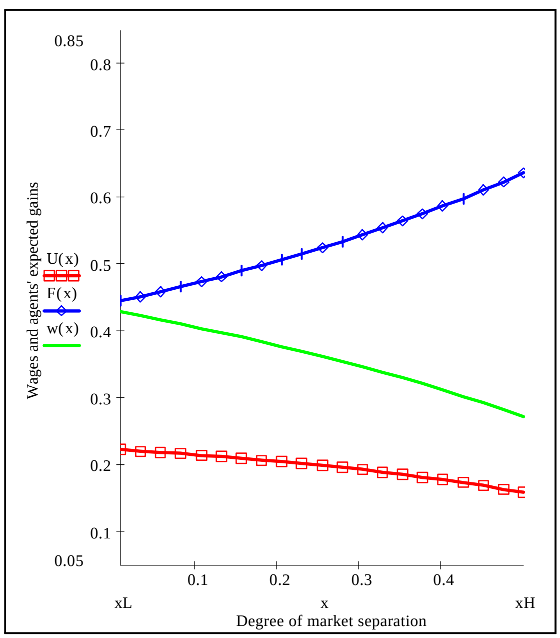

Figure 1.2. The case $r = 1.5$

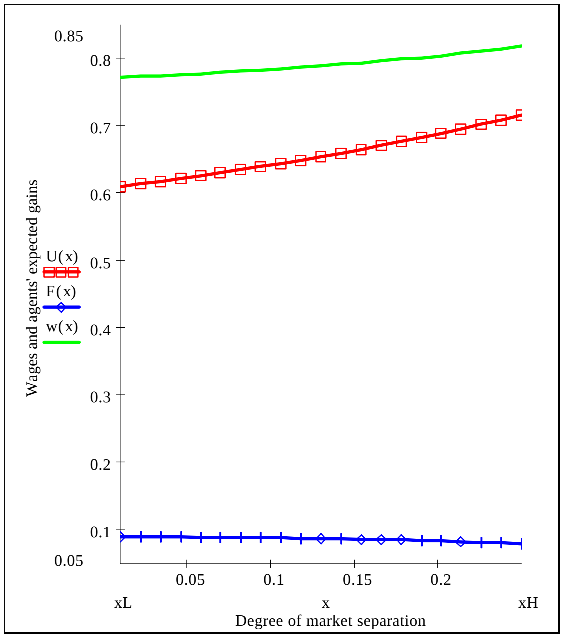

Figure 1.3. The case $r = 0.5$

4 Bounds on the Welfare Loss

Since our analysis omits some important benefits of market integration, such as improved match qualities, our welfare result about market integration is only suggestive of the negative consequences of integration. Nevertheless, the result is useful for providing a yardstick against which one can measure other benefits of market integration. For this purpose we ask: What is the upper bound on the welfare loss from increased matching difficulty due to market integration? Also, realistic markets such as the labour market are typically large. For our results to be useful, we need to know the answer to the following question: How does the welfare loss from increased matching difficulty behave when the market gets large?

The two questions are related and we start with the second question. Let us compare the welfare level where the market is separated into k sub-markets with the one where the market is fully integrated. Denoting $x_1 = 1/M$ and using $g(\cdot)$ defined in (11) one can calculate the per-capita welfare loss from integration as:

$$\Delta V(k) \equiv V(\frac{k}{M}) - V(\frac{1}{M}) = \frac{1}{1+r}\left[e^{-rg(x_1)} - e^{-rg(kx_1)}\right]. \tag{13}$$

The limit of this loss depends on the way in which the market expands. One way is that k

is fixed while $M \to \infty$. That is, the number of sub-markets is fixed and the expansion of the market simply adds more workers and firms to each sub-market. In this case $x_1 \to 0$ and $kx_1 \to 0$, which imply $\Delta V(k) \to 0$. Thus, when each sub-market expands to infinity at the same rate as the total market does, the difference in per-capita welfare between the integrated market and k sub-markets vanishes. This is not surprising because in the limit each sub-market has infinitely many firms and workers, just as the integrated market does.[10]

The second way that the market expands is that k/M is fixed while $M \to \infty$. That is, the numbers of workers and firms in each market do not change and the expansion of the economy simply adds more sub-markets. In this case, $x_1 \to 0$ but kx_1 is constant. Because the expansion does not change the numbers of workers and firms in each sub-market, the per-capita welfare loss from integration does not vanish in the limit. Instead,

$$\Delta V(k) \to \frac{e^{-r} - e^{-rg(kx_1)}}{1+r}. \tag{14}$$

Proposition 5 *Per-capita welfare loss from market integration in the limit is bounded above by $L(r)$ where*

$$L(r) = \begin{cases} \frac{e^{-r}-(1-\frac{r}{2})^2}{1+r} & \text{if } r < 1 \\ \frac{e^{-r}-4^{-r}}{1+r} & \text{if } r \geq 1. \end{cases} \tag{15}$$

Proof. Per-capita welfare loss from market integration is the highest when market expansion does not change the size of each sub-market. Thus, the upper bound on the welfare loss can be found by maximizing (14) over kx_1. The maximum is attained at $kx_1 = \bar{x}$. Since $\bar{x} = \frac{r}{2}$ when $r < 1$ and $\bar{x} = \frac{1}{2}$ when $r \geq 1$ (see (4)), then $g(kx_1) \leq g(\frac{r}{2})$ when $r < 1$ and $g(kx_1) \leq g(\frac{1}{2}) = 2\ln 2$ when $r \geq 1$. Calculating $e^{-rg(r/2)}$ leads to the upper bound in (15). ■

[10]Note that the total welfare loss from integration does not vanish: $(N+M) \cdot \Delta V(k) \to \frac{k-1}{2} re^{-r}$, which is maximized at $r = 1$.

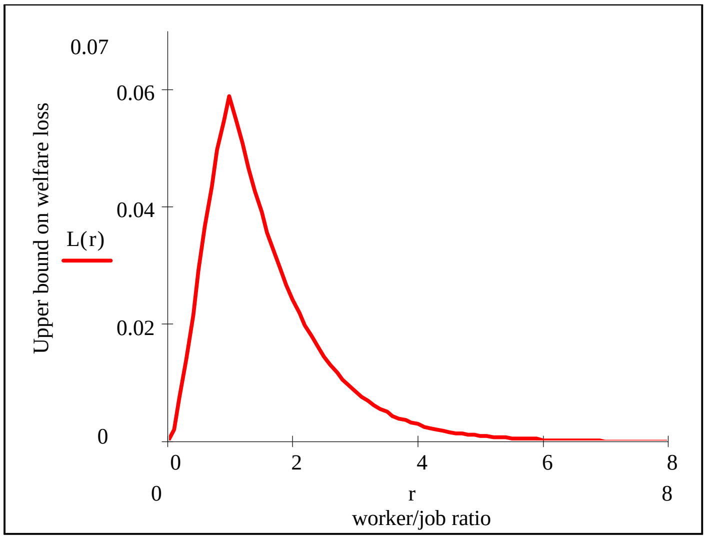

Figure 2. The upper bound on the welfare loss

The upper bound $L(\cdot)$ is plotted in Figure 2. There are two interesting details about the upper bound. First, the maximum of $L(r)$ is $L(1) \approx 0.06$. That is, when the number of workers is equal to the number of jobs, the increased matching difficulty caused by market integration is most severe and the upper bound of such welfare loss is about 12% of the total value of output. Second, the function $L(r)$ decreases very rapidly when r deviates from 1, with $L(r) \to 0$ for $r \to 0$ or $r \to \infty$. Thus, in markets where one side is much shorter than the other side, the increase in the matching difficulty generated by market integration is very limited, in which case the welfare loss is small relative to other benefits of market integration omitted here.[11]

The upper bound $L(r)$ is obtained by comparing the fully integrated market with the extremely separated markets where there are only two firms or two workers in each sub-market. In realistic discussions markets have already been integrated to some degree and one is interested in the coordination cost of further integration. The welfare cost of such further integration is much smaller than the upper bound provided above. For example, if $M = 100$ and there are two sub-

[11] If each sub-market can have only one firm or one worker, a situation ruled out by (4), the upper bound on the welfare loss from market integration is larger than in (15). In this case, $kx_1 \leq \min\{1, r\}$. Noting that $g(1) = \infty$ and $rg(r) = -\ln(1-r)$, the upper bound $L(r)$ is now given by

$$L(r) = \frac{e^{-r} - (1-r)}{1+r} \text{ if } r < 1; \quad = \frac{e^{-r}}{1+r} \text{ if } r \geq 1.$$

This function has the same shape as the one in (15) and the maximum is $L(1) \approx 0.183$.

markets (i.e., $k = 2$), then integrating the two sub-markets into one increases the coordination cost by at most 0.25% of the total value of output.

In general, if the worker/job ratio is not too high, the marginal increase in the matching difficulty generated by integration is decreasing as the market becomes increasingly integrated. To see this, begin with k_a sub-markets and let $x_a = k_a/M$. The cost of integrating the sub-markets into k_a/t ($t \geq 2$) sub-markets can be obtained by modifying (13):

$$\frac{1}{1+r}\left[e^{-rg(x_a/t)} - e^{-rg(x_a)}\right].$$

For fixed x_a, this cost is increasing and concave in t, provided $r \leq 2t/x_a$. Since $x_a \leq 1/2$ and $t \geq 2$, successive integration increases the cost by smaller and smaller amounts, provided $r \leq 8$.

5 Conclusion

When it is costly for agents to find a match, integrating small markets into a large one reduces the number of matches. We have focused on this dependence of the matching difficulty on the market size by explicitly analyzing how firms' wage decisions affect the number of matches and how they respond to market integration. It is shown that integration reduces the relative market power of agents on the much shorter side of the market. Thus, if the worker/job ratio is high, integration increases wages, but if the worker/job ratio is low, integration reduces wages. Regardless of the nature of the wage response, market integration reduces social welfare when everyone is weighted equally. This marginal reduction in welfare shrinks as the market becomes increasingly integrated.

The social welfare loss from the increased matching difficulty might be outweighed by other benefits of market integration, which are deliberately abstracted from the analysis here. As shown in Section 4, the upper bound on the social welfare loss from increased matching difficulty falls very rapidly when the total demand in the market deviates (in either direction) from the total supply. Thus, when the labour market is characterized by a shortage of skilled workers, the increased matching difficulty is likely to be overwhelmed by other benefits of market integration. For transitional economies the high unemployment rate typically associated with increased labour mobility might be small in comparison with the benefit from better matches between skills and jobs.

To conclude the paper, we comment on two assumptions made in the model. The first is that each firm has only one job to offer. This assumption is not necessary for the negative dependence of the aggregate number of matches on the market size. Appendix B extends the analysis to the case where each firm offers more than one job and obtains similar results. The second assumption is that the wage posting game ends after one-period play. In reality, agents who fail to get matched in one period can try to get matched in the future. Incorporating this repeated play will reduce the extent to which market integration reduces the number of matches. However, the negative effect will not vanish, as long as there always are positive measures of unmatched agents on both sides of the market. In the stationary equilibrium of such an economy, firms' wage decisions and workers' trade-off between a wage and the match probability will be qualitatively similar to the ones in the one-period game. The restriction to a one-period setting is thus a useful simplification for markets, such as the labour market, that exhibit high turnovers and persistent unemployment.

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Appendix

A Proof of Proposition 4

The function $f(x)$ defined in (12) is an increasing function for all $x \in (0, \bar{x}]$, as shown later. With $f'(x) > 0$, consider the three cases in the proposition:

Case (i): $r \leq f(\frac{1}{M})$. In this case (12) is satisfied and so $w'(x) > 0$ for all $x \in [\frac{1}{M}, \bar{x}]$.

Case (ii): $r \geq f(f(\frac{1}{2})/2)$. Since $f(f(\frac{1}{2})/2) < f(\frac{1}{2})$, either $r \geq f(\frac{1}{2})$ or $f(f(\frac{1}{2})/2) \leq r < f(\frac{1}{2})$. If $r \geq f(\frac{1}{2})$ then (12) is violated for all $x \leq 1/2$ and so $w'(x) < 0$ for all $x \leq \bar{x}$ (note $\bar{x} \leq 1/2$). If $r < f(\frac{1}{2})$ then $\bar{x} \leq r/2 < f(\frac{1}{2})/2$ and so $r \geq f(f(\frac{1}{2})/2)$ implies $w'(x) < 0$ for all $x \in [\frac{1}{M}, \bar{x}]$.

Case (iii): $f(\frac{1}{M}) < r < f(f(\frac{1}{2})/2)$. Then $f(\bar{x}) > r$ and so there exists $x_0 \in (\frac{1}{M}, \bar{x})$ such that $r > f(x)$ for $x \in [\frac{1}{M}, x_0)$, in which case $w'(x) < 0$, and $r < f(x)$ for $x \in (x_0, \bar{x}]$, in which case $w'(x) > 0$.

Let us now show $f'(x) > 0$ for $x \in (0, \bar{x}]$. Calculate $f'(x) = g'(x)f1(x)/[g(x)]^2$ where

$$f1(x) = \frac{2(g(x))^2}{g'(x)} + \ln(g'(x)) - \frac{g(x)g''(x)}{(g'(x))^2}.$$

Since $g(x) > 0$ and $g'(x) > 0$ for all $x \in (0, 1)$ (see the proof of Proposition 3), $f'(x) > 0$ iff $f1(x) > 0$. It can be computed that $f1(\frac{1}{2}) > 0$. Since $\bar{x} \leq 1/2$, $f1(x) > 0$ for all $x \in (0, \bar{x}]$ if $f1'(x) < 0$ in this range. Compute:

$$g'(x) = \frac{1}{x}\left(\frac{1}{1-x} - g(x)\right); \quad g''(x) = \frac{1}{x^2}\left[\frac{3x-2}{(1-x)^2} + 2g(x)\right];$$

$$g'''(x) = \frac{1}{x^3}\left[\frac{11x^2 - 15x + 6}{(1-x)^3} - 6g(x)\right].$$

Then,

$$f1'(x) = \frac{g(x)}{x^4(1-x)^2[g'(x)]^3}\left[\frac{2-3x}{1-x} - (4-x)g(x) + 2\left[g(x)\right]^2\right].$$

Thus $f1'(x) < 0$ iff $g(x) \in (g_1(x), g_2(x))$ where

$$g_1(x) = 1 - \frac{x}{4}\left[1 + \sqrt{\frac{9-x}{1-x}}\right], \; g_2(x) = 1 - \frac{x}{4}\left[1 - \sqrt{\frac{9-x}{1-x}}\right].$$

It is easy to show that the function $[xg(x) - xg_1(x)]$ is an increasing function for $x \in (0, 1)$ and has a value 0 at $x = 0$. Thus $g(x) > g_1(x)$. To show $g(x) < g_2(x)$, consider the function

$f2(x) \equiv xg_2(x) - xg(x)$. Then $f2(0) = 0$ and

$$f2'(x) \sim (1-x)(9-x) + 2x^2 - (3-x)\sqrt{(1-x)(9-x)}.$$

It can be verified that the expression on the right-hand side is negative for all $0 < x \leq 1/2$. Thus $f2'(x) < 0$ and so $f2(x) \geq f2(\frac{1}{2}) > 0$ for all $0 < x \leq 1/2$. This shows $g(x) < g_2(x)$ and so $f1(x) > 0$ for all $0 < x \leq 1/2$, yielding $f'(x) > 0$. ■

B The Case When Each Firm Has Multiple Vacancies

The symbols (M, N, m, n, x) have the same meanings as in Section 3. Let each firm have $b \geq 2$ jobs. Since the case where one firm can satisfy all the workers in the sub-market is not interesting, let us assume $b < n$. In the symmetric, mixed-strategy equilibrium, all firms post a wage $w \in (0,1)$ and each worker applies to each firm with probability $1/m$. If a firm gets b or fewer workers, each worker gets a job with probability one; if the firm gets $t > b$ applicants, only b applicants will be chosen randomly and so each applicant will be chosen with probability b/t.

To determine w, consider a single firm's deviation to a wage $w^d \in (0,1)$. Observing the deviation, each worker applies to the deviator with probability α^d and applies to each of the non-deviators with probability $\hat{\alpha} = (1-\alpha^d)/(m-1)$. If a worker applies to the deviator, the probability that he gets a job is

$$q(\alpha^d) \equiv \sum_{t=0}^{b-1} C_{n-1}^t (\alpha^d)^t (1-\alpha^d)^{n-1-t} + \sum_{t=b}^{n-1} \frac{b}{t+1} C_{n-1}^t (\alpha^d)^t (1-\alpha^d)^{n-1-t}.$$

The first summation deals with cases where the firm has at most $(b-1)$ other applicants; the second summation deals with cases where the firm has at least b other applicants. The probability $q(\alpha^d)$ can be rewritten as

$$q(\alpha^d) = b \cdot \frac{1-(1-\alpha^d)^n}{n\alpha^d} - \sum_{t=0}^{b-2} \left(\frac{b}{t+1} - 1\right) C_{n-1}^t (\alpha^d)^t (1-\alpha^d)^{n-1-t}.$$

Similarly, when a worker applies to a non-deviator, the probability that he gets a job is $q(\hat{\alpha})$. For the worker to be indifferent between the two firms, we must have:

$$w^d \cdot q(\alpha^d) = w \cdot q\left(\frac{1-\alpha^d}{m-1}\right). \tag{16}$$

Again, this defines a relationship $\alpha^d = \alpha^d(w^d, w)$.

The deviator chooses w^d to maximize the expected profit, taking w as given and facing the constraint $\alpha^d = \alpha^d(w^d, w)$. The deviator's expected profit is

$$(1 - w^d)\left[\sum_{t=1}^{b-1} tC_n^t(\alpha^d)^t(1-\alpha^d)^{n-t} + b \cdot \sum_{j=b}^{n} C_n^t(\alpha^d)^t(1-\alpha^d)^{n-t}\right].$$

The expression in $[\cdot]$ can be shown to be $n\alpha^d q(\alpha^d)$. Using (16) to eliminate w^d, the deviator's expected profit is:

$$n\alpha^d\left[q(\alpha^d) - wq\left(\frac{1-\alpha^d}{m-1}\right)\right].$$

Deriving the first-order condition for α^d and setting $\alpha^d = \alpha = 1/m$, one obtains:

$$w = \frac{q(\frac{1}{m}) - \alpha\delta}{q(\frac{1}{m}) + \frac{\alpha}{m-1}\delta}, \qquad \text{where } \delta = -q'(\alpha^d)\mid_{\alpha^d=1/m}.$$

Let H now be the probability that a firm successfully fills each vacancy and F be a firm's expected profit per vacancy. Define (U, V) accordingly. To find how (w, H, V, U, F) respond to market integration, consider an example: $M = 20$, $b = 5$, $N = 100$. Initially the market is separated into five sub-markets so that $m = 4$ and $n = 20$. Integrating the five sub-markets into one yields the following changes:

$$\Delta w \approx 0.047,\ \Delta H \approx -0.019,\ \Delta V \approx -0.016,\ \Delta F \approx -0.049,\ \Delta U \approx 0.030.$$

In this example, market integration increases wages, increases workers' surplus but reduces firms' surplus and reduces the social welfare level.

Workers can also be worse off. For example, if $N = 80$, integrating the five sub-markets into one yields the following changes:

$$\Delta w \approx 0.013,\ \Delta H \approx -0.017,\ \Delta V \approx -0.017,\ \Delta F \approx -0.015,\ \Delta U \approx -0.003.$$