Centre de recherche sur l'emploi et les fluctuations économiques (CREFÉ)
Center for Research on Economic Fluctuations and Employment (CREFE)

Université du Québec à Montréal

Cahier de recherche/Working Paper No. 141

Collective Female Labor Supply: Theory and Application *

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Septembre 2001

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* Financial support from the TMR Marie Curie Research Training Grant (Contract n° ERBFMBICT983433) is gratefully acknowledged. This paper has been partly written when I was working in DELTA, whose hospitality is gratefully acknowledged. Preliminary versions have been presented in Paris, Lille, Québec and Montréal. We thank François Bourguignon, Pierre-André Chiappori, Bernard Fortin, François Gardes, Guy Lacroix, Thierry Magnac and Costas Meghir for useful comments and suggestions. We also thank Bénédicte Sabatier for her research assistance in the data processing. We bear the sole responsability for any remaining errors.

Résumé:
Dans cet article, nous traitons la question de l'offre féminine de travail dans le cadre de l'approche collective (Chiappori, Journal of Political Economy (1992)). Nous étudions des couples mariés et partons de l4observation empirique que l'offre de travail du mari est généralement déterminée par des contraintes exogènes. Nous montrons alors que, dans ce cas, des éléments structurels du processus de décision, tels que les préférences individuelles or la règle qui détermine la répartition intra-familiale du bien-être peut-être identifiée á condition que la demande pour au moins un bien est observée conjointement avec l'offre de travail. Ces considérations théoriques sont suivies par une application empirique utilisant des données françaises.

Abstract:
In this paper, we deal with female labour supply in the collective framework (Chiappori, Journal of Political Economy (1992)). We study married couples and start from the empirical observation that the husband's labour supply is generally determined by exogenous constraints. We then show that, in this case, structural elements of the decision process, such as individual preferences or the rule that determines the intra-household distribution of welfare, can be identified if household demand for at least one commodity, together with the wife's labour supply, is observed. These theoretical considerations are followed by an empirical application using French data.

Keywords:
Collective Decisions, Female labour Supply, Commodity Demands, Intrahousehold Distribution
JEL classification: D12, J22

1 Introduction

Traditionally, the household, as a whole, is considered as the elementary decision unit ; in particular, consumption and labour supply decisions are modeled as though household members were maximizing a unique utility function under a budget constraint. However, recent dissatisfaction with this so-called unitary model arose in a large part from the weakness of its theoretical foundations. We must admit, at least since Arrow's famous impossibility theorem, that a household comprising several adult members does not necessarily behave as a single rational agent. Furthermore, the specific restrictions imposed by the unitary model have received little empirical support, if any. In particular, the *Income Pooling Hypothesis* — according to which only total exogenous family income, and not its distribution across members, matters for labour supply and consumption decisions — has been strongly rejected in many recent studies (see Lundberg and Pollak (1996) for a survey).

For these reasons, Chiappori (1988, 1992) has proposed a model of labour supply based upon a collective representation of household behaviour. In this framework, each person is characterized by specific preferences, and decisions are only assumed to result in Pareto-efficient outcomes. He demonstrates that these simple assumptions are sufficient to generate testable restrictions on household labour supply under the form of partial differential equations. He also shows that, if these restrictions are satisfied, some elements of the decision process, such as individual preferences and the rule that determines the distribution of welfare within the household, can be retrieved from the observation of both labour supplies. More recently, Donni (2000) has extended this theoretical model to incorporate the possibility of non-participatory decisions and non-linear budget sets. Apps and Rees (1997) and Chiappori (1997) have yielded the theoretical basis for introducing household production. Fong and Zhang (2001) have studied a collective model of labour supply where there are two distinct types of leisure : one type is each person's independent (or private) leisure, and the other type is spousal (or public) leisure.

This setting turns out to be profitable as shown by several recent empirical applications. For example, Fortin and Lacroix (1996) closely follow Chiappori's initial framework. They use a functional form that nests both the unitary and the collective model as particular cases and find, using Canadian data, that the restrictions implied by the unitary setting are strongly rejected, while the collective ones are not. Chiappori et al. (2001) extend the collective models to allow for 'distribution factors', defined as being any variable that is exogenous with respect to preferences but may influence the decision process. Using the Panel Study of Income Dynamics and choosing the sex ratio and an indicator of the divorce legislation as distribution factors, they find that the restrictions implied by the collective model are not rejected. Blundell et al. (1998) further develop the collective model to cover the possibility of discrete choices and unobserved heterogeneity. They analyze household labour supply using United Kingdom data from 1978 to 1983 and notably find that the estimated wage elasticities are not at odds with intuition.

Still, it appears that, in most studies, the structural parameters describing the intra-household decision process are not estimated with precision (apart from the estimates given by Chiappori et al. (2001)). One reason is that, in most countries, male labour supply is rigid and largely determined by exogenous constraints. For example, only 13 per cent of the working population in European Union countries have a part-time job: 67 per cent of these are married women, and the remaining 33 per cent are divided almost equally between men and single women (Eurostat, 1988). Similar trends, even if less marked, are observed in the United States: among married couples, about 11 per cent of working men have a part-time job, three times less than working women (Bureau of Labor Statistics, 2000). These facts are the starting point for the current paper where we focus on the wife's labour supply and assume that the husband's hours of work are constrained at an upper bound.

Understanding the source of such a difference between a wife and husband's behaviour is a very important research topic but is beyond the scope of this paper. At this stage, we only admit that, for sociological reasons, the preferences for leisure are generally lower for the husband than for the wife while his market wage is higher. This implies that, in most data set, the husband's labour supply is generally *observed* at the current legal (or sociocultural) maximum. If so, the usual proof of identifiability and testability of the collective approach is no longer valid. A solution is to use additional information on household behaviour. We thus suppose that the wife's labour supply and, at least, one commodity demand are jointly observed. We demonstrate that this setting allows us to identify some elements of the intra-household decision process and generate testable restrictions on household behaviour. We also consider the possibility of non-participation for the wife and show, in this case, that commodity demands endogenously switch regimes. This result extends to collective models the theory of household behaviour under rationing developed by Neary and Roberts (1980). Finally, we estimate and test this collective model using French data for couples in which the husband is working full-time. These data are especially suitable for our purpose because the French labour market is characteristically rigid. In the estimation process, we explicitly take into account the possibility of non-participation for the wife with a Full Information Maximum Likelihood Method applied to a five-equation system. We find that the main structural parameters are fairly well estimated in view of the small size of our sample.

This paper is structured as follows. Section 2 discusses the assumptions of our framework and Section 3 presents the main theoretical results. Section 4 provides an analysis of our econometric strategy and Section 5 gives a brief description of the data and the empirical results. Section 6 concludes.

2 The Collective Approach

2.1 Basic Framework

We consider only the case of married couples (m and f) in a single period setting.[1] The wife's and husband's labour supply are respectively denoted by L^f and L^m with market wages w_f and w_m. The wife's and husband's demand for commodity n ($n = 1, \ldots, N$) are respectively denoted by C^{fn} and C^{mn} with prices set to one. Non-labour income is denoted by y. For convenience, the spouses' total time endowment is normalized at one.[2] Let $C^i = (C^{i1}, \ldots, C^{iN})'$ be the vector of member i's consumptions ($i = m, f$). We adopt the following assumption on preferences.

Assumption A1 Each household member is characterized by specific preferences. These can be represented by utility functions of the form: $u^i(1 - L^i, C^i)$ that are both strongly concave, infinitely differentiable and strictly increasing in all their arguments, with $\lim_{C^i \to 0} u^i(1 - L^i, C^i) = -\infty$.

The household members are said to be 'egoistic' in the sense that their utility only depends on their own consumption and leisure. However, all the

[1] Of course, the fact that the household members are married is not important. The terminology is just for convenience.

[2] This upper bound for members' hours of work can alternatively be seen as a legal or socio-cultural norm.

results immediately extend to the case of 'altruistic' agents in a Beckerian sense, with utilities represented by the form:

$$W_i[u^m(1 - L^m, C^m), u^f(1 - L^f, C^f)],$$

where $W_i(\cdot)$ is a strictly increasing function. The crucial hypothesis is the existence of some type of separability in the preferences of the two household members. Finally, let us note that the condition on limits in A1 lets us rule out the cases where consumption is equal to zero.

We implicitly assume that there is no public consumption and no domestic production. The budget constraint is then written as follows:

$$y + L^f \cdot w_f + L^m \cdot w_m \geqslant \textstyle\sum_n^N (C^{fn} + C^{mn}).$$

Let us remark that, typically, we only observe household purchases of commodities within a certain period. Even if we equate these purchases with consumption, we generally do not observe the individual consumptions for private commodities. We only observe the aggregate consumption $C^n = C^{fn} + C^{mn}$.

The main originality of the efficiency approach lies in the fact that the household decisions result in Pareto-efficient outcomes and that no additional assumption is made about the process. This is formally expressed in the following assumption.

Assumption A2 The outcome of the decision process is Pareto efficient; that is, for any wage-income bundle (w_f, w_m, y), the labour-consumption bundle (L^f, L^m, C^f, C^m) chosen by the household is such that no other bundle $(L^{f*}, L^{m*}, C^{f*}, C^{m*})$ in the budget set could make both members better off.

This assumption has a good deal of intuitive appeal. First of all, the household is one of the preeminent examples of a repeated game. Then, given the symmetry of information, it is plausible that agents find mechanisms to support efficient outcomes since cooperation often emerges as a long term equilibrium of repeated noncooperative relations. A second point is that axiomatic models of bargaining with symmetric information, such as Nash or Kalai-Smorodinsky bargaining, which have been previously used to analyze negotiation within the household (Manser and Brown (1980) and McElroy and Horney (1981)), assume efficient outcomes.

Pareto-efficiency essentially means that there exists a scalar μ such that the household behaviour is a solution to the following program:

$$\max_{\{L^f,L^m,C^f,C^m\}} (1-\mu)\cdot u^f(1-L^f,C^f)+\mu\cdot u^m(1-L^m,C^m) \qquad (\bar{\mathrm{P}})$$

with respect to

$$y+L^f\cdot w_f+L^m\cdot w_m \geqslant \textstyle\sum_n^N (C^{fn}+C^{mn}),$$

$$1\geqslant L^i\geqslant 0 \quad \text{and} \quad C^i\geqslant 0 \qquad i=f,m.$$

The parameter μ has an obvious interpretation as a 'distribution of power' index. If $\mu=0$, then the household behaves as though the wife always gets her way whereas, if $\mu=1$, it is as if the husband is the effective dictator.

Practically, some structure is added to the decision process. To obtain well-behaved labour supplies and commodity demands, we assume that the scalar $\mu\in\ ]0,1[$ is a *single-valued* and infinitely differentiable function of w_f,w_m and y. The underlying idea is that, within a bargaining context, the threat point is expected to depend on non-labour income and the wage that the spouses receive when they work. If so, most cooperative equilibrium concepts imply that μ is a function of w_f,w_m and y.

2.2 Decentralization and Rationing

We say that a pair of labour supplies and a pair of systems of commodity demands are consistent with collective rationality if A1 and A2 are jointly fulfilled. The next step is to define what we call the sharing rule. To do this, we use the following lemma.

Lemma *A pair of labour supplies $\bar{L}^i(w_f,w_m,y)$ and a pair of systems of N commodity demands $\bar{C}^i(w_f,w_m,y)$ are consistent with collective rationality if and only if there exists a pair of functions $\rho_i(w_f,w_m,y)$ with $\sum\rho_i=y$ such that $(\bar{L}^i,\bar{C}^i)$ is a solution of*

$$\max_{\{L^i,C^i\}} u^i(1-L^i,C^i) \quad \text{subject to} \quad \textstyle\sum_n^N C^{in}=\rho_i+L^i\cdot w_i$$

$$\text{and} \quad 1\geqslant L^i\geqslant 0, \quad C^i\geqslant 0\ ,$$

for any $(w_f,w_m,y)\in \mathrm{IR}^2_{++}\times \mathrm{IR}$.

Proof. An application of the First and the Second Theorems of Welfare Economics, which state the equivalence between efficiency and a decentralized equilibrium when there are no externalities. ■

This lemma has several important consequences. Specifically, it determines the functional structure of labour supplies and commodity demands. When both spouses are unrationed on the labour market, we have :

$$\begin{aligned} \bar{L}^f(w_f, w_m, y) &= \lambda^f(w_f, \rho), \quad (1)\\ \bar{L}^m(w_f, w_m, y) &= \lambda^m(w_m, y-\rho), \quad (2) \end{aligned}$$

and

$$\bar{C}^n(w_f, w_m, y) = \zeta^{fn}(w_f, \rho) + \zeta^{mn}(w_m, y-\rho), \quad (3)$$

where $\rho = \rho_f$ and $y - \rho = \rho_m$. The functions $\lambda^f, \lambda^m, \zeta^{fn}$ and ζ^{mn} are traditional Marshallian labour supplies and commodity demands. In particular, the labour supplies satisfy Slutsky Positivity :

$$\lambda^i_{w_i} - \lambda^i_{\rho_i} \cdot L^i > 0.$$

For convenience, the wife's share ρ is called the sharing rule. The latter is generally a function of all the exogenous variables.

For treating corner solutions, we must develop a 'collective' theory of household behaviour under (endogenous) rationing.[3] We follow the procedure used by Neary and Roberts (1980) for standard systems of demands. When the wife is rationed on the labour market, her actual wage is replaced in commodity demands by a shadow wage ω_f implicitly defined by

$$\lambda^f(\omega_f, \rho_f + L^f_R \cdot (w_f - \omega_f)) = L^f_R \Leftrightarrow \omega_f = \omega_f(L^f_R \cdot w_f + \rho_f, L^f_R), \quad (4)$$

where L^f_R is the level of rationing. Thus, the nth commodity demand becomes :

$$\begin{aligned} \bar{C}^n(w_f, w_m, y) &= \zeta^{fn}(\omega_f, L^f_R \cdot (w_f - \omega_f) + \rho_f) + \zeta^{mn}(w_m, \rho_m)\\ &= \zeta^{fn}_\star(L^{fn}_R \cdot w_f + \rho, L^f_R) + \zeta^{mn}(w_m, y-\rho), \quad (5) \end{aligned}$$

[3] We may expect that, in the case of exogenous rationing, the sharing rule is a function of the 'scale' of rationing. Since we are only interested in dealing with corner solutions, we exclude this possibility. This does, however, provide an interesting direction for future research.

where the definition (4) is used in the second line. This expression (5) means that, in the case of rationing, commodity demands switch regimes : an increase in the wife's wage only has an income effect on household consumption. This is in line with the traditional setting where a single utility function is assumed. However, we must note that, in collective models, an increase in the wife's (market) wage generally has an impact on household consumption through the sharing rule, even when the wife does not work, i.e., $L_R^f = 0$.

We also have the following cases. When the husband is rationed on the labour market, the nth commodity demand becomes :

$$\bar{C}^n(w_f, w_m, y) = \zeta^{fn}(w_f, \rho) + \zeta_\star^{mn}(y + L_R^m \cdot w_m - \rho, L_R^m). \tag{6}$$

Finally, when both spouses are rationed on the labour market, the nth commodity demand becomes :

$$\bar{C}^n(w_f, w_m, y) = \zeta_\star^{fn}(w_f \cdot L_R^f + \rho, L_R^f) + \zeta_\star^{mn}(y + L_R^m \cdot w_m - \rho, L_R^m). \tag{7}$$

In this case, we obtain the model of pure consumption studied in a recent working paper by Bourguignon et al. (1995).

2.3 The Benchmark Case : Both Household Members Are Unrationed.

Traditionally, both household members are assumed to be unrationed. Precisely, it is implicitly assumed that $(w_f, w_m, y) \in R$ where R is a partition of $\mathbb{R}^2_{++} \times \mathbb{R}$ defined by

$$(w_f, w_m, y) \in R \quad \text{iff} \quad 1 > \bar{L}^f > 0 \quad \text{and} \quad 1 > \bar{L}^m > 0.$$

As pointed out by Chiappori (1988, 1992), such a framework, where labour supplies are flexible and characterized by (1) and (2), has two interesting properties for empirical implementations. First, it is sufficient for generating testable restrictions on labour supplies under the form of partial differential equations. These equations can be viewed as analogous, in the collective setting, to Slutsky relations in the traditional model and they can be translated into restrictions on parameters which may in turn either be tested statistically or be used a priori for reducing the estimation task. Second, it allows us to recover, from the observed behaviour, individual preferences and the outcome of the decision process — the sharing rule. This setting is slightly extended in Donni (2000) to cover the cases where one person in the household does not participate to the labour market.

However, the main theoretical results on collective labour supply always suppose that the set R is observed by the economist. This is excessively restrictive. In most countries, the husband's labour supply is fixed at the upper bound ($\bar{L}^m = 1$, in our notation) for the majority of households and equal to zero for a minority of them. Moreover, there is obviously nothing to say about the testability or the identifiability of the collective approach if only the wife's labour supply is taken into account.[4] Blundell et al. (1998) accept as a fact these rigidities in the husband's behaviour but they show that the sharing rule can be identified from observation of wife's labour supply and husband's participation decision. However, they do not explain why the husband's choices (contrary to wife's choices) are discrete. In addition, they suppose that husband's non-participation to the labour market is voluntary. This assumption is possible in the United-Kingdom, where the labour market is competitive, but more debatable in other countries. In what follows, we deal with this problem in another way.

3 Collective Female Labour Supply

3.1 Preliminary Considerations

In our approach, we do not reappraise the fact that the husband's labour supply is generally fixed at the upper bound. This is consistent with empirical evidence as well as classical work based on the unitary setting (e.g., Heckman (1974) or Mroz (1987)). It is plausible that, for sociological reasons, the preferences for leisure are lower for the husband than for the wife while his market wage is higher. These considerations lead us to assume that $(w_f, w_m, y) \in P$ where P is a partition of $\mathbb{R}^2_{++} \times \mathbb{R}$ defined by

$$(w_f, w_m, y) \in P \quad \text{iff} \quad 1 > \bar{L}^f > 0 \quad \text{and} \quad \bar{L}^m = 1.$$

In other words, we assume that husbands are endogenously rationed on the labour market. Practically, the few households remaining where the husband

[4] This is not true for the unitary approach. We have testable constraints on the wife's behaviour: $\bar{L}^f_y = \bar{L}^f_{w_m}$ (Income Pooling) and $\bar{L}^f_{w_f} - \bar{L}^f \cdot \bar{L}^f_y > 0$ (Slutsky Positivity). Furthermore, the 'household' preferences between the wife's leisure and consumption can be identified from observation of the wife's labour supply.

does not work are explained by an exogenous rationing, e.g., involuntary unemployment, and are neglected.

The idea of this paper is to use information on household consumption to identify the sharing rule and derive testable restrictions. Therefore, we study the wife's labour supply and household commodity demands in a unified framework.

3.2 Identifying the Sharing Rule

First, we assume that only one commodity demand, together with the wife's labour supply, is observed. We recall that when the husband's labour supply is fixed at one, these functions are written as:

$$\begin{aligned}\bar{L}^f(w_f, w_m, y) &= \lambda^f(w_f, \rho),\\ \bar{C}^n(w_f, w_m, y) &= \zeta^{fn}(w_f, \rho) + \zeta_\star^{mn}(y + w_m - \rho, 1),\end{aligned}$$

where the sum of non-labour income and husband's wage is the 'exogenous' household income. Moreover, under our assumptions, $\bar{L}^f(w_f, w_m, y)$ and $\bar{C}^n(w_f, w_m, y)$ as well as $\rho(w_f, w_m, y)$ are infinitely differentiable in all their arguments on $\text{int}(P)$. To simplify the derivation of the results, let us now introduce the following definitions:

$$\alpha^n = \frac{\bar{C}_y^n \cdot \bar{L}_{w_m}^f - \bar{L}_y^f \cdot \bar{C}_{w_m}^n}{\bar{L}_{w_m}^f - \bar{L}_y^f} \quad \text{and} \quad \beta^n = \frac{\bar{L}_{w_m}^f - \bar{L}_y^f}{\alpha_y^n \cdot \bar{L}_{w_m}^f - \alpha_{w_m}^n \cdot \bar{L}_y^f},$$

if denominators are different from zero, where the notation F_x stands for the partial differential of function F with respect to variable x. As we will show below (see the proof of Proposition 1), α^n corresponds to the slope of the husband's nth commodity demand while β^n is the inverse of the derivative of this slope. Let us now assume that the functions that we consider satisfy some regularity conditions.

Assumption R1 The wife's labour supply and the nth commodity demand are such that

$$\bar{L}_{w_m}^f \neq \bar{L}_y^f, \quad \alpha_{w_m}^n \neq \alpha_y^n \quad \text{and} \quad \alpha_y^n \cdot L_{w_m}^f \neq \alpha_{w_m}^n \cdot L_y^f,$$

for any $(w_f, w_m, y) \in \text{int}(P)$.

These conditions can be interpreted as follows. The first condition obviously implies the absence of income pooling in the wife's labour supply, i.e., the impact of the husband's wage and the impact of non-labour income on the wife's labour supply have to be *different*. The second condition can be seen, in a certain sense, as a generalization of this assumption for the husband's nth commodity demand since α^n corresponds to the slope of this demand. The third condition is slightly more complicated. It implies, in view of the definition of β^n, that the second derivative of the husband's nth commodity demand is different from zero. Finally, let us remark that these conditions are generically true in the usual sense. However, they exclude the demand for the commodity which is implicitly defined by the budget constraint

$$\sum_n (C^{fn} + C^{mn})$$

and the demand for commodities which are exclusively consumed by the wife.

We can now put forward the next result which says that, in the present setting, some elements of the wife's preferences and the sharing rule can be retrieved.

Proposition 1 *Let $\bar{L}^f(w_f, w_m, y)$ be the wife's labour supply and $\bar{C}^n(w_f, w_m, y)$ the nth commodity demand. Let us assume Collective Rationality and R1. Then, the sharing rule can be retrieved on P up to a constant ϵ. Specifically, the derivatives of the sharing rule on int(P) are given by*

$$\rho_{w_f} = -\alpha^n_{w_f} \cdot \beta^n, \quad \rho_{w_m} = 1 - \alpha^n_{w_m} \cdot \beta^n, \quad \text{and} \quad \rho_y = 1 - \alpha^n_y \cdot \beta^n.$$

Moreover, for each choice of ϵ, the wife's preferences between total consumption and leisure (i.e., the marginal rate of substitution) are uniquely defined. Finally, the individual nth commodity demands can also be retrieved up to a constant ϵ^n.

Proof. See the Appendix.

We briefly sketch the basic steps of the proof. The idea is that changes either in non-labour income or in the husband's wage can have only an income effect; specifically, they will affect the wife's labour supply only insofar as her share of exogenous income, as defined by the sharing rule, is modified. This means that any simultaneous change in non-labour income and the husband's wage that leaves the wife's labour supply unchanged must keep her share constant as well. From this idea, it is possible to measure the effect of exogenous income on commodity demand, keeping the wife's share unchanged. This yields the husband's Engel curve α^n. Once the husband's Engel curve is retrieved, it is possible to obtain other structural elements, such as the sharing rule or the wife's Engel curve, by differentiation of this Engel curve and resolution of the resulting system of partial differential equations. Knowing the sharing rule allows us to write down the wife's actual budget constraint and to compute her preferences in the usual way.

3.3 Testing Collective Rationality

3.3.1 One Commodity Demand

The next result gives a set of testable restrictions that the wife's labour supply and the commodity demand have to satisfy.

Proposition 2 *Let $\bar{L}^f(w_f, w_m, y)$ be the wife's labour supply and $\bar{C}^n(w_f, w_m, y)$ the nth commodity demand. Let us assume Collective Rationality and R1. Then,*

a)

$$\bar{L}^f_{w_f} - \frac{\bar{L}^f_{w_m} - \bar{L}^f_y}{(\alpha^n_{w_m} - \alpha^n_y) \cdot \beta^n} \cdot \left(\bar{L}^f - \alpha^n_{w_f} \cdot \beta^n\right) > 0,$$

b)

$$\alpha^n_{w_f} \cdot \beta^n_{w_m} = \alpha^n_{w_m} \cdot \beta^n_{w_f} \quad \text{and} \quad \alpha^n_{w_f} \cdot \beta^n_y = \alpha^n_y \cdot \beta^n_{w_f},$$

for any $(w_f, w_m, y) \in int(P)$.

Proof. See the Appendix.

These restrictions provide a joint test of collective rationality under specific assumptions, namely, egoistic (or altruistic in a Beckerian sense) agents and absence of public consumption and domestic production. The first condition corresponds to the Slutsky Positivity translated in the collective approach. The second condition results from the separability property of the behavioural functions: husband's wage and non-labour income affect the household behaviour only through the sharing rule.

3.3.2 Several Commodity Demands

More can be obtained when the demand for several commodities, rather than a single one, is observed. This is formally expressed in the following proposition.

Proposition 3 *Let $\bar{L}^f(w_f, w_m, y)$ be the wife's labour supply and $\bar{C}^{n_1}(w_f, w_m, y)$ and $\bar{C}^{n_2}(w_f, w_m, y)$ two commodity demands. Let us assume Collective Rationality and R1. Then,*

$$\alpha_{w_f}^{n_1} \cdot \alpha_{w_m}^{n_2} = \alpha_{w_f}^{n_2} \cdot \alpha_{w_m}^{n_1} \quad \text{and} \quad \alpha_{w_f}^{n_1} \cdot \alpha_{y}^{n_2} = \alpha_{w_f}^{n_2} \cdot \alpha_{y}^{n_1},$$

for any $(w_f, w_m, y) \in int(P)$.

Proof. See the Appendix.

The difference with the case of only one commodity is that the constraints here are based on a second, rather than a third order partial differential equation, which is more restrictive. In particular, for the functional form that we use in the empirical application of this paper, the second condition of Proposition 2 is automatically satisfied and the test of collective rationality is based only on the condition of Proposition 3.

3.4 Extension : The Wife's Rationing

One of the main limitations of the preceding results is the assumption that the wife is free to vary the hours she works. However, this is a fact : many wives choose not to work at all or to work full-time, two cases ruled out in the earlier discussion. This is the motivation for this section. We only consider the wife's decision to participate in the labour market but the results can easily be extended to the converse case of working full-time.

To begin with, we note that the existence of a well-behaved participation frontier does not stem from the theoretical set-up as in standard labour supply models, but has to be postulated.[5] We, therefore, use the following

[5] A formal discussion of this point is given in Donni (2000). The underlying idea is that in the collective approach, when the wife is indifferent between working and not-working, an increase in the wife's wage also has an income effect on labour supply (through the sharing rule). Therefore, the traditional argument for the uniqueness of the reservation wage is no longer valid. To exclude the possibility of multiple reservation wages, we can use a convenient assumption on preferences and the sharing rule. However, in what follows, we adopt a more direct approach.

assumption:

Assumption R2 There exists a positive function $\gamma(w_m, y)$ defined on $\mathrm{IR}_{++} \times \mathrm{IR}$ such that the wife does not participate in the labour market if and only if $w_f \leqslant \gamma(w_m, y)$.

We assume that, although the wife does not work, we observe her market wage w_f. We now consider the set N where N is a connected partition of $\mathrm{IR}^2_{++} \times \mathrm{IR}$ defined by

$$(w_f, w_m, y) \in N \quad \text{iff} \quad \bar{L}^f = 0 \quad \text{and} \quad \bar{L}^m = 1.$$

We also define $I \equiv \{(w_m, y) | (w_f, w_m, y) \in N \text{ and } w_f = \gamma(w_m, y)\}$. An illustration of these sets is given, for a fixed y, in Figure 1. In this figure, two points must be stressed. First, the wife does not participate in the labour market when her wage is below its reservation value. Second, the husband's labour supply falls below 1 for some wages which are particularly low or high but these extreme values are not of interest to us. Finally, we assume that we jointly observe the nth commodity demand, given by

$$\bar{C}^n(w_f, w_m, y) = \zeta_\star^{fn}(\rho, 0) + \zeta_\star^{mn}(y + w_m - \rho, 1),$$

on N and the participation frontier, defined as $w_f = \gamma(w_m, y)$, on I.

As we show below, this framework yields sufficient information to identify the sharing rule and generate testable restrictions. First, let us note that, under our assumptions, commodity demands and the participation frontier are infinitely differentiable respectively on $\mathrm{int}(N)$ and on I. Moreover, along the participation frontier, the nth commodity demand can be written as follows:

$$\hat{C}^n(w_m, y) = \bar{C}^n(\gamma(w_m, y), w_m, y).$$

This function is also infinitely differentiable on I. The next step is to define the following functions:

$$A^n = \frac{\hat{C}^n_y \cdot \gamma_{w_m} - \gamma_y \cdot \hat{C}^n_{w_m}}{\gamma_{w_m} - \gamma_y} \quad \text{and} \quad B^n = \frac{\gamma_{w_m} - \gamma_y}{\gamma_{w_m} \cdot A^n_y - \gamma_y \cdot A^n_{w_m}},$$

if the denominators are different from zero. The interpretation is now familiar: A^n corresponds to the slope of the husband's commodity demand along the participation frontier while B^n is the inverse of the derivative of this slope. Let us now assume that the behavioural functions at stake satisfy some regularity conditions.

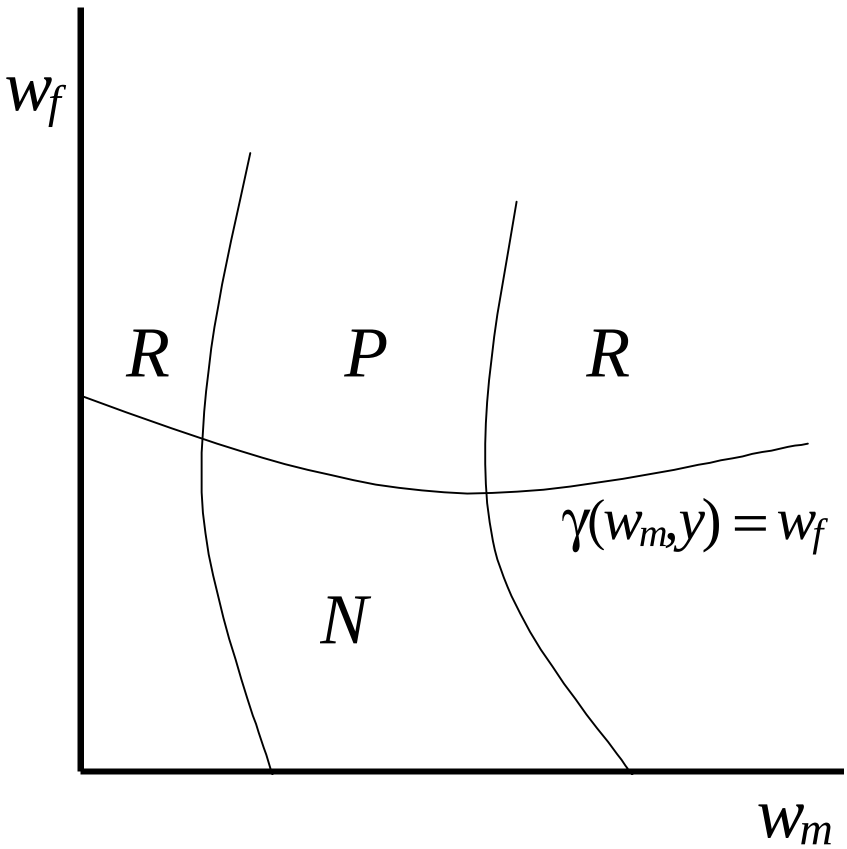

Figure 1: Non-participation set and participation frontier

Assumption R3 The wife's participation frontier and the nth commodity demand are such that

$$\gamma_{w_m} \neq \gamma_y, \quad A^n_{w_m} \neq A^n_y \quad \text{and} \quad \gamma_{w_m} \cdot A^n_y \neq \gamma_y \cdot A^n_{w_m},$$

$$\hat{C}^n_{w_m} \neq \hat{C}^n_y,$$

for any $(w_m, y) \in I$.

The first three equations are required to identify the sharing rule along the participation frontier. The last is required to identify the sharing rule on $\text{int}(C)$.

The next result says, first, that the sharing rule can be retrieved on N up to a constant and, second, that testable restrictions are generated.

Proposition 4 *Let $\gamma(w_m, y)$ be the wife's participation frontier and $\bar{C}^n(w_f, w_m, y)$ the nth commodity demand. Let us assume Collective Rationality, R2 and R3. Then, the sharing rule can be retrieved on N up to a constant ϵ. Specifically, let $v(w_m, y) = \rho(\gamma(w_m, y), w_m, y)$ be the sharing rule along the participation frontier, then the derivatives of the sharing rule on I are given by*

$$v_{w_m} = 1 - A^n_{w_m} \cdot B^n \quad \text{and} \quad v_y = 1 - A^n_y \cdot B^n.$$

Moreover, the individual nth commodity demand can be retrieved up to a constant ϵ^n. Finally, we have the following constraint:

$$A^n_y \cdot B^n_{w_m} = A^n_{w_m} \cdot B^n_y,$$

for any $(w_m, y) \in I$.

Proof. See the Appendix.

At this stage, some precisions are necessary. First, the proof of this proposition does not require the complete specification of the underlying wife's labour supply but only the specification of the participation frontier. This result is very useful since the number of hours of work is not always provided in data sets. Often the information contained in consumer surveys is only concerned with the participation decision of household members. Second, considering additional commodity demands obviously creates new constraints. These are not formally examined in this paper. In fact, this setting with double rationing can be interpreted as a model of pure consumption. This kind of model has received great attention in Bourguignon et al. (1995). These authors derive the whole set of testable constraints. They also show that the sharing rule can be identified from any triplet of commodity demands. Therefore, a test of continuity of the sharing rule consists in comparing the sharing rule obtained from three commodity demands and the one given in Proposition 4. Finally, this result completes the identification property given in Proposition 1 and assures us that the sharing rule is identified on the entire set of w_f, w_m and y which is relevant for empirical applications.

4 Econometric Analysis

We consider in this section the empirical implementation of the model described above. First, we propose a functional form for the labour supply and the system of commodity demands. Second, we introduce stochastic terms and derive the log-likelihood function.

4.1 Functional Form

In this section and those that follow, we adopt the following conventions: w_{fh} denotes the wife's hourly wage in household h, w_{mh} the husband's hourly wage, y_h the monthly non-labour income, L_{fh} the observed number of worked hours per month, C_h^n the expenditure on commodity n per month and T the (legal or socio-cultural) maximum number of hours per month.

4.1.1 Labour Supply and Commodity Demands

In order to estimate and test this model, we adopt the linear functional form, initially proposed by Hausman (1981), for the wife's *latent* labour supply. If we neglect the stochastic terms at this stage, we have:

$$L_{fh}^* = \alpha_h + \beta \cdot w_{fh} + \gamma \cdot \rho_h,$$

where L_{fh}^* is the wife's latent number of worked hours per month. The wife is rationed on the labour market if the latent variable is either greater than T or lower than 0:

$$L_{fh} = T \quad \text{if} \quad L_{fh}^* \geqslant T, \qquad L_{fh} = 0 \quad \text{if} \quad L_{fh}^* \leqslant 0,$$

$$\text{and} \quad L_{fh} = L^*_{fh} \quad \text{otherwise.}$$

Moreover, the intercept α_h is assumed to depend on a set of variables:

$$\alpha_h = \sum_{j=1}^{J} \alpha_j \cdot z_{jfh},$$

where z_{jfh} are socio-demographic characteristics relevant for explaining the wife's behaviour (e.g., the wife's age, the region of the household, the number of children and so on). Finally, Slutsky Positivity is *globally* fulfilled if and only if $\beta \geqslant 0$ and $\gamma \leqslant 0$.

This specification has several desirable properties. First, the linear form for labour supply has been frequently used in empirical studies and is suitable for French data (see Bourguignon and Magnac (1990) for another application).[6] Second, the wife's preferences between leisure and consumption have a well-known form (see Hausman (1980) or Stern (1986), for examples). Precisely, they are given by the following indirect utility function:

$$V(w_{fh}, \rho_h) = \exp(\gamma \cdot w_{fh}) \cdot (\rho_h + \frac{\beta}{\gamma} \cdot w_{fh} - \frac{\beta}{\gamma^2} + \frac{\alpha_h}{\gamma}). \tag{8}$$

Third, this specification permits us to recover a closed form for shadow wages which are used to compute rationed commodity demands. Of course, the main limitation of the linear functional form is its lack of flexibility; in particular, it implies that the labour supply curve is either upward sloping everywhere or backward bending everywhere.

We now consider the functional form of the commodity demands. To begin with, let us recall that the previous theory states that identification of the sharing rule relies on the non-linearity of the husband's commodity demands. We thus assume that, when $0 < L^*_{fh} < T$, the nth commodity demand has a quadratic form as follows:

$$\begin{aligned} C^n_h &= a^n_{fh} + b^n_f \cdot w_{fh} + c^n_f \cdot \rho_h + d^n_f \cdot {\rho_h}^2 \\ &\quad + a^n_{mh} + c^n_m \cdot (y_h + w_{mh}T - \rho_h) + d^n_m \cdot (y_h + w_{mh}T - \rho_h)^2. \end{aligned}$$

This functional form switches regimes in the case of rationing. Two cases must be considered. First, when $L^*_{fh} \leqslant 0$, we compute the shadow wage as follows:

$$\omega_{fh} = -\frac{\alpha_{fh} + \gamma_f \cdot \rho_h}{\beta_f},$$

[6] Other specifications for the wife's labour supply have unsuccessfully been tried. In particular, we introduced a square term for the wife's share in the labour equation but it proved to be insignificant.

and introduce this expression in the nth commodity demand to obtain:

$$\begin{aligned} C_h^n &= a_{fh}^n + b_f^n \cdot \omega_{fh} + c_f^n \cdot \rho_h + d_f^n \cdot \rho_h{}^2 \\ &\quad + a_{mh}^n + c_m^n \cdot (y_h + w_{mh}T - \rho_h) + d_m^n \cdot (y_h + w_{mh}T - \rho_h)^2. \end{aligned}$$

Second, when $L_{fh}^* \geqslant T$, the shadow wage becomes:

$$\omega_{fh} = \frac{T - \alpha_{fh} - \gamma_f \cdot (w_{fh}T + \rho_h)}{\beta_f - \gamma_f \cdot T},$$

and we introduce this expression in the nth commodity demand. Since this substitution also influences the shadow income, we obtain:

$$\begin{aligned} C_h^n &= a_{fh}^n + b_f^n \cdot \omega_{fh} + c_f^n \cdot ((w_{fh} - \omega_{fh})T + \rho_h) + d_f^n \cdot ((w_{fh} - \omega_{fh})T + \rho_h)^2 \\ &\quad + a_{mh}^n + c_m^n \cdot (y_h + w_{mh}T - \rho_h) + d_m^n \cdot (y_h + w_{mh}T - \rho_h)^2. \end{aligned}$$

This specification reflects the facts that (i) commodity demands are continuous everywhere, in particular, along the frontiers where the regime switches, and (ii) an increase in the wife's wage, in the case of rationing, only has an income effect on consumption. Finally, since individual intercepts cannot be both identified, we reset the parameters $a_h^n = a_{fh}^n + a_{mh}^n$ with

$$a_h^n = \textstyle\sum_{j=1}^{J} a_j^n \cdot z_{jh},$$

where z_{jh} are socio-demographic characteristics relevant for explaining both spouses' behaviour.

4.1.2 The Sharing Rule and the Collective Constraints

As in Fortin and Lacroix (1997) and Blundell et al. (1998), we adopt a linear specification for the sharing rule:

$$\rho_h = A \cdot w_{fh}T + B \cdot w_{mh}T + C \cdot y_h. \tag{9}$$

Although the sign of the parameters is not formally determined by the model above, the intuition suggests that

$$0 < B < C < 1.$$

However, the coefficient A can be either positive or negative since its sign results from two opposite effects. For one, we may expect that the wife transfers part of her welfare gains to her partner if her wage rises and she is working. This is what we call a 'welfare effect'. However, she probably improves her bargaining power at the same time. This is a 'distribution effect'. Therefore, she could also obtain money from her husband. The latter situation is likely prevalent if the welfare gains resulting from the wage increase are small enough (notably if the wife does not work).[7] Precisely, we can show, using (8) and (9), that the wife takes advantage of an increase in her wage, for the present specification, only if

$$L_{fh} + A \cdot T > 0. \tag{10}$$

To test collective rationality, we must derive the unconstrained model that corresponds to the present specification. We adopt the approach followed by Browning et al. (1994). The crucial point here is that there is an equivalence between collective rationality and the existence of a single sharing rule in all the equations. Moreover, at least one commodity demand, together with the wife's labour supply, is required to identify the sharing rule. Precisely, let us write the sharing rule as follows:

$$\rho_h = A \cdot w_{fh}T + B \cdot (w_{mh}T + \Phi \cdot y_h),$$

where $\Phi = C/B$. First, we identify Φ from the estimation of the wife's labour supply. Then, we identify A and B from the estimation of one commodity demand (say $n = 1$, without loss of generality). Thus, the strategy for testing the collective rationality consists in writing the sharing rule as follows:

$$\rho_h^n = A^n \cdot w_{fh}T + B^n \cdot (w_{mh}T + \Phi \cdot y_h),$$

for each additional commodity demand, and checking the following equalities:

$$A^n = A \quad \text{and} \quad B^n = B,$$

for any $n > 1$.

4.2 Stochastic Specification and Likelihood Function

We have to make some allowance for stochastic terms in the right-hand side of these equations. There are several potential source of such randomness:

[7] These two effects could be modelled by assuming that the sharing rule switches regimes in the case of rationing. However, this raises further problems with the logical consistency of this model.

unobservable heterogeneity in preferences or in the sharing rule and optimization/measurement errors in observations. The most satisfactory treatment would be to allow for each of these and then to develop a full stochastic model (see Blundell et al. (1998) for such an attempt in another context). Still this raises issues related to the identification of these terms. We adopt a much more conventional approach of simply adding error terms to each equation: υ_h for the wife's labour supply and ε_h^n for the nth commodity demand.

We assume that the vector $(\upsilon_h, \varepsilon_h^1, \ldots, \varepsilon_h^N)$ follows a multidimensional normal distribution with mean zero and a covariance matrix given by

$$\begin{array}{lcll} \Sigma & & & \text{if} \quad 0 < L_{fh}^* < T, \\ \Sigma_0 & = & \Gamma_0 \cdot \Sigma \cdot \Gamma_0' & \text{if} \quad L_{fh}^* \leqslant 0, \\ \Sigma_1 & = & \Gamma_1 \cdot \Sigma \cdot \Gamma_1' & \text{if} \quad L_{fh}^* \geqslant T, \end{array}$$

where Σ is a matrix of free parameters and Γ_0 and Γ_1 are identity matrices with free parameters instead of zeros in the first column. This specification is a convenient approximation when stochastic terms in the wife's labour supply result from a mix of unobservable taste heterogeneity and optimization/measurement errors (see Kooreman et Kapteyn (1986) on this point). This reflects the fact that, in the case of rationing, the taste heterogeneity in the wife's labour supply is introduced into commodity demands through shadow wages.

These assumptions, with the relationships defined above, directly induce a distribution on hours of work and quantities of commodity. There are three states of the world. Wives are either unrationed, or rationed at 0 or rationed at T. Let us denote $\underline{R}_h = -\alpha_h - \beta \cdot w_{fh} - \gamma \cdot \rho_h$ and $\bar{R}_h = T - \alpha_h - \beta \cdot w_{fh} - \gamma \cdot \rho_h$ and

$$g : (L_{fh}, C_h^1, \ldots, C_h^N) \to (\upsilon_h, \varepsilon_h^1, \ldots, \varepsilon_h^N)$$

the relationship between observations and stochastic terms. The density for wives who are unrationed is given by

$$\begin{aligned} f(L_{fh}, C_h^1, \ldots, C_h^N) &= \phi_\Sigma(\upsilon_h, \varepsilon_h^1, \ldots, \varepsilon_h^N) \\ &= \phi_\Sigma(g(L_{fh}, C_h^1, \ldots, C_h^N)), \end{aligned}$$

where ϕ_Σ denotes the multidimensional normal density with a mean of zero and a matrix Σ of covariances and where the determinant of the Jacobian matrix for the variable transformation is equal to one. Then the contribution to the likelihood for wives who are rationed at 0 is given by

$$\begin{aligned} F_0(C_h^1,\ldots,C_h^N) &= \int_{-\infty}^{\underline{R}_h} \phi_{\Sigma_0}(\upsilon_h,\varepsilon_h^1,\ldots,\varepsilon_h^N)\cdot \mathrm{d}\upsilon_h \\ &= \int_{-\infty}^{0} \phi_{\Sigma_0}(g(L_{fh}^*,C_h^1,\ldots,C_h^N))\cdot \mathrm{d}L_{fh}^*. \end{aligned}$$

Similarly, the contribution to the likelihood for wives who are rationed at T is given by

$$\begin{aligned} F_1(C_h^1,\ldots,C_h^N) &= \int_{\overline{R}_h}^{+\infty} \phi_{\Sigma_1}(\upsilon_h,\varepsilon_h^1,\ldots,\varepsilon_h^N)\cdot \mathrm{d}\upsilon_h \\ &= \int_{T}^{+\infty} \phi_{\Sigma_1}(g(L_{fh}^*,C_h^1,\ldots,C_h^N))\cdot \mathrm{d}L_{fh}^*. \end{aligned}$$

Finally, combining these expressions provides the log-likelihood function of the econometric model:

$$L(L_{fh},C_h^1,\ldots,C_h^N)=\textstyle\sum_{L_{fh}=0}\ln F_0+\sum_{L_{fh}=T}\ln F_1+\sum_{0<L_{fh}<T}\ln f.$$

This function can be estimated by numerical algorithms. A critical assumption maintained in the above analysis concerns the observability of the wage for all wives. This is of course not the case for unemployed women. The approach used thus constructs a fitted value $\hat{w}_{fh}$ for the wage using familiar censored regression techniques and interprets $\hat{w}_{fh}$ as the wage faced by all workers. In comparison with the procedure which consists in replacing only wages for unemployed women, this method has the advantage of limiting the endogeneity problems associated with w_{fh}. If commodity demands are non-linear, however, this method may lead to inconsistent estimators.

5 Data and Empirical Results

In this section we present the main results. First we describe the data set, then we give the estimated coefficients and the statistics for the test of the collective constraints.

5.1 The Data

Data are drawn from the household survey “Budget des Familles” conducted by the national institute of economic and statistical information of France in a sample of 12000 French households in 1984–85. The survey was designed to the analyze living standards and contains detailed information on earnings and income from property and transfers, on expenditures for nondurable as well as durable commodities, on most socio-demographic characteristics of individuals and households and, finally, on the number of hours and the work status of individuals. Several studies have exploited this survey over the last 10 years (e.g., Bourguignon et al. (1993)) and it permits some interesting comparisons to be made.

From the original sample, we select a subsample of married couples with the husband working full-time (whose monthly labour supply is comprised between 140 and 180 hours, but normalized at $T = 160$) and, at the most, one child between 3 and 18 years old. We also restrict the sample to couples where the husband and the wife (if working) are not self-employed. These selection rules and the exclusion of missing data leave us with a total of 739 cases for the empirical analysis. Ideally, we should have selected a subsample of couples without children. Children and expenditures on them may actually be considered as public commodities for both parents, whereas the model considered above only allows for private commodities. Moreover, children are expected to increase problems related to household production. It turns out however that considering only childless couples restricts the size of our sample too much.

The wife's labour supply is the number of hours worked per month. It is computed by multiplying the number of reported weekly hours by 4.2. Expenditures on nondurable commodities are recorded in the survey on diaries covering two-week periods and extrapolated for the year.[8] In the empirical application, we calculate monthly expenditures on food (at home and away), clothing (for male, female and children), recreation (including books, disks, vacations and sporting goods) and transport (excluding purchases of vehicles) respectively. Practically, there may be problems due to the infrequency of purchases. However, this must not be overestimated because the commodities that we consider are very aggregated and the lumpiness in these expenditures is negligible.

The wife's hourly wage is computed as the monthly wage net of social contributions but including overtime, premiums, pensions, and a monetary evaluation of benefits in kind divided by the number of hours worked. This

[8] Expenditures on clothing are recorded over a two-month period, but this difference with other nondurables is not taken into account.

Table 1: Descriptive Statistics of the Sample

	Mean	**St. Dev.**
Endogenous Variables		
Wife's Monthly Hours of Labour (LAB)	106	74
Percentage of zeros	31	
Monthly Food Expenditures* (FOO)	2431	1119
Percentage of zeros	0	
Monthly Clothing Expenditures* (CLO)	655	683
Percentage of zeros	8	
Monthly Recreation Expenditures* (REC)	601	707
Percentage of zeros	1	
Monthly Transport Expenditures* (TRA)	878	959
Percentage of zeros	6	
Exogenous Variables		
Wife's Hourly Wage (Actual) $\times$ 160*	5021	1707
Wife's Hourly Wage (Predicted) $\times$ 160*	4696	964
Husband's Hourly Wage $\times$ 160*	6658	3134
Monthly Nonlabour Income*	299	1435
Wife's Age	39	11
Husband's Age	41	11
Paris Region (0–1)	0.15	0.36
Rural Region (0–1)	0.25	0.43
Presence of a child (0–1)	0.44	0.49
Number of observations	739	

Note: *In French francs.

wage is then replaced for all observations by the fitted values derived from a conventional wage equation estimated for participating wives with a correction for the selection bias.[9] The husband's hourly wage is defined in the same way as the wife's hourly wage. The monthly non-labour income includes various transfers and income from different types of assets (including child benefits) and the virtual income of home owner-occupiers, certainly the most important asset return. The latter is not directly observed but computed as the fitted values of an equation estimated on renting households. Table 1 gives descriptive statistics of the sample.

5.2 Parameter Estimates

We have estimated 54 structural parameters; 7 for the wife's labour supply, 11 for each commodity demand and 3 for the sharing rule. These are presented in Table 2. We first note that only 15 parameters are statistically significant at the 5% level. We then compute the statistics for the score test, using the unconstrained model previously derived, to check collective rationality. This statistic, which follows a χ^2 distribution with 6 degrees of freedom, is equal to 8.804 with a p-value of 0.185. In other words, the data that we consider do not reject the efficiency hypothesis. This confirms tests previously performed with the same data set by Bourguignon et al. (1993).

We can now consider the estimated parameters of the wife's labour supply. All the coefficients have the expected sign (apart perhaps from those of the region dummies), and the Slutsky Positivity is globally satisfied. Nevertheless, the scale of both the wage coefficient (β) and the share coefficient (γ) strongly contrasts with the one provided by traditional studies with French data (see for example Bourguignon and Magnac (1990)). Precisely, the impact of the wife's wage is positive, but insignificant and unusually small, with a corresponding elasticity, computed at the average point of the sample, equal to 0.226. On the contrary, the impact of the wife's share is especially large in terms of absolute value (the corresponding elasticity cannot be calculated since we do not observe the share of each household member). There are at least two explanations for these features. First, these coefficients are obviously the structural parameters of a collective model and they have to be

[9] This regression includes, among the explanatory variables, the wife's education (measured in years), the square and the cube of this variable, the wife's age, a cross-term of education and age, an indicator of the labour market tension, dummies for the wife's nationality, for the region and for households with a telephone.

Table 2: FIML Parameter Estimates

Parameters of the behavioural Equations					
	LAB	**FOO**	**CLO**	**REC**	**TRA**
Intercept	267.4	136.5	384.5	60.2	664.0
	(60.3)	(499.8)	(256.7)	(314.2)	(587.2)
Wife's Wage	0.816	1.929	1.281	0.745	9.024
	(2.529)	(5.928)	(4.180)	(2.676)	(27.801)
Wife's Share	-96.5	-154.4	-143.4	-82.1	-692.9
	(43.9)	(188.4)	(149.5)	(148.2)	(438.0)
Wife's Share exp2	—	57.2	33.7	-7.7	-46.1
		(52.3)	(38.8)	(36.0)	(73.5)
Husband's Share	—	283.3	97.9	124.8	173.6
		(69.8)	(40.7)	(51.7)	(75.0)
Husband's Share exp2	—	-6.0	-2.0	-1.9	-3.7
		(2.8)	(1.7)	(2.3)	(3.2)
Socio-demographics					
One Child 4–18 years	-15.7	322.4	-21.8	-36.0	-50.4
	(20.5)	(99.8)	(74.4)	(75.8)	(160.7)
Paris Region	-45.1	168.9	-177.6	163.2	-523.0
	(34.2)	(160.6)	(122.5)	(123.7)	(242.9)
Rural Region	12.1	84.8	-35.8	-107.9	91.8
	(25.4)	(131.4)	(95.8)	(110.6)	(210.0)
Wife's Age	-6.0	0.2	-22.5	-1.0	-34.9
	(1.1)	(14.0)	(10.1)	(11.5)	(14.5)
Husband's Age	—	-0.6	9.9	-7.2	-8.0
		(12.0)	(8.0)	(10.7)	(9.6)
Parameters of the Sharing Rule					
Wife's Wage × 160	-0.468				
	(0.205)				
Husband's Wage × 160	0.146				
	(0.075)				
Nonlabour Income	0.464				
	(0.200)				

Note: 1) Standard deviations are in brackets,.

2) The variables in the sharing rule are in thousands of francs.

interpreted in view of the estimated parameters of the sharing rule. Second, we assume here that the number of hours worked has an upper bound, defined by a legal or socio-cultural maximum, and is fixed at 160.

We do not dwell on the estimated parameters of the commodity demands since these cannot be directly interpreted (the corresponding elasticities cannot be calculated either). Still, it is worth remarking that, in these equations, the parameters related to the husband's behaviour are fairly well estimated (most of them are significant at the 5% level) in comparison with those related to the wife's behaviour. A possible explanation is suggested by the proof of Proposition 2 where we show that the identification of the husband's Engel curves relies on the first derivatives of the wife's labour supply and the household commodity demands whereas the wife's Engel curves are derived as a by-product of the derivative of the husband's Engel curves. As for the control variables, let us note that the child dummy is significant (with a positive sign) in the food equation while the Paris dummy is significant (with a negative sign) in the transport equation. On the other hand, the wife's age is significant (with a negative sign) in the clothing and the transport equation.

We now turn to the estimated parameters of the sharing rule. These are statistically significant at the 5% level and can be interpreted as follows. First, a one thousand franc increase in the wife's potential wage (normalized at 160 hours) decreases the wife's share by 468 francs. This means that the wife partially transfers her gains in utility to her husband. This also explains why the wage coefficient in the wife's labour supply is so small. Actually, we can show, using (10), that the wife gains from a wage increase only if her labour supply is greater than 74.9 hours per month. Second, a one thousand franc increase in the husband's wage (normalized at 160 hours) increases the wife's share by only 146 francs. That is, only a small part of the husband's wage goes to the wife. Consequently, the share coefficient in the wife's labour supply is unusually great (in terms of absolute value) compared to estimates given by unitary models. Third, a one thousand franc increase in non-labour income increases the wife's share by 464 francs and the husband's share by 536 francs. We can say that non-labour income is almost equally divided between household members. Chiappori et al. (2001) provide estimates of the same order for the parameters of the sharing rule. Still, the most remarkable fact here is the precision of the estimates given in Table 2 considering the small size of our sample. Previous studies which use the husband's behaviour to identify the sharing rule yield standard deviations for structural parameters which are much greater. Finally, the estimated parameters of the sharing rule can be used to test the income pooling hypothesis. The latter implies that the husband's wage and non-labour income have the same impact on the sharing rule. We can use the Wald test to check this assumption. This statistic, which follows a χ^2 distribution with 1 degree of freedom, is equal to 4.494 with a p-value of 0.034. Therefore, the income pooling hypothesis is rejected as required for identifying this model.

6 Conclusion

In this paper, we start from the assumption that the husband's hours of work are fixed at a legal maximum and we consider the wife's labour supply and household commodity demands in a unified framework. We first show that structural elements of the decision process can be identified with a single commodity demand. Second, we generate a set of conditions under the form of partial differential equations which can be used to statistically test the collective setting. Finally, we conclude with an empirical illustration using French data. These empirical results show that the structural parameters describing the intra-household decision process and the husband's behaviour are fairly well estimated.

The rigidity of the husband's labour supply is certainly a good approximation for most countries. Moreover, we expect that using information on household consumption will generally provide precise and robust estimates of the main structural parameters. The collective approach to household consumption is indeed a promising research program as shown by preliminary investigations by Bourguignon et al. (1993) or Browning et al. (1994). Still, several theoretical extensions (e.g., fixed costs of participation, non-linear income taxation, or involuntary unemployment) are necessary to properly assess the present setting. Above all, future research should stress the stochastic specification and the functional form that we have adopted. Specifically, the linearity of the wife's labour supply is certainly a severe limitation. However, a more flexible specification does not allow us to recover a closed form for the shadow wages which are used to incorporate rationings in the model.

Finally, we should stress one point in particular. The collective model of female labour supply, under the specific assumptions that we have adopted, encompasses the corresponding unitary model with labour supply and commodity demands. Therefore, the potential drawbacks of our theoretical set-up (e.g., an absence of public consumption or domestic production) are somehow or other in the unitary model of female labour supply. This generalization of the unitary model is convenient since the collective constraints are not rejected by the data.

Appendix

A.1 Proof of Proposition 1

If we differentiate the wife's labour supply (1) with respect to y and w_m and eliminate $\lambda^f_{\rho_f}$, we obtain:

$$\bar{L}^f_y \cdot \rho_{w_m} = \bar{L}^f_{w_m} \cdot \rho_y. \tag{11}$$

Then we use this equation to differentiate the nth commodity demand along the locus defined by $\mathrm{d}\rho = 0$. We obtain, after simplification, the husband's Engel curve:

$$\zeta^{mn}_{\star\rho_m} = \alpha^n.$$

If we differentiate this expression again with respect to y, w_m and w_f, we obtain:

$$\begin{aligned} \zeta^{mn}_{\star\rho_m\rho_m} \cdot (1-\rho_y) &= \alpha^n_y, & (12)\\ \zeta^{mn}_{\star\rho_m\rho_m} \cdot (1-\rho_{w_m}) &= \alpha^n_{w_m}, & (13)\\ -\zeta^{mn}_{\star\rho_m\rho_m} \cdot \rho_{w_f} &= \alpha^n_{w_f}. & (14) \end{aligned}$$

Solving this system of partial differential equations with (11) yields:

$$\rho_{w_f} = -\alpha^n_{w_f} \cdot \beta^n, \quad \rho_{w_m} = 1 - \alpha^n_{w_m} \cdot \beta^n, \quad \rho_y = 1 - \alpha^n_y \cdot \beta^n, \tag{15}$$

and

$$\zeta^{mn}_{\star\rho_m\rho_m} = (\beta^n)^{-1}. \tag{16}$$

To retrieve $\lambda^f_{\rho_f}$ and $\lambda^f_{w_f}$, we differentiate the wife's labour supply with respect to y, w_m and w_f, use (15), and rearrange to obtain :

$$\lambda^f_{\rho_f} = \frac{\bar{L}^f_{w_m} - \bar{L}^f_y}{(\alpha^n_{w_m} - \alpha^n_y) \cdot \beta^n} \quad \text{and} \quad \lambda^f_{w_f} = \bar{L}_{w_f} + \frac{\bar{L}^f_{w_m} - \bar{L}^f_y}{(\alpha^n_{w_m} - \alpha^n_y)} \cdot \alpha^n_{w_f}. \tag{17}$$

Similarly, we can differentiate the nth commodity demand with respect to y, w_m and w_f, use (15), and rearrange demand to obtain:

$$\zeta_{\rho_f}^{fn} = \frac{\alpha_{w_m}^n \cdot \bar{C}_y^n - \alpha_y^n \cdot \bar{C}_{w_m}^n}{\alpha_{w_m}^n - \alpha_y^n} \quad \text{and} \quad \zeta_{w_f}^{fn} = \bar{C}_{w_f}^n - \frac{\bar{C}_{w_m}^n - \bar{C}_y^n}{\alpha_{w_m}^n - \alpha_y^n} \cdot \alpha_{w_f}^n.$$

Finally, knowing the sharing rule allows us to write down the wife's actual budget constraint and her preferences can be computed in the usual way. ■

A.2 Proof of Proposition 2

If we introduce the derivatives of the wife's labour supply, given by (17), in the Slutsky Positivity condition, we obtain the first condition in the statement of the proposition. If we differentiate (16) with respect to y, w_m and w_f, and simplify, we obtain the second condition. ■

A.3 Proof of Proposition 3

If we consider any pair of commodity demands, say n_1 and n_2, we have a corresponding pair of systems of equations (12)–(14). If we simplify, we obtain the condition in the statement of the proposition. ■

A.4 Proof of Proposition 4

The proof follows in stages. We prove that: A) the sharing rule can be retrieved on the participation frontier, B) this identification can be extended on int(N), and C) testable restrictions are generated.

A) Frontier solution: Along the participation frontier, by the continuity of the wife's labour supply, we have the following identity:

$$\lambda^f(\gamma(w_m, y), \upsilon(w_m, y)) = 0,$$

where $\upsilon(w_m, y)$ is infinitely differentiable. If we differentiate this identity with respect to w_m and y and eliminate $\lambda_{\rho_f}^f$ and $\lambda_{w_f}^f$, we obtain:

$$\gamma_y \cdot \upsilon_{w_m} = \gamma_{w_m} \cdot \upsilon_y. \tag{18}$$

Then we use this expression and differentiate the nth commodity demand along the direction $\mathrm{d}\upsilon = 0$. We obtain the husband's Engel curve: $\zeta_{\star\rho_m}^{mn} =$

A^n. We differentiate this expression again to obtain:

$$\zeta^{mn}_{\star\rho_m\rho_m} \cdot (1 - \upsilon_{w_m}) = A^n_{w_m}, \tag{19}$$
$$\zeta^{mn}_{\star\rho_m\rho_m} \cdot (1 - \upsilon_y) = A^n_y. \tag{20}$$

Finally, using (18) and solving this system yields

$$\upsilon_{w_m} = 1 - A^n_{w_m} \cdot B^n \quad \text{and} \quad \upsilon_y = 1 - A^n_y \cdot B^n,$$

and

$$\zeta^{mn}_{\star\rho_m\rho_m} = B^n. \tag{21}$$

These equations define the sharing rule up to an additive constant along the participation frontier.

B) Interior solution: We differentiate the nth commodity demand with respect to w_m, w_f and y and eliminate $\zeta^{mn}_{\star\rho_m}$ and $\zeta^{fn}_{\star\rho_f}$ to obtain a partial differential equation in ρ_{w_f}, ρ_{w_m} and ρ_y:

$$(\bar{C}^n_{w_m} - \bar{C}^n_y) \cdot \rho_{w_f} - \bar{C}^n_{w_f} \cdot \rho_{w_m} + \bar{C}^n_{w_f} \cdot \rho_y = 0. \tag{22}$$

>From standard theorems of the partial differential equation theory (e.g., John (1983)), the partial differential equation (22) together with the specification of the sharing rule on the boundary completely determines the wife's share ρ for any $(w_f, w_m, y) \in N$, provided that a regularity condition is satisfied. First, let us remark that the partial differential equation (22) can be written as

$$u \cdot \nabla\rho = 0,$$

where $\nabla\rho$ is the gradient of ρ and u is the vector $(\bar{C}^n_{w_m} - \bar{C}^n_y, -\bar{C}^n_{w_f}, \bar{C}^n_{w_f})$. Now, the condition is that the vector u is tangent to the participation frontier. Since the equation of this frontier is $w_f - \gamma(w_m, y)$ and given that, on the frontier, $\bar{C}$ coincides with $\hat{C}$, this condition becomes:

$$\hat{C}^n_{w_m} \neq \hat{C}^n_y.$$

Formally, this result is local rather than global. Additional conditions are required to identify the sharing rule on the entire set N. These conditions are not specified here.

C) Constraints: Differentiating (21) with respect to y and w_m and simplifying yields the condition in the statement of the proposition. There exists another testable constraint if we examine the differentiability in the direction of the non-participation set. The latter has a more complex form and is not specified here. ■

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