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Collective Household Labor Supply: Nonparticipation and Income Taxation *

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Résumé:
Dans cet article, nous adoptons les hypothèses habituelles de l'approche collective, à savoir, l'individualisme et l'efficacité, pour étudier l'offre de travail du ménage. L'innovation théorique est double. D'une part, nous incorporons dans le cadre initial la décision de participer au marché du travail. D'autre part, nous abandonnons l'hypothèse de linéarité de la contrainte budgetaire. Nous montrons alors que (i) des éléments structurels du processus de décision peuvent être retrouvés, et (ii) des restrictions testables sont générées de l'observation des offres de travail du ménage. Nous examinons également, pour ce modèle, comment faire des simulations de réformes fiscales.

Abstract:
In this paper, we adopt the usual assumptions of the collective approach, i.e. individualism and efficiency, to study household labor supply. The theoretical innovation is twofold. First, we incorporate the decision to participate in the labor market in the initial setting. Second, we abandon the assumption of linearity of the budget constraint. We show that (i) structural elements such as preferences or the outcome of the decision process can be recovered, and (ii) testable restrictions are generated from the observation of the household labor supplies. We also examine, for this model, how to simulate the incidence of fiscal reforms.

Keywords:
Collective Models, Labor Supply, Income Tax, Corner Solutions

JEL classification: H31, J22

1 Introduction

Traditionally, in micro-economics, household behavior is derived from the maximization of a unique well-behaved utility function. However, this 'unitary' approach has been severely criticized in recent years. At the theoretical level, it is not based on methodological individualism, which requires individuals to be characterized by their own preferences. Moreover, it cannot generally be used to study the intra-household distribution of welfare. At the empirical level, the specific restrictions imposed by the unitary model have received little empirical support, if any. In particular, the income pooling hypothesis — according to which only total exogenous income, and not its distribution across household members, matters for labor supply and consumption decisions — has been strongly rejected in many studies (see Lundberg and Pollak (1998) for a survey of this literature).

Chiappori (1988, 1992) has proposed an alternative model of labor supply based upon a 'collective' representation of household behavior. He assumes that each household member is characterized by his (her) own utility function and that decisions result in Pareto-efficient outcomes. He shows that when agents are egoistic and consumption is private, these simple assumptions allow to generate testable restrictions and recover, from observed behavior, certain elements of the decision process, such as individual preferences and the rule that determines the distribution of utilities within the household.[1] This opens a very interesting area of empirical research on the intra-household impact of policy reforms — an issue which is completely disregarded by the unitary approach. However these results are derived for the simplest possible case. The possibility of nonparticipation in the labor market is neglected and the budget constraint is assumed to be linear. To properly assess the collective framework as a useful tool for policy analysis, the future development of the theory must address these issues. This is the motivation for the current paper which may be used as a theoretical kit for applied econometricians.

In Section 2, we discuss the main assumptions that are made. Besides individualism and Pareto-efficiency, we suppose that agents are egoistic and consumption is exclusively private.

In Section 3, we postulate that each member chooses not to work if his

[1] Moreover, Fortin and Lacroix (1997), Chiappori et al. (2001), and Blundell et al. (2001) provide empirical results which are consistent with the collective approach.

(her) respective market wage is below a reservation value. This property does not stem from the theoretical background, as for the unitary model, but has to be explicitly assumed. We then show that Chiappori's conclusions are still valid if either the husband or the wife (but not both) does not work. This result completes and clarifies in some respects a proposition given in Blundell et al. (2001) who consider that the husband's choices are discrete. It is expected to be particularly useful for future empirical applications. The widespread practice for treating the nonparticipation problem indeed consists in leaving households with nonworking persons out of the sample. This explains why the estimates of the structural parameters often lack precision and may be subject to selection biases. The theoretical results of this section can be used to implement the Full Information Maximum Likelihood (FIML) method on a sample with jointly working and nonworking persons. This should also increase the number of observations used in the estimation procedure.

In Section 4, we generalize the previous result to the case where the budget constraint is nonlinear. Specifically, the model that we develop can be used to account for the progressiveness of income tax. This feature is particularly attractive judging by the quantity of literature during the last two decades on the disincentive effects of income tax. However, past studies are always based on the unitary approach which is empirically rejected. Consequently, they may be seriously misleading. This is also underlined by Apps and Rees (1988) and more recently, Brett (1998) who demonstrate the importance of considering the intra-household effects of fiscal reforms in welfare analysis. In this section, we also show that, using the present framework, the impact of changes in tax parameters on the intra-household distribution of welfare and on the household labor supply can be measured using current data.

In Section 5, we conclude with a summary and some general considerations.

2 Collective Household Labor Supply

We consider the case of a married couple (m and f) in a single period setting. The husband's and the wife's labor supply are respectively denoted by L^m and L^f with market wages w_m and w_f. Aggregate (Hicksian) consumption of each spouse is respectively denoted by C^m and C^f with prices set to one. Nonlabor income is denoted by y. For convenience, the spouses' total time endowment is normalized at one. Finally, we adopt the following assumption on preferences.

Assumption A1 Each household member is characterized by specific utility functions of the form: $u^i(1-L^i, C^i)$. These functions are both strongly concave, infinitely differentiable and strictly increasing in all their arguments on $\mathbb{R}^3_{++}$, with $\lim_{C^i \to 0} u^i(1-L^i, C^i) = \lim_{L^i \to 1} u^i(1-L^i, C^i) = -\infty$.

Three remarks are in order. First, we suppose that there is no public consumption and no domestic production. Second, the household members are said to be "egoistic" in that their utility only depends on their own consumption and leisure.[2] Third, the two conditions on limits permit ruling out the cases where individual consumption or leisure is equal to zero, as shown below.

The main originality of the efficiency approach lies in the fact that the household decisions result in Pareto efficient outcomes and that no additional assumption is made about the process. Explicitly, we say:

Assumption A2 The outcome of the decision process is Pareto efficient.

This assumption has a good deal of intuitive appeal. The household is one of the preeminent examples of a repeated game. Therefore, it is plausible that agents find mechanisms to support efficient outcomes since cooperation often emerges as a long term equilibrium of repeated noncooperative relations.

Provided that the budget set is convex, efficiency means that there exists a scalar μ such that the household behavior is a solution to the following program:

$$\max_{(L^f, L^m, C^f, C^m)} (1-\mu) \cdot u^f(1-L^f, C^f) + \mu \cdot u^m(1-L^m, C^m) \qquad (\bar{\mathrm{P}})$$

$$\text{subject to } (L^m, L^f, C^m + C^f) \in S,$$

where S is the household budget set which is assumed to be convex and compact. It generally depends on wages and income.

[2] However, all the results immediately extend to the case of "altruistic" agents in a Beckerian sense, with utilities represented by $W^i(u^m(1-L^m, C^m), u^f(1-L^f, C^f))$, where $W^i(\cdot)$ is a strictly increasing function. See Chiappori (1992) for a discussion of this point.

To obtain well-behaved labor supply functions and consumption functions (instead of correspondences), we assume that the scalar $\mu \in]0,1[$ is a *single-valued* and infinitely differentiable function of (w_m, w_f, y). The underlying idea is that, within a bargaining context, the threat point may well depend on nonlabor income and the wage that the spouses receive when they work. If so, most cooperative equilibrium concepts will imply that μ is a function of w_m, w_f and y.

We denote the solutions to $(\bar{\mathrm{P}})$ by $\bar{L}^m$, $\bar{L}^f$, $\bar{C}^m$ and $\bar{C}^f$ as functions of w_m, w_f and y. Let us remark that $\bar{C}^m$ and $\bar{C}^f$ are treated as unobservable, and we can observe the aggregate consumption only at the household level. We then say that a pair of labor supplies $\bar{L}^m$ and $\bar{L}^f$ is consistent with collective rationality conditionally on S if and only if there exist a pair of functions $\bar{C}^m$ and $\bar{C}^f$ and some function μ, such that, for any (w_m, w_f, y), $(\bar{L}^m, \bar{L}^f, \bar{C}^m, \bar{C}^f)$ is a solution to $(\bar{\mathrm{P}})$. This definition does not postulate a particular form for the budget set S.[3]

3 The Linear Case

In this section, we consider the case of a linear budget constraint and extend the main conclusions of Chiappori (1988, 1992) to corner solutions.

3.1 Basic Framework

To begin with, we introduce the following assumption on the budget set.

Assumption S1 The budget set S is given by $w_m L^m + w_f L^f + y \geqslant C^m + C^f$, $1 \geqslant L^m \geqslant 0$, $1 \geqslant L^f \geqslant 0$, $C^m \geqslant 0$ and $C^f \geqslant 0$.

Because of A1, $(\bar{\mathrm{P}})$ always has an interior solution for consumption and leisure: $C^i > 0$ and $L^i < 1$ for any i (if the budget set is not empty).

[3]Nevertheless, the collective rationality of a pair of labor supplies can be assessed only for a given budget set. In fact, any pair of labor supplies $\bar{L}^m$ and $\bar{L}^f$ can be collectively rationalized by an infinity of budget sets. To show this, it is sufficient to define a budget set S^* which consists, for any bundle (w_m^*, w_f^*, y^*), of a single point equal to $\bar{L}^m(w_m^*, w_f^*, y^*)$ and $\bar{L}^f(w_m^*, w_f^*, y^*)$, and any value for the household consumption. If so, the pair of labor supplies $\bar{L}^m$ and $\bar{L}^f$ is obviously consistent with collective rationality conditionally on S^*.

This seems realistic since consumption is aggregated and leisure is arbitrarily defined.

The next step is to define what we call the sharing rule. Indeed, when the agents are egoistic and the budget constraint is linear, efficiency implies that the allocation of resources can be decentralized.

Lemma 1 *Let $(\bar{L}^m, \bar{L}^f)$ be a pair of labor supplies consistent with collective rationality conditionally on S1. Then, there exist a pair of functions $(\bar{C}^m, \bar{C}^f)$ and a pair of functions (ρ^m, ρ^f), with $\sum_i \rho^i = y$, such that $(\bar{L}^i, \bar{C}^i)$ is a solution to*

$$\max_{\{L^i, C^i\}} u^i(1 - L^i, C^i) \text{ subject to } C^i - L^i \cdot w_i = \rho^i \text{ and } L^i \geqslant 0, \qquad (\mathrm{P}_i')$$

for any $(w_m, w_f, y) \in \mathbb{R}^3_{++}$.

Proof. An application of the Second Theorem of Welfare Economics. Q.E.D.

This lemma states that the decision process can be represented in two stages. First, members divide the nonlabor income according to some predetermined rule which is a function of w_m, w_f and y. Second, they independently choose consumption and labor supply subject to their own budget constraint. The spouses' labor supplies, for an interior solution, then have the following functional structure:

$$\begin{aligned} \bar{L}^m &= \lambda^m(w_m, \rho(w_m, w_f, y)), & (1) \\ \bar{L}^f &= \lambda^f(w_f, y - \rho(w_m, w_f, y)), & (2) \end{aligned}$$

where $\rho = \rho^m$ and $\rho^f = y - \rho$. In the remainder of this paper, the husband's share ρ is called the 'sharing rule'. It can be either positive or negative.

3.2 Identification and Testability

A preliminary remark to make is that, in what follows, wages are assumed to always be observed by the economist, even when the wife or the husband does not work. This is not realistic but, in practical terms, wages can generally be estimated by an auxiliary equation. This point is discussed below.

In the standard unitary framework, the participation decision is modeled in terms of a reservation wage defined by the fact that, at this wage, the agent is indifferent between working and not working. In the present context, we can naturally define a pair of reservation wages in the same way.[4] Nevertheless, we need additional assumptions to ensure the existence of a well-behaved participation frontier. To show this, we introduce the following definition for member i :

$$\varpi^i(w_m, w_f, y) = -\frac{u_L^i(1, \rho^i(w_m, w_f, y))}{u_C^i(1, \rho^i(w_m, w_f, y))},$$

where F_x is the partial differential of function F with respect to variable x. This equation is the marginal rate of substitution between leisure and consumption computed along the axis $L^i = 0$ for a given endowment ρ^i (and equal to C^i). Then, the reservation wage is implicitly defined, for member i, by

$$w_i = \varpi^i(w_m, w_f, y)$$

as a function of the partner's wage and nonlabor income. Therefore, the uniqueness of the reservation wage does not result from the theoretical framework that we have adopted, but must be explicitly postulated.[5] We use the following assumption.

Assumption R1 Preferences and the sharing rule are such that

$$\max_{i=m,f}(|\varpi^i(w_m^*, w_f^*, y) - \varpi^i(w_m^\circ, w_f^\circ, y)|) \leqslant \max_{i=m,f}(|w_i^* - w_i^\circ|)$$

for any (w_m^*, w_f^*, y) and $(w_m^\circ, w_f^\circ, y) \in \mathrm{IR}_{++}^3$.

This condition is satisfied if the impact of w_m and w_f on the individual shares is "small enough" in absolute value. It is not expected to be very restrictive and it greatly simplifies the analysis. Specifically, R1 means that the system of equations ϖ^m and ϖ^f is a contraction with respect to the variables w_m and w_f. There are two corollaries. First, for any y, the pair of functions ϖ^m and ϖ^f has a unique fixed point with respect to w_m and

[4] Blundell et al. (2001) underline that, if a member is indifferent between working and not working, his (her) partner must be indifferent as well. This obviously stems from efficiency. Let us assume, for example, that at the reservation wage, the husband is indifferent between working and not working. If his participation entails a strict gain for the wife, then the husband's nonparticipation is clearly Pareto-inefficient.

[5] However, the existence of the reservation wage is always assured because, for fixed w_j and y, the function ϖ^i is upper-bounded: $\varpi^i(w_i, w_j, y) < \infty$ for any w_i.

w_f. Then, for any y, there exists one and only one pair of wages, denoted by $\hat{w}_m(y)$ and $\hat{w}_f(y)$, such that both household members are indifferent between working and not working. Second, for any w_j and y, each function ϖ^i has a unique fixed point with respect to w_i.[6] Then there exists, for each member i, a function $\gamma^i(w_j, y)$ defined on $\mathbb{R}^2_{++}$ such that member i participates in the labor market if and only if $w_i > \gamma^i(w_j, y)$. Consequently, $\mathbb{R}^3_{++}$ is partitioned into four connected sets as shown in Figure 1.

In this figure, we show that the spouses do not work when their respective market wage is below a reservation value. Furthermore, for a given y, the participation frontiers γ^i have only one intersection (according to R1) and are convex with respect to the partner's wage. This convexity is not formally implied by the present setting. A possible interpretation is as follows. When the husband (wife) does not work, an increase in his (her) wage probably has a positive effect on his (her) bargaining power. If so, the wife's (husband's) share decreases and, if leisure is normal, her (his) reservation wage declines. When the husband (wife) works, an increase in his (her) wage also has a positive effect on the household labor income which may compensate for the increase in the husband's (wife's) bargaining power : her (his) share is expected to rise at some point.

To simplify the analysis, we introduce some definitions. The spouses' participation set, i.e. the set of (w_m, w_f, y) such that both spouses choose to work, is denoted by P. The wife's nonparticipation set, i.e. the set of (w_m, w_f, y) such that the wife (and only the wife) chooses not to work, is denoted by N_f. Similarly, the husband's nonparticipation set is denoted by N_m. The spouses' nonparticipation set, i.e. the set of (w_m, w_f, y) such that both spouses choose not to work, is denoted by N.

[6] This comes from the fact that, for any w_j and y, the function ϖ^i is a contraction with respect to w_i because $|\varpi^i(w_i^*, w_j, y) - \varpi^i(w_i^\circ, w_j, y)| \leqslant |w_i^* - w_i^\circ|$ (a simple consequence of R1).

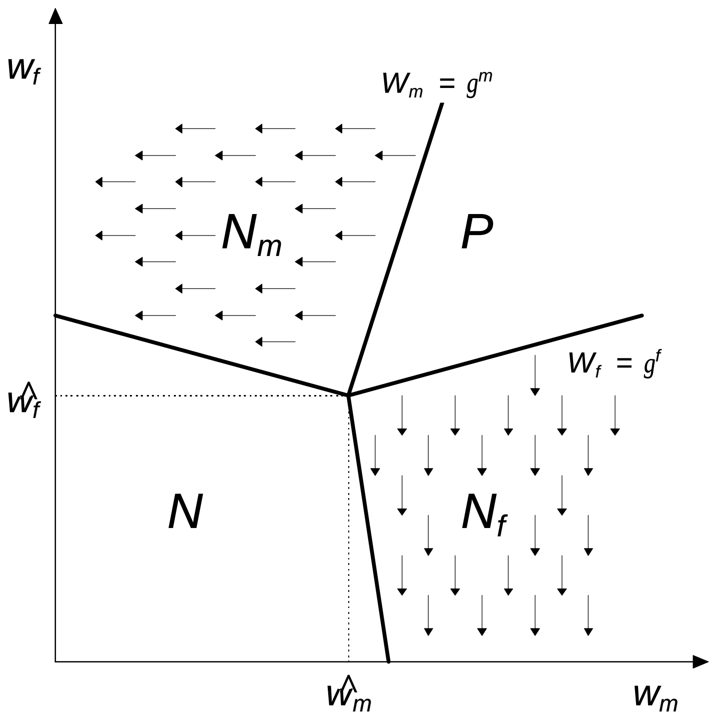

Figure 1: Participation and Non-participation Sets

At this stage, two remarks are in order. First, it can be shown that the labor supplies $\bar{L}^m$ and $\bar{L}^f$ as well as the sharing rule ρ are infinitely differentiable in all their arguments on P, $\text{int}(N_i)$ and $\text{int}(N)$.[7] However, these functions are generally nondifferentiable along the participation frontiers. Second, when both partners do not participate in the labor market, there is nothing that can be said about identification or testability : the only information at the economist's disposal concerns the household expenditure which is exogenously given by nonlabor income. Consequently, we assume that $(w_m, w_f, y) \in R$ where R is a partition of $\mathbb{R}^3_{++}$ defined by

$$(w_m, w_f, y) \in R \quad \text{iff} \quad \bar{L}^f \neq 0 \quad \text{or} \quad \bar{L}^m \neq 0.$$

That is, at least one of the labor supplies corresponds to an interior solution

[7] This result comes from the smoothness of u_i and μ and the application of the Implicit Function Theorem on the first order conditions of $(\bar{\text{P}})$. See Magnus and Neudecker (1988, p. 143, Th. A3) for an appropriate version of this theorem.

of ($\bar{\text{P}}$). Identification of the sharing rule and other structural elements can possibly be attained on R, but not outside.[8]

The following theorem establishes that some structural elements of the decision process can be retrieved from the observation of both labor supplies.

Theorem 1 *Let $(\bar{L}^m, \bar{L}^f)$ be a pair of labor supplies, satisfying regularity conditions listed in Lemma 2–4, consistent with collective rationality conditionally on S1. Let us assume R1. Then the sharing rule ρ is identified up to an additive constant ϵ on R. Moreover, for each choice of ϵ, preferences represented by u^m and u^f are uniquely identified. Finally, the labor supplies have to satisfy testable restrictions under the form of partial differential equations on P.*

The proof of this theorem follows in stages. First, we consider the identification of the sharing rule in the spouses' participation set. This is followed by a proof of identification in the nonparticipation sets of each household member. Finally, identification of preferences follows.

3.2.1 Identification in the Spouses' Participation Set

This is the only case which is implicitly considered by Chiappori (1988, 1992). For any $(w_m, w_f, y) \in P$ such that $\bar{L}_y^m \cdot \bar{L}_y^f \neq 0$, we introduce the following definitions:

$$A(w_m, w_f, y) = \frac{\bar{L}_{w_f}^m(w_m, w_f, y)}{\bar{L}_y^m(w_m, w_f, y)}, \qquad B(w_m, w_f, y) = \frac{\bar{L}_{w_m}^f(w_m, w_f, y)}{\bar{L}_y^f(w_m, w_f, y)}.$$

The first result, which is well-known, is that the sharing rule can be identified and testable constraints are generated on P if regularity conditions are satisfied.

Lemma 2 *Let us assume that $\bar{L}_y^m \cdot \bar{L}_y^f \neq 0$ and $AB_y - B_{w_f} \neq A_y B - A_{w_m}$ for any $(w_m, w_f, y) \in P$. Then the sharing rule is identified on P up to a constant. In addition, labor supplies have to satisfy some restrictions under the form of partial differential equations on P.*

[8] Of course, this is no longer true if additional information is used. For example, Donni (2001) uses the observation of a system of commodity demands to identify the sharing rule when both household members are rationed.

Proof. See Chiappori (1988, 1992). Q.E.D.

We only give a sketch of the proof. From the functional structure of the labor supplies (1) and (2), we obtain two partial differential equations:

$$\rho_{w_f} - A\rho_y = 0 \qquad \text{and} \qquad \rho_{w_m} - B\rho_y = -B.$$

This is a system of partial differential equations in ρ which can be solved if it is differentiated with respect to w_m, w_f and y and if the cross-derivative restrictions are taken into account; it defines the sharing rule up to a constant.

We now consider the identification when one member does not work.

3.2.2 Identification in the Wife's Nonparticipation Set

We start with the case where only the husband works, i.e. $w_f \leqslant \gamma^f(w_m, y)$. As before, we have the following definition on int(N_f), if $\bar{L}^m_y \neq 0$:

$$A(w_m, w_f, y) = \frac{\bar{L}^m_{w_f}(w_m, w_f, y)}{\bar{L}^m_y(w_m, w_f, y)}.$$

In addition, along the wife's participation frontier, we have the following definition by continuity, if $\lim_{w_f \uparrow \gamma^f} \bar{L}^m_y \neq 0$:

$$a(w_m, y) = A(w_m, \gamma^f(w_m, y), y).$$

This function is defined on the set I_f of (w_m, y) such that $w_m \geqslant \hat{w}_m(y)$.

The following lemma states that, under regularity conditions, the sharing rule is defined on N_f.

Lemma 3 *Let us assume that* $\lim_{w_f \uparrow \gamma^f} \bar{L}^m_y \neq 0$ *and* $a \cdot \gamma^f_y \neq -1$ *for any* $(w_m, y) \in I_f$ and $\bar{L}^m_y \neq 0$ *for any* $(w_m, w_f, y) \in$ *int(*N_f*). Then the sharing rule is identified up to a constant on* N_f*.*

Proof. The basic idea is that the derivatives of the sharing rule from the preceding lemma provide boundary conditions for the partial differential equation:

$$\rho_{w_f} - A\rho_y = 0. \tag{3}$$

From standard theorems in partial differential equation theory, this defines the sharing rule (up to an additive constant) provided that the following condition is fulfilled. First, we write (3) as $\bigtriangledown\rho \cdot \vec{u} = 0$ where $\bigtriangledown\rho$ is the gradient of ρ and $\vec{u}$ is the vector $(0, 1, -A)$. Then, the condition is that $\vec{u}$ is not tangent to the wife's participation frontier. Since the equation of this frontier is $w_f - \gamma^f(w_m, y) = 0$ and, given that on the frontier A coincides with a, this condition states that $1 + a \cdot \gamma_y^f \neq 0$, for all $(w_m, y) \in I_f$. If this condition is fulfilled on the frontier, then the partial differential equation together with the boundary condition defines ρ up to an additive constant. Q.E.D.

The intuition of the reasoning is illustrated in Figure 1. From Lemma 2, the values of the partials are identified on the frontier of participation, represented by the lower curve in bold. These values provide boundary conditions for the partial differential equation (3). The latter, characterized by the vector field $\vec{u}$ in N_f, indicates the direction in which the sharing rule is constant. That is, it defines the indifference surfaces of the sharing rule which, under the regularity conditions of Lemma 3, pass *through* the wife's participation frontier. An important remark is that this identification result is local rather than global. Additional conditions are required to recover the sharing rule on the whole of the wife's nonparticipation set. However, this local result is certainly sufficient for empirical applications.

3.2.3 Identification in the Husband's Nonparticipation Set

We now consider the case where only the wife works, i.e. $w_m \leqslant \gamma^m(w_f, y)$. The reasoning is exactly the same as before. We have the following definition on $\operatorname{int}(N_m)$, if $\bar{L}_y^f \neq 0$:

$$B(w_m, w_f, y) = \frac{\bar{L}_{w_m}^f(w_m, w_f, y)}{\bar{L}_y^f(w_m, w_f, y)}.$$

In addition, along the husband's participation frontier, we have the following definition by continuity, if $\lim_{w_m \uparrow \gamma^m} \bar{L}_y^f \neq 0$:

$$b(w_f, y) = B(\gamma^m(w_f, y), w_f, y).$$

This function is defined on the set I_m of (w_f, y) such that $w_f \geqslant \hat{w}_f(y)$.

The following lemma states that, under regularity conditions, the sharing rule is defined on N_m.

Lemma 4 *Let us assume that* $\lim_{w_m \uparrow \gamma^m} \bar{L}_y^f \neq 0$ *and* $b \cdot \gamma_y^m \neq -1$ *for any* $(w_f, y) \in I_m$ *and* $\bar{L}_y^f \neq 0$ *for any* $(w_m, w_f, y) \in int(N_m)$*. Then the sharing rule is identified up to a constant on* N_m.

Proof. The proof is exactly the same as the preceding one. Q.E.D.

Finally, knowing the rule allows us to determine each member's actual budget constraint and to compute preferences in the usual way.

This theorem must be related to a previous result given by Blundell et al. (2001). Using a similar proof, those authors show that the sharing rule can be retrieved up to a constant when the wife's labor supply is continuous and the husband's labor supply is discrete (working or not working).

3.3 A Simple Parametric Example

To illustrate the preceding results, we consider a very simple example of functional form. We assume that when both spouses work, the labor supplies are as follows:

$$\begin{aligned} L^m &= a_0 + a_1 w_f + a_2 w_m + a_3 w_f w_m + a_4 y + a_5 y^2 \qquad (4) \\ &= \mathbf{a}'\mathbf{w} \text{ (say)}, \\ L^f &= b_0 + b_1 w_f + b_2 w_m + b_3 w_f w_m + b_4 y + b_5 y^2 \qquad (5) \\ &= \mathbf{b}'\mathbf{w} \text{ (say)}. \end{aligned}$$

We also assume that if the wife does not work, the husband's labor supply switches regime, i.e. the parameters change:

$$\begin{aligned} L^m &= A_0 + A_1 w_f + A_2 w_m + A_3 w_f w_m + A_4 y + A_5 y^2 \\ &= \mathbf{A}'\mathbf{w} \text{ (say)}. \end{aligned}$$

However, the parameters must satisfy certain restrictions for the labor supply to be continuous along the frontier. We must have the following relation:

$$\mathbf{A}'\mathbf{w} = \mathbf{a}'\mathbf{w} + s \cdot (\mathbf{b}'\mathbf{w}), \qquad (6)$$

where s is a free parameter. Indeed, along the participation frontier, by definition, the last term of (6) vanishes, and consequently $\mathbf{A}' \cdot \mathbf{w} = \mathbf{a}' \cdot \mathbf{w}$, i.e. the labor supply is continuous. Strictly speaking, the constraints implied by (6) do not constitute a test of the collective approach but rather a test of the auxiliary assumptions (in particular, the continuity of μ and the convexity of S).

We consider the identification on the spouses' participation set and use the definitions given in Chiappori (1992). First, we retrieve the sharing rule:

$$\begin{aligned}\rho &= k_0 + k_1 w_f + k_2 w_m + k_3 w_f w_m + k_4 y + k_5 y^2 \\ &= \mathbf{k}'\mathbf{w} \text{ (say)},\end{aligned} \tag{7}$$

where

$$k_1 = \frac{a_1 b_3}{\Delta}, \quad k_2 = \frac{a_3 b_2}{\Delta}, \quad k_3 = \frac{a_3 b_3}{\Delta}, \quad k_4 = \frac{a_4 b_3}{\Delta}, \quad k_5 = \frac{a_5 b_3}{\Delta}, \tag{8}$$

with $\Delta = a_4 b_3 - a_3 b_4$, and k_0 is an unknown constant. Furthermore, the collective rationality implies a constraint: $a_5 b_3 = a_3 b_5$. Finally, the structural form of the labor supplies is linear. Precisely, we have:

$$\lambda^m(w_m, \rho) = Z_0^m + Z_1^m w_m + Z_2^m \rho \tag{9}$$

$$\lambda^f(w_f, y - \rho) = Z_0^f + Z_1^f w_f + Z_2^f (y - \rho), \tag{10}$$

where the parameters Z_0^i, Z_1^i, Z_2^i can be uniquely identified (once the constant k_0 has been chosen), with $Z_1^i \geqslant 0$ and $Z_2^i \leqslant 0$ by Slutsky Negativity. The utility function is well-known and can be retrieved (e.g., Hausman (1981)).

We now consider the identification on the wife's nonparticipation set. On this set, the sharing rule is defined by a partial differential equation and the fact that it is continuous along the participation frontier. We assume that the solution of this problem is given by a sharing rule with the following functional form:

$$\begin{aligned}\rho &= K_0 + K_1 w_f + K_2 w_m + K_3 w_f w_m + K_4 y + K_5 y^2 \\ &= \mathbf{K}'\mathbf{w} \text{ (say)}.\end{aligned} \tag{11}$$

This function must be equal to (7) along the participation frontier. It means that the parameters of (11) have to satisfy constraints:

$$\mathbf{K}'\mathbf{w} = \mathbf{k}'\mathbf{w} + r \cdot (\mathbf{b}'\mathbf{w}), \tag{12}$$

where r is a free parameter.[9] This unique degree of freedom vanishes if we use the partial differential equation (3) which is valid on this nonparticipation

[9] For the model to be logically consistent, it may be useful to suppose that this parameter is nonnegative. Otherwise, the sharing rule (11), combined with the 'structural' labor supply (9), may lead to a positive supply of labor for the wife.

set:

$$\frac{K_1 + K_3 w_m}{K_4 + 2K_5 y} = \frac{A_1 + A_3 w_m}{A_4 + 2A_5 y}.$$

This yields three algebraic equations. If we use (6), (8) and (12), we show that these equations are redundant and we obtain: $r = b_3 \cdot s/\Delta$. Since the theory establishes that the model is exactly identified, we may conclude that this solution is unique. Finally, the same analysis can be performed in the husband's nonparticipation set. This is left to the reader.

To estimate this model, the next step is to make some allowance for unobservable heterogeneity in the labor supplies (4) and (5).[10] Since wages are unobserved for nonparticipants, we must also specify a stochastic model for explaining market wages. In this case, the proof of the identification raises further theoretical difficulties which are beyond the scope of this paper. Blundell et al. (2001) investigate this issue within the context of a linear model. They show that the presence of unobservable heterogeneity does not invalidate the main conclusions. Broadly speaking, a necessary condition for the identification in this context is that there exists a variable which influences the wife's (husband's) wage without affecting the sharing rule and the husband's (wife's) wage. If we accept this result, the reduced model, given by (4) and (5), can be estimated using the usual techniques (FIML or similar methods), the structural model can be retrieved and the constraint can be tested.

4 The Nonlinear Case

In this section, we consider the case of a nonlinear budget constraint. This generalization is particularly attractive for analyzing the incidence of income tax on household labor supply.

4.1 Basic Framework

We maintain the essential assumptions of the collective setting but we give up the linearity of the budget constraint. Precisely, we adopt the following assumption:

[10] Such heterogeneity may come from the preferences and from the sharing rule.

Assumption S2 The budget set S is given by $h(L^m, L^f\,; w_m, w_f, y) \geqslant C^m + C^f$, $1 \geqslant L^m \geqslant 0$, $1 \geqslant L^f \geqslant 0$, $C^m \geqslant 0$ and $C^f \geqslant 0$, where the function $h(\cdot)$ is infinitely differentiable, increasing in all its arguments and concave in L^m and L^f.

Two remarks are in order. First, we assume that the budget set is perfectly observed by the economist. This is essential for deriving the results of this section. Second, the concavity of h implies that the budget set is convex. Consequently, the solutions of ($\bar{\text{P}}$) are well-behaved functions (instead of correspondences).

The next step is to define a generalization of the sharing rule. When the budget set is convex, the decision process can be decentralized as previously. To do this, we define a pair of shadow wages as follows:

$$\omega^m(w_m, w_f, y) = h_{L^m}(\bar{L}^m, \bar{L}^f\,; w_m, w_f, y), \tag{13}$$
$$\omega^f(w_m, w_f, y) = h_{L^f}(\bar{L}^m, \bar{L}^f\,; w_m, w_f, y). \tag{14}$$

They coincide with the marginal rates of transformation between leisure and consumption *computed at the equilibrium.* We also define a shadow income as follows:

$$\eta(w_m, w_f, y) = h(\bar{L}^m, \bar{L}^f\,; w_m, w_f, y) - \textstyle\sum_i \bar{L}^i \cdot \omega^i. \tag{15}$$

This function gives the necessary income such that, at the equilibrium, the budget constraint would be saturated if wages was given by ω^m and ω^f. Finally, we can put forward the following lemma.

Lemma 5 *Let $(\bar{L}^m, \bar{L}^f)$ be a pair of labor supplies consistent with collective rationality conditionally on S2. Then, there exist a pair of functions $(\bar{C}^m, \bar{C}^f)$ and a pair of functions (ρ^m, ρ^f), with $\sum_i \rho^i = \eta$, such that $(\bar{L}^i, \bar{C}^i)$ is a solution to*

$$\max_{\{L^i, C^i\}} u_i(1 - L^i, C^i) \text{ subject to } C_i - \omega^i L_i = \rho^i,\ L^i \geqslant 0, \tag{P$''_{\text{i}}$}$$

for any $(w_m, w_f, y) \in \mathrm{IR}^3_{++}$.

Proof. An application of the Second Theorem of Welfare Economics. Q.E.D.

As before, the decision process can be seen as a two-stage budgeting one. In the first step, the members agree on some shadow wages and some intrahousehold distribution of the shadow income. In the second step, they freely choose their own labor supply and consumption subject to their specific budget constraint. For an interior solution, the labor supplies have the following functional structure:

$$\begin{aligned}\bar{L}^m &= \lambda^m(\omega^m(w_m, w_f, y), \rho(w_m, w_f, y)), \quad (16)\\ \bar{L}^f &= \lambda^f(\omega^f(w_m, w_f, y), \eta(w_m, w_f, y) - \rho(w_m, w_f, y)), \quad (17)\end{aligned}$$

where $\rho = \rho^m$ and $\eta - \rho = \rho^f$. The function ρ can be seen as the natural generalization of the sharing rule introduced in Section 3. We also note that the shadow wages and income are observed by the economist, since the budget set is supposed to be known.

4.2 Identification and Testability

To simplify, we suppose that both spouses work, i.e. $(w_m, w_f, y) \in P$. However, the generalization to nonworking spouses is elementary. Then, we impose a regularity condition on labor supplies and the budget constraint:

Assumption R2 Labor supplies $\bar{L}^m(w_m, w_f, y)$ and $\bar{L}^f(w_m, w_f, y)$ and the budget constraint $h(L^m, L^f; w_m, w_f, y)$ are such that

$$\begin{vmatrix} \omega^m_{w_m} & \omega^f_{w_m} & \eta_{w_m} \\ \omega^m_{w_f} & \omega^f_{w_f} & \eta_{w_f} \\ \omega^m_y & \omega^f_y & \eta_y \end{vmatrix} \neq 0$$

for any $(w_m, w_f, y) \in P$.

This condition excludes some very unusual budget constraints[11] but is generically fulfilled in most useful cases; an example with income tax is given below.

If R2 is satisfied, by the Theorem of Implicit Functions, we can locally express the actual variables w_f, w_m and y as a function of the shadow variables ω_f, ω_m and η. Therefore, we can write the labor supplies (16) and (17) as follows:

$$\begin{aligned}\hat{L}^m(\omega_m, \omega_f, \eta) &= \lambda^m(\omega_m, \varphi(\omega_m, \omega_f, \eta)), \quad (18)\\ \hat{L}^f(\omega_m, \omega_f, \eta) &= \lambda^f(\omega_f, \eta - \varphi(\omega_m, \omega_f, \eta)), \quad (19)\end{aligned}$$

[11] A trivial example of inappropriate budget constraint is given by the following form $h(L^m + L^f, w_m, w_f, y)$. In this case, the husband and the wife are characterized by the same shadow wage and R2 is obviously not satisfied.

where the φ – rule is implicitly defined by

$$\varphi(\omega^m(w_m, w_f, y), \omega^f(w_m, w_f, y), \eta(w_m, w_f, y)) = \rho(w_m, w_f, y).$$

The functions $\hat{L}^m$ and $\hat{L}^f$ can be computed from the traditional labor supplies and the budget constraint. Furthermore, under our assumptions, they are infinitely differentiable in all their arguments. For any $(\omega_m, \omega_f, \eta)$ such that $\hat{L}^m_\eta \cdot \hat{L}^f_\eta \neq 0$, we introduce the following definitions:

$$\hat{A}(\omega_m, \omega_f, \eta) = \frac{\hat{L}^m_{\omega_f}(\omega_m, \omega_f, \eta)}{\hat{L}^m_\eta(\omega_m, \omega_f, \eta)}, \qquad \hat{B}(\omega_m, \omega_f, \eta) = \frac{\hat{L}^f_{\omega_m}(\omega_m, \omega_f, \eta)}{\hat{L}^f_\eta(\omega_m, \omega_f, \eta)}.$$

The next theorem establishes that some structural elements can be retrieved and testable constraints are generated from the observation of both labor supplies.

Theorem 2 *Let $(\bar{L}^m, \bar{L}^f)$ be a pair of labor supplies consistent with collective rationality conditionally on S2. Let us assume R2 and, for any $(\omega_m, \omega_f, \eta)$, $\hat{L}^m_\eta \cdot \hat{L}^f_\eta \neq 0$ and $\hat{A}\hat{B}_\eta - \hat{B}_{\omega_f} \neq \hat{B}\hat{A}_\eta - \hat{A}_{\omega_m}$. Then the sharing rule ρ is identified up to an additive constant ϵ on P. Moreover, for each choice of ϵ, preferences represented by u^m and u^f are uniquely identified. Finally, the labor supplies must satisfy testable restrictions under the form of partial differential equations.*

Proof. The φ – rule is retrieved from (18) and (19) and Lemma 2. Then, the usual sharing rule can be obtained with the definitions (13), (14) and (15). Q.E.D.

4.3 Labor Supply and Income Taxation

As an illustration, we consider a model of labor supply with income taxation. In this case, after tax wages depend on the tax law and total hours worked.

4.3.1 Characteristics of Tax Systems

In the United-States and several other countries, the base of the tax system is the household income as a whole. Disposable income is then given by

$$h(w_m \cdot L^m + w_f \cdot L^f + y), \tag{20}$$

with $h' > 0$.[12] For this form, R2 is generically satisfied. To illustrate this, let us define the total household income as $E(w_f, w_m, y) = w_f \bar{L}_f + w_m \bar{L}_m + y$. Then, using a simple computation, it can be shown that R2 is satisfied if

$$h' \neq h'' \cdot (E_y \cdot (E - y) - E_{w_f} w_f - E_{w_m} w_m).$$

Moreover, the tax schedule is to a large extent progressive (i.e. $h'' \leqslant 0$). If not, the budget set is often approximated in empirical applications, by its convex hull. Finally, the tax rates fit a step function.[13] If the household income is in the kth bracket, the shadow wages and income are then given by

$$\omega_f = w_f \cdot (1 - t_k), \quad \omega_m = w_m \cdot (1 - t_k), \tag{21}$$

$$\eta = B_k - T(B_k) - (B_k - y) \cdot (1 - t_k), \tag{22}$$

where t_k is the marginal tax rate in the kth bracket, B_k is the lower limit of that bracket and $T(B_k)$ is the amount of income tax corresponding to B_k.

4.3.2 Measuring the Incidence of Income Taxation

A large quantity of literature in both public and labor economics is interested in evaluating the disincentive and distributional effects of income tax. However, these studies suppress the analysis of intra-household resource allocation by adopting the unitary approach. Apps and Rees (1988) and Brett (1998) underline that this is allowable only if the household allocates income exactly in accordance with the social welfare function of the planner. If not, traditional analysis with a single utility function might be seriously misleading.

Theoretically, the collective approach permits us to analyze the intra-household redistributive effects of fiscal reforms. The basic question is whether the impact of tax parameters on the sharing rule, and consequently on labor supply, can be retrieved from current data. In fact, the answer depends on the structure of ρ. To begin with, we define δ as a set of tax parameters (e.g.,

[12] There are other types of tax systems. For example, the base of the tax system may also be the spouses' income. The disposable income is then given by $h(w_m \cdot L^m) + h(w_f \cdot L^f) + y$.

[13] Since the Second Theorem of Welfare Economics is not based on the smoothness of the budget constraint, a pair of shadow wages can be defined everywhere. But, at the step points, these wages are unobserved for the economist and the preceding results are thus invalidated. However, this problem is likely negligible in empirical applications.

one or several tax rates which seem especially important or some statistics which summarize the tax system) and recall that, under regularity conditions, the sharing rule can be written as a function of ω_m, ω_f and η. If δ does not operate in φ as a specific argument,

$$\rho = \varphi(\tilde{\omega}^m(L^m, L^f, \delta), \tilde{\omega}^f(L^m, L^f, \delta), \tilde{\eta}(L^m, L^f, \delta)), \tag{23}$$

where $\tilde{\omega}^m, \tilde{\omega}^f$ and $\tilde{\eta}$ have obvious definitions, we say that the intra-household redistributive impact of the tax system is neutral. If so, a change in the tax parameters influences household behavior only through a change in the shadow variables. Since the derivatives of the $\varphi -$ rule are identifiable, it is clear that the impact of the tax parameters on the intra-household distribution of income and on household labor supply can be empirically evaluated if we use (23) and apply the Implicit Functions Theorem to (18) and (19).

Still, the assumption of neutrality is not insignificant. For instance, let us consider the bargaining model of McElroy and Horney (1981). In this model, the intra-household allocation is determined by the Nash solution of a cooperative game where the threat points are given by the utility levels obtained when divorce is involved. Then, if the base of the tax system is household income, it is plausible that an increase in the progressiveness of the tax schedule, because it smooths the respective financial situations of each spouse in the case of divorce, tends to decrease inequalities within the household as well as between households. But this implication of the progressiveness cannot be reduced to a modification of the shadow wages or the shadow income. In this case, φ directly depends on δ:

$$\rho = \varphi(\tilde{\omega}^m(L^m, L^f, \delta), \tilde{\omega}^f(L^m, L^f, \delta), \tilde{\eta}(L^m, L^f, \delta), \delta). \tag{24}$$

If so, we may wonder whether the effect of tax parameters on the intra-household distribution of income is retrievable. Actually, the answer is positive if we have data where some variability in the tax parameters is observed. Precisely, from the relations (18), (19) and (24), we easily obtain:

$$\varphi_\delta = \frac{\hat{L}^m_\delta}{\hat{L}^m_{\omega_f}} \cdot \varphi_{\omega_f} = \frac{\hat{L}^f_\delta}{\hat{L}^f_{\omega_m}} \cdot \varphi_{\omega_m}, \tag{25}$$

where $\hat{L}^m_\delta$ and $\hat{L}^f_\delta$ can be estimated using the usual techniques. Moreover, the second equality is a testable constraint induced by the presence of tax parameters. Finally, the intra-household redistributive neutrality can be tested as $\varphi_\delta = 0$. In a sense, this may also be seen as a test of the income pooling hypothesis since these parameters are not expected to have a direct impact under the unitary approach.

In principle, such data, with some variability in the tax parameters, can be constructed. It suffices to have a panel or a time-series of cross-sections provided that the tax law changes during the period of observation or a set of cross-sections coming from various countries characterized by different fiscal legislations. In particular, the PSID is especially convenient for that purpose because it contains these two kinds of variations. Nevertheless, it might be difficult, in practice, to choose the most relevant parameters of the tax system since we do not have a theoretical model which explains the effects of tax on household bargaining. This is essentially an empirical issue open for future investigation.

4.3.3 A Simple Parametric Example

To illustrate the results of this section, we consider the functional form used in Section 3.3 for the labor supplies, expressed in terms of the shadow variables, and introduce an additive term for the tax parameters:

$$L^m = a_0 + a_1\omega_f + a_2\omega_m + a_3\omega_f\omega_m + a_4\eta + a_5\eta^2 + a_6\delta, \tag{26}$$

$$L^f = b_0 + b_1\omega_f + b_2\omega_m + b_3\omega_f\omega_m + b_4\eta + b_5\eta^2 + b_6\delta, \tag{27}$$

where ω_f, ω_m and η are defined by (21) and (22). Disposable income is thus assumed to be given by (20). If we condition on a particular tax system (characterized by the vector of tax parameters $\delta = \delta^*$), we can redefine the constants: $a_0^* = a_0 + a_6\delta^*$ and $b_0^* = b_0 + b_6\delta^*$. Therefore, using the preceding results, we can show that the sharing rule has the following functional form:

$$\varphi = k_0^* + k_1\omega_f + k_2\omega_m + k_3\omega_f\omega_m + k_4\eta + k_5\eta^2,$$

where k_1, k_2, k_3, k_4 and k_5 are defined by (8) and $k_0^* = k_0 + k_6\delta^*$. If δ is variable, k_6 can be identified. In particular, if we apply (25), we obtain:

$$k_6 = \frac{a_6 \cdot b_3}{\Delta} = \frac{b_6 \cdot a_3}{\Delta}.$$

Moreover, this relation yields a testable constraint: $a_6 \cdot b_3 = b_6 \cdot a_3$. Finally, the neutrality of the tax system can be tested by verifying $k_6 = 0$.

A straightforward method for estimating this model is to introduce unobservable heterogeneity in both labor supplies and follow the FIML framework developed by Hausman (1981). Still, this may be cumbersome. A simpler approach is to account for the endogeneity of the regressors by the Conditional FIML method. Natural instruments for ω_f, ω_m and η are then given by w_f, w_m and y.

5 Conclusion

In this paper, we present two closely related extensions of the collective model of labor supply. First, we take into account participatory decisions. Second, we consider nonlinear budget constraints. We show that, in both cases, the main conclusions of Chiappori can be generalized. Finally, we study how to measure the incidence of income tax on household labor supply.

We show that, in the collective approach, simulations of fiscal reforms pose additional problems. The incidence of a change in tax parameters on labor supply is generally indeterminate when usual cross-section data are used. This indeterminacy can intuitively be explained by the fact that the collective model is not completely structural. By this we mean that the process which leads to intra-household distribution is not explicitly specified. Theoretically, this indeterminacy could be solved by a model which explains how the resources are actually shared within the household. Such models exist in family economics (e.g., McElroy and Horney (1981)) but are not easily implemented empirically. The alternative approach is to use richer data where changes in tax parameters are observed. This provides a very interesting direction for future empirical research.

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