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ABSTRACT

Dynamics of Intrahousehold Bargaining

This paper studies the dynamics of bargaining in an intrahousehold context. To explore long-term partner relationships, we analyse bilateral bargaining by considering that spouses take decisions sequentially. We conclude that a greater valuation of the present, rather than the future, for the spouse who takes the second decision, increases the set of possible sustainable agreements, as well as the proportion of time that this agent devotes to a family good.

JEL Classification: C71, C72, C62, J12

Keywords: family bargaining, Stackelberg game, family good

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Introduction

This paper examines the dynamic aspects of partner relationships. Specifically, we analyse the influence of the valuation of the current situation on both the time that each individual devotes to the provision of a family good, and the gains of well-being derived from cooperation.

The theoretical study of family decision-making, and its relationship with consumption and labor supply, has produced a significant body of literature. Some of these works take as reference the theory of bilateral bargaining. In this framework, the results of the bargaining depend on the threat point that is fixed, that is to say, the status quo. Family bargaining models have mainly identified this threat point with divorce (Manser and Brown, 1980; McElroy and Horney, 1981). In this case, it is assumed that the agents can communicate freely and that the fulfillment of agreements is guaranteed by an external contract or institution. Nevertheless, divorce does not necessarily constitute the only possible threat point in a bargaining process. Lundberg and Pollak (1993) and Chen and Woolley (2001) consider a noncooperative equilibrium: Cournot-Nash equilibrium. In this situation, the status quo is defined as an internal situation in the bargaining process and the agents take their decisions simultaneously at the threat point.

These family bargaining models do not take into account the dynamic aspects of the bargaining process. An exception is the work of Andaluz and Molina (2007), who develop a repeated noncooperative game in which both members of a family can contribute voluntarily to the provision of a family public good. They study how individual preferences, the degree of altruism between agents, and the bargaining power of spouses, influence the sustainability of agreements.

In our work, we extend the analysis of the dynamic aspects of the family bargaining process using a model of bilateral bargaining in which the status quo is defined as a noncooperative solution. We assume that in this stage the spouses take their decisions sequentially (equilibrium of Stackelberg). Those situations in which one of the spouses takes a decision, knowing the decision already taken by the other spouse, have not previously been considered in the literature. Additionally, the analysis of family decisions, from the perspective of the noncooperative game, allows us to capture dynamic elements of the models of family bargaining. In this framework, Pareto-superior solutions can arise as a result of repeated games in the absence of institutions that require the agents to fulfill the agreements. In accordance with the folk theorem, cooperation can be derived from the equilibrium of a repeated game, whenever the players have a greater valuation of the present, and when there exists some internal mechanism that punishes all deviation from the cooperative solution.

As regards the main results, we should mention that a greater valuation of the present, rather than the future, implies an increase in the possible sustainable agreements derived from the bargaining, as well as an increase in the proportion of time devoted to the family good for the spouse who decides second.

I Framework

We develop a supergame in an intrahousehold framework in which spouses may contribute voluntarily to the provision of a family good whose consumption is nonrival. We suppose that the agents do not know the moment of the dissolution of the marriage, and that the objective of each is to maximize the discounted value of their current utilities:

$$\sum_{t=1}^{\infty} \delta^t u_j(x_j, Q); (j = 1, 2)$$

where δ denotes the discount factor, x_j indicates the private consumption of agent j, $j = 1, 2$; Q represents the family good, $Q = q_1 + q_2$, with q_j being the proportion of hours that agent j devotes to the provision of this good. The family good, Q, can include any situation which requires the joint performance of the spouses. For instance, the quality of the children, the maintenance of the home, or the result of elder care.

We suppose that both spouses have identical preferences, represented by the following functional form (see Konrad and Lommerud 2000):

$$u_1 = x_1 + Q - q_1^{\beta}; u_2 = x_2 + Q - q_2^{\alpha} \tag{1}$$

where $x_1 = w_1(1 - q_1)$ and $x_2 = w_2(1 - q_2)$, w_j represents the wage rate for agent j and the maximum time available for each spouse is normalized to one.

We assume that the contribution to the family good not only reduces the time available to the labor market, but also has a psychological cost, represented by an increasing and convex function in each of these arguments $\left(q_1^{\beta}, q_2^{\alpha}\right)$, being $\alpha, \beta > 1$.

In what follows, we first solve the one-shot game and then determine the optimum levels of consumption and contribution to the family good among all the multiple stationary paths.

II The one-shot game

We formulate a noncooperative equilibrium in which the result is determined sequentially (see Espinosa and Rhee 1989). In our work, the provision of a family good is the outcome of a Stackelberg game in which the leader (spouse 1) commits to a certain quantity of provision, while anticipating the optimal contribution of the follower (spouse 2). Spouse 1 chooses his own private consumption and his provision of the family good, after knowing the provision of the family good decided by spouse 2.

We could consider a situation where only one of the agents (spouse 2) has acquired the skills for providing a family good. In this situation, spouses cannot take decisions simultaneously, since the other (spouse 1) can anticipate the optimal contribution of spouse 2 to decide his contribution (See Buchholz et al. 1997). Therefore, spouse 1 acts as a leader and spouse 2 acts as a follower.

Applying the backward induction procedure, we begin by obtaining the equilibrium corresponding to spouse 2 (the follower). Formally,

$$\begin{aligned}
&\underset{x_2,q_2}{Max}\ u_2 = x_2 + Q - q_2^{\alpha} \\
&s.to\ x_2 = w_2(1-q_2) \\
&\qquad q_1 = \bar{q}_1
\end{aligned} \tag{2}$$

From here, we deduce the levels of consumption and the provision of the family good,

$$q_2^* = \left(\frac{1-w_2}{\alpha}\right)^{1/\alpha-1};\ x_2^* = w_2\left[1-\left(\frac{1-w_2}{\alpha}\right)^{1/\alpha-1}\right] \tag{3}$$

and the utility level,

$$u_2^d = w_2 + \bar{q}_1 + (\alpha-1)\left(\frac{1-w_2}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{4}$$

For spouse 1 (the leader) we formulate the following maximization problem,

$$\begin{aligned}
&\underset{x_1,q1}{Max}\ u_1 = x_1 + Q - q_1^{\beta} \\
&s.to\ x_1 = w_1(1-q_1) \\
&\qquad q_2 = q_2^*
\end{aligned} \tag{5}$$

and we obtain the level of private consumption and the provision of the household good made by spouse 1,

$$q_1^* = \left(\frac{1-w_1}{\beta}\right)^{1/\beta-1} \; ; \; x_1^* = w_1 \left[1 - \left(\frac{1-w_1}{\beta}\right)^{1/\beta-1}\right] \tag{6}$$

Therefore, the levels of utility in the noncooperative solution for both spouses are,

$$u_1^* = w_1 + (\beta - 1)\left(\frac{1-w_1}{\beta}\right)^{\beta/\beta-1} + \left(\frac{1-w_2}{\alpha}\right)^{1/\alpha-1} \tag{7}$$

$$u_2^* = w_2 + (\alpha - 1)\left(\frac{1-w_2}{\alpha}\right)^{\alpha/\alpha-1} + \left(\frac{1-w_1}{\beta}\right)^{1/\beta-1} \tag{8}$$

In an intrahousehold framework, spouses can tacitly achieve Pareto-superior levels of private consumption and family good, since they are able to implicitly create a strategy that punishes all deviations from a cooperative solution. Therefore, the choice of a cooperative solution among the possible equilibria is a reasonable result in long term intrahousehold relationships. However, the equilibrium of the one shot game can be the result in a supergame, since the repetition of the game is not sufficient to eliminate the static noncooperative equilibrium. To guarantee the achievement of a cooperative solution, we introduce a punishment strategy. We adopt the so-called trigger strategy (Friedman, 1971), so that when there is a deviation of the cooperative solution, the levels of private consumption and the provision of the family good revert to those of noncooperative equilibrium.

For the sake of simplicity, we only consider the case of stationary paths for all t. A stationary path is sustainable in a subgame perfect equilibrium if it satisfies the following conditions:

$$u_j(x_j, Q) - u_j^* \geq 0; j = 1, 2 \tag{9}$$

$$\frac{u_2(x_2, Q)}{1-\delta} \geq u_2^d(q_1) + \frac{\delta u_2^*}{1-\delta} \tag{10}$$

The condition (9) establishes that both spouses have incentives to cooperate, since the well-being these agents obtain in the cooperative solution is greater or equal to the well-being obtained in the noncooperative solution. The condition (10) determines that the spouse who decides second has not incentive to deviate from the efficient solution. Therefore, the maintenance of the cooperative equilibrium depends on the agent who decides second. Given the sequential nature of this game, if spouse 1 observes that spouse 2 does not deviate, nor will he.

In Figure 1, we represent the curves of indifference of the spouses in the noncooperative solution. For spouse 1, the slope of the curve of indifference in the noncooperative equilibrium is zero in (q_1^*, q_2^*) and is increasing and convex if $q_1 > q_1^*$ and $q_2 > q_2^*$, $\left.\frac{dq_2}{dq_1}\right|_{u_1^*} > 0$, $\left.\frac{d^2q_2}{dq_1^2}\right|_{u_1^*} > 0$.

Analogously, for spouse 2, the slope of the curve of indifference that contains the solution of the one shot game is equal to minus infinity in the combination (q_1^*, q_2^*), and is increasing and concave when $q_1 > q_1^*$ and $q_2 > q_2^*$, being $\left.\frac{dq_2}{dq_1}\right|_{u_2^*} > 0$; $\left.\frac{d^2q_2}{dq_1^2}\right|_{u_2^*} < 0$.

All the points located inside the area formed by both curves of indifference are Pareto superior to the equilibrium of the one shot game. Those located in the contract curve *CC*' are efficient solutions. However, as we have mentioned above, the sustainability of an efficient solution as a result of repeated interaction is not guaranteed if no strategy of punishment is introduced. To incorporate this strategy of punishment, we can define the function $g = g(q_1, q_2, \delta)$, representing all the combinations of the provision of the family good, (q_1, q_2) that satisfy the restriction (10) with equality given the discount factor. Formally,

$$g(q_1, q_2, \delta) = (1 - w_2)q_2 - q_2^{\alpha} + \delta q_1 - (\alpha - 1)\left(\frac{1 - w_2}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \delta\left(\frac{1 - w_1}{\beta}\right)^{\frac{1}{\beta-1}} \tag{11}$$

From (11), we deduce that the set of Pareto-superior solutions to the equilibrium of the one shot game, sustainable by way of the repeated interaction, is greater when the discount factor is higher. We observe that when $q_1 > q_1^*$ and $q_2 > q_2^*$, the function g is increasing and concave, with the value of its slope being minus infinity in the noncooperative solution (q_1^*, q_2^*) and increasing in δ.

As shown in Figure 1, among the Pareto-superior combinations (q_1, q_2), we can identify a subset of sustainable solutions which can be achieved by way of repeated interaction. In particular, all the combinations of (q_1, q_2) located to the right of the broken line.

Figure 1. Set of Pareto-Superior Solutions.

(Figure 1 here)

III Bargaining solution

In order to determine an equilibrium among the multiple stationary paths, it is necessary to specify how the levels of consumption and contribution to the family good are chosen among all possible solutions. Focusing on the study of sustainable solutions, we suppose that there exists a bargaining process in which both spouses take their decisions by way of the symmetric Nash bargaining solution. That is to say, they choose the stationary paths of private consumption and family good provision that maximize the product of the utilities, after being normalized by the utility levels of the noncooperative solution, and within the set of sustainable equilibria. Formally,

$$\underset{x_1,x_2,q_1,q_2}{Max} J(x_1, x_2, q_1, q_2) = (u_1 - u_1^*)(u_2 - u_2^*)$$

$$s.to \quad (9) \text{ and } (10) \tag{12}$$

The solution of the previous problem depends on the discount factor. In fact, when δ takes value zero, the noncooperative solution satisfies the restrictions (9) and (10). Alternatively, if this factor takes value one, all the Pareto-superior solutions are indeed sustainable and consequently, the bargaining agreement constitutes an efficient solution. In both cases, the bargaining solution is determined by way of the tangency between an Iso-J line and an Iso-g line, as shown in Figure 2.

Figure 2. Set of possible sustainable bargaining solutions

(Figure 2 here)

Proposition

The contribution to the family good of the spouse who decides second (follower) is increasing with respect to the discount factor: $\frac{dq_2}{d\delta} > 0$.

The influence of the discount factor on the contribution to the family good of the spouse who decides first (leader) is ambiguous: $\frac{dq_1}{d\delta} \gtreqless 0$.

Proof.

We assume that $J(x_1, x_2, q_1, q_2)$ is strictly concave with the slopes of the Iso-J curves being monotones: $\frac{\partial(\frac{J_1}{J_2})}{\partial q_1} < 0$.

Differentiating the first order conditions of (12) with respect to δ, in an interior solution and applying Cramer's rule, we find:

$$sign\left(\frac{dq_2}{d\delta}\right) = sign\left[-\lambda\delta(1-w_2-\alpha q_2^{\alpha-1}) + (q_1-q_1^*)(J_{11}\left(1-w_2-\alpha q_2^{\alpha-1}\right) - \delta J_{12})\right] \tag{13}$$

From $(1-w_2-\alpha q_2^{\alpha-1}) < 0$, $q_1 > q_1^*$, and the above assumptions, we deduce that $\frac{dq_2}{d\delta} > 0$.

Differentiating the restriction with respect to δ, we find,

$$sign\left(\frac{dq_1}{d\delta}\right) = sign\left[(q_1-q_1^*) + (1-w_2-\alpha q_2^{\alpha-1})\frac{dq_2}{d\delta}\right]$$

which can be positive or negative, $\frac{dq_1}{d\delta} \lesseqgtr 0.\square$

From this result, it is possible to deduce that the agent who makes his decisions second, will devote more time to the provision of the family good when he places more value on the present. However, the path of the contribution to the family good made by the spouse who decides first can be increased or decreased, depending on the discount factor. An increasing evolution implies that the difference between the hours that this agent devotes to the family good in the cooperative solution, and the hours determined in the noncooperative equilibrium, is not very significant.

Knowing the evolution of the paths of the provision of the family good, we can deduce the influence of the discount factor on the level of utility derived from the cooperation.

Given that

$\frac{dU_1}{d\delta} = \frac{-(1-w_1-\beta q_1^{\beta-1})\left(q_1-q_1^*\right)}{\delta} + \left[1 - \frac{(1-w_1-\beta q_1^{\beta-1})(1-w_2-\alpha q_2^{\alpha-1})}{\delta}\right]\frac{dq_2}{d\delta} \lesseqgtr 0$

and $\frac{dU_2}{d\delta} = \left(1-\frac{1}{\delta}\right)(1-w_2-\alpha q_2^{\alpha-1})\frac{dq_2}{d\delta} - \frac{\left(q_1-q_1^*\right)}{\delta} \lesseqgtr 0.$

For both spouses, an increase in the discount factor can increase or reduce the level of utility in the bargaining solution. Concretely, if the bargaining agreement implies a significant increase in the contribution to the family good made by the leader, a greater discount factor gives rise to a greater level of utility for the leader, and a lower level of utility for the follower. The opposite occurs if the difference between the time devoted to the provision of the household good in cooperation, and in the one-shot game, is not very significant.

IV Conclusions

This paper has focused on the dynamic aspects of bargaining processes. We have set up a supergame in an intrahousehold framework in which both spouses may contribute voluntarily to the provision of a family good. We have adopted a punishment strategy, namely the trigger strategy, to guarantee the achievement of a cooperative solution in a supergame. Thus, both spouses can punish all deviations from a cooperative solution and can tacitly achieve Pareto-superior levels of private consumption and family good. We have assumed that in the noncooperative equilibrium, spouses decide sequentially the levels of private consumption and the time devoted to the provision of a family good. The sequential characterization of the decision making implies that the maintenance of all possible cooperative solutions will be exclusively conditional on the behavior of the spouse who decides in second (the follower), which facilitates the identification of the set of possible sustainable bargaining agreements.

Spouses choose the stationary paths of private consumption and family good provision by way of the symmetric Nash bargaining solution. After introducing this process of bargaining, we are able to deduce the influence of the valuation of the present on the time that each individual devotes to the provision of a family good and, its effects on the gains of well-being derived from cooperation.

In particular, the following conclusions are obtained:

Firstly, the set of possible sustainable agreements derived from bargaining is greater when the discount factor of the spouse who decides second is higher.

Secondly, the contribution of the follower to the family good increases with respect to the discount factor. That is to say, a greater valuation of the present implies an increase in the proportion of time that this agent devotes to the family good.

Thirdly, the gains of well-being derived from the bargaining show an ambiguous relationship to the discount factor. The effect of the discount factor will be positive or negative for both spouses, depending on the increase in the time devoted to the family good in the bargaining situation when the spouse acts as leader.

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Figure 1. Set of Pareto-Superior Solutions

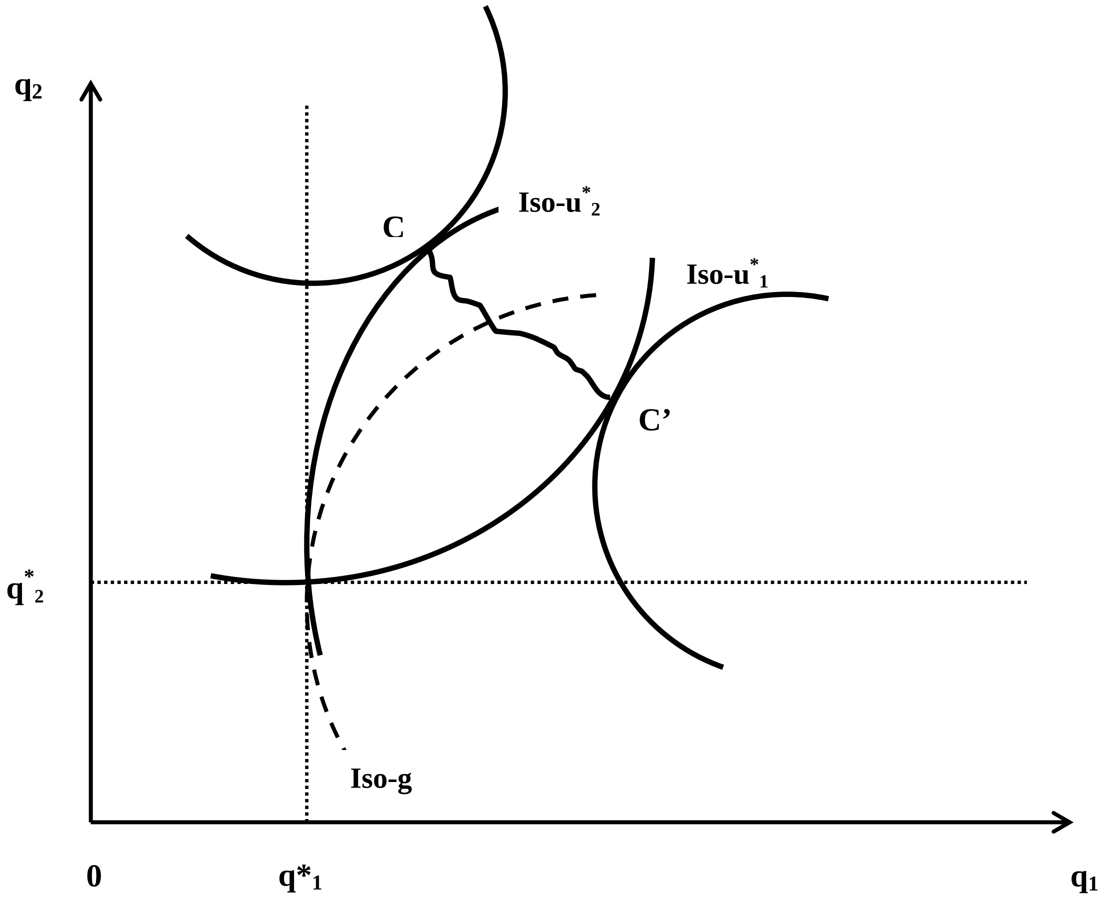

Figure 2. Set of possible sustainable bargaining solutions