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Animal Spirits meets Creative Destruction *

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Résumé:
Nous montrons comment un processus de destruction créatrice Schumpeterien peut entraîner un comportement rationnel de masse ("moutonnier") des entrepreneurs à travers les différents secteurs de l'économie, un comportement qui peut apparaître comme provenant d'effets non fondamentaux ("esprits animaliers"). En conséquence, une économie avec des secteurs multiples dans laquelle les améliorations de productivité sont apportées par des entrepreneurs indépendants cherchant un profit peut être caractérisée par des expansions, des ralentissements et des baisses de façon régulière tout en étant une partie intégrante du processus de croissance à long terme. L'équilibre cyclique que nous étudions possède certes un taux de croissance moyen supérieur mais également un niveau de bien-être inférieur par rapport à l'équilibre acyclique correspondant. Nous trouvons que les cycles générés par notre modèle présentent plusieurs caractéristiques des cycles économiques observés dans les données et qu'une relation négative émerge entre la volatilité et la croissance parmis les économies caractérisées par des cycles.

Abstract:
We show how a Schumpeterian process of creative destruction can induce rational, herd- behavior by entrepreneurs across diverse sectors of the economy that may look like it is fuelled by "animal spirits". Consequently, a multi-sector economy, in which sector-specific, productivity improvements are made by independent, profit-seeking entrepreneurs, can exhibit regular booms, slowdowns and downturns as an inherent part of the long-run growth process. The cyclical equilibrium that we study has a higher average growth rate but lower welfare than the corresponding acyclical one. We find that the cycles generated by our model exhibit several features of actual business cycles, and that across cycling economies, a negative relationship emerges between volatility and growth.

Keywords:
Entrepreneurship, innovation, endogenous business cycles, endogenous growth

JEL classification: E0, E3, O3, O4

"*The recurring periods of prosperity of the cyclical movement are the form progress takes in capitalistic society.*" (Joseph Schumpeter, 1927)

1 Introduction

Are business cycles simply random shocks around a deterministic trend, or are there more fundamental linkages between short–run fluctuations and long–run growth? Although, in recent times, macroeconomists have tended to study the sources of fluctuations and the determinants of growth separately, there are several reasons to question this standard dichotomy. First, post war cross–country evidence (e.g. Ramey and Ramey, 1995) suggests a significant negative partial correlation between volatility and growth, after controlling for standard growth correlates. This correlation is economically significant even amongst OECD countries. Second, while it is clear that some portion of aggregate volatility is the result of exogenous disturbances, the recurring asymmetry between the responses of the economy during upturns and downturns, is suggestive of an endogenously determined component (see also Freeman, Hong and Peled, 1999). Third, there is also increasing evidence that the strength of cyclical upturns are related to the depth of preceding downturns (see Beaudry and Koop 1993 and Altissimo and Violante 2001). Finally, even for fluctuations that are typically associated with obvious aggregate shocks, the causal links are not clear.[1]

The view that growth and cycles are intimately linked is often associated with Schumpeter (1927). He argued that growth occurs through a process of "creative destruction" — competition amongst entrepreneurs in the search for new ideas that will render their rivals' ideas obsolete. This idea is central to modern theories of endogenous long run growth starting with Aghion and Howitt (1992), Grossman and Helpman (1991) and Sergestrom, Anant and Dinopolous (1990). However, Schumpeter also argued that this process of entrepreneurial innovation is responsible for the regular short–run fluctuations in economic activity, which he termed the "normal" business cycle.[2]

[1] For example, Zarnowitz (1998) argues that the US downturn in the early 1970s predates the 1973 oil price hike, suggesting that this shock simply made an underlying cyclical movement worse.

[2] Aghion and Howitt (1992) develop an identical engine of growth to Grossman and Helpman (1991) but differ in focusing on a model with a single innovating sector. Lumpy growth in Aghion and Howitt is possible, since there is no reason that single sectors should experience smooth growth. However, except by coincidence, this cannot explain the diffused productivity improvements observed over the business cycle. See Phillips and Wrase (1999) for further discussion.

The key to explaining such business cycles, he argued, was to understand why entrepreneurial activity would be clustered over time.

One source of clustering was suggested by Schumpeter himself: "... as soon as any step in a new direction has been successfully made it, at once and thereby, becomes easy to follow... the first success draws other people in its wake and finally crowds of them, which is what the boom consists of." (Schumpeter, 1927). He further argued that there would be a downturn prior to the boom as resources are allocated to learning and imitation. Recently, several authors have formalized these "Schumpeterian cycles" in an attempt to understand their linkages to long–run growth.[3] However, these theories rely on the arrival of major technological breakthroughs that influence all sectors — a General Purpose Technology (GPT). While the GPT story may be consistent with "long waves", most studies find little evidence to support the notion that such economy–wide advances can explain high frequency business cycles (see for example Jovanovic and Lach 1997, Andolfatto and MacDonald, 1998).

An alternative theory of why activity in diverse sectors of the economy may be clustered is developed by Shleifer (1986). He shows that, when imitation limits the longevity of monopoly profits, a strategic complementarity arises that could lead entrepreneurs to implement innovations at the same time, even if the innovations themselves arrive uniformly through time. The clustering of implementation results in a boom in labor demand, which in turn generates the high demand for output necessary to support the boom. The temporary nature of the associated monopoly profits induces entrepreneurs to delay implementation until demand is maximized, so that a self–reinforcing cycle arises. Shleifer interprets his theory as a formalization of Keynes' (1936) notion of "animal spirits".[4]

There are, however, several important limitations to Shleifer's theory of implementation cycles. Firstly, since innovations arrive exogenously, long–run growth is exogenous, so the theory has no implications for the impact of cycles on growth. Secondly, because of the multiplicity of equilibria that arise in his model, it is not possible to obtain precise predictions even for the effect of growth on cycles. Thirdly, the temporary nature of profits relies on the assumption of drastic, but costless

[3] See, for example, Jovanovic and Rob (1990), Cheng and Dinopoulos (1992), Helpman and Trajtenberg (1998) and Li (2000). The literature on Schumpeterian cycles is discussed by Aghion and Howitt (1998), who note that GPTs are suited to generating Schumpeterian long waves. Though they discuss research on the high frequency business cycle, they fall short of advocating GPTs as a method to understand it. The examples they emphasize further support the long view: the steam engine, the electric dynamo, the laser, the computer.

[4] The expressions "animal spirits" is often associated with stochastic changes in the expectations of investors that turn out to be self–fulfilling. In the cyclical equilibrium that we study, however, the behavior of entrepreneurs may have the *appearance* of being fuelled by animal spirits, but in fact expectations are *deterministic*.

imitation. It is not clear how robust the results would be to a less abrupt erosion of profits. Finally, Shleifer's theory depends critically on the impossibility of storage. If they could, innovators would choose to produce when costs are low (i.e. before the boom), store the output and then sell it when demand is high (i.e. in the boom). Such a pattern of production would undermine the existence of cycles.[5]

In this article, we draw on the insights of Schumpeter (creative destruction) and Shleifer (animal spirits), to develop a simple theory of endogenous, cyclical growth. We show how a multi–sector economy, in which sector–specific, productivity improvements are made by independent, profit–seeking entrepreneurs, can exhibit regular booms, slowdowns and downturns in economic activity as an inherent part of the long–run growth process. We establish the existence of a unique cyclical growth path along which the growth rate and the length and amplitude of cycles are endogenously determined. Our theory does not rely on the arrival of GPTs nor on drastic imitation, and allows for the possibility of storage. Specifically, we show that the process of creative destruction itself can induce endogenous clustering of implementation *and* innovation.

Creative destruction implies that, even if a patent or the fear of price competition dissuades imitation, the dissemination of knowledge caused by implementation eventually leads to improvements that limit a successful entrepreneur's time of incumbency. Anticipating this, entrants will optimally time implementation to ensure that their profits arrive at a time of non-depressed aggregate activity and that they maximize the length of their incumbency.[6] It is these effects which lead to clustering in entrepreneurial implementation and, hence, to an aggregate level boom. If an entrepreneur implements before the boom, he reveals the information underlying his productivity improvement to potential rivals who may use this information in designing their own productivity improvements. By delaying implementation until the boom he delays reaping the rewards but maximizes his expected reign of incumbency. During the delay, entrepreneurs rely on maintaining

[5] Since questions of the timing of production and implementation clearly play an important role in producers' minds, we believe the clustering of innovations underlying the theory should at least be robust to the possibility of storage. For many goods, there is no reason to limit production to occuring only at the time of sale.

[6] For example, consider an entrepreneur's decision to open a new branch outlet in a previously untried location. The resources required in such an undertaking are not generally measured in official statistics as being separate from directly productive activities, but are substantial nonetheless (e.g. planning, market surveying, financing, hiring, contracting, negotiating, etc.). Moreover, the tacit knowledge so created in this process is not protected. The knowledge that a branch outlet in a product line at a particular location is profitable is valuable to future entrepreneurs. Though careful to avoid setting up the same line and quality of store as the initial entrant, the entrant's profits will induce some to search for alternative lines, and perhaps, higher qualities, that will allow them to tap into this market. The fruits of these searches will eventually end the initial entrant's reign of high profits.

secrecy regarding the nature of the innovations that they hold.[7]

Our cycle not only features clustering of implementation, but also endogenous clustering of innovation. It is this feature which generates the endogenous interactions between long–run growth and short–run fluctuations. After the boom, wage costs are so high that it is initially not profitable to undertake new entrepreneurial activities. As the next boom approaches, however, the present value of new innovations grows until at some point it becomes profitable to allocate entrepreneurial effort to innovation. As labor effort is withdrawn from production, per capita output (and measured productivity) gradually decline. Eventually it becomes profitable to implement the stock of innovations that have accumulated during the downturn, and the cycle begins again.

We adopt a broad interpretation of innovation to include any improvement that is the outcome of purposive design in search of profit. Entrepreneurs are the source of refinements to process, organization and product improvements that increase productivity within narrowly defined sectors. The knowledge created by such entrepreneurial activity is both tacit and sector-specific. Unlike R & D, or scientific knowledge, the improvements created may not be formally expressible (as in a blueprint or design) and need not lend themselves to protection by patent. It is our view that such mundane entrepreneurial decisions are the major source of high frequency productivity improvements, not the patentable R & D improvements of a laboratory, which are often the focus in the growth literature.[8] Our interpretation of entrepreneurship is thus not unlike the similarly Schumpeterian interpretation advanced by Aghion and Saint Paul (1998) or even that of Hall (2000) based on entrepreneurship as "reorganization" in a recession. It is similar in effect to the benefits stemming from recessions, emphasized by Caballero and Hammour (1994), though for different reasons.[9]

Although our model is rather stylized, it has clear predictions for the interactions of long run growth and short run fluctuations. Firstly, the cycle in our model shows a positive feedback from both the duration and depth of downturns to the magnitude of succeeding upturns. This feature

[7]As Cohen, Nelson and Walsh (2000) document, firms do indeed view secrecy as the best form of protection — patenting is a less desired means of protecting knowledge.

[8]This view was shared by Schumpeter: "...The function of entrepreneurs is to reform or revolutionize the pattern of production by exploiting an invention or, more generally, an untried technological possibility for producing a new commodity or producing an old one in a new way, by opening up a new source of supply of materials or a new outlet for products by reorganizing industry and so on. ... This function does not essentially consist in either inventing anything or otherwise creating the conditions which the enterprise exploits. It consists in getting things done" Schumpeter (1950, p. 132).

[9]The distinction between entrepreneurship and R&D is an important one. In our model, entrepreneurial effort is countercyclical, whereas in the data, R&D expenditures tend to be acyclical or even procyclical.

is consistent with the evidence of Beaudry and Koop (1993), Pesaran and Potter (1997) and Altissimo and Violante (2001). Secondly, the cycles generated by our model exhibit asymmetries in upturns and downturns, that have some features in common with the evidence of Emery and Koenig 1992, Sichel 1993 and Balke and Wynne 1995. In particular, business cycles typically exhibit rapid growth in output at the beginning of the boom, a gradual slowdown and then a decline which occurs at a fairly constant rate. Thirdly, consistent with the evidence of Ramey and Ramey (1995), variation in the productivity of entrepreneurship induces a negative relationship between long run growth and output volatility.

Recently several authors have developed related, non–GPT models of endogenous growth and cycles. Francois and Shi (1999) modify the Grossman and Helpman (1991) growth model by allowing exogenous, drastic imitation (as in Shleifer 1986), by introducing a technological innovation process requiring accumulated inputs through time, and by treating the interest rate as exogenous.[10] That model also inherits Shleifer's (1986) non-robustness to storage. In Matsuyama (1999) the clustering of innovations also results from the short–term nature of monopoly rents, though through a different channel. In his framework growth arises due to increasing product variety. Thus the upsurge in growth there arises through drastic innovations that represent wholly new (though partially substitutable) products, and is driven by a few leading sectors. This again lends itself more easily to a long cycle interpretation rather than the decentralized growth that we observe in the high volatility cycle. Freeman, Hong and Peled (1999) develop a model of cycles featuring a "time to build" component in innovation. As they emphasize, this technology describes "big" research or infrastructural projects, once again suggesting a long wave application of the cycle. However, the resulting dynamics of the economy are, at least superficially similar to those reported here.

The present paper proceeds as follows. Section 2 presents the economy's fundamentals and defines a general equilibrium, and, in Section 3, we show that one equilibrium of the model is an acyclical growth path that is qualitatively identical to that studied by Grossman and Helpman (1991). Section 4 presents the main results of the paper. We posit a cycle and derive the equilibrium behavior of households, firms and entrepreneurs that would be consistent with such a cycle. We then derive the sufficient conditions required for a unique cyclical equilibrium to

[10] The exogeneity of the interest rate and choice of technology in Francois and Shi (1999) are related. A non-memoryless research technology ensures labour allocations to R&D through the length of the cycle. In the present paper, this is achieved endogenously by adjustment in the endogenous interest rate. We show that movements in the interest rate play a crucial role in supporting the cycle.

exist, and show that the cyclical equilibrium is stable. Section 5 examines the implications of our equilibrium growth process for the endogenous relationship between long–run growth and short–run volatility, and for the impacts of a counter–cyclical fiscal policy. We also compare the long run growth and welfare in the acyclical and cyclical equilibria. In our conclusion, we discuss possible extensions of the model that would help to match business cycle facts more closely. Technical details of proofs and derivations are relegated to the appendix.

2 The Model

2.1 Assumptions

Time is continuous and indexed by t. We consider a closed economy with no government sector. Households have isoelastic preferences

$$U(t) = \int_t^\infty e^{-\rho(s-t)} \frac{c(s)^{1-\sigma}}{1-\sigma} ds \tag{1}$$

where ρ denotes the rate of time preference and we assume that $\sigma \in (0,1)$. Each household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]} c(\tau) d\tau \leq B(t) + \int_t^\infty e^{-[R(\tau)-R(t)]} w(\tau) d\tau \tag{2}$$

where $w(t)$ denotes wage income, $B(t)$ denotes the household's stock of assets at time t and $R(t)$ denotes the discount factor from time zero to t.

Final output is produced by competitive firms according to a Cobb-Douglas production function utilizing intermediates, k, indexed by i, over the unit interval:

$$y(t) = \exp\left(\int_0^1 \ln k_i(t) di\right). \tag{3}$$

Final output is storable (at an arbitrarily small cost), but cannot be converted back into an input for use in production. We let p_i denote the price of intermediate i.

Output of intermediate i depends upon the state of technology in sector i, $A_i(t)$, and the labor resources devoted to production, l_i, in a linear manner:

$$k_i^s(t) = A_i(t) l_i(t). \tag{4}$$

Labor receives the equilibrium wage $w(t)$. There is no imitation, so the dominant entrepreneur in each sector undertakes all production and earns monopoly profits by limit pricing until displaced by a higher productivity rival. We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

Competitive entrepreneurs in each sector attempt to find ongoing marginal improvements in productivity by diverting labor effort away from production and towards innovation.[11] They finance their activities by selling equity shares to households. The probability of an entrepreneurial success in instant t is $\delta x_i(t)$, where δ is a parameter, and x_i is the labor effort allocated to entrepreneurship in sector i. At any point in time, entrepreneurs decide whether or not to allocate labor effort to innovation, and if they do so, how much. The aggregate labour effort allocated to entrepreneurship is given by $X(t) = \int_0^1 x_i(t)dt$.

New innovations dominate old ones by a factor e^{γ}. Entrepreneurs with innovations, must choose whether or not to implement their innovation immediately or delay implementation until a later date. Once they implement, the knowledge associated with the innovation becomes publicly available, and can be built upon by rival entrepreneurs. However, prior to implementation, the knowledge is privately held by the entrepreneur. We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of periods in which innovations are implemented in sector i is denoted by Ω_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing an innovation at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

Finally, we assume the existence of arbitrageurs who instantaneously trade assets to erode any profit opportunities. There are three potential assets in our economy: claims to the profits of intermediate firms, stored intermediate output and stored final output. As we shall see, in all of the equilibria discussed below, only claims to the profits of intermediate firms will be traded — intermediate and final output are never stored. However, the potential for stored output to be traded imposes restrictions on the possible equilibria that can emerge.

In summary, our model is formally identical to that developed by Grossman and Helpman (1991), but with an elasticity of intertemporal substitution, $1/\sigma$, that exceeds unity. However, we have expanded the set of possible strategies by divorcing the realization of innovations from

[11] This process can equivalently be thought of as a search for product improvements, process improvements, organizational advances or anything else in the form of new knowledge which creates a productive advance over the existing state of the art.

the decision to implement them (as in Shleifer, 1986) and by allowing intermediate output to be potentially storable.

2.2 Definition of Equilibrium

Given an initial stock of implemented innovations represented by a cross–sectoral distribution of productivities $\{A_i(0)\}_{i=0}^1$ and an initial distribution of unimplemented innovations, $\{Z_i(0)\}_{i=0}^1$, an equilibrium for this economy satisfies the following conditions:

• Households allocate consumption over time to maximize (1) subject (2). The first–order conditions of the household's optimization require that

$$c(t)^\sigma = c(s)^\sigma e^{R(t)-R(s)-\rho(t-s)} \qquad \forall \; t, s, \tag{5}$$

and that the transversality condition holds

$$\lim_{s\to\infty} e^{-R(s)} B(s) = 0 \tag{6}$$

• Final goods producers choose intermediates to maximize profits. The derived demand for intermediate i is then

$$k_i^d(t) = \frac{y(t)}{p_i(t)} \tag{7}$$

• Intermediate producers set prices. It follows that the price of intermediate i is given by

$$p_i(t) = \frac{w(t)}{e^{-\gamma} A_i(t)} \tag{8}$$

and the instantaneous profit earned is

$$\pi_i(t) = (1 - e^{-\gamma}) y(t). \tag{9}$$

Note crucially that firm profits are proportional the aggregate demand.

• Labour market clearing:

$$\int_0^1 l_i(t) di + X(t) = 1 \tag{10}$$

Labour market equilibrium also implies

$$w(t)(1 - X(t)) = e^{-\gamma} y(t) \tag{11}$$

• Free entry into arbitrage. For all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{R(s)-R(t)}{s-t}$.

• There is free entry into innovation. Entrepreneurs select the sector in which they innovate so as to maximize the expected present value of the innovation. Also

$$\delta \max[V_i^D(t), V_i^I(t)] \leq w(t), \quad x_i(t) \geq 0 \qquad \text{with at least one equality} \tag{12}$$

• In periods where there is implementation, entrepreneurs with innovations must prefer to implement rather that delay until a later date

$$V_i^I(t) \geq V_i^D(t) \quad \forall\ t \in \Omega_i \tag{13}$$

• In periods where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } Z_i(t) &= 0, \\ \text{or if } Z_i(t) &= 1,\ V_i^I(t) \leq V_i^D(t) \quad \forall\ t \notin \Omega_i. \end{aligned} \tag{14}$$

In what follows we characterize two types of equilibria that satisfy these conditions. The first mirrors the familiar acyclical growth path analyzed by Grossman and Helpman (1991). However, the second is a growth path featuring regular downturns and upsurges in economic activity.

3 The Acyclical Equilibrium

Along an acyclical growth path, the rate of innovation is constant and output grows at a constant rate. The key feature of this equilibrium is that innovation occurs every period and implementation occurs immediately, so that $Z_i(t) = 0\ \forall\ i, t$. Although, this growth path is well understood, it is useful to briefly outline the equilibrium and, in particular, to see why implementation of innovations is never delayed.

In the acyclical equilibrium, consumption is a continuous function of time and its growth rate can be described by the familiar differential equation

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\sigma}. \tag{15}$$

where $r\,(t) = \dot{R}$ denotes the instantaneous interest rate. Since all innovations are implemented immediately, the aggregate rate of productivity growth is

$$g(t) = \delta\gamma X(t) \tag{16}$$

No–arbitrage implies that

$$r(t) + \delta X(t) = \frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} \tag{17}$$

Since, innovation occurs in every period, free entry into R&D implies that

$$\delta V(t) = w(t). \tag{18}$$

Putting these conditions together yields

Proposition 1 : *If*

$$1 - e^{-\gamma} < \frac{\rho}{\delta\gamma(1-\sigma)} < \frac{e^{\gamma} - 1}{\gamma(1-\sigma)}, \tag{19}$$

then there exists an acyclical equilibrium with a constant growth rate given by

$$g^a = \frac{[\delta(1 - e^{-\gamma}) - \rho e^{-\gamma}]\gamma}{1 - \gamma\,(1-\sigma)\,e^{-\gamma}}. \tag{20}$$

Along this equilibrium growth path the first inequality in (19) implies that $r(t) > g^a(t)$ at every moment.[12] Along a balanced growth path, this condition must hold for the transversality condition to be satisfied and hence for utility to be bounded. However, this condition also ensures both that no output is stored, and that the implementation of any innovation is never delayed. The return on storage is the growth in the price of the intermediate good in non–innovating sectors, which in turn equals $g^a(t)$. Thus, since $r(t) > g^a(t)$, it never pays to store the intermediate.[13] That delay is never optimal in this equilibrium can be seen by considering the extreme case where obsolescence is certain after implementation. In this case the gain from delay is the growth in profits equal to $g^a(t)$. However, since this gain is discounted at the rate $r(t)$, immediate implementation is always optimal. If obsolescence is not certain, the relative gain from immediate implementation is even greater.

4 The Cyclical Equilibrium

In this section we posit a cyclical growth path along which innovations are implemented in clusters rather than in a smooth fashion. We derive the optimal behavior of agents in such a cyclical

[12] The second condition in Proposition 1 ensures that entrepreneurs are sufficiently profitable to warrant investment, when $\sigma < 1$. Otherwise growth would be zero.

[13] Obviously, since $r > 0$, final output is never stored either.

equilibrium and the evolution of the key variables under market clearing. We derive sufficient conditions for the existence of such a cyclical equilibrium and show that market clearing implies a unique positive cycle length and long run growth rate.

Suppose that the implementation of entrepreneurial innovations occurs at discrete intervals. An implementation period is denoted by T_ν where $v \in \{1, 2, ..., \infty\}$, and we adopt the convention that the vth cycle starts in period T_{v-1} and ends in period T_ν. The evolution of output during a typical cycle between implementation is depicted in Figure 1. A boom occurs when accumulated innovations are implemented at T_{v-1}. After that there is an interval during which no entrepreneurial effort is devoted to improvement of existing technologies and consequently where all resources are used in production. During this interval, no new innovations are implemented so that growth slows to zero. At some time T_v^E innovation commences again, but successful entrepreneurs withhold implementation until time T_v. Entrepreneurial activity occurs throughout the interval $[T_v^E, T_v]$ and causes a decline in the economy's production, as resources are diverted away from production towards the search for improvements. At T_v all successful entrepreneurs implement, and the $(v+1)$th cycle starts with a boom.

Over intervals during which the discount factor does not jump, consumption is allocated as described by (15). However, as we will demonstrate here, along the cyclical growth path, the discount rate jumps at the boom, so that consumption exhibits a discontinuity during implementation periods.[14] We therefore characterize the optimal evolution of consumption from the beginning of one cycle to the beginning of the next by the difference equation

$$\sigma \ln \frac{c_0(T_v)}{c_0(T_{v-1})} = R(T_v) - R(T_{v-1}) - \rho \left(T_v - T_{v-1}\right). \tag{21}$$

where the 0 subscript is used to denote values of variables the instant after the implementation boom. Note that a sufficient condition for the boundedness of the consumer's optimization problem is that $\ln \frac{c_0(T_v)}{c_0(T_{v-1})} < R(T_v) - R(T_{v-1})$ for all v, or that

$$\frac{1}{T_v - T_{v-1}} \ln \frac{c_0(T_v)}{c_0(T_{v-1})} < \frac{\rho}{1-\sigma} \quad \forall\, v. \tag{22}$$

In our analysis below, it is convenient to define the discount factor that will be used to discount from some time t during the cycle to the beginning of the next cycle. This discount factor is given

[14] Discontinuities in consumption can only be ruled out if the discount factor evolves smoothly. Note further that only upward jumps in the discount factor are possible.

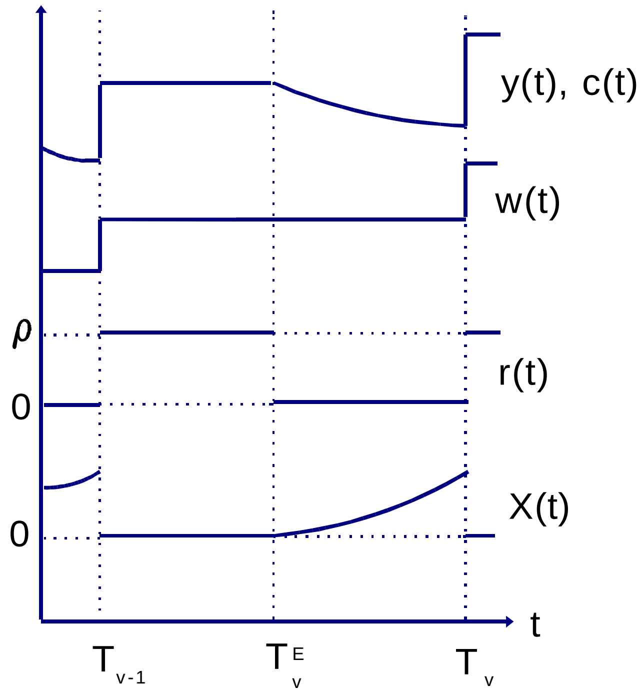

Figure 1: The Cyclical Growth Path

by

$$\beta(t) = R(T_v) - R(t) = R(T_v) - R(T_{v-1}) - \int_{T_{v-1}}^{t} r(s)ds. \tag{23}$$

4.1 Entrepreneurship

Let $P_i(s)$ denote the probability that, since time T_v, no entrepreneurial success has been made in sector i by time s. It follows that the probability of there being no innovation by time T_{v+1} conditional on there having been none by time t, is given by $P_i(T_{v+1})/P_i(t)$. Hence, the value of an incumbent firm in a sector where no innovation has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \int_t^{T_{v+1}} e^{-\int_t^\tau r(s)ds} \pi_i(\tau)d\tau + \frac{P_i(T_{v+1})}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_{v+1}). \tag{24}$$

The first term here represents the discounted profit stream that accrues to the entrepreneur with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter.

In the acyclical equilibrium, the role of secrecy is not relevant because innovators would always prefer to implement even if it were possible that, by delaying, they could protect their knowledge. Since simultaneous innovation can only occur with a second order probability in that equilibrium, it is assumed away. In the cyclical equilibrium considered here, secrecy (i.e. protecting the knowledge embodied in a new innovation by delaying implementation) can be a valuable option.[15] Innovations are withheld until a common implementation time, so that simultaneous implementation is a possibility. However, as the following Lemma demonstrates, such duplications do not arise in the cycling equilibrium:

Lemma 1 *In a cyclical equilibrium, successful entrepreneurs can credibly signal a success immediately and all research in their sector will stop until the next round of implementation.*

If an entrepreneur's announcement is credible, other entrepreneurs will exert their efforts in sectors where they have a better chance of becoming the dominant entrepreneur. One might imagine that unsuccessful entrepreneurs would have an incentive to mimic successful ones by falsely announcing success to deter others from entering the sector. However, in fact, there is no advantage to this strategy relative to the alternative of allocating effort to the sector until, with some probability, another entrepreneur is successful, and then switching to another sector.[16] This result stems from the memoryless nature of the Poisson process governing innovation — there is no advantage to having previously exerted effort in any given sector. An alternative assumption that will imply the same shutting down of innovation after a success, is that the allocation of entrepreneurial effort is directly observable so that success can be inferred directly.

In the cyclical equilibrium, entrepreneurs have conjectures that ensure no more allocation of entrepreneurship to a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v), \tag{25}$$

Since no implementation occurs during the cycle, the entrepreneur is assured of incumbency until at least T_{v+1}. Incumbency beyond that time depends on the probability that there has not been

[15] As Cohen, Nelson and Walsh (2000) document, delaying implementation to protect knowledge is a widely followed practice in reality.

[16] Readers concerned with the robustness of the equilibrium should note that, if we assume an arbitrarily small but positive signalling cost, the equilibrium would involve strictly dominant strategies.

another successful innovation in that sector up until then.[17] This depends on the amount of entrepreneurship conducted in that sector within the cycle. The symmetry of sectors implies that innovative effort is allocated evenly over all sectors that have not yet experienced an innovation within the cycle. Thus the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp\left(-\int_{T_v^E}^{T_v} \delta \tilde{x}_i(\tau) d\tau\right) \tag{26}$$

where $\tilde{x}_i(\tau)$ denotes the quantity of labor that would be allocated to entrepreneurship if no innovation had been discovered prior to time τ in sector i, recalling that T_v^E denotes the time at which entrepreneurship re-commences within the cycle. The amount of entrepreneurship varies over the cycle. However at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

4.2 Within–cycle dynamics

Within a cycle, $t \in [T_{v-1}, T_v]$, the state of technology in use is unchanging. A critical variable is the amount of labor devoted to entrepreneurship, the opportunity cost of which is production. In order to determine this, we first characterize wages paid to labour in production.

Lemma 2 *The wage for $t \in [T_{v-1}, T_v]$ is pinned down by the level of technology*

$$w(t) = e^{-\gamma} \exp\left(\int_0^1 \ln A_i(T_{v-1}) di\right) = w_v. \tag{27}$$

The wage is completely pinned down by the technology given competition between the producing firms in attempting to hire labour. This competition does not drive the wage up to labor's marginal product because firms earn monopolistic rents in their sectors. However, it does ensure that labor benefits proportionately from productivity advancements. We denote the improvement in aggregate productivity during implementation period T_v (and, hence, the growth in the wage) by e^{Γ_v}, where

$$\Gamma_v = \int_0^1 \left[\ln A_i(T_v) - \ln A_i(T_{v-1})\right] di \tag{28}$$

Since wages are determined by the level of technology in use, and since this does not change within the cycle, wages are constant within the cycle.

[17] A signal of further entrepreneurial success submitted by an incumbent is not credible in equilibrium. This is for the standard reason that innovation in other sectors is always more profitable than innovation in one's own, so that an incumbent's success signals do not dissuade innovation. Note also that though there is no patent protection and hence the possibility of imitation, this is weakly dominated given the Bertrand interaction between imitators and incumbents. With arbitrarily small costs to imitation this strategy is strictly dominated by non-imitation.

Following an implementation boom, the economy passes through two distinct phases:

The Slowdown:

As a result of the boom, wages rise rapidly. Since the next implementation boom is some time away, the present value of engaging in innovation falls below the wage, $\delta V^D(t) < w(t)$. During this phase, no labour is allocated to entrepreneurship and no new innovations come on line. Since technology is unchanging, final output must be constant

$$g(t) = \frac{\dot{w}(t)}{w(t)} = 0 \tag{29}$$

With zero growth, the demand side of the economy dictates that the interest rate just equal the discount rate,

$$r(t) = \sigma g(t) + \rho = \rho. \tag{30}$$

Since the economy is closed and there is no incentive to store either intermediate or final output when $r(t) \geq 0$, it must be the case that:

$$c(t) = y(t). \tag{31}$$

During the slowdown, the expected value of entrepreneurship, $\delta V^D(t)$, need not be equal across periods — it can be changing provided that entrepreneurship continues to be dominated by production. In fact, since the interest rate is positive over this phase, the value of entrepreneurship is necessarily growing at the rate ρ. Since the wage is constant during the cycle, $\delta V^D(t)$, must eventually equal $w(t)$. At this point, the entrepreneurship commences. The following Lemma demonstrates that it does so smoothly:

Lemma 3 *At time T_v^E, when entrepreneurship first commences in a cycle, $w_v = \delta V^D(t)$ and $X\left(T_v^E\right) = 0$.*

Proof: : See Appendix. ■

The Downturn:

For positive entrepreneurship to occur under free entry, it must be that $w_v = \delta V^D(t)$. Since the wage is constant throughout the cycle, the value of entrepreneurship, $\delta V^D(t)$, must also be constant during this phase. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits because implementation is delayed, the instantaneous interest rate must be zero.

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)} = 0. \tag{32}$$

With a positive discount rate, $\rho > 0$, a zero interest rate implies that consumption must be declining. Since the economy is closed, it follows once again that because there is no incentive to store output (31) holds.[18] Hence, per capita output must also decline:

$$g(t) = \frac{r(t) - \rho}{\sigma} = -\frac{\rho}{\sigma}. \tag{33}$$

This occurs during the downturn because labour flows out of production and into entrepreneurship (knowledge capital is being built). Using (11), (33) and the fact that $X(T_v^E) = 0$, yields the following expression for aggregate entrepreneurship at time t:

$$X(t) = 1 - e^{-\frac{\rho}{\sigma}[t - T_v^E]}. \tag{34}$$

The proportion of sectors that have not yet experienced an entrepreneurial success by time $t \in (T_v^E, T_v)$ is given by

$$P(t) = \exp\left(-\int_{T_v^E}^{t} \delta x(\tau) d\tau\right). \tag{35}$$

Recalling that labor is only devoted to entrepreneurship in sectors which have not innovated since the start of the cycle, the labor allocated to entrepreneurship in each sector is then

$$x(t) = \frac{X(t)}{P(t)}. \tag{36}$$

Differentiating (35), and substituting in (36), we thus obtain the aggregate rate of entrepreneurial success,

$$\dot{P}(t) = -\delta x(t) P(t) = -\delta X(t). \tag{37}$$

Observe that although the rate of decline in the proportion of sectors that have not yet innovated, $P(t)$, is proportional to the amount of entrepreneurship in each sector, the level reductions in P are proportional to the aggregate amount of entrepreneurship. This reflects the fact that as new

[18] Although $r = 0$, strict preference for zero storage results from arbitrarily small storage costs.

innovations arise, the aggregate labor effort is allocated across fewer and fewer sectors. It follows that if the cycle is sufficiently long it is possible that all sectors will innovate.

The dynamic movement of variables implied by our hypothesized cycle is sketched in Figure 1. The resulting allocation of labor to entrepreneurship (34) determines the size of the output boom at the end of the cycle. Denote the interval over which there is positive entrepreneurship by

$$\Delta_v^E = T_v - T_v^E. \tag{38}$$

Then we have:

Proposition 2 *In an equilibrium where there is positive entrepreneurship only over the interval* $(T_v^E, T_v]$*, the growth in productivity during the succeeding boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right). \tag{39}$$

Equation (39) tells us how the size of the productivity boom depends positively on the amount of time the economy is in the entrepreneurship phase, Δ_v^E. The amount of innovation in that phase is determined by the movements in the interest rate, so once the length of the entrepreneurship phase is known, the growth rate over the cycle is pinned down. The size of the boom is convex in Δ_v^E, reflecting the fact that as the boom approaches, the labor allocated towards innovation is increasing. This also implies that the boom size is increasing in the depth of the downturn, since from (34) the longer the downturn the greater the allocation of entrepreneurial effort and hence the larger the decline in output. This is a feature which the model here shares with GPT type Schumpeterian models of the cycle, such as described in Aghion and Saint Paul (1998) and which has received considerable empirical support (see Beaudry and Koop 1993, Pesaran and Potter 1997, and Altissimo and Violante 2001). Note that the size of the boom does not depend directly on the cycle length $T_v - T_{v-1}$. That is, the growth spur from entrepreneurship depends only on the amount of time that entrepreneurial effort was exerted, and not the amount of time between implementations of entrepreneurial success.

4.3 Market Clearing During the Boom

For an entrepreneur who is holding an innovation, $V^I(t)$ is the value of implementing immediately. During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that

$$V_0^I(T_v) > V_0^D(T_v) \tag{40}$$

Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = w_v. \tag{41}$$

From (40), the return to innovation at the boom is the value of immediate (rather than delayed) incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq w_{v+1} \tag{42}$$

The opportunity cost to financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no innovations have occurred

$$\beta(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right). \tag{43}$$

Note that since the short–term interest rate is zero over this phase, $\beta(t) = \beta(T_v)$, $\forall\ t \in (T_v^E, T_v)$. Combined with (41) and (42) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log\left(\frac{w_{v+1}}{w_v}\right) = \Gamma_v. \tag{44}$$

Free entry into innovation ensures that $\beta(T_v) > \Gamma_v$ cannot obtain in equilibrium.

Provided that $\beta(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next either because it is dominated by the long–run rate of return on claims to future profits. However, unlike final output, the return on stored intermediate output in sectors with no innovations, is strictly positive because of the increase in its price that occurs as a result of the boom. Even though there is a risk that the intermediate becomes obsolete at the boom, if the anticipated price increase is sufficiently large, households may choose to purchase claims to intermediate output rather than claims to firm profits.[19]

[19] Note that incumbent entrepreneurs will not be able to use storage to dissuade further innovation in their sector.

If innovative activities are to be financed at time t, it cannot be the case that households are strictly better off buying claims to stored intermediate goods. There are two types of storage that could arise, but the return to each is the same. In sectors with unimplemented innovations, entrepreneurs who hold innovations have the option of implementing immediately but not actually selling until the boom. The best way to do so is to hire labour and produce an instant prior to the boom; producing any earlier will not be any cheaper and will yield a higher probability of displacement. Also, the best time to sell is an instant after the boom, since after the boom interest rates are positive and demand is flat. Since the revenue is the same, the difference between producing an instant before the boom and an instant after the boom, is that the former involves the current wage and the latter involves the higher future wage. Thus, the return on claims to stored intermediates is $\log w_{v+1}/w_v = \Gamma_v$. In sectors with no innovation, incumbent firms could sell such claims, use them to finance greater current production and then store the good to sell at the beginning of the next boom when the price is higher. In this case, since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1} / p_{i,v} = \Gamma_v$.

It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$\beta(T_v) \geq \Gamma_v \tag{45}$$

Free–entry into arbitrage ensures that $\beta(T_v) < \Gamma_v$ cannot obtain in equilibrium. Because there is a risk of obsolescence, this condition implies that at any time prior to the boom the expected rate of return on claims to stored intermediates is strictly less than $\beta(t)$.

Combining this (44) and (45) yields the following implication of market clearing during the boom for the long–run growth path:

Proposition 3 *Long run asset market clearing requires that*

$$\Gamma_v = \frac{\rho \Delta_v^E}{1-\sigma}. \tag{46}$$

Since returns to innovation are identical across sectors, one may suppose that incumbents have an incentive to store intermediate production and threaten to use it to undercut any future innovator in their sector. If credible, such a threat would lead outside entrepreneurs to search for innovations in other sectors. However, such a threat is not credible. If faced with an innovator holding a productive advantage that will be implemented at time T, an incumbent would always have incentive to sell stockpiled intermediates before time T since by doing so they would obtain a higher price than by delaying and selling it in competition with the new innovator.

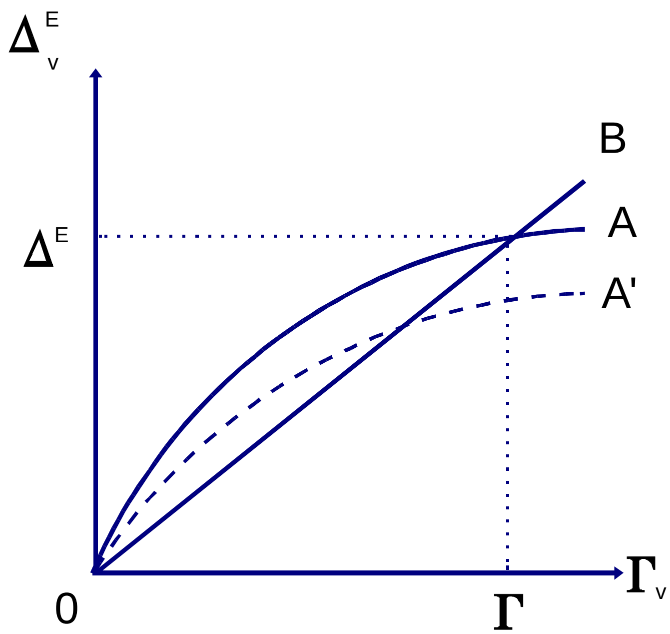

Figure 2: Equilibrium Recession length and Boom Size

Since the short term interest rate during the downturn is zero, asset market clearing requires that the long term interest rate at the end of the downturn is equal to its value at the beginning. The value at the end must equal the size of the productivity boom in equilibrium; the value at the beginning reflects the size of the future boom and the time until it occurs. It follows that asset market–clearing yields a unique relationship between the length of the downturn and the size of the subsequent productivity boom.

Figure 2 depicts the two conditions (39) and (46) graphically. As can be seen, combining the two conditions yields a unique (non–zero) equilibrium pair (Γ, Δ^E) that is consistent with the within–cycle dynamics and the asset market clearing condition. Substituting out Γ from (39) using (46) implies that Δ^E must satisfy

$$\left(1 - \frac{\rho}{\delta\gamma(1-\sigma)}\right)\Delta^E = \frac{1 - e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma} \tag{47}$$

Note that although we did not impose any stationarity on the cycles, the equilibrium conditions imply stationarity of the size of the boom and the length of the downturn. For a unique positive value of Δ^E that satisfies this condition to exist it is sufficient that $\frac{\rho}{\delta\gamma(1-\sigma)} < 1$.

4.4 Optimal Entrepreneurial Behavior

It has thus far been assumed that entrepreneurs are willing to follow the innovation and implementation sequence hypothesized in the cycle. Firstly, the equilibrium conditions that we have considered so far effectively assume that entrepreneurs who plan to delay implementation until the boom, are willing to just start engaging in innovative activities at exactly T_v^E. However, the willingness of entrepreneurs do this depends crucially on the expected value of monopoly rents resulting from innovation, relative to the current labour costs. This is a forward looking condition: given Γ and Δ^E, the present value of these rents depend crucially on the length of the subsequent cycle, $T_{v+1} - T_v$.

Since Lemma 3 implies that entrepreneurship starts smoothly at T_v^E, free entry into entrepreneurship, requires that

$$\delta V^D(T_v^E) = \delta e^{-\beta(T_v^E)} V_0^I(T_v) = w_v \tag{48}$$

Since the increase in the wage across cycles reflects only the improvement in productivity: $w_{v+1} = e^{\Gamma} w_v$, and since from the asset market clearing conditions, we know that $\beta(T_v^E) = \Gamma$, it immediately follows that the increase in the present value of monopoly profits from the beginning of one cycle to the next must, in equilibrium, reflect only the improvements in aggregate productivity:

$$V_0^I(T_{v+1}) = e^{\Gamma} V_0^I(T_v). \tag{49}$$

Equation (49) implies that given some initial implementation period and stationary values of Γ and Δ^E, the next implementation periods is determined. Notice, once again that this stationarity is not imposed, but is an implication of the equilibrium conditions. Letting $\Delta_v = T_v - T_{v-1}$, we therefore have the following result:

Proposition 4 *Given the boom size, Γ, and the length of the entrepreneurial innovation phase, Δ^E, there exists a unique cycle length, Δ, such that entrepreneurs are just willing to commence innovation, Δ^E periods prior to the boom.*

In the appendix we show that the implied cycle length is given by

$$\Delta = \Delta^E + \frac{1}{\rho} \ln \left[1 + \left(\frac{\frac{\rho}{\delta\gamma(1-\sigma)} - (1 - e^{-\gamma})}{\frac{1-e^{-\gamma}}{\rho} - \frac{e^{-\gamma}}{\delta}} \right) \Delta^E \right], \tag{50}$$

Note that for the equilibrium value of Δ to strictly exceed Δ^E (which it must) requires that $\frac{\rho}{\delta\gamma(1-\sigma)} > 1 - e^{-\gamma}$.

In addition, the equilibrium conditions (12), (13) and (14) on entrepreneurial behavior also impose the following requirements on our hypothesized cycle:

• Successful entrepreneurs at time $t = T_v$, must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{51}$$

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{52}$$

• No entrepreneur wants to innovate during the slowdown of the cycle. Since in this phase of the cycle $\delta V^D(t) < w(t)$, this condition requires that

$$\delta V^I(t) < w(t) \qquad \forall\, t \in (0, T_v^E) \tag{53}$$

Figure 3 illustrates the evolution of the relevant value functions in the cyclical equilibrium, and the productivity adjusted wage w_v/δ. At the beginning of the cycle $w_v = \delta V^I(T_v) > \delta V^D(T_v)$. Since the wage is constant, $\delta V^D(t)$ grows and $\delta V^I(t)$ declines during the first phase of the cycle, this condition implies that $\delta V^D(t)$ and $\delta V^I(t)$ must intersect before $\delta V^D(t)$ reaches $w(t)$. It follows that when entrepreneurship starts, it is optimal to delay implementation, $V^D(T_v^E) > V^I(T_v^E)$. Over time, during the entrepreneurship phase, the probability of not being displaced at the boom if you implement early declines so that $V^I(t)$ rises over time. Eventually, an instant prior to the boom, $V^I(T_{v+1}) = V^D(T_{v+1})$, but until that point it continues to be optimal to delay. At the boom, the value of immediate implementation rises by more than the value of delay, so that all existing innovations are implemented. However, since the wage increases by at least as much as $V^I(t)$, entrepreneurship ceases and the cycle begins again.[20]

[20] Generally it is necessary to have a value of $\sigma < 1$. Estimates of the elasticity of substitution, $1/\sigma$, based on aggregate consumption data are typically smaller than 1. However, as Beaudry and van Wincoop (1996) document, these estimates are biased towards zero, and that when one uses more disaggregate data, values above 1 cannot be rejected.

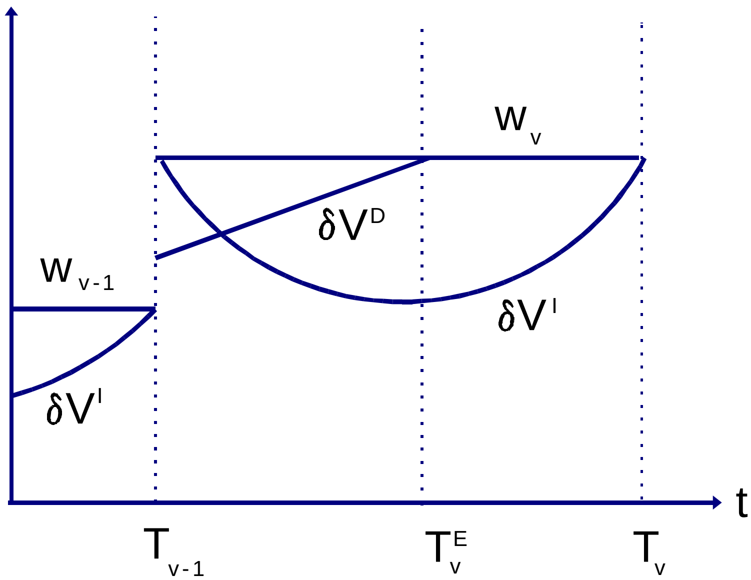

Figure 3: Evolution of Value Functions

4.5 Existence

The equilibrium conditions are satisfied under remarkably simple sufficient conditions:

Proposition 5 *If*

$$\frac{1-e^{-\gamma}}{e^{-\gamma}} < \frac{\rho}{\delta\gamma(1-\sigma)} < 1, \tag{54}$$

and if the values of $(\Delta^E, \Delta, \Gamma)$ *solving (39), (46) and (50), are such that* $\Gamma < \gamma$*, then there exists a unique cyclical equilibrium growth path.*

The first inequality in (54) is sufficient to ensure that the long run interest rate exceeds the long run growth rate as in (22), so that the transversality condition is satisfied. It also implies that during the cycle the short–run interest rate always exceeds the short–run growth rate. This implies that at the beginning of a cycle, implementation is never delayed, because any gain in profits from delay is less the rate at which it is discounted. However, during the downturn, this condition also implies that implementation is delayed until the next boom. To understand this, note that the boom is the only time during the cycle at which the increment in output exceeds the increment in the discount factor. Although, the increment in *productivity,* Γ, exactly equals the increment in the discount factor, β, the reallocation of labour resources back into production implies that output increases by more than the increment in productivity. Thus, the increase in profits at the boom exceeds the rate at which they are discounted. If the probability of being displaced is sufficiently low (which it will be towards the end of the cycle), there is an incentive to delay implementation.

The second inequality in (54) is necessary for there to exist a downturn length Δ^E, such that the resulting boom is consistent with asset market equilibrium. It is also sufficient to ensure that value of immediate implementation declines monotonically during the slowdown. It is straightforward to show that the two inequalities in (54) are also sufficient for the existence of the acyclical equilibrium, but not vice versa. However, provided γ is small, the conditions for the cyclical equilibrium are not much more demanding. The additional requirement that $\Gamma < \gamma$, ensures that not all sectors innovate during the cycle.

Entrepreneurship in our cycle increases through the downturn when its opportunity cost is low. In this sense, the downturn of our cycle creates a beneficial effect for the economy, as in Caballero and Hammour (1995) where the downturn cleans out inefficient firms. The benefits of downturns in our model are very different, however, as they are intimately linked to the economy's growth process. The actions of entrepreneurs here also resemble the "reorganization" activity emphasized by Hall (2000) in his model of the cycle. Though entrepreneurial activity through the cycle is difficult to measure, it is likely to be positively correlated with other, more measurable, forms of effort re-allocation through the cycle. In this regard it is interesting to note that, consistent with our model, US post-secondary educational investments are found to be counter-cyclical, see Dellas and Sakellaris (1997). This counter-cyclicality of enrolments is also found in a broader group of OECD countries by Sakellaris and Spilimbergo (2000).

4.6 Stability

4.6.1 Stability of the Instantaneous equilibrium

We first assess the stability of the instantaneous equilibrium is in response to small perturbations in the behavior of agents. That is, as a result of errors made by a small measure of agents, will ensuing market prices reinforce those errors and thereby signal to other agents optimal behavior which is further inconsistent with equilibrium behavior? In every instant of the cycle, labor chooses between entrepreneurship and production, entrepreneurs choose between implementing today or delaying until tomorrow, and incumbents must decide whether to sell now or store. We now consider the stability of the instantaneous equilibrium with respect to each of these decisions in turn:

Stability in the labour market: Suppose first that too few agents engage in entrepreneurship than is implied by our equilibrium. In the no entrepreneurship phase, production strictly dominates entrepreneurial effort, $w(t) > \delta \max[V^D(t), V^I(t)]$. Small errors of this kind, will thus not affect others' optimal behavior. The same is not true, however, in the positive entrepreneurship phase, when $w(t) = \delta V^D(t)$. Consider a small perturbation of the equilibrium such that at t, insufficient labor flows into entrepreneurship. Then from (33) the short–run growth rate is given by $g(t) = -\frac{\rho}{\sigma} + \varepsilon$. It follows from (15) that $r(t) > 0$, so that $\frac{\dot{V}^D(t)}{V^D(t)} > 0$, which, since $\frac{\dot{w}(t)}{w(t)} = 0$, implies more labour will be allocated to entrepreneurship. Thus, since fewer than the equilibrium number of individuals flowing into entrepreneurship results in greater incentives for entrepreneurship, the relationship is stable.

Stability of entrepreneurial delay: Suppose now that some measure of entrepreneurs erroneously implement their innovations immediately during the downturn rather than delaying. In this phase $w(t) = \delta V^D(t) > \delta V^I(t)$. Such unanticipated implementation leaves $V^D(t)$ unchanged because the technology in use at the time of the boom is unchanged; some of it has just been implemented earlier. However, it raises $V^I(t)$ due to the unexpected increasing in productivity of technology $\tau \in [t, T_v]$. Since $V^D(\tau)$ is discretely bigger than $V^I(\tau)$ over this phase, a small deviation from the equilibrium has no effect on incentives to delay. This is even true the instant before implementation, since $\lim_{\tau \to T_v} V^D(\tau) - V^I(\tau) = 0$.[21] This limit still converges following a surprise implementation, since the two terms are equivalent at T_v. Thus, though earlier implementation raises V^I relative to V^D, it cannot alter the relative ranking of the two.

Stability of No–Storage: Long–run asset market clearing in a cyclical equilibrium requires that the return on claims to firm profits equals the return to storage in the last instant of the cycle, $\beta(t) = \Gamma$. Suppose that someone mistakenly offers to finance the production of extra intermediate output for storage, by buying claims to the stored output. This act will effectively draw some labour effort out of entrepreneurship causing the anticipated value of Γ to decline so

[21] To see this compute the limit in equation (??) in the appendix.

that $\beta(t) > \Gamma$. In the next instant the buyer will be better off selling the intermediate output and using the proceeds to finance entrepreneurs, which will restore the equilibrium.

4.6.2 Dynamic Stability

A second notion of stability relates to the dynamic convergence of the economy to its long–run growth path. Like the acyclical growth path, the cyclical equilibrium is "jump stable". As our analysis demonstrates, there is a unique triple $\left(\Gamma, \Delta^E, \Delta\right)$ that is consistent with equilibrium. Thus the economy's dynamics necessarily involve jumps to this long run path since no other $\left(\Gamma, \Delta^E, \Delta\right)$ triple can hold, even in the short run, without violating the equilibrium conditions. In principle, the economy could jump to the acyclical equilibrium if expectations regarding which equilibrium the economy is in were to change in a coordinated way. However, it should be emphasized that the cycles generated by our model are *not* the result of exogenous shifts in expectations, as in Evans, Honkapohja and Romer (1998) for example. In Section 5, we discuss in more detail the dynamic adjustment in response to changes in the models parameters. Note finally that, although $\left(\Gamma, \Delta^E, \Delta\right)$ are unique, strictly speaking there are multiple cyclical equilibria exhibiting identical cyclical properties and long run growth. The reason is that the length of the first cycle Δ_0 is indeterminate on the interval $[\Delta^E, \Delta]$.

5 Implications for Long–Run Growth and Volatility

In this section we compare the long–run growth rates in the cycling and acyclical economies and discuss some of the implications of the equilibrium that we have characterized for the impact of parameter and policy changes on cycles, growth and the relationship between them.

5.1 Growth and welfare in cyclical and acyclical economies

Let the average growth rate in the cycling equilibrium be denoted

$$g^c = \Gamma/\Delta, \tag{55}$$

and recall the acyclical equilibrium growth, g^a given in (20). Then we have

Proposition 6 *The long run growth rate in the cyclical equilibrium g^c exceeds that in the acyclical equilibrium, g^a.*

The cyclical equilibrium yields higher average growth because all entrepreneurship occurs in the downturn when growth is negative and the interest rate is low relative to the economy's long run average. Thus compared with the acyclical economy where the interest rate is constant, the same expected flow of profits for the same expected length of incumbency has higher value in the cycling economy, thereby inducing more entrepreneurship and higher growth.

Although the long–run growth rate is higher in the cyclical equilibrium, the same is not true of welfare. Consider two economies that start with an identical stock of implemented technologies and zero unimplemented innovations. Suppose one of the economies is in a cyclical equilibrium at the beginning of a cycle and the other is in an acyclical equilibrium. There are three key differences that determine relative welfare in the two economies: (1) the long–run growth rate in the cyclical economy is higher, (2) the initial consumption in the cyclical economy is higher because some labor is allocated to production in the acyclical economy, whereas none is during this phase of the cyclical equilibrium, and (3) until the next boom, the short–run growth rate in the cyclical economy is zero or negative, whereas it is positive in the acyclical one. As we illustrate below, this last factor tends to dominate so that welfare is lower in the cyclical economy.[22]

Table 1 compares the welfare consequences of moving from the cyclical to the acyclical equilibrium under a broad range of parameters that are consistent with the model's existence conditions. It turns out that the welfare benefits of removing cycles are relatively large, around 3%. The reason for this is that the growth rate differences are relatively small and the labor allocated to entrepreneurship in the acyclical economy is also small, so that the third effect on welfare, discussed above, swamps the other two. Note, however, that welfare in the cycling economy is sensitive to the point in the cycle where welfare is computed. Since technology is fixed and there is no innovation over the slowdown, we could start at any point in the interval $\left[T_{v-1}, T_v^E\right]$. If welfare is computed at the end of a slowdown, T_v^E, as opposed to at its start, T_{v-1}, the first boom arrives earlier and hence welfare is higher.

[22]Lucas (1987) performs a related comparison but without an underlying structural model of the economy. He simply computes the welfare improvement from eradicating the cycle while maintaining the same average growth rate and finds this welfare gain to be extremely small.

				Growth Rate Cycles	Growth Rate No-cycles	Welfare Increase	Δ
Benchmark	Parameters						
$\gamma = .12$	$\rho = .025$	$\sigma = .25$	$\delta = 2$	2.666%	2.660%	2.98%	3.83
$\gamma = .115$				2.440%	2.429%	2.82%	4.4
$\gamma = .125$				2.910%	2.902%	3.13%	3.4
	$\rho = .022$			2.704%	2.695%	3.02%	3.31
	$\rho = .028$			2.640%	2.625%	2.94%	4.4
		$\sigma = .2$		2.681%	2.676%	2.39%	2.67
		$\sigma = .27$		2.664%	2.654%	3.21%	4.4
			$\delta = 2.4$	3.258%	3.250%	3.03%	2.58
			$\delta = 1.8$	2.382%	2.365%	2.942%	4.85

Table 1: Growth and Welfare Differences across equilibria

5.2 Impact of Entrepreneurial Productivity

Consider the impact of an increase in entrepreneurial productivity δ on the cyclical growth path.[23] One variable of particular interest, aside from the long–run growth rate, is the economy's volatility. We measure this as the mean squared deviation of log consumption from its trend:

$$\Sigma^2 = \frac{1}{\Delta}\int_0^{\Delta-\Delta^E} [\Gamma - g^c t]^2\, dt + \frac{1}{\Delta}\int_{\Delta-\Delta^E}^{\Delta} \left[\Gamma - \frac{\rho}{\sigma}(t-(\Delta-\Delta^E)) - g^c t\right]^2 dt. \tag{56}$$

Proposition 7 *An increase in δ results in shorter cycles, smaller booms, shallower recessions and, hence, lower volatility.*

To understand these results first consider Figure 2. For a given cycle length and downturn length (Δ, Δ^E), an increase in δ causes the size of the boom to be larger because entrepreneurship is now more productive. This is illustrated by the outward shift in OA to OA'. However, now the economy would be to the right of OB, so that the asset market is out of equilibrium, with $\beta < \Gamma$ just prior to the boom, so that there is an incentive to store. Arbitrageurs would be willing to offer incumbents and entrepreneurs incentives to produce more intermediate output than needed to supply current demand. In particular, entrepreneurs with unimplemented innovations would respond by bringing production forward slightly from the boom. But if all entrepreneurs do this, the boom would actually occur earlier and the incentive to store would disappear. Applying this argument recursively, one can see that the length of the downturn (and hence the entire)

[23]In our model human capital is normalized to unity. However, in a more general set up, varying the amount of human capital would be equivalent to varying δ.

cycle would fall until it is just short enough to ensure that for the (smaller) size of the boom that results, the incentive to produce early and store has been removed (i.e. $\beta = \Gamma$ just prior to the boom). Thus, as noted in the proposition, the cycle length, recession length and boom size, would all fall. Although the adjustment process for this economy is simply a jump to the new equilibrium, forcing more gradual adjustment by altering the model's dynamics would yield a similar outcome.[24]

The implication for long run growth, $g^c = \frac{\Gamma}{\Delta}$, depends on how much Γ falls relative to Δ. It turns out, however, that deriving the effect of changes in δ on the growth rate is not analytically straightforward. Numerical simulations suggest that increasing δ increases the average growth rate, as one would expect. In fact, after extensive simulations we have not been able to find a single exception to this. Essentially, an increase in δ causes the length of the downturn to fall proportionately less than the entire cycle length. Combining this with the impacts on volatility implies that:

Across economies with different values of δ, there exists a negative relationship between long–run growth and volatility.

Thus, the cyclical equilibrium is, at least superficially, consistent with the results of Ramey and Ramey (1995). Note however that this relationship does not represent the impact of volatility on growth, nor the impact of growth on volatility. Rather it is an induced relationship due to variation in the productivity of entrepreneurship.

5.3 Impact of a Tax on Producers

Shleifer (1986) shows that a tax on intermediate producers' profits could be used as a method of dampening fluctuations. Consider the impact of such a tax, z, (or equivalently a subsidy to entrepreneurs) that is redistributed back to households in a lump–sum fashion.

Proposition 8 *A tax on production reduces average consumption volatility, but also reduces long run growth.*

[24] Although we do not explicitly allow this here, under more gradual adjustment (say if Δ^E were initially unable to change) the higher value of Γ provides incentives to implement earlier, thus shrinking the entrepreneurial phase, Δ^E, and the size of the cycle, so that, with gradual adjustment, the economy converges to the new less volatile steady state.

The tax on producers affects neither the asset market equilibrium nor the impact of the length of the innovation phase on the size of the boom. Thus, the equilibrium values of Γ and Δ^E remain unchanged. However, an increase in the tax rate means that the length of subsequent cycles must be longer to induce the same rate of entrepreneurship. The length of the cycle now satisfies

$$\Delta = \Delta^E + \frac{1}{\rho} \ln \left[1 + \left(\frac{\frac{\rho}{\delta\gamma(1-\sigma)} - (1 - e^{-\gamma})}{(1 - z)\frac{1-e^{-\gamma}}{\rho} - \frac{e^{-\gamma}}{\delta}} \right) \Delta^E \right]. \tag{57}$$

It follows that the equilibrium value of Δ increases, so that long–run growth, Γ/Δ, declines. The longer cycles with no increase in recession depth or boom size, result in lower volatility. Not surprisingly then a dampening tax comes at a cost, which did not appear in Shleifer since growth was exogenous there.

6 Concluding Remarks

This paper has established the existence of cycles along a balanced growth path of a completely standard multi–sectoral Schumpeterian growth model, that allows for the possibility of delayed implementation and storage. Specifically, we show that: even with multiple sectors, in general equilibrium, with reasonable assumptions on preferences, technology and market competition, no static increasing returns to scale, no stochastic expectations, no threshold effects, and rational forward looking behavior, there exists a business cycle that is interlinked with the economy's growth process. Moreover, we establish conditions under which a unique cycling equilibrium arises. The equilibrium cycle's necessary and sufficient conditions, though conceptually complicated, are relatively simple, and are only marginally more restrictive than those required to generate the non-cycling (standard) equilibrium.

The endogenous cycles generated by our model have several features that we believe are crucial to understanding actual business cycles. First and foremost, the cyclical fluctuations are the result of independent actions by decentralized decision–makers. They are not the result of economy–wide shocks or economy–wide technological breakthroughs, but emerge as a result of pecuniary demand externalities that induce coordination. This is true of both the boom, which reflects Shleifer's formalization of "animal spirits" in the joint implementation of innovations, and of the downturn, which reflects the common incentives of entrepreneurs in anticipation of the upcoming boom. Second, as in our cycle, the quantitative analyses of Emery and Koenig (1992), Sichel (1993) and Balke and Wynne (1995), suggest that the average cycle starts with a growth spurt which is then followed by a growth slowdown before the economy enters a period of relatively constant decline during the recessionary phase. Thirdly, as is consistent with the findings of Beaudry and Koop (1993), Pesaran and Potter (1997) and Altissimo and Violante (2001) there is a positive feedback from downturns to subsequent cyclical upturns. Finally, the equilibrium relationship between growth and volatility is negative, which is consistent with the cross–country evidence of Ramey and Ramey (1995).

While we believe our analysis provides a useful step in understanding the endogenous linkages between growth and business cycles, the basic model must be extended along several dimensions before it can be compared with the data in any meaningful way. In particular, the nature of the cycle generated by our simple model does not map very well into the data. Nevertheless, we believe that the central mechanism we have described here is robust to various extensions which we are currently developing and which we briefly outline below:

• Smoothing the cycle — The growth spurt and the start of the slowdown are unrealistically abrupt. In reality expansions tend to be spread out over time, so that positive growth is more common than zero or negative growth. However, the expansion can be made longer and smoother by allowing for a period of learning–by–doing in sectors with newly implemented innovations, such that maximum productivity is not achieved immediately.[25] In contrast to Shleifer (1986), innovations are not immediately imitated upon implementation and incumbents retain their position for the duration of the cycle. So long as firms learn quickly enough to ensure that the initial wage exceeds the value of entrepreneurship, the cycle continues to exist.

• Unskilled Labour — The downturn in our cycle results from the allocation of labour to entrepreneurship in anticipation of the upcoming boom. Although measured labor productivity falls during the downturn, real wages do not, so in this sense workers are not made worse off. Introducing unskilled workers, workers who can only be used in production, yields more realistic implications. In particular, although the unit cost of production remains constant over the cycle, the wages of the skilled rise and the wages of the unskilled fall as the relative demand for skilled workers rises in response to the approaching boom.

• Physical Investment — Although we allow for the possibility of storage we assume away physical capital as a vehicle for smoothing aggregate consumption over time. In Shleifer's (1986) model introducing physical capital in a standard way would destroy the cyclical equilibrium because

[25] Alternatively one could introduce adjustment costs in the reallocation of labor across sectors.

households would try to consume the benefits of the boom in advance by dissaving. This would not arise in our equilibrium. The reason is that a decline in production prior to the boom is necessary in our model to free up resources for growth–promoting activities. Although it cannot be optimal for consumption to jump discontinuously at the boom, output and investment would.

• Aggregate uncertainty — The length and other characteristics of actual business cycles, vary from cycle to cycle and look rather different from the deterministic equilibrium cycle described here. However, introducing a degree of aggregate uncertainty would be possible without changing the basic analysis. For example, the stochastic arrival of GPTs that raises productivity in all sectors, say, would cause the size and length of booms and recessions between GPTs to vary over time.

A valuable feature of the model developed here is its parsimony. Apart from a slight generalization of preferences, the model is identical to Grossman and Helpman (1991, Ch. 4). The ultimate value of theoretical endeavors aimed at understanding the interactions between growth and cycles will be in their ability to provide a convincing account of the high frequency data. While the model does fit some features of high frequency cycles well, we do not claim to have done that yet. However, the model's simplicity allows it to be used as a platform for these more empirically motivated extensions.

6.1 Appendix

Proof of Proposition 1: From the household's Euler equation we have

$$\sigma g(t) + \rho = r(t). \tag{58}$$

Differentiating (18) yields

$$\frac{\dot{V}(t)}{V(t)} = \frac{\dot{w}(t)}{w(t)} = g(t) \tag{59}$$

Substituting into (17) using gives

$$r(t) = \frac{\delta(1-e^{-\gamma})y(t)}{w(t)} + g(t) - \delta X(t) \tag{60}$$

$$r(t) = \frac{\delta(1-e^{-\gamma})(1-X(t))}{e^{-\gamma}} + \delta\gamma X(t) - \delta X(t) \tag{61}$$

In equating (58) and (61) and solving for the stationary allocation of labor to entrepreneurship thus yields

$$X(t) = X^* = \frac{\delta(1-e^{-\gamma}) - \rho e^{-\gamma}}{\delta - (1-\sigma)e^{-\gamma}\delta\gamma}. \tag{62}$$

Substituting into (16) gives (20). Note that with $\sigma < 1$, the existence of a positive growth path requires that $\delta(1-e^{-\gamma}) > \rho e^{-\gamma}$ which rearranges to the second inequality in (19). Also for utility to bounded and the transversality condition to hold requires that $r(t) > g(t)$. Using (20) and (58) a sufficient condition given by the first inequality in (19).

Proof of Lemma 1 We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1) entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the existing production methods. If another entrepreneur continues to innovate in that sector, with positive probability they will also develop a productive advantage of e^{γ}, the expected profit from implementing such an innovation is 0, since, in developing their improvement, they have not been able to observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to this are strictly less than attempting to innovate in another sector where there has been no signal of success, or from simply working in production, $w(t) > 0$.

Part (2): Assuming success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease from Part (1) by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.

Proof of Lemma 2: From the production function we have

$$\ln y(t) = \int_0^1 \ln \frac{y(t)}{p_i(t)} di \tag{63}$$

Substituting for the prices $p_i(t)$ using (8) yields

$$0 = \int_0^1 \ln \frac{w(t)e^{\gamma}}{A_i(T_{v-1})} di \tag{64}$$

which re-arranges to (27).

Proof of Lemma 3: First note that in any preceding no-entrepreneurship phase, $r(t) = \rho$. Thus, since, in a cycling equilibrium, the date of the next implementation is fixed at T_v, the expected value of entrepreneurship, δV^D, also grows at the rate $\rho > 0$. Thus, if under $X(T_v^E) = 0$, $\delta V^D(T_v^E) > w_v$, then the same inequality is also true the instant before, i.e. at $t \to T_v^E$, since w_v is constant within the cycle. But this violates the assertion that entrepreneurship commences at T_v^E. Thus necessarily, $\delta V^D(T_v^E) = w_v$ at $X\left(T_v^E\right) = 0$.

Proof of Proposition 2: Long–run productivity growth is given by

$$\Gamma_{v+1} = \int_0^1 \ln A_i(T_{v+1}) di - \int_0^1 \ln A_i(T_v) di = (1 - P(T_v))\gamma \tag{65}$$

Integrating (37) over the entrepreneurship phase yields the economy's stock of accumulated knowledge capital over the cycle:

$$1 - P(T_v) = \int_{T_{v+1}^E}^{T_{v+1}} \delta X(t) dt \tag{66}$$

Substituting for $X(\cdot)$ using (34) yields

$$1 - P(T_v) = \delta \int_{T_{v+1}^E}^{T_{v+1}} \left(1 - e^{-\frac{\rho}{\sigma}[t - T_{v+1}^E]}\right) d\tau \tag{67}$$

which upon integration gives (39).

Proof of Proposition 3: The increase in output from the beginning of one cycle to the beginning of the next reflects only the improvement in productivity $y_0(T_v) = e^{\Gamma_v} y_0(T_{v-1})$. Moreover, since all output is consumed it follows that $c_0(T_v) = e^{\Gamma_v} c_0(T_{v-1})$. This implies that the long run discount factor is given by

$$\beta(t) = \sigma\Gamma_v + \rho\left(T_{v+1} - T_v\right) - \int_{T_v}^{t} r(s)ds. \tag{68}$$

In particular, since $r(t) = 0$ during the downturn,

$$\beta(t) = \sigma\Gamma_v + \rho\Delta_v^E \quad \forall\, t \in (T_v^E, T_v). \tag{69}$$

Combining this with (44) and (45) yields (46).

Proof of Proposition 4: The discounted monopoly profits from owning an innovation at time T_v is given by

$$V_0^I(T_v) = (1 - e^{-\gamma}) \int_{T_v}^{T_{v+1}} e^{-\int_{T_v}^{\tau} r(s)ds} y(\tau)d\tau + P(T_v)e^{-\beta(T_v)} V_0^I(T_{v+1}). \tag{70}$$

Substituting for $V_0^I(T_{v+1})$ using (49), it follows that

$$V_0^I(T_v) = \left(\frac{(1-e^{-\gamma})y_0(T_v)}{1 - P(T_v)e^{\Gamma - \beta(T_v)}}\right) \int_{T_v}^{T_{v+1}} e^{\int_{vT}^{\tau}[g(s)-r(s)]ds} d\tau \tag{71}$$

Integrating yields

$$V_0^I(T_v) = \left(\frac{(1-e^{-\gamma})y_0(T_v)}{1 - P(T_v)e^{\Gamma - \beta(T_v)}}\right) \left[\frac{1 - e^{-\rho(T_{v+1}^E - T_v)}}{\rho} + e^{-\rho(T_{v+1}^E - T_v)} \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)\right]. \tag{72}$$

Asset market clearing during the boom and the fact that $X(T_v) = 0$ implies (using (11)) that

$$\delta V_0^I(T_v) = w_{v+1} = e^{-\gamma} y_0(T_v). \tag{73}$$

Substituting into (72) we have

$$(1 - e^{-\gamma})\delta \left[\frac{1 - e^{-\rho(T_{v+1}^E - T_v)}}{\rho} + e^{-\rho(T_{v+1}^E - T_v)} \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)\right] = e^{-\gamma}\left(1 - P(T_v)e^{\Gamma - \beta(T_v)}\right). \tag{74}$$

But $\beta(T_v) = \rho(T_{v+1}^E - T_v) + \Gamma$, so that multiplying through by $e^{\rho(T_{v+1}^E - T_v)}$ yields

$$(1 - e^{-\gamma})\delta \left[\frac{e^{\rho(T_{v+1}^E - T_v)} - 1}{\rho} + \left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)\right] = e^{-\gamma}\left(e^{\rho(T_{v+1}^E - T_v)} - 1 + 1 - P(T_v)\right). \tag{75}$$

Collecting terms we have

$$\left(\frac{(1-e^{-\gamma})\delta}{\rho} - e^{-\gamma}\right)\left(e^{\rho(T_{v+1}^E - T_v)} - 1\right) = e^{-\gamma}\left(1 - P(T_v)\right) - (1 - e^{-\gamma})\delta\left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right). \tag{76}$$

From (??) and (46) we have that $1 - P(T_v) = \frac{\rho\Delta^E}{\gamma(1-\sigma)}$ and using (47) to substitute out the second term on the r.h.s. we get

$$\left(\frac{(1-e^{-\gamma})\delta}{\rho} - e^{-\gamma}\right)\left(e^{\rho(T^E_{v+1}-T_v)} - 1\right) = e^{-\gamma}\frac{\rho\Delta^E}{\gamma(1-\sigma)} - (1-e^{-\gamma})\left(\delta - \frac{\rho}{\gamma(1-\sigma)}\right)\Delta^E \tag{77}$$

Rearranging yields

$$e^{\rho(T^E_{v+1}-T_v)} = 1 + \left(\frac{\frac{\rho}{\gamma(1-\sigma)} - (1-e^{-\gamma})\delta}{\frac{(1-e^{-\gamma})\delta}{\rho} - e^{-\gamma}}\right)\Delta^E \tag{78}$$

Taking logs and noting that $T^E_{v+1} - T_v = T_{v+1} - T_v - \Delta^E = \Delta_v - \Delta^E$ yields (50).

Proof of Proposition 5: It is easily verified that under the conditions in (54) there does exist a unique triple $(\Delta^E, \Delta, \Gamma) > 0$ which solves (39), (46) and (50). In addition, as described in the text, the following three conditions must also be satisfied for this to be an equilibrium:

(E1) $V^I_0(T_v) > V^D_0(T_v)$.
(E2) $V^I(t) < V^D(t) \qquad \forall\, t \in (T^E_v, T_v)$
(E3) $\delta V^I(t) < w(t) \qquad \forall\, t \in (0, T^E_v)$.

We prove that each of these conditions hold under (54) in turn:

(E1) Since $V^I_0(T_{v+1}) = e^{\Gamma} V^I_0(T_v)$, we can write

$$V^D_0(T_v) = e^{-\beta(T_v)+\Gamma} V^I_0(T_v). \tag{79}$$

Since $\beta(T_v) = \rho\Delta + \sigma\Gamma$, condition (E1) requires that $\rho\Delta > (1-\sigma)\Gamma$, which must be true for the consumer's optimization problem to be bounded. Using (46), this condition simply requires that $\Delta > \Delta^E$, which, from (50) is true as long as $\frac{\rho}{\delta\gamma(1-\sigma)} > 1 - e^{-\gamma}$. Since $e^{-\gamma} < 1$, this strictly holds if the first inequality in (54) is satisfied.

(E2) This inequality can be written as

$$V^I(t) = \int_t^{T_v} e^{-\int_t^\tau r(s)ds}\pi(\tau)d\tau + \frac{P(T_v)}{P(t)} V^D(t) < V^D(t) \qquad \forall\, t \in (T^E_v, T_v) \tag{80}$$

During the downturn we know that $V^D(t) = w_{v-1}/\delta = e^{-\gamma} y_0/\delta$ and $r(t) = 0$. Thus, we can write the condition as

$$(1-e^{-\gamma})y_0 \int_t^{T_v} e^{-\frac{\rho}{\sigma}(\tau - t)} d\tau + \frac{P(T_v)}{P(t)} e^{-\gamma} y_0/\delta < e^{-\gamma} y_0/\delta \tag{81}$$

Rearranging yields

$$e^{-\gamma}\left(1-\frac{P(T_v)}{P(t)}\right)-(1-e^{-\gamma})\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}(T_v-t)}}{\rho/\sigma}\right)>0, \tag{82}$$

where

$$P(t)=1-\int_{T_v^E}^{t}\delta\left(1-e^{-\frac{\rho}{\sigma}[\tau-T_v^E]}\right)d\tau=1-\delta[t-T_v^E]+\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}[t-T_v^E]}}{\rho/\sigma}\right). \tag{83}$$

When $t=T_v$, this becomes

$$P(T_v)=1-\delta\Delta^E+\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right). \tag{84}$$

First observe that $\log P(t)$ is decreasing and convex in t:

$$\frac{d\log P}{dt}=-\frac{\delta(1-e^{-\frac{\rho}{\sigma}(t-T_v^E)})}{1-\delta(t-T_v^E)+\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}(t-T_v^E)}}{\rho/\sigma}\right)}<0 \tag{85}$$

$$\frac{d^2\log P}{dt^2}=-\frac{P(t)\delta\frac{\rho}{\sigma}e^{-\frac{\rho}{\sigma}(t-T_v^E)}+\delta(1-e^{-\frac{\rho}{\sigma}(t-T_v^E)})^2}{P(t)^2}<0. \tag{86}$$

It follows that

$$-\frac{\log P(T_v)-\log P(t)}{T_v-t}\geq\left.\frac{d\log P(t)}{dt}\right|_{t=T_v} \tag{87}$$

Let

$$q=-\frac{\sigma}{\rho}\left.\frac{d\log P(t)}{dt}\right|_{t=T_v}=\frac{\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)}{1-\delta\Delta^E+\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)}. \tag{88}$$

Since $\delta\Delta^E>1$, it follows that $q>1$. Hence

$$\log P(T_v)-\log P(t)\leq -q\frac{\rho}{\sigma}(T_v-t) \tag{89}$$

Rearranging gives

$$1-\frac{P(T_v)}{P(t)}\geq 1-e^{-q\frac{\rho}{\sigma}(T_v-t)} \tag{90}$$

It follows that a sufficient condition for (82) is that

$$e^{-\gamma}\left(1-e^{-q\frac{\rho}{\sigma}(T_v-t)}\right)-\left(1-e^{-\gamma}\right)\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}(T_v-t)}}{\rho/\sigma}\right)\geq 0 \tag{91}$$

We know that (82), and hence (91), holds with equality at $t=T_v$. It follows that a sufficient condition is that the l.h.s. of (91) declines monotonically with $t<T_v$. That is

$$-e^{-\gamma}q\frac{\rho}{\sigma}e^{-q\frac{\rho}{\sigma}(T_v-t)}+\left(1-e^{-\gamma}\right)\delta e^{-\frac{\rho}{\sigma}(T_v-t)}<0 \qquad \forall\, t\in[T_v^E,T_v] \tag{92}$$

Since $q > 1$, $e^{-q\frac{\rho}{\sigma}(T_v - t)} \leq e^{-\frac{\rho}{\sigma}(T_v - t)}$, and so a sufficient condition is

$$q > \frac{\sigma(1-e^{-\gamma})\delta}{\rho e^{-\gamma}}. \tag{93}$$

Substituting in for q from (88) yields

$$\frac{\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)}{1-\delta\Delta^E+\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)} > \frac{\sigma(1-e^{-\gamma})\delta}{\rho e^{-\gamma}} \tag{94}$$

Since the l.h.s is increasing in Δ^E, this condition says that Δ^E must be sufficiently large. Since the denominator here is less than 1, a sufficient condition is that given by

$$1-e^{-\frac{\rho}{\sigma}\Delta^E} > \frac{1-e^{-\gamma}}{e^{-\gamma}}. \tag{95}$$

In Figure 2, at the positive intersection of (39) and (46), the former must be steeper than the latter. Differentiating these two curves one can see that Δ^E must satisfy

$$1-e^{-\frac{\rho}{\sigma}\Delta^E} > \frac{\rho}{\delta\gamma(1-\sigma)}. \tag{96}$$

It follows that a globally sufficient condition for (E2) is again given by the first inequality in (54).

(E3) Long–run market clearing implies that

$$\delta V^I(T_{v-1}) = w_v. \tag{97}$$

It follows that a sufficient condition for (E3) is

$$\frac{dV^I(t)}{dt} < 0, \forall\, t \in (0, T_v^E). \tag{98}$$

The value of immediate implementation can be expressed as

$$V^I(t) = (1-e^{-\gamma})\int_t^{T_v^E} e^{-\int_t^\tau r(s)ds} y(\tau)d\tau + e^{-\int_t^{T_v^E} r(s)ds} V^I(T_v^E) \;\; \forall\, t \in (0, T_v^E) \tag{99}$$

Since during this phase $r(t) = \rho$ and $g = 0$, this is

$$V^I(t) = (1-e^{-\gamma})\left(\frac{1-e^{-\rho(T_v^E - t)}}{\rho}\right) y_0(T_{v-1}) + e^{-\rho(T_v^E - t)} V^I(T_v^E) \tag{100}$$

Differentiating w.r.t. to t yields

$$\frac{dV^I(t)}{dt} = -(1-e^{-\gamma})e^{-\rho(T_v^E - t)} y_0(T_{v-1}) + \rho e^{-\rho(T_v^E - t)} V^I(T_v^E) \tag{101}$$

If (54) holds then from (E2), we have that $V^I(T_v^E) < w_v/\delta = e^{-\gamma}y_0(T_{v-1})/\delta$, and so

$$\frac{dV^I(t)}{dt} < -(1-e^{-\gamma})e^{-\rho(T_v^E-t)}y_0 + \rho e^{-\rho(T_v^E-t)}e^{-\gamma}y_0/\delta \tag{102}$$

$$= -\frac{e^{-\rho(T_v^E-t)}y_0}{\delta}\left[(1-e^{-\gamma})\delta - \rho e^{-\gamma}\right] < 0. \tag{103}$$

Where the last inequality follows from (54).

Proof of Proposition 6: Growth in the acyclical economy is given by g^a in (20). In the cyclical economy, from (46) the average long run growth rate can be expressed as

$$g^c = \frac{\Gamma}{\Delta} = \frac{\rho}{1-\sigma}\frac{\Delta^E}{\Delta}. \tag{104}$$

Using (50) and the fact that for any $x > 0$, $\ln(1+x) < x$ we have

$$\Delta < \Delta^E + \frac{1}{\rho}\left(\frac{\frac{\rho}{\delta\gamma(1-\sigma)} - (1-e^{-\gamma})}{\frac{1-e^{-\gamma}}{\rho} - \frac{e^{-\gamma}}{\delta}}\right)\Delta^E. \tag{105}$$

It follows that

$$g^c > \frac{\rho}{1-\sigma}\frac{\Delta^E}{\left(\Delta^E + \frac{1}{\rho}\left(\frac{\frac{\rho}{\delta\gamma(1-\sigma)}-(1-e^{-\gamma})}{\frac{1-e^{-\gamma}}{\rho}-\frac{e^{-\gamma}}{\delta}}\right)\Delta^E\right)} \tag{106}$$

$$g^c > \frac{\rho}{1-\sigma}\left(\frac{1}{\frac{\rho-\gamma(1-\sigma)\rho e^{-\gamma}}{\gamma(1-\sigma)((1-e^{-\gamma})\delta-\rho e^{-\gamma})}}\right) = \frac{[\delta(1-e^{-\gamma})-\rho e^{-\gamma}]\gamma}{1-(1-\sigma)\gamma e^{-\gamma}} = g^a. \tag{107}$$

Proof of Proposition 8: Equation (39), computes the growth effect of a given entrepreneurial length and is thus independent of z. The same is true of the asset market clearing condition, equation (46), since the tax is time invariant. However equation (50), the labour market clearing condition, now becomes:

$$(1-z)\,\delta V^D(t) \le w_v. \tag{108}$$

The corresponding changes in the proof of Proposition (4) directly yields equation (57) . It follows that the equilibrium value of Δ increases, so that long–run growth, Γ/Δ, declines.

Welfare Calculations

Here we outline the welfare comparisons used in Table 1. The calculations assume that the economies starts at T_0 with identical distributions of implemented innovations and no unimplemented innovations. Hence the maximum level of output, $\overline{y}(T_0)$, that could be produced if all labor were being used in manufacturing is the same in both equilibria.

The Acyclical Equilibrium: Household welfare is given by

$$\begin{aligned} W^A(T_0) &= \int_{T_0}^{\infty} e^{-\rho(t-T_0)} \frac{c(t)^{1-\sigma}}{1-\sigma} dt = \frac{c(0)^{1-\sigma}}{1-\sigma} \int_{T_0}^{\infty} e^{-[\rho-(1-\sigma)g^a](t-T_0)} dt \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \left(\frac{1}{\rho-(1-\sigma)g^a} \right) \end{aligned} \tag{109}$$

Since some labor is allocated to entrepreneurship $c(T_0) = (1 - X^*)\overline{y}(T_0)$,where X^* is given by (62).

The Cyclical Equilibrium: Household welfare at the beginning of the first cycle is

$$\begin{aligned} W^C(T_0) &= \int_{T_0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt = \sum_{v=0}^{\infty} e^{-\rho(T_v-T_1)} \int_{T_v}^{T_{v+1}} e^{-\rho(t-T_v)} \frac{c(t)^{1-\sigma}}{1-\sigma} dt \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \sum_{v=0}^{\infty} e^{-[\rho-(1-\sigma)\frac{\Gamma}{\Delta}]v\Delta} \left\{ \int_{T_v}^{T_{v+1}^E} e^{-\rho(t-T_v)} dt + e^{-\rho(T_{v+1}^E-T_v)} \int_{T_{v+1}^E}^{T_{v+1}} e^{-\frac{\rho}{\sigma}(t-T_{v+1}^E)} dt \right\} \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \sum_{v=0}^{\infty} e^{-\rho(\Delta-\Delta^E)v} \left\{ \frac{1-e^{-\rho(T_{v+1}^E-T_v)}}{\rho} + e^{-\rho(T_{v+1}^E-T_v)} \left(\frac{1-e^{-\frac{\rho}{\sigma}(T_{v+1}-T_{v+1}^E)}}{\rho/\sigma} \right) \right\} \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \left(\frac{1}{1-e^{-\rho(\Delta-\Delta^E)}} \right) \left\{ \frac{1-e^{-\rho(\Delta-\Delta^E)}}{\rho} + e^{-\rho(\Delta-\Delta^E)} \left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma} \right) \right\} \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \left\{ \frac{1}{\rho} + \left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma} \right) \frac{1}{e^{\rho(\Delta-\Delta^E)}-1} \right\} \\ &= \frac{c(T_0)^{1-\sigma}}{1-\sigma} \left\{ \frac{1}{\rho} + \frac{\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)\left(\frac{1-e^{-\gamma}}{\rho} - \frac{e^{-\gamma}}{\delta}\right)}{\frac{\rho}{\delta\gamma(1-\sigma)} - (1-e^{-\gamma})} \right\} \end{aligned} \tag{110}$$

Since no labour is allocated to entrepreneurship at the beginning of the cycle, $c(T_0) = \overline{y}(T_0)$.

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